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Time harmonic interactions in an orthotropic media in the context of fractional order theory of thermoelasticity

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Abstract. The present investigation deals with the thermomechanical interactions in an orthotropic thermoelastic homogeneous body in the context of fractional order theory of thermoelasticity due to time harmonic sources. The application of a time harmonic concentrated and distributed sources has been considered to show the utility of the solution obtained. Assuming the disturbances to be harmonically time dependent, the expressions for displacement components, stress components and temperature change are derived in frequency domain. Numerical inversion technique has been used to determine the results in physical domain. The effect of frequency on various components has been depicted through graphs.

Keywords: orthotropic medium; fractional calculus; frequency domain; fourier transform; concentrated and uniformly distributed loads

1. Introduction

Thermoelasticity is the theory in which changes are produced in the body due to its heat exchange and mechanical work on the body. Nowadays more attention has been given to non-classical theories as they overcome the drawbacks which occurred in the classical theories. These theories of generalized thermoelasticity give finite speed of propagation of heat signals. The classical theory of thermoelasticity has aroused much interest in recent times due to its numerous applications in engineering discipline such as nuclear reactor design, high energy particle accelerators, geothermal engineering, advanced aircraft structure design etc. Moreover, the fractional order theory of generalized thermoelasticity is an important branch of solid mechanics. This theory is used by many researchers and scientists to find the solution of many problems, which contains differential equations of non-integer order. Fractional calculus has been applied in many fields like quantum mechanics, nuclear physics, chemistry, astrophysics, control theory etc.

The growing popularity of fractional calculus is due to its global dependency, which is more appropriate to solve some particular problems of physical processes. Caputo (1967) was the one who gave the definition of fractional derivative of order ‘α’ where $0 < \alpha \leq 1$. Some other definitions can be found in Miller and Ross (1993).

Abbas (2015) had studied the thermoelastic interactions in an infinite fiber-reinforced anisotropic medium by using dual-phase-lag model of generalized thermoelasticity. Biswas *et al*. (2017a and b) had examined the thermal shock response of propagation of electro-magneto-thermoelastic disturbances in an orthotropic medium with three phase-lags. Lata (2018a and b) had studied the Effect of energy dissipation on plane waves in sandwiched layered thermoelastic medium. Marin (1997a and b) had studied the domain of influence in thermoelasticity of bodies with voids. Ezzat and Ezzat (2016) had formulated fractional thermoelasticity applications for porous asphaltic materials. Lata and Kaur (2019a-c) had studied the effect of inclined load on transversely isotropic magneto thermoelastic rotating solid with time harmonic source. Kumar *at al*. (2016) had study the effects of Hall current in a transversely isotropic magnetothermoelastic two temperature medium with rotation and with and without energy dissipation due to normal force. Lata (2018a and b) had analyzed reflection and refraction of plane waves in layered nonlocal elastic and anisotropic thermoelastic medium. Lata *et al*. (2016) had studied the Plane waves in anisotropic thermoelastic medium. Xiong and Ying (2016) had studied the effect of variable properties and moving heat source on magneto-thermoelastic body using fractional order theory. Jiang and Xu (2010) obtained a fractional heat conduction equation with fractional derivative in the general orthogonal curvilinear coordinate and also in the other orthogonal coordinate system. Lata and Kaur (2019a-c) had investigated a two dimensional problem of transversely isotropic thick plate with two temperature and GN type-III in frequency domain. Abbas (2016) had investigated the problem of a thermoelastic infinite body with spherical cavity in the context of fractional order thermoelasticity. Karami *et al*. (2019a and b) had analyzed the wave propagation of functionally graded anisotropic nanoplates resting on winkler-pasternak foundation. Boulefrakh *et al*. (2019) had studied the effect of parameters of visco-pasternak foundation on the bending and vibration properties of a thick FG plate. Karami *et al*.

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(2019a and b) had studied the Galerkin's approach for buckling analysis of functionally graded anisotropic nanoplates for different boundary conditions. Boukhlif *et al*. (2019) had investigated a simple quasi-3D HSDT for the dynamics analysis of FG thick plate on elastic foundation. Boutaleb *et al*. (2019) had studied the dynamic analysis of nanosize FG rectangular plates based on simple nonlocal quasi 3D HSDT. Bourada *et al*. (2019) had investigated the porous functionally graded beam using a sinusoidal shear deformation theory. Kumar and Gupta (2013) had studied the plane wave propagation in anisotropic thermoelastic medium with fractional order derivative and voids in the context of the theory of two-phase-lag and three-phase-lag model of thermoelasticity. Tripathi *et al*. (2018) studied fractional order generalized thermoelastic response in a half space due to a periodically varying heat source. Abbas *et al*. (2018) had studied the effect of fractional order derivative on photo-thermoelastic process in an infinite semi-conducting medium with a cylindrical hole.

Ying and Yun (2015) formed a fractional dual-phase-lag model and the corresponding bio-heat transfer equation. Alimirzaei *et al*. (2019) had investigated nonlinear analysis of viscoelastic micro-composite beam with geometrical imperfection using FEM: MSGT electro magneto-elastic bending, buckling and vibration solutions.

Medani *et al*. (2019) had analyzed the Static and dynamic behavior of (FG-CNT) reinforced porous sandwich plate. Zarga *et al*. (2019) had studied the thermomechanical bending for functionally graded sandwich plates using a simple quasi-3D shear deformation theory. Chaabane *et al*. (2019) had studied the bending and free vibration responses of functionally graded beams resting on elastic foundation. Marin *et al*. (2017) had studied the effect of micro temperature for micropolar thermoelastic bodies. Xiong and Niu (2017) established fractional order generalized thermoelastic diffusion theory for anisotropic and linearly thermoelastic diffusion medium. Lata (2019) had investigated time harmonic interactions in an axisymmetric thick circular plate using fractional order theory. Lata and Kaur (2019a-c) had studied the effect of inclined load in transversely isotropic magneto thermoelastic solid with two temperature and without energy dissipation in generalized thermoelasticity. Abbas (2017) had analyzed the generalized thermoelastic interactions in a hollow cylinder with temperature dependent material properties.

Marin (2010) had also proved some basic theorems for microstretch thermoelastic materials by using Lagrange's identity. Marin and Craciun (2017) had proved the uniqueness results for a boundary value problem in dipolar thermoelasticity to model composite materials. Lata and Kaur (2018) had studied the effect of hall current in transversely isotropic magneto thermoelastic rotating medium with fractional order heat transfer due to normal force. Marin and Othman (2017) had examined the effect of thermal loading due to laser pulse on thermoelastic porous medium under G-N theory. Marin *et al*. (2016) had studied the mixed initial-boundary value problems for micropolar porous bodies. Marin (1997a and b) had proved uniqueness of solutions of initial-boundary value problem in thermoelasticity for bodies with voids.

Inspite of all investigations no attempt has been made yet to study the response of thermomechanical interactions in orthotropic medium using fractional order theory with phase lags due to time harmonic sources. The purpose of present paper is to examine the thermomechanical interactions in an orthotropic thermoelastic solid by using fractional order theory with three phase lags in frequency domain. The expression for components of displacement, normal stress, tangential stress and temperature change are derived, when the time harmonic mechanical or thermal source is applied. The components of stress, displacement and temperature change subjected to uniformly distributed load and concentrated load are obtained. The effect of frequency on various components has been depicted through graphs.

2. Basic equations

Following Chawla and Kumar (2014) the constitutive relations and basic governing equations of anisotropic three phase lag thermoelastic model in the absence of body forces and heat sources are the following.

$$\sigma_{ij} = c_{ijkm} e_{km} - \beta_{ij} \mathrm{T}, \tag{1}$$

$$\sigma_{ij,j} = \rho \ddot{u}_i, \tag{2}$$

$$\begin{aligned} K_{ij}\left(1+\frac{\tau_t^{\alpha}}{\alpha!}\frac{\partial}{\partial t^{\alpha}}\right)\dot{T}_{,ji} + K_{ij}^{*}\left(1+\frac{\tau_v^{\alpha}}{\alpha!}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)T_{,ji} \\ = \left[\left(1+\frac{\tau_q^{\alpha}}{\alpha!}\right)+\frac{\tau_q^{2\alpha!}}{2\alpha!}\right]\left[\rho C_E \ddot{T} + \beta_{ij} T_0 \ddot{e}_{ij}\right]. \end{aligned} \tag{3}$$

In Eqs. (1)-(3) $c_{ijkm}\,(= c_{kmij} = c_{jikm} = c_{ijmk})$ is the tensor of elastic constant, ρ is the density, T_0 is the reference temperature such that $\left|\frac{T}{T_0}\right| \ll 1$, u_i are the components of displacement vector u, C_E is the specific heat at constant strain, u_i are the components of displacement vector u, $\sigma_{ij} = (\sigma_{ji})$ and $e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$ are the components of stress and strain tensors respectively. T (x, y, z, t) is the temperature distribution from the reference temperature T_0. Also τ_q, τ_t and τ_v are respectively, the phase lag of the heat flux, the phase lag of the temperature gradient and the phase lag of the thermal displacement, β_{ij} are tensor of thermal moduli, $K_{ij}\,(= K_{ji})$ and $K_{ij}^{*}\,(= K_{ji}^{*})$ are the components of thermal conductivity and material characteristic constant respectively.

The basis of these symmetries of C_{ijkm} is due to

(1) The stress tensor is symmetric, which is only possible if $(C_{ijkm} = C_{jikm})$
(2) If a strain energy density exists for the material, the elastic stiffness tensor must satisfy $C_{ijkm} = C_{kmij}$
(3) From stress tensor and elastic stiffness tensor symmetries infer $(C_{ijkm} = C_{ijmk})$ and $C_{ijkm} = C_{kmij} = C_{jikm} = C_{ijmk}$

In all above equations dot (.) represents the partial derivative w.r.t time and (,) denote the partial derivative w.r.t spatial coordinate.

The Eq. (1) for an orthotropic media in Cartesian coordinate system (x, y, z) in component form can be written as

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{yz}\\ \sigma_{xz}\\ \sigma_{xy}\end{bmatrix}=\begin{bmatrix}C_{11} & C_{12} & C_{13} & 0 & 0 & 0\\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0\\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & C_{66}\end{bmatrix}\begin{bmatrix}e_{xx}\\ e_{yy}\\ e_{zz}\\ 2e_{yz}\\ 2e_{xz}\\ 2e_{xy}\end{bmatrix}\begin{bmatrix}\beta_1\\ \beta_2\\ \beta_3\\ 0\\ 0\\ 0\end{bmatrix}T, \quad (4)$$

$$\begin{aligned}&\sigma_{xx}=C_{11}e_{xx}+C_{12}e_{yy}+C_{13}e_{zz}-\beta_1 T,\\ &\sigma_{yy}=C_{12}e_{xx}+C_{22}e_{yy}+C_{23}e_{zz}-\beta_2 T,\\ &\sigma_{zz}=C_{13}e_{xx}+C_{23}e_{yy}+C_{33}e_{zz}-\beta_3 T,\\ &\sigma_{yz}=2C_{44}e_{yz},\quad \sigma_{xz}=2C_{55}e_{xz},\quad \sigma_{xy}=2C_{66}e_{xy},\end{aligned} \quad (5)$$

$$\begin{aligned}&C_{11}\frac{\partial^2 u}{\partial x^2}+C_{66}\frac{\partial^2 u}{\partial y^2}+C_{55}\frac{\partial^2 u}{\partial z^2}+(C_{12}+C_{66})\frac{\partial^2 v}{\partial x\partial y}\\ &+(C_{13}+C_{55})\frac{\partial^2 w}{\partial x\partial z}-\beta_1\frac{\partial T}{\partial x}=\rho\frac{\partial^2 u}{\partial t^2},\end{aligned}$$

$$\begin{aligned}&(C_{12}+C_{66})\frac{\partial^2 u}{\partial x\partial y}+C_{66}\frac{\partial^2 v}{\partial x^2}+C_{22}\frac{\partial^2 v}{\partial y^2}+C_{44}\frac{\partial^2 v}{\partial z^2}\\ &+(C_{23}+C_{44})\frac{\partial^2 w}{\partial y\partial z}-\beta_2\frac{\partial T}{\partial y}=\rho\frac{\partial^2 v}{\partial t^2},\end{aligned} \quad (6)$$

$$\begin{aligned}&(C_{13}+C_{55})\frac{\partial^2 u}{\partial x\partial z}+(C_{23}+C_{44})\frac{\partial^2 v}{\partial y\partial z}+C_{55}\frac{\partial^2 w}{\partial x^2}\\ &+C_{44}\frac{\partial^2 w}{\partial y^2}+C_{33}\frac{\partial^2 w}{\partial z^2}-\beta_3\frac{\partial T}{\partial z}=\rho\frac{\partial^2 w}{\partial t^2},\end{aligned} \quad (7)$$

$$\begin{aligned}&K_1\left(1+\frac{\tau_t^\alpha}{\alpha!}\frac{\partial}{\partial t^\alpha}\right)\dot{T}_{,11}+K_2\left(1+\frac{\tau_t^\alpha}{\alpha!}\frac{\partial}{\partial t^\alpha}\right)\dot{T}_{,22}\\ &+K_3\left(1+\frac{\tau_t^\alpha}{\alpha!}\frac{\partial}{\partial t^\alpha}\right)\dot{T}_{33}+K_1^*\left(1+\frac{\tau_v^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)T_{,11}\\ &+K_2^*\left(1+\frac{\tau_v^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)T_{,22}+K_3^*\left(1+\frac{\tau_v^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)T_{,33}\\ &=\left[1+\frac{\tau_q^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}+\frac{\tau_q^{2\alpha!}}{2\alpha!}\frac{\partial^{2\alpha}}{\partial t^{2\alpha}}\right]\\ &\quad\left[\rho C_E\ddot{T}+T_0\left(\beta_1\ddot{u}_{1,1}+\beta_2\ddot{u}_{2,2}+\beta_3\ddot{u}_{3,3}\right)\right].\end{aligned} \quad (8)$$

3. Formulation of the problem

We consider two dimensional an orthotropic thermoelastic homogeneous body initially at temperature T_0 with and without energy dissipation in generalized thermoelasticity using three phase lag model. We take a rectangular coordinate axis (x, y, z) with z-axis as axis of symmetry. The components of displacement vector $\vec{u}$, $\vec{v}$ and $\vec{w}$ and temperature change T for the two dimensional problem have the form

$$\vec{u}=u(x,z,t),\quad \vec{v}=0,\quad \vec{w}=(x,z,t),$$
$$\text{and}\quad T=T(x,z,t),$$

With the aid of (9), Eqs. (5)-(8) reduce to the form

$$C_{11}\frac{\partial^2 u}{\partial x^2}+C_{55}\frac{\partial^2 u}{\partial z^2}+(C_{13}+C_{55})\frac{\partial^2 w}{\partial x\partial z}-\beta_1\frac{\partial T}{\partial x}=\rho\frac{\partial^2 u}{\partial t^2}, \quad (10)$$

$$(C_{13}+C_{55})\frac{\partial^2 u}{\partial x\partial z}+C_{55}\frac{\partial^2 w}{\partial x^2}+C_{33}\frac{\partial^2 w}{\partial z^2}-\beta_3\frac{\partial T}{\partial z}=\rho\frac{\partial^2 w}{\partial t^2}, \quad (11)$$

$$\begin{aligned}&K_1\left(1+\frac{\tau_t^\alpha}{\alpha!}\frac{\partial}{\partial t^\alpha}\right)\dot{T}_{,11}+K_3\left(1+\frac{\tau_t^\alpha}{\alpha!}\frac{\partial}{\partial t^\alpha}\right)\dot{T}_{33}\\ &+K_1^*\left(1+\frac{\tau_v^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)T_{,11}+K_3^*\left(1+\frac{\tau_v^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)T_{,33}\\ &=\left[1+\frac{\tau_q^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}+\frac{\tau_q^{2\alpha!}}{2\alpha!}\frac{\partial^{2\alpha}}{\partial t^{2\alpha}}\right]\rho C_E\ddot{T}\\ &+T_0(\beta_1\ddot{u}_{1,1}+\beta_3\ddot{u}_{3,3})].\end{aligned} \quad (12)$$

Also

$$\sigma_{11}=C_{11}e_{11}+C_{13}e_{33}-\beta_1 T, \quad (13)$$

$$\sigma_{33}=C_{13}e_{11}+C_{33}e_{33}-\beta_3 T, \quad (14)$$

$$\sigma_{13}=2C_{55}e_{13}, \quad (15)$$

Where $e_{11}=\frac{\partial u}{\partial x}$, $e_{33}=\frac{\partial w}{\partial z}$, $e_{13}=\frac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)$, $\beta_{ij}=\beta_i\delta_{ij}$, $K_{ij}=K_i\,\delta_{ij}$, $K_{ij}^*=K_i^*\delta_{ij}$, i is not summed where i = 1, 2, 3 and δ_{ij} is kronecker delta.

To facilitate the solution the following dimensionless quantities are used

$$\begin{aligned}&x'=\frac{x}{L},\quad z'=\frac{z}{L},\quad u'=\frac{\rho c_1^2}{L\,T_0\beta_1}u,\quad w'=\frac{\rho c_1^2}{L\,T_0\beta_1}w,\\ &t'=\frac{C_1}{L}t,\quad \sigma_{33}'=\frac{\sigma_{33}}{T_0\beta_1},\quad \sigma_{31}'=\frac{\sigma_{31}}{T_0\beta_1},\quad T'=\frac{T}{T_0}.\end{aligned} \quad (16)$$

Where $c_1^2=c_{11}/\rho$ and L is a constant of dimension of length. Using dimensionless quantities given by (16) in Eqs. (10)-(12) and suppressing the primes for convenience yield

$$\frac{\partial^2 u}{\partial x^2}+\delta_1\frac{\partial^2 u}{\partial z^2}-\delta_2\frac{\partial^2 w}{\partial x\partial z}-\frac{\partial T}{\partial x}=\frac{\partial^2 u}{\partial t^2}, \quad (17)$$

$$\delta_3\frac{\partial^2 w}{\partial z^2}+\delta_1\frac{\partial^2 w}{\partial x^2}+\delta_2\frac{\partial^2 u}{\partial x\partial z}-\frac{\beta_3}{\beta_1}\frac{\partial T}{\partial z}=\frac{\partial^2 w}{\partial t^2}, \quad (18)$$

$$\begin{aligned}&\epsilon_1\tau_t\frac{\partial}{\partial t}\left(\frac{\partial^2 T}{\partial x^2}\right)+\epsilon_2\tau_t\frac{\partial}{\partial t}\left(\frac{\partial^2 T}{\partial z^2}\right)+\epsilon_3\tau_v\left(\frac{\partial^2 T}{\partial x^2}\right)\\ &+\epsilon_4\tau_v\left(\frac{\partial^2 T}{\partial z^2}\right)=\tau_q\left[\frac{\partial^2 T}{\partial z^2}+\beta_1^2\epsilon_5\frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial x}+\frac{\beta_3}{\beta_1}\frac{\partial w}{\partial z}\right)\right],\end{aligned} \quad (19)$$

Where

$$\delta_1=\frac{c_{55}}{c_{11}},\qquad \delta_2=\frac{c_{13}+c_{15}}{c_{11}},\qquad \delta_3=\frac{c_{33}}{c_{11}},$$
$$\tau_t=\left(1+\frac{\tau_t^\alpha}{\alpha!}\frac{\partial}{\partial t^\alpha}\right),\qquad \tau_v=\left(1+\frac{\tau_v^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right),$$
$$\tau_q=\left(1+\frac{\tau_q^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}+\frac{\tau_q^{2\alpha!}}{2\alpha!}\frac{\partial^{2\alpha}}{\partial t^{2\alpha}}\right),$$
$$\epsilon_1=\frac{K_1}{\rho\,L\,C_1C_E},\qquad \epsilon_2=\frac{K_3}{\rho\,LC_1C_E},\qquad \epsilon_3=\frac{K_1^*}{\rho\,c_1^2C_E},$$
$$\epsilon_4=\frac{K_3^*}{\rho\,c_1^2C_E},\qquad \epsilon_5=\frac{T_0}{\rho^2\,c_1^2C_E}.$$

Assuming the harmonic behaviour as

$$(u, w, T)\ (x, z, t) = (u, w, T)\ (x, z)\ e^{i\omega t}, \tag{20}$$

Where ω is angular frequency
The Eqs. (17)-(19) with the help of (20) takes the form

$$\frac{\partial^2 u}{\partial x^2} + \delta_1 \frac{\partial^2 u}{\partial z^2} + \delta_2 \frac{\partial^2 w}{\partial x \partial z} - \frac{\partial T}{\partial x} = -\omega^2\ u, \tag{21}$$

$$\delta_3 \frac{\partial^2 w}{\partial z^2} + \delta_1 \frac{\partial^2 w}{\partial x^2} + \delta_2 \frac{\partial^2 u}{\partial x \partial z} - \frac{\beta_3}{\beta_1}\frac{\partial T}{\partial z} = -\omega^2\ w, \tag{22}$$

$$\epsilon_1 \tau_t' i\omega \left(\frac{\partial^2 T}{\partial x^2}\right) + \epsilon_2 \tau_t' i\omega \left(\frac{\partial^2 T}{\partial z^2}\right) + \epsilon_3 \tau_v' \left(\frac{\partial^2 T}{\partial x^2}\right) + \epsilon_4 \tau_v' \left(\frac{\partial^2 T}{\partial z^2}\right) = \tau_q' \left[-\omega^2 T - \beta_1^2 \omega^2 \epsilon_5 \left(\frac{\partial u}{\partial x} + \frac{\beta_3}{\beta_1}\frac{\partial w}{\partial z}\right)\right], \tag{23}$$

Where

$$\tau_t' = 1 + \frac{\tau_t^\alpha}{\alpha!}(i\omega)^\alpha,$$
$$\tau_v' = 1 + \frac{\tau_v^\alpha}{\alpha!}(i\omega)^\alpha,$$
$$\tau_q' = 1 + \frac{\tau_q^\alpha}{\alpha!}(i\omega)^\alpha + \frac{\tau_q^{2\alpha}}{2\alpha!}(i\omega)^{2\alpha}.$$

Apply the Fourier transform defined by

$$\hat{f}(\xi, z, \omega) = \int_{-\infty}^{\infty} \bar{f}\,(x, z, \omega) e^{i\xi x}\, dx, \tag{24}$$

On Eqs. (21)-(23), we obtain a system of three homogeneous equations. These resulting equations have non- trivial solutions if the determinant of the coefficient ($\hat{u}$, $\hat{w}$, $\hat{T}$) vanishes, which yield to the following characteristic equation.

$$(\mathrm{P}D^6 + QD^4 + RD^2 + S)\left(\hat{u}, \hat{w}, \hat{T}\right) = 0, \tag{25}$$

Where

$$D = \frac{d}{dz},$$
$$P = \{\tau_v' \delta_3 \delta_1 \epsilon_4 + \epsilon_2 vv \delta_1 \delta_3 i\omega \tau_t'\},$$
$$\begin{aligned}\mathrm{Q} = \tau_t'[&\delta_3 i\epsilon_2 \omega^3 - i\omega\delta_3\epsilon_2\xi^2 - i\epsilon_1\delta_3\delta_1\omega\xi^2 + i\epsilon_2\delta_1\omega^3 \\ &-\epsilon_2 i\omega\xi^2\delta_1^2 - i\epsilon_2\omega\delta_2^2\xi + \tau_v'[\delta_3\epsilon_4\omega^2 - \delta_3\epsilon_4\xi^2 \\ &-\delta_1\delta_3\epsilon_3\xi^2 + \delta_1\epsilon_4\omega^2 + \epsilon_4\xi^2\delta_2^2] \\ &+\tau_q'[\delta_1\delta_3\omega^2 + \beta_3^2\epsilon_5\delta_1\omega^2],\end{aligned}$$
$$\begin{aligned}R = \tau_t'[&-i\delta_3\epsilon_1\omega^3\xi^2 + i\epsilon_2\omega^5 - i\delta_1\epsilon_2\xi^2\omega^3 + i\delta_3\epsilon_1\omega\xi^4 \\ &-i\epsilon_2\xi^2\omega^3 + i\delta_1\epsilon_2\omega\xi^4 - i\delta_1\epsilon_1\xi^2\omega^3 + i\delta_1^2\epsilon_1\omega\xi^4 \\ &-i\ \delta_2^2\ \epsilon_1\omega\ \xi^4] + \tau_v'[-\delta_3\epsilon_3\omega^2\xi^2 + \epsilon_4\omega^4 - \omega^2\xi^2\delta_1\epsilon_4 \\ &+\epsilon_3\xi^4\delta_3 - \epsilon_4\omega^2\xi^2 + \delta_1\epsilon_4\xi^4 - \delta_1\omega^2\xi^2\epsilon_3 - \delta_2^2\epsilon_3\xi^4 \\ &+\delta_1^2\epsilon_3\xi^4] + \tau_q'[\delta_3\omega^4 + \beta_3^2\epsilon_5\omega^4 - \delta_3\omega^2\xi^2 \\ &-\beta_3^2\epsilon_5\omega^2\xi^2 + \omega^4\delta_1 + \delta_2^2\omega^2\xi^2 - \beta_1\beta_3\delta_2\epsilon_5\omega^2\xi^2 \\ &+\omega^2\xi^2\delta_3\beta_1^2\epsilon_5 - \xi^2\omega^2\delta_1^2]\end{aligned}$$
$$\begin{aligned}S = \tau_t'[&i\delta_1\epsilon_1\xi^4\omega^3 + i\epsilon_1\xi^4\omega^3 - i\epsilon_1\xi^2 s^5 - i\epsilon_1\omega\delta_1\xi^6] \\ &+\tau_v'[-\xi^2\omega^4\epsilon_3 + \xi^4\omega^2\delta_1\epsilon_3 + \omega^2\ \xi^4\epsilon_3 - \delta_1\xi^6\epsilon_3] \\ &+\tau_q'[\omega^6 - \xi^2\omega^4\delta_1 - \omega^4\xi^2 + \delta_1\xi^4\omega^2 + \xi^2\omega^4\beta_1^2\epsilon_5 \\ &-\beta_1^2\xi^4\omega^2\delta_1\epsilon_5],\end{aligned}$$

Where

$$\tau_t' = \left(1 + \frac{\tau_t^\alpha}{\alpha!}(i\omega)^\alpha\right),$$
$$\tau_v' = \left(1 + \frac{\tau_v^\alpha}{\alpha!}(i\omega)^\alpha\right),$$
$$\tau_q' = \left(1 + \frac{\tau_q^\alpha}{\alpha!}(i\omega)^\alpha + \frac{\tau_q^{2\alpha!}}{2\alpha!}(i\omega)^{2\alpha}\right).$$

The roots of the Eq. (25) are $\pm\lambda_i$ (i = 1, 2, 3); the solution of the equation satisfying the radiation conditions can be written as

$$\tilde{u} = A_1\ e^{-\lambda_1 z} + A_2\ e^{-\lambda_2 z} + A_3\ e^{-\lambda_3 z}, \tag{26}$$

$$\tilde{w} = d_1\ A_1 e^{-\lambda_1 z} + d_2 A_2\ e^{-\lambda_2 z} + d_3\ A_3 e^{-\lambda_3 z}, \tag{27}$$

$$\tilde{T} = l_1\ A_1\ e^{-\lambda_1 z} + l_2\ A_2\ e^{-\lambda_2 z} + l_3\ A_3\ e^{-\lambda_3 z}, \tag{28}$$

Where

$$d_i = \frac{\lambda_i^4\ A^* + \lambda_i^2\ B^* + C^*}{\lambda_i^4\ A' + \lambda_i^2\ B' + C'}; \quad i = 1, 2, 3, \tag{29}$$

$$l_i = \frac{\lambda_i^4\ P' + \lambda_i^2\ Q' + R'}{\lambda_i^4\ A' + \lambda_i^2\ B' + C'}; \quad i = 1, 2, 3 \tag{30}$$

Where

$$\begin{aligned}A^* &= \tau_t'\delta_1\,[-i\epsilon_2\ \omega] - \tau_v'[\delta_1\ \epsilon_4],\\ B^* &= \tau_t'[i\epsilon_2\omega^3 - i\xi^2\epsilon_2\omega - i\delta_1\epsilon_1\omega\xi^2] \\ &\quad +\tau_v'[\epsilon_4\omega^2 - \epsilon_4\xi^2 - \delta_1\xi^2\epsilon_3] + \tau_q'[\delta_1\omega^2],\\ C^* &= \tau_t'[-i\epsilon_1\xi^2\omega^3 + i\xi^4\epsilon_1\omega] + \tau_v'[-\epsilon_3\xi^2\omega^2 + \xi^4\epsilon_3] \\ &\quad + \tau_q'[\omega^4 - \xi^2\omega^2 - \beta_1^2\epsilon_5\xi^2\omega^2],\\ A' &= \delta_3[\tau_t' i\epsilon_2\omega + \tau_v'\epsilon_4],\\ B' &= \tau_t'[-i\epsilon_1\omega\delta_3\xi^2 + i\epsilon_2\omega^3 - \delta_1 i\epsilon_2\omega\xi^2] \\ &\quad + \tau_v'[-\delta_3\epsilon_3\xi^2 + \epsilon_4\omega^2 - \delta_1\epsilon_4\xi^2] \\ &\quad + \tau_q'[\delta_3\omega^2 + \beta_3^2\epsilon_5\omega^2],\\ C' &= \tau_t'[-i\epsilon_1\omega^3\xi^2 - i\delta_1\epsilon_1\omega\xi^4] \\ &\quad +\tau_v'[-\epsilon_3\omega^2\xi^2 + \delta_1\epsilon_3\xi^4] + \tau_q'[\omega^4 - \delta_1\xi^2\omega^2],\\ P' &= [\delta_1\delta_3],\\ Q' &= [\delta_3\omega^2 - \xi^2\delta_3 + \omega^2\delta_1 - \xi^2\delta_1^2 + \xi^2\delta_2^2],\\ R' &= [\omega^4 - \delta_1\xi^2\omega^2 - \xi^2\omega^2 + \delta_1\xi^4].\end{aligned}$$

4. Boundary conditions

We apply a normal force and thermal source on the surface of boundary, which are assumed to be time harmonic. The boundary conditions are given by

$$\sigma_{33} = -F_1\ \psi_1\ (x)\ e^{i\omega t}, \tag{31}$$

$$\sigma_{31} = 0, \tag{32}$$

$$\frac{\partial T}{\partial z} = F_2\ \psi_2(x)\ e^{i\omega t}\ \ at\ \ z = 0, \tag{33}$$

Where F_1 is the magnitude of force applied, F_2 is the constant temperature applied on the boundary, $\psi_1(x)$ and $\psi_2(x)$ is the source distribution function along x-axis.

By applying the harmonic behaviour and Fourier transform defined by Eqs. (20) and (24) on the boundary conditions (31)-(33) and with the help of Eqs. (1), (13)-(16), (26)-(28), we obtain components of displacement, normal stress, tangential stress and temperature change as

$$\tilde{u} = -\frac{F_1\,\tilde{\psi}_1(\xi)}{\Delta}\left(\Delta_1 e^{-\lambda_1 z} + \Delta_2 e^{-\lambda_2 z} + \Delta_3 e^{-\lambda_3 z}\right)e^{i\omega t} + \frac{F_2\,\tilde{\psi}_2(\xi)}{\Delta}\left(\Delta_1^* e^{-\lambda_1 z} + \Delta_2^* e^{-\lambda_2 z} + \Delta_3^* e^{-\lambda_3 z}\right)e^{i\omega t}, \tag{34}$$

$$\tilde{w} = -\frac{F_1\,\tilde{\psi}_1(\xi)}{\Delta}\left(\mathrm{d}_1\Delta_1 e^{-\lambda_1 z} + \mathrm{d}_2\Delta_2 e^{-\lambda_2 z} + \mathrm{d}_3\Delta_3 e^{-\lambda_3 z}\right)e^{i\omega t} + \frac{F_2\,\tilde{\psi}_2(\xi)}{\Delta}\left(\mathrm{d}_1\Delta_1^* e^{-\lambda_1 z} + \mathrm{d}_2\Delta_2^* e^{-\lambda_2 z} + \mathrm{d}_3\Delta_3^* e^{-\lambda_3 z}\right)e^{i\omega t}, \tag{35}$$

$$\tilde{T} = -\frac{F_1\,\tilde{\psi}_1(\xi)}{\Delta}\left(\mathrm{l}_1\Delta_1 e^{-\lambda_1 z} + \mathrm{l}_2\Delta_2 e^{-\lambda_2 z} + \mathrm{l}_3\Delta_3 e^{-\lambda_3 z}\right)e^{i\omega t} + \frac{F_2\,\tilde{\psi}_2(\xi)}{\Delta}\left(\mathrm{l}_1\Delta_1^* e^{-\lambda_1 z} + \mathrm{l}_2\Delta_2^* e^{-\lambda_2 z} + \mathrm{l}_3\Delta_3^* e^{-\lambda_3 z}\right)e^{i\omega t}, \tag{36}$$

$$\widetilde{\sigma_{33}} = -\frac{F_1\,\tilde{\psi}_1(\xi)}{\Delta}\left(\Delta_{11}\Delta_1 e^{-\lambda_1 z} - \Delta_{12}\Delta_2 e^{-\lambda_2 z} + \Delta_{13}\Delta_3 e^{-\lambda_3 z}\right)e^{i\omega t} + \frac{F_2\,\tilde{\psi}_2(\xi)}{\Delta}\left(\Delta_{11}\Delta_1^* e^{-\lambda_1 z} + \Delta_{12}\Delta_2^* e^{-\lambda_2 z} + \Delta_{13}\Delta_3^* e^{-\lambda_3 z}\right)e^{i\omega t}, \tag{37}$$

$$\widetilde{\sigma_{13}} = -\frac{F_1\,\tilde{\psi}_1(\xi)}{\Delta}\left(\Delta_{21}\Delta_1 e^{-\lambda_1 z} + \Delta_{22}\Delta_2 e^{-\lambda_2 z} + \Delta_{23}\Delta_3 e^{-\lambda_3 z}\right)e^{i\omega t} + \frac{F_2\,\tilde{\psi}_2(\xi)}{\Delta}\left(\Delta_{21}\Delta_1^* e^{-\lambda_1 z} + \Delta_{22}\Delta_2^* e^{-\lambda_2 z} + \Delta_{23}\Delta_3^* e^{-\lambda_3 z}\right)e^{i\omega t}, \tag{38}$$

Where

$$\Delta = \Delta_{11}(\Delta_{22}\Delta_{33} - \Delta_{32}\Delta_{23}) - \Delta_{12}(\Delta_{21}\Delta_{33} - \Delta_{23}\Delta_{31}) + \Delta_{13}(\Delta_{21}\Delta_{32} - \Delta_{22}\Delta_{31}),$$
$$\Delta_1^* = (\Delta_{12}\Delta_{23} - \Delta_{13}\Delta_{22}),$$
$$\Delta_2^* = (\Delta_{13}\Delta_{21} - \Delta_{11}\Delta_{23}),$$
$$\Delta_3^* = (\Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21}).$$
$$\Delta_{1j} = \frac{C_{13}\,\xi\mathrm{i}}{\rho c_1^2} - \frac{C_{33} d_j \lambda_j}{\rho c_1^2} - \frac{\beta_3}{\beta_1} l_j; \quad j = 1,2,3,$$
$$\Delta_{2j} = \frac{C_{55}}{\rho c_1^2}\left[-\lambda_j + {}_{\mathrm{i}}\xi d_j\right]; \quad j = 1,2,3,$$

4.1 Mechanical force on the surface of half-space

Taking $F_2 = 0$ in Eqs. (34)-(38), we obtain the components of tangential stress, normal stress, displacement, temperature change due to mechanical force.

4.2 Thermal source on the surface of half-space

Taking $F_1 = 0$ in Eqs. (34)-(38), we obtain the components of tangential stress, normal stress, displacement and temperature change due to thermal source.

5. Applications

5.1 Concentrated force

The solution due to concentrated normal force is obtained by setting

$$\psi_1(x) = \delta(x), \qquad \psi_2(x) = \delta(x), \tag{39}$$

Where δ(x) is the Dirac delta function. By applying Laplace and Fourier transformations defined in Eqs. (19)-(20) on (35), we get

$$\widehat{\psi_1}(\xi) = 1, \qquad \widehat{\psi_2}(\xi) = 1, \tag{40}$$

Using (40) in (34)-(38), we obtain the components of tangential stress, normal stress, displacement and thermodyanamical temperature.

5.2 Uniformly distributed force

The solution due to uniformly distributed force is obtained by setting

$$\{\psi_1(x),\,\psi_2(x)\} = \begin{Bmatrix} 1\ if\ |x| \le m \\ 0\ if\ |x| > m \end{Bmatrix}, \tag{41}$$

The Laplace and Fourier transforms of $\psi_1(x)$ and $\psi_2(x)$ with respect to the pair (x,ξ) in case of uniformly distributed load of non-dimensional width 2 m applied at origin of co-ordinate system $x = z = 0$ is given by

$$\{\,\widehat{\psi_1}(\xi),\ \widehat{\psi_2}(\xi)\,\} = [2\,sin(\xi m)/\,\xi]\ ,\xi \ne 0. \tag{42}$$

Using (42) in (34)-(38), we get the components of tangential stress, normal stress, displacement, thermo-dynamical temperature.

6. Inversion of transformation

To obtain the solution of the problem in physical domain, we must invert the transformations in Eqs. (34)-(38). Here the displacement components, tangential and normal stresses and temperature change are functions of z and the parameters of Fourier transforms ξ and hence are of the form f (ξ, z). To obtain the function f (x, z) in the physical domain, we first invert the Fourier transform used by Sharma *et al.* (2008).

$$\mathrm{f}\,(x,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\xi x_1}\ \hat{f}(\xi,z)\,\mathrm{d}\xi = \frac{1}{2\pi}\int_{-\infty}^{\infty} |\cos(\xi x)\,f_e - isin(\xi x) f_0|\,\mathrm{d}\xi, \tag{43}$$

Where f_0 and f_e are respectively the odd and even parts of $\hat{f}\,(\xi\ ,\ \mathrm{z})$. The method for evaluating this integral is described in Press *et al*. (1986). It involves the use of

Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

7. Numerical results and discussion

For numerical computations, we take the following values of the relevant parameter for an orthotropic thermoelastic material (Biswas *et al.* 2017a and b and Kumar and Chawla 2014)

$$c_{11} = 18.78 \times 10^{10}\ Kgm^{-1}s^{-2},$$
$$c_{13} = 8.0 \times 10^{10}\ Kgm^{-1}s^{-2},$$
$$c_{33} = 10.2 \times 10^{10}\ Kgm^{-1}s^{-2},$$
$$c_{55} = 10.06 \times 10^{10}\ Kgm^{-1}s^{-2},$$
$$T_0 = 0.293 \times 10^{3}\ K,$$
$$C_E = 4.27 \times 10^{2}\ J/KgK,$$
$$\beta_1 = 1.96 \times 10^{-5}\ K^{-1},$$
$$\beta_3 = 1.4 \times 10^{-5}\ K^{-1},$$
$$\rho = 8.836 \times 10^{3}\ Kgm^{-3},$$
$$K_1 = .12 \times 10^{3}\ Wm^{-1}K^{-1},$$
$$K_3 = .33 \times 10^{3}\ Wm^{-1}K^{-1},$$
$$K_1^* = 1.313 \times 10^{2}\ W/s,$$
$$K_3^* = 1.54 \times 10^{2}\ W/s,$$
$$\tau_t = 1.5 \times 10^{-7}s,$$
$$\tau_v = 1.0 \times 10^{-8}s,$$
$$\tau_q = 2.0 \times 10^{-7}\ s.$$

The values of tangential stress, normal stress, tangential and normal displacements and temperature change T for an orthotropic body are presented graphically for the non-dimensional frequencies $\omega = .25$, $\omega = .5$ and $\omega = .75$ respectively. Also, the value of fractional parameter is assumed to be constant, here we take $\alpha = 0.8$

(1) The red dashed line with centre symbol diamond (◊) for an orthotropic thermoelastic body corresponds to $\omega = .25$
(2) The purple dashed line with centre symbol plus (+) for an orthotropic thermoelastic body corresponds to $\omega = .5$
(3) The black dashed line with centre symbol circle (o) for an orthotropic thermoelastic body corresponds to $\omega = .75$

8. Mechanical forces on the surface of half space

8.1 Concentrated mechanical force

Figs. 1 and 2 shows the variation of tangential and normal displacements with distance x. The values of tangential and normal displacements follows an oscillatory pattern for $\omega = 0.25$, $\omega = 0.5$ and $\omega = 0.75$ respectively. Fig. 3 depicts the value of normal stress σ_{33}. It can be seen that nature of normal stress is also oscillatory. First it decreases gradually then increases for $\omega = 0.25$, $\omega = 0.5$ and $\omega = 0.75$, Figs. 5 and 6 gives the variation of temperature change and tangential stress with respect to

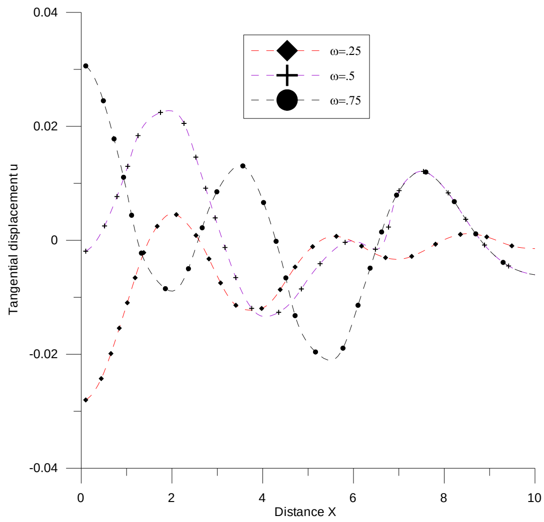

Fig. 1 Variation of tangential displacement u with distance $\boldsymbol{x}$ (concentrated mechanical force)

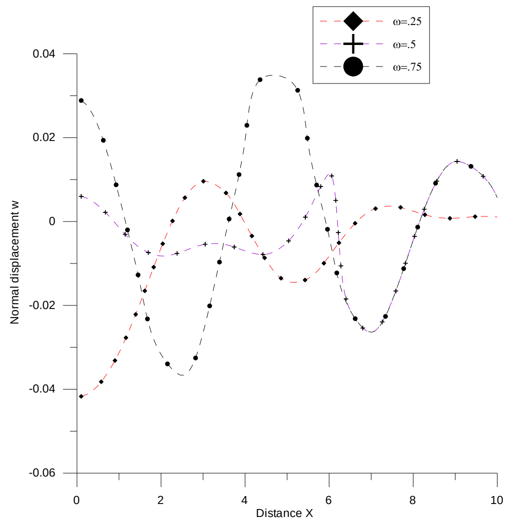

Fig. 2 Variation of normal displacement w with distance $\boldsymbol{x}$ (concentrated mechanical force)

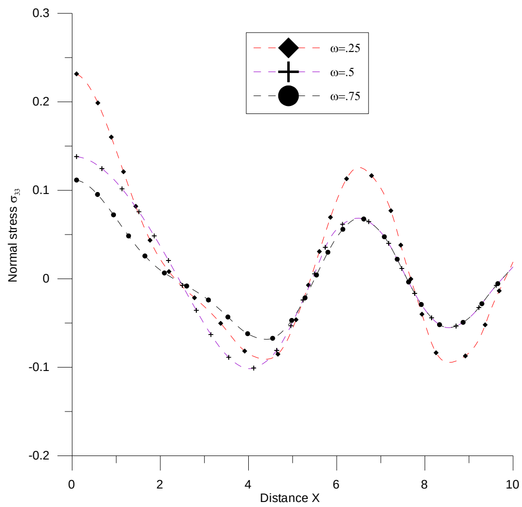

Fig. 3 Variation of normal stress $\boldsymbol{\sigma_{33}}$ with distance $\boldsymbol{x}$ (concentrated mechanical force)

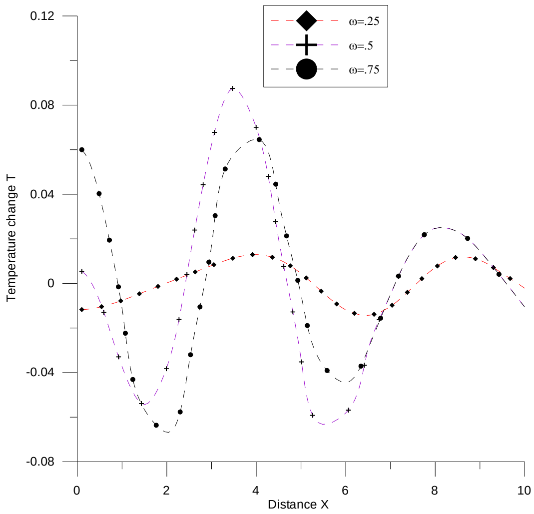

Fig. 4 Variation of temperature change T with distance $\boldsymbol{x}$ (concentrated mechanical force)

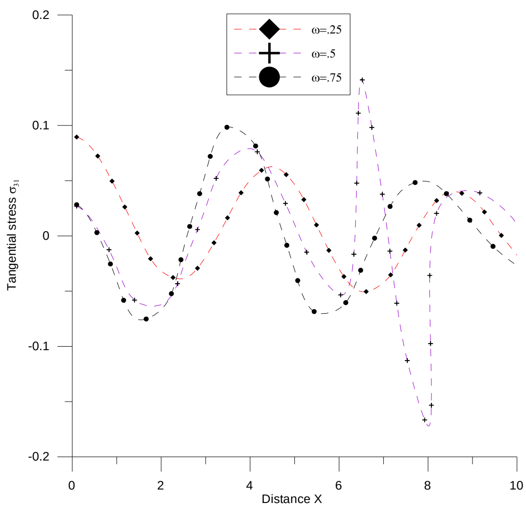

Fig. 5 Variation of tangential stress $\boldsymbol{\sigma_{31}}$ with distance $\boldsymbol{x}$ (concentrated mechanical force)

distance x corresponding to $\omega = 0.25$, $\omega = 0.5$ and $\omega = 0.75$ respectively. We see that in the range $0 \leq x \leq 2$ value of temperature change T for $\omega = 0.5$ and $\omega = 0.75$ first decreases then show an oscillatory behaviour in the whole range. Fig. 5 display the behaviour of tangential stress σ_{31}, where in the range $0 \leq x \leq 6$ behaviour is oscillatory for $\omega = 0.25$, $\omega = 0.5$ and $\omega = 0.75$ respectively then there is a sharp increase in the range $6 \leq x \leq 7$ for $\omega = 0.5$ afterwards the pattern is oscillatory.

8.2 uniformly distributed force

In uniformly distributed mechanical force, Fig. 6 as in case of concentrated force displays the variation of tangential displacement u with distance x, it can be seen that in the whole range $0 \leq x \leq 10$ for $\omega = 0.25$, $\omega = 0.5$ and $\omega = 0.75$, pattern is oscillatory. Fig. 7 interprets the variations of normal displacement w with distance x. The values of normal displacement first increase then decreases

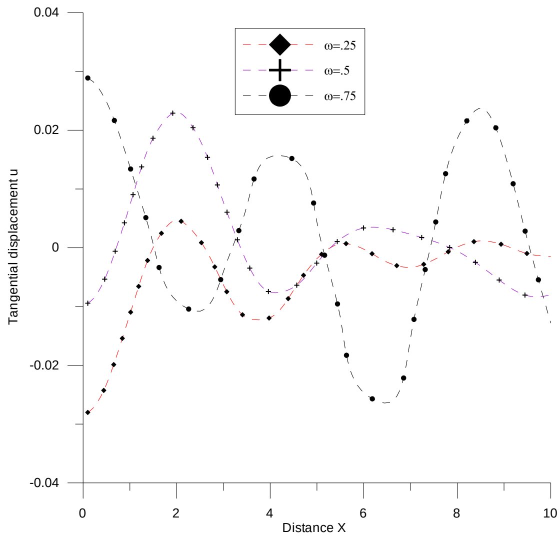

Fig. 6 Variation of tangential displacement u with distance $\boldsymbol{x}$ (uniformly distributed mechanical force)

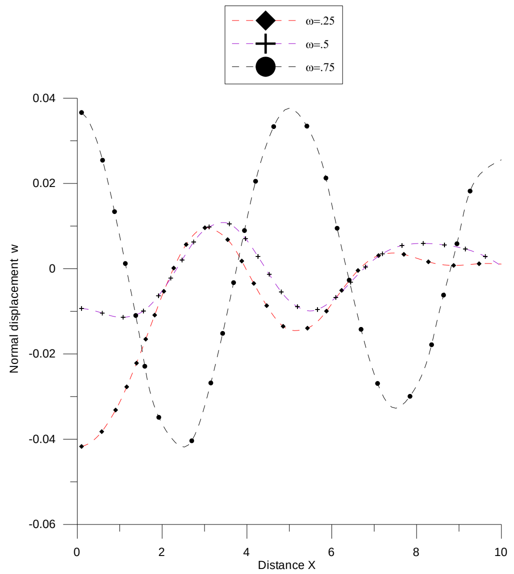

Fig. 7 Variation of normal displacement w with distance x (uniformly distributed mechanical force)

for $\omega = 0.25$ and $\omega = 0.5$, for $\omega = 0.75$ it decreases first then increases i.e. somehow oscillates. Figs. 8 and 9 gives the change in the magnitude of normal stress σ_{33} and temperature change T with increasing value of distance x. It is depicted that the variations for both follows an oscillatory pattern corresponding to three frequencies $\omega = 0.25$, $\omega = 0.5$ and $\omega = 0.75$, Fig 10 displays the variations of tangential stress σ_{31}, here we see that in the whole range pattern is oscillatory.

9. Thermal source on the surface of half space

9.1 Concentrated thermal source

Fig. 11 shows variation of tangential displacement u with distance x corresponding to three different frequencies ω

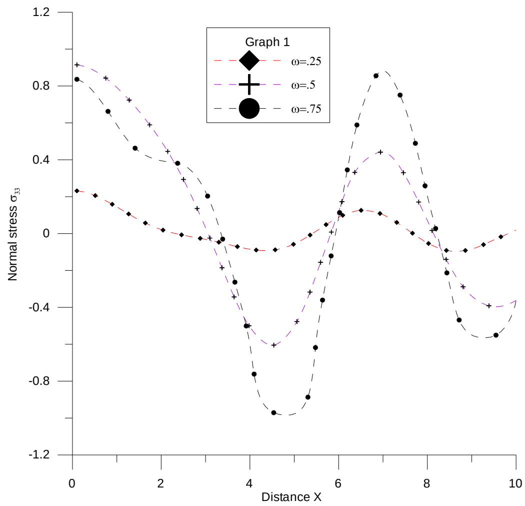

Fig. 8 Variation of normal stress $\boldsymbol{\sigma_{33}}$ with distance $\boldsymbol{x}$ (uniformly distributed mechanical force)

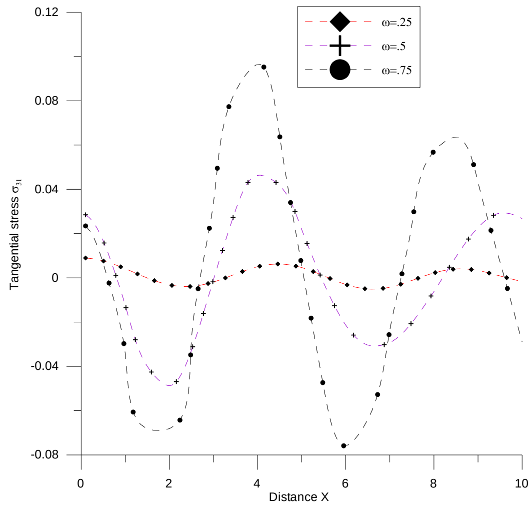

Fig. 10 Variation of normal stress $\boldsymbol{\sigma_{33}}$ with distance $\boldsymbol{x}$ (uniformly distributed mechanical force)

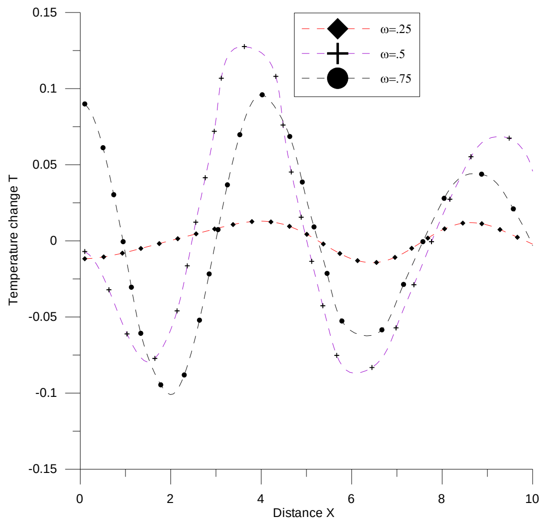

Fig. 9 Variation of temperature change T with distance $\boldsymbol{x}$ (uniformly distributed mechanical force)

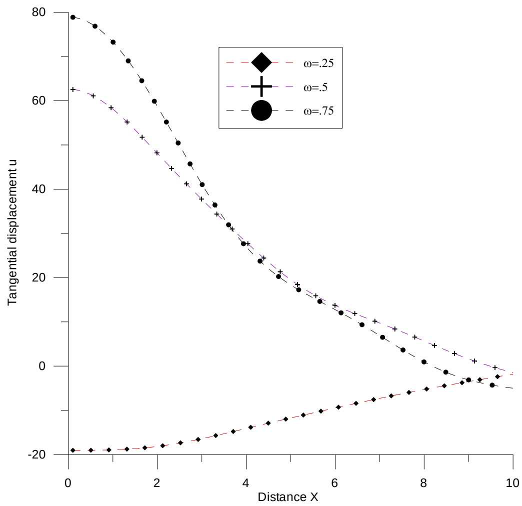

Fig. 11 Variation of tangential displacement u with distance x (concentrated thermal source)

$= 0.25$, $\omega = 0.5$ and $\omega = 0.75$ respectively. We see that for $\omega = 0.25$ near the loading surface value of tangential displacement increases with increase in the value of x, whereas for $\omega = 0.5$ and $\omega = 0.75$ it decreases with increase in the value of, it can be seen that all the three curves corresponding to three different frequencies meet each other at $x = 9.5$. Also, Fig. 12 display the change in the behaviour of normal displacement w here the value of w for all the three different frequencies increases near the loading surface, it follows an oscillatory pattern for $\omega = 0.2$, $\omega = 0.5$ and $\omega = 0.75$ respectively. Fig 13 interprets the behaviour of normal stress σ_{33}, it can be seen that for $\omega = 0.5$ it decreases first then increase sharply in the range $5 \leq x \leq 8$ i.e., oscillates. Whereas for $\omega = 0.25$, $\omega = 0.75$ it decreases with increase in the magnitude of x. Fig. 14 display the variation of temperature change T which is also oscillatory in the whole range. Fig. 15 gives the variation of tangential stress σ_{13}, here for $\omega = 0.25$ value of σ_{13} increases first then decreases while for $\omega = 0.5$

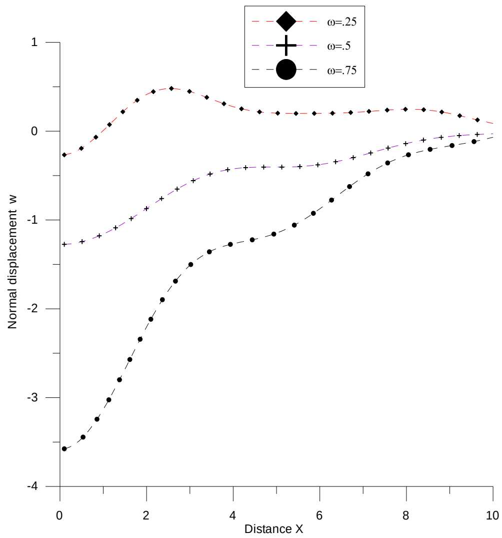

Fig. 12 Variation of normal displacement w with distance $\boldsymbol{x}$ (concentrated thermal source)

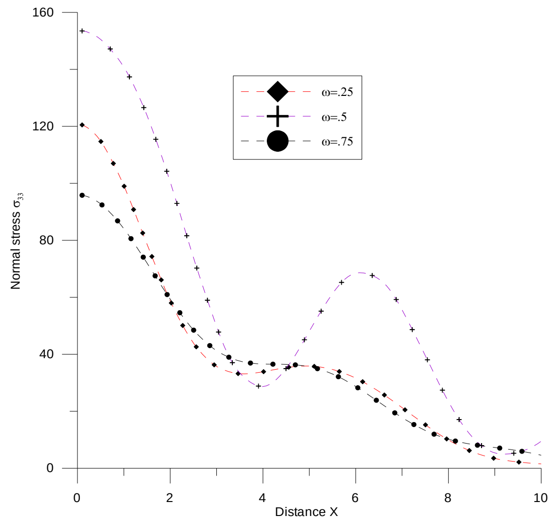

Fig. 13 Variation of normal stress σ_{33} with distance x (concentrated thermal source)

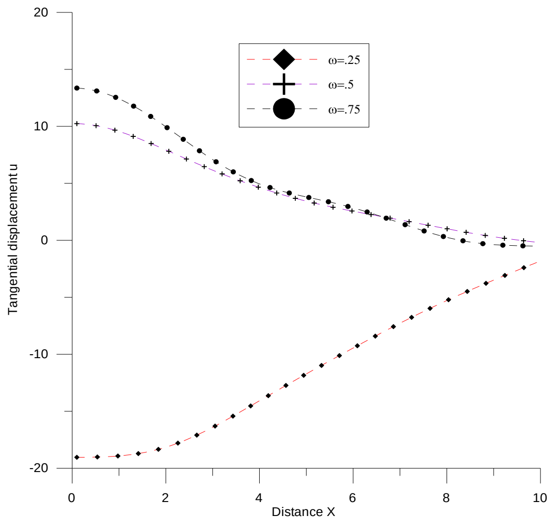

Fig. 16 Variation of tangential displacement u with distance $\boldsymbol{x}$ (uniformly distributed thermal source)

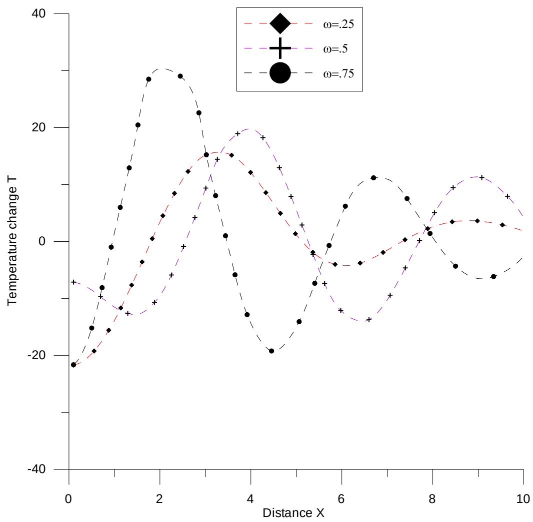

Fig. 14 Variation of temperature change T with distance x (concentrated thermal source)

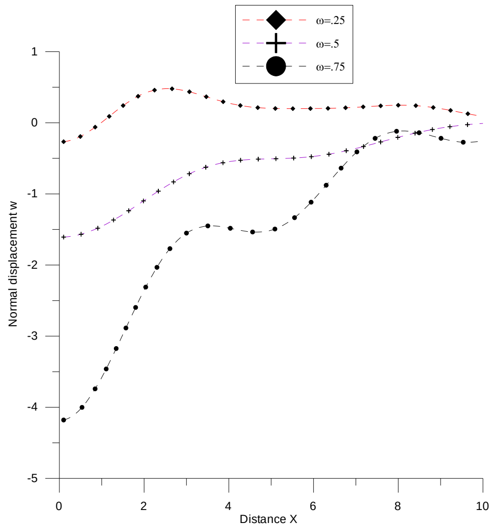

Fig. 17 Variation of normal displacement w with distance x (uniformly distributed thermal source)

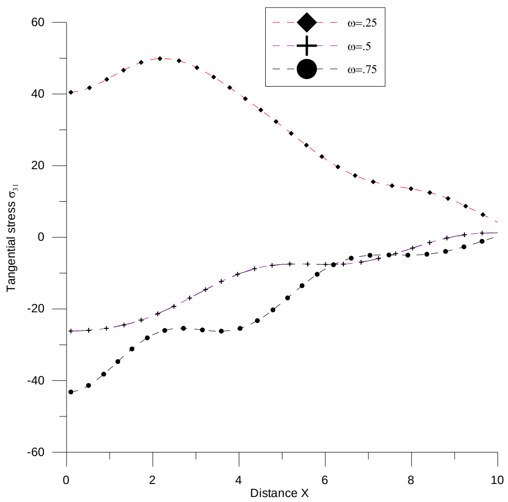

Fig. 15 Variation of tangential stress $\boldsymbol{\sigma_{31}}$ with distance $\boldsymbol{x}$ (concentrated thermal source)

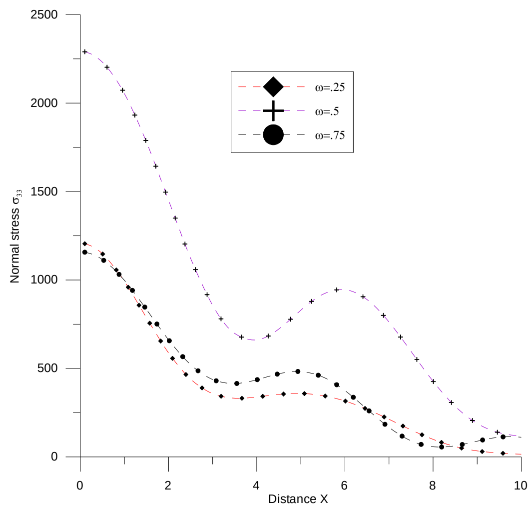

Fig. 18 Variation of normal stress σ_{33} with distance x (uniformly distributed thermal source)

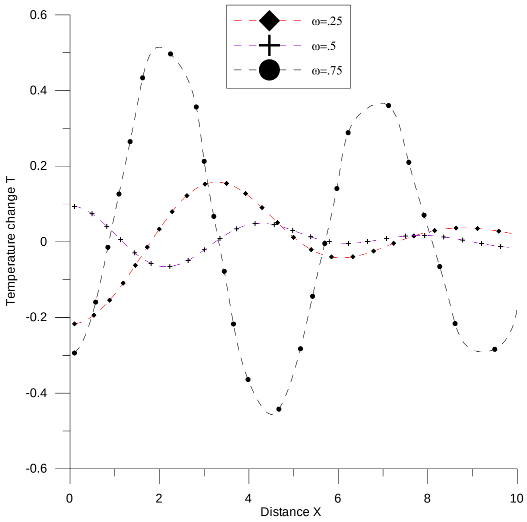

Fig. 19 Variation of temperature change T with distance x (uniformly distributed thermal source)

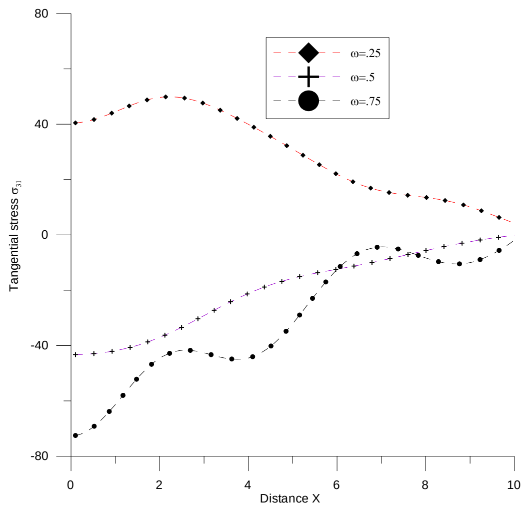

Fig. 20 Variation of tangential stress σ_{31} with distance x (uniformly distributed thermal source)

and $\omega = 0.75$ it increases in the whole range.

9.2 Uniformly distributed thermal source

Figs. 16-20 shows the characteristics for uniformly distributed thermal source. It is depicted from Figs. 16-20 that the distribution curves for tangential displacement u, normal displacement w, normal stress σ_{33}, temperature change T and tangential stress σ_{13} for uniformly distributed thermal source, follow same trends as in case of concentrated thermal source for the three different nosn-dimensional frequencies $\omega = 0.25$ and $\omega = 0.5$, for $\omega = 0.75$ respectively.

10. Conclusions

From the above investigation it can be seen that the effect of frequency on various components play an important role in the deformation of an orthotropic homogeneous thermoelastic body. We examine that change in the value of non-dimensional frequency has a significant impact on displacement components, stress components and temperature change. When the time harmonic mechanical and thermal source is applied, the variations in all the components are almost oscillatory and it describes a big difference in the magnitudes of all the components.

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