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Transfer matrix formulations and single variable shear deformation theory for crack detection in beam-like structures

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Abstract. This study aims to estimate crack location and crack length in damaged beam structures using transfer matrix formulations, which are based on analytical solutions of governing equations of motion. A single variable shear deformation theory (SVSDT) that considers parabolic shear stress distribution along beam cross-section is used, as well as, Timoshenko beam theory (TBT). The cracks are modelled using massless rotational springs that divide beams into segments. In the forward problem, natural frequencies of intact and cracked beam models are calculated for different crack length and location combinations. In the inverse approach, which is the main concern of this paper, the natural frequency values obtained from experimental studies, finite element simulations and analytical solutions are used for crack identification via plots of rotational spring flexibilities against crack location. The estimated crack length and crack location values are tabulated with actual data. Three different beam models that have free-free, fixed-free and simple-simple boundary conditions are considered in the numerical analyses.

Keywords: crack detection; single variable shear deformation theory; transfer matrix formulations

1. Introduction

Cracks located in structural members may cause catastrophic failures of civil engineering and mechanical engineering applications. Presence of cracks lead to stiffness reduction of structural elements. Thus, the dynamic behaviour of a structure is changed when one or more structural members are damaged. For cracked beams, the forward problem that consists of free vibration analysis considering known crack location and properties was investigated in many studies (Ostachowicz and Krawczuk 1991, Khaji *et al.* 2009, Zhou *et al.* 2016, Zhou *et al.* 2016, Liu *et al.* 2017, Zhou and Abdel Wahab 2017, Rajasekaran and Khaniki 2018, Gillich *et al.* 2019). Besides forward approach, crack detection of beam-like structures has been an attractive research area for many years as identification of cracks plays a very important role in structural health monitoring (SHM). One of the effective non-destructive tests for SHM is a vibration-based method such as modal analysis. Rizos *et al*. (1990) used an analytical approach for crack detection of a cantilever Euler-Bernoulli beam considering the crack as a massless rotational spring. Narkis (1994) applied analytical solution for inverse problem of cracked simply supported Euler-Bernoulli beams. Nandwana and Maiti (1997) obtained crack location and size of damaged stepped cantilever beams using finite element based natural frequencies. In this study, the inverse approach was based on plotting variation of spring stiffness of crack with crack location for first two or three modes. Lele and Maiti (2002) studied inverse problem of cracked cantilever Timoshenko beams. Nahvi and Jabbari (2005) performed crack detection analysis of a cantilever using experimental modal data and finite element model. Dansheng *et al*. (2007) used TBT for detection of cracks on a simply supported beam via anti-resonant frequencies. Barad *et al*. (2013) calculated crack locations and magnitudes of a cantilever Euler-Bernoulli beam by using plots of variation between crack/depth ratio and crack location. Kindowa-Petrova (2014) performed a crack detection analysis using Euler-Bernoulli beam theory (EBT) and analytical solution. In this study, it was seen that the accuracy of crack detection was strictly related to spring stiffness calculation approach. Fekrazadeh and Khaji (2017) applied a crack detection approach to simply supported Timoshenko beams using a test mass. Jena and Parhi (2015) used a modified particle swarm optimization (PSO) technique for crack detection of cantilever beams according to EBT. Mungla *et al*. (2016) investigated inverse problem of cracked Euler-Bernoulli beams under clamped-clamped boundary condition. Frequency-based analytical method was used in this study. Khiem and Huyen (2017) proposed a solution to detect single crack in functionally graded Timoshenko beam. Moezi *et al*. (2018) applied Cuckoo-Nelder-Mead optimization method for crack detection of cantilever beams. Khatir *et al*. (2016) applied BAT algorithm and Particle Swarm Optimization (PSO)

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technique for damage identification and localization of single and multiple cracked simply supported beams. Khatir *et al*. (2018) performed a crack identification analysis of a cantilever beam and plane frame structure using PSO method. In this study, the natural frequencies of intact and damaged beam model are calculated experimentally as well as finite element method (FEM). Rosales *et al*. (2009) applied crack detection approach to beam-like structures using power series technique and artificial neural networks (ANN). Khnaijar and Benamar (2017) proposed a new discrete model for forward and inverse problem of cracked beam vibrations. The popularity of non-destructive evaluation is not limited to beam-like elements. Nanthakumar *et al*. (2013) proposed an iterative method for inverse problem for detecting cracks in piezoelectric structures. Nanthakumar *et al*. (2016) solved inverse problem of detecting inclusion interfaces in a piezoelectric structure using level set method. Inverse analysis techniques are not limited to crack detection in structures. A potential modelling approach, which can be used for damaged structures, is based on IGA, e.g. Phung Van *et al*. (2019), Le *et al*. (2018) and Phung Van *et al*. (2018). Vu-Bac *et al*. (2018) applied NURBS-based inverse analysis for recovering the applied loads and deformations of thin shell structures considering kinematic and constitutive nonlinearities. Ghasemi *et al*. (2017) presented a methodology which was a combination of isogeometric analysis (IGA), level set and point wise mapping technique for topology optimization of flexoelectric materials. Ghasemi *et al*. (2018) proposed a design methodology for topology optimization of multi-material based flexoelectric composites. In recent studies, the extended IGA (XIGA), which is the combination of IGA and extended FEM was applied to crack detection and quantification of plate structures (Khatir and Abdel Wahab 2019). Khatir *et al*. (2019) presented results of a combined IGA-FEM approach for damage assessment of a free-free beam structure.

In the most of the studies concerned with vibrations of cracked beam-like structures, the beams are modelled according to EBT and TBT. It is known that the natural frequencies of Euler-Bernoulli beams are overestimated due to the assumption that a cross-section remains rigid and perpendicular to the axis of the beam. TBT, which considers shear deformation and rotational inertia provides more realistic results when compared to EBT. It should be noted that TBT needs a parameter called shear coefficient to reduce the error arised from assumption of constant shear stress distribution of cross-section (Han *et al.* 1999). Therefore, some researchers studied high-order beam theories that based on considering a realistic shear stress distribution because of cross-section of beams does not remain plane after bending (Levinson 1981, Bickford 1982, Reddy 1984, Heyliger and Reddy 1988). Due to complicated formulations and time consuming solutions of high-order beam and plate theories, a research area that focuses on simple and realistic beam and plate theories is arised. Shimpi (2002) presented a refined plate theory that aimed to consider shear and bending components of displacements. Shimpi *et al*. (2006) studied on two displacement based shear deformation theories involving two functions for bending of plates. Klouche *et al*. (2017) studied on an original SVSDT for buckling analysis of isotropic plates. Abdelbari *et al*. (2018) investigated bending analysis of thick beams using a single variable shear deformation model. Shimpi *et al*. (2017) presented a SVSDT that had parabolic shear stress distribution along cross-section of beam element. The SVSDT did not need a shear correction factor and provided a fourth order governing equation of motion. Bozyigit and Yesilce (2018) investigated natural frequencies of multi-bay and multi-storey frames using SVSDT. The SVSDT was applied to harmonic response analysis of fixed supported multi-storey frame structures by Bozyigit and Yesilce (2018). One of the key assumptions of SVSDT is considering transverse displacement as assembly of bending and shearing components. Based on this assumption, a nonlocal shear deformation beam theory was applied to bending, buckling and vibrations of FG nanobeams (Zemri *et al.* 2015). A nonlocal quasi-3D theory including shear deformation and thickness stretching effects was used for free flexural vibrations of FG nanobeams (Bouafia *et al.* 2017). Free vibration analysis of nanoscale beams was investigated by a general beam model based on Gurtin-Murdoch continuum surface elasticity theory (Youcef *et al.* 2018). A two-unknown trigonometric shear deformation beam theory was used for investigating the effects of moisture and temperature on free vibrations of FG nanobeams on elastic foundation (Mouffoki *et al.* 2017). Free vibration analysis of simply supported porous FG beam was investigated by using a higher order trigonometric shear deformation theory (Bourada *et al.* 2019). A nonlocal trigonometric shear deformation theory based on neutral surface position was developed for bending, buckling and vibration of FG nanobeams (Ahouel *et al.* 2016). A nonlocal hyperbolic refined plate theory was used for free vibrations of FG plates (Belkorissat *et al.* 2015). The wave propagation of FG plates was investigated using various high-order shear deformation plate theories(Ait Yahia *et al.* 2015). The free vibration analysis of FG sandwich plates was performed using a 3-unknown hyperbolic shear deformation theory which does not require a shear correction factor (Belabed *et al.* 2018). It should be noted that high order shear deformation theories would be more suitable in some cases such as laminated structures. However, SVSDT which has significantly simple formulations when compared to high order shear deformation theories, is an important alternative for a realistic vibration analysis of beam-like structures.

The research area about simple, accurate and effective beam theories like SVSDT is not limited to vibration analysis of structures. A novel simple two unknown beam theory which presents a parabolic shear stress distribution without a shear correction factor was used for post-buckling analysis of composite beams (Abdelhakim *et al.* 2018). The bending analysis of FG beams was investigated by using a simple two unknown hyperbolic shear deformation theory (Zidi *et al.* 2017). A simple three-unknown shear deformation theory was used for bending analysis of FG plate (Mohammed Sid Ahmed *et al.* 2016, Hachemi *et al.* 2017) and buckling analysis of graphene sheet (Mohammed Sid Ahmed *et al.* 2018).

The transfer matrix method (TMM) is an effective tool for free vibration analysis of beams and beam-assembly structures. The results of TMM are exact as the method uses analytical mode shapes. After obtaining local transfer matrices of beam segments, the global transfer matrix of whole vibrating system is constructed. The TMM has been used for free vibrations of different beam models using different beam theories (Lin and Chang 2005, Attar 2012, Wu and Chang 2013, Lee and Lee 2016, Lee and Lee 2017, Lee and Lee 2018, Lee and Lee 2018).

In this study, transfer matrix formulations are adapted to an analytical crack detection approach based on plots of variation of rotational spring flexibility with crack location using the first three natural frequencies. The SVSDT is applied to dynamic analysis of cracked beams by means of both forward and inverse problem. At first, natural frequencies of cracked beams are calculated using TMM and SVSDT. The relative errors of free vibration analysis using SVSDT and TBT are tabulated according to results in literature. Then, crack detection procedure is performed using SVSDT comparatively with TBT. Various numerical examples including beams have free-free, fixed-free and simple supported boundary conditions are given to demonstrate the effectiveness of proposed method. The accuracy of analytical based crack detection approaches are strictly related to beam theories. From this point of view, the authors are encouraged to study on analytical based crack detection of structures using a beam theory which considers a realistic shear stress distribution along cross-section unlike previous studies. The novelty of this study is based on application of SVSDT for crack detection in beam-like structures. Moreover, TMM is combined with SVSDT for an analytical based inverse problem for detecting cracks.

2. Theoretical model and formulation

One of the effective crack modelling approach in beam-like structures is using a linear rotational spring that divides beams into segments. By this way, the methods that use analytical formulations such as TMM becomes effectively applicable to vibration problems of cracked beams. A representation of a single cracked beam element using a linear rotational spring is presented in Fig. 1, where C_R is spring flexibility, b is width of the cross section, h is height of the cross section, L^* and L are location of crack and length of the beam, respectively. The spring flexibility C_R can be obtained by using Eqs. (1)-(2) as (Ostachowicz and Krawczuk 1991):

$$C_R = \frac{72\pi f(\alpha)}{Ebh^2} \tag{1}$$

$$f(\alpha) = 0.6384\alpha^2 - 1.035\alpha^3 + 3.7201\alpha^4 - 5.1773\alpha^5 + 7.553\alpha^6 - 7.332\alpha^7 + 2.4909\alpha^8 \tag{2}$$

where α is crack ratio (l_c/h), l_c is crack length, E is elastic modulus and $f(\alpha)$ is local compliance function calculated according to linear elastic fracture mechanics (Kindowa-Petrova 2014).

The following assumptions are considered in this study:
1. The material of beam is homogeneous and isotropic.
2. The cross-section of beam is uniform.
3. The behavior of beam is linear elastic.
4. The crack remains open under bending.
5. The damping is neglected.

It should be noted that crack representation in Fig. 1 does not limit the study for edge cracks located lower part of cross-sections. Whether the crack is at the top or bottom of cross-section does not change the result as equivalent spring flexibility is calculated by means of decrease in the elastic deformation energy at cross-section.

The total transverse displacement function of a beam in free vibration is defined as assembly of bending and shearing components according to SVSDT formulations as (Shimpi *et al.* 2017):

$$W^S = W_b + W_s \tag{3}$$

where W^S is total transverse displacement, W_b and W_s are displacement components of bending and shearing, respectively. The governing equation of motion of a beam in free vibration according to SVSDT is written as (Shimpi *et al.* 2017):

$$EI\frac{\partial^4 W_b}{\partial x^4} - \frac{\bar{m}I}{A}\left(1+\frac{12(1+\mu)}{5}\right)\frac{\partial^4 W_b}{\partial x^2 \partial t^2} + \bar{m}\frac{\partial^2 W_b}{\partial t^2} + \frac{\bar{m}^2 I}{A^2 E}\frac{12(1+\mu)}{5}\frac{\partial^4 W_b}{\partial t^4} = 0 \tag{4}$$

where x is beam coordinate, A is cross-sectional area, μ is Poisson's ratio, $\bar{m}$ is mass per unit length, I is area moment of inertia, t is time. It should be noted that the details of derivation of Eq.(4) are not presented herein to improve the readability of paper. The further details of SVSDT can be investigated in some papers about different applications of theory (Shimpi *et al.* 2017, Bozyigit and Yesilce 2018, Bozyigit and Yesilce 2018).

It is seen from Eq.(4) that the governing motion equation of SVSDT is constructed in terms of only bending component of total displacement. The solution of Eq.(4) gives the W_b function. Using separation of variables approach with the assumption of $W_b(x,t) = W_b(x)e^{i\omega t}$, Eq.(5) is obtained, where ω is natural angular frequency.

$$a_0\frac{d^4 W_b}{dz^4} + b_0\omega^2\frac{d^2 W_b}{dz^2} - c_0\omega^2 W_b(z) + d_0\omega^4 W_b(z) = 0 \tag{5}$$

where

$$a_0 = \frac{EI}{L^4}, b_0 = -\frac{\bar{m}I}{AL^2}\left(1+\frac{12(1+\mu)}{5}\right),$$

$$c_0 = \bar{m}, d_0 = \frac{\bar{m}^2 I}{A^2 E}\frac{12(1+\mu)}{5}, z = x/L$$

The solution of $W_b(z)$ is written as:

$$W_b(z) = \{D\}e^{isz} \tag{6}$$

where $\{D\}$ represents vector of integration constants.

$W_b(z)$ function can be rewritten as Eq.(7) using Eq.(6):

$$W_b(z) = (D_1 e^{is_1 z} + D_2 e^{is_2 z} + D_3 e^{is_3 z} + D_4 e^{is_4 z}) \tag{7}$$

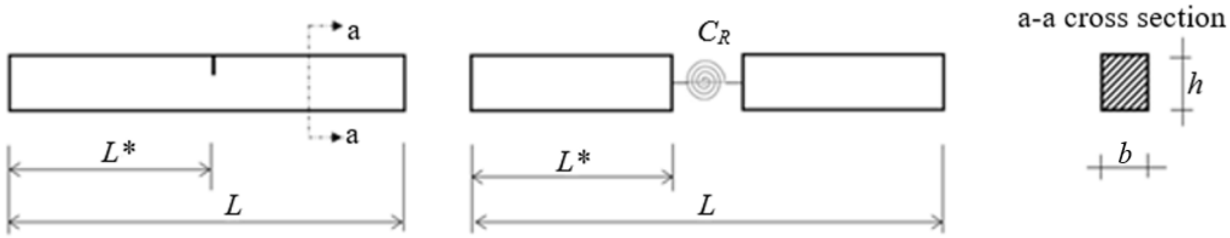

Fig. 1 Representation of a single cracked beam element by means of a linear rotational spring

where s_1, s_2, s_3, s_4 are characteristic roots of the equation that can be calculated by substituting Eq.(6) into Eq.(5).The bending component of slope function can be written as follows:

$$\frac{dW_b}{dz} = (is_1D_1e^{is_1z} + is_2D_2e^{is_2z} + is_3D_3e^{is_3z} + is_4D_4e^{is_4z}) \quad (8)$$

The bending moment function $M^S(z)$ and shear force function $Q^S(z)$ are defined in SVSDT as Eq.(9) and (10), respectively (Shimpi *et al*. 2017).

$$M^S(z) = -\frac{EI}{L^2}\frac{d^2W_b}{dz^2} \quad (9)$$

$$Q^S(z) = -\frac{EI}{L^3}\frac{d^3W_b}{dz^3} - \frac{\bar{m}I\omega^2}{AL}\frac{dW_b}{dz} \quad (10)$$

By using Eq. (7), $M^S(z)$ and $Q^S(z)$ functions can be rewritten as:

$$\begin{aligned} M^S(z) = (&Hs_1^2D_1e^{is_1z} + Hs_2^2D_2e^{is_2z} \\ &+ Hs_3^2D_3e^{is_3z} + Hs_4^2D_4e^{is_4z}) \end{aligned} \quad (11)$$

$$\begin{aligned} Q^S(z) = &(Jis_1^3 - Kis_1)D_1e^{is_1z} + (Jis_2^3 - Kis_2)D_2e^{is_2z} \\ &+(Jis_3^3 - Kis_3)D_3e^{is_3z} + (Jis_4^3 - Kis_4)D_4e^{is_4z} \end{aligned} \quad (12)$$

where $H = EI/L^2, J = EI/L^3, K = \left(\bar{m}I\omega^2\right)/(AL)$

W_S and W^S functions can be given in final form as Eqs. (13) and (14), respectively.

$$W_s = T\left(-H\frac{d^2W_b}{dz^2} - PW_b(z)\right) \quad (13)$$

$$\begin{aligned} W^S = &\left(THs_1^2 - TP + 1\right)D_1e^{is_1z} + \left(THs_2^2 - TP + 1\right)D_2e^{is_2z} \\ &+\left(THs_3^2 - TP + 1\right)D_3e^{is_3z} + \left(NHs_4^2 - TP + 1\right)D_4e^{is_4z} \end{aligned} \quad (14)$$

where $T = \dfrac{12(1+\mu)}{5AE}; P = \left(\bar{m}I\omega^2/A\right)$

The total slope function is achieved as assembly of bending and shearing components as:

$$\begin{aligned} \frac{dW^S}{dz} = &\left(is_1 + TJis_1^3 - TRis_1\right)D_1e^{is_1z} + \left(is_2 + TJis_2^3 - TRis_2\right)D_2e^{is_2z} \\ &+\left(is_3 + TJis_3^3 - TRis_3\right)D_3e^{is_3z} + \left(is_4 + TJis_4^3 - TRis_4\right)D_4e^{is_4z} \end{aligned} \quad (15)$$

where $R=P/L$

3. Transfer matrix method (TMM) formulations

The transfer matrix of a beam element is constructed using force and displacement relations of two end ($z=0$ and $z=1$). The state vector (Z) of left-hand side ($z=0$) is written as:

$$\begin{Bmatrix} W^S \\ \dfrac{dW^S}{dz} \\ Q^S \\ M^S \end{Bmatrix}_{z=0} = \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \\ \eta_1 & \eta_2 & \eta_3 & \eta_4 \\ \kappa_1 & \kappa_2 & \kappa_3 & \kappa_4 \\ \varsigma_1 & \varsigma_2 & \varsigma_3 & \varsigma_4 \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{Bmatrix} \quad (16)$$

where $\lambda_n = \left(THs_n^2 - TP + 1\right), \eta_n = \left(is_n + TJis_n^3 - TRis_n\right)$,
$\kappa_n = (Jis_n^3 - Kis_n), \varsigma_n = Hs_n^2, (n = 1,2,3,4)$

Eq. (16) can be expressed in closed form by

$$\{Z\}_{z=0} = [T_0]\{D\} \quad (17)$$

where

$$[T_0] = \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \\ \eta_1 & \eta_2 & \eta_3 & \eta_4 \\ \kappa_1 & \kappa_2 & \kappa_3 & \kappa_4 \\ \varsigma_1 & \varsigma_2 & \varsigma_3 & \varsigma_4 \end{bmatrix}$$

The state vector of right-hand side ($z=1$) of beam element is given in Eq. (18).

$$\begin{Bmatrix} W^S \\ \dfrac{dW^S}{dz} \\ Q^S \\ M^S \end{Bmatrix}_{z=1} = \begin{bmatrix} e^{is_1}\lambda_1 & e^{is_2}\lambda_2 & e^{is_3}\lambda_3 & e^{is_4}\lambda_4 \\ e^{is_1}\eta_1 & e^{is_2}\eta_2 & e^{is_3}\eta_3 & e^{is_4}\eta_4 \\ e^{is_1}\kappa_1 & e^{is_2}\kappa_2 & e^{is_3}\kappa_3 & e^{is_4}\kappa_4 \\ e^{is_1}\varsigma_1 & e^{is_2}\varsigma_2 & e^{is_3}\varsigma_3 & e^{is_4}\varsigma_4 \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{Bmatrix} \quad (18)$$

Eq. (17) can be written in a simple form as:

$$\{Z\}z=1 = [T1]\{D\} \quad (19)$$

where

$$[T_1] = \begin{bmatrix} e^{is_1}\lambda_1 & e^{is_2}\lambda_2 & e^{is_3}\lambda_3 & e^{is_4}\lambda_4 \\ e^{is_1}\eta_1 & e^{is_2}\eta_2 & e^{is_3}\eta_3 & e^{is_4}\eta_4 \\ e^{is_1}\kappa_1 & e^{is_2}\kappa_2 & e^{is_3}\kappa_3 & e^{is_4}\kappa_4 \\ e^{is_1}\varsigma_1 & e^{is_2}\varsigma_2 & e^{is_3}\varsigma_3 & e^{is_4}\varsigma_4 \end{bmatrix}$$

The arbitrary constant vector $\{D\}$ can be derived as Eqs.(20) and (21) by using Eqs.(17) and (19), respectively.

$$\{D\} = [T_0]^{-1}\{Z\}_{z=0} \tag{20}$$

$$\{D\} = [T_1]^{-1}\{Z\}_{z=1} \tag{21}$$

The relationship between state vectors $\{Z\}_{z=0}$ and $\{Z\}_{z=1}$ can be obtained using Eqs. (20) and (21) as follows:

$$\{Z\}_{z=1} = [T_1]\,[T_0]^{-1}\{Z\}_{z=0} \tag{22}$$

$$\{Z\}_{z=1} = [T^*]\{Z\}_{z=0} \tag{23}$$

where $[T^*]=[T_1]\,[T_0]^{-1}$ and $[T^*]$ represents transfer matrix of beam element.

If the beam is divided into m sub-segments along its length, the global transfer matrix of the system can be obtained by a chain multiplication of transfer matrices of beam segments as:

$$[T_G^*] = [T^*]_m\,[T^*]_{m-1} \ldots [T^*]_2[T^*]_1 \tag{24}$$

where $[T_G^*]$ is global transfer matrix of whole vibrating system.

In this study, due to using a rotational spring for crack modelling, a discontinuity occurs between slopes of beam segments. Therefore, an additional matrix that represent the discontinuity of slope at crack location must be embedded in Eq.(24). The global transfer matrix of the system is expressed in Eq.(25) for a single cracked beam element (Attar 2012) where $[C^*]$ represents the discontinuity of slope at crack location.

$$[T_G^*] = [T^*]_2\,[C^*][T^*]_1 \tag{25}$$

where

$$[C^*] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & C_R \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

According to characteristics of TMM, Eq. (25) can be extended for multi-span beams or multi-stepped beams having different material properties in each section. However, the material of each section must be homogeneous. Any flexibility jumps due to local non-homogeneities may cause false crack detections.

It should be noted that the formulations of TMM for cracked Timoshenko beams can be derived without any difficulties using same approach. The details of analytical formulations and TMM approach for Timoshenko beams are not presented in this study to shorten the paper. Further details of TMM formulations for different types of Timoshenko beams can be found in open literature (Lin and Chang 2005, Wu and Chang 2013, Al Rjoub and Hamad 2017, El-Sayed and Farghaly 2017).

3.1 Solution of forward problem

After obtaining the global transfer matrix of system, a matrix reduction procedure is applied. The reduced global transfer matrices of beams having various boundary conditions are presented in Table 1.

Table 1 Reduced global transfer matrices for different boundary conditions

		Reduced Global Transfer Matrix
Boundary condition	Free-Free	$\begin{bmatrix} T_G^*(3,1) & T_G^*(3,2) \\ T_G^*(4,1) & T_G^*(4,2) \end{bmatrix}$
	Fixed-Free	$\begin{bmatrix} T_G^*(3,3) & T_G^*(3,4) \\ T_G^*(4,3) & T_G^*(4,4) \end{bmatrix}$
	Simple-Simple	$\begin{bmatrix} T_G^*(1,2) & T_G^*(1,3) \\ T_G^*(4,2) & T_G^*(4,3) \end{bmatrix}$

The natural frequencies of cracked beam models are calculated by equating the determinant of reduced global transfer matrices to zero. A root finding algorithm that based on a trial and error on interpolation can be used for calculation of roots. The sign change between trial values means that there must be root lying in this interval.

3.2 Solution of inverse problem

The measured first three natural frequencies of cracked beams are used to plot the variation of C_R versus crack location. Since the spring flexibility representing the crack is irrespective of the vibration modes, the plots should intersect at one point that represents actual crack location and rotational spring flexibility (Lele and Maiti 2002). After the crack is located with C_R, the crack length can be calculated by using Eqs. (1) and (2). There may be two intersection points representing possible crack positions if the boundary, loading and geometry conditions of beam model are symmetrical. In this case, for real applications, both symmetrical crack locations must be checked carefully to avoid false crack detection. It should be noted that the three plots cannot intersect on a point in some cases due to graphical procedures. To vanish this errors, origin of the intersection region should be chosen as actual crack location (Kindowa-Petrova 2014). However, in some cases, the plots of variation of C_R with crack location of first two natural frequencies are intersected at one common point. For this type of graphs, there is no need to add plot of third mode. Moreover, elastic modulus value must be updated using Eq. (26) for each mode to perform an accurate crack detection. This calibration procedure is also known as "zero-setting" (Nandwana and Maiti 1997, Lele and Maiti 2002, Kindowa-Petrova 2014).

$$E_n^{updated} = \left(\frac{\omega_{measured}}{\omega_{analytical}}\right)^2 E \tag{26}$$

where $\omega_{measured}$ is measured natural frequency of intact beam, $\omega_{analytical}$ is calculated natural frequency of intact beam by TMM and $E_n^{updated}$ is calibrated elastic modulus value, where n is mode number (n= 1, 2, 3).

The proposed crack detection approach can be used effectively in most of the common beam systems if number of crack is one. For multiple cracked beam systems, the analytical crack detection procedures using plots of C_R with crack location become unmanagable as the main idea is obtaining C_R for each simulated position of crack along beam axis. It should be noted that the proposed inverse problem solution is not suitable for multiple crack detection approaches contrary to optimization methods such as Particle Swarm Optimization (PSO) (Khatir *et al.* 2018) and Cuckoo-Nelder-Mead Optimization Algorithm (COA-NM) (Moezi *et al.* 2018). Moreover, due to need of chain multiplication of member global transfer matrices of structures in TMM, the proposed approach is inapplicable for crack detection of multi-bay and multi-story frames. However, TMM can be applied to both forward and inverse problem of single cracked one-bay one-story frame structures.

4. Experimental validation

For experimental validation, free vibration analysis and crack detection procedure is performed for a free-free beam using following data: L = 0.75 m, b = 0.025 m, h = 0.006 m, $E = 2\times10^8$ kN/m^2, μ = 0.3, $\overline{m}$ = 0.0012 kNs^2/m^2. The experiment was accomplished using a Bruel and Kjaer type 8206–001 impact hammer, a Bruel and Kjaer DeltaTron™ type 4507 accelerometer and a National Instruments NI 9234 analog-to-digital converter. The free-free boundary conditions were modelled by very flexible springs. A saw cut was used for damage generation on the beam (Khatir *et al.* 2018).

First four natural frequencies of the beam obtained from experiment (Khatir *et al.* 2018) and using SVSDT and TBT via TMM are presented in Table 2. The relative error of using SVSDT and TBT for intact and cracked beams can be seen from Fig. 2 where the relative errors of SVSDT are relatively higher for α = 0.5 (3 mm crack). Theoretically, SVSDT should provide more realistic results when compared to TBT as SVSDT considers a parabolic shear stress distribution along cross-section. However, the cross-section of beam used in experimental validation is very small. Therefore, for the thin beam used in the experiment, relatively high α values like 0.5 may provide imperfect results for SVSDT as effects of non-constant shear stress distribution on cross-sections are remarkable for thick beams.

It can be seen from Table 2 that TBT and SVSDT provide same natural frequencies for intact beam. However, the effects of SVSDT becomes observable for cracked beams by providing lower natural frequencies when compared to TBT. As it can be seen from Table 2, the 2nd and 4th frequencies are not effected by cracks. This situation is a result of location of crack as there is a node at the middle of free-free beam for 2nd and 4th modes, theoretically. Thus, experimentally measured natural frequencies (Khatir *et al.* 2018) are decreased by increasing crack depth for all modes. The relative error between experimental modal analysis and TMM results are acceptable as experiment conditions cannot ensure ideal boundary conditions, exact material properties, geometrical properties and crack length. The crack detection analysis of free-free beam model is performed using experimentally

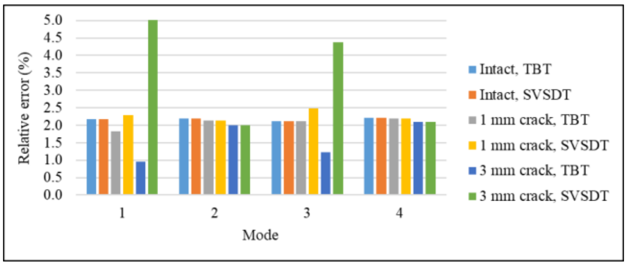

Fig. 2 The relative error between transfer matrix formulations and experimental study

Table 2 First four natural frequencies of beam model with free-free boundary conditions

Crack length (mm)	Crack location (m)	Natural frequency	f_1 (Hz)	f_2 (Hz)	f_3(Hz)	f_4(Hz)
Intact		TBT	55.5173	152.9684	299.6888	494.9719
		SVSDT	55.5173	152.9684	299.6888	494.9719
		Khatir *et al.* (2018)	**56.75**	**156.39**	**306.19**	**506.15**
1	0.375	TBT	55.3566	152.9684	299.0044	494.9719
		SVSDT	55.0918	152.9684	297.9062	494.9719
		Khatir *et al.* (2018)	**56.3835**	**156.3035**	**305.4699**	**506.0238**
3	0.375	TBT	53.7884	152.9684	292.6859	494.9719
		SVSDT	51.2230	152.9684	283.3435	494.9719
		Khatir *et al.* (2018)	**54.3088**	**156.0788**	**296.3372**	**505.5442**

measured natural frequency values in Table 2. The relations of spring flexibility values and crack locations are plotted for the first three natural frequencies (Fig. 3).

Using Fig. 3, the cracks of free-free beam model are localized by graphically detecting intersection of curves C_R vs L^* of first three modes. For the curves that do not a have a unique intersection point, the origin of intersection area is considered at crack location. Then, crack lengths are calculated using Eqs. (1) and (2) without any difficulty. The results of crack detection procedure according to TBT and SVSDT are presented in Table 3.

According to Table 3, SVSDT is more accurate in comparison with TBT for 1 mm crack length. However, TBT provides better crack detection results for 3 mm crack length. It is clearly seen from Table 3 that TMM formulations provides significantly better results for crack localization when compared to crack length prediction according to experimental modal analysis results of free-free beam model (Khatir *et al.* 2018). It should be noted that analytically calculated natural frequencies of modes 2 and 4 are not influenced by crack length as crack is located on a node for 2nd and 4th mode. However, especially for symmetrical beams by means of boundary conditions, the proposed analytical based crack detection approach needs C_R vs L^* plots for at least three modes to prevent false crack localization. Thus, 2nd natural frequency values are taken

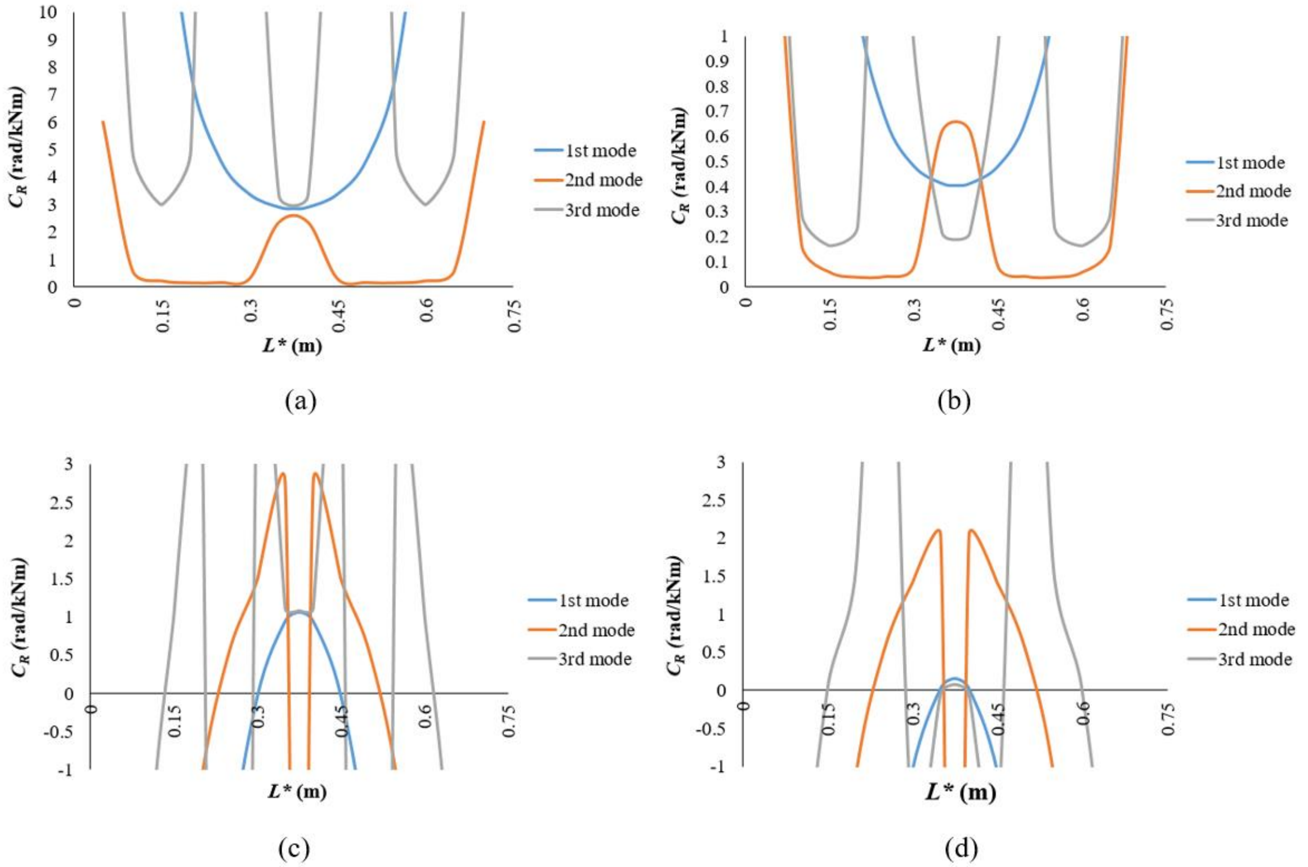

Fig. 3 a) Plots of C_R vs L^* using TBT (3 mm crack); b) Plots of C_R vs L^* using TBT (1 mm crack); c) Plots of C_R vs L^* using SVSDT (3 mm crack); d) Plots of C_R vs L^* using SVSDT (1 mm crack)

into account even the change due to crack is negligibly small or zero. The plot of C_R vs L^* for 4th mode is unnecessary in this case as the intersection point of three curves is revealed. For unsymmetrical beams like cantilevers, even C_R vs L^* plots for first two modes intersect on a common point. Plotting similar curves for higher modes is not necessary.

5. Numerical case studies

5.1 Simply supported beam

In the first numerical case study, a simply supported beam model is considered for free vibrations and crack detection approach using the following beam properties (Kindowa-Petrova 2014): L = 3 m, b = 0.18 m, h = 0.30 m, $E = 2.1\times10^8$ kN/m^2, μ = 0.3, $\bar{m}$ = 0.424 kNs^2/m^2.

The first three natural frequencies of the beam calculated using finite element method (FEM) (Kindowa-Petrova 2014) and using SVSDT and TBT via TMM are presented in Table 4. The relative error between TMM formulations and FEM can be seen in Fig. 4 for simply supported beam model.

Table 4 shows that the agreement between SVSDT and FEM are better in comparison with agreement between TBT and FEM (Kindowa-Petrova 2014). Natural frequencies of simply supported beam are decreased with cracks. For intact beam, there is perfect agreement between TBT and SVSDT. However, SVSDT provides more realistic result when cracks are considered.

Table 3 Actual and predicted crack properties using TBT and SVSDT for free-free beam model

		Crack Properties			
Case	Theory	Actual length (mm)	Predicted length (mm)	Actual location (mm)	Predicted location (mm)
1	TBT	1	1.38	375	330
2		3	3.2	375	375
1	SVSDT	1	0.82	375	360
2		3	2.3	375	360

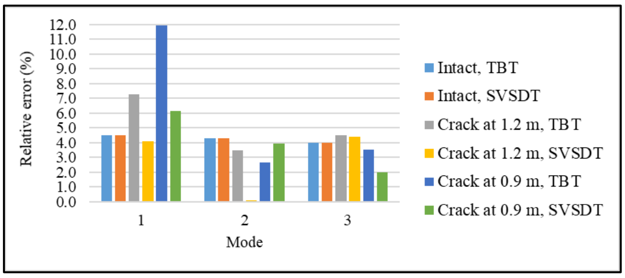

Fig. 4. The relative error between transfer matrix formulations and FEM (Kindowa-Petrova 2014) for simply supported beam

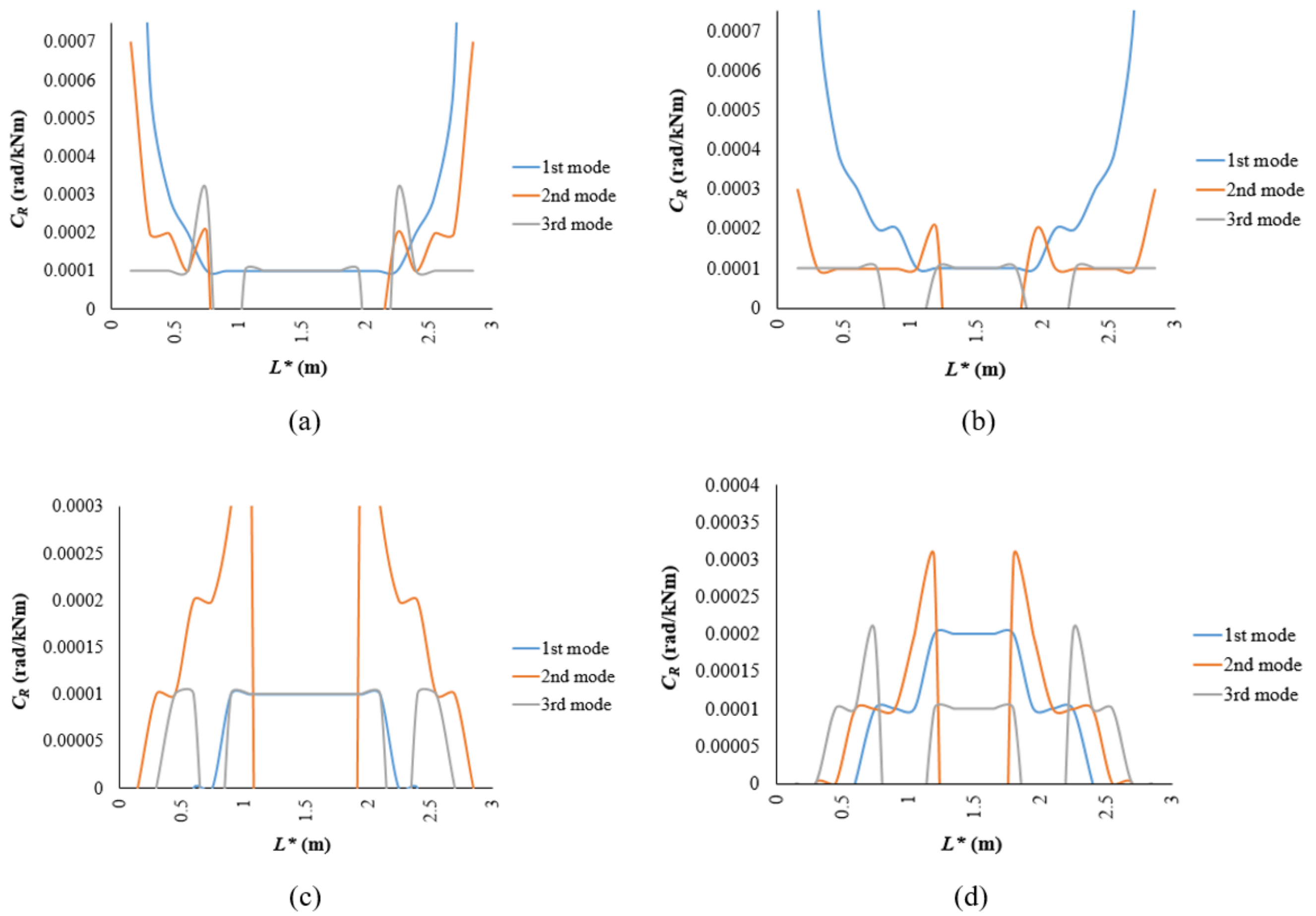

Fig. 5 a) Plots of C_R vs L^* using TBT (crack located at 0.9 m) b) Plots of C_R vs L^* using TBT (crack located at 1.2 m) c) Plots of C_R vs L^* using SVSDT (crack located at 0.9 m) d) Plots of C_R vs L^* using SVSDT (crack located at 1.2 m)

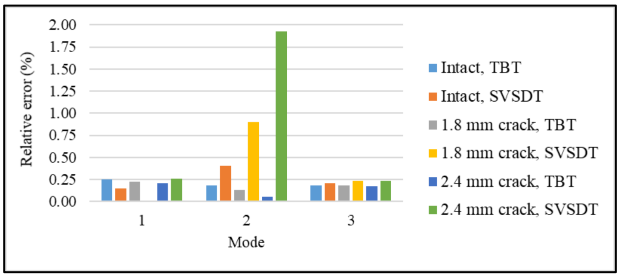

Fig. 6 The relative error between transfer matrix formulations and analytical solution (Jena and Parhi 2015) for cantilever beam

The relative errors presented in Fig.4 may be a result of modelling approach of FEM. An 8-node solid element model is used and the cross section is divided into 48 parts. The length of the beam is divided into 20 parts (Kindowa-Petrova 2014). Increasing element numbers of FEM modelling may decrease the relative error between results of TMM and FEM.

The natural frequency values presented in Table 4 are considered in the crack detection procedures of simply supported beam model. Fig. 5 represents the plots of spring flexibility vs crack location for the first three natural frequencies of simply supported beam. Using Fig. 5, the cracks of beam with simply supports are detected. The results of crack detection procedure according to TBT and SVSDT are presented in Table 5.

According to Table 5, SVSDT is more accurate in comparison with TBT for crack length identification of simply supported beams. On the other hand, the prediction of crack location using TBT provides better results for cracks located near middle of the simply supported beam.

5.2 Cantilever beam

The second numerical case study is based on a cantilever beam having the following properties (Jena and Parhi 2015): L = 0.8 m, b = 0.05 m, h = 0.006 m, E = 72.4×10^6 kN/m^2, μ = 0.33, $\overline{m}$ = 0.00084 kNs^2/m^2.

The calculated first three natural frequencies of the cantilever beam using analytical formulations according to EBT (Jena and Parhi 2015) and using SVSDT and TBT via TMM are presented in Table 6. The relative errors between TMM formulations for TBT, SVSDT and analytical solutions of EBT are presented in Fig. 6. Table 6 reveals that SVSDT provides lower natural frequencies compared with TBT. Increasing crack length cause a decrement in natural frequencies for all theories. As the results of reference study (Jena and Parhi 2015) are analytical, there is a perfect agreement between the results of TBT, SVSDT and EBT.

Using the natural frequency values in Table 6, the crack detection procedures of cantilever beam model is

Table 4 First three natural frequencies of beam model with simple-simple boundary conditions

Crack length (m)	Crack location (m)	Natural frequency	f_1 (Hz)	f_2 (Hz)	f_3(Hz)
Intact		**Present study (TBT)**	76.9066	293.8955	619.1764
		Present study (SVSDT)	76.9066	293.8955	619.1764
		Kindowa-Petrova (2014)	**80.5483**	**307.1372**	**644.9117**
0.15	1.2	**Present study (TBT)**	61.2250	276.0065	585.3560
		Present study (SVSDT)	63.3129	285.6195	585.7857
		Kindowa-Petrova (2014)	**66.0079**	**285.8582**	**612.8261**
0.15	0.9	**Present study (TBT)**	60.4433	250.3985	610.8526
		Present study (SVSDT)	64.4228	267.2371	611.6406
		Kindowa-Petrova (2014)	**68.6451**	**257.1785**	**633.2934**

Table 5 Actual and predicted crack properties using TBT and SVSDT for simply supported beam model

Case	Theory	**Crack Properties**			
		Actual length (cm)	Predicted length (cm)	Actual location (cm)	Predicted location (cm)
1	TBT	15	14.56	90	78
2		15	14.36	120	124
1	SVSDT	15	14.86	90	78
2		15	14.86	120	107

Table 6 First three natural frequencies of beam model with fixed-free boundary conditions

Crack length (mm)	Crack location (m)	Natural frequency	f_1 (Hz)	f_2 (Hz)	f_3(Hz)
Intact		**Present study (TBT)**	7.7019	48.2536	135.0521
		Present study (SVSDT)	7.6717	47.9709	134.5313
		Jena and Parhi (2015)	**7.6829**	**48.1669**	**134.8128**
1.8	0.4	**Present study (TBT)**	7.6891	47.9033	135.0513
		Present study (SVSDT)	7.6707	47.4121	135.1296
		Jena and Parhi (2015)	**7.6717**	**47.8427**	**134.8120**
2.4	0.4	**Present study (TBT)**	7.6772	47.5860	135.0506
		Present study (SVSDT)	7.6413	46.6634	135.1281
		Jena and Parhi (2015)	**7.6612**	**47.5604**	**134.8120**

Table 7 Actual and predicted crack scenarios using TBT and SVSDT for cantilever beam model

Case	Theory	**Crack Properties**			
		Actual length (mm)	Predicted length (mm)	Actual location (mm)	Predicted location (mm)
1	TBT	1.8	1.7	400	400
2		2.4	2.3	400	400
1	SVSDT	1.8	1.5	400	400
2		2.4	2.0	400	400

performed. Fig. 7 shows the variation of spring flexibility and crack locations for first two natural frequencies of cantilever beam. The comparison of actual and predicted crack properties can be seen from Table 7.

It can be seen from Fig. 7 that, for cantilever beam model, the plots of spring flexibility vs crack location of first two modes are good enough for crack detection as they are exactly intersected on a point unlike other numerical examples.

Table 7 shows that the TMM formulations perform a perfect crack localization for the cantilever beam model. For the crack length prediction, the error of SVSDT is high in comparison with TBT for both crack length values. The accuracy of crack detection in cantilever beam model is significantly better when compared to free-free and simply supported models. This result is not about boundary conditions. As proposed approach is analytical based, the difference between natural frequencies calculated analytically and natural frequencies used for detection (obtained values from experiment, FEM or analytical solution) is an important sign of accuracy of crack detection.

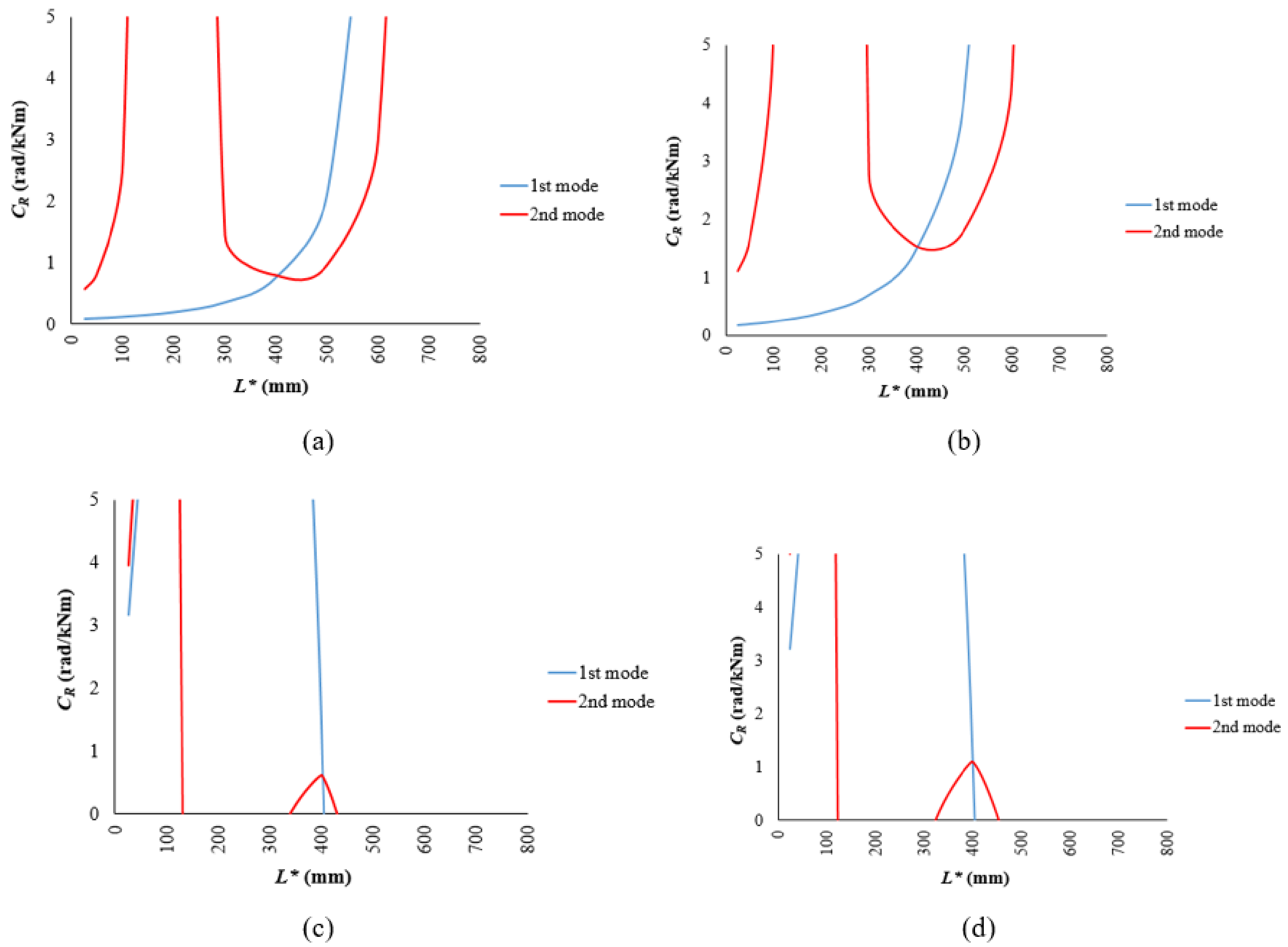

Fig. 7 a) Plots of C_R vs L^* using TBT (1.8 mm crack) b) Plots of C_R vs L^* using TBT (2.4 mm crack c) Plots of C_R vs L^* using SVSDT (1.8 mm crack) d) Plots of C_R vs L^* using SVSDT (2.4 mm crack)

6. Conclusions

The transfer matrix formulations are combined with an analytical crack detection approach based on plots of variation of crack location and spring flexibilities for cracked beam structures. The SVSDT is applied to both forward and inverse vibration problems of cracked beams. The results of SVSDT are presented comparatively with TBT results. Experimental data from literature were used to validate the proposed approach. Furthermore, numerical case studies were presented. It is seen that relative error of TMM on forward problem can be used a sign of accuracy for the inverse approach. The results indicate that proposed crack detection approach can be used effectively for different types of boundary conditions for beam-like structures. Moreover, it can be extended for beam assembly structures such as multi-span beams and simple frame structures.

The SVSDT provides more realistic results in comparison with TBT, theoretically. However, to observe the effects of parabolic shear stress distribution along cross-section, the beams should be relatively thick. The computational time of SVSDT is better than TBT for both forward and inverse problem. Moreover, EBT results can be obtained from SVSDT directly by ignoring shear deformation related parts of governing equation of motion. This advantage of SVSDT would be important for crack detection in thin beam-like structures.

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