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A refined HSDT for bending and dynamic analysis of FGM plates

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Abstract. In this work, a novel higher-order shear deformation theory (HSDT) for static and free vibration analysis of functionally graded (FG) plates is proposed. Unlike the conventional HSDTs, the proposed theory has a novel displacement field which includes undetermined integral terms and contains fewer unknowns. Equations of motion are obtained by using Hamilton's principle. Analytical solutions for the bending and dynamic investigation are determined for simply supported FG plates. The computed results are compared with 3D and quasi-3D solutions and those provided by other plate theories. Numerical results demonstrate that the proposed HSDT can achieve the same accuracy of the conventional HSDTs which have more number of variables.

Keywords: bending; vibration; functionally graded plate; shear deformation theory

1. Introduction

Functionally graded materials (FGMs) are fabricated from a mixture of two materials to accomplish a composition that produces a specific functionality. These materials are generally isotropic but nonhomogeneous. The reason for interest in FGMs is that it may be possible to provide certain kinds of FGM structures capable of adapting to operating conditions. Presenting novel characteristics, FGMs have also attracted intensive research interests, which were mainly focused on their bending, buckling, dynamic and vibration characteristics of FG structures (Akbaş, 2016, Eltaher *et al.* 2013, Ebrahimi *et al*. 2009a, b, Zaoui *et al.* 2017a, Guerroudj *et al.* 2018). Since the shear deformation has considerable influences on the responses of FG plates, shear deformation models are employed to capture such shear deformation influences. The first-order shear deformation theory (FSDT) considers the shear deformation influences by the way of linear distribution of in-plane displacements within the thickness (Mindlin, 1951, Meksi *et al.* 2015, Bouderba *et al.* 2016, Bellifa *et al.* 2016, Kolahchi *et al.* 2016a, b, Madani *et al.* 2016, Zamanian *et al.* 2017). Since the first-order shear deformation theory violates the conditions of zero transverse shear stresses on the upper and lower surfaces of the plate, a shear correction coefficient which is related to many parameters is required to compensate for the error due to a constant shear strain supposition within the thickness. The HSDTs consider the shear deformation influences, and respect the zero transverse shear stresses on the upper and lower surfaces of the plate; hence, a shear correction coefficient is not needed. By employing HSDT, Mahapatra *et al*. (2017) studied nonlinear thermoelastic deflection of temperature-dependent FGM curved shallow shell under nonlinear thermal loading. Using the simple higher-order shear deformable kinematics including the temperature dependent properties of each constituent, Mehar and Panda (2017) studied the thermoelastic nonlinear frequency analysis of CNT reinforced FG sandwich structure. HSDTs are also used to study laminated composites structures with different loads cases (Kolahchi, 2017, Kolahchi *et al.* 2017a, b, Mahapatra *et al.* 2016a, b, c, d, e, Sahoo *et al.* 2016, Mahapatra and Panda, 2016, Mahapatra *et al.* 2015, Mahapatra and Panda, 2015, Kar *et al.* 2015). Kolahchi and Moniri Bidgoli (2016) used the sinusoidal shear deformation beam theory for dynamic instability of single-walled carbon nanotubes. Arani and Kolahchi employed HSDT to study buckling response of embedded concrete columns armed with carbon nanotubes. Bilouei *et al*. (2016) investigated the buckling of concrete columns retrofitted with Nano-Fiber Reinforced Polymer using HSDT.

In general, HSDTs are proposed based on the supposition of higher-order distributions of in-plane displacements (Reddy, 2000, Zenkour, 2006, Pradyumna and Bandyopadhyay, 2008, Ait Atmane *et al.* 2010, Benyoucef *et al.* 2011, Xiang *et al.* 2011, Bouderba *et al.* 2013, Thai and Kim, 2013, Tounsi *et al.* 2013, Ait Amar Meziane *et al.* 2014, Ait Yahia *et al.* 2015, Mahi *et al*. 2015, Kar *et al.* 2016a, b, Baseri *et al.* 2016, Beldjelili *et al.* 2016, Raminnea *et al.* 2016, Bousahla *et al.* 2016, Aldousari, 2017, Rahmani *et al.* 2017) or both axial and vertical displacements within the thickness (Matsunaga, 2008, Chen

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et al. 2009, Talha and Singh, 2010, Reddy, 2011, Neves *et al.* 2012a, b, Hebali *et al.* 2014, Swaminathan and Naveenkumar, 2014, Bourada *et al.* 2015, Hamidi *et al.* 2015, Akavci, 2016, Bennoun *et al.* 2016). Some of these models are computational costs because with each additional power of the thickness coordinate, an additional variable is added to the theory (e.g., theories by Pradyumna and Bandyopadhyay (2008) and Neves *et al.* (2012a, b) with nine variables, Reddy (2011) with eleven unknowns, Talha and Singh (2010) with thirteen unknowns). Although some well-known HSDTs have the same five variables (e.g., third-order shear deformation theory (Reddy, 2000), sinusoidal shear deformation theory (Zenkour, 2006), hyperbolic shear deformation theory (Ait Atmane *et al.* 2010, Benyoucef *et al.* 2011)), their governing equations are 4 much more complicated than those of FSDT. Thus, needs exist for the development of HSDTs which are simple to utilize.

The objective of this work is to construct a simple HSDT for FG plates. The proposed theory has only four variables and four equations of motion, but it respects the stress-free boundary conditions on the upper and lower surfaces of the plate without introducing any shear correction factors. The displacement field of the developed theory is chosen based on a constant vertical displacement and trigonometric variation of in-plane displacements within the thickness. The addition of the integral term into the in-plane displacements leads to a reduction in the number of variables and governing equations, hence makes the model simple to utilize. Analytical solutions for static and dynamic are obtained. Numerical examples are presented to check the accuracy of the proposed theory.

2. Theoretical formulation

2.1 Kinematics

The displacement field of the proposed theory is constructed based on the following suppositions: (1) the in-plane displacements are partitioned into bending and shear components; (2) the bending parts of the in-plane displacements are similar to those used by CPT; (3) the shear parts of the in-plane displacements are formulated by using the integral term and give rise to the trigonometric distributions of shear strains and hence to shear stresses within the thickness of the plate in such a way that the shear stresses vanish on the upper and lower surfaces of the plate; and (4) the transverse displacement is independent to the thickness coordinate. Based on these considerations, the following displacement field can be constructed

$$u(x,y,z,t)=u_0(x,y,t)-z\frac{\partial w_0}{\partial x}+k_1\, f(z)\int\theta(x,y,t)\,dx \tag{1a}$$

$$v(x,y,z,t)=v_0(x,y,t)-z\frac{\partial w_0}{\partial y}+k_2\, f(z)\int\theta(x,y,t)\,dy \tag{1b}$$

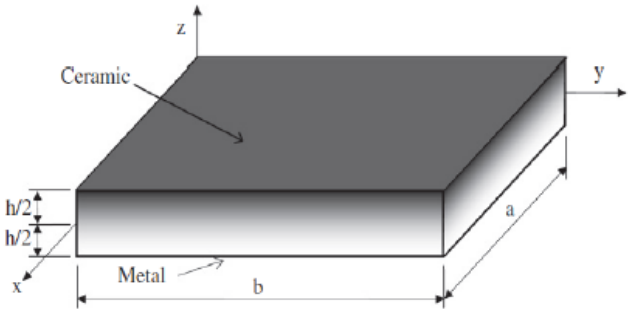


Fig. 1 Geometry and coordinates of FG plates

$$w(x,y,z,t)=w_0(x,y,t) \tag{1c}$$

where $u_0(x,y,t)$, $v_0(x,y,t)$, $w_0(x,y,t)$ and $\theta(x,y,t)$ are the four variables displacement functions of middle surface of the plate. Note that the integrals do not have limits. In the present study, it is employed terms with integrals instead of terms with derivatives. The constants k_1 and k_2 depends on the geometry.

In this work, the present HSDT is obtained by setting:

- Model 1

$$f(z)=\frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right) \tag{2a}$$

- Model 2

$$f(z)=\frac{z}{2\pi}\left(\pi+2\cos\frac{\pi z}{h}\right) \tag{2b}$$

It should be noted that unlike the FSDT, this model does not require shear correction coefficients. The kinematic relations can be obtained as follows:

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}+f(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix}, \tag{3a}$$

$$\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=g(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} \tag{3b}$$

Where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial y}\\ \dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial x}\end{Bmatrix},\quad\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}=\begin{Bmatrix}-\dfrac{\partial^2 w_0}{\partial x^2}\\ -\dfrac{\partial^2 w_0}{\partial y^2}\\ -2\dfrac{\partial^2 w_0}{\partial x\partial y}\end{Bmatrix}, \tag{4a}$$

$$\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix}=\begin{Bmatrix}k_1\theta\\ k_2\theta\\ k_1\dfrac{\partial}{\partial y}\int\theta\,dx+k_2\dfrac{\partial}{\partial x}\int\theta\,dy\end{Bmatrix}, \tag{4b}$$

Table 1 Material properties employed in the FG plates

Properties	Metal	Ceramic	
	Aluminum (Al)	Alumina (Al_2O_3)	Zirconia (ZrO_2)
$E(Gpa)$	70	380	200
ν	0.3	0.3	0.3
ρ (kg/m^3)	2702	3800	5700

$$\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix}=\begin{Bmatrix}k_2\int\theta\,dy\\ k_1\int\theta\,dx\end{Bmatrix},\quad g(z)=\frac{df(z)}{dz} \tag{4c}$$

The integrals defined in the above equations shall be resolved by a Navier type method and can be written as follows:

$$\frac{\partial}{\partial y}\int\theta\,dx=A'\frac{\partial^2\theta}{\partial x\partial y},\quad \frac{\partial}{\partial x}\int\theta\,dy=B'\frac{\partial^2\theta}{\partial x\partial y} \tag{5a}$$

$$\int\theta\,dx=A'\frac{\partial\theta}{\partial x},\quad \int\theta\,dy=B'\frac{\partial\theta}{\partial y} \tag{5b}$$

where the coefficients A' and B' are expressed according to the type of solution employed, in this case via Navier. Therefore, A', B', k_1 and k_2 are expressed as follows:

$$A'=-\frac{1}{\alpha^2},\quad B'=-\frac{1}{\beta^2},\quad k_1=\alpha^2,\quad k_2=\beta^2 \tag{6}$$

where α and β are defined in eq. (22).

2.2 Constitutive relations

The material characteristics of FG plates are considered to change continuously within the thickness. Three homogenization procedures employed to compute the Young's modulus $E(z)$ are: (1) the power law variation, (2) the exponential variation, and (3) the Mori–Tanaka scheme.

For the power law variation, the Young's modulus is expressed as (Reddy 2000, Birman and Byrd 2007, Zidi *et al.* 2014, Zemri *et al.* 2015, Zidi *et al.* 2017)

$$E(z)=E_m+(E_c-E_m)\left(\frac{1}{2}+\frac{z}{h}\right)^p \tag{7a}$$

where p is the gradient index; and the subscripts m and c represent the metallic and ceramic constituents, respectively.

For the exponential variation, the Young's modulus is written as (Belabed *et al.* 2014, Meradjah *et al.* 2018)

$$E(z)=E_0e^{p(z/h+1/2)} \tag{7b}$$

where E_0 is the Young's modulus of the homogeneous plate; and p is a parameter that indicates the material distribution across the thickness of the plate.

For Mori-Tanaka scheme, the Young's modulus is defined as (Benveniste 1987, Mori and Tanaka 1973, Younsi *et al.* 2018)

$$E(z)=E_m+(E_c-E_m)\frac{V_c}{1+(1-V_c)\left(\frac{E_c}{E_m}-1\right)\frac{1+\nu}{3-3\nu}} \tag{7c}$$

where V_c = $(0.5+z/h)^p$ is the volume fraction of the ceramic. Since the influences of the variation of Poisson's ratio v on the response of FG plates are very small (Yang *et al.* 2005, Kitipornchai *et al.* 2006), it is supposed to be constant for convenience.

The linear constitutive relations of an FG plate can be expressed as

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}Q_{11}&Q_{12}&0&0&0\\ Q_{12}&Q_{22}&0&0&0\\ 0&0&Q_{44}&0&0\\ 0&0&0&Q_{55}&0\\ 0&0&0&0&Q_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{Bmatrix} \tag{8}$$

where

$$\begin{gathered}Q_{11}(z)=Q_{22}(z)=\frac{E(z)}{1-\nu^2},\ Q_{12}(z)=\nu Q_{11}(z)\\ Q_{44}(z)=Q_{55}(z)=Q_{66}(z)=\frac{E(z)}{2(1+\nu)}\end{gathered} \tag{9}$$

2.3 Equations of motion

Equations of motion are employed herein using Hamilton's principle (Zaoui *et al.* 2019, Chaabane *et al.* 2019). The principle can be stated in an analytical form as

$$\int_0^T(\delta U+\delta V-\delta K)dt=0 \tag{10}$$

where δU is the variation of strain energy; δV is the variation of the external work done by external load applied to the FG plate; and δK is the variation of kinetic energy.

The variation of strain energy of the plate is given by

$$\begin{aligned}\delta U&=\int_V\left[\sigma_x\,\varepsilon_x+\sigma_y\,\varepsilon_y+\tau_{xy}\gamma_{xy}+\tau_{yz}\,\gamma_{yz}+\tau_{xz}\,\gamma_{xz}\right]dV\\ &=\int_A\Big[N_x\delta\,\varepsilon_x^0+N_y\delta\,\varepsilon_y^0+N_{xy}\delta\,\gamma_{xy}^0+M_x^b\delta\,k_x^b\\ &\quad+M_y^b\delta\,k_y^b+M_{xy}^b\delta\,k_{xy}^b+M_x^s\delta\,k_x^s+M_y^s\delta\,k_y^s\\ &\quad+M_{xy}^s\delta\,k_{xy}^s+S_{yz}^s\delta\,\gamma_{yz}^s+S_{xz}^s\delta\,\gamma_{xz}^0\Big]dA\end{aligned} \tag{11}$$

Where A is the top surface and the stress resultants N, M and S are defined by

$$\begin{gathered}(N_i,M_i^b,M_i^s)=\int_{-h/2}^{h/2}(1,z,f)\sigma_i dz,\quad (i=x,y,xy)\\ (S_{xz}^s,S_{yz}^s)=\int_{-h/2}^{h/2}g(z)(\tau_{xz},\tau_{yz})\,dz\end{gathered} \tag{12}$$

The variation of work done by external forces can be expressed as

$$\delta V = -\int_0^L q\ \delta\, w_0\, dx \tag{13}$$

where q is the transverse load.

The variation of kinetic energy of the plate can be given as

$$\begin{aligned}\delta K &= \int_V \left(\dot{u}\,\delta\dot{u} + \dot{v}\delta\dot{v} + \dot{w}\delta\dot{w}\right)\rho(z)dV \\ &= \int_A \{I_0\left(\dot{u}\delta\dot{u} + \dot{v}\delta\dot{v} + \dot{w}\delta\dot{w}\right) \\ &- I_1\left(\dot{u}_0\frac{\partial\delta\dot{w}_0}{\partial x} + \frac{\partial\dot{w}_0}{\partial x}\delta\dot{u}_0 + \dot{v}\frac{\partial\delta\dot{w}_0}{\partial y} + \frac{\partial\dot{w}_0}{\partial y}\delta\dot{v}_0\right) \\ &- I_2\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\dot{w}_0}{\partial x} + \frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\dot{w}_0}{\partial y}\right) + K_2\left(\frac{\partial\dot{\varphi}}{\partial x}\frac{\partial\delta\dot{\varphi}}{\partial x} + \frac{\partial\dot{\varphi}}{\partial y}\frac{\partial\delta\dot{\varphi}}{\partial y}\right) \\ &- J_1\left(\dot{u}_0\frac{\partial\delta\dot{\varphi}}{\partial x} + \frac{\partial\dot{\varphi}}{\partial x}\delta\dot{u}_0 + \dot{v}_0\frac{\partial\delta\dot{\varphi}}{\partial y} + \frac{\partial\dot{\varphi}}{\partial y}\delta\dot{v}_0\right) \\ &+ J_2\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\dot{\varphi}}{\partial x} + \frac{\partial\dot{\varphi}}{\partial x}\frac{\partial\delta\dot{w}_0}{\partial x} + \frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\dot{\varphi}}{\partial y} + \frac{\partial\dot{\varphi}}{\partial y}\frac{\partial\delta\dot{w}_0}{\partial y}\right)\Big\}dA\end{aligned} \tag{14}$$

where dot-superscript convention represents the differentiation with respect to the time variable t, $\rho(z)$ is the masse density and the moments of inertia (I_i,J_i,K_i) are expressed by

$$(I_0, I_1, I_2) = \int_{-h/2}^{h/2}\rho(z)\left[1, z, z^2\right]dz \tag{15a}$$

$$(J_1, J_2, K_2) = \int_{-h/2}^{h/2}\rho(z)\left[f, z\,f, f^2\right]dz \tag{15b}$$

Substituting the relations δU, δV and δK from Eqs. (11), (13) and (14) into Eq. (10), integrating by parts and collecting the coefficients of δu_0, δv_0, δw_0, $\delta\theta$, one obtains the following equations of motion

$$\delta u_0 : \frac{N_x}{\partial x} + \frac{N_{xy}}{\partial y} = I_0\,\ddot{u}_0 - I_1\frac{\partial\ddot{w}_0}{\partial x} + k_1A'J_1\frac{\partial\ddot{\varphi}}{\partial x} \tag{16a}$$

$$\delta v_0 : \frac{N_{xy}}{\partial x} + \frac{N_y}{\partial y} = I_0\,\ddot{v}_0 - I_1\frac{\partial\ddot{w}_0}{\partial y} + k_2B'J_1\frac{\partial\ddot{\varphi}}{\partial y} \tag{16b}$$

$$\begin{aligned}\delta w_0 &: \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x\partial y} + \frac{\partial M_y^b}{\partial y^2} + q \\ &= I_0\,\ddot{w}_0 + I_1\left(\frac{\partial\ddot{u}_0}{\partial x} + \frac{\partial\ddot{v}_0}{\partial y}\right) - I_2\nabla^2\ddot{w}_0 - J_2\nabla^2\ddot{\theta}\end{aligned} \tag{16c}$$

$$\begin{aligned}\delta\theta &: -k_1M_x^s - k_2M_y^s - \left(k_1A' + k_2B'\right)\frac{\partial^2 M_{xy}^s}{\partial x\partial y} + k_1A'\frac{\partial S_{xz}^s}{\partial x} + k_2B'\frac{\partial S_{yz}^s}{\partial y} \\ &= -J_1\left(k_1A'\frac{\partial\ddot{u}_0}{\partial x} + k_2B'\frac{\partial\ddot{v}_0}{\partial y}\right) - J_2\left(k_1A'\frac{\partial^2\ddot{w}_0}{\partial^2 x} + k_2B'\frac{\partial^2\ddot{w}_0}{\partial^2 y}\right) \\ &- K_2\left(\left(k_1A'\right)^2\frac{\partial^2\ddot{\theta}}{\partial^2 x} + \left(k_2B'\right)^2\frac{\partial^2\ddot{\phi}}{\partial^2 y}\right)\end{aligned} \tag{16d}$$

Substituting Eq. (8) into Eq. (12) and integrating within the thickness of the plate, the stress resultants are expressed as

$$\begin{Bmatrix}\{N_x\}\\ \{N_y\}\\ \{N_{xy}\}\\ \{M_x^b\}\\ \{M_y^b\}\\ \{M_{xy}^b\}\\ \{M_x^s\}\\ \{M_y^s\}\\ \{M_{xy}^s\}\end{Bmatrix} = \begin{bmatrix}\begin{bmatrix}A_{11} & A_{12} & 0\\ A_{12} & A_{22} & 0\\ 0 & 0 & A_{66}\end{bmatrix} & \begin{bmatrix}B_{11} & B_{12} & 0\\ B_{12} & B_{22} & 0\\ 0 & 0 & B_{66}\end{bmatrix} & \begin{bmatrix}B_{11}^s & B_{12}^s & 0\\ B_{12}^s & B_{22}^s & 0\\ 0 & 0 & B_{66}^s\end{bmatrix}\\ \begin{bmatrix}B_{11} & B_{12} & 0\\ B_{12} & B_{22} & 0\\ 0 & 0 & B_{66}\end{bmatrix} & \begin{bmatrix}D_{11} & D_{12} & 0\\ D_{12} & D_{22} & 0\\ 0 & 0 & D_{66}\end{bmatrix} & \begin{bmatrix}D_{11}^s & D_{12}^s & 0\\ D_{12}^s & D_{22}^s & 0\\ 0 & 0 & D_{66}^s\end{bmatrix}\\ \begin{bmatrix}B_{11}^s & B_{12}^s & 0\\ B_{12}^s & B_{22}^s & 0\\ 0 & 0 & B_{66}^s\end{bmatrix} & \begin{bmatrix}D_{11}^s & D_{12}^s & 0\\ D_{12}^s & D_{22}^s & 0\\ 0 & 0 & D_{66}^s\end{bmatrix} & \begin{bmatrix}H_{11}^s & H_{12}^s & 0\\ H_{12}^s & H_{22}^s & 0\\ 0 & 0 & H_{66}^s\end{bmatrix}\end{bmatrix}\begin{Bmatrix}\{\varepsilon_x^0\}\\ \{\varepsilon_y^0\}\\ \{\gamma_{xy}^0\}\\ \{k_x^b\}\\ \{k_y^b\}\\ \{k_{xy}^b\}\\ \{k_x^s\}\\ \{k_y^s\}\\ \{k_{xy}^s\}\end{Bmatrix} \tag{17a}$$

$$\begin{Bmatrix}S_{yz}^s\\ S_{xz}^s\end{Bmatrix} = \begin{bmatrix}A_{44}^s & 0\\ 0 & A_{55}^s\end{bmatrix}\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} \tag{17b}$$

and stiffness components are expressed as

$$\left(A_{ij}, B_{ij}, D_{ij}, B_{ij}^s, D_{ij}^s, H_{ij}^s\right) = \int_{-h/2}^{h/2}\left[1, z, z^2, f(z), zf(z), f^2(z)\right]Q_{ij}(z)dz \tag{18a}$$

$$(i, j) = (1,2,6)$$

$$A_{44}^s = A_{55}^s = \int_{-h/2}^{h/2}[g(z)]^2\,Q_{44}(z)dz \tag{18b}$$

2.4 Equations of motion in terms of displacements

By substituting Eq. (17) into Eq. (16), the equations of motion can be expressed in terms of displacements (u_0, v_0, w_0, θ) as

$$\begin{aligned}&\delta u_0 : A_{11}\,d_{11}u_0 + A_{66}\,d_{22}u_0 + (A_{12} + A_{66})\,d_{12}v_0 \\ &- B_{11}\,d_{111}w_0 - (B_{12} + 2B_{66})\,d_{122}w_0 + (k_1B_{11}^s + k_2B_{12}^s)\,d_1\theta \\ &+ (k_1A' + k_2B')B_{66}^s d_{122}\theta = I_0\ddot{u}_0 - I_1d_1\ddot{w}_0 + (k_1A')J_1d_1\ddot{\theta}\end{aligned} \tag{19a}$$

$$\begin{aligned}&\delta v_0 : (A_{12} + A_{66})\,d_{12}u_0 + A_{66}\,d_{11}v_0 + A_{22}\,d_{22}v_0 \\ &- (B_{12} + 2B_{66})\,d_{112}w_0 - B_{22}\,d_{222}w_0 + (k_1B_{12}^s + k_2B_{22}^s)\,d_2\theta \\ &+ (k_1A' + k_2B')B_{66}^s d_{112}\theta = I_0\ddot{v}_0 - I_1d_2\ddot{w}_0 + \left(k_2B'\right)J_1d_2\ddot{\theta}\end{aligned} \tag{19b}$$

$$\begin{aligned}&\delta w_0 : B_{11}\,d_{111}u_0 + (B_{12} + 2B_{66})\,d_{122}u_0 + (B_{12} + 2B_{66})\,d_{112}v_0 \\ &+ B_{22}d_{222}v_0 - D_{11}\,d_{1111}w_0 - D_{22}d_{2222}w_0 - 2(D_{12} + 2D_{66})\,d_{1122}w_0 \\ &+ (k_1D_{11}^s + k_2D_{12}^s)\,d_{11}\theta + (k_1D_{12}^s + k_2D_{22}^s)\,d_{22}\theta + 2(k_1A' + k_2B')\,D_{66}^s \\ &d_{1122}\theta + q = I_0\ddot{w}_0 + J_0\ddot{\varphi} + I_1\left(d_1\ddot{u}_0 + d_2\ddot{v}_0\right) - I_2\nabla^2\ddot{w}_0 + \left(k_1A'\right)J_2d_{11}\ddot{\theta} \\ &+ \left(k_2B'\right)J_2d_{22}\ddot{\theta}\end{aligned} \tag{19c}$$

$$\begin{aligned}&\delta\theta : -(k_1B_{11}^s + k_2B_{12}^s)\,d_1u_0 - (k_1B_{12}^s + k_2B_{22}^s)\,d_2v_0 \\ &- B_{66}^s(k_1A' + k_2B')(d_{122}u_0 + d_{112}v_0) + (k_1D_{11}^s + k_2D_{12}^s)\,d_{11}w_0 \\ &+ (k_1D_{12}^s + k_2D_{22}^s)d_{22}w_0 + 2D_{66}^s(k_1A' + k_2B')\,d_{1122}w_0 \\ &- (k_1{}^2H_{11}^s + k_2{}^2H_{22}^s + 2k_1k_2H_{12}^s)\,\theta + (k_1A')^2\,A_{55}^s d_{11}\theta \\ &+ (k_2B')^2\,A_{44}^s d_{22}\theta - H_{66}^s(k_1A' + k_2B')^2\,d_{1122}\theta \\ &= -J_1\left(k_1A'd_1\ddot{u}_0 + k_2B'd_2\ddot{v}_0\right) + J_2\left(k_1A'd_{11}\ddot{w}_0 + k_2B'd_{22}\ddot{w}_0\right) \\ &- K_2\left((k_1A')^2\,d_{11}\ddot{\theta} + (k_2B')^2\,d_{22}\ddot{\theta}\right)\end{aligned} \tag{19d}$$

Table 2 Non-dimensional stress and deflection of Al_2/AlO_3 square plates

p	Theory	$\bar{\sigma}_x(h/3)$			$\bar{w}$		
		$a/h=4$	$a/h=10$	$a/h=100$	$a/h=4$	$a/h=10$	$a/h=100$
1	Quasi-3D (Neves *et al*. 2012a)	0.5925	1.4945	14.9690	0.6997	0.5845	0.5624
	Quasi-3D (Neves *et al*. 2012b)	0.5910	1.4917	14.9440	0.7020	0.5868	0.5648
	Quasi-3D (Neves *et al*. 2013)	0.5911	1.4917	14.9450	0.7020	0.5868	0.5647
	Quasi-3D (Carrera *et al*. 2008)	0.6221	1.5064	14.9690	0.7171	0.5875	0.5625
	Quasi-3D (Carrera *et al*. 2011)	0.6221	1.5064	14.9690	0.7171	0.5875	0.5625
	TSDT (Thai and Kim, 2013)	0,5812	1,4808	14,9676	0,7284	0,589	0,5625
	CPT	0.8060	2.0150	20.1500	0.5623	0.5623	0.5623
	FSDT	0.8060	2.0150	20.1500	0.7291	0.5889	0.5625
	Model 1	0,5796	1,4891	14,9675	0,7272	0,5888	0,5625
	Model 2	0.5803	1.4894	14.9675	0.7280	0.5889	0.5625
4	Quasi-3D (Neves *et al*. 2012a)	0.4404	1.1783	11.9320	1.1178	0.8750	0.8286
	Quasi-3D (Neves *et al*. 2012b)	0.4340	1.1593	11.7380	1.1095	0.8698	0.8241
	Quasi-3D (Neves *et al*. 2013)	0.4330	1.1588	11.7370	1.1108	0.8700	0.8240
	Quasi-3D (Carrera *et al*. 2008)	0.4877	1.1971	11.9230	1.1585	0.8821	0.8286
	Quasi-3D (Carrera *et al*. 2011)	0.4877	1.1971	11.9230	1.1585	0.8821	0.8286
	TSDT (Thai and Kim, 2013)	0,4449	1,1794	11,9209	1,1599	0,8815	0,8287
	CPT	0.6420	1.6049	16.0490	0.8281	0.8281	0.8281
	FSDT	0.6420	1.6049	16.0490	1.1125	0.8736	0.8286
	Model 1	0,4402	1,1774	11,9207	1,1626	0,882	0,8287
	Model 2	0.4424	1.1783	11.9208	1.1619	0.8818	0.8287
10	Quasi-3D (Neves *et al*. 2012a)	0.3227	1.1783	11.9320	1.3490	0.8750	0.8286
	Quasi-3D (Neves *et al*. 2012b)	0.3108	0.8467	8.6013	1.3327	0.9886	0.9228
	Quasi-3D (Neves *et al*. 2013)	0.3097	0.8462	8.6010	1.3334	0.9888	0.9227
	Quasi-3D (Carrera *et al*. 2008)	0.3695	0.8965	8.6077	1.3745	1.0072	0.9361
	Quasi-3D (Carrera *et al*. 2011)	0.3695	0.8965	8.6077	1.3745	1.0072	0.9361
	TSDT (Thai and Kim, 2013)	0,3259	0,8785	8,906	1,3909	1,0087	0,9362
	CPT	0.4796	1.1990	11.9900	0.9354	0.9354	0.9354
	FSDT	0.4796	1.1990	11.9900	1.3178	0.9966	0.9360
	Model 1	0,3214	0,8767	8,9058	1,3909	1,0089	0,9362
	Model 2	0.3235	0.8775	8.9059	1.3917	1.0089	0.9362

where *dij*, *dijl* and *dijlm* are the following differential operators

$$d_i = \frac{\partial}{\partial x_i}, \quad d_{ij} = \frac{\partial^2}{\partial x_i \partial x_j}, \quad d_{ijl} = \frac{\partial^3}{\partial x_i \partial x_j \partial x_l}, \quad \partial_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m} \quad (i, j, l, m = 1,2) \tag{20}$$

3. Analytical solutions

Consider a simply supported rectangular FG plate with length *a* and width *b* under transverse load *q* as shown in Fig. 1. Based on the Navier approach, the solutions are considered as

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn} e^{i\omega t} \cos(\alpha x)\sin(\beta y) \\ V_{mn} e^{i\omega t} \sin(\alpha x)\cos(\beta y) \\ W_{mn} e^{i\omega t} \sin(\alpha x)\sin(\beta y) \\ X_{mn} e^{i\omega t} \sin(\alpha x)\sin(\beta y) \end{Bmatrix} \tag{21}$$

where ω is the frequency of free vibration of FG plate, $\sqrt{i} = -1$ the imaginary unit. The coefficients α and β are given by

$$\alpha = m\pi/a, \quad \beta = n\pi/b \tag{22}$$

The transverse load *q* is also expanded in the double-Fourier sine series as

Table 3 Non-dimensional stresses and displacements of Al_2/AlO_3 square plates (a/h=10)

p	Theory	$\bar{u}(-h/4)$	$\bar{w}$	$\bar{\sigma}_x(h/3)$	$\bar{\sigma}_{xy}(-h/3)$	$\bar{\sigma}_{xz}(h/6)$
1	Quasi-3D (Carrera *et al*. 2008)	0.6436	0.5875	1.5062	0.6081	0.2510
	Quasi-3D (Wu *et al*. 2011)	0.6436	0.5876	1.5061	0.6112	0.2511
	SSDT (Zenkour, 2006)	0.6626	0.5889	1.4894	0.6110	0.2622
	HSDT (Mantari *et al*. 2012)	0.6398	0.5880	1.4888	0.6109	0.2566
	TSDT (Wu and Li, 2010)	0.6414	0.5890	1.4898	0.6111	0.2599
	TSDT (Thai and Kim, 2013)	0,6414	0,5890	1,4898	0,6111	0,2608
	Model 1	0,6407	0,5888	1,4891	0,611	0,2626
	Model 2	0.6410	0.5889	1.4894	0.6110	0.2621
2	Quasi-3D (Carrera *et al*. 2008)	0.9012	0.7570	1.4147	0.5421	0.2496
	Quasi-3D (Wu *et al*. 2011)	0.9013	0.7571	1.4133	0.5436	0.2495
	SSDT (Zenkour, 2006)	0.9281	0.7573	1.3954	0.5441	0.2763
	HSDT (Mantari *et al*. 2012)	0.8957	0.7564	1.3940	0.5438	0.2741
	TSDT (Wu and Li, 2010)	0.8984	0.7573	1.3960	0.5442	0.2721
	TSDT (Thai and Kim, 2013)	0,8984	0,7573	1,396	0,5442	0,2737
	Model 1	0,8972	0,7573	1,3949	0,544	0,2778
	Model 2	0.8977	0.7573	1.3954	0.5441	0.2763
4	Quasi-3D (Carrera *et al*. 2008)	1.0541	0.8823	1.1985	0.5666	0.2362
	Quasi-3D (Wu *et al*. 2011)	1.0541	0.8823	1.1841	0.5671	0.2362
	SSDT (Zenkour, 2006)	1.0941	0.8819	1.1783	0.5667	0.2580
	HSDT (Mantari *et al*. 2012)	1.0457	0.8814	1.1755	0.5662	0.2623
	TSDT (Wu and Li, 2010)	1.0502	0.8815	1.1794	0.5669	0.2519
	TSDT (Thai and Kim, 2013)	1,0502	0,8815	1,1794	0,5669	0,2537
	Model 1	1,0484	0,882	1,1774	0,5666	0,261
	Model 2	1.0492	0.8818	1.1783	0.5667	0.2580
8	Quasi-3D (Carrera *et al*. 2008)	1.0830	0.9738	0.9687	0.5879	0.2262
	Quasi-3D (Wu *et al*. 2011)	1.0830	0.9739	0.9622	0.5883	0.2261
	SSDT (Zenkour, 2006)	1.1340	0.9750	0.9466	0.5856	0.2121
	HSDT (Mantari *et al*. 2012)	1.0709	0.9737	0.9431	0.5850	0.2140
	TSDT (Wu and Li, 2010)	1.0763	0.9747	0.9477	0.5858	0.2087
	TSDT (Thai and Kim, 2013)	1,0763	0,9746	0,9477	0,5858	0,2088
	Model 1	1,0741	0,9751	0,9456	0,5854	0,2143
	Model 2	1.0751	0.9749	0.9466	0.5856	0.2121

$$q(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Q_{mn}\ \sin(\alpha x)\ \sin(\beta y) \tag{23}$$

where Q_{mn}= q_0 for sinusoidally distributed load.

Substituting Eqs. (21) and (23) into Eq. (19), the following problem is obtained

$$\left(\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & 0 & m_{13} & m_{14} \\ 0 & m_{22} & m_{23} & m_{24} \\ m_{13} & m_{23} & m_{33} & m_{34} \\ m_{14} & m_{24} & m_{34} & m_{44} \end{bmatrix}\right)\begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ Q_{mn} \\ 0 \end{Bmatrix} \tag{24}$$

where

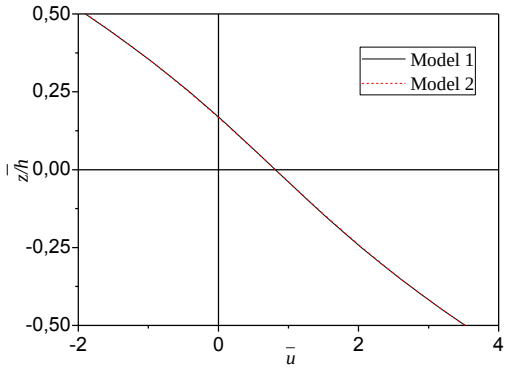
$$\begin{aligned} &S_{11}=-(\alpha^2 A_{11}+\beta^2 A_{66}),\quad S_{12}=-\alpha\beta(A_{12}+A_{66}),\\ &S_{13}=\alpha(\alpha^2 B_{11}-\beta^2(B_{12}+2B_{66})),\\ &S_{14}=\alpha(k_1 B_{11}^s+k_2 B_{12}^s-(k_1 A'+k_2 B')\beta^2 B_{66}^s),\\ &S_{22}=-(\alpha^2 A_{66}+\beta^2 A_{22}), \end{aligned} \tag{25}$$

$$\begin{aligned} &S_{23}=\beta^3 B_{22}+\alpha^2\beta(B_{12}+2B_{66})\\ &S_{24}=\beta(k_1 B_{22}^s+k_2 B_{12}^s-(k_1 A'+k_2 B')\alpha^2 B_{66}^s),\\ &S_{33}=-(\alpha^4 D_{11}+2\alpha^2\beta^2(D_{12}+2D_{66})+\beta^4 D_{22}),\\ &S_{34}=-k_1(\alpha^2 D_{11}^s+\beta^2 D_{12}^s)+2(k_1 A'+k_2 B')\alpha^2\beta^2 D_{66}^s-k_2(\beta^2 D_{22}^s+\alpha^2 D_{12}^s),\\ &S_{44}=-k_1(k_1 H_{11}^s+k_2 H_{12}^s)-(k_1 A'+k_2 B')^2\alpha^2\beta^2 H_{66}^s-k_2(k_1 H_{12}^s+k_2 H_{22}^s)\\ &-(k_1 A')^2\alpha^2 A_{55}^s-(k_2 B')^2\beta^2 A_{44}^s \end{aligned} \tag{25}$$

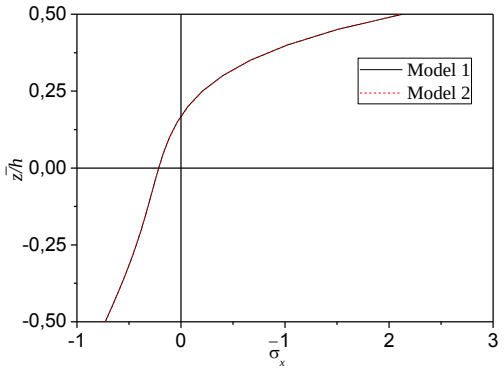
$$\begin{aligned} &m_{11}=-I_0,\quad m_{13}=\alpha I_1,\quad m_{14}=-k_1 A'\alpha J_1,\\ &m_{22}=-I_0,\quad m_{23}=\beta I_1,\quad m_{24}=-k_2 B'\beta J_1\\ &m_{33}=-I_0-(\alpha^2+\beta^2)I_2,\quad m_{34}=(k_1 A'\alpha^2+k_2 B'\beta^2)J_2,\\ &m_{44}=-K_2((k_1 A')^2\alpha^2+(k_2 B')^2\beta^2) \end{aligned}$$

4. Numerical results

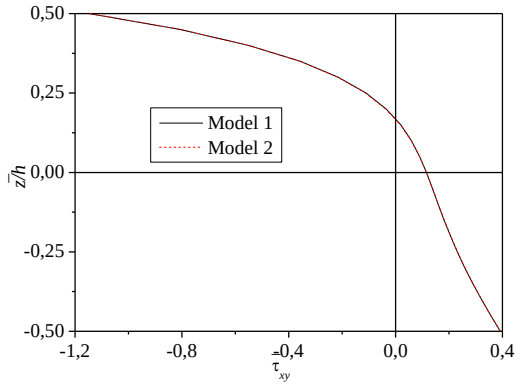
In this section, various numerical examples are examined and discussed to check the accuracy of the



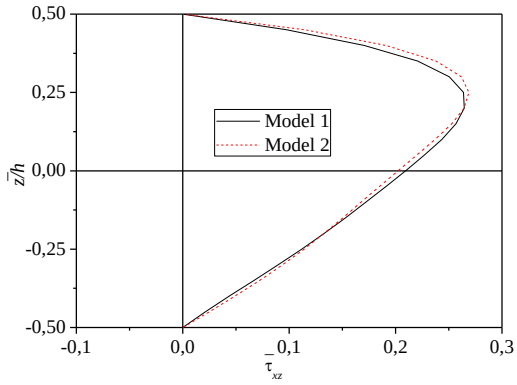
(a) In-plane displacement $\bar{u}$



(b) In-plane stress $\bar{\sigma}_x$



(c) Transverse normal stress $\bar{\tau}_{xy}$



(d) Transverse shear stress $\bar{\tau}_{xz}$

Fig. 2 Variation of non-dimensional displacement and stresses through the thickness of Al/Al_2O_3 square plates (a/h=5, p=4)

proposed theory in predicting the bending and dynamic responses of simply supported FG plates. Two types of FGMs are studied: Al/Al_2O_3 and Al/ZrO_2. The material properties of FG plates are presented in Table 1. The Young's modulus and density of FG plates, unless mentioned otherwise, are calculated by employing the power law variation (see Eq. (7a)). For static investigation, a plate under sinusoidal load is considered. For convenience, the following dimensionless forms are employed

$$\begin{aligned}
&\bar{u}=\frac{100E_c h^3}{q_0 a^4}u\left(0,\frac{b}{2},z\right), \qquad \bar{w}=\frac{10E_c h^3}{q_0 a^4}w\left(\frac{a}{2},\frac{b}{2}\right)\\
&\bar{\sigma}_x(z)=\frac{h}{q_0 a}\sigma_x\left(\frac{a}{2},\frac{b}{2},z\right), \quad \bar{\tau}_{xy}(z)=\frac{h}{q_0 a}\tau_{xy}(0,0,z)\\
&\bar{\tau}_{xz}(z)=\frac{h}{q_0 a}\tau_{xz}\left(0,\frac{b}{2},z\right), \quad \hat{\omega}=\omega h\sqrt{\rho_c/E_c}, \quad \bar{\omega}=\frac{\omega a^2}{h}\sqrt{\rho_c/E_c}\\
&\bar{\beta}=\omega h\sqrt{\rho_m/E_m}
\end{aligned} \tag{26}$$

4.1 Bending investigation

Example 1: The first example is established for thin and thick Al/Al_2O_3 square plates. Three different values of the gradient index p = 1, 4 and 10 are taken into account. The computed results are compared with quasi-3D solutions reported by Neves *et al*. (2012a, b and 2013) and Carrera *et al*. (2008, 2011) and those given by third shear deformation theory (TSDT) of Thai and Kim (2013), CPT (Carrera *et al.* 2008) and FSDT (Carrera *et al.* 2008). Table 2 presents non-dimensional normal stress $\bar{\sigma}_x$ and vertical displacement $\bar{w}$. Noted that the studies of Neves *et al.* (2012a, b and 2013) and Carrera *et al.* (2008, 2011) are implemented via Carrera's unified formulation. However, the quasi-3D solutions of Neves *et al*. are obtained based on sinusoidal (Neves *et al.* 2012a), hyperbolic (Neves *et al.* 2012b), and cubic (Neves *et al.* 2013) distributions of in-plane displacement and a quadratic distribution of vertical displacement within the thickness, while the analytical (Carrera *et al.* 2008) and finite element (Carrera *et al.* 2011) quasi-3D solutions of Carrera *et al*. are determined based on fourth-order distributions of both in-plane and vertical displacement within the thickness. In addition, the quasi-3D solutions consider the thickness stretching influence (i.e., $\varepsilon_z \neq 0$), which is not introduced in FSDT and the two proposed models. It can be observed that the computed results are in good agreement with quasi-3D solutions particularly with those given by Neves *et al*. (2012b). In general, the two proposed models give a good prediction of both displacement and stress even in thick FG plates (a/h = 4)

Table 4 Non-dimensional vertical displacement $\bar{w}$ of Al_2/AlO_3 rectangular plates.

a/h	b/a	Theory	Gradient index (p)					
			0.1	0.3	0.5	0.7	1	1.5
		3D (Zenkour, 2007)	0.5769	0.5247	0.4766	0.4324	0.3727	0.2890
		Quasi-3D (Zenkour, 2007)	0.5731	0.5181	0.4679	0.4222	0.3612	0.2771
		Quasi-3D (Mantari and Soares, 2012a)	0.5776	0.5222	0.4716	0.4255	0.3640	0.2792
	1	HSDT (Mantari and Soares, 2012b)	0.6363	0.5752	0.5195	0.4687	0.4018	0.3079
		TSDT (Thai and Kim, 2013)	0.6362	0.5751	0.5194	0.4687	0.4011	0.3079
		Model 1	0.6315	0.5709	0.5157	0.4654	0.3984	0.3061
		Model 2	0.6343	0.5734	0.5179	0.4674	0.3999	0.3072
		3D (Zenkour, 2007)	1.1944	1.0859	0.9864	0.8952	0.7727	0.6017
		Quasi-3D (Zenkour, 2007)	1.1880	1.0740	0.9701	0.8755	0.7494	0.5758
		Quasi-3D (Mantari and Soares, 2012a)	1.1938	1.0790	0.9748	0.8797	0.7530	0.5785
2	2	HSDT (Mantari and Soares, 2012b)	1.2776	1.1553	1.0441	0.9431	0.8093	0.6238
		TSDT (Thai and Kim, 2013)	1.2775	1.1553	1.0441	0.9431	0.8086	0.6238
		Model 1	1.2716	1.1500	1.0394	0.9389	0.8053	0.6216
		Model 2	1.2753	1.1532	1.0423	0.9415	0.8074	0.6231
		3D (Zenkour, 2007)	1.4430	1.3116	1.1913	1.0812	0.9334	0.7275
		Quasi-3D (Zenkour, 2007)	1.4354	1.2977	1.1722	1.0580	0.9057	0.6962
		Quasi-3D (Mantari and Soares, 2012a)	1.4419	1.3035	1.1774	1.0626	0.9096	0.6991
	3	HSDT (Mantari and Soares, 2012b)	1.5341	1.3874	1.2540	1.1329	0.9725	0.7506
		TSDT (Thai and Kim, 2013)	1.5340	1.3873	1.2540	1.1329	0.9719	0.7506
		Model 1	1.5277	1.3817	1.2490	1.1285	0.9684	0.7484
		Model 2	1.5316	1.3852	1.2521	1.1313	0.9706	0.7499
		3D (Zenkour, 2007)	0.3490	0.3168	0.2875	0.2608	0.2253	0.1805
		Quasi-3D (Zenkour, 2007)	0.3475	0.3142	0.2839	0.2563	0.2196	0.1692
		Quasi-3D (Mantari and Soares, 2012a)	0.3486	0.3152	0.2848	0.2571	0.2203	0.1697
	1	HSDT (Mantari and Soares, 2012b)	0.3602	0.3259	0.2949	0.2668	0.2295	0.1785
		TSDT (Thai and Kim, 2013)	0.3602	0.3259	0.2949	0.2668	0.2295	0.1785
		Model 1	0.3595	0.3253	0.2943	0.2663	0.2292	0.1783
		Model 2	0.3599	0.3257	0.2947	0.2666	0.2294	0.1785
		3D (Zenkour, 2007)	0.8153	0.7395	0.6708	0.6085	0.5257	0.4120
		Quasi-3D (Zenkour, 2007)	0.8120	0.7343	0.6635	0.5992	0.5136	0.3962
		Quasi-3D (Mantari and Soares, 2012a)	0.8145	0.7365	0.6655	0.6009	0.5151	0.3973
4	2	HSDT (Mantari and Soares, 2012b)	0.8325	0.7534	0.6819	0.6173	0.5319	0.4150
		TSDT (Thai and Kim, 2013)	0.8325	0.7534	0.6819	0.6173	0.5319	0.4150
		Model 1	0.8315	0.7525	0.6812	0.6167	0.5314	0.4148
		Model 2	0.8321	0.7531	0.6817	0.6172	0.5318	0.4150
		3D (Zenkour, 2007)	1.0134	0.9190	0.8335	0.7561	0.6533	0.5121
		Quasi-3D (Zenkour, 2007)	1.0094	0.9127	0.8248	0.7449	0.6385	0.4927
		Quasi-3D (Mantari and Soares, 2012a)	1.0124	0.9155	0.8272	0.7470	0.6404	0.4941
	3	HSDT (Mantari and Soares, 2012b)	1.0325	0.9345	0.8459	0.7659	0.6601	0.5154
		TSDT (Thai and Kim, 2013)	1.0325	0.9345	0.8459	0.7659	0.6601	0.5154
		Model 1	1.0315	0.9335	0.8451	0.7652	0.6595	0.5151
		Model 2	1.0322	0.9342	0.8457	0.7657	0.6600	0.5154

where the influences of shear deformation are considerable. Whereas, the FSDT gives a good prediction of displacement only, but the stress is no better than those given by CPT. Noted that the two proposed models have only four variables, while the number of variables in FSDT and quasi-3D (Neves *et al.* 2012b) is five and eleven, respectively. Also, the two proposed models do not required shear correction coefficients as in the case of FSDT.

Table 5 Non-dimensional fundamental frequencies $\bar{\beta}$ of Al/ZrO_3 square FG plates

Theory	$p=0$		$p=1$			$a/h=5$		
	$a/h=\sqrt{10}$	$a/h=10$	$a/h=5$	$a/h=10$	$a/h=20$	$p=2$	$p=3$	$p=5$
3D (Vel and Batra, 2004)	0.4658	0.0596	0.2192	0.0578	0.0153	0.2211	0.2197	0.2225
Quasi-3D (Neves *et al*. 2012a)	–	–	0.2193	0.0596	0.0153	0.2198	0.2212	0.2225
Quasi-3D (Neves *et al*. 2012b)	–	–	0.2193	0.0596	0.0153	0.2201	0.2216	0.2230
Quasi-3D (Neves *et al*. 2013)	–	–	0.2193	–	–	0.2200	0.2215	0.2230
TSDT (Thai and Kim, 2013)	0.4623	0.0577	0.2169	0.0592	0.0152	0.2178	0.2193	0.2206
Model 1	0.4624	0.0577	0.2277	0.0619	0.0158	0.2257	0.2262	0.2271
Model 2	0.4628	0.0577	0.2277	0.0618	0.0158	0.2257	0.2262	0.2271

Table 6 non-dimensional frequency $\hat{\omega}$ of Al/Al_2O_3 square FG plates

a/h	Mode (m,n)	Theory	Gradient index (p)				
			0	0.5	1	4	10
5	1(1,1)	TSDT (Hosseini *et al.* 2011a)	0.2113	0.1807	0.1631	0.1378	0.1301
		FSDT (Hosseini *et al.* 2011b)	0.2112	0.1805	0.1631	0.1397	0.1324
		TSDT (Thai and Kim, 2013)	0.2113	0.1807	0.1631	0.1378	0.1301
		Model 1	0.2113	0.1807	0.1631	0.1377	0.1300
		Model 2	0.2113	0.1808	0.1632	0.1377	0.1300
	2(1,2)	TSDT (Hosseini *et al.* 2011a)	0.4623	0.3989	0.3607	0.2980	0.2771
		FSDT (Hosseini *et al.* 2011b)	0.4618	0.3978	0.3604	0.3049	0.2856
		TSDT (Thai and Kim, 2013)	0.4623	0.3989	0.3607	0.2980	0.2771
		Model 1	0.4624	0.3990	0.3608	0.2977	0.2770
		Model 2	0.4628	0.3992	0.3610	0.2976	0.2771
	3(2,2)	TSDT (Hosseini *et al.* 2011a)	0.6688	0.5803	0.5254	0.4284	0.3948
		FSDT (Hosseini *et al.* 2011b)	0.6676	0.5779	0.5245	0.4405	0.4097
		TSDT (Thai and Kim, 2013)	0.6688	0.5803	0.5254	0.4284	0.3948
		Model 1	0.6693	0.5806	0.5257	0.4280	0.3948
		Model 2	0.6701	0.5812	0.5262	0.4280	0.3952
10	1(1,1)	TSDT (Hosseini *et al.* 2011a)	0.0577	0.0490	0.0442	0.0381	0.0364
		FSDT (Hosseini *et al.* 2011b)	0.0577	0.0490	0.0442	0.0382	0.0366
		TSDT (Thai and Kim, 2013)	0.0577	0.0490	0.0442	0.0381	0.0364
		Model 1	0.0577	0.0490	0.0442	0.0381	0.0364
		Model 2	0.0577	0.0490	0.0442	0.0381	0.0364
	2(1,2)	TSDT (Hosseini *et al.* 2011a)	0.1377	0.1174	0.1059	0.0903	0.0856
		FSDT (Hosseini *et al.* 2011b)	0.1376	0.1173	0.1059	0.0911	0.0867
		TSDT (Thai and Kim, 2013)	0.1377	0.1174	0.1059	0.0903	0.0856
		Model 1	0.1377	0.1174	0.1059	0.0902	0.0856
		Model 2	0.1377	0.1174	0.1059	0.0902	0.0856
	3(2,2)	TSDT (Hosseini *et al.* 2011a)	0.2113	0.1807	0.1631	0.1378	0.1301
		FSDT (Hosseini *et al.* 2011b)	0.2112	0.1805	0.1631	0.1397	0.1324
		TSDT (Thai and Kim, 2013)	0.2113	0.1807	0.1631	0.1378	0.1301
		Model 1	0.2113	0.1807	0.1631	0.1377	0.1300
		Model 2	0.2114	0.1808	0.1632	0.1377	0.1300
20	1(1,1)	TSDT (Hosseini *et al.* 2011a)	0.0148	0.0125	0.0113	0.0098	0.0094
		FSDT (Hosseini *et al.* 2011b)	0.0148	0.0125	0.0113	0.0098	0.0094
		TSDT (Thai and Kim, 2013)	0.0148	0.0125	0.0113	0.0098	0.0094
		Model 1	0.0148	0.0125	0.0113	0.0098	0.0094
		Model 2	0.0148	0.0125	0.0113	0.0098	0.0094

Table 7 The first two non-dimensional frequencies $\hat{\omega}$ of Al/Al_2O_3 square FG plates

Mode	a/h	Theory	Gradient index (p)				
			0	0.5	1	4	10
1	2	Quasi-3D (Matsunaga, 2008)	0.9400	0.8233	0.7477	0.5997	0.5460
		TSDT (Thai and Kim, 2013)	0.9297	0.8110	0.7356	0.5924	0.5412
		Model 1	0.9307	0.8118	0.7363	0.5921	0.5414
		Model 2	0.9323	0.8130	0.7374	0.5923	0.5423
	5	Quasi-3D (Matsunaga, 2008)	0.2121	0.1819	0.1640	0.1383	0.1306
		TSDT (Thai and Kim, 2013)	0.2113	0.1807	0.1631	0.1378	0.1301
		Model 1	0.2113	0.1807	0.1631	0.1377	0.1300
		Model 2	0.2114	0.1808	0.1632	0.1377	0.1300
	10	Quasi-3D (Matsunaga, 2008)	0.0578	0.0492	0.0443	0.0381	0.0364
		TSDT (Thai and Kim, 2013)	0.0577	0.0490	0.0442	0.0381	0.0364
		Model 1	0.0577	0.0490	0.0442	0.0381	0.0364
		Model 2	0.0577	0.0490	0.0442	0.0380	0.0364
2	2	Quasi-3D (Matsunaga, 2008)	1.7406	1.5425	1.4078	1.1040	0.9847
		TSDT (Thai and Kim, 2013)	1.7233	1.5192	1.3844	1.0919	0.9807
		Model 1	1.7283	1.5231	1.3880	1.0934	0.9836
		Model 2	1.7345	1.5279	1.3926	1.0963	0.9876
	5	Quasi-3D (Matsunaga, 2008)	0.4658	0.4040	0.3644	0.3000	0.2790
		TSDT (Thai and Kim, 2013)	0.4623	0.3989	0.3607	0.2980	0.2771
		Model 1	0.4624	0.3990	0.3608	0.2977	0.2770
		Model 2	0.4628	0.3992	0.3610	0.2976	0.2771
	10	Quasi-3D (Matsunaga, 2008)	0.1381	0.1180	0.1063	0.0905	0.0859
		TSDT (Thai and Kim, 2013)	0.1376	0.1174	0.1059	0.0903	0.0856
		Model 1	0.1376	0.1174	0.1059	0.0902	0.0856
		Model 2	0.1376	0.1174	0.1059	0.0901	0.0856

Example 2: In the next example, a moderately thick Al/Al_2O_3 square plate with a/h = 10 is investigated. Table 3 shows non-dimensional displacements and stresses of plates for different values of gradient index p The computed results are compared with the quasi-3D solutions reported by Carrera *et al*. (2008) and Wu *et al*. (2011) and those generated by the sinusoidal shear deformation theory (SSDT) (Zenkour, 2006), higher-order shear deformation theory (HSDT) (Mantari *et al.* 2012) and third-order shear deformation theory (TSDT) (Wu and Li, 2010, Thai and Kim, 2013). Since the influence of thickness stretching is omitted in 2D plate models (Zenkour, 2006, Mantari *et al.* 2012, Wu and Li, 2010, Thai and Kim, 2013) and the two proposed models, they lead to the solutions close to each other, and their solutions are also in good agreement with the quasi-3D solutions (Carrera *et al.* 2008, Wu *et al.* 2011). The through thickness variation for displacement and stresses are also presented and compared in Fig. 2 for a thick plate with a/h = 5 and p = 4. In general, a good agreement between the results is observed, except in the case of the transverse shear stress $\bar{\tau}_{xz}$ where a small difference between the results is found (see Fig. 2(d)).

Example 3: In this example, an Al/Al_2O_3 plates with thickness ratios a/h= 2 and 4 are investigated to check accuracy of the two proposed models for very thick FG plates. The Young's modulus is calculated via the exponential distribution (Eq. 7b). The computed results are compared with 3D solutions (Zenkour, 2007), quasi-3D solutions (Zenkour, 2007, Mantari and Guedes Soares, 2012a), and those predicted by HSDT (Guedes Soares, 2012b) and TSDT (Thai and Kim, 2013) in Table 4. It

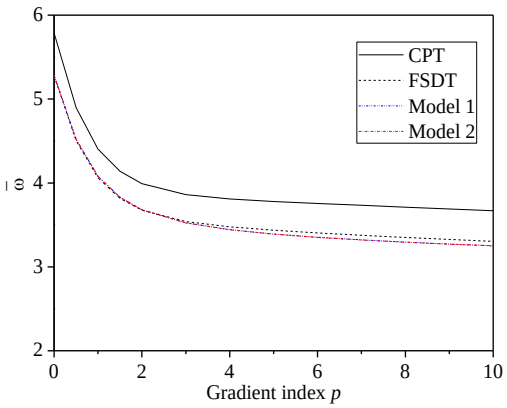


Fig. 3 Variation of non-dimensional fundamental natural frequency $\bar{\omega}$ versus gradient index p of Al/Al_2O_3 square FG plates (a/h = 5)

should be noted that the quasi-3D solutions reported by Zenkour (2007) are determined based on a sinusoidal distribution of both axial and vertical displacements, while the quasi-3D solutions provided by Mantari and Guedes Soares (2012a) are determined based on a cubic distribution of axial displacement and a quadratic distribution of vertical displacement. In general, the computed results are almost identical with those given by Mantari *et al*. (2012b) based on HSDT for all cases. Compared to 3D solutions (Zenkour, 2007) and quasi-3D solutions (Zenkour, 2007, Mantari and Guedes Soares, 2012a), the two proposed models slightly overestimate the vertical displacement of very thick FG plates with *a/h* = 2. This might be due to the thickness stretching influence in very thick plates which is neglected in the two proposed models, HSDT (Guedes Soares 2012b) and TSDT (Thai and Kim 2013). However, for thick plates with *a/h* = 4, good results are observed. It should be noted that the two proposed models contain four variables as against five in the case of HSDT (Mantari *et al*. 2012b) and six in the case of quasi-3D solutions (Zenkour 2007, Mantari and Guedes Soares 2012a). It can be concluded that the two proposed models are not only accurate but also efficient in predicting the responses of FG plates.

4.1 Free vibration investigation

Example 4: In this example, a verification procedure is performed for thin and thick *Al/ZrO₂* square plates. The obtained results are compared with those generated with 3D solutions of Vel and Batra (2004), quasi-3D solutions of Neves *et al*. (2012a, b and 2013) and TSDT of Thai and Kim (2013). The Young's modulus is predicted via Mori–Tanaka scheme. The non-dimensional fundamental frequency $\bar{\beta}$ is given in Table 5 for different values of thickness ratio and gradient index. It can be observed that the computed results agree well with the 3D solutions (Vel and Batra 2004) and quasi-3D solutions (Neves *et al.* 2012a, b and 2013).

Example 5: In the next example, thin and thick Al/Al_2O_3 square plates with thickness ratio varied from 5 to 20 and gradient index varied from 0 to 10 are investigated using different plate models. The non-dimensional frequencies $\hat{\omega}$ generated by TSDT (Hosseini-Hashemi *et al.* 2011a), FSDT (Hosseini-Hashemi *et al.* 2011b), TSDT (Thai and Kim, 2013), and the two proposed models are compared in Table 6. A good agreement between the results given by TSDT (Hosseini-Hashemi *et al.* 2011a) and the two proposed models is found for all modes of vibration. It should be noted that the TSDT (Hosseini-Hashemi *et al.* 2011a) contains more number of variables than the two proposed models. To further demonstrate the accuracy of the two proposed models for wide range of gradient index *p* and thickness ratio *a/h*, Figs. 3 and 4 illustrate the variation of non-dimensional frequency $\bar{\omega}$ versus the gradient index *p* and thickness ratio *a/h*, respectively. It can be observed that the two proposed models generated almost identical results for all values of gradient index *p* (see Fig. 3) and thickness ratio *a/h* (see Fig. 4), whereas FSDT slightly overestimates the frequency of FG plates at high value of gradient index *p*. This is due to the employ of a constant

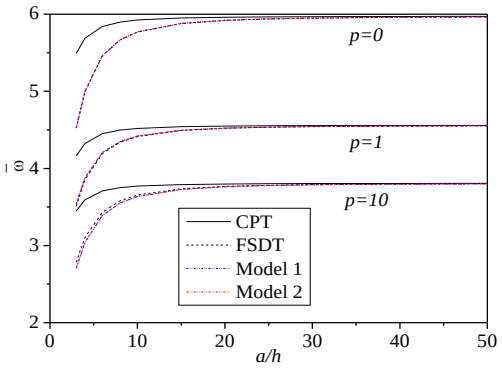


Fig. 4 Variation of non-dimensional fundamental natural frequency $\bar{\omega}$ versus side-to-thickness ratio (*a*/*h*) of Al/Al_2O_3 square FG plates

shear correction coefficient for any values of gradient index *p*. Since CPT neglects the shear deformation influences, it overestimates frequency of thick plates (see Fig. 3).

Example 6: In the last example, Al/Al_2O_3 square plates with thickness ratio *a/h* varied from 2 to 10 are examined. The aim of this example is to check the accuracy of the two proposed models for very thick FG plates. Table 7 presents the first two non-dimensional frequencies $\hat{\omega}$ for various values of thickness ratio and gradient index. The obtained results are compared with those given by the quasi-3D theory (Matsunaga 2008) and TSDT (Thai and Kim 2013). Again, the computed results are shown to be in excellent agreement with quasi-3D solutions (Matsunaga 2008) for all modes of vibration even in very thick plates (*a/h* = 2).

5. Conclusions

A simple HSDT for static and dynamic behaviors of FG plates is presented in this study. The theory accounts for the shear deformation influences without using a shear correction coefficient. By expressing the shear parts of the in-plane displacements with the integral term, the number of variables and governing equations of the two proposed models is reduced to four as against five in the FSDT and common HSDTs. Numerical results demonstrate that the two proposed models can be comparable with the existing HSDTs with a larger number of variables. An improvement of present approach will be considered in the future work to consider the thickness stretching effect by employing quasi-3D shear deformation models (Bessaim *et al.* 2013, Bousahla *et al.* 2014, Belabed *et al.* 2014, Fekrar *et al.* 2014, Hebali *et al.* 2014, Meradjah *et al.* 2015, Larbi Chaht *et al.* 2015, Hamidi *et al*. 2015, Bourada *et al.* 2015, Bennoun *et al.* 2016, Draiche *et al.* 2016, Benbakhti *et al.* 2016, Benahmed *et al.* 2017, Ait Atmane *et al.* 2017, Benchohra *et al.* 2017, Bouafia *et al.* 2017,) and the wave propagation problem (Mahmoud *et al.* 2015, Ait Yahia *et al.* 2015, Boukhari *et al.* 2016, Meftah *et al*.2017, Zaoui *et al.* 2017b).

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