

Non-Linear dynamic pulse buckling of laminated composite curved panels

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Abstract. In this paper, non-linear dynamic buckling behaviour of laminated composite curved panels subjected to dynamic in-plane axial compressive loads is studied using finite element methods. The work is carried out using the finite element software ABAQUS. The curved panels are modelled with S4R element and the nonlinear dynamic equilibrium equations are solved using the ABAQUS/Explicit algorithm. The effect of aspect ratio, radius of curvature and thickness are studied. The importance of orientation of plies in the direction of loading is also reiterated in this study. Vol'mir's criterion is used to calculate the dynamic buckling loads. The panels are subjected to rectangular pulse load of various amplitude and durations and the responses are observed. For particular loading amplitude, a critical value of loading duration is observed beyond which the variation of dynamic buckling load is insignificant. It is also observed that, the value of dynamic buckling load reduces as the loading duration is increased though the reduction is not much after a particular loading duration.

Keywords: dynamic buckling; composite laminates; curved panels; axial loads

1. Introduction

Laminated composite curved panels are used in structures like aircrafts, space shuttles, high-performance machines, etc. Today they are being used in civil engineering, biomedical automobile fields as well. Structures made of composite materials are costly but, due to their high strength to weight ratio, they are essential. Thus, these structures have to be analysed and designed carefully. Thin structures made up of composite materials are susceptible to stability loss. Analysing the whole structure is time consuming, so small parts of the structures are taken up for analysis and design. The laminated composite curved panels are quite essential parts of various complex structures. During the operation period of these complex structures, the panels are subjected to dynamic loads along their edges from their neighbouring components. At critical loads, instability may arise in these panels which can further damage the whole structure. This instability caused due to dynamic loads must be considered while designing a curved panel which is operating in dynamic environment. In addition, emergencies like accidents, blast loading, etc. can render the panel unstable. Hence these components should be designed properly according to the requirements. Dynamic stability of structures is a very vast area which includes dynamic buckling due to pulsating loads and impulsive loads. Jansen (2005) described the difference between two types of dynamic instabilities: parametric excitation and dynamic buckling. In dynamic buckling, the loading is an impulse load or step load, and in parametric excitation, the structure is subjected to vibratory

loads or pulsating loads. In the present investigation the stability of the laminated composite curved panels subjected to suddenly applied load along their boundaries are studied.

A number of articles have been published in field of static buckling of composite laminates. Gerard and Becker (1957) reported many tables and charts considering effects of various parameters on static buckling of composite plates. Chamis (1969) studied buckling of simply supported composite plates using Galerkin method and reported results for various aspect ratios and loading conditions. Leissa (1985) studied static buckling of laminated composite plates and shells and reported the effects of various parameters on the static buckling loads. Leissa (1990) studied the static buckling of single-layered simply supported square composites having non-uniformly spaced fibres. Rajasekaran (2017) studied isotropic and orthotropic plates with various boundary conditions and loading functions subjected to in-plane loads to analyse static buckling and free vibration using element-based differential quadrature method. Kiran and Kattiman (2017) studied the static buckling of magneto-electro-elastic plates subjected to in-plane compressive loads using first order shear deformation theory. The effects of thickness, load factor, aspect ratio and boundary conditions have been reported. Topal (2017) studied the static buckling load optimisation of stepped columns constituted of symmetric angle-ply laminates. Results of optimisation for various parameters like fillet radius, boundary conditions have been presented. Zerín *et al.* (2017) studied a large span self supporting roofing structure assuming it to be a laminated composite plate structure using finite element method to calculate the static buckling load. Various lamination sequence, cutout shapes, free-edge forms have been considered in linear buckling analysis in ANSYS software.

Many investigators have undertaken the static buckling study of shell structures as well. Karman (1941) studied the

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static buckling of thin cylindrical isotropic shells subjected to axial loads using non-linear large deformation theory. Hutchinson *et al.* (1971) studied the effect of imperfection on static buckling of cylindrical shells. Zhang and Matthews (1983) studied the cylindrically curved panels subjected to both axial and shear forces for static buckling. Di Sciuva and Carrera (1990) developed linearized equations for static buckling of cylindrical shells using analytical method and using finite element analysis. Bisagni (1999) reported the experimental methodology and results of static buckling of cylinders made up of carbon fiber of various layup sequences. Jaunky and Knight Jr. (1999) calculated static buckling loads of laminated composite cylindrical panels using various shell theories and boundary conditions. Hilburger *et al.* (2001) conducted experimental and numerical study on quasi-isotropic curved panels with central cutout to calculate the static buckling loads. The effect of parameters like cutout size and initial imperfection have been taken into consideration for pre-buckling, buckling and post-buckling response. Kidane *et al.* (2003) studied the static buckling of grid stiffened composite cylindrical shells subjected to axial loads experimentally and using analytical methods and reported results for various winding angles and stiffener spacing. Moon *et al.* (2010) studied experimentally and using finite element analysis, the static buckling of composite cylinders subjected to hydrostatic pressure.

The parametric instability of composite plates has also been studied by many authors. Moorthy *et al.* (1990) studied the dynamic instability of composite plates subjected to in-plane loads. The effects of damping, orthotropy ply orientation have been studied by the authors using finite element analysis. Chattopadhyay and Radu (2000) studied the dynamic instability of composite plates using finite element analysis. The authors have used higher-order shear deformation theory considering both transverse and rotary inertia effects. Ramachandra and Panda (2012) studied the dynamic instability of composite plate subjected to non-uniform in-plane loads for various boundary conditions. Wang *et al.* (2013) studied vibrational instability of laminated composite plates subjected to periodic loads in hygrothermal atmosphere. The authors reported the regions of dynamic instability in the plate. Kumar *et al.* (2015) studied parametric instability of composite skew plates subjected to in-plane loads. The authors have reported results considering various parameters like aspect ratio, thickness, skew angle, loading function and boundary conditions. Darabi and Ganesan (2017) studied dynamic instability of internally tapered composite plates subjected to harmonic in-plane loads.

The parametric instability of shell structures has been studied by some authors. Sahu and Datta (2001) studied the dynamic instability of doubly curved panels subjected to in-plane load using finite element analysis. The authors have reported the effects of static load factors, boundary conditions, shallowness ratio. Qatu (2002) reported a very extensive literature survey on the works undertaken in the field of vibration of shells till 2002. This research article is in two parts. Patel *et al.* (2006) studied static buckling and dynamic instability of various kinds of stiffened shell panels.

The authors have reported results for various parameters like shell geometry, stiffening scheme lamination scheme for laminated composite shells. Alijani and Amabili (2014) conducted an extensive literature survey on the stability and vibration of isotropic, composite and functionally graded material shells from 2003 till 2013. Dey and Ramachandra (2014) studied the dynamic instability of composite curved panels subjected to transverse patch and partial edge loading conditions.

Dynamic pulse buckling of laminated composite plate and shell structures have been studied by few authors. Gilat and Aboudi (1995) studied the dynamic buckling of plates and cylindrical shells. The authors reported that the effect of axial inertia is more pronounced when the pulse durations were short. Ari-Gur and Simonetta (1997) studied dynamic buckling of composite plates and reported that dynamic buckling loads can be static buckling loads if the applied pulse frequency is near to the fundamental frequency of the plate. Kubiak (2005) studied thin-walled rectangular composite plates in which, the fiber volume fraction was varied width-wise. The analytical-numerical method proposed by the author did not account for the changes in buckling modes, which occurs during the analysis. Bisagni (2005) studied the dynamic buckling of composite cylinders using finite element analysis and checked the effect of loading duration and imperfection on dynamic buckling loads. Kowal-Michalska and Mania (2008) calculated the deflection of isotropic and orthotropic plates and studied the effects of loading function and initial imperfection. The authors gave a buckling criterion for orthotropic plate by comparing the uniaxial limiting stress in the direction of loading with that of the stress obtained from Hill's criterion. Shariyat (2011) reported a theory for viscoelastic composite plates that are subjected to thermoplastic loadings, calibrated with the help of nonlinear strain-displacement expressions. Priyadarsini *et al.* (2012) studied the dynamic buckling of composite cylinders experimentally and using finite element analysis and checked the effects of geometric properties and layup sequence. Gao and Fatt (2012) studied composite single curvature panels subjected to external blasts using Lagrange's equation of motion and Budiansky-Roth criterion. Azarboni *et al.* (2015) studied the dynamic buckling of isotropic plates subjected to various loading functions, boundary conditions, imperfections. Ovesy *et al.* (2015) studied the dynamic buckling of delaminated composite plates using semi-analytical finite strip method. Yang and Wang (2016) studied the dynamic buckling of isotropic stiffened plates with elastically restrained edges subjected to in-plane loads and proposed a new criterion for dynamic buckling.

From the literature, it is revealed that the static buckling and dynamic instability with pulsating load of composite plate/shell is given considerable importance by the earlier investigators. However, the instability study of laminated composite curved panels with suddenly applied load is very few, though it is vital domain of research. So, still there exists a lot of scope to study the dynamic buckling behaviour of laminated composite curved panels which are subjected to impulsive loading in detail. In the current study, the dynamic buckling behaviour of laminated composite

curved panels are studied considering aspect ratio, radius of curvature, thickness and ply orientation using finite element method. Vol'mir's criterion is used to calculate the dynamic buckling loads. According to this criterion, the critical transverse deflection value is assumed to be equal to the thickness of plate (Kubiak (2013)). The results reported here will help a designer to design the plates which are operating in dynamic environment so that these will be dynamically stable, safe and economical. All the results reported in the study are numerical, some experiments are quite necessary in order to understand the behaviour in depth.

2. Theory and formulation

The non-linear dynamic equilibrium equation solved by Abaqus/Explicit is given in Eq. (1).

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K(\{u\})]\{u\} = \{F(t)\} \quad (1)$$

Where, $[M]$ is the mass matrix, $[C]$ the damping matrix, $[K(\{u\})]$ the stiffness matrix, which depends on the deformations due to the geometric non-linearity, $\{u\}$ the nodal displacement vector, $\{\dot{u}\}$ the nodal velocity vector, $\{\ddot{u}\}$ the nodal acceleration vector and $\{F(t)\}$ the load vector. In the current study, the damping effect is not considered. The non-linear dynamic equilibrium equation solved is shown in Eq. (2).

$$[M]\{\ddot{u}\} + [K(\{u\})]\{u\} = \{F(t)\} \quad (2)$$

The current study is carried out in Abaqus/Explicit in which, shells which having thickness less than $1/15^{\text{th}}$ of that of the characteristic length, are called thin shells which are modelled using Kirchhoff shell theory. Abaqus/Explicit consists of general-purpose shell elements which provides solution to both thin and thick shells. In the current study, the laminated composite curved panel is modelled with S4R element. S4R is a general-purpose conventional shell element. The non-linear dynamic equation is solved by keeping the time step automatic as the numerical stability of the dynamic explicit analysis depends on the time-step.

3. Results and discussion

Convergence and validation study are carried out using some of the results available in the literature. Then the current problem is discussed.

3.1 Convergence and validation study

3.1.1 Convergence Study for laminated composite curved panel

The convergence study is carried out on two problems. Non-dimensional buckling load is evaluated for a square planed, simply supported, uniformly loaded symmetric cross-ply panel ($0^\circ/90^\circ/0^\circ/90^\circ/0^\circ$) as shown in Fig. 1. The support conditions are shown in Fig. 2. The material properties and other parameters are taken as: $b/a = 1$, $R/a = 20$, $E_{11} = 40E_{22}$, $G_{12} = G_{13} = 0.5E_{22}$, $G_{23} = 0.6E_{22}$ and $\nu_{12} =$

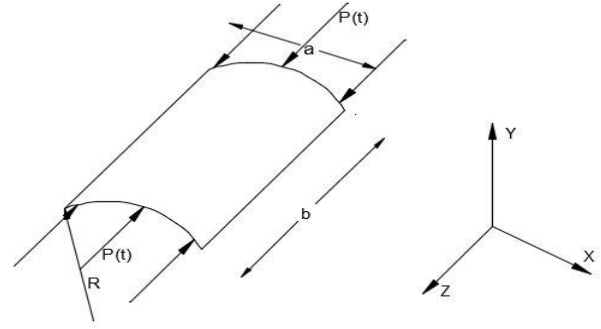


Fig. 1 Geometry of the panel with uniform loading

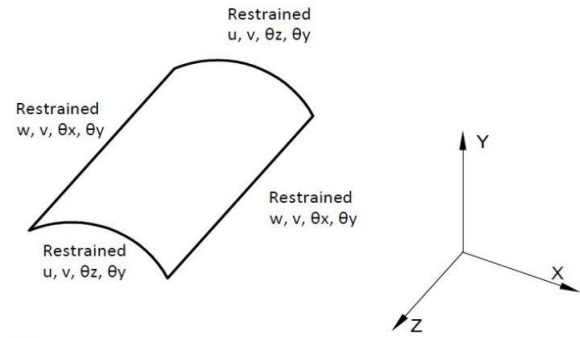


Fig. 2 Boundary conditions for the panel

Table 1 Non-dimensional static buckling load for panel with $a/h=100$. $b/a=1$ and $R/a=100$

Analysis	Mesh size	Buckling load	
		kN/m	Non-dimensional
Present	40×40	337.51	36.4875
Di Sciuva and Carrera (1990)	5×5	-	36.86
Patel <i>et al.</i> (2003)	8×8	-	36.8452

0.25 as reported by Di Sciuva and Carrera (1990). The non-dimensional buckling load is calculated using Eq. (3). The convergence study of the non-dimensional static buckling loads for the curved panel of $a/h = 100$ for different mesh sizes is shown in Fig. 3. For the same material properties with $a/h = 50$, the convergence study is again carried out and the results are shown in Fig. 4.

$$\overline{P_{cr}} = \frac{P_{cr} b^2}{E_{22} h^3} \quad (3)$$

In order to perform dynamic analysis during the next phase of the study, an adequate density value is to be considered. The material properties presented by Priyadarsini *et al.* (2012) are used. $E_{22} = 9250$ MPa and density = 1700 Kg/m^3 for $a/h=100$ and $a/h=50$ cases. The value of E_{22} is substituted in the ratios given above and the static buckling loads are calculated. The static buckling loads for curved panels with $a/h=100$ and $a/h=50$ are presented in Tables 1-2, respectively.

It is observed that in both the cases, the results are almost converged with mesh size of 40×40 . Further, for validation of the present results, they are compared with

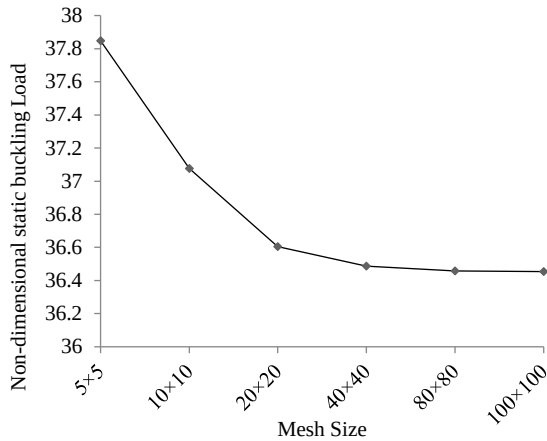


Fig. 3 Non-dimensional buckling load with different mesh sizes for curved panel with $a/h=100$

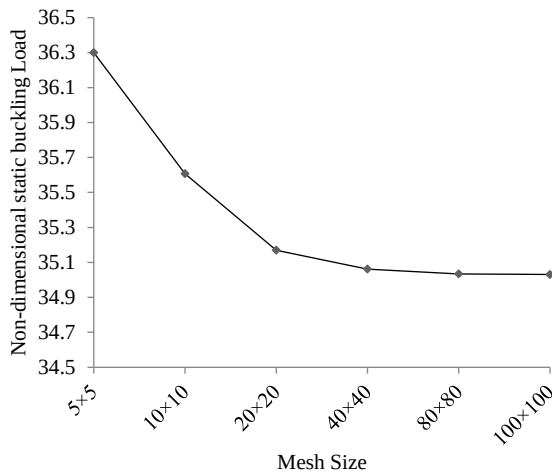


Fig. 4 Non-dimensional buckling load with different mesh sizes for curved panel with $a/h=50$

Table 2 Non-dimensional static buckling load for panel with $a/h=50$, $b/a=1$ and $R/a=100$

Analysis	Mesh Size	Buckling Load	
		kN/m	Non-dimensional
Present	40x40	2594.5	35.0607
Di Sciuva and Carrera (1990)	5x5	-	35.42

finite element results reported by Di Sciuva and Carrera (1990) and Patel *et al.* (2003) for the case with $a/h=100$ in Table 1. The results with $a/h=50$ are shown in Table 2, along with the finite element results of Di Sciuva and Carrera (1990).

The convergence and validation study carried out on the laminated composite curved panel show that the results with mesh size 40×40 match well with the available results.

3.1.2 Nonlinear dynamic response of Aluminium Plate subjected to axial load

To check the response of a plate to dynamic loading, a square planed aluminium plate is subjected to dynamic axial load and the central deflection of the plate is observed. The

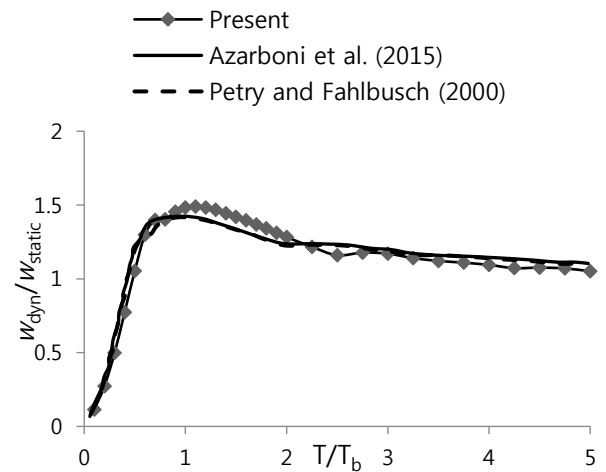


Fig. 5 Ratio of dynamic to static displacement with ratio of applied time(T) to first time period (T_b) of aluminium plate with $a/b=1$, $a=0.5m$, $h= 0.005a$ and imperfection= $0.2h$

present results are compared with the results from Azarboni *et al.* (2015) and Petry and Fahlbusch (2000). The geometric properties of the plate are taken from Azarboni *et al.* (2015), but the material properties are taken from Kowal-Michalska and Mania (2008). $E = 70\text{GPa}$, $\nu = 0.33$, $\rho = 2950 \text{ kg/m}^3$; $a/b = 1$, $a = 0.5m$, $h = 0.005a$. The geometry of the plate is shown in Fig. 1. Simply supported boundary conditions taken for both pre-buckling and buckling stages are same as shown in Fig. 2. The initial imperfection is taken as $0.2h$ ($= 20\%$ of the thickness of the plate). The imperfection of first buckling mode is incorporated in the plate. The effect of loading duration is observed. The static buckling load is calculated using linear static instability analysis (Eigenvalue), which in this case is 21648 N/m . Then, first natural period of the plate is calculated ($T_b = 21.36 \times 10^{-3} \text{ s}$). Next, for calculating the static transverse displacement with the in-plane load, the non-linear static Riks analysis is performed. The central transverse displacement corresponding to three times static buckling load ($3 \times P_{\text{static}}$) is calculated while incorporating imperfection in the plate ($w_{\text{static}} = 0.0057m$). For different values of durations of loading (T), the dynamic central transverse displacements (w_{dyn}) are calculated using non-linear dynamic analysis (Abaqus/Explicit). The loading function is a sinusoidal function given by Eq. (4).

$$P_{\text{dyn}} = \begin{cases} 3P_{\text{static}} \sin \frac{\pi t}{T}, & 0 \leq t \leq T \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

The results (ratio of $w_{\text{dyn}}/w_{\text{static}}$ vs. ratio of T/T_b) of the dynamic analysis are presented along with the FEM results of Azarboni *et al.* (2015) and Petry and Fahlbusch (2000) in Fig. 5.

3.2 Dynamic buckling

The laminated composite curved panel used in the convergence and validation study is again used in the

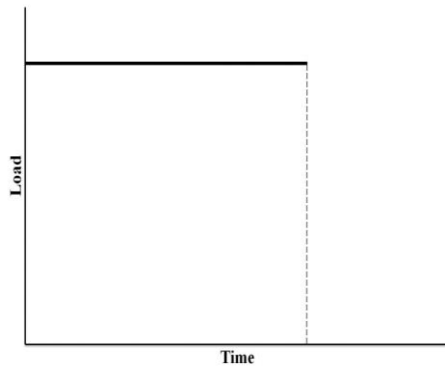


Fig. 6 Rectangular pulse load

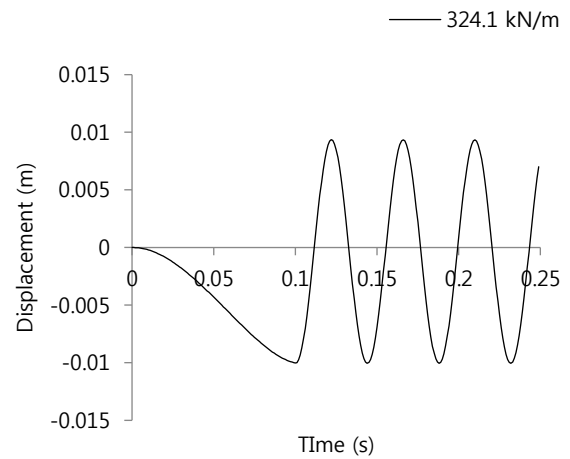


Fig. 7(c) Transverse Displacement vs. Time plot of loading duration of 0.1s

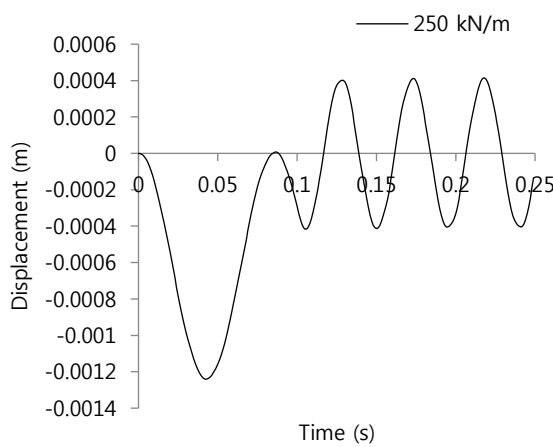


Fig. 7(a) Transverse Displacement vs. Time plot of loading duration of 0.1s

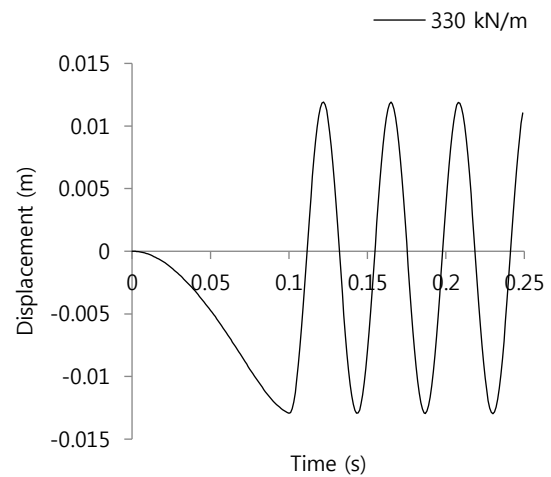


Fig. 7(d) Transverse Displacement vs. Time plot of loading duration of 0.1s

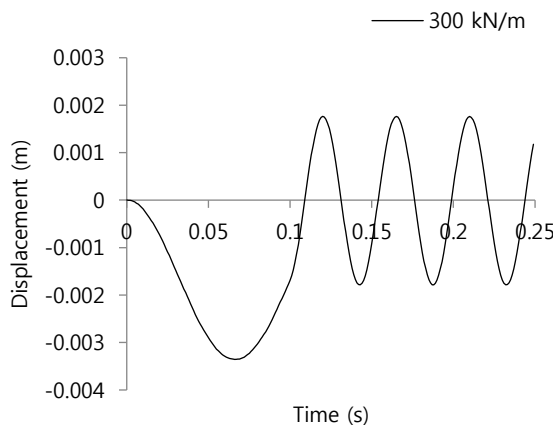


Fig. 7(b) Transverse Displacement vs. Time plot of loading duration of 0.1s

dynamic buckling study. The material properties used are: $b/a = 1$, $R/a = 20$, $a/h = 100$, $E_{11} = 40E_{22}$, $G_{12} = G_{13} = 0.5E_{22}$, $G_{23} = 0.6E_{22}$ and $\nu_{12} = 0.25$. Dynamic rectangular pulse loads as shown in Fig. 6 are applied for various durations. As described earlier, the material properties reported by

Priyadarsini *et al.* (2012) are used with $E_{22} = 9250$ MPa and density = 1700 Kg/m^3 . The value of E_{22} is substituted in the ratios given above. Table 3 shows the calculated dynamic buckling loads.

Figures 7(a)-7(d) show the transverse displacement vs. time plots for calculating the dynamic buckling load for a loading duration of 0.1s. For various magnitude of loads (250 kN/m, 300 kN/m, 324.1 kN/m and 330 kN/m), transverse displacements are observed. The loads are applied for 0.1s and the responses are observed till 0.25s. The target transverse displacement is set in accordance with Vol'mir's criterion as the value of thickness of the curved panel = 0.01m (Kubiak (2013)). The load which gives a displacement equal to or almost reaches the target value is the dynamic buckling load according to Vol'mir's criterion. According to Vol'mir's criterion, thin-walled structures when subjected to dynamic loads become unstable when the transverse displacement reaches the magnitude of thickness of the structure (Vol'mir (1974)). In certain structures, the abrupt change in displacement with respect to Budiansky-

Table 3 Dynamic buckling loads for laminated composite curved panel

Time (s)	Load (kN/m)	Displacement (m)
1	324	0.01010
0.1	324.1	0.01005
0.05	376.5	0.01001
0.01	1467.1	0.01001

Hutchinson criterion is not observed (Patel *et al.*, (2011)). In the present investigation also the un-bounded transverse or in-plane displacement is not observed. Moreover, the results obtained by Vol'mir criterion are within 10 percent variation with the results obtained by Budiansky-Hutchinson as per Table 6.13 of the monograph authored by Kubiak (2013). Hence in the current study, Vol'mir's criterion is used.

It is important to mention here is that according to Vol'mir's criterion during the transverse vibration of the plate due to the application of in-plane dynamic load, the plate dynamically buckles if the amplitude of transverse vibration reaches the magnitude of thickness even once, however additional criteria in the literature might not support this conclusion. Hence, the results obtained using Vol'mir's criterion may further be validated by other criteria to be sure, whether the dynamic buckling is actually occurring or not. Further, for in-depth understanding, experimental investigations are quite necessary. The authors will focus on these points in their future work.

The static buckling load for this panel is 337.51 kN/m, whereas the dynamic buckling load is 324.1 kN/m with rectangular pulse load of duration 0.1s.

It is observed from Table 3 that for short durations of application of load (like 0.01 s), the dynamic buckling load is higher than static buckling load (337.51 kN/m). However, as the loading duration is increased, the dynamic buckling load starts decreasing.

3.2.1 Effect of aspect ratio

The effect of aspect ratio is studied on the same laminated composite curved panel in this section. The ratio b/a is increased, keeping a constant and non-linear dynamic buckling loads are calculated. The pulse loads are applied for various durations. The dynamic buckling loads for laminated composite curved panels with different aspect ratios and various loading durations are shown in Fig. 8.

It is observed that the variation in dynamic buckling loads is significant till $b/a = 2$. Further increase in this ratio does not affect the dynamic buckling load much. The dynamic buckling loads for loading durations of 1s and 0.1s are very close to the static buckling load signifying quasi-static load. Figure 9 shows the deformed shape of the laminated composite curved panel with $b/a = 2$ at critical point of loading and deformation scale factor = 10. The scale factor is chosen so as to visualise the deformation in a better way.

3.2.2 Effect of radius of curvature

The effect of radius of curvature is studied on the same laminated composite curved panel for non-linear dynamic

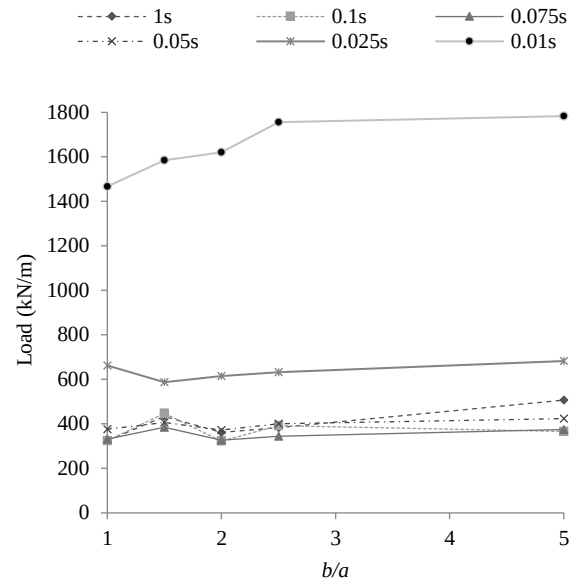


Fig. 8 Non-linear dynamic buckling load vs b/a plot for various loading durations. $R/a = 20$ and $a/h = 100$

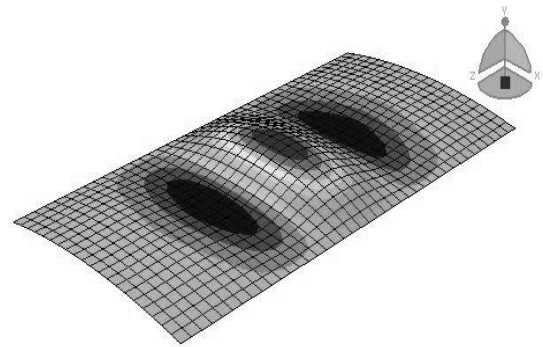


Fig. 9 Deformed shape of laminated composite curved panel with $b/a = 2$, $R/a = 20$ and $a/h = 100$

buckling behaviour in this section. The ratio R/a is varied along with loading duration. Figure 10 shows the non-linear dynamic buckling loads for laminated composite curved panels with different radius of curvatures and various loading durations.

It is observed that for a particular loading duration, the variation in dynamic buckling load is not significant. It is significant to note that for large R/a ratios, the non-linear dynamic buckling loads are very close to the static buckling loads. Figure 11 shows the deformed shape of laminated composite curved panel with $R/a = 5$, at critical point of loading and deformation scale factor = 12.

3.2.3 Effect of thickness of the panel

The effect of thickness of the panel is studied in this section. Keeping the number of plies same (5 nos.); the thickness is varied. The ratio a/h is varied keeping a constant. In order to have a better understanding of the effect of thickness, the analysis is carried out for two panels with different radius of curvatures. One with $R/a = 20$ and other with $R/a = 5$. In both the cases, aspect ratio (b/a) is

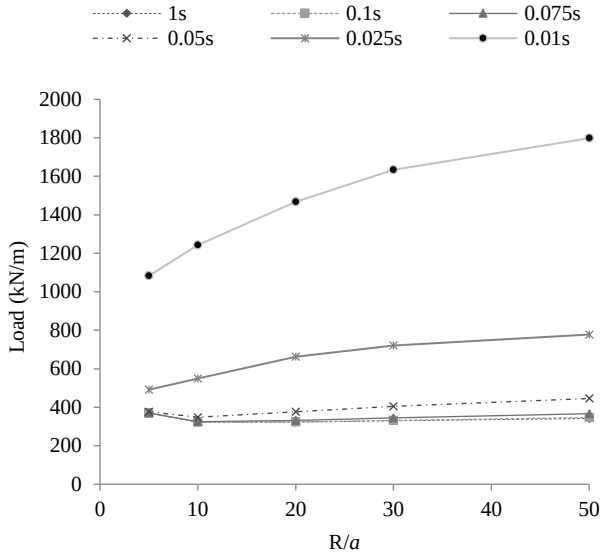


Fig. 10 Non-linear dynamic buckling load vs R/a plot for various durations of loading. $a/b=1$ and $a/h=100$

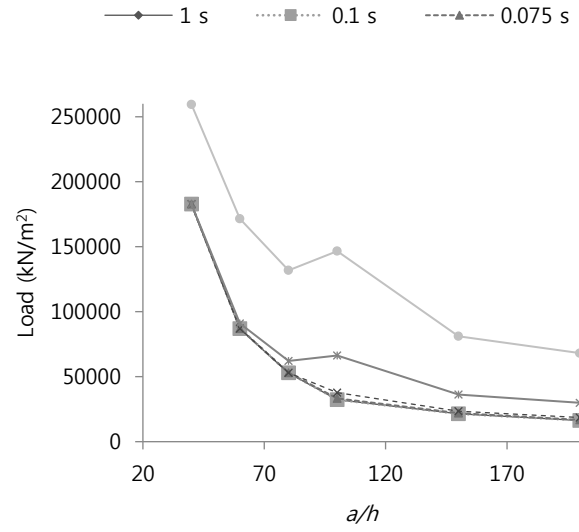


Fig. 13 Non-linear dynamic buckling load vs a/h plot of laminated composite curved panel with $R/a = 5$ for various loading durations and $a/b=1$

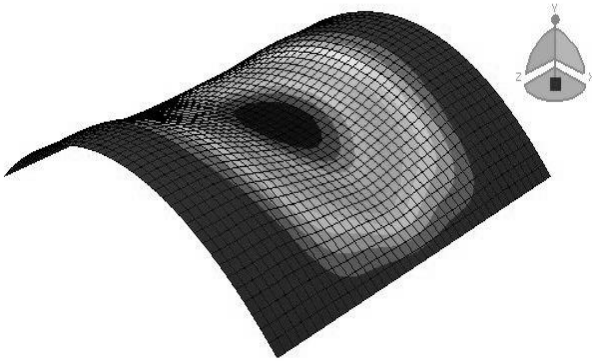


Fig. 11 Deformed shape of laminated composite curved panel with $R/a = 5$ and $b/a = 1$

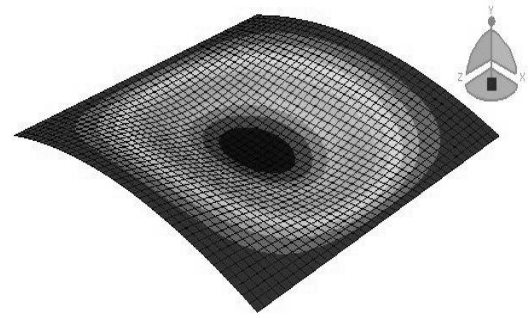


Fig. 14 Deformed shape of laminated composite curved panel with $R/a = 20$, $a/h = 40$ and $a/b = 1$

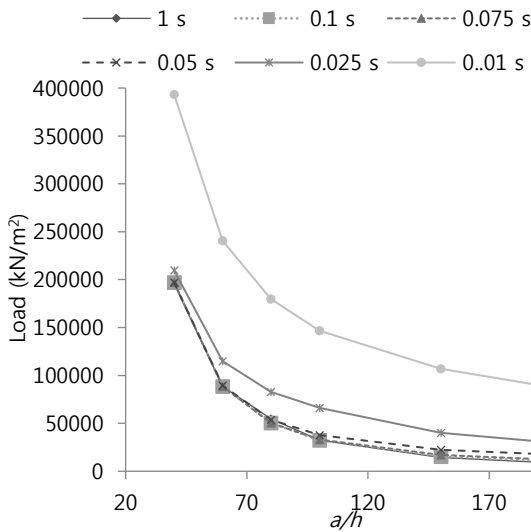


Fig. 12 Non-linear dynamic buckling load vs a/h plot of laminated composite curved panel with $R/a = 20$ for various loading durations and $a/b=1$

kept 1. Figure 12 shows the variation of load for different a/h ratios and for various loading durations for $R/a = 20$. Figure 13 shows the variation of load for different a/h ratios and for various loading durations for $R/a = 5$.

The applied load is a line load (kN/m). To be able to compare the results, the loads are divided with their corresponding thicknesses. Therefore, we calculate the dynamic buckling pressure (expressed in kN/m^2). The panel with $a/h = 40$ is a very thick and hence the non-linear dynamic buckling loads are very high even for longer durations of loading. As the ratio increases, the dynamic buckling loads reduce gradually. It is seen from Fig.12 that decrease in the thickness of the curved panel, the non-linear dynamic buckling load also decreases for all loading durations in a uniform manner. However, for the panel with $R/a=5$, irregular variation in dynamic buckling load is observed for loading durations 0.025s and 0.01s (Fig. 13). This signifies that panels with greater curvature need to be designed carefully. Figure 14 shows the deformed shape of laminated composite curved panel with $R/a = 20$ and $a/h = 40$, at critical time of loading and deformation scale factor =15. Figure 15 shows the deformed shape of laminated composite curved panel with $R/a = 5$ and $a/h = 40$, at critical time of loading and deformation scale factor =15.

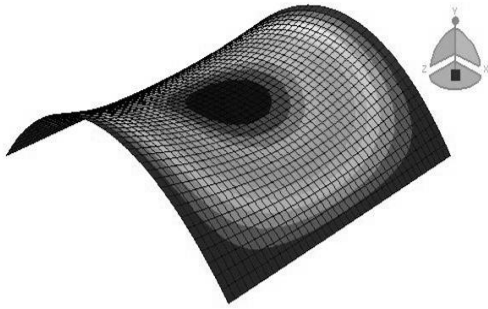


Fig. 15 Deformed shape of laminated composite curved panel with $R/a = 5$, $a/h = 40$ and $a/b = 1$

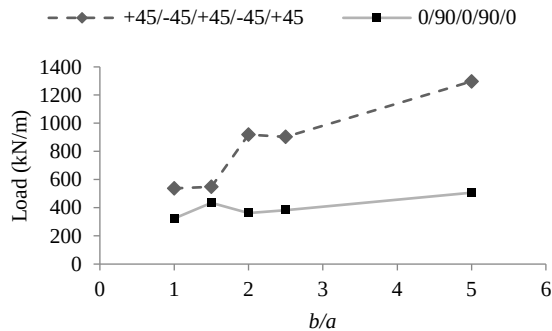


Fig. 16(a): Non-linear dynamic buckling load vs b/a plot for balanced and cross-ply laminates. Loading duration = 1 s. $R/a = 20$ and $a/b = 1$

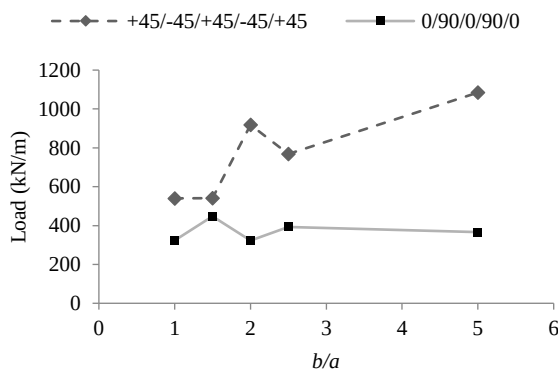


Fig. 16(b): Non-linear dynamic buckling load vs b/a plot for balanced and cross-ply laminates. Loading duration = 0.1 s. $R/a = 20$ and $a/b = 1$

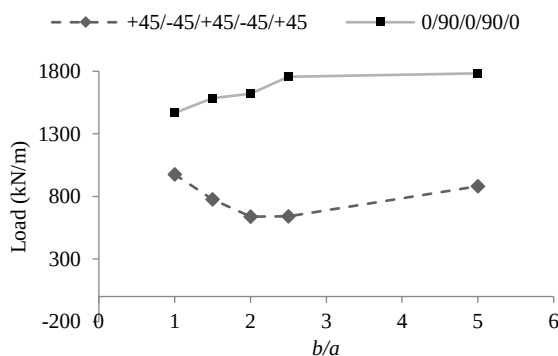


Fig. 16(c) Non-linear dynamic buckling load vs b/a plot for balanced and cross-ply laminates. Loading duration = 0.01 s. $R/a = 20$ and $a/b = 1$.

3.2.4 Effect of ply orientation

In this section, the effect of ply orientation is studied. Instead of cross-ply laminates, balanced laminates are used. The configuration is taken as $(+45^\circ/-45^\circ/+45^\circ/-45^\circ/+45^\circ)$. Since the loading is in the axial direction, this study will show the significance of orientation of plies for better response of the laminated composite curved panel. Figures 16(a)–16(c) show non-linear dynamic buckling loads for balanced laminates and cross-ply laminates for different b/a ratios and loading durations of 1 s, 0.1s, and 0.01s respectively. The R/a ratio is 20 and $a/h = 100$.

It is seen from the Fig. 16(c) that the dynamic buckling loads are relatively smaller when the ply orientation is changed. A peak is observed for $b/a = 2$, with 1s and 0.1s loading duration but for 0.01s loading duration, it is the lowest value. It is observed that for shorter durations of loading, balanced laminates have lower dynamic buckling loads. However, this trend changes once the loading duration is increased. The non-linear dynamic buckling loads are close for loading durations 1s and 0.1s.

Further, the effect of ply orientation on dynamic buckling load is studied keeping b/a ratio constant and varying R/a ratio for different loading durations. The aspect ratio (b/a) is kept 1 and the R/a ratios 5, 20 and 50 are taken. Figures 17(a)–17 (c) show the non-linear dynamic buckling loads for balanced laminates and cross-ply laminates for different loading durations with R/a ratios 5, 20 and 50 respectively.

With increase in R/a ratio, the dynamic buckling load keeps increasing. This is because the curvature keeps decreasing with increase in R/a ratio. Comparing panels made up of balanced laminates and panels made up of cross-ply laminates, balanced laminates have higher dynamic buckling load when the loading duration is greater than 0.025s, but have a lower value when loading duration is lesser than 0.025s. Panel made up of balanced laminates with $R/a = 5$ is a cylindrical panel, it shows a negligible variation in dynamic buckling load for various durations of loading.

4. Conclusion

The conclusions of this study can be made as follows,

- The non-linear dynamic buckling loads can be lower than static buckling loads for longer durations of loading.
- Panels with aspect ratio (b/a) greater than 1.5 have negligible effect on non-linear dynamic buckling loads.
- The effect of aspect ratio and effect of curvature is not prominent on the dynamic buckling load when the loading durations is more than 0.025s.
- With increase in R/a ratio, the panel becomes relatively flat, so the non-linear dynamic buckling load increases steadily. This effect is more prominent when the loading duration is lower than 0.025s.
- With decrease in thickness of the panel, the dynamic buckling pressure also decreases. However, panels with higher curvature need to be designed carefully when the loading duration is 0.025s and lower.
- Balanced laminates with different aspect ratios (b/a) have higher dynamic buckling load than cross-ply laminates when the loading duration is high.

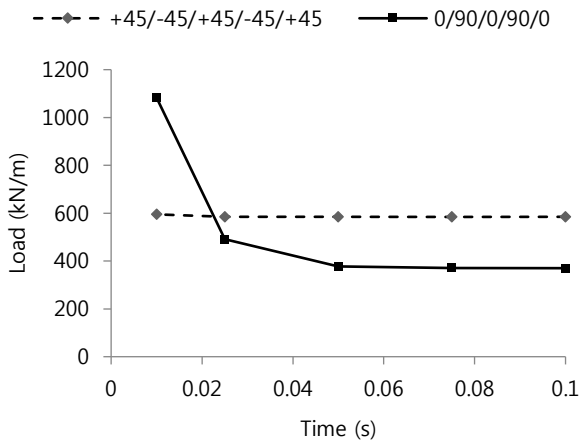


Fig. 17(a): Non-linear dynamic buckling load vs Time plot for balanced and cross-ply laminates for different loading durations. $R/a = 5$. $b/a = 1$ and $a/h=100$.

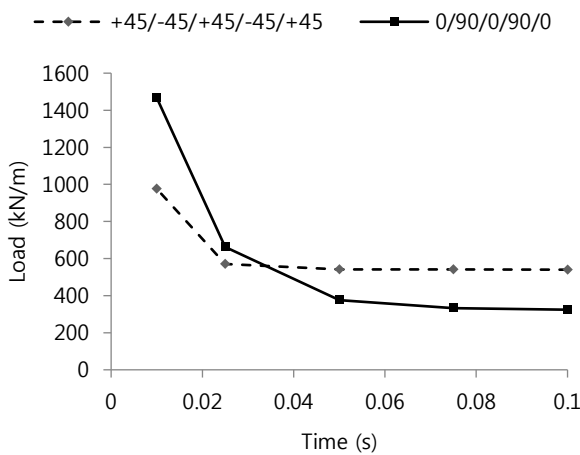


Fig. 17(b): Non-linear dynamic buckling load vs Time plot for balanced and cross-ply laminates for different loading durations. $R/a = 20$. $b/a = 1$ and $a/h=100$

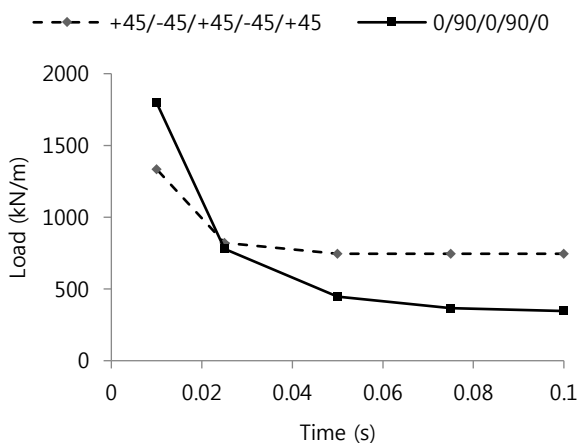


Fig. 17(c) Non-linear dynamic buckling load vs Time plot for balanced and cross-ply laminates for different loading durations. $R/a = 50$. $b/a = 1$ and $a/h=100$

- When the loading duration is 0.025s, both balanced and cross-ply laminates have very close dynamic buckling loads

- Laminated composite curved panels comprising of balanced laminates have no variation in non-linear dynamic buckling load when the R/a ratio is 5.

- In the present investigation, all the results reported are based on Vol'mir's criterion, however additional criteria in the literature might not support these results. In addition to this, the experimental investigations are quite necessary for in-depth understanding of the dynamic buckling behaviour of curved panels. The authors will focus on these points in their future work.

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