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A GN-based modified model for size-dependent coupled thermoelasticity analysis in nano scale, considering nonlocality in heat conduction and elasticity: An analytical solution for a nano beam with energy dissipation

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Abstract. This investigation deals with a size-dependent coupled thermoelasticity analysis based on Green-Naghdi (GN) theory in nano scale using a new modified nonlocal model of heat conduction, which is based on the GN theory and nonlocal Eringen theory of elasticity. In the analysis based on the proposed model, the nonlocality is taken into account in both heat conduction and elasticity. The governing equations including the equations of motion and the energy balance equation are derived using the proposed model in a nano beam resonator. An analytical solution is proposed for the problem using the Laplace transform technique and Talbot technique for inversion to time domain. It is assumed that the nano beam is subjected to sinusoidal thermal shock loading, which is applied on the one of beam ends. The transient behaviors of fields' quantities such as lateral deflection and temperature are studied in detail. Also, the effects of small scale parameter on the dynamic behaviors of lateral deflection and temperature are obtained and assessed for the problem. The proposed GN-based model, analytical solution and data are verified and also compared with reported data obtained from GN coupled thermoelasticity analysis without considering the nonlocality in heat conduction in a nano beam.

Keywords: nano-sized resonator; nonlocal heat conduction; Green-Naghdi theory; analytical solution; nonlocal coupled thermoelasticity; energy dissipation

1. Introduction

Recently, the micro-electro-mechanical systems (MEMS) and nano-electro-mechanical systems (NEMS) such as nanoresonators with ultra-high resonant frequencies are widely used in various industries and applications. Some of the main applications of MEMS/NEMS can be listed as: ultra-high sensitive sensing, molecular transportation, molecular separation, high-frequency signal processing, and biological imaging (Arash *et al.* 2015). The temperature and thermal effects have a main role in the operation of micro/nano structures such as micro/nano electromechanical systems (MEMS/NEMS). In micro/nano scale, when a device (such as resonators) is working, the temperature of body may be changed by the generated internal heat. Also, in some engineering applications, the micro/nano structures are subjected to thermal shock loading such as high rate laser pulses. It means that it is needed to consider the interactions between temperature field and elasticity field to have a realistic analysis of the structures. Regarding the thermoelasticity analysis in solids, there is a very important phenomenon, which is called the second sound effect in solids. This effect arises the thermal wave propagation with finite speed in solids. In this regards, there are some very well-known coupled thermoelasticity theories such as Lord-Shulman (LS) theory (Lord and Shulman 1967), Green-Lindsay (GL) theory (Green and Lindsay 1972) and Green-Naghdi (GN) theory (with and without energy dissipation) (Green and Naghdi 1992, Green and Naghdi, 1993). The concepts behind the above mentioned theories were explained in detail by Hetnarski and Eslami (2009) and Ignaczak and Ostoja-Starzewski (2010) in their books. The above mentioned theories are valid in macroscopic scale and there are some published works on the application of the above mentioned theories in macro scale such as application of GN theory (Hosseini *et al.* 2011, Abbas 2014, Hosseini 2014a,b, Ezzat and El-Bary 2017, Moradi-Dastjerdi and Payganeh 2017, Fang *et al.* 2017, Abouelregal and Zenkour 2017, Hosseini and Zhang 2018, Hosseini 2018). So, it is very important that the small scale effects are taken into account in heat conduction and also in thermoelasticity analysis.

There are so many published works with considering the small scale effects in the engineering calculations such as bending analysis of micro/nano beams (Akbaş 2016a, Akbaş 2018c), free and forced vibrations of micro/nano beams (Akbaş 2016b, Zakeri *et al.* 2016, Akbaş 2017a,b, Akbaş 2018a,b, Akbaş 2019), linear dynamic analysis in a nano beam (Abbondanza *et al.* 2016) and buckling analysis of axially pressurized nanotubes (Malikan 2019). Some advanced theories of beams and plates have been developed and used to illustrate the static, dynamic and thermal buckling problem in macro scale and also in micro and nano structures considering the small scale effects in the

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calculations (Bensaid *et al*. 2017, Bensaid and Guenanou 2017). The nonlocal strain gradient based higher order refined beam models has also developed for size dependent dynamic analysis taking into account the interaction with elastic foundation (Bensaid *et al*. 2018a). A new and simple HSDT was proposed by Bensaid *et al*. (2018a) for nonlinear thermal stability responses in FG plates. Thermal stability analysis of nano beams with considering small scale effects and thermal buckling analysis in FG-CNT reinforced composites resting on elastic foundations have been studied in 2018 and 2019 using an effective solution method (Bensaid and Bekhadda 2018, Bensaid and Kerboua 2019).

Some of the existing researches have been focused on the considering the small scale effect in the heat conduction. It is very important to know the behavior of heat flow and thermal wave in solids in nano scale. In this regards, there are some research works in which some heat conduction models were proposed with considering the small scale effects such as (Guyer and Krumhansl 1966, Sobolev 1994, Tzou 1996, Tzou and Guo 2010, Ma 2012). In some of the previous works, the small scale effect was consider in heat conduction by modification the classical heat conduction equations (Sobolev 1994, Tzou 1996), the dual-phase-lag (DPL) model of heat conduction (Tzou and Guo 2010), by developing a model for prediction of the effective thermal conductivity (Ma 2012), a model based on the phonon gas dynamics (Dong *et al.* 2014) and using Suykens's nonlocal-in-time kinetic energy method (El-Nabulsi 2018) or deriving the laws of thermodynamics in terms of internal powers for nonlocal materials (Fabrizio 2011). Recently, the heat conduction of nanofluid has been studied by some researchers (Rana *et al*. 2016, Olofinkua 2018, Ansari *et al.* 2018) using the above mentioned theory and methods. By reviewing the previous published works and also the above mentioned papers and the cited works by them, it can be clearly concluded that the heat conduction models with considering the size effects have been developed for employing in the analysis of nano structures.

Eringen (1974) proposed the linear theory of nonlocal thermoelasticity based on his theory on nonlocal elasticity in 1972 (Eringen and Edelen 1972) and then Balta and Suhubi (1977) modified Eringen's model for formulating the nonlocal thermoelasticity with considering the temperature rate. The nonlocal theory in thermoelasticity was developed by researchers for various uncoupled and coupled thermoelasticity analysis such as sensitivity analysis of nonlocal classical coupled thermoelasticity (Meriç 1988), nonlocal thermoelastic wave propagation in plates (Inan and Eringen 1991), proving a reciprocity theorem and a uniqueness theorem for nonlocal uncoupled and classical coupled thermoelasticity (Dhaliwal and Jun, 1994), proposing the strategies based on a unified thermodynamic framework for nonlocal thermoelasticity analysis (Polizzotto 2003) and also based on the irreversible thermodynamic (Jou *et al.* 2010a,b).

Some coupled thermoelasticity analysis have been carried out in nano scale by some researchers considering the small scale effect only in elasticity field (Bougoffa *et al.* 2010, Zenkour *et al.* 2014, Berezovski *et al.* 2014, Zenkour and Abouelregal 2014, Yu *et al.* 2015, Zenkour *et al.* 2015, Rezazadeh *et al.* 2015, Zenkour 2017, Ebrahimi and Haghi 2017, Zenkour 2018, Bostani and Karami Mohammadi 2018). The analysis were based on the application of some coupled thermoelasticity theories such as classical theory (Bougoffa *et al.* 2010), DPL (Zenkour *et al.* 2014), Piola–Kirchhoff formulation (Berezovski *et al.* 2014), LS theory for FG nanobeams (Zenkour and Abouelregal 2014), Caputo fractional derivative (Yu *et al.* 2015), GN theory without energy dissipation for a nano beam (Zenkour *et al.* 2015). In the above mentioned published works, the size effect was considered only in elasticity field. Some of them employed the Eringen nonlocal elasticity theory to take into account the small scale effect as nonlocality in the formulations (Zenkour *et al.* 2014, Zenkour and Abouelregal 2014, Zenkour *et al.* 2015).

For coupled thermoelasticity analysis of nano structures such as nano beam resonators subjected to shock loading, some theories such as the LS and GN (without energy dissipation) theories without considering small scale effect in heat conduction were employed to assess the thermoelastic wave and thermoelastic damping in nano structures (Rezazadeh *et al.* 2015, Zenkour 2017, Ebrahimi and Haghi 2017, Zenkour 2018, Bostani and Karami Mohammadi 2018, Tan and Chen 2019, Barretta *et al.* 2018, Ebrahimi *et al.* 2017). It is very important to note that the nano size effect has a main role in the thermoelastic wave propagation analysis in nano structures. Ardito *et al*. (2008a, b) studied on the effects of small scale on the thermoelastic damping in MEMS by introducing the nonlocal coupled thermoelasticity based on the Zener's formula and nonlocal Eringen theory of elasticity. They concluded that the considering nonlocality in elasticity not sufficient to quantitatively explain the damping MEMS. So, it means that the considering of small scale effect only in elasticity field doesn't give a realistic behaviors of temperature field influenced by small scale effect. It is needed to propose a heat conduction model to take into account the size effect.

There are some very well-known model for considering the small scale effect in elasticity field to derive the governing equations. Some of them have been successfully used for coupled and uncoupled thermoelasticity of nano structures such as Eringen nonlocal theory (Zenkour *et al.* 2014, Zenkour and Abouelregal 2014, Zenkour *et al.* 2015, Rezazadeh *et al.* 2015, Zenkour 2017, Ebrahimi and Haghi 2017, Zenkour 2018, Bostani and Karami Mohammadi 2018, Ardito *et al.* 2008a,b, Zenkour and Abouelregal 2019), modified strain gradient elasticity (Bostani and Karami Mohammadi 2018), the modified couple stress theory (Ebrahimi *et al.* 2017, Tan and Chen 2019, Kumar and Devi 2017) and stress-driven nonlocal integrals (Barretta *et al.* 2018). Using the existing models for considering the small scale effect in elasticity, some researchers proposed some models for considering the small scale effect in uncoupled and coupled thermoelasticity. Recently, Yu *et al*. (2016, 2017) developed the proposed heat conduction model considering the small scale effect by Guyer and Krumhansl (1966) to derive the formulations for uncoupled and classical coupled thermoelasticity (Yu *et al.* 2016) and the

Zener, Lifshitz, and Roukes' damping models for the analysis of Size-dependent damping in a nanobeam (Yu *et al.* 2017).

By reviewing the published literatures such as the above mentioned papers on the coupled thermeolasticity analysis in nano scale, it can be clearly found that the size effect has been commonly considered only in elasticity. There are a few cases in which the size effect was taken into account in both elasticity and heat conduction based on the coupled thermoelasticity theories except the GN theory. The GN theory of coupled thermoelasticity was employed for the analysis of coupled thermoelasticity in nanobeam by considering the small scale effect only in elasticity (Zenkour *et al.* 2015, Zenkour 2017, Zenkour and Abouelregal 2019). In this regards, Hosseini (2018) proposed an analytical solution for coupled thermoelasticity analysis in a heat-affected nano beam resonator based on GN theory with considering the nonlocality only in elasticity. So, the coupled thermoelasticity analysis in nano scale based on GN theory with considering small scale effect in both elasticity and heat conduction can be considered as a new research topic.

By reviewing the published papers in this subject that some of them were reviewed in the previous paragraphs, it can be concluded that some analytical solutions have been used for size dependent elasticity and thermoelasticity analysis in which the small scale effect has been taken into account only in elasticity field. The employed analytical solutions based on some theories for considering small scale effect such as nonlocal Eringen theory, modified couple stress theory and other ones are effective methods for the assumed boundary conditions. In the thermoelasticity analysis of micro/nano sized structures, there is a research gap on the size dependent coupled thermoelasticity analysis based on Green–Naghdi (GN) theory. In other words, the carried out studies on the size dependent coupled thermoelasticity analysis based on the Green-Naghdi theory take into account the size effect only in the equations of motion (elasticity field) without considering the size effect in heat conduction (without considering the size effect in energy balance equation).

In this paper, a new modified model based on GN theory and Eringen elasticity theory is proposed for nonlocal coupled thermoelasticity in nano scale with considering the nonlocality in both elasticity and heat conduction. The proposed model is employed for coupled thermoelasticity analysis in a nanobeam resonator subjected to thermal shock loading using an analytical solution. The effects of small scale parameter on the transient behaviors of lateral deflection and temperature fields have been studied in detail. In other words, the main differences and novel points of the proposed GN-based model in this research can be summarized as: a) considering the small scale effect in heat conduction by proposing a nonlocal model based on the GN theory with energy dissipation, b) developing an analytical solution to find the temperature and lateral deflection in the closed forms for a nano beam, c) studying on the effects of some parameters such as small scale parameter, height of nano beam and the intensity value of thermal shock loading on the transient behaviors of fields' quantities.

2. Governing equations based on nonlocal GN theory

The fundamental equations of the generalized coupled thermoelasticity based on Green – Naghdi theory (with energy dissipation) without considering nonlocality in the formulations can be written as (Green and Naghdi 1992, Green and Naghdi 1993),

The equation of motion:

$$\nabla.\boldsymbol{\sigma}+\rho\mathbf{b}=\rho\ddot{\mathbf{U}} \tag{1}$$

The strain-displacement relation:

$$\varepsilon=\frac{1}{2}\left(\nabla\mathbf{U}+(\nabla\mathbf{U})'\right) \tag{2}$$

The stress-strain relation (without considering size effects):

$$\boldsymbol{\sigma}=\mathbf{C}\varepsilon-\beta\theta \tag{3}$$

The energy balance equation:

$$\nabla.\mathbf{q}=R-T_0\dot{S} \tag{4}$$

The entropy relationship:

$$S=\left(\frac{\rho c}{T_0}\right)T+\beta:\varepsilon \tag{5}$$

The employed heat conduction equation (without considering size effects) in GN theory (type III) was given as (Green and Naghdi 1992, 1993, Hetnarski and Eslami 2009):

$$\dot{\mathbf{q}}=-\mathbf{K}\nabla\dot{\theta}-\mathbf{K}^*\nabla\theta \tag{6}$$

where ρ is mass density, σ is stress tensor, $\mathbf{U}$ is the displacement vector, $\mathbf{b}$ is the body force vector per unit mass, $\mathbf{q}$ is the heat flux vector, T_0 is the reference temperature, θ is defined as $\theta=\hat{T}-T_0$, which $\hat{T}$ is the absolute temperature, S is the entropy per unit volume, R is the internal heat source per unit volume per unit time, ε is the strain tensor, β is the second order tensor of stress-temperature moduli, $\mathbf{K}$ and $\mathbf{K}^*$ are the second order tensors of thermal conductivity and GN theory new material constant, $\mathbf{C}$ is the forth order tensor of elastic moduli and c is the specific heat. The del operator (∇), the superscript dot (.), (∇.) and the superscript prime ($'$) indicate the gradient of a function, the differentiation with respect to time, the divergence operator and the transpose of a matrix, respectively.

To have a realistic analysis of coupled thermoelasticity in micro/nano scale, it is very important to take into account the size effects in the calculations. In this regards, there are some models that have been proposed to consider the small scale effects in elasticity analysis such as nonlocal stress gradient elasticity model (Eringen 2002, Polizzotto 2014), strain gradient elasticity model (Aifantis 1999) and the modified couple stress theory Yang *et al*. (2002). One of the very well-known theories for considering the small scale

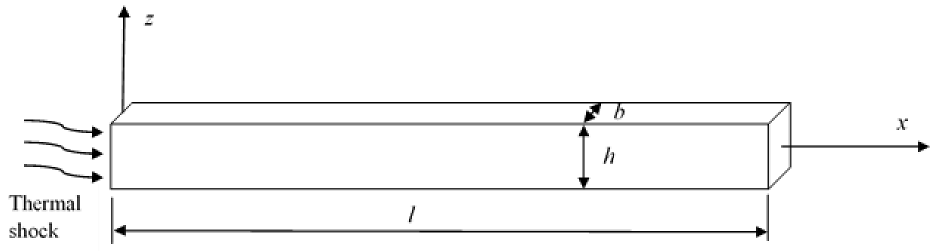

Fig. 1 The assumed nano beam in the problem, which is subjected to thermal shock loading

effects is nonlocal elasticity theory, which is based on the stress gradient and proposed by Eringen (2002). Also, it is very necessary to consider the small scale effects in heat conduction for coupled thermoelasticity analysis. Some researchers proposed some models for nonlocal heat conduction, which the small scale effects are taken into account by considering the nonlocality in heat conduction.

A modified model based on Green-Naghdi theory is proposed for the generalized coupled thermoelasticity in micro/nano scale by considering the nonlocality in both heat conduction and elasticity. To derive the governing equations of nonlocal coupled thermoelasticity, the Eqs. (3) and (6) should be rewritten as follows by considering small scale parameter (e_0a) (Eringen 2002).

$$\left[1-(e_0a)^2\nabla^2\right]\boldsymbol{\sigma}=\mathbf{C}\,\varepsilon-\beta\,\theta \tag{7}$$

The Eq. (7) was proposed by Eringen for nonlocal elasticity analysis (Eringen 2002). A new modified model based on the presented model for nonlocal heat conduction in the higher order simple material by Guyer and Krumhansl (1966), Polizzotto (2003) and Yu *et al*. (2016) is proposed and extended here for heat conduction in Green-Naghdi theory with energy dissipation as

$$\left[1-(e_0a)^2\nabla^2\right]\dot{\mathbf{q}}-2(e_0a)^2\nabla\nabla.\dot{\mathbf{q}}=-\mathbf{K}\nabla\dot{\theta}-\mathbf{K}^*\nabla\theta \tag{8}$$

For the stress and heat flux gradient type, which the condition $tr(\mathbf{Q})=0$ is satisfied (Yu *et al.* 2016) (the term **Q** is the flux of heat flux and $tr(\mathbf{Q})$ is the trace of matrix **Q**), the proposed model can be written as:

$$\left[1-(e_0a)^2\nabla^2\right]\dot{\mathbf{q}}=-\mathbf{K}\nabla\dot{\theta}-\mathbf{K}^*\nabla\theta \tag{9}$$

The Eqs. (8) and (9) are the new proposed model for heat conduction in GN theory of coupled thermoelasticity with considering small scale effect. The proposed model can be written for GN theory without energy dissipation for the higher order simple material as

$$\left[1-(e_0a)^2\nabla^2\right]\dot{\mathbf{q}}-2(e_0a)^2\nabla\nabla.\dot{\mathbf{q}}=-\mathbf{K}\nabla\dot{\theta} \tag{10}$$

or,

$$\left[1-(e_0a)^2\nabla^2\right]\mathbf{q}-2(e_0a)^2\nabla\nabla.\mathbf{q}=-\mathbf{K}\nabla\theta \tag{11}$$

and for the stress and heat flux gradient type as

$$\left[1-(e_0a)^2\nabla^2\right]\dot{\mathbf{q}}=-\mathbf{K}\nabla\dot{\theta} \tag{12}$$

or,

$$\left[1-(e_0a)^2\nabla^2\right]\mathbf{q}=-\mathbf{K}\nabla\theta \tag{13}$$

The Eqs. (11) and (13) are the proposed and extended models by Guyer and Krumhansl (1966), Polizzotto (2003) and Yu *et al*. (2016) for heat conduction with considering small scale effects.

3. Nonlocal GN-based coupled thermoelasticity for micro/nano beam resonator

The governing equations of coupled thermoelasticity for a micro/nano beam resonator with considering nonlocality only in elasticity was proposed by Hosseini (2018). In the presented problem (Hosseini 2018), a micro/nano beam resonator with length l, width b and thickness h was considered with the x, y and z axes, which are defined along the longitudinal, width and thickness directions as $0\le x\le l$, $-\frac{b}{2}\le y\le\frac{b}{2}$ and $-\frac{h}{2}\le z\le\frac{h}{2}$, respectively (see Fig. 1). So, in this paper, the governing equations are derived for coupled thermoelasticity based on GN theory for the above mentioned micro/nano beam resonator with considering nonlocality in both heat conduction and elasticity. The displacement field based on the nonlocal Ryleigh beam theory (NRBT) is given by Hosseini (2018):

$$\begin{aligned} &u_x(x,y,z)=-z\frac{\partial w(x,t)}{\partial x}\quad, \\ &u_y(x,y,z)=0\quad,\quad u_z(x,y,z)=w(x,t) \end{aligned} \tag{14}$$

where t stands for the time. The governing equation of motion in terms of the nonlocal bending moment $M^{nl}(x,t)$ and deflection is (Kiani 2015, Hosseini 2018):

$$\rho\left(A\frac{\partial^2 w(x,t)}{\partial t^2}-I\frac{\partial^4 w(x,t)}{\partial t^2\partial x^2}\right)-\frac{\partial^2\left(M^{nl}(x,t)\right)}{\partial x^2}=0 \tag{15}$$

The nonlocal constitutive stress-displacement relation can be obtained from Eq. (7) as follow (Kiani 2015):

$$\begin{aligned} &\sigma_{xx}^{nl}(x,z,t)-\left(e_0a\right)^2\frac{\partial^2\sigma_{xx}^{nl}(x,z,t)}{\partial x^2} \\ &=-Ez\frac{\partial^2 w(x,t)}{\partial x^2}-\gamma_T\theta(x,z,t) \end{aligned} \tag{16}$$

where

$$\gamma_T = E\,\alpha_T \big/ (1-2\nu),$$

$$\theta(x,z,t) = \hat{T}(x,z,t) - T_0$$

and α_T is the thermal expansion coefficient. The term $\sigma_{xx}^{nl}(x,z,t)$ stands for the nonlocal stresses. The terms *A*, *I*, *ρ*, *E* and ***v*** are the cross-sectional area, second area moment of inertia, density, elastic modulus and Poison's ratio of the assumed micro/nano beam resonator, respectively. The Eq. (16) can be written in terms of thermal and bending moment as (Hosseini 2018):

$$M^{nl}(x,t) - (e_0 a)^2 \frac{\partial^2 M^{nl}(x,t)}{\partial x^2} = -EI \frac{\partial^2 w(x,t)}{\partial x^2} - \gamma_T M_T(x,t) \quad (17)$$

where,

$$M_T(x,t) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \theta(x,z,t).z.dA \quad (18)$$

Using Eqs. (11) and (13), it is obtained that:

$$\begin{aligned} &EI\frac{\partial^4 w}{\partial x^4} - \rho\left[A(e_0 a)^2 + I\right]\frac{\partial^4 w}{\partial t^2 \partial x^2} + \rho (e_0 a)^2 I \frac{\partial^6 w}{\partial t^2 \partial x^4} \\ &+\rho A \frac{\partial^2 w}{\partial t^2} + \gamma_T \frac{\partial^2 M_T}{\partial x^2} = 0 \end{aligned} \quad (19)$$

From Eq. (4), the energy balance equation with the absence of *R* (the internal heat source per unit volume per unit time) is:

$$\nabla.\mathbf{q} = -T_0\,\dot{S} \quad (20)$$

or,

$$\nabla.\dot{\mathbf{q}} = -T_0\,\ddot{S} \quad (21)$$

It is assumed that the micro/nano beam resonator is very thin and there is no any heat flow across the upper and lower surfaces of the beam and also the heat flux is only along longitudinal direction without any variations along other directions. Substituting the Eq. (5) into (21) and using the Eqs. (9), the energy balance equations based on GN theory with energy dissipation for the assumed micro/nano beam resonators with considering the nonlocality in heat conduction can be obtained as

$$\begin{aligned} &\kappa^*\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial z^2}\right) + k\left(\frac{\partial^2\dot{\theta}}{\partial x^2} + \frac{\partial^2\dot{\theta}}{\partial z^2}\right) \\ &= \rho c\left[\ddot{\theta} - (e_0 a)^2 \frac{\partial^2\ddot{\theta}}{\partial x^2}\right] - \gamma_T T_0\, z\left[\frac{\partial^2 \ddot{w}}{\partial x^2} - (e_0 a)^2 \frac{\partial^4 \ddot{w}}{\partial x^4}\right] \end{aligned} \quad (22)$$

where

$$\mathbf{K} = k\,\mathbf{I} \qquad \text{or,} \qquad \mathrm{K}_{ij} = k\,\delta_{ij} \quad (23)$$

$$\mathbf{K}^* = \kappa^*\mathbf{I} \qquad \text{or,} \qquad \mathrm{K}^*_{ij} = \kappa^*\,\delta_{ij} \quad (24)$$

The term **I** is the identity tensor. The temperature increment θ (*x*,*z*,*t*) can be assumed to vary along the thickness direction of very thin nano beam in term of sin(*pz*) (Elsibai 2011), where $p = \frac{\pi}{h}$:

$$\theta(x,z,t) = T(x,t)\sin(pz) \quad (25)$$

By substitution the Eq. (25) into Eqs. (19) and (22) and integration form both equations, the following equations can be obtained as:

$$\begin{aligned} &EI\frac{\partial^4 w}{\partial x^4} - \rho\left[A(e_0 a)^2 + I\right]\frac{\partial^2 \ddot{w}}{\partial x^2} + \rho(e_0 a)^2 I \frac{\partial^4 \ddot{w}}{\partial x^4} \\ &+\rho A\ddot{w} + \gamma_T \frac{\partial^2 T}{\partial x^2}\int_{-\frac{h}{2}}^{\frac{h}{2}} z\sin(pz)\,dA = 0 \end{aligned} \quad (26)$$

$$\begin{aligned} &\kappa^*\frac{\partial^2 T}{\partial x^2}\int_{-\frac{h}{2}}^{\frac{h}{2}} z\sin(pz)\,dA - \kappa^* T\,p^2\int_{-\frac{h}{2}}^{\frac{h}{2}} z\sin(pz)\,dA \\ &+k\frac{\partial^2 \dot{T}}{\partial x^2}\int_{-\frac{h}{2}}^{\frac{h}{2}} z\sin(pz)\,dA - k\dot{T}\,p^2\int_{-\frac{h}{2}}^{\frac{h}{2}} z\sin(pz)\,dA \\ &= \rho c\ddot{T}\int_{-\frac{h}{2}}^{\frac{h}{2}} z\sin(pz)\,dA - \rho c(e_0 a)^2\frac{\partial^2\ddot{T}}{\partial x^2}\int_{-\frac{h}{2}}^{\frac{h}{2}} z\sin(pz)\,dA \\ &-\gamma_T T_0\frac{\partial^2 \ddot{w}}{\partial x^2}\int_{-\frac{h}{2}}^{\frac{h}{2}} z^2\,dA + \gamma_T T_0 (e_0 a)^2\frac{\partial^4 \ddot{T}}{\partial x^4}\int_{-\frac{h}{2}}^{\frac{h}{2}} z^2\,dA \end{aligned} \quad (27)$$

or,

$$\begin{aligned} &EI\frac{\partial^4 w}{\partial x^4} - \rho\left[A(e_0 a)^2 + I\right]\frac{\partial^2 \ddot{w}}{\partial x^2} \\ &+\rho(e_0 a)^2 I\frac{\partial^4 \ddot{w}}{\partial x^4} + \rho A\ddot{w} + \frac{2\gamma_T b}{p^2}\frac{\partial^2 T}{\partial x^2} = 0 \end{aligned} \quad (28)$$

$$\begin{aligned} &\kappa^*\frac{\partial^2 T}{\partial x^2} - \kappa^* T\,p^2 + k\frac{\partial^2 \dot{T}}{\partial x^2} - k\dot{T}\,p^2 \\ &= \rho c\ddot{T} - \rho c(e_0 a)^2\frac{\partial^2 \ddot{T}}{\partial x^2} \\ &-\frac{p^2\gamma_T T_0 I}{2b}\frac{\partial^2 \ddot{w}}{\partial x^2} + (e_0 a)^2\frac{p^2\gamma_T T_0 I}{2b}\frac{\partial^4 \ddot{w}}{\partial x^4} \end{aligned} \quad (29)$$

The Eqs. (28) and (29) are the governing equations of size-dependent coupled thermoelasticity in a micro/nano beam resonator based on the GN theory with considering nonlocality in both heat conduction and elasticity. The Eq.

(29) is derived using the proposed new modified model for GN-based heat conduction with considering small scale effects.

To find an analytical solution, the Laplace transformation technique is employed in respect to time. The homogenous initial conditions for all of field's variables are supposed and the transformed governing equations are:

$$EI\frac{\partial^4\bar{w}}{\partial x^4}-\rho\left[A\left(e_0a\right)^2+I\right]s^2\frac{\partial^2\bar{w}}{\partial x^2}+\rho\left(e_0a\right)^2 I\,s^2\frac{\partial^4\bar{w}}{\partial x^4}+\rho As^2\bar{w}+\frac{2\gamma_T b}{p^2}\frac{\partial^2\bar{T}}{\partial x^2}=0 \tag{30}$$

$$\kappa^*\frac{\partial^2\bar{T}}{\partial x^2}-\kappa^* p^2\bar{T}+k\,s\frac{\partial^2\bar{T}}{\partial x^2}-k\,p^2 s\bar{T}=\rho c\,s^2\bar{T}-\rho c\left(e_0a\right)^2 s^2\frac{\partial^2\bar{T}}{\partial x^2}-\frac{p^2\gamma_T T_0 I}{2b}s^2\frac{\partial^2\bar{w}}{\partial x^2}+\frac{p^2\gamma_T T_0 I}{2b}\left(e_0a\right)^2 s^2\frac{\partial^4\bar{w}}{\partial x^4} \tag{31}$$

where the terms $\bar{w}$ and $\bar{T}$ can be defined by the Laplace operator $\Im(\ )$ as:

$$\Im\left[T\left(x,t\right)\right]=\bar{T}\left(x,s\right)=\int_0^\infty T\left(x,t\right)e^{-st}dt \tag{32}$$

$$\Im\left[w\left(x,t\right)\right]=\bar{w}\left(x,s\right)=\int_0^\infty w\left(x,t\right)e^{-st}dt \tag{33}$$

Using the proposed non-dimensional parameters by Hosseini (2018), the governing equations can be rewritten in dimensionless forms.

$$\hat{x}=\frac{x}{l},\ \hat{z}=\frac{z}{h},\ \hat{\bar{w}}=\frac{\bar{w}}{h},\ \hat{\bar{u}}=\frac{\bar{u}}{l},\ \hat{t}=\frac{V^*t}{l},\quad \hat{M}_T=\frac{M_T}{T_0\,b\,h^2},\ \hat{\bar{T}}=\frac{\bar{T}}{T_0},\ \hat{\bar{\sigma}}=\frac{\bar{\sigma}}{E} \tag{34}$$

The term V^* stands for the reference velocity, which can be defined as $V^*=\sqrt{\frac{E}{\rho}}$. The Eqs. (30) and (31) in the dimensionless forms are:

$$\frac{\partial^4\hat{\bar{w}}}{\partial\hat{x}^4}+B_1 s^2\frac{\partial^4\hat{\bar{w}}}{\partial\hat{x}^4}+B_2 s^2\frac{\partial^2\hat{\bar{w}}}{\partial\hat{x}^2}+B_3 s^2\hat{\bar{w}}+B_4\frac{\partial^2\hat{\bar{T}}}{\partial\hat{x}^2}=0 \tag{35}$$

$$\varepsilon\left(\frac{\partial^2\hat{\bar{T}}}{\partial\hat{x}^2}-B_5\hat{\bar{T}}\right)+\left(s\frac{\partial^2\hat{\bar{T}}}{\partial\hat{x}^2}-s\,B_5\hat{\bar{T}}\right)=B_6 s^2\hat{\bar{T}}-B_7 s^2\frac{\partial^2\hat{\bar{T}}}{\partial\hat{x}^2}-B_8 s^2\frac{\partial^2\hat{\bar{w}}}{\partial\hat{x}^2}+B_9 s^2\frac{\partial^4\hat{\bar{w}}}{\partial\hat{x}^4} \tag{36}$$

$$B_1=\frac{\left(e_0a\right)^2}{l^2},\ B_2=-\left(1+12\frac{\left(e_0a\right)^2}{h^2}\right),\quad B_3=\frac{12l^2}{h^2},\ B_4=\frac{24l^2\gamma_T T_0}{h^2\pi^2E},\ \varepsilon=\frac{l\kappa^*}{V^*k}, \tag{37}$$

$$B_5=\frac{\pi^2l^2}{h^2},\ B_6=\frac{\rho c_v V^* l}{k},\ B_7=\frac{\rho c_v V^*}{k}\frac{\left(e_0a\right)^2}{l},\quad B_8=\frac{h^2\pi^2\gamma_T V^*}{24kl},\ B_9=\frac{h^2\pi^2\gamma_T V^*}{24kl^3}\left(e_0a\right)^2 \tag{38}$$

For the sake of brevity, the hat (^) is omitted for dimensionless variables in Eqs. (35) and (36) as:

$$\frac{\partial^4\bar{w}}{\partial x^4}+B_1 s^2\frac{\partial^4\bar{w}}{\partial x^4}+B_2 s^2\frac{\partial^2\bar{w}}{\partial x^2}+B_3 s^2\bar{w}+B_4\frac{\partial^2\bar{T}}{\partial x^2}=0 \tag{39}$$

$$\varepsilon\left(\frac{\partial^2\bar{T}}{\partial x^2}-B_5\bar{T}\right)+\left(s\frac{\partial^2\bar{T}}{\partial x^2}-s\,B_5\bar{T}\right)=B_6 s^2\bar{T}-B_7 s^2\frac{\partial^2\bar{T}}{\partial x^2}-B_8 s^2\frac{\partial^2\bar{w}}{\partial x^2}+B_9 s^2\frac{\partial^4\bar{w}}{\partial x^4} \tag{40}$$

4. Analytical solution

Hosseini (2018) proposed an analytical solution for coupled thermoelasticity in a micro/nano beam resonator with considering only the nonlocality in elasticity. Using the proposed approach by Hosseini (2018), it is possible to find another analytical solution for the current problem. From Eq. (40), it is possible to obtain the second derivative of temperature in terms of temperature and the second and fourth derivatives of deflection as:

$$\frac{\partial^2\bar{T}}{\partial x^2}=\eta_1\left(s\right)\bar{T}-\eta_2\left(s\right)\frac{\partial^2\bar{w}}{\partial x^2}+\eta_3\left(s\right)\frac{\partial^4\bar{w}}{\partial x^4} \tag{41}$$

where

$$\eta_1\left(s\right)=\frac{\varepsilon B_5+sB_5+s^2B_6}{\varepsilon+s+s^2B_7},\quad \eta_2\left(s\right)=\frac{s^2B_8}{\varepsilon+s+s^2B_7},\quad \eta_3\left(s\right)=\frac{s^2B_9}{\varepsilon+s+s^2B_7} \tag{42}$$

It can be seen that the second derivatives of temperature depends to temperature, the second derivative of deflection and the fourth derivative of deflection. Comparing to the research work of Hosseini (2018) (considering nonlocality only in elasticity), the fourth derivative of deflection is added to Eq. (41) when the nonlocality is considered in both heat conduction and elasticity. Additionally, the coefficients $\eta_1(s)$, $\eta_2(s)$ and $\eta_3(s)$ are influenced by nonlocal parameter (e_0a).

By substitution the Eq. (41) into Eq. (39), it is obtained:

$$\begin{aligned}&\gamma_1(s)\frac{\partial^4\bar{w}}{\partial x^4}-\gamma_2(s)\frac{\partial^2\bar{w}}{\partial x^2}+\gamma_3(s)\bar{w}\\&+\gamma_4(s)\left\{\eta_1(s)\bar{T}-\eta_2(s)\frac{\partial^2\bar{w}}{\partial x^2}+\eta_3(s)\frac{\partial^4\bar{w}}{\partial x^4}\right\}=0\end{aligned}\tag{43}$$

where

$$\begin{aligned}\gamma_1(s)&=1+B_1s^2\\\gamma_2(s)&=B_2s^2\\\gamma_3(s)&=B_3s^2\\\gamma_4(s)&=B_4\end{aligned}\tag{44}$$

The temperature can be obtained in the term of deflection as:

$$\begin{aligned}\bar{T}=&-\left\{\frac{\gamma_1(s)}{\gamma_4(s)\eta_1(s)}+\frac{\eta_3(s)}{\eta_1(s)}\right\}\frac{\partial^4\bar{w}}{\partial x^4}\\&+\left\{\frac{\gamma_2(s)}{\gamma_4(s)\eta_1(s)}+\frac{\eta_2(s)}{\eta_1(s)}\right\}\frac{\partial^2\bar{w}}{\partial x^2}-\frac{\gamma_3(s)}{\gamma_4(s)\eta_1(s)}\bar{w}\end{aligned}\tag{45}$$

Also, the second derivation of T respect to x takes the following form:

$$\begin{aligned}\frac{\partial^2\bar{T}}{\partial x^2}=&-\left\{\frac{\gamma_1(s)}{\gamma_4(s)\eta_1(s)}+\frac{\eta_3(s)}{\eta_1(s)}\right\}\frac{\partial^6\bar{w}}{\partial x^6}\\&+\left\{\frac{\gamma_2(s)}{\gamma_4(s)\eta_1(s)}+\frac{\eta_2(s)}{\eta_1(s)}\right\}\frac{\partial^4\bar{w}}{\partial x^4}-\frac{\gamma_3(s)}{\gamma_4(s)\eta_1(s)}\frac{\partial^2\bar{w}}{\partial x^2}\end{aligned}\tag{46}$$

Substitution of Eqs. (45) and (46) into Eq. (41) gives a governing equation in term of lateral deflection as:

$$A^*\frac{\partial^6\bar{w}}{\partial x^6}+B^*\frac{\partial^4\bar{w}}{\partial x^4}+C^*\frac{\partial^2\bar{w}}{\partial x^2}+D^*\bar{w}=0\tag{47}$$

$$\begin{aligned}A^*&=A^*(s)=-\frac{\gamma_1(s)}{\gamma_4(s)\eta_1(s)}-\frac{\eta_3(s)}{\eta_1(s)}\\B^*&=B^*(s)=\frac{\gamma_2(s)}{\gamma_4(s)\eta_1(s)}+\frac{\eta_2(s)}{\eta_1(s)}+\frac{\gamma_1(s)}{\gamma_4(s)}+\eta_3(s)\\C^*&=C^*(s)=-\left\{\frac{\gamma_2(s)}{\gamma_4(s)}+\frac{\gamma_3(s)}{\gamma_4(s)\eta_1(s)}\right\}\\D^*&=D^*(s)=\frac{\gamma_3(s)}{\gamma_4(s)}\end{aligned}\tag{48}$$

An analytical solution can be proposed as:

$$\begin{aligned}\bar{w}(x,s)=&C_1(s)e^{-P_1(s)x}+C_2(s)e^{P_1(s)x}+C_3(s)e^{-P_2(s)x}\\&+C_4(s)e^{P_2(s)x}+C_5(s)e^{-P_3(s)x}+C_6(s)e^{P_3(s)x}\end{aligned}\tag{49}$$

where

$$\begin{aligned}P_1(s)&=\frac{\left\{-3A^*\psi^{1/3}\begin{bmatrix}i\left(\psi^{2/3}\sqrt{3}+12\sqrt{3}A^*C^*-4\sqrt{3}\left(B^*\right)^2\right)+\psi^{2/3}\\+4B^*\psi^{1/3}-12A^*C^*+4\left(B^*\right)^2\end{bmatrix}\right\}^{1/2}}{6A^*\psi^{1/3}}\\P_2(s)&=\frac{\left\{3A^*\psi^{1/3}\begin{bmatrix}i\left(\psi^{2/3}\sqrt{3}+12\sqrt{3}A^*C^*-4\sqrt{3}\left(B^*\right)^2\right)-\psi^{2/3}\\-4B^*\psi^{1/3}+12A^*C^*-4\left(B^*\right)^2\end{bmatrix}\right\}^{1/2}}{6A^*\psi^{1/3}}\\P_3(s)&=\frac{\left\{6A^*\psi^{1/3}\left[\psi^{2/3}-2B^*\psi^{1/3}-12A^*C^*+4\left(B^*\right)^2\right]\right\}^{1/2}}{6A^*\psi^{1/3}}\end{aligned}\tag{50}$$

$$\begin{aligned}\psi=&12\sqrt{3}A^*\begin{bmatrix}27\left(A^*\right)^2\left(D^*\right)^2-18A^*B^*C^*D^*\\+4A^*\left(C^*\right)^3+4D^*\left(B^*\right)^3-\left(B^*\right)^2\left(C^*\right)^2\end{bmatrix}^{1/2}-\\&-108D^*\left(A^*\right)^2+36A^*B^*C^*-8\left(B^*\right)^3\end{aligned}$$

Substitution Eq. (49) into Eq. (45), the following analytical solution can be found for temperature, too.

$$\begin{aligned}\bar{T}(x,s)=&K_1(s)e^{-P_1(s)x}+K_2(s)e^{P_1(s)x}+K_3(s)e^{-P_2(s)x}\\&+K_4(s)e^{P_2(s)x}+K_5(s)e^{-P_3(s)x}+K_6(s)e^{P_3(s)x}\end{aligned}\tag{51}$$

where

$$\begin{aligned}K_1(s)&=\left\{\lambda_1(s)\left[P_1(s)\right]^4+\lambda_2(s)\left[P_1(s)\right]^2+\lambda_3(s)\right\}C_1=\Gamma_1(s)C_1\\K_2(s)&=\left\{\lambda_1(s)\left[P_1(s)\right]^4+\lambda_2(s)\left[P_1(s)\right]^2+\lambda_3(s)\right\}C_2=\Gamma_2(s)C_2\\K_3(s)&=\left\{\lambda_1(s)\left[P_2(s)\right]^4+\lambda_2(s)\left[P_2(s)\right]^2+\lambda_3(s)\right\}C_3=\Gamma_3(s)C_3\\K_4(s)&=\left\{\lambda_1(s)\left[P_2(s)\right]^4+\lambda_2(s)\left[P_2(s)\right]^2+\lambda_3(s)\right\}C_4=\Gamma_4(s)C_4\\K_5(s)&=\left\{\lambda_1(s)\left[P_3(s)\right]^4+\lambda_2(s)\left[P_3(s)\right]^2+\lambda_3(s)\right\}C_5=\Gamma_5(s)C_5\\K_6(s)&=\left\{\lambda_1(s)\left[P_3(s)\right]^4+\lambda_2(s)\left[P_3(s)\right]^2+\lambda_3(s)\right\}C_6=\Gamma_6(s)C_6\end{aligned}\tag{52}$$

The terms $\lambda_1(s)$, $\lambda_2(s)$ and $\lambda_3(s)$ are defined as

$$\begin{aligned}\lambda_1(s)&=-\frac{\gamma_1(s)}{\gamma_4(s)\eta_1(s)}-\frac{\eta_3(s)}{\eta_1(s)}\\\lambda_2(s)&=\left\{\frac{\gamma_2(s)}{\gamma_4(s)\eta_1(s)}+\frac{\eta_2(s)}{\eta_1(s)}\right\}\\\lambda_3(s)&=-\frac{\gamma_3(s)}{\gamma_4(s)\eta_1(s)}\end{aligned}\tag{53}$$

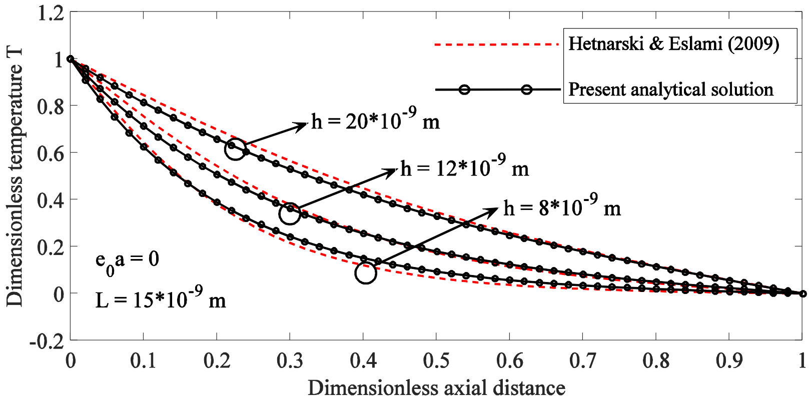

Fig. 2 The comparison between the obtained results for long times and e_0a=0 with those obtained by Hetnarski and Eslami (2009) for steady state heat conduction analysis.

The unknown coefficients $C_1(s)$, $C_2(s)$, $C_3(s)$, $C_4(s)$, $C_5(s)$ and $C_6(s)$ should be determined by the boundary conditions of the problem. The same initial and boundary conditions with those assumed by Hosseini (2018) are considered for the problem as:

$$w(\mathrm{x},0)=\frac{\partial w}{\partial t}(\mathrm{x},0)=0\ ,\ \ T(\mathrm{x},0)=\frac{\partial T}{\partial t}(\mathrm{x},0)=0 \qquad (54)$$

and,

$$w(0,t)=\frac{\partial w}{\partial x}(0,t)=0 \qquad T(0,t)=T^*(t) \qquad (55)$$

After using Laplace transform, the above boundary conditions take the forms as:

$$\bar{w}(0,s)=\frac{\partial \bar{w}}{\partial x}(0,s)=0 \qquad \bar{T}(0,\mathrm{s})=\bar{T}^*(s) \qquad (56)$$

The boundary conditions at the other end of the micro/nano beam $x=l$ take the form in Laplace domain as:

$$\bar{w}(l,s)=\frac{\partial \bar{w}}{\partial x}(l,s)=0\ ,\ \ \bar{T}(l,s)=0 \qquad (57)$$

By substituting the Eqs. (49) and (51) into boundary conditions, the following set of equations are obtained to calculate the unknown coefficients $C_1(s)$, $C_2(s)$, $C_3(s)$, $C_4(s)$, $C_5(s)$ and $C_6(s)$.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ P_1^2 & P_1^2 & P_2^2 & P_2^2 & P_3^2 & P_3^2 \\ \Gamma_1 & \Gamma_2 & \Gamma_3 & \Gamma_4 & \Gamma_5 & \Gamma_6 \\ e^{-P_1l} & e^{P_1l} & e^{-P_2l} & e^{P_2l} & e^{-P_3l} & e^{P_3l} \\ P_1^2e^{-P_1l} & P_1^2e^{P_1l} & P_2^2e^{-P_2l} & P_2^2e^{P_2l} & P_3^2e^{-P_3l} & P_3^2e^{P_3l} \\ \Gamma_1e^{-P_1l} & \Gamma_2e^{P_1l} & \Gamma_3e^{-P_2l} & \Gamma_4e^{P_2l} & \Gamma_5e^{-P_3l} & \Gamma_6e^{P_3l} \end{bmatrix} \begin{Bmatrix} C_1(s) \\ C_2(s) \\ C_3(s) \\ C_4(s) \\ C_5(s) \\ C_6(s) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \bar{T}^*(s) \\ 0 \\ 0 \\ 0 \end{Bmatrix} \qquad (58)$$

or,

$$\begin{Bmatrix} C_1(s) \\ C_2(s) \\ C_3(s) \\ C_4(s) \\ C_5(s) \\ C_6(s) \end{Bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ P_1^2 & P_1^2 & P_2^2 & P_2^2 & P_3^2 & P_3^2 \\ \Gamma_1 & \Gamma_2 & \Gamma_3 & \Gamma_4 & \Gamma_5 & \Gamma_6 \\ e^{-P_1l} & e^{P_1l} & e^{-P_2l} & e^{P_2l} & e^{-P_3l} & e^{P_3l} \\ P_1^2e^{-P_1l} & P_1^2e^{P_1l} & P_2^2e^{-P_2l} & P_2^2e^{P_2l} & P_3^2e^{-P_3l} & P_3^2e^{P_3l} \\ \Gamma_1e^{-P_1l} & \Gamma_2e^{P_1l} & \Gamma_3e^{-P_2l} & \Gamma_4e^{P_2l} & \Gamma_5e^{-P_3l} & \Gamma_6e^{P_3l} \end{bmatrix}^{-1} \begin{Bmatrix} 0 \\ 0 \\ \bar{T}^*(s) \\ 0 \\ 0 \\ 0 \end{Bmatrix} \qquad (59)$$

After calculating the unknown coefficients $C_1(s)$, $C_2(s)$, $C_3(s)$, $C_4(s)$, $C_5(s)$ and $C_6(s)$, it is possible to calculate the coefficients $K_1(s)$, $K_2(s)$, $K_3(s)$, $K_4(s)$, $K_5(s)$ and $K_6(s)$ using Eqs. (52). The fields' variables $\bar{w}(x,s)$ and $\bar{T}(x,s)$ are obtained from Eqs. (49) and (51) in Laplace domain. Using the following approach, which is based on Talbot inversion Laplace transformation, the deflection and temperature are computed in time domain.

$$\begin{aligned} w(x,t) &= \frac{2}{5t}\sum_{k=0}^{M-1}\mathrm{Re}(\gamma_k\bar{w}(x,s)) \\ T(x,t) &= \frac{2}{5t}\sum_{k=0}^{M-1}\mathrm{Re}(\gamma_k\bar{T}(x,s)) \end{aligned} \qquad (60)$$

where the term "M" stands for number of samples and

$$\begin{aligned} & s_k=\frac{\delta_k}{t}\ ,\ \delta_0=\frac{2M}{5},\ \ \gamma_0=0.5e^{\delta_0} \\ & \delta_k=\frac{2k\pi}{5}\left(\cot\left(\frac{k\pi}{M}\right)+i\right)\quad , \\ & \gamma_k=\left[1+i\left(\frac{k\pi}{M}\right)\left(1+\left[\cot\left(\frac{k\pi}{M}\right)\right]^2\right)-i\cot\left(\frac{k\pi}{M}\right)\right]e^{\delta_k} \qquad 0<k<M \end{aligned} \qquad (61)$$

5. Numerical results and discussion

The verification of the proposed method was carried out by Hosseini (2018) with comparing between the obtained results from proposed analytical solution for e_0a=0 and the presented method and data by Hetnarski and Eslami (2009) for a 2D domain at steady state conditions. If the small scale parameter e_0a is assumed to be zero, the above mentioned verification can be also valid for the current problem. So, the obtained results in this problem (with considering nonlocality in both heat conduction and elasticity) at long times are compared with those reported by Hetnarski and Eslami (2009), which can be observed in Fig. 2. When the small scale parameter e_0a is assumed to be e_0a = 0, the obtained results are the same with those reported by Hosseini (2018). In long times, the behaviors of temperature field are converged to temperature distribution in steady state. A good agreement can be found in Fig. 2.

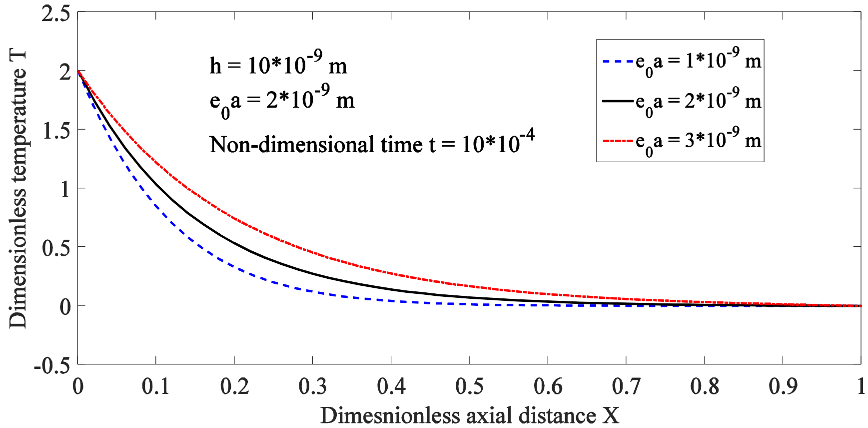

Fig. 3 Effects of small scale parameter on the distribution of dimensionless temperature along dimensionless axial distance for first example

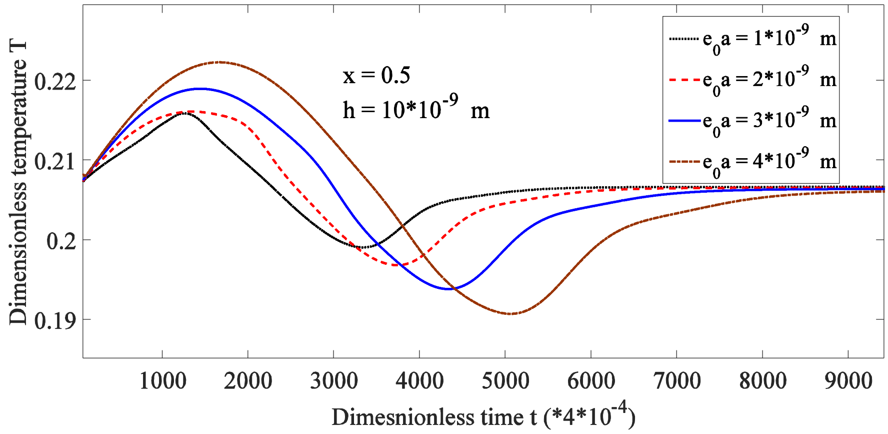

Fig. 4 Effects of small scale parameter on the time history of dimensionless temperature for first example

A gold (*Au*) nano beam resonator is assumed for the problem to show the ability of the proposed method for the nonlocal coupled thermoelasticity analysis. So, the same mechanical properties with those assumed by Elsibai, *et al.* (2011) and Hosseini (2018) are considered for the analysis.

$$\alpha_T = 14.2 \times 10^{-6}\mathrm{K}^{-1}, \rho = 1930\mathrm{kg/m}^3, T_0 = 293\ \mathrm{K},$$
$$\mathrm{k} = 317\mathrm{Wm}^{-1}\mathrm{K}^{-1} \tag{62}$$
$$C_v = 130\mathrm{Jkg}^{-1}, E = 180\ \mathrm{Gpa}, v = 0.44$$

The dimensions of the nano beam are assumed to be:

$$\begin{aligned} &l = 20*10^{-9}\,m, \\ &8*10^{-9}\,m < h < 12*10^{-9}\,m, \quad b = 0.5h \end{aligned} \tag{63}$$

In the research work of Hosseini (2018), the function $T^*(t) = 2H(t)$ or $\bar{T}^*(s) = \frac{2}{s}$ was assumed in the boundary conditions. So, this boundary conditions are employed in this article as first example. It was shown in a research work by Hosseini (2018) that the variation of small scale parameter (e_0a) didn't have any significant effects on the distribution of temperature because the nonlocality in heat conduction was not considered. So, in this paper, the nonlocality is assumed in the heat conduction using a new modified model based on GN theory. By assuming the function $T^*(t) = 2H(t)$ or $\bar{T}^*(s) = \frac{2}{s}$ in boundary conditions, the dimensionless temperature distributions along longitudinal direction are drawn in Fig. 3 for various values of small scale parameter e_0a. As it can be clearly observed in Fig. 3, the variation in value of e_0a creates a significant effect in the distribution diagrams of dimensionless temperature.

Also, the variation of e_0a influences the time history of dimensionless temperature, which can be seen in Fig. 4. It means that the considering of nonlocality in both heat conduction and elasticity influences the transient behaviors of dimensionless deflection and temperature. In other words, the proposed new modified model based on the GN theory has a high capability for the coupled thermoelasticity analysis in nano scale with considering small scale effects.

As second example, it is assumed that one of the beam ends is excited by sinusoidal thermal shock loading. So, the function $T^*(t)$ is assumed in the boundary conditions as follow:

$$T^*(t) = \begin{cases} F^* \sin(\frac{\pi}{t_0}t) \;, & t \le t_0 \\ 0 \;, & t > t_0 \end{cases} \tag{64}$$

Using Laplace transformation, the $\bar{T}^*(s)$ can be obtained as:

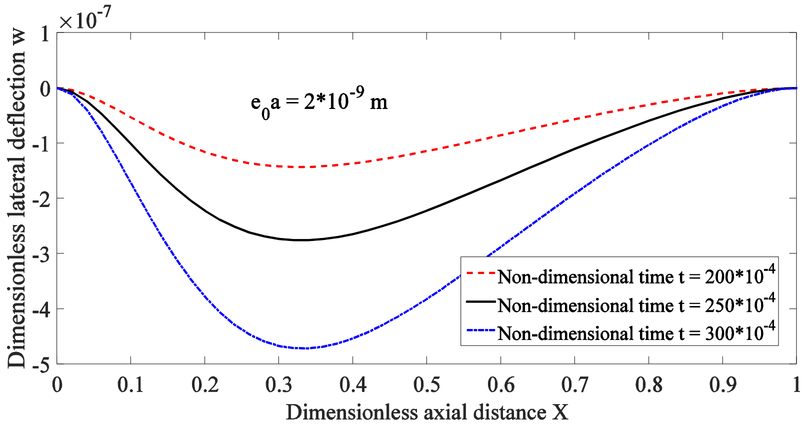

Fig. 5 The distribution of dimensionless lateral deflection along dimensionless axial distance at various time instants for second example

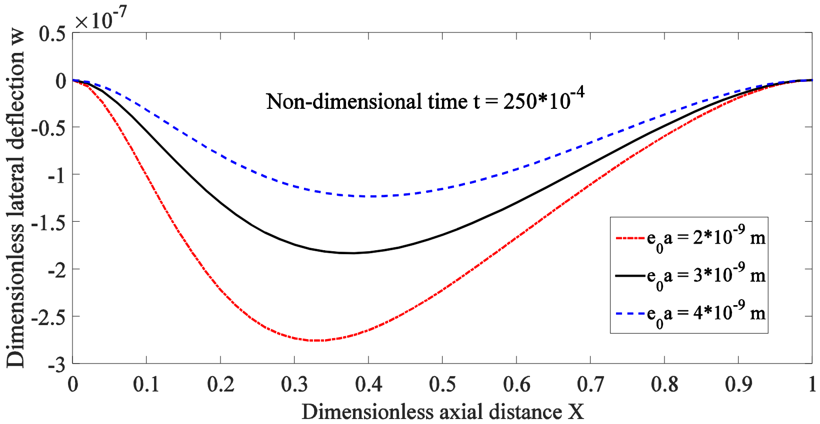

Fig. 6 Effects of small scale parameter on the distribution of dimensionless lateral deflection along dimensionless axial distance for second example

$$\bar{T}^*(s) = \begin{cases} F^* \dfrac{(\frac{\pi}{t_0})}{(\frac{\pi}{t_0})^2 + s^2}, & t \leq t_0 \\ 0, & t > t_0 \end{cases} \tag{65}$$

It this paper, it is assumed that $t_0 = 1$.

5.1 Lateral deflection's analysis

The Figs. 5-8 show the transient behaviors of lateral deflection of the assumed micro/nano beam resonator subjected to sinusoidal thermal shock loading with considering F^*=1. Fig. 5 depicts the distributions of dimensionless lateral deflection along axial distance for certain value of e_0a at various time instants. The effects of variation in the value of e_0a on the dimensionless lateral deflection (elasticity field) can be observed in Figs. 6 and 7, which the distribution along longitudinal direction and time history of dimensionless lateral deflection are shown, respectively. It can be clearly found that the maximum values in the diagrams of distribution of dimensionless lateral deflection are decreased, when the value of e_0a is increased. A similar behavior can be also observed in Fig. 7 for time histories of dimensionless lateral deflection. It means that the small scale parameter e_0a influences the elasticity fields. The proposed new modified GN model for nonlocal coupled thermoelasticity with considering nonlocality in both elasticity and heat conduction simulates transient behaviors of elasticity in small scales. The same results were obtained in the research work of Hosseini (2018) for elasticity field. But the main difference of the proposed new GN-based model is for the analysis of temperature field. Other parameters such as the geometrical physical factors of the structure, the thermal and mechanical boundary conditions, type of thermal effect as well as its intensity value influence the transient behaviors of lateral deflection of the assumed nano beam. For example, Fig. 8 shows the distributions of dimensionless lateral deflection along axial distance for certain value of e_0a at a certain time for various values of beam height. It can be observed when the value of beam height is increased the maximum value of dimensionless lateral deflection is decreased. In the next section, the effect of beam height on the temperature field will be discussed in detail. The effect of intensity value of thermal loading as thermal boundary conditions on the transient behaviors of dimensionless lateral deflection can be found in Fig. 9 in which the distributions of dimensionless lateral deflection along axial distance are drawn for various values of F^*. When the value of $F^*$is

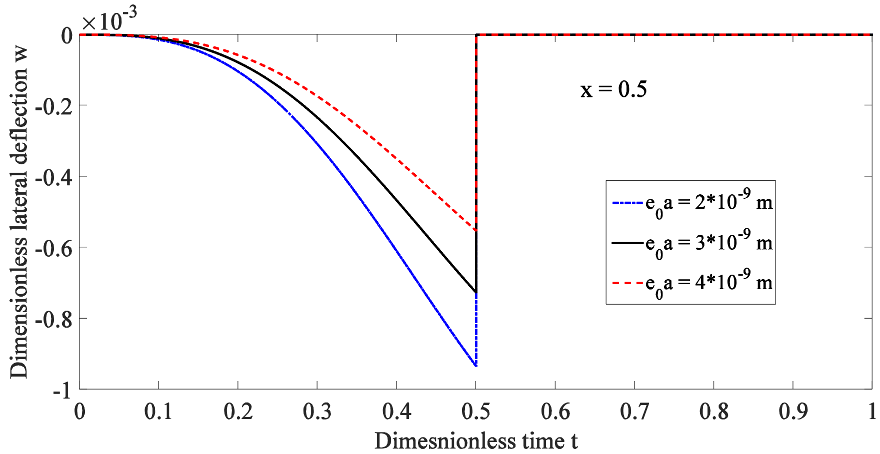

Fig. 7 Effects of small scale parameter on the time history of dimensionless lateral deflection for second example

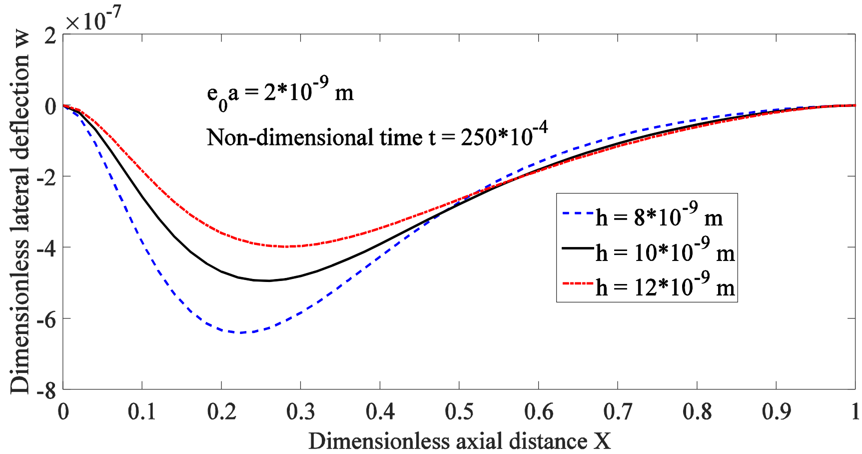

Fig. 8 Effects of the nano beam height on the distribution of dimensionless lateral deflection along dimensionless axial distance for second example.

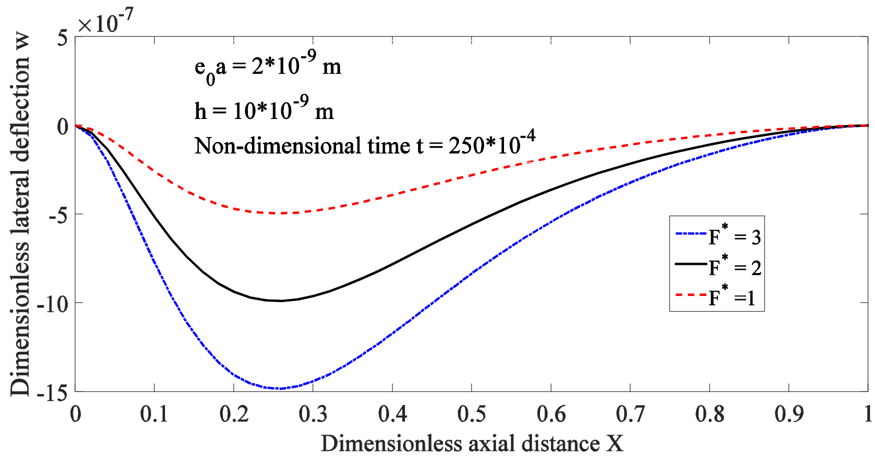

Fig. 9 Effects of the thermal loading intensity on the distribution of dimensionless lateral deflection along dimensionless axial distance for second example

increased, the values of dimensionless lateral deflection are increased. Based on the coupling effects between temperature and displacement fields in GN theory of coupled thermoelasticity, the temperature field is influenced by changing in displacement field. So, it is needed to discuss the temperature field based on the obtained results using the proposed new GN-based model of coupled thermoelasticity.

5.2 Temperature field's analysis

In Figs. 10-12, the temperature field in the assumed micro/nano beam resonator is assesses for various values of e_0a to find the small scale effects on the temperature field. The distribution of dimensionless temperature along longitudinal direction of the micro/nano beam resonator are drawn in Fig. 10 at various time instants for a certain value of small scale parameter as $e_0a = 2^*10^{-9}$. In the proposed

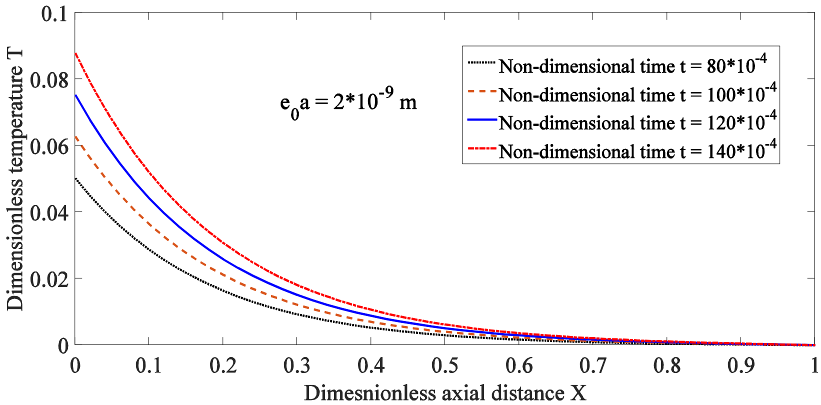

Fig. 10 The distribution of dimensionless temperature along dimensionless axial distance at various time instants for second example

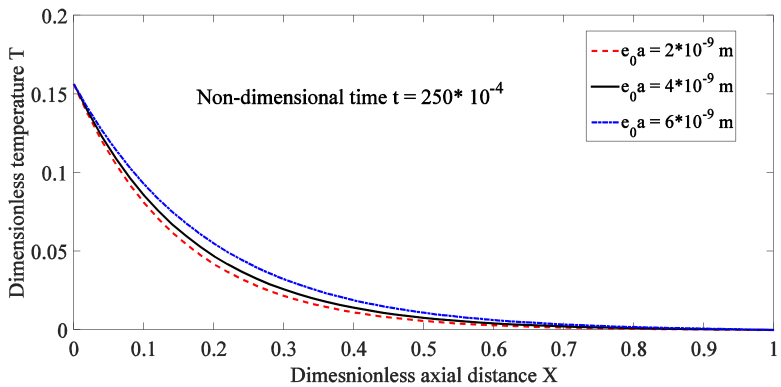

Fig. 11 Effects of small scale parameter on the distribution of dimensionless temperature along dimensionless axial distance for second example

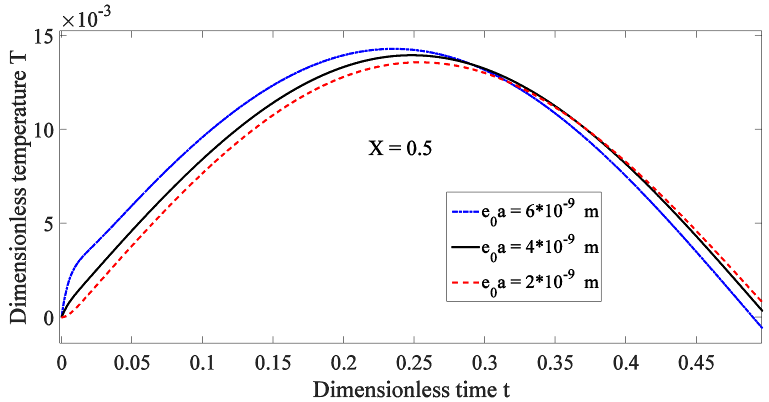

Fig. 12 Effects of small scale parameter on the time history of dimensionless temperature for second example

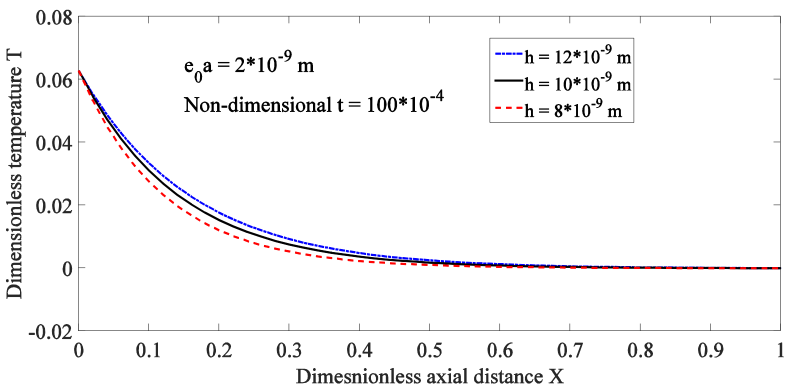

Fig. 13 Effects of the nano beam height on the distribution of dimensionless temperature along dimensionless axial distance for second example

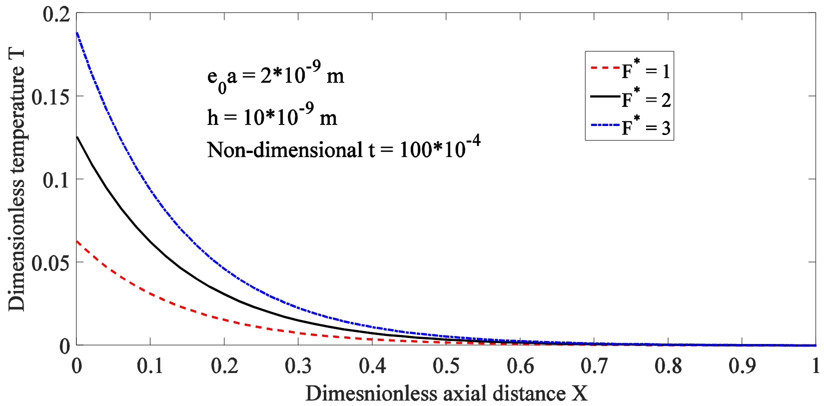

Fig. 14 Effects of the thermal loading intensity on the distribution of dimensionless temperature along dimensionless axial distance for second example

new modified model based on GN theory, the small scale parameter e_0a e_0a is taken into accounts in both elasticity and heat conduction. So, the transient behaviors of dimensionless temperature should be affected by variation in the value of e_0a. To show the effects of e_0a on the temperature field, the distribution of dimensionless temperature along longitudinal direction of the assumed beam resonator are illustrated in Fig. 11 for various values of e_0a. It can be concluded that the value of e_0a influences the temperature field employing the proposed new modified model. The time histories of dimensionless temperature for x=0.5 are presented in Fig. 12 for different values of e_0a. When the value of e_0a is increased, the maximum value of dimensionless temperature is increased. To find the effects of geometrical parameters such as height of nano beam on the transient behaviors of temperature field, the dimensionless temperature distributions along axial distance are illustrated in Fig. 13 for various values of nano beam height. It can be clearly observed that nano beam height influences the transient behaviors of temperature field but not as much as the effect it has on the elasticity field. The effects of intensity value of thermal loading on the dimensionless temperature field can be assessed using Fig. 14 in which the dimensionless temperature distributions along axial distance are depicted for various values of F^*. The dimensionless temperature at every point on nano beam is increased by increasing the value of F^*. It is very important to note that the obtained transient behaviors of dimensionless temperature and elasticity fields are based on the applied thermal and mechanical boundary conditions in the problem (sinusoidal thermal shock loading). By comparing between the presented results in this paper and the reported results based on other type of thermal and mechanical boundary conditions such as reported data by Hosseini (2018) based on a similar thermal shock loading in first example of this article, it can be concluded that the transient behaviors of temperature and elasticity field are different for both problems. It means that the type of thermal loading and also thermal and mechanical boundary conditions influence the transient behaviors of both temperature and elasticity fields. The proposed new model based on GN theory with energy dissipation has a high capability to show the effects of small scale on the transient behaviors of both lateral deflection and temperature, considering various thermal and mechanical boundary conditions and also various types of thermal shock loading.

6. Conclusion

A new modified model based on GN theory with energy dissipation is proposed for nonlocal coupled thermoelasticity analysis of micro/nano beam resonator considering the nonlocality in both elasticity and heat conduction. The governing equatiosn are derived using GN theory and Eringen nonlocal theory. An analytical solution is presented to study on the transient behaviors of lateral deflection and temperature fields. The main results of the research can be outlined as:

- The governing equations of size-dependent coupled thermoelasticity with enegy dissipation in micro/nano scale are formulated considering the small scale effects in both elasticity and heat conduction.
- Employing the proposed analytical solution, it is possible to present the dimensionless temperature and lateral deflection in the closed forms. The presented closed forms for fields' variables can be used for other analysis of micro/nano beam resonators such as band structure analysis of thermoelastic wave propagation in nano metamaterials.
- The effects of small scale parameters on the transient behaviors of both dimensionless lateral deflection and temperature are obtained and studied in detail when the energy is dissipated.
- Two types of thermal shock loading are assessed in the problem using the proposed new modified model and analytical solution. The solution can be developed for other coupled problems such as coupled photo-thermoelasticity based on the nonlocal GN theory with energy dissipation.
- The outcome of this study can provide beneficial background for the application of the proposed GN-based model and analytical solution for other coupled problems in engineering such as nonlocal coupled diffusion-thermoelasticity analysis or nonlocal coupled photo-thermoelasticity analysis.

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