Structural Engineering and Mechanics, *Vol. 73, No. 5 (2020) 585-598*
DOI: https://doi.org/10.12989/sem.2020.73.5.585

A study of fracture of a fibrous composite

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(Received September 1, 2019, Revised October 24, 2019, Accepted October 30, 2019)

Abstract. We develop design model within which nucleation and propagation of crack in a fibrous composite is described. It is assumed that under loading, crack initiation and fracture of material happens in the composite. The problem of equilibrium of a composite with embryonic crack is reduced to the solution of the system of nonlinear singular integral equations with the Cauchy type kernel. Normal and tangential forces in the crack nucleation zone are determined from the solution of this system of equations. The crack appearance conditions in the composite are formed with regard to criterion of ultimate stretching of the material's bonds. We study the case when near the fiber, the binder has several arbitrary arranged rectilinear prefracture zones and a crack with interfacial bonds. The proposed computational model allows one to obtain the size and location of the zones of damages (prefracture zones) depending on geometric and mechanical characteristics of the fibrous composite and applied external load. Based on the suggested design model that takes into account the existence of damages (the zones of weakened interparticle bonds of the material) and cracks with end zones in the composite, we worked out a method for calculating the parameters of the composite, at which crack nucleation and crack growth occurs.

Keywords: binder; fiber; prefracture zone; crack nucleation; cohesive forces; crack growth

1. Introduction

The creation of composite materials with high-strength and reliability is achieved by reinforcing the binder with rigid and high-strength component (fiber). At the design stage of a new composite, it is necessary to take into account that at different parts of a composite, there may arise cracks.

We consider a composite consisting of elastic medium (matrix) and inclusions (fibers) made of different elastic material and distributed in it. The fracture process of such composite materials is determined from interaction of a fiber and binder. Fibers made of another material (reinforcing elements) compose by weight a relatively small part and significantly effect on its strength. As a rule, the constructions made of composites work under complex stress state. There is a great amount of works, (Li *et al.* 2005, Lü *et al.* 2007, Mishnaevsky and Brøndsted 2009, Chaudhuri 2011, Távara *et al.* 2011, Greco *et al.* 2013, Lü *et al.* 2011, Liu *et al.* 2012, Brighenti *et al.* 2013, Ko and Ju 2013, Bouhala *et al.* 2013, Mirsalimov and Hasanov 2014a,b, Hao *et al.* 2015, Mirsalimov and Hasanov 2015, Ibraheem *et al.* 2015, Mokhtari *et al.* 2015, Afshar *et al.* 2015, Usal 2015, Ju and Wu 2016, Le *et al.* 2016, Mirsalimov and Askarov 2016, Cameselle-Molares *et al.* 2017, Woo 2017, Babaei and Farrokhabadi 2017, Bakhshan *et al.* 2018, Aveiga and Ribeiro 2018) and others devoted to fracture of a composite. In the work (Li *et al.* 2005) a cohesive-zone model for a fiber-reinforced polymer–matrix composite is proposed. The analytical solution of an asymmetrical propagation crack of composite materials under the action of variable moving loads and unit-step moving loads is presented in the paper (Lü *et al.* 2007). An overview of methods of the mathematical modeling of deformation, damage and fracture in fiber reinforced composites is given in the paper (Mishnaevsky and Brøndsted 2009). In the study (Chaudhuri 2011) the three-dimensional asymptotic stress field in the vicinity of the front of a semi-infinite through-thickness crack weakening an infinite transversely isotropic unidirectional fiber reinforced composite plate, of finite thickness and subjected to far-field mode I/II loadings, is obtained. The eigenfunction expansion technique is used. In the work (Távara *et al.* 2011) the behavior of the fiber–matrix interface under transverse tension is investigated using a new linear elastic–brittle interface model. In the study (Greco *et al.* 2013), a numerical multi-scale failure analysis of locally periodic fiber-reinforced composites is conducted. In the research (Lü *et al.* 2011) an asymmetrical dynamic crack model of bridging fiber pull-out of composite materials is proposed for analyzing the distributions stress and displacement under the loading conditions of an applied nonhomogenous stress and the traction forces. The study (Liu *et al.* 2012) adopts the acoustic emission technique to study the failure mechanisms and damage evolution of carbon fiber/epoxy composite laminates. In the work (Brighenti *et al.* 2013) two computational models for the simulation of the cracking behavior of fiber-reinforced brittle–matrix composites – based on a continuous finite element approach and on a lattice approach, respectively – are presented. Crack nucleation and growth in long Fibre Reinforced

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http://www.techno-press.com/journals/sem&subpage=7
ISSN: 1225-4568 (Print), 1598-6217 (Online)

Composites are investigated using the extended finite element method and the cohesive zone model in the work (Bouhala *et al*. 2013). In the paper (Mirsalimov and Hasanov 2014a), an elastic plane with a periodic system of circular filled holes and two periodic systems of rectilinear cracks with bonds between faces at the tip regions is considered. The holes are filled with elastic plugs with the surface was covered by a homogenous cylindrical film. In the study (Ko and Ju 2013), a micromechanical multi-level elastoplastic evolutionary damage framework is presented to predict the overall transverse mechanical behavior and damage evolutions of cylindrical fiber-reinforced ductile composites. In the work (Mirsalimov and Hasanov 2014b), a plane problem of fracture mechanics for an isotropic medium with a periodic system of round holes filled with absolutely rigid inclusions soldered along the contour and rectilinear cracks with interfacial bonds at the end zones collinear to the coordinate axes of unequal length under transverse shear is considered. In the paper (Hao *et al*. 2015) the stress singularity of the matrix crack perpendicular to the fiber bundles is investigated by the method of optical caustics. A model of cracks nucleation in composite with periodic structures is given in the research (Mirsalimov and Hasanov 2015). The model is based on the analysis of the cracking zone. In the publication (Ibraheem *et al*. 2015) an experimental investigation is conducted to examine the behavior and cracking of steel fiber reinforced concrete beams subjected to combined torsion, bending, and shear. In the article (Mokhtari *et al*. 2015), the compression failure of thermoplastic composites made of jute/polypropylene was studied by experimental and numerical investigations. It was found that the loading conditions laminates and the fibers orientation affect the compression failure load of the thermoplastic composites. The extended finite element method is generalized to study the fiber bridging phenomenon in fracture analysis of unidirectional composites by Afshar *et al.* (2015). In the paper (Usal 2015) a continuum damage model is developed for the linear viscoelastic behavior of composites with microcracks consisting of an isotropic matrix reinforced by two arbitrarily independent and inextensible fiber families. In the work (Ju and Wu 2016) a computational stochastic micromechanics-based framework is presented to study the overall mechanical behavior of longitudinal continuous fiber-reinforced composites considering progressive fiber breaking evolution. Two stochastic risk-competing models are presented to simulate the fiber breaking evolution in an inhomogeneous fashion considering the local load sharing mechanisms. The overall stress–strain responses and the fiber breaking evolution are satisfactorily predicted, and validations are performed and compared with experimental data. The paper (Le *et al*. 2016) presents a 3D simulation of damages and cracks growth in fibrous composite material using Discrete Element Method. In the work (Mirsalimov and Askarov 2016) a thin elastic isotropic binder with inclusions of another material and rectilinear cracks arbitrarily placed near the inclusion is considered. The matrix (binder) subjected to bending. To determine the optimum form of inclusion which minimizes the stress intensity factors (moments) near crack tips a fracture mechanics problem is solved. A quasi-static progressive damage model for prediction of the fracture behavior and strength of adhesively bonded fiber-reinforced polymer joints is introduced in the paper (Cameselle-Molares *et al*. 2017). The fracture behavior of plain weave textile composite was studied numerically by finite element analysis and cohesive zone modeling in the work (Woo 2017). The study (Babaei and Farrokhabadi 2017) focuses on a computational constitutive model which predicts the matrix cracking evolution and fiber breakage in cross-ply composite laminates with open hole under in-plane loading. The work (Bakhshan *et al*. 2018) aims to obtain failure loads for open-hole unidirectional composite plates under tensile loading. For this purpose, a user-defined material model in the finite element analysis package is developed to predict the failure load of the open-hole composite laminates. A good agreement between experimental results and numerical predictions is observed. The work (Aveiga and Ribeiro 2018) aims at the development of a delamination propagation model to estimate a progressive interlaminar delamination failure in fiber reinforced composite materials and to allow the prediction of material's degradation. Also works on the shear and bending of the composite plates (Mahi *et al*. 2015, Draiche *et al*. 2016, Kaci *et al*. 2018) should be noted.

Development of a design model permitting to predict the stress-strain state at the prefracture stage (crack nucleation) in a composite is of great importance.

In contrast to all above-listed works, this article provides a new solution methodology for the problems of the nucleation and propagation of cracks in a fibrous composite. The computational model proposed in present work makes it possible to find in the process of solving the problem of fracture mechanics the size and location of the zones of damages (prefracture zones) depending on the geometric and mechanical characteristics of the fibrous composite and the applied external load. One original feature of the present work is that the solution is not based on the finite element solution but relies on discretization of a system of singular integral equations. This approach is most appealing and convenient for practical use in engineering calculations.

In connection with the known observations of physical process of fracture, we take the following natural succession of analysis of the process of nucleation and propagation of a bridged crack in the composite. a) studying prefracture zone appearance in a composite and formation of arbitrary size cracks with interfacial bonds b) analysis of crack propagation at the end zones. The goal of this work is to develop a mathematical model for a composite body "a binder-fiber" allowing calculation of critical external loads under which initiation and growth of a crack with bonds in the end zone, happens.

2. Problem statement

Let an unbounded body made of elastic isotropic medium (matrix) and the fibers distributed in it under the plane deformation conditions be subjected to stretching by mean stresses (stretching at infinity) $\sigma_x = \sigma_x^\infty$, $\sigma_y = \sigma_y^\infty$, $\tau_{xy} = \tau_{xy}^\infty$.

As the composite is loaded, the prefracture zones will appear in the binder. The prefracture zones are simulated as areas of weakened interparticle bonds of the material (Mirsalimov 2007, Mirsalimov and Kalantarly 2015a, b, Mirsalimov and Hasanov 2014). Interaction of prefracture zone faces is simulated by introducing between their faces the bonds with the given diagram of deformation (Mirsalimov 2007, Mirsalimov and Kalantarly 2015a, b, Mirsalimov and Hasanov 2014).

The physical nature of such bonds, location and sizes of prefracture zones depend on the form of the material.

As the mentioned zones (the layers of the overstrained material) are small compared with the remaining part of the binder of the composite, we can mentally remove them and replace by cuts whose surfaces interact between themselves by some law corresponding to the action of the removed material. Account of these effects in problems of nonlinear fracture mechanics is an important and very difficult issue.

In the case under consideration, appearance of cracks is a process of passage of prefracture zone to the domain of broken bonds between the surfaces of the material of the binder (Mirsalimov 2007). In this case, location and the size of prefracture zone are not known in advance and should be determined in the process of solving the fracture mechanics problem.

The investigations (Mirsalimov 1987, Panasyuk 1991, Rusinko and Rusinko 2011) of appearance of domains with violated structure of the material, show that at the initial stage of embedding, the prefracture zone is a narrow-elongated layer, and then as the load increases, there suddenly appears a secondary system of zones containing a material with partially violated bonds.

Analysis of interaction of the binder and fibers is carried out on the basis of a model with one fiber. The remaining fibers are “smeared”, and the material outside of the isolated fiber appears to be uniform and isotropic with corresponding effective elastic constants (according to the rule of “mixtures”) (Mirsalimov and Askarov 2016, Mirsalimov 2018). With such approach, the interaction of other “smeared” fibers and prefracture zones is realized by the corresponding effective elastic constants. Herewith, there are no restrictions on configuration and relative sizes of fibers and prefracture zones. It is assumed that the prefracture zones do not intersect between themselves and fiber.

The origin of the system of coordinates is compatible with geometrical center of fibers (Fig. 1). It is accepted that an elastic fiber from another material is inserted into the circular hole of the binder.

It is assumed that rigid adhesion of different materials holds everywhere on the conjunction boundary L ($\tau = R\exp(i\theta)$). At the center of rectilinear prefracture zones we locate the origin of local systems of coordinates $x_kO_ky_k$ whose axes x_k coincide with the prefracture zone lines and make the angles α_k with the axis x (Fig. 1).

The prefracture zone faces interact so that this interaction (bonds between faces) restrains nucleation of a crack. For mathematical description of interaction of the prefracture zone faces, it is considered that between them there are bonds with the given deformation law.

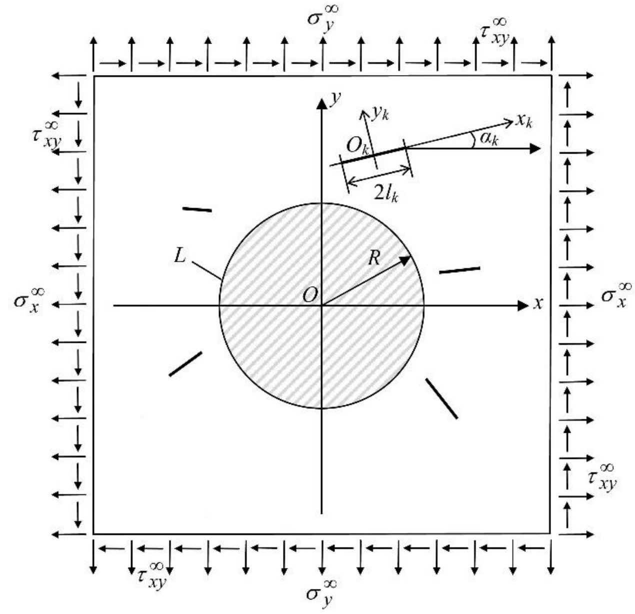

Fig. 1 Calculation scheme of the fracture mechanics problem for case of arbitrary number of prefracture zones in a fibrous composite

Under the action of external tensile loads on the composite, in the bonds connecting the prefracture bands faces there will arise normal q_{y_k} and tangential $q_{x_ky_k}$ forces. Consequently, normal and tangential stresses numerically equal to $q_{y_k}(x_k)$ and $q_{x_ky_k}(x_k)$ will be applied to prefracture zone faces. The values of these stresses are unknown in advance and should be determined in the process of solving the boundary value problem of fracture mechanics.

In the problem under consideration, the boundary conditions are of the form

$$\sigma_r - i\tau_{r\theta} = \sigma_r^0 - i\tau_{r\theta}^0 \quad \text{on } L$$
$$v_r - iv_\theta = v_r^0 - iv_\theta^0 \tag{1}$$

$$\sigma_{y_k} = q_{y_k}, \qquad \tau_{x_ky_k} = q_{x_ky_k}$$
$$(k = 1,2,\ldots,N) \quad \text{on the prefracture zone faces} \tag{2}$$

Here σ_r, $\tau_{r\theta}$, σ_r^0, $\tau_{r\theta}^0$ are stress tensor components in the binder and fiber, respectively; v_r, v_θ, v_r^0, v_θ^0 are radial and tangential of the vector displacements components in the binder and fiber, respectively; $i^2 = -1$; N is the number of prefracture zones in the binder.

Conditions (1)-(2) should be complemented by a relation that connects the opening of prefracture zone faces and forces in binders. Without loss of generality, we represent this relation in the form

$$\left(v_k^+ - v_k^-\right) - i\left(u_k^+ - u_k^-\right) = \\ = \Pi_y(x_k, \sigma_k) q_{y_k}(x_k) - i\Pi_x(x_k, \sigma_k) q_{x_k y_k}(x_k) \quad (3)$$

$$(k = 1, 2, \ldots, N)$$

The functions $\Pi_y(x_k, \sigma_k)$ and $\Pi_x(x_k, \sigma_k)$ are effective compliances of bonds dependent on their tension; $\sigma_k = \sqrt{q_{y_k}^2 + q_{x_k y_k}^2}$ is the modulus of force vector in bonds, $\left(v_k^+ - v_k^-\right)$ and $\left(u_k^+ - u_k^-\right)$ are normal and tangential component of the opening of the faces of the k-th prefracture zone, respectively.

Under constant values Π_y, Π_x in (3) we have linear law of deformation. In the general case, the deformation law is nonlinear and given (Mirsalimov 2007).

To determine the values of external tensile load under which crack nucleation happens, the problem statement should be complemented by the condition (criterion) of appearance of a crack (rupture of interparticle bonds of the material). As such a condition we accept the criterion of critical opening of the prefracture zone faces

$$\left|\left(v_k^+ - v_k^-\right) - i\left(u_k^+ - u_k^-\right)\right| = \delta_{cr} \quad (k = 1, 2, \ldots, N) \quad (4)$$

where δ_{cr} is the characteristics of the resistance of the binder's material to crack nucleation.

This additional condition allows to determine the parameters of the composite, under which the crack appears in the binder.

3. Case of a single prefracture zone

Assume that the binder has one rectilinear prefracture zone (Fig. 2).

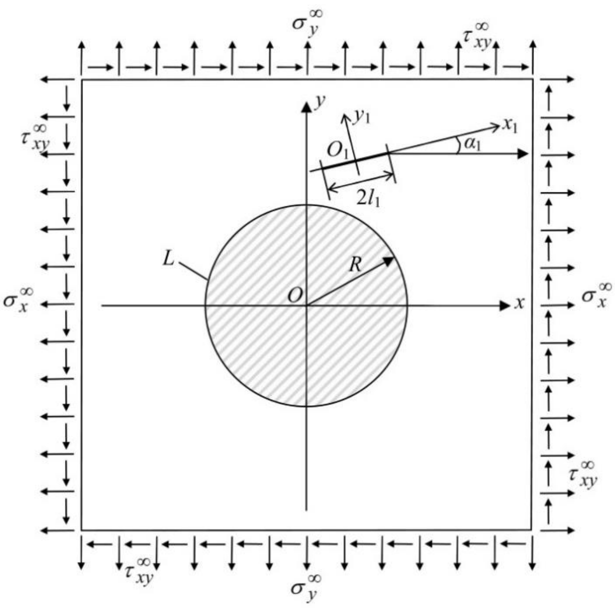

Fig. 2 Calculation scheme of the fracture mechanics problem for case of a single prefracture zone in a fibrous composite

The stress tensor components σ_x, σ_y, τ_{xy} and displacement vector components u, v in the plane problem of elasticity theory may be represented by the analytic functions $\varphi(z)$, $\psi(z)$ and Kolosov-Muskhelishvili formulas (Muskhelishvili 1977)

$$\sigma_x + \sigma_y = \sigma_r + \sigma_\theta = 2\left[\varphi'(z) + \overline{\varphi'(z)}\right]$$

$$\sigma_y - \sigma_x + 2i\tau_{xy} =$$

$$= (\sigma_\theta - \sigma_r + 2i\tau_{r\theta})e^{-2i\theta} = 2\left[\bar{z}\phi''(z) + \psi'(z)\right]$$

$$2\mu(u + iv) = \kappa\varphi(z) - z\overline{\varphi'(z)} - \overline{\psi(z)}$$

where μ is the shear modulus; κ is the Muskhelishvili's constant: $\kappa = 3 - 4\nu$ for plane strain and $\kappa = (3 - \nu)/(1 + \nu)$ for plane stress; ν is the Poisson ratio.

By means of these formulas, on the interface of media L we have

$$\varphi(z) + z\overline{\varphi'(z)} + \overline{\psi(z)} = \varphi_0(z) + z\overline{\varphi_0'(z)} + \overline{\psi_0(z)} \quad (z \in L) \quad (5)$$

$$\kappa\varphi(z) - z\overline{\varphi'(z)} - \overline{\psi(z)} = \frac{\mu}{\mu_0}\left[\kappa_0\varphi_0(z) - z\overline{\varphi_0'(z)} - \overline{\psi_0(z)}\right] \quad (6)$$

where κ, μ and κ_0, μ_0 are elastic constants of the material of the binder and fiber, respectively; $\varphi(z)$, $\psi(z)$ and $\varphi_0(z)$, $\psi_0(z)$ are analytic functions for the binder and fiber, respectively.

On the faces of rectilinear prefracture zone we have the boundary condition

$$\Phi(x_1) + \overline{\Phi(x_1)} + x_1\overline{\Phi'(x_1)} + \overline{\Psi(x_1)} = q_{y_1} - iq_{x_1 y_1} \quad (7)$$

where x_1 is the affix of the points of the rectilinear prefracture zone.

We look for the complex potentials $\varphi_0(z)$, $\psi_0(z)$ describing the stress-strain state of the fiber in the form (Muskhelishvili 1977)

$$\varphi_0(z) = \sum_{k=1}^{\infty} a_k z^k, \qquad \psi_0(z) = \sum_{k=1}^{\infty} b_k z^k \quad (8)$$

We denote the left-hand side of boundary condition (5) by $f_1 + if_2$ and accept that on the contour L the function $f_1 + if_2$ expands in Fourier series. The Fourier series for the function $f_1 + if_2$ has the form

$$f_1 + if_2 = \sum_{k=-\infty}^{\infty} A_k e^{ik\theta} \quad (9)$$

Based on boundary condition (8), (9) and relations (5), using the method of power series, we have

$$a_n = \frac{A_n}{R^n} \quad (n > 1), \qquad \mathrm{Re}\, a_1 = \frac{A_1}{2R} \quad (10)$$

$$b_n=\frac{\overline{A}_{-n}}{R^n}-(n+2)\frac{A_{n+2}}{R^n}\qquad (n\geq 0)$$

To determine the coefficients A_k, it is necessary to consider the solution of the problem of elasticity theory for the binder with a prefracture zone. Using the analytic functions $\varphi_0(z)$ and $\psi_0(z)$, after some elementary transformations the boundary conditions on the contour L for finding complex potentials $\varphi(z)$ and $\psi(z)$ will be written in the form

$$\varphi(\tau)+\tau\overline{\varphi'(\tau)}+\overline{\psi(\tau)}=\sum_{k=-\infty}^{\infty}A_k e^{ik\theta} \tag{11}$$

$$\begin{gathered}\kappa\varphi(\tau)-\tau\overline{\varphi'(\tau)}-\overline{\psi(\tau)}=\frac{\mu}{\mu_0}\left\{\kappa_0\sum_{k=1}^{\infty}a_kR^ke^{ik\theta}-\overline{a}_1Re^{i\theta}-\right.\\ \left.-\sum_{k=0}^{\infty}(k+2)\overline{a}_{k+2}R^{2k+2}e^{-ik\theta}-\sum_{k=0}^{\infty}\overline{b}_kR^ke^{-ik\theta}\right\}\end{gathered} \tag{12}$$

We sought for the solution of boundary value problem (7), (11) and (12) in the form

$$\Phi(z)=\Phi_1(z)+\Phi_2(z)\,,\quad \Psi(z)=\Psi_1(z)+\Psi_2(z)$$

$$\begin{gathered}\Phi_1(z)=\sum_{k=0}^{\infty}c_kz^{-k}\,,\qquad \Psi_1(z)=\sum_{k=0}^{\infty}d_kz^{-k}\\ \varphi'(z)=\Phi(z)\,,\qquad \psi'(z)=\Psi(z)\end{gathered} \tag{13}$$

$$c_0=\frac{1}{4}\left(\sigma_x^{\infty}+\sigma_y^{\infty}\right),\qquad d_0=\frac{1}{2}\left(\sigma_y^{\infty}-\sigma_x^{\infty}\right)+i\,\tau_{xy}^{\infty}$$

$$\begin{gathered}\Phi_2(z)=\frac{1}{2\pi}\int_{-l_1}^{l_1}\frac{g_1(t)dt}{t-z_1}\\ \Psi_2(z)=\frac{1}{2\pi}e^{-2i\alpha_1}\int_{-l_1}^{l_1}\left[\frac{g_1(t)}{t-z_1}-\frac{\overline{T}_1g_1(t)}{(t-z_1)^2}\right]dt\end{gathered} \tag{14}$$

where $T_1=te^{i\alpha_1}+z_1^0$; $z_1=e^{-i\alpha_1}\left(z-z_1^0\right)$; l_1 is the half-length of the prefracture zone; $g_1(x_1)$ is the desired function characterizing the opening of prefracture zone faces

$$\begin{gathered}g_1(x_1)=\frac{2\mu}{i(1+\kappa)}\times\\ \times\frac{d}{dx_1}\left[u_1^+(x_1,0)-u_1^-(x_1,0)+i\left(v_1^+(x_1,0)-v_1^-(x_1,0)\right)\right]\end{gathered} \tag{15}$$

Using the functions (13)-(14), we represent the boundary conditions (11)-(12) in the following form

$$\varphi_1(\tau)+\tau\overline{\varphi_1'(\tau)}+\overline{\psi_1(\tau)}=\sum_{k=-\infty}^{\infty}A_ke^{ik\theta}+f_1^0(\tau) \tag{16}$$

$$\begin{gathered}\kappa\varphi_1(\tau)-\tau\overline{\varphi_1'(\tau)}-\overline{\psi_1(\tau)}=\\ =\frac{\mu}{\mu_0}\left\{\kappa_0\sum_{k=1}^{\infty}a_kR^ke^{ik\theta}-\overline{a}_1Re^{i\theta}-\right.\\ \left.-\sum_{k=0}^{\infty}(k+2)\overline{a}_{k+2}R^{2k+2}e^{-ik\theta}-\sum_{k=0}^{\infty}\overline{b}_kR^ke^{-ik\theta}\right\}+f_2^0(\tau)\end{gathered}$$

where

$$f_1^0(\tau)=-\left[\varphi_2(\tau)+\tau\overline{\varphi_2'(\tau)}+\overline{\psi_2(\tau)}\right]$$

$$f_2^0(\tau)=-\left[\kappa\varphi_2(\tau)-\tau\overline{\varphi_2'(\tau)}-\overline{\psi_2(\tau)}\right]$$

Satisfying by functions (13)-(14) the boundary conditions (16) and comparing the coefficients for the identical powers $\exp(i\theta)$ in the both hand sides, we get a system of algebraic equations for determining the coefficients c_k, d_k and A_k. These equations allow to obtain formulas in the explicit form for c_k, d_k and A_k by the desired function $g_1(x_1)$.

Satisfying by the complex potentials (13)-(14) the boundary conditions on the prefracture zone faces (7), after some transformations we get a complex integral equation with respect to the unknown function $g_1(x_1)$:

$$\begin{gathered}\int_{-l_1}^{l_1}\left[R_{11}(t,x_1)g_1(t)+S_{11}(t,x_1)\overline{g_1(t)}\right]=\pi F_1(x_1)\\ |x_1|\leq l_1\end{gathered} \tag{17}$$

where

$$\begin{gathered}F_1(x_1)=q_{y_1}-iq_{x_1y_1}-\\ -\left[\Phi_1(x_1)+\overline{\Phi_1(x_1)}+x_1\overline{\Phi'(x_1)}+\overline{\Psi_1(x_1)}\right]\end{gathered}$$

x_1, t, z_1^0 and l_1 are dimensionless variables referred to R; R_{nk} and S_{nk} (n = k = 1) are determined by the known formulas (VI. 62) of the book (Panasyuk *et al.* 1976).

To the singular integral equation, we should add the condition of uniqueness of displacements when tracing the prefracture zone contour

$$\int_{-l_1}^{l_1}g_1(t)dt=0 \tag{18}$$

Under condition (18), the complex singular integral equation (17), by means of algebraization procedure (Panasyuk *et al.* 1976, Mirsalimov 1987) is reduced to the system of M algebraic equations for determining M unknowns $g_1(t_m)$ (m = 1,2,…,M):

$$\begin{gathered}\frac{1}{M}\sum_{k=1}^{M}l_1\left[g_1(t_m)R_{11}(l_1t_m,l_1x_r)+\overline{g_1(t_m)}S_{11}(l_1t_m,l_1x_r)\right]=\\ =F_1(x_r)\end{gathered} \tag{19}$$

$$\sum_{m=1}^{M} g_1(t_m) = 0$$

where

$$t_m = \cos\frac{2m-1}{2M}\pi \qquad m = 1,2,\ldots,M$$

$$x_r = \cos\frac{\pi r}{M} \qquad r = 1,2,\ldots,M-1$$

If in the system (19) we pass to complex conjugated variables, we get one more M algebraic equations. The right-hand sides of the system (19) contain unknown values of $q_{y_1}(x_1)$ and $q_{x_1y_1}(x_1)$ tractions at the nodal points of prefracture zone.

The additional relation (3) for $k = 1$ is the condition determining the unknown stresses in the bonds between the prefracture zone faces. In the problem under consideration, it is convenient to write this condition for the derivative of the opening of displacements of prefracture zone faces

$$\begin{aligned}&\frac{\partial}{\partial x_1}\left[v_1^+(x_1,0) - v_1(x_1,0) - i\left(u_1^+(x_1,0) - u_1^-(x_1,0)\right)\right] = \\ &= \frac{\partial}{\partial x_1}\left[\Pi_y(x_1,\sigma_1)q_y(x_1) - i\Pi_x(x_1,\sigma_1)q_{x_1y_1}(x_1)\right]\end{aligned} \qquad (20)$$

Using the obtained solution, we can write

$$\begin{aligned}&g_1(x_1) = \frac{2\mu}{1+\kappa}\times \\ &\times\frac{d}{dx_1}\left[\Pi_y(x_1,\sigma_1)q_{y_1}(x_1) - \Pi_x(x_1,\sigma_1)q_{x_1y_1}(x_1)\right]\end{aligned} \qquad (21)$$

where x_1 is the affix of the points of prefracture zone faces.

This complex differential equation helps to find the stresses q_{y_1} and $q_{x_1y_1}$ in the bonds between the prefracture zone faces. To construct missing algebraic equations for determining approximate values of stresses $q_{y_1}(t_m)$ and $q_{x_1y_1}(t_m)$ at the nodal points we require fulfillment of conditions (21) at the nodal points t_m ($m = 1,2,\ldots,M$) contained in prefracture zone. Herewith, the finite differences method is used. As a result, we obtain a complex algebraic system of M equations for determining approximate values $q_{y_1}(t_m)$, $q_{x_1y_1}(t_m)$ ($m = 1,2,\ldots,M$) at the nodal points of prefracture zone. Herewith the boundary conditions

$$q_{y_1}(\pm l_1) = 0, \qquad q_{x_1y_1}(\pm l_1) = 0$$

corresponding to the conditions

$$v_1^+(\pm l_1,0) - v_1^-(\pm l_1,0) = 0$$

$$u_1^+(\pm l_1,0) - u_1^-(\pm l_1,0) = 0$$

were taken into account.

For completeness of the obtained algebraic equations we need two complex equations determining the coordinates of vertices (location) of prefracture zone. As the stresses in the composite are everywhere bounded, the solution of the singular integral equation should be sought in the class of everywhere bounded functions (stresses). Therefore, it is necessary to add to the system (19) the conditions of stress boundedness at the ends of prefracture band $x_1 = \pm l_1$. These conditions are of the form

$$\begin{aligned}&\sum_{m=1}^{M}(-1)^{M+m} g_1(t_m)\tan\frac{2m-1}{4m}\pi = 0 \\ &\sum_{m=1}^{M}(-1)^{m} g_1(t_m)\cot\frac{2m-1}{4m}\pi = 0\end{aligned} \qquad (22)$$

Because of unknown size of the prefracture zone, the obtained joined system of algebraic equations with respect to c_k, d_k, A_k, $g_1(t_m)$, $q_{y_1}(t_m)$, $q_{x_1y_1}(t_m)$ is nonlinear.

Its numerical solution allows under the given external load to obtain the coordinates of vertices (location) and the size of prefracture zone, the stress-strain state of the composite. Obviously, having determined the coordinates of the vertices of prefracture zone, by the known formulas of analytic geometry we can find the coordinates of the center z_1^0 of prefracture zone, the angle α_1 with the axis x (Fig. 2) and the length l_1 of prefracture zone. The physical condition for the finiteness of the stresses at the vertices of prefracture zone of is used.

Even under linear elastic bonds, the joined system of equations because of unknown quantity l_1, becomes nonlinear. The method of successive approximations is used for solving it. The essence of this method is in the following. We solve the joined algebraic system under some definite value of l_1^* with respect to the remaining unknowns c_k, d_k, A_k, $g_1(t_m)$, $q_{y_1}(t_m)$, $q_{x_1y_1}(t_m)$. In the case of linear-elastic bonds, the remaining unknowns enter into the joined system linearly. The values of l_1^* and the values of remaining constants corresponding to it will not, generally speaking, satisfy equations (22). Therefore, selecting the values of the parameter l_1 we will many times repeat calculations until equations (22) will be satisfied with given accuracy.

In each approximation, the joined algebraic system was solved by the Gauss method with the choice of the principal element.

In the case of nonlinear law of deformation of bonds, for determining forces in prefracture zone, the iterative method similar to the method of elastic solutions (Il'yushin 2003) is used. It is assumed that the law of deformation of interparticle bonds in prefracture zone is linear for $V_1 = \sqrt{u_1^2 + v_1^2} \le V_*$. The first step of the iterative process of calculation is in solving the system of resolving equations for linear-elastic bonds. The next iterations are fulfilled only if the inequality $V_1(x_1) > V_*$ holds on a part of prefracture zone (Mirsalimov 2007). For such iterations the system of resolving equations is solved for quasielastic

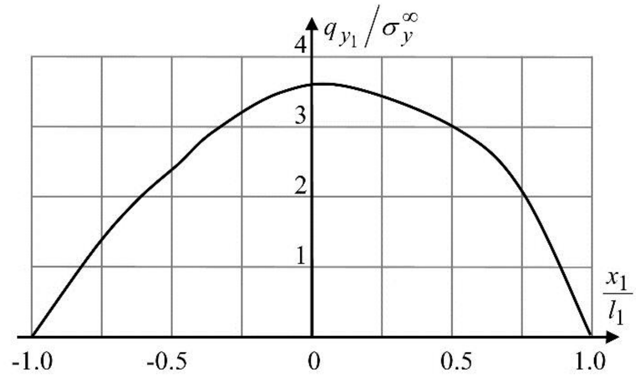

Fig. 3 Distribution of normal forces q_{y_1}/σ_y^∞ in prefracture zone

bonds (cohesion forces) with effective compliance variable along the prefracture zone and dependent on the value of the modulus of forces vector in bonds obtained at the previous stage of calculation. The calculation of effective compliance is carried out as in finding the secant modulus in the method of variable parameters of elasticity (Birger 1965). It is accepted that the successive approximation process ends as the forces in prefracture zone obtained at two successive steps differ a little.

The nonlinear part of the curve of deformation of bonds was represented in the form of bilinear dependence whose ascendant part corresponds to elastic deformation of bonds ($0 < V_1(x_1) < V_*$) with maximum tension of bonds. For $V_1(x_1) > V_*$, the deformation law was described by nonlinear dependence determined by two points (V_*, σ_*) and $(\delta_{cr}, \sigma_{cr})$, and for $\sigma_{cr} \geq \sigma_*$ we have the increasing linear dependence (linear strengthening corresponding to elastic deformation of bonds).

In Fig. 3 we give distribution of normal forces q_{y_1}/σ_y^∞ in prefracture zone. The compliances of bonds in normal and tangential stresses were accepted equal and constant along prefracture zone. The law of change of tangential stresses along the prefracture zone is similar to the change of normal forces with a difference that absolute values of tangential forces are significantly lower. And the maximum values of tangential stresses are achieved for small sizes of prefracture bands.

The graph of distribution of maximum value of vector forces modulus $\sigma = \sqrt{q_{y_1}^2 + q_{x_1 y_1}^2}$ depending on relative size $\lambda = l_1 / R$ of prefracture zone, is depicted in Fig. 4. The vector forces modulus σ/σ_y^∞ reaches its maximum value when the relative size λ = 0.04. The numerical calculation was carried out for the fiber $\nu = 0.30$; $\mu_0 = 4.5 \cdot 10^5$ MPa; and binder $\nu = 0.32$; $\mu = 2.6 \cdot 10^5$ MPa; $\sigma_* = 35$ MPa; $\sigma_{cr} / \sigma_* = 2$; $\delta_{cr} = 2.1 \cdot 10^{-6}$ m; $\Pi_B = 1.5 \cdot 10^{-7}$ m/MPa (Π_B is effective compliance of bonds).

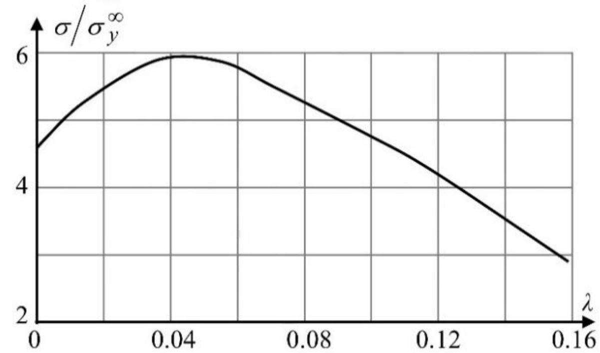

Fig. 4 Distribution of maximum value of vector forces modulus σ depending on relative size of the prefracture zone

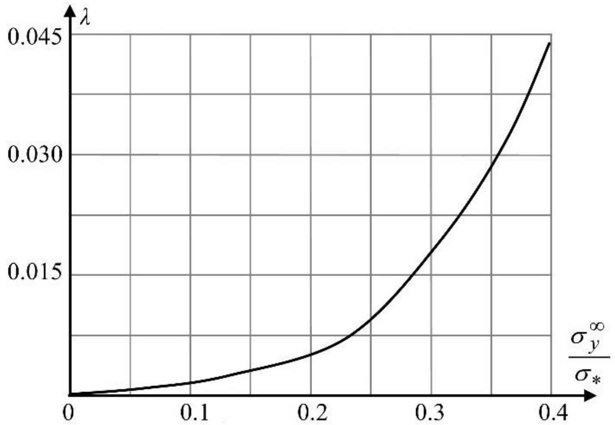

Fig. 5 Dependence of the prefracture zone length on external tensile load $\sigma_y^\infty / \sigma_*$

The largest values of the modulus of force vector in bonds, as a rule, are attained in the middle of the prefracture zone. As the size of prefracture zone increases, the level of the stresses q_{y_k} and $q_{x_k y_k}$ in the bonds decreases and, respectively, the value of the modulus of force vector in bonds decreases.

The graph of dependence of the prefracture zone length λ on external tensile load $\sigma_y^\infty / \sigma_*$ is given in Fig. 5. For this case we find: $\alpha_1 = 38^\circ$, $z_1^0 = 1.23 R e^{i\pi/11}$.

Using the solution of the problem, we find displacements on the prefracture zone faces

$$-\frac{1+\kappa}{2\mu} \int_{-l_1}^{x_1} g_1(x_1) dx_1 = v_1(x_1, 0) - i u_1(x_1, 0) \qquad (23)$$

Assuming $x_1 = x_1^0$, applying the change of variable and replacing the integral by the sum, we find

$$-\frac{1+\kappa}{2\mu} \frac{\pi l_1}{M} \sum_{m=1}^{M_1} g_1(t_m) = v_1(x_1^0, 0) - i u_1(x_1^0, 0) \qquad (24)$$

Here M_1 is the number of nodal points contained in the interval $(-l_1, x_1^0)$.

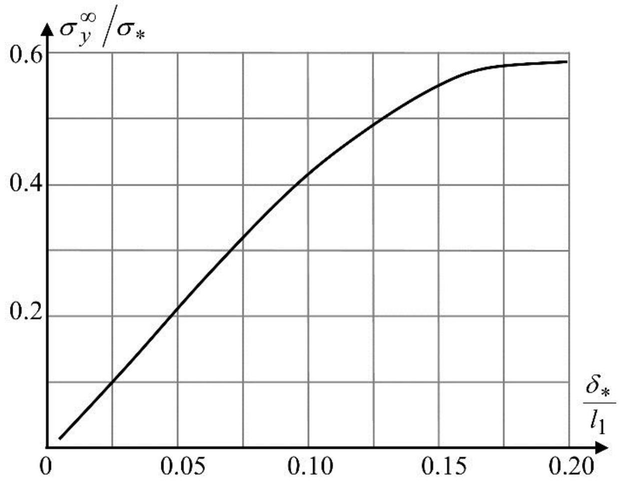

Fig. 6 Dependence of critical tensile load $\sigma_y^\infty / \sigma_*$ on relative opening δ_* / l_1 at the centre of prefracture zone

Taking into account $g_1(t_m) = v_1^0(t_m) - iu_1^0(t_m)$, from relation (24) we find $v_1(x_1^0, 0)$, $u_1(x_1^0, 0)$ and displacement vector modulus on prefracture zone faces for

$$V_0 = \sqrt{u_1^2 + v_1^2} = \frac{1+\kappa}{2\mu} \frac{\pi l_1}{M} \sqrt{A^2 + B^2}$$

$$A = \sum_{m=1}^{M_1} v_1^0(t_m), \qquad B = \sum_{m=1}^{M_1} u_1^0(t_m) \tag{25}$$

To determine the critical state under which a crack happens, we use limit condition (4). Then the condition that determines the critical value of the external tensile load, will be the relation

$$\frac{1+\kappa}{2\mu} \frac{\pi l_1}{M} \sqrt{A^2 + B^2} = \delta_{cr} \tag{26}$$

The joint solution of the joined algebraic system and condition (26) allows (under the given characteristics of crack resistance) to find critical value of the external tensile load, the coordinates of vertices, and the size of prefracture zone for limit equilibrium state under which a crack appears in the composite.

The graph of dependence of critical tensile load $\sigma_y^\infty / \sigma_*$ on relative opening δ_* / l_1 at the center of prefracture zone, where $\delta_* = \frac{\pi \delta_{cr} \mu}{(1+\kappa)\sigma_*}$, is given in Fig. 6.

The obtained algebraic system of equations allows solution of the problem with any accuracy given beforehand. The convergence of the procedure of numerical solution using singular equations is discussed in various works (Panasyuk *et al*. 1976, Ladopoulos 2000, Savruk and Kazberuk 2017). Many studies show that since $M = 20$, the values of the coefficients of the function of normal displacement are not significantly changed. In the calculations M was equal to 30.

4. Case of arbitrary number of prefracture zones

Now assume that in the next composite, near the fiber there are N rectilinear prefracture zones of length $2l_k$ ($k = 1,2,\ldots,N$). At the center of prefracture bands we locate the origin of local systems of coordinates $x_kO_ky_k$ whose axes x_k coincide with prefracture band and form the angles α_k with the axis x (Fig. 1).

In the case under investigation, appearance of embryonic cracks in the composite is the process of breaking of material's bonds between prefracture zone faces of the binder. The location and sizes of prefracture zone are not known in advance and should be determined.

As in the case of a single prefracture zone, the considered boundary value problem of mechanics of composite materials turns to be an elasticity theory problem with an unknown boundary and it is required to solve it in the course of solving the boundary value problem.

The solution of the problem for this case is similar to the solution in the case of a single prefracture zone. Relations (11)-(13) remain valid also for the considered case of arbitrary number of prefracture zones. According to condition (2), on rectilinear prefracture zone faces we have

$$\Phi(x_k) + \overline{\Phi(x_k)} + x_k \overline{\Phi'(x_k)} + \overline{\Psi(x_k)} = q_{y_k} - iq_{x_k y_k} \tag{27}$$

$$(k = 1,2,\ldots,N)$$

where x_k is the affix of the points of the k-th prefracture zone.

The complex potentials $\Phi_2(z)$ and $\Psi_2(z)$ are generalized for the case of arbitrary number of prefracture zones:

$$\Phi_2(z) = \frac{1}{2\pi} \sum_{k=1}^{N} \int_{-l_k}^{l_k} \frac{g_k(t)dt}{t - z_k}$$

$$\Psi_2(z) = \frac{1}{2\pi} \sum_{k=1}^{N} e^{-2i\alpha_k} \int_{-l_k}^{l_k} \left[\frac{\overline{g_k(t)}}{t - z_k} - \frac{\overline{T}_k e^{i\alpha_k}}{(t - z_k)^2} g_k(t) \right] dt \tag{28}$$

where $T_k = te^{i\alpha_k} + z_k^0$, $z_k = e^{-i\alpha_k}(z - z_k^0)$, $g_k(x_k)$ ($k = 1,2,\ldots,N$) are the desired functions characterizing the opening of prefracture zone faces

$$g_k(x_k) = \frac{2\mu}{i(1+\kappa)} \times$$

$$\times \frac{d}{dx_k} \left[u_k^+(x_k,0) - u_k^-(x_k,0) + i\left(v_k^+(x_k,0) - v_k^-(x_k,0) \right) \right] \tag{29}$$

Satisfying by the functions (13) and (28) the boundary conditions on prefracture zone faces (27), we get the system of N complex integral equations with respect to unknown functions $g_k(x_k)$ ($k = 1,2,\ldots,N$):

$$\sum_{k=1}^{N} \int_{-l_k}^{l_k} \left[R_{nk}(t,x) g_n(t) + S_{nk}(t,x) \overline{g_n(t)} \right] dt = \pi F_n(x) \tag{30}$$

$$|x| \le l_n \qquad (n = 1,2,\ldots,N)$$

where

$$F_n(x) = q_{y_n} - iq_{x_n y_n} -$$

$$-\left[\Phi_1(x_n) + \overline{\Phi_1(x_n)} + x_n \overline{\Phi_1'(x_n)} + \overline{\Psi_1(x_n)}\right]$$

and x, t, z_n^0 and l_n are dimensionless variables referred to the radius R of the fiber.

To the system of integral equations we should add the conditions

$$\int_{-l_k}^{l_k} g_k(t)dt = 0 \qquad (k = 1,2,\ldots,N) \tag{31}$$

Under additional conditions (31) by means of algebraization procedure the system of singular integral equations (30) is reduced to the system of $N\times M$ complex algebraic equations for determining $N\times M$ unknowns $g_k(t_m)$ $(k = 1,2,\ldots,N;\ m = 1,2,\ldots,M)$

$$\frac{1}{M}\sum_{m=1}^{M}\sum_{k=1}^{N} l_k\left[g_k(t_m)R_{nk}(l_n t_m, l_n x_r) + \right.$$

$$\left. + \overline{g_n(t_m)}S_{nk}(l_n t_m, l_n x_r)\right] = F_n(x_r) \tag{32}$$

$$\sum_{m=1}^{M} g_n(t_m) = 0$$

$$n = 1,2,\ldots,N; \qquad r = 1,2,\ldots,M-1$$

If in equations (32) we pass to complexly-conjugated values, we get one more $N\times M$ algebraic equations.

By the load function $F_n(x_r)$ $(n = 1,2,\ldots,N)$ on prefracture zone faces, the right hand sides of algebraic equations contains the desired coefficients c_k, d_k, A_k, the unknown values of q_{y_n} and $q_{x_n y_n}$ tractions at nodal points of the corresponding prefracture zone.

Using the obtained solution and equation (3), we find

$$g_k(x_k) = \frac{2\mu}{1+\kappa}\frac{d}{dx_k}\times$$

$$\times\frac{d}{dx_k}\left[\Pi_y(x_k,\sigma_k)q_{y_k}(x_k) - i\Pi_x(x_k,\sigma_k)q_{x_k y_k}(x_k)\right] \tag{33}$$

$$(k = 1,2,\ldots,N)$$

where x_k is the affix of the faces of the k-th prefracture zone.

These complex differential equations help to determine the forces q_{y_k} and $q_{x_k y_k}$ $(k = 1,2,\ldots,N)$ in the bonds between the faces of corresponding prefracture zones.

To construct the missing algebraic equations for finding approximate values of forces $q_{y_k}(t_m)$ and $q_{x_k y_k}(t_m)$ at the nodal points, we behave as in the case of a single prefracture zone. As a result we get a complex algebraic system of $N\times M$ equations for determining the values $q_{y_k}(t_m)$, $q_{x_k y_k}(t_m)$ $(k = 1,2,\ldots,N;\ m = 1,2,\ldots,M)$ at the nodal points of the prefracture zone. For completeness of the obtained algebraic equations we need $2\times N$ complex equations determining the coordinates of vertices of prefracture bands.

The solution of the system of singular integral equations is sought in the class of everywhere bounded functions (stresses). Therefore, it is necessary to add to the system (32), the stress boundedness conditions at the ends of prefracture bands $x_k = \pm l_k$. These $2N$ conditions are of the form

$$\sum_{m=1}^{M}(-1)^{M+m} g_k(t_m)\tan\frac{2m-1}{4M}\pi = 0$$

$$(k = 1,2,\ldots,N) \tag{34}$$

$$\sum_{m=1}^{M}(-1)^{m} g_k(t_m)\cot\frac{2m-1}{4M}\pi = 0$$

The obtained resolving joined system of algebraic equations with respect to c_k, d_k, A_k, $g_k(t_m)$, $q_{y_k}(t_m)$, $q_{x_k y_k}(t_m)$ $(m = 1,2,\ldots,M)$ allows under the given external load to obtain stress-strain state of the composite in the presence of arbitrary number of prefracture bands in the binder.

The joined resolving system of equations, even for linear elastic bonds because of unknown quantities l_k $(k = 1,2,\ldots,N)$ turned to be nonlinear. To solve it, we use the method of successive approximations.

To study the critical equilibrium, when a crack appears, we use relation (4) and have

$$\frac{1+\kappa}{2\mu}\frac{\pi l_k}{M}\sqrt{A_k^2 + B_k^2} = \delta_{cr}$$

$$(k = 1,2,\ldots,N)$$

where

$$A_k = \sum_{m=1}^{M_{1k}} v_k^0(t_m), \qquad B_k = \sum_{m=1}^{M_{1k}} u_k^0(t_m) \tag{35}$$

M_{1k} is the number of nodal points contained in the interval $\left(-l_k, x_k^0\right)$.

Analysis of the model of crack nucleation in the binder of the composite in the loading process is reduced to parametric joint investigation of resolving algebraic system of equations, finite-difference analogue of condition (33) and crack nucleation criterion (35) for different values of free parameters of the composite. These are different geometrical and mechanical characteristics of the materials of the binder and fiber.

5. Interaction of prefracture zones and crack with interfacial bonds

At some stage of loading, simultaneous existence in the binder the prefracture zones and cracks with interfacial bonds, is possible.

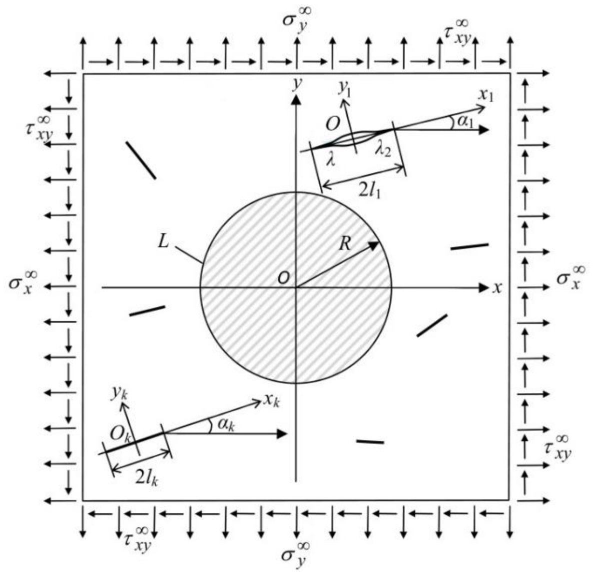

Fig. 7 Calculation scheme of the interaction of prefracture zones and a single crack in a fibrous composite

Assume that the binder near the fiber has a crack of length $2l_{01}$ and $(N-1)$ rectilinear prefracture zones. Interaction of prefracture zones in the vicinity of the crack may lead to loss of stability of the crack and appearance of new cracks with interfacial bonds. It is accepted that the sizes of end zones of the crack are comparable with the crack's length.

At the centers of prefracture zones and bridged crack we locate the origin of local systems of coordinates $x_kO_ky_k$, the axes x_k $(k = 1,2,\ldots,N)$ coincide with prefracture zones and make angles α_k with the axis x (Fig. 7).

The axis x_1 coincides with the line of the bridged crack and makes the angle α_1 with the axis x. Based on what has been said earlier, the conditions on the interface of the materials have the form (1), and on the faces of bridged crack and prefracture zones:

$$\sigma_{y_1} = 0\,, \qquad \tau_{x_1y_1} = 0\,, \qquad \lambda_1 < x_1 < \lambda_2$$

$$\sigma_{y_1} = q_{y_1}\,, \qquad \tau_{x_1y_1} = q_{x_1y_1}\,, \qquad -l_1 \le x_1 \le \lambda_1 \quad \text{and} \quad \lambda_2 \le x_1 \le l_1 \tag{36}$$

$$\sigma_{y_k} = q_{y_k}\,, \quad \tau_{x_ky_k} = q_{x_ky_k} \quad \text{on} \quad L_k \quad (k = 2,3,\ldots,N)$$

where L_k are the faces of the k-th prefracture zone.

We write the boundary conditions (36) on the crack faces with end zones and prefracture zones, by means of the Kolosov-Muskhelishvili unknown formulas (Muskhelishvili 1977) for complex potentials $\Phi(z)$ and $\Psi(z)$ in the form

$$\Phi(x_k) + \overline{\Phi(x_k)} + x_k\overline{\Phi'(x_k)} + \overline{\Psi(x_k)} = F_k \qquad (k = 1,2,\ldots,N) \tag{37}$$

where

$$F_1 = \begin{cases} 0 & \text{on the crack faces} \\ q_{y_1} - iq_{x_1y_1} & \text{on the faces of end zones of crack} \end{cases} \tag{38}$$

$$F_k = q_{y_k} - iq_{x_ky_k} \qquad (k = 2,3,\ldots,N)$$

x_k is the affix of the faces of the k-th prefracture zone.

We look for complex potentials $\Phi(z)$ and $\Psi(z)$ in the form (13) and (28). Satisfying by the function (13) and (28) the boundary conditions (37) and (38), we get the system of N complex singular integral equations of type (30), where $F_n(x)$ are determined by formulas (38). It is necessary to add to the system of singular integral equations of type (30), additional equalities of type (31), expressing condition of uniqueness of displacements when tracing the contour of the crack with end zones and prefracture zones. Under additional conditions of type (31), the system of singular integral equations of type (30), by means of algebraization procedure (Panasyuk *et al*. 1976, Mirsalimov 1987) is reduced to the system of $N\times M$ complex algebraic equations for finding $N\times M$ unknowns $g_k(t_m)$ $(k = 1,2,\ldots,N;\ m = 1,2,\ldots,M)$ of type (32). The right hand sides of algebraic system of equations (32) contain unknown values of normal q_{y_k} and tangential $q_{x_ky_k}$ forces at the nodal points contained in prefracture zones and end zones of crack. By means of the obtained solution we have N complex differential equations of type (33). They help to determine the forces q_{y_k} and $q_{x_ky_k}$ in the bonds of corresponding zones. To construct missing algebraic equations for determining the forces $q_{y_k}(t_m)$ and $q_{x_ky_k}(t_m)$ at the nodal points, we behave as in the previous case, i.e. we use the finite differences method. As a result we get a complex algebraic system of $N\times M$ equations for determining $q_{y_k}(t_m)$, $q_{x_ky_k}(t_m)$ at the nodal points of the crack's end zones and prefracture zones. Herewith we take into account the boundary conditions

$$q_{y_k}(\pm l_k) = 0\,, \qquad q_{x_ky_k}(\pm l_k) = 0$$

corresponding to the conditions

$$v_k^+(\pm l_k, 0) - v_k^-(\pm l_k, 0) = 0\,,$$

$$u_k^+(\pm l_k, 0) - u_k^-(\pm l_k, 0) = 0$$

In the case under consideration, for completeness of the obtained algebraic equations we need $2\times N$ complex equations allowing to determine the coordinates of vertices (location) of prefracture zones and crack's end zones. As the stresses in the composite are everywhere bounded, then the solution of the system of singular integral equations should be sought in the class of everywhere bounded functions (stresses). Therefore, to the system it is necessary to add the conditions of stress boundeness on the ends of end zones of the crack and prefracture zone of type (34). These $2N$ complex equations help to determine the coordinates of vertices of prefracture zones and end zones of crack.

Because of unknown sizes of prefracture zones and crack's end zones even for linear-elastic bonds the algebraic system (32)-(36) is nonlinear. To solve it, the method of successive approximations is used.

The obtained joined algebraic system with respect to c_k,

d_k, A_k, $g_k(t_m) = v_k^0(t_m) - iu_k^0(t_m)$ (k = 1,2,…,N; m = 1,2,…,M) allows under the given external load to find the stress-strain state of the composite in the presence of bridged crack with end zones and arbitrary number of prefracture zones in the binder. The numerical solution of the obtained joined system of algebraic equations allows to find approximate values of coefficients c_k, d_k, A_k, the values of functions $v_k^0(t_m)$, $u_k^0(t_m)$, $q_{y_k}(t_m)$, $q_{x_k y_k}(t_m)$ and coordinates (location) of crack with end zones and prefracture zones. The algorithm for solving algebraic systems is similar to the previous case.

Using the solution of the problem, we calculate the opening of the faces of a crack with end zones and of prefracture zones

$$v_1^+(x_1,0) - v_1^-(x_1,0) = \Pi_y(x_1,\sigma_1) q_{y_1}(x_1)$$
$$u_1^+(x_1,0) - u_1^-(x_1,0) = \Pi_x(x_1,\sigma_1) q_{x_1 y_1}(x_1) \tag{39}$$

$$-\frac{1+\kappa}{2\mu}\frac{\pi d_k}{M}\sum_{m=1}^{M_{1k}} g_k(t_m) = v_k(x_{0k},0) - iu_k(x_{0k},0) \tag{40}$$
$$(k = 1,2,\ldots,N)$$

where M_{1k} is the number of nodal points contained in the interval $(-l_k, x_{0k})$.

The condition of critical opening of crack faces at the base of the end zone, will be

$$\left|\Pi_y(\lambda_1,\sigma_1(\lambda_1)) q_{y_1}(\lambda_1) - i\Pi_x(\lambda_1,\sigma_1(\lambda_1)) q_{x_1 y_1}(\lambda_1)\right| = \delta_c$$
$$\text{for } x_1 = \lambda_1$$
$$\left|\Pi_y(\lambda_2,\sigma_1(\lambda_2)) q_{y_1}(\lambda_2) - i\Pi_x(\lambda_2,\sigma_1(\lambda_2)) q_{x_1 y_1}(\lambda_2)\right| = \delta_c \tag{41}$$
$$\text{for } x_1 = \lambda_2$$

where δ_c is a characteristic of the fracture toughness of the binder's material.

The obtained solution of the problem enables to predict appearance of new cracks in the binder's material. For that, as before, it is necessary to complement the problem statement with criterion of crack initiation (breaking of interparticle bonds of the material) of type (4).

Using the obtained solution, the limit condition may be written in the form

$$V_{0\kappa}(x_k^*) = \frac{1+k}{2\mu}\frac{\pi d_k}{M}\sqrt{A_k^2 + B_k^2} = \delta_{cr}$$
$$(k = 1,2,\ldots,N) \tag{42}$$
$$A_k = \sum_{m=1}^{M_{1k}} v_k(t_m), \quad B_k = \sum_{m=1}^{M_{1k}} u_k(t_m)$$

where x_k^* is the coordinate of the prefracture zone point at which breaking of the material's bonds occurs; M_{1k} is the number of nodal points in the interval $(-l_k, x_k^*)$.

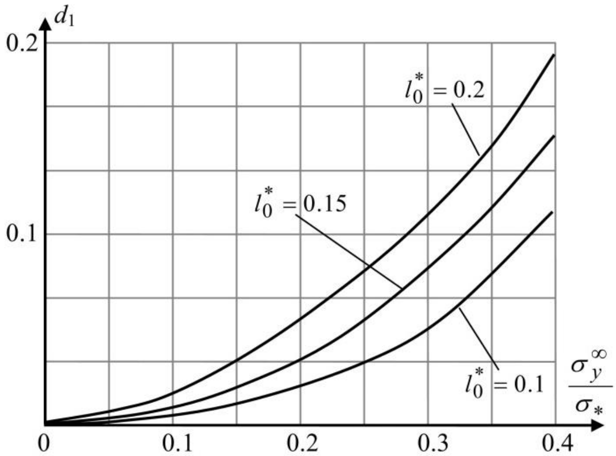

Fig. 8 Dependence of the length of crack end zone on value of the external load for different values of crack length

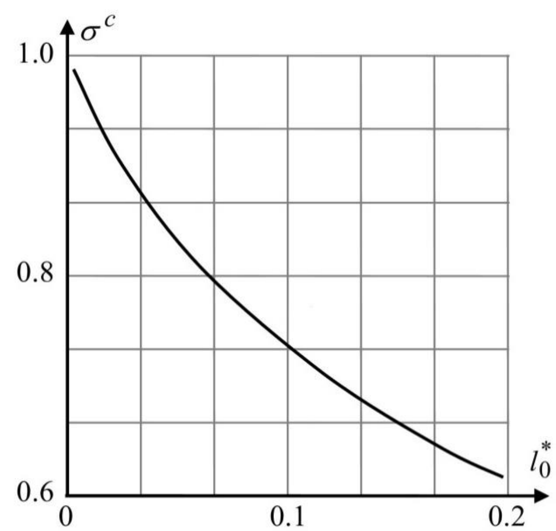

Fig. 9 Dependence of critical tensile load σ^c on dimensionless crack length under the found angle of orientation $\alpha_1 = 45^\circ$

These additional conditions allow to establish the parameters of the composite, under which new cracks appear in the binder.

Dependence of the length of the end zone of the crack $d_1 = (l_1 - \lambda_1)/R$ on the value of the external load $\sigma_y^\infty / \sigma_*$ for different values of crack length $l_0^* = (\lambda_2 - \lambda_1)/R$ for $\alpha_1 = 45^\circ$, $z_1^0 = 1.3Re^{i\pi/8}$ is depicted in Fig. 8.

Fig. 9 represents the dependence of critical tensile load $\sigma^c = \sigma_y^\infty / \sigma_*$ on dimensionless length of the crack l_0^* under the found angle of orientation $\alpha_1 = 45^\circ$.

Analysis of the model of a crack with bonds between the faces in the binder, weakened with arbitrary number of prefracture zones (damages) in the loading process, is reduced to parametric joint investigation of resolving algebraic system and crack growth criterion (41) for different values of free parameters of the fibrous composite. Mechanical and geometrical characteristics of materials of the binder and fiber serve as free parameters of the composite.

6. Conclusions

The experience of using fibrous composites in constructions convincingly shows that at the design stage of composites it is necessary to take into account the cases when there may arise damages and cracks in the binder. The existing methods of strength analysis of constructions made of fibrous composite ignore this circumstance. This situation makes it impossible to design a composite with minimal material consumption with guaranteed reliability and durability. So, it is necessary to conduct limit analysis of the composite in order to establish the critical loads at which crack nucleation and crack growth in the binder occurs. The size of critical minimal prefracture zone at which a crack appears, is recommended to consider as design characteristics of the binder's material. Based on the suggested design model that takes into account the existence of damages (the zones of weakened interparticle bonds of the material) and cracks with end zones in the composite, we worked out a method for calculating the parameters of the composite, at which crack nucleation and crack growth occurs. Knowing the principal values of critical parameters of crack nucleation, crack growth, and influence of the properties of materials on them, one can reasonably manage the crack nucleation and crack growth phenomena through design and technological solutions at the design stage.

Numerical realization of the obtained resolving equations allows to solve practically important problems of design of composites:

- to assess guaranteed resource of a fibrous composite with regard to expected defects and loading conditions;
- to set up admissible level of defects and maximum value of workloads providing sufficient margin of reliability;
- to carry out the choice of material with necessary complex of static and cyclic characteristics of the fracture toughness.

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Nomenclatures

$g_k(x_k)$	Desired function characterizing opening of prefracture zone faces;
$i^2=$	-1;
l_k	Half length of k-th prefracture zone;
N	Number of prefracture zones in binder;
q_{y_k}, $q_{x_k y_k}$	Cohesive forces in interfacial bonds;
R	Fiber radius;
V_*	Value of opening of crack faces, at which a transition linear to nonlinear bonds deformation law takes place;
v_r, v_θ	Radial and tangential of vector displacements components in binder;
v_r^0, v_θ^0	Radial and tangential of vector displacements components in fiber;
$(v_k^+ - v_k^-)$, $(u_k^+ - u_k^-)$	Normal and tangential component of opening of faces of k-th prefracture zone;
x_k^*	Coordinate of prefracture zone point at which breaking of material's bonds occurs;
z_k^0	Complex coordinate of k-th prefracture zone;
α_k	Angle of k-th prefracture zone with axis x;
δ_{cr}	Characteristics of resistance of binder's material to crack nucleation;
δ_c	Characteristic of fracture toughness of binder's material;
κ, μ	Elastic constants of material of binder;

κ_0, μ_0	Elastic constants of material of fiber;
v	Poisson ratio;
σ_x, σ_y, τ_{xy}	Components of stress tensor;
σ_r, σ_θ, $\tau_{r\theta}$	Components stress tensor in binder;
σ_r^0, σ_θ^0, $\tau_{r\theta}^0$	Components stress tensor in fiber;
$\sigma_k = \sqrt{q_{y_k}^2 + q_{x_k y_k}^2}$	Modulus of force vector in bonds;
σ_*	Maximum elastic stresses in bonds;
σ^c	Tension of bonds corresponding to limit opening crack faces;
$\Pi_y(x_k,\sigma_k)$, $\Pi_x(x_k,\sigma_k)$	Effective compliances of bonds;
$\varphi(z)$, $\Psi(z)$ and $\varphi_0(z)$, $\Psi_0(z)$	Analytic functions for binder and fiber, respectively.