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Plane waves in generalized magneto-thermo-viscoelastic medium with voids under the effect of initial stress and laser pulse heating

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Abstract. The present paper aims to study the influence of the magnetic field and initial stress on the 2-D problem of generalized thermo-viscoelastic material with voids subject to thermal loading by a laser pulse in the context of the Lord-Shulman and the classical dynamical coupled theories. The analytical expressions for the physical quantities are obtained in the physical domain by using the normal mode analysis. These expressions are calculated numerically for a specific material and explained graphically. Comparisons are made with the results predicted by the Lord-Shulman and the coupled theories in the presence and absence of the initial stress and the magnetic field.

Keywords: Lord-Shulman; thermo-viscoelasticity; initial stress; magnetic field; laser pulse; voids

1. Introduction

The heat conduction equations for the classical linear uncoupled and coupled thermoelasticity theories are of the diffusion type predicting the infinite speed of propagation of heat wave contrary to physical observations. To eliminate this paradox inherent in the classical theories, generalized theories of thermoelasticity were developed. The generalized thermoelasticity theories admit so-called second-sound effects, that is, they predict the finite velocity of propagation for the heat field. The first attempt towards the introduction of generalized thermoelasticity was headed in Lord and Shulman (1967). The Lord-Shulman theory introduces a new physical concept which called relaxation time. Since the heat conduction equation of this theory is of the wave-type, it automatically ensures the finite speed of propagation of heat wave. The second generalization was developed in Green and Linsay (1972). This theory contains two constants that act as relaxation times and modifies all the equations of coupled theory, not the heat conduction equation only. The linear viscoelasticity remains an important area of research. The mechanical model representation of linear viscoelastic behavior results was explored in Othman and Abbas (2012). The solution of the boundary value problems for linear viscoelastic materials, including temperature variations in both quasi-static and dynamic problems made great strides in the last decades, in Biot (1954). The solution of linear viscoelasticity problems with corresponding linear elastic solutions was linked in Bland (2016).

The theory of elastic materials with voids is one of the most important generalizations of the classical theory of elasticity. This theory is concerned with elastic materials consisting of a distribution of small pores (voids, which contain nothing of mechanical or energetic significance) in which the void volume is included among the kinematic variables. Practically, this theory is useful for investigating various types of geological and biological materials for which elastic theory is inadequate. The behavior of plane waves in a linear elastic material with voids was studied in Cowin (2013). The domain of influence theorem in the linear theory of elastic materials with voids was discussed in Dhaliwal and Wang (1994). The visco-elastic behavior of linear elastic materials with voids was studied by Singh *et al.* (2018). While a nonlinear and linear theory of thermo-visco-elastic materials with voids was studied in Iesan (2015). The mathematical simulation of the mechanism of acoustic drying of porous materials was investigated by Feodorov *et al.* (2012). One can find some work on thermo-elasticity with voids in the kinds of literature Othman and Atwa (2012), Sharma and Kumar (2016), Othman and Marin (2017), (Marin *et al.* 2017), Marin (1997), Marin (1999).

Investigation of the interaction between the magnetic field and stress and strain in a thermoelastic solid is very important due to its many applications in the fields of geophysics and plasma physics. Especially in nuclear fields, the extremely high temperature and temperature gradients, as well as the magnetic fields originating inside nuclear reactors influence their design and operations. The effect of two-temperature and gravity on the 2-D problem of thermo-viscoelastic material under the three-phase-lag model was investigated in Othman and Zidan (2015). During the second half of the twentieth century, great attention has been devoted to the study of electro magneto-thermoelastic coupled problems based on the generalized thermo-elasticity and MHD by Ellahi (2013), Yousif *et al.* (2019), Fetecau *et al.* (2018). The initial stresses are developed in

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the medium due to many reasons, resulting from the difference in temperature, the process of quenching, shot pinning and cold working, slow the process of creep, differential external of forces, gravity variations, etc. The Earth is supposed to be under high initial stresses. It is therefore of great interest to study the effect of these stresses on the propagation of stress waves. During the last five decades, considerable attenuation has been directed towards this phenomenon. The acoustic propagation under initial stresses would be fundamentally different from that under stress-free state was depicted in Biot (1965). Several problems with the theory of thermoelasticity have been solved by Hassan *et al.* (2018), Othman and Atwa (2014), Othman and Said (2015), Othman *et al.* (2015).

The classical Fourier's law of the heat conduction is no longer valid because it leads to an infinite propagation speed of the thermal energy. The stress wave induced by Pico and Femto-second laser pulses in a semi-infinite metal by expressing the laser pulse energy as a Fourier series was studied by Wang and Xu (2002). The ultra-short lasers are those with pulse duration ranging from nanoseconds to Femto-seconds. In the case of ultra-short-pulsed laser heating, the high-intensity energy flux and ultra-short duration laser beam has introduced situations, where very large thermal gradients or an ultra-high heating rate may exist on the boundaries was studied by Sun *et al.* (2008). They have proposed several models to describe the mechanism of heat conduction during the heating of the short-pulse; it has been found that the parabolic and the hyperbolic models are useful for modification materials as thin films. When a metal film is heated by a laser pulse, a thermoelastic wave is generated due to thermal expansion near the surface.

The present work is to obtain the physical quantities in a homogenous, isotropic, thermo-visco-elastic material with voids subject to thermal loading by a laser pulse in the case of absence and presence of initial stress and magnetic field. The model is illustrated in the context of (CT) and (L-S) theories. The normal mode analysis is used to obtain the expressions for physical quantities. The distributions of considered variables are represented graphically.

2. Formulation of the problem

Consider a homogeneous, isotropic, thermally conducting viscoelastic half-space ($z \geq 0$) with voids. For the two− dimensional problem, we assume the dynamic displacement vector as $\boldsymbol{u}=(u,0,w)$. All quantities considered will be functions of the time variable t and of the coordinates x and z The whole body is at a constant temperature $T_0$and it is acting on throughout by a constant magnetic field $H=(0,H_0,0)$, which is oriented towards the positive direction of the y-axis. Due to the application of this magnetic field, their results an induced magnetic field $\boldsymbol{h}$ and an induced electric field $\boldsymbol{E}$. We assume that both $\boldsymbol{h}$ and $\boldsymbol{E}$ are small in magnitude in accordance with the assumptions of the linear theory of thermo-viscoelasticity. The electric intensity vector is normal to both the magnetic intensity and the displacement vector. Thus, it has the components $\boldsymbol{E}=(E_1,0,E_3)$, the current density vector $\boldsymbol{J}$ is parallel to $\boldsymbol{E}$, thus $\boldsymbol{J}=(J_1,0,J_3)$.

The variation of the magnetic and electric fields for a finitely conducting slowly moving medium, are given by Maxwell's equations as Othman and Abd-Elaziz (2017).

$$\operatorname{curl}\boldsymbol{h} = \boldsymbol{J} + \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}, \tag{1}$$

$$\boldsymbol{E} = -\mu_0 (\frac{\partial \boldsymbol{u}}{\partial t} \times \boldsymbol{H}), \tag{2}$$

$$\operatorname{curl} \boldsymbol{E} = -\mu_0 \frac{\partial \boldsymbol{h}}{\partial t}, \tag{3}$$

$$\operatorname{div}\boldsymbol{h} = 0. \tag{4}$$

From Eqs. (1)-(4) we can obtain

$$\boldsymbol{E} = \mu_0 H_0 (\frac{\partial w}{\partial t}, 0, -\frac{\partial u}{\partial t}), \qquad \boldsymbol{h} = -H_0(0, e, 0),$$

$$\boldsymbol{J} = (H_0 \frac{\partial e}{\partial z} - \varepsilon_0 \mu_0 H_0 \frac{\partial^2 w}{\partial t^2}, 0, -H_0 \frac{\partial e}{\partial x} + \varepsilon_0 \mu_0 H_0 \frac{\partial^2 u}{\partial t^2}).$$

The basic governing equations for a linear generalized thermo-viscoelastic media with voids under the effect of initial stress and magnetic field in the absence of body forces are written in (Lord-Shulman 1967, Cowin 1985).

$$(\mu^* + p)\nabla^2 \boldsymbol{u} + (\lambda^* + \mu^*)\nabla(\nabla \cdot \boldsymbol{u}) - \beta^* \nabla T + b^* \nabla \phi + \boldsymbol{F} = \rho \frac{\partial^2 \boldsymbol{u}}{\partial t^2}, \tag{5}$$

$$A^* \nabla^2 \phi - \xi_1 \phi - \xi_2 \dot{\phi} - B^* (\nabla \cdot \boldsymbol{u}) + (\tau \nabla^2 + m) T = \rho \chi \frac{\partial^2 \phi}{\partial t^2}, \tag{6}$$

$$\begin{gathered}(1 + \tau_0 \frac{\partial}{\partial t})[\rho C_e \frac{\partial T}{\partial t} + \beta^* T_0 \frac{\partial e}{\partial t} + m T_0 \frac{\partial \phi}{\partial t} - \rho Q] \\ - \varsigma \nabla^2 \frac{\partial \phi}{\partial t} = K \nabla^2 T.\end{gathered} \tag{7}$$

And the constitutive relations are given by

$$\sigma_{ij} = (\mu^* + p) u_{i,j} + \mu^* u_{j,i} + (\lambda^* u_{k,k} - \beta^* T + b^* \phi)\delta_{ij}, \tag{8}$$

The parameters λ^*, μ^*, β^*, A^*, B^* and b^* are defined as

$$\begin{gathered}\lambda^* = \lambda(1 + \alpha_0 \frac{\partial}{\partial t}), \quad \mu^* = \mu(1 + \alpha_1 \frac{\partial}{\partial t}), \quad \beta^* = \beta(1 + \beta_0 \frac{\partial}{\partial t}), \\ A^* = A(1 + \alpha_3 \frac{\partial}{\partial t}), \ B^* = b(1 + \alpha_4 \frac{\partial}{\partial t}), \quad b^* = b(1 + \alpha_2 \frac{\partial}{\partial t}),\end{gathered} \tag{9}$$

where, $\beta_0 = \frac{1}{\beta}(3\lambda\alpha_0 + 2\mu\alpha_1)\alpha_t$, $\beta = (3\lambda + 2\mu)\alpha_t$, the dot notation is used to denote time differentiation. The strain tensor is $e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \ i, j = 1,3$ and $\boldsymbol{F}$ is the Lorentz force given by $\boldsymbol{F}= \mu0(\boldsymbol{J}\times\boldsymbol{H})$.

The components of the Lorentz force will be

$$F_x = \mu_0 H_0 (H_0 \frac{\partial e}{\partial x} - \varepsilon_0 \mu_0 H_0 \frac{\partial^2 u}{\partial t^2}), \quad F_y = 0,$$

$$F_z = \mu_0 H_0 (H_0 \frac{\partial e}{\partial z} - \varepsilon_0 \mu_0 H_0 \frac{\partial^2 w}{\partial t^2}).$$

The plate surface is illuminated by laser pulse given by the heat input

$$\lambda^* = \lambda(1+\alpha_0 \frac{\partial}{\partial t}), \quad \mu^* = \mu(1+\alpha_1 \frac{\partial}{\partial t}), \quad \beta^* = \beta(1+\beta_0 \frac{\partial}{\partial t}),$$
$$A^* = A(1+\alpha_3 \frac{\partial}{\partial t}),\ B^* = b(1+\alpha_4 \frac{\partial}{\partial t}), \quad b^* = b(1+\alpha_2 \frac{\partial}{\partial t}), \tag{10}$$

For x–z plane, Eq. (5) gives rise to the following two equations

$$[\mu(1+\alpha_1 \frac{\partial}{\partial t})+p]\nabla^2 u + [\lambda(1+\alpha_0 \frac{\partial}{\partial t}) + \mu(1+\alpha_1 \frac{\partial}{\partial t}) + \mu_0 H_0^2]\frac{\partial e}{\partial x}$$
$$-\beta(1+\beta_0 \frac{\partial}{\partial t})\frac{\partial T}{\partial x} + b(1+\alpha_2 \frac{\partial}{\partial t})\frac{\partial \phi}{\partial x} = (\rho+\varepsilon_0\mu_0^2 H_0^2)\frac{\partial^2 u}{\partial t^2}, \tag{11}$$

$$[\mu(1+\alpha_1 \frac{\partial}{\partial t})+p]\nabla^2 w + [\lambda(1+\alpha_0 \frac{\partial}{\partial t}) + \mu(1+\alpha_1 \frac{\partial}{\partial t}) + \mu_0 H_0^2]\frac{\partial e}{\partial z}$$
$$-\beta(1+\beta_0 \frac{\partial}{\partial t})\frac{\partial T}{\partial z} + b(1+\alpha_2 \frac{\partial}{\partial t})\frac{\partial \phi}{\partial z} = (\rho+\varepsilon_0\mu_0^2 H_0^2)\frac{\partial^2 w}{\partial t^2}. \tag{12}$$

For simplifications we shall use the following non-dimensional variables:

$$x_i' = \frac{\varpi}{c_1}x_i,\ u_i' = \frac{\rho c_1 \varpi}{\beta T_0}u_i,\ T' = \frac{T}{T_0},\ \phi' = \frac{\varpi^2\chi}{c_1^2}\phi,\ t' = \varpi t,$$
$$\sigma_{ij}' = \frac{\sigma_{ij}}{\beta T_0},\ p' = \frac{p}{\beta T_0},\ c_1^2 = \frac{\lambda+2\mu}{\rho},\ \varpi = \frac{C_e(\lambda+2\mu)}{K}, \tag{13}$$
$$Q' = \frac{Q}{\varpi T_0 C_e}, \quad c_2^2 = \frac{\mu}{\rho},$$
$$\{\alpha_0', \alpha_1', \alpha_2', \alpha_3', \alpha_4', \tau_0'\} = \varpi\{\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \tau_0\}.$$

where, ϖ is the characteristic frequency of the material and c_1, c_2 are the longitudinal and shear wave velocities in the medium, respectively.

Using Eq. (13), then, Eqs. (11), (12), (6) and (7) become respectively (dropping the dashed for convenience).

$$[\delta^2(1+\alpha_1 \frac{\partial}{\partial t})+a_0 p]\nabla^2 u + [(1-2\delta^2)(1+\alpha_0 \frac{\partial}{\partial t}) + \delta^2(1+\alpha_1 \frac{\partial}{\partial t})$$
$$+a_1]\frac{\partial e}{\partial x} - (1+\beta_0 \frac{\partial}{\partial t})\frac{\partial T}{\partial x} + a_2(1+\alpha_2 \frac{\partial}{\partial t})\frac{\partial \phi}{\partial x} = a_3\frac{\partial^2 u}{\partial t^2}, \tag{14}$$

$$[\delta^2(1+\alpha_1 \frac{\partial}{\partial t})+a_0 p]\nabla^2 w + [(1-2\delta^2)(1+\alpha_0 \frac{\partial}{\partial t}) + \delta^2(1+\alpha_1 \frac{\partial}{\partial t})$$
$$+a_1]\frac{\partial e}{\partial z} - (1+\beta_0 \frac{\partial}{\partial t})\frac{\partial T}{\partial z} + a_2(1+\alpha_2 \frac{\partial}{\partial t})\frac{\partial \phi}{\partial z} = a_3\frac{\partial^2 w}{\partial t^2}, \tag{15}$$

$$(1+\alpha_3 \frac{\partial}{\partial t})\nabla^2\phi - a_4(\phi+\xi\frac{\partial \phi}{\partial t}) - a_5(1+\alpha_4 \frac{\partial}{\partial t})e$$
$$+(a_6\nabla^2 + a_7)T = \frac{1}{\delta_1^2}\frac{\partial^2 \phi}{\partial t^2}, \tag{16}$$

$$(1+\tau_0 \frac{\partial}{\partial t})[\frac{\partial T}{\partial t} + \varepsilon(1+\beta_0 \frac{\partial}{\partial t})\frac{\partial e}{\partial t} + a_8\frac{\partial \phi}{\partial t} - Q] - a_9\nabla^2\frac{\partial \phi}{\partial t} = \nabla^2 T, \tag{17}$$

where,

$$a_0 = \frac{\beta T_0}{\rho c_1^2},\ a_1 = \frac{c_A^2}{c_1^2},\ a_2 = \frac{b c_1^2}{\varpi^2\chi\beta T_0},\ a_3 = 1+\frac{c_A^2}{c^2},\ c_A^2 = \frac{\mu_0 H_0^2}{\rho},$$

$$c^2 = \frac{1}{\varepsilon_0\mu_0},\ a_4 = \frac{\xi_1 c_1^2}{A\varpi^2},\ a_5 = \frac{b\chi\beta T_0}{A\rho c_1^2},\ a_6 = \frac{\tau\varpi^2\chi T_0}{A c_1^2},$$
$$a_7 = \frac{m\chi T_0}{A},\ a_8 = \frac{m c_1^4}{K\varpi^3\chi},\ a_9 = \frac{\varsigma c_1^2}{K\varpi\chi T_0},\ \delta^2 = \frac{c_2^2}{c_1^2},\ c_3^2 = \frac{A}{\rho\chi},$$
$$\delta_1^2 = \frac{c_3^2}{c_1^2},\ \xi = \frac{\xi_2\varpi}{\xi_1},\ \varepsilon = \frac{T_0\beta^2}{\rho C_e(\lambda+2\mu)},\ i, j = 1, 3.$$

The non-dimensional constitutive relations are given by

$$\sigma_{ij} = [\delta^2(1+\alpha_1 \frac{\partial}{\partial t}) + a_0 p]u_{i,j} + \delta^2(1+\alpha_1 \frac{\partial}{\partial t})u_{j,i}$$
$$+[(1-2\delta^2)(1+\alpha_0 \frac{\partial}{\partial t})u_{k,k} - (1+\beta_0 \frac{\partial}{\partial t})T + a_2(1+\alpha_2 \frac{\partial}{\partial t})\phi]\delta_{ij}. \tag{18}$$

The expressions relating displacement components $u(x,z,t)$ $w(x,z,t)$ to the potentials are

$$u = \Phi_{,x} + \Psi_{,z},\ w = \Phi_{,z} - \Psi_{,x},\ e = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = \nabla^2\Phi,$$
$$\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = \nabla^2\Psi. \tag{19}$$

Substituting from Eq. (19) into Eqs. (14)-(17), we obtain

$$[\delta^2(1+\alpha_1 \frac{\partial}{\partial t}) + a_0 p]\nabla^2\Psi = a_3\frac{\partial^2\Psi}{\partial t^2}, \tag{20}$$

$$(1+a_0 p + a_1 + \delta_0 \frac{\partial}{\partial t})\nabla^2\Phi - (1+\beta_0 \frac{\partial}{\partial t})T + a_2(1+\alpha_2 \frac{\partial}{\partial t})\phi = a_3\frac{\partial^2\Phi}{\partial t^2}, \tag{21}$$

$$(1+\alpha_3 \frac{\partial}{\partial t})\nabla^2\phi - a_4(\phi+\xi\frac{\partial \phi}{\partial t}) - a_5(1+\alpha_4 \frac{\partial}{\partial t})\nabla^2\Phi$$
$$+(a_6\nabla^2 + a_7)T = \frac{1}{\delta_1^2}\frac{\partial^2 \phi}{\partial t^2}, \tag{22}$$

$$(1+\tau_0 \frac{\partial}{\partial t})[\frac{\partial T}{\partial t} + \varepsilon(1+\beta_0 \frac{\partial}{\partial t})\nabla^2\frac{\partial \Phi}{\partial t} + a_8\frac{\partial \phi}{\partial t} - Q]$$
$$-a_9\nabla^2\frac{\partial \phi}{\partial t} = \nabla^2 T. \tag{23}$$

where $\delta_0 = \alpha_0 + 2\delta^2(\alpha_1 - \alpha_0)$.

3. Normal mode analysis

The solution of the considered physical variable can be decomposed in terms of normal modes as the following form:

$$[\Phi, \Psi, T, \phi, \sigma_{ij}](x, z, t) = [\bar{\Phi}, \bar{\Psi}, \bar{T}, \bar{\phi}, \bar{\sigma}_{ij}](z)\exp[\omega t + \mathrm{i}a x] \tag{24}$$

where, ω is the frequency, a is the wave number in the $x-$ direction and $\mathrm{i} = \sqrt{-1}$.

Eqs. (20)-(23) with the aid of Eq. (24) become respectively,

$$(\mathrm{D}^2 - k_1^2)\bar{\Psi} = 0, \tag{25}$$

$$(b_1\mathrm{D}^2-b_2)\bar{\Phi}-b_3\bar{T}+b_4\bar{\phi}=0, \tag{26}$$

$$(b_5\mathrm{D}^2-b_6)\bar{\Phi}-(a_6\mathrm{D}^2-b_7)\bar{T}-(b_8\mathrm{D}^2-b_9)\bar{\phi}=0, \tag{27}$$

$$(b_{10}\mathrm{D}^2-b_{11})\bar{\Phi}-(\mathrm{D}^2-b_{12})\bar{T}-(b_{13}\mathrm{D}^2-b_{14})\bar{\phi}=Q_0 f\,(x,t)\,\mathrm{e}^{-\gamma z}. \tag{28}$$

where

$$\mathrm{D}=\frac{\mathrm{d}}{\mathrm{dz}},\ k_1^2=a^2+\frac{a_3\omega^2}{\delta^2(1+\alpha_1\omega)+a_0 p},\ \ b_1=1+a_0p+a_1+\omega\delta_0,$$

$$b_2=b_1a^2+a_3\omega^2, b_3=1+\omega\beta_0,\ b_4=a_2(1+\omega\alpha_2),$$

$$b_5=a_5(1+\alpha_4\omega), b_6=b_5a^2,\ b_7=a_6a^2-a_7, b_8=1+\alpha_3\omega,$$

$$b_9=b_8a^2+a_4(1+\xi\omega)+\frac{\omega^2}{\delta_1^2},\ b_{10}=\varepsilon(1+\omega\beta_0)(1+\omega\tau_0)\omega,\ b_{11}=b_{10}a^2,$$

$$b_{12}=a^2+(1+\omega\tau_0)\omega, b_{13}=a_9\omega,\ \ b_{14}=b_{13}a^2+a_8(1+\omega\tau_0)\omega,$$

$$Q_0=\frac{I_0\gamma}{2\pi r^2t_0^2}, f\,(x,t)=[1+\tau_0(1-\frac{t}{t_0})]\exp(-\frac{t}{t_0}-\frac{x^2}{r^2}-\omega t-iax).$$

Eliminating $\bar{T}$ and $\bar{\phi}$ between Eqs. (26)-(28) we get the following ordinary differential equation satisfied with $\bar{\Phi}$

$$(\mathrm{D}^6-d_1\mathrm{D}^4+d_2\mathrm{D}^2-d_3)\bar{\Phi}=Q_0N_1 f\,(x,t)\,\mathrm{e}^{-\gamma z}. \tag{29}$$

In a similar manner

$$(\mathrm{D}^6-d_1\mathrm{D}^4+d_2\mathrm{D}^2-d_3)\{\bar{T},\bar{\phi}\}=Q_0\{N_2,N_3\}f\,(x,t)\,\mathrm{e}^{-\gamma z}, \tag{30}$$

where,

$$d_1=\frac{f_1}{f_0}, d_2=\frac{f_2}{f_0}, d_3=\frac{f_3}{f_0},\quad f_0=b_1(b_8-a_6b_{13}),$$

$$f_1=a_6(b_1b_{14}+b_2b_{13}-b_4b_{10})-b_1(b_9+b_8b_{12}-b_7b_{13})$$
$$+b_5(b_3b_{13}+b_4)-b_8(b_2+b_3b_{10}),$$

$$f_2=b_1(b_7b_{14}-b_9b_{12})+b_2(a_6b_{14}+b_7b_{13}-b_8b_{12}-b_9)$$
$$+b_3(b_5b_{14}+b_6b_{13}-b_8b_{11}-b_9b_{10})+b_4(b_5b_{12}+b_6-a_6b_{11}-b_7b_{10}),$$

$$f_3=b_2(b_7b_{14}-b_9b_{12})+b_3(b_6b_{14}-b_9b_{11})+b_4(b_6b_{13}-b_7b_{11}),$$

$$N_1=-(a_6b_4+b_3b_8)\gamma^2+b_4b_7+b_3b_9,$$

$$N_2=-b_1b_8\gamma^4+(b_1b_9-b_4b_5+b_2b_8)\gamma^2+b_4b_6-b_2b_9,$$

$$N_3=a_6b_1\gamma^4-(a_6b_2+b_1b_7+b_3b_5)\gamma^2+b_2b_7+b_3b_6.$$

Eq. (29) can be factored as:

$$(\mathrm{D}^2-k_2^2)(\mathrm{D}^2-k_3^2)(\mathrm{D}^2-k_4^2)\bar{\Phi}=Q_0N_1 f\,(x,t)\,\mathrm{e}^{-\gamma z}, \tag{31}$$

where, $k_j^2\ (j=2,3,4)$ are the roots of the characteristic equation of Eq. (31).

The solution of Eq. (25) bound as $z\to\infty$, can be written as

$$\bar{\Psi}(z)=R_1e^{-k_1z}. \tag{32}$$

The solution of Eq. (31), bound as $z\to\infty$, is given by

$$\bar{\Phi}(z)=\sum_{j=2}^{4}R_je^{-k_jz}+L_1Q_0N_1 f\,(x,t)\,\mathrm{e}^{-\gamma z}. \tag{33}$$

Similarly, the solution of Eq. (30), can be written as

$$\{\bar{T}(z),\bar{\phi}(z)\}=\sum_{j=2}^{4}\{S_{1j},S_{2j}\}R_je^{-k_jz}+L_1Q_0\{N_2,N_3\}f\,(x,t)\,\mathrm{e}^{-\gamma z}, \tag{34}$$

where

$$S_{1j}=\frac{b_1b_8k_j^4-(b_1b_9+b_2b_8-b_4b_5)k_j^2+b_2b_9-b_4b_6}{(b_3b_8+b_4a_6)k_j^2-(b_3b_9+b_4b_7)},$$

$$S_{2j}=\frac{-b_1a_6k_j^4+(b_1b_7+b_2a_6+b_3b_5)k_j^2-(b_2b_7+b_3b_6)}{(b_3b_8+b_4a_4)k_j^2-(b_3b_9+b_4b_7)},$$

$$L_1=\frac{1}{\gamma^6-d_1\gamma^4+d_2\gamma^2-d_3},\quad j=2,3,4.$$

Substituting Eqs. (32)-(34) in Eq. (24) we get

$$\Psi=R_1e^{-k_1z+\omega t+iax}, \tag{35}$$

$$\{\Phi,T,\phi\}=\sum_{j=2}^{4}\{1,S_{1j},S_{2j}\}R_je^{-k_jz+\omega t+iax}+L_1Q_0\{N_1,N_2,N_3\}f_0(x,t)\,\mathrm{e}^{-\gamma z}. \tag{36}$$

Inserting Eqs. (35) and (36) in Eq. (19), the displacement components u and w, bound as $z\to\infty$ are obtained as

$$u=(\sum_{j=2}^{4}iaR_je^{-k_jz}-k_1R_1e^{-k_1z})e^{\omega t+iax}-\frac{2x}{r^2}L_1Q_1N_1\,\mathrm{e}^{-\gamma z}, \tag{37}$$

$$w=-(\sum_{j=2}^{4}k_jR_je^{-k_jz}+iaR_1e^{-k_1z})e^{\omega t+iax}-\gamma L_1Q_1N_1\,\mathrm{e}^{-\gamma z}. \tag{38}$$

The stress components are of the form

$$\begin{aligned}\sigma_{xx}&=iak_1[(1-2\delta^2)(1+\alpha_0\omega)-(1+a_0p+\delta_0\omega)]R_1e^{-k_1z+\omega t+iax}\\&+L_1L_2Q_1\,\mathrm{e}^{-\gamma z}+\sum_{j=2}^{4}[(1-2\delta^2)(1+\alpha_0\omega)k_j^2-(1+a_0p+\delta_0\omega)a^2\\&-b_3S_{1j}+b_4S_{2j}]R_je^{-k_jz+\omega t+iax},\end{aligned} \tag{39}$$

$$\begin{aligned}\sigma_{zz}&=iak_1[(1+a_0p+\delta_0\omega)-(1-2\delta^2)(1+\alpha_0\omega)]R_1e^{-k_1z+\omega t+iax}\\&+L_1L_3Q_1\,\mathrm{e}^{-\gamma z}+\sum_{j=2}^{4}[(1+a_0p+\delta_0\omega)k_j^2-(1-2\delta^2)(1+\alpha_0\omega)a^2\\&-b_3S_{1j}+b_4S_{2j}]R_je^{-k_jz+\omega t+iax},\end{aligned} \tag{40}$$

$$\begin{aligned}\sigma_{xz}&=[\delta^2(1+\alpha_1\omega)(a^2+k_1^2)+a_0pk_1^2]R_1e^{-k_1z+\omega t+iax}\\&+L_1L_4Q_1\,\mathrm{e}^{-\gamma z}-\mathrm{i}a[2\delta^2(1+\alpha_1\omega)+a_0p]\sum_{j=2}^{4}k_jR_je^{-k_jz+\omega t+iax},\end{aligned} \tag{41}$$

where

$$Q_1=Q_0f_0(x,t),\ f_0(x,t)=[1+\tau_0(1-\frac{t}{t_0})]\exp(-\frac{t}{t_0}-\frac{x^2}{r^2}),$$

$$L_2 = -(1+a_0 p+\delta_0 t_1)\left(\frac{2}{r^2}-\frac{4x^2}{r^4}\right)N_1 + (1-2\delta^2)(1+\alpha_0 t_1)\gamma^2 N_1 - (1+\beta_0 t_1)N_2$$

$$L_3 = [-(1+a_0 p+\delta_0 t_1)\gamma^2 + (1-2\delta^2)(1+\alpha_0 t_1)\left(\frac{2}{r^2}-\frac{4x^2}{r^4}\right)]N_1 - (1+\beta_0 t_1)N_2$$

$$L_4 = \frac{2x\gamma}{r^2}[2\delta^2(1+\alpha_1 t_1)+a_0 p]N_1,$$

$$t_1 = -\frac{1}{t_0}\left[1+\frac{\tau_0 t_0}{t_0+\tau_0(t_0-t)}\right].$$

4. The boundary conditions

In order to determine the parameters $R_j(j=1,2,3,4)$ we need to consider the boundary condition at $z=0$ as follows:

The mechanical boundary conditions:

$$\sigma_{zz} = -p_1 N(x,t), \quad \sigma_{xz} = 0, \quad \frac{\partial\phi}{\partial z} = 0. \tag{42}$$

The thermal boundary condition: the surface of the half-space is subjected to a thermal shock

$$T = p_2 N(x,t). \tag{43}$$

where, p_1, p_2 are the magnitude of the mechanical force and $N(x,t)$ is known function.

Substituting from the expressions of the variables considered into the boundary conditions, (42) and (43) respectively, we can obtain the following equations:

$$h_{11}R_1 + \sum_{j=2}^{4} h_{1j}R_j = -p_1, \tag{44}$$

$$h_{21}R_1 + \sum_{j=2}^{4} h_{2j}R_j = 0, \tag{45}$$

$$\sum_{j=2}^{4} h_{3j}R_j = 0, \tag{46}$$

$$\sum_{j=2}^{4} S_{1j}R_j = p_2, \tag{47}$$

where,

$$h_{11} = iak_1[(1+a_0 p+\delta_0\omega)-(1-2\delta^2)(1+\alpha_0\omega)],$$

$$h_{21} = \delta^2(1+\alpha_1\omega)(a^2+k_1^2)+a_0 p k_1^2,$$

$$h_{1j} = (1+a_0 p+\delta_0\omega)k_j^2-(1-2\delta^2)(1+\alpha_0\omega)a^2-b_3 S_{1j}+b_4 S_{2j},$$

$$h_{2j} = -ia[2\delta^2(1+\alpha_1\omega)+a_0 p]k_j, \quad h_{3j} = -k_j S_{2j}, \quad j=2,3,4$$

Solving Eqs. (44)-(47) for $R_j(j=1,2,3,4)$ by using the inverse of matrix method as follows:

$$\begin{pmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ 0 & h_{32} & h_{33} & h_{34} \\ 0 & S_{12} & S_{13} & S_{14} \end{pmatrix}^{-1} \begin{pmatrix} -p_1 \\ 0 \\ 0 \\ p_2 \end{pmatrix}. \tag{48}$$

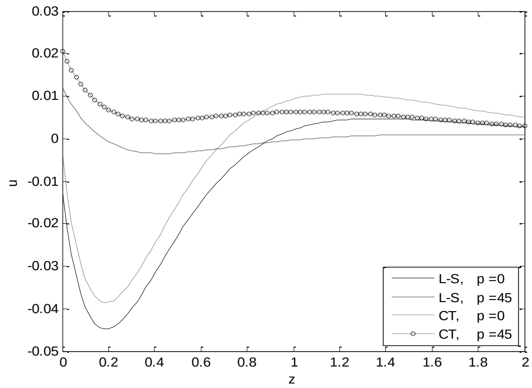

Fig. 1 Variation of the displacement u with z in the presence and absence of initial stress

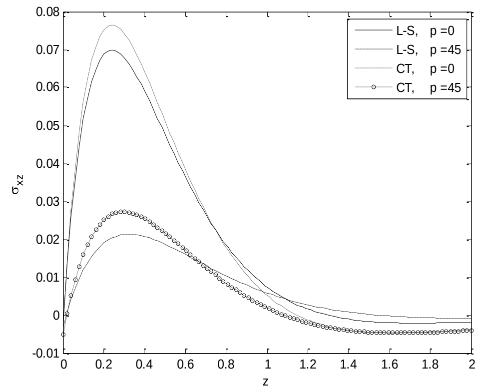

Fig. 2 Variation of the stress σ_{xz} with z in the presence and absence of initial stress

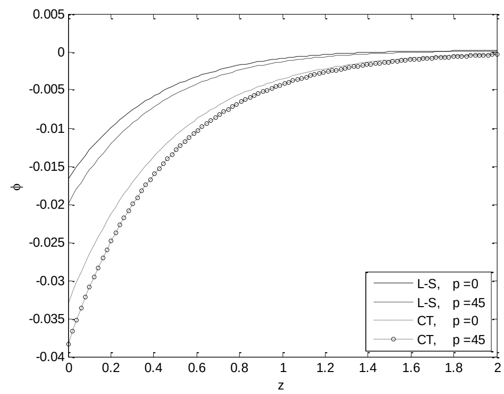

Fig. 3 Variation of the volume fraction field ϕ with z in the presence and absence of initial stress

5. Numerical results and discussion

We will present some numerical results to illustrate the problem. The material chosen for the purpose of numericalcomputation is copper, the physical data for which are given in Othman and Zidan (2015), in SI units:

$$\lambda = 7.76\times10^{10}\,\mathrm{kg\,m^{-1}\,s^{-2}}, \quad \mu = 3.86\times10^{10}\,\mathrm{kg\,m^{-1}s^{-2}},$$

$$K = 386\,\mathrm{W\,m^{-1}K^{-2}}, T_0 = 293\mathrm{K}, \rho = 8954\,\mathrm{kg\,m^{-3}},$$

$$\alpha_t = 1.78\times10^{-5}\,\mathrm{K^{-1}}, C_e = 383.1\,\mathrm{J\,kg^{-1}K^{-1}}.$$

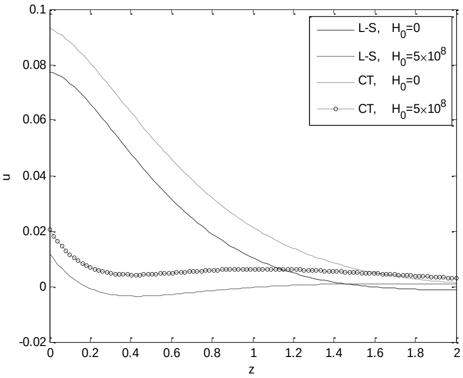

Fig. 4 Variation of the displacement u with z in the presence and absence of magnetic field

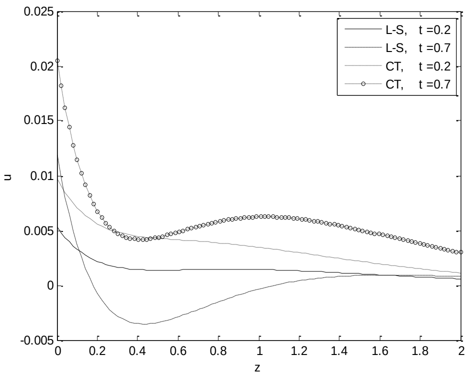

Fig. 7 Variation of the displacement u with z in two values of time

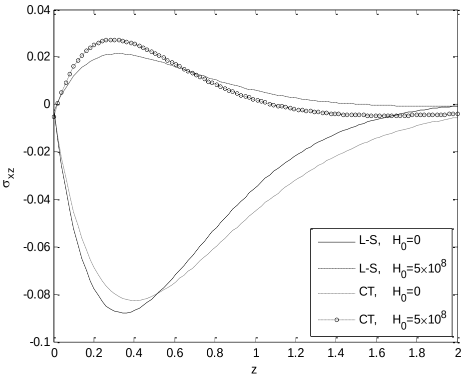

Fig. 5 Variation of the stress σ_{xz} with z in the presence and absence of magnetic field

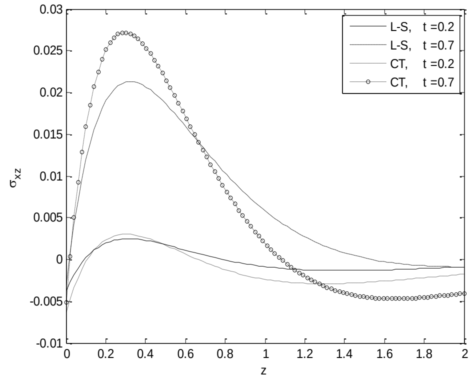

Fig. 8 Variation of the stress σ_{xz} with z in two values of time

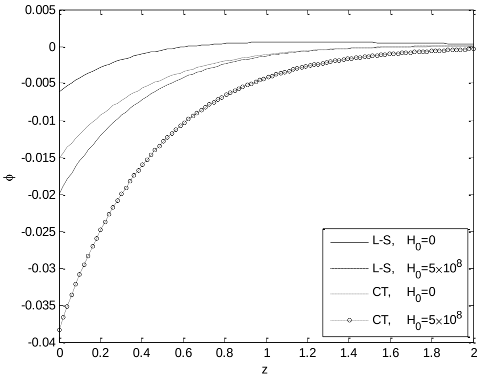

Fig. 6 Variation of the volume fraction field ϕ with z in the presence and absence of magnetic field

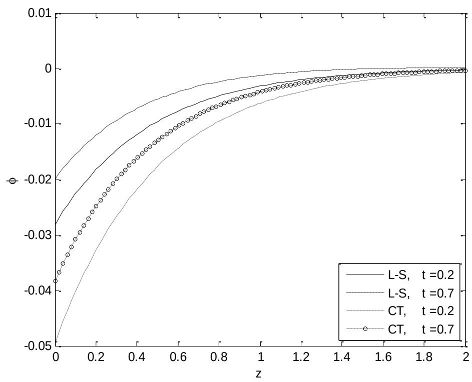

Fig. 9 Variation of the volume fraction field ϕ with z in two values of time

The voids parameters are

$$A = 1.688\times10^{-5}\,\mathrm{kg\,m\,s^{-2}}, b = 1.139\times10^{10}\,\mathrm{kg\,m^{-1}s^{-2}},$$

$$m = 2\times10^{5}\,\mathrm{kg\,m^{-1}s^{-2}K^{-1}}, \chi = 1.75\times10^{-15}\,\mathrm{m^{2}},$$

$$\xi_1 = 1.475\times10^{10}\,\mathrm{kg\,m^{-1}s^{-2}}, \xi_2 = 3.8402\times10^{-4}\,\mathrm{kg\,m^{-1}\,s^{-3}}$$

$$\tau = 0.2\times10^{-5}\,\mathrm{kg\,m^{-1}s^{-2}K^{-1}}, \varsigma = 0.1\times10^{-4}\,\mathrm{kg\,m^{-1}s^{-2}}$$

The magnetic field parameters are

$$\varepsilon_0 = \frac{10^{-9}}{36\pi}\,\mathrm{Fm^{-1}}, \mu_0 = 4\pi\times10^{-7}\,\mathrm{Hm^{-1}}, H_0 = 5\times10^{8}\,\mathrm{Am^{-1}},$$

The laser pulse parameters are

$$I_0 = 10\,J\,m^{-2},\ r = 10\mu m,\ \gamma = 1m^{-1},\ t_0 = 5ns.$$

The comparisons were carried out for

$$p_1 = 0.2,\ p_2 = 0.5,\ t = 0.7,\ x = 0.2,$$

$$\omega = 2 + 0.5\mathrm{i},\ a = 1.5,\ 0 \le z \le 2,\ \ p = 45,$$

$$\alpha_0 = 3.25\times10^{-2},\ \alpha_1 = 3.91\times10^{-2},\ \alpha_2 = 6.51\times10^{-2},$$

$$\alpha_3 = 1.02\times10^{4},\ \alpha_4 = 1.95\times10^{-2}.$$

The above numerical technique was used for the distribution of the real parts of the displacement component and the change in the volume σ_{xz} the stress component u, and the change in the volume fraction field ϕ with distance z for (CT) and (L-S) theories with and without initial stress and magnetic field effect also with two values of time which are shown graphically in the 2-D figures 1-9. At the absence of the (initial stress - magnetic field) effect, also at t=0.2, the solid line represents the solution in the context of the (L-S) and the dot line represents the solution for the (CT), while at t=0.7, the dashed line represents the solution in the context of the (L-S) and the dot line with circles represents the solution for the (CT). Here all the variables are taken in non-dimensional form.

Fig. 1 shows that the distribution of the displacement component u increases with the increase of the initial stress in the range $0<z<0.9$, then conversely in the other ranges for the both theories. Fig. 2 explains that the distribution of the stress component σ_{xz} in the context of (CT), decreases with the increase of the initial stress. However, in the context of (L-S), it decreases with the increase of the initial stress in the range $0<z<1.1$, then conversely in the other ranges. Fig. 3 depicts that the distribution of the change in the volume fraction field ϕ decreases with the increase of the initial stress for the both theories. It explains that all the curves converge to zero, and the initial stress has a significant role on the distributions of all physical functions.

Fig. 4 shows that the distribution of the displacement component u decreases with the increase of the magnetic field in the range $0<z<1.7$ for (CT) theory and in the range $0<z<1.9$ for (L-S) theory, then conversely in the other ranges from the both theories. Fig. 5 explains that the distribution of the stress component σ_{xz} increases with the increase of the magnetic field for the both theories. Fig. 6 depicts that the distribution of the change in the volume fraction field ϕ decreases with the increase of the magnetic field for the both theories. It explains that all the curves converge to zero, and the magnetic field has a dual role on the distributions of all physical functions. Fig. 7 shows that the distribution of the displacement component u increases with the increase of the time for (CT) theory. However, in the context of (L-S) theory, it decreases with context of (L-S) theory, it increases with the increase of the time the increase of the time in the range $0.1<z<1.7$, then conversely in the other ranges. Fig. 8 explains that the distribution of the stress component σ_{xz} increases with the increase of the time in the range $0<z<1.3$ for (CT) theory, then conversely in the other ranges. However, in the context of (L-S) theory, it increases with the increase of the time. Fig. 9 shows that the distribution of the change in the volume fraction field ϕ increases with the increase of the time for the both theories. It explains that all the curves converge to zero, and the laser pulse has a significant role on the distributions of all physical functions.

6. Conclusions

According to the above analysis, we can conclude that:

1. The magnetic field effect plays an important role on all the physical quantities.
2. The presence and absence of the initial stress in the current model has a significant effect.
3. The normal mode analysis has been used is applied to a wide range of problems in bthermo-viscoelasticity.
4. The value of all physical quantities converges to zero with the increase of the distance and all of them are continuous.
5. It noticed that the thermo-viscoelastic materials with voids subject to thermal loading by a laser pulse has an important role in the distribution of the field quantities, since the amplitude of these quantities is varying (increasing or decreasing) with the changes of the initial stress and the magnetic field effect.

Conflicts of interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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Nomenclature

σ_{ij}	Components of stress tensor
r	the beam radius
e_{ij}	Components of strain tensor
ϕ	the volume fraction field
$e=e_{kk}$	Cubic dilatation
λ, μ	Lame' constants
u, w	Displacement components
δ_{ij}	Kronecker's delta
T	Thermodynamic temperature
C_e	Specific heat at constant strain
α_t	Coefficient of linear thermal expansion
ρ	Density
K	Coefficient of thermal conductivity
p	the initial stress
τ_0	the thermal relaxation time
ε_0	the electric permittivity
μ_0	the magnetic permeability
Q	the heat input of the laser pulse
I_0	the energy absorbed
t_0	the pulse rise time

α_0, α_1, α_2, α_3, α_4	the viscoelastic parameters
T_0	Reference temperature $\lvert (T - T_0)/T_0 \rvert < 1$
A_1, ξ_1, ξ_2, B, τ, ζ, m, χ	the material constants due to presence of voids