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Numerical analysis of the receding contact problem of two bonded layers resting on an elastic half plane

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Abstract. The present study deals with the numerical analysis of the symmetric contact problem of two bonded layers resting on an elastic half plane compressed with a rigid punch. In this context, Finite Element Method (FEM) based software called ANSYS and ABAQUS are used. It is assumed that the elastic layers have different elastic constants and heights and the external load is applied to the upper elastic layer by means of a rigid stamp. The problem is solved under the assumptions that the contact between two elastic layers, and between the rigid stamp are frictionless, the effect of gravity force is neglected. To validate the constructed model and obtained results a comparison is performed with the analytical results in literature. The numerical results for normal stresses and shear stresses are obtained for various parameters of load, material and geometry and are tabulated and illustrated.

Keywords: contact mechanics; numerical analysis; normal stress; shear stress

1. Introduction

With the rapidly developing industrial technology, the demand for structural and mechanical elements formed from new kind of materials is increasing rapidly. Therefore, the studies on the mechanical behaviors of structural and mechanical elements with different solution techniques such as analytical, numerical and experimental methods have been the focus of interest of scientists (Avcar 2015 and 2016, Akbas 2016, Kolahchi *et al*. 2016, Abdelaziz *et al.* 2017, Kolahchi 2017, Zarei *et al*. 2017, Amnieh *et al*. 2018, Hajmohammad *et al*. 2018, Tigdemir *et al*. 2018, Achouri *et al*. 2019, Avcar 2019, Jassas *et al*. 2019). Most of these structural and mechanical components are in contact with each other which is the one of the methods of load transferring. The characteristics of the contact, the types of conduction of the stresses of bodies to the each other, strains occurred in contacting bodies, contact lengths and the distribution of the contact stress field plays vital role on the general behavior of the structure. Hence, the contact problem is the one of the most important problems encountered in engineering fields; metal forming processes, drilling problems, bearings, crash analysis of vehicles, tires, cooling of electronic devices, brakes, clutches, internal combustion engines, bush and ball bearings, hinges, gaskets are the some of the engineering applications. Besides, the contact problem has importance for biomechanics where human joints, implants or teeth are of consideration. Therefore, the large number of researches and efforts devoted for examining the contact problems for long years due to the technical importance (Wriggers 2003). The first investigation on the contact problem is performed by Hertz (1881) in which an analytical solution for the contact problem of two elastic bodies with curved surfaces is presented. In recent years contact problem still keeps its importance especially in engineering field and with the development of computer technologies, the numerical methods to solve contact problems have been developed with the aid of software. One of these methods is FEM which offers efficient results for contact problems considering a real geometry of interacting bodies, complex physical and mechanical properties of materials, mixed boundary conditions. Because of FEM is simple, easily adaptable to computations, applicable to large classes of geometries, materials and loading conditions, and presents quite accurate results, several studies on the contact problems have been examined using FEM. Chan and Tuba (1971a, b) presented a modified FEM for the problems of contacting elastic bodies and examined the effects of clearance, friction and load on the stresses in turbine blade fastenings. Fredricksson (1976) suggested a FEM of structural mechanics problems with surface nonlinearities arising from contact problems. Okamoto and Nakazawa (1979) dealed with the development of a theoretical method which gives a solution for nonlinear contact problems with irreversibility resulting from stick slip phenomenon. Oden and Pires (1984) presented the numerical analysis of contact problems with non-classical friction laws. Bathe and Chaudhary (1985) gave a solution procedure for the analysis of planar and axisymmetric contact-problems involving sticking, frictional sliding and separation under large deformations. Peric and Owen (1992) proposed a model based on the penalty method for 3D contact problems with friction. Klarbring and Björkman (1992) concerned with the formulation and numerical realization of large displacement contact problems with friction. Simo and

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Laursen (1992) described the augmented Lagrangian technique to frictional problems which is well-suited to finite element implementation. Yan and Fischer (2000) compared the contact pressure solution obtained using the FEM with that of the Hertz theory for the case of a standard rail, a crane rail and a switch. Hildi and Laborde (2002) concerned with the frictionless unilateral contact problem between two elastic bodies in a bidimensional context. Mohamed *et al.* (2006) developed a finite element model to simulate the frictional contact of elastic bodies. Ozsahin (2007) investigated the contact problem for an elastic layer resting on an elastic half plane according to the theory of elasticity with integral transformation technique. Wiest *et al.* (2008) studied the contact pressure distribution between a wheel and a rail crossing nose using the FEM. Liu *et al*. (2010) treated two-dimensional elastic contact problems, including normal, tangential, and rolling contacts using the FEM. Roncevic and Siminiati (2010) gave a description of the receding contact problem using NX Nastran software. Birinci (2011) examined the plane crack-contact problem for an infinite elastic layer with two symmetric rectangular rigid stamps on its upper and lower surfaces. Ayatollahi *et al*. (2014) simulated a four-layer road structure consisting of an edge transverse crack using three-dimensional FEM in order to capture the influence of a single-axle wheel load on the crack propagation through the asphalt concrete layer. Gandhi *et al*. (2014) considered the elastic-plastic frictionless spherical indentation analysis via the finite element analysis and experimentally. Stress intensity factors are numerically examined for interfacial edge crack between two dissimilar composite plates jointed with single side composite patch by Cetisli and Kaman (2014). Gandhi *et al.* (2015) investigated the impact of material dependency in elastic-plastic contact models by contact analysis of sphere and flat contact model and wheel rail contact model by considering the material properties without friction. Salem *et al.* (2015) investigated J integral for repaired cracks in plates with bonded composite patch and stiffeners with three-dimensional FEM. A numerical modeling of crack propagation in the cement mantle of the reconstructed acetabulum is examined by Benouis *et al.* (2015). Effects of temperature dependent material properties on mixed mode fracture parameters of functionally graded materials subjected to thermal loading are studied by Rajabi *et al* (2016). Turan *et al.* (2016) analyzed the functionally graded elastic layer resting on homogeneous elastic substrate under axisymmetric static loading. Belaasilia *et al.* (2017) studied on the application of the asymptotic numerical method for solving problems in solid mechanics. Deng *et al.* (2017) examined the FEM to investigate the accuracy and applicability of half-space-based methods. Karabulut *et al.* (2017) considered a receding contact problem for an elastic layer resting on a half plane in which the layer is pressed by two rectangular stamps placed symmetrically. Static frictional contact problems of double cantilever beam are analyzed by mathematical programming in the framework of scaled boundary FEM, in which the contact conditions can be expressed as the B-differential equations by Chaolei *et al.* (2018). Çömez *et al.* (2018) examined the plane receding contact problem for a functionally graded layer resting on two quarter-planes using the theory of linear elasticity. An enriched FEM technique is investigated for thermo-mechanical contact problem based on the extended FEM by Khoei and Bahmani (2018). Liu *et al.* (2018) considered the smooth receding contact problem between a homogeneous half-plane and a composite laminate composed of an inhomogeneously coated elastic layer. Rončević *et al.* (2018) presented a numerical analysis for a frictionless receding contact problem of a perfect-fit pin and bushing in a uniaxially loaded plate. Çömez (2019) considered the frictional contact problem of an orthotropic layer indented by a rigid moving cylindrical punch.

As distinct from above mentioned studies, this paper deals with the numerical analysis of the symmetric contact problem of two bonded layers resting on an elastic half plane compressed with a rigid punch using FEM based ANSYS and ABAQUS software. For this aim, the plane symmetric double receding contact problem of a rigid stamp and two infinite elastic layers with different elastic constants and heights is investigated. The external load is applied to the upper elastic layer by means of a rigid stamp. It is assumed that the contact surfaces are frictionless, the effect of gravity force is neglected. The obtained results are verified by comparing them with analytical results in literature. The numerical results for normal stresses and shear stresses are obtained for various quantities and tabulated and illustrated.

2. Numerical model and solution of problem

The plane symmetric double receding contact problem of a rigid punch and two infinite elastic layers with different elastic constants and heights is shown in Fig 1. Here, the stamp and upper layer are in contact in the interval $(-a, a)$ while the lower layer and half plane are in contact in the interval $(-b, b)$, and the layers are fully bonded to each other. The contact between the layer and the half-plane is assumed to be frictionless. The thickness of the upper layer and lower layer are h_1 and h_2, respectively and μ_i and ν_i $(i = 1,2,3)$ are elastic constants of the layers and half plane. The subscript $i(i = 1,2,3)$ refers to the layers and half planes, respectively. The loading is provided by a rigid punch with radius R subjected to concentrated normal force 2P. Thickness in z-direction is taken to be unit.

The problem is assumed as a two-dimensional contact problem. The material properties of the layers of the model are taken to be elastic and isotropic. The system is physically symmetrical in terms of geometrical, material properties and loading. Therefore, half of the problem geometry is modeled. In the analyses, geometric properties are taken as $L = 50m$ (length of the layer in x direction), $h_1 = 1m$ (thickness of the lower layer in y direction), the load $P = 120000N$ and material properties are taken as $E_1 = 3 \cdot 10^4 MPa$, $\nu_2 = \nu_1 = 0.25$. Other parameters are chosen such that $(G1/P/h_1))$, (R/h_1), $m=(G_2/G_1)$ and $n=(G_3/G_2)$ ratios are coherent with dimensionless values obtained from analytical solution, where G is shear modulus, R is punch radius. The geometry of model and the application of the load are shown schematically in Fig. 2.

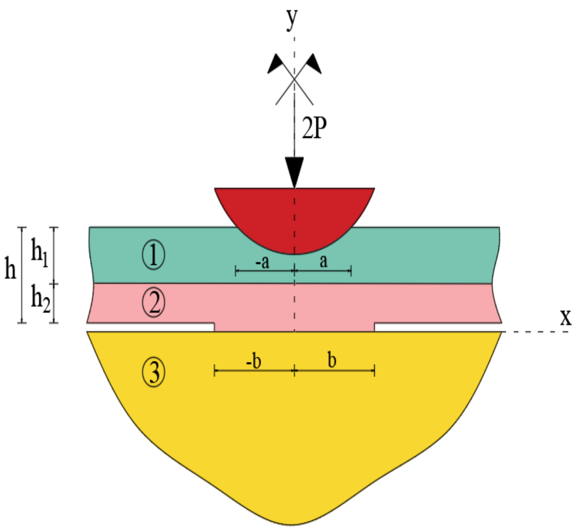

Fig. 1 Geometry of the contact problem

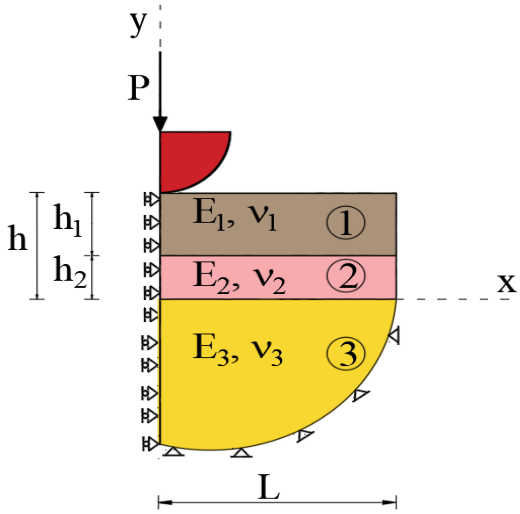

Fig. 2 The geometry for the analysis

Contact problems include two boundaries. One of these is defined as the contact surface and the other as the target surface. Surface-surface contact for those problems is very convenient. The steps of the contact analysis can be summarized as: (1) Modelling of 2D geometry model; (2) Identification of material properties; (3) Meshing; (4) Identification of contact pairs, identification of target surface and contact surface; (5) Application of boundary conditions and load steps; (6) Identification of solution options; (7) solution of contact problem; (8) obtaining and interpreting of results.

In the analysis, PLANE 183 is chosen as the element type for the creation of models for ANSYS software. The PLANE 183 element has 8 nodes and 2 degrees of freedom. The interaction between contact surfaces is modeled with surface to surface. CONTA172 and TARGE169 elements is used for contact modeling. Stiffness of punch has been created by defining a very large modulus of elasticity.

In analysis, element type RAX2 rigid element, which is 2-node linear axisymmetric rigid link is chosen for

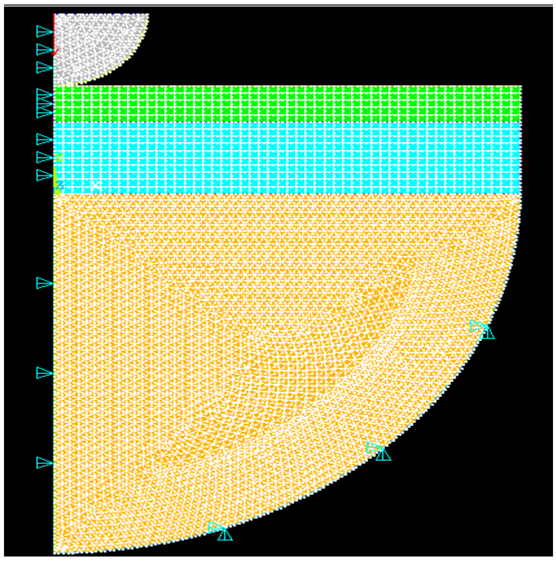

Fig. 3 The finite element mesh in ANSYS

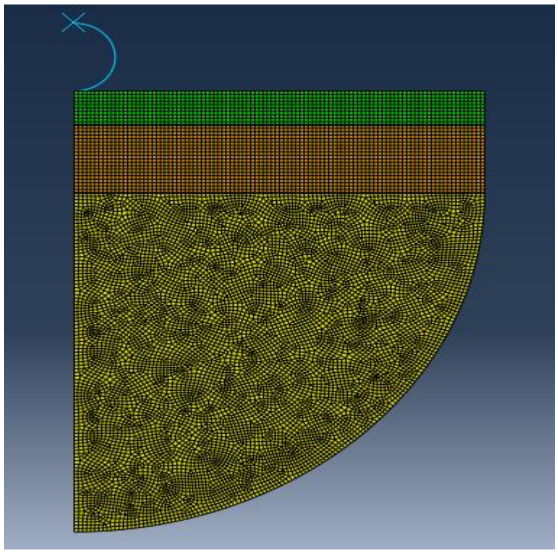

Fig. 4 The finite element mesh in ABAQUS

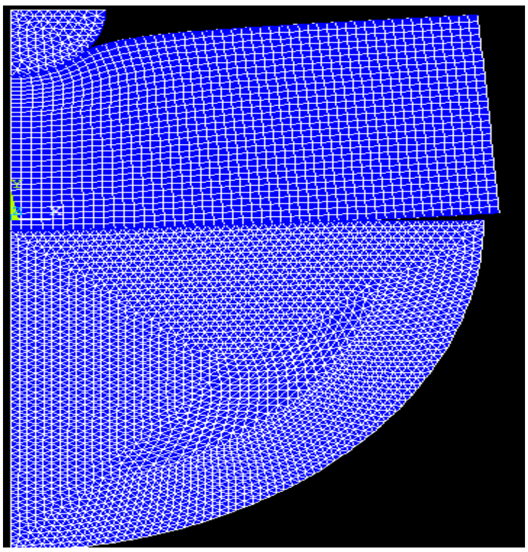

Fig. 5 Deformed shape of model in ANSYS

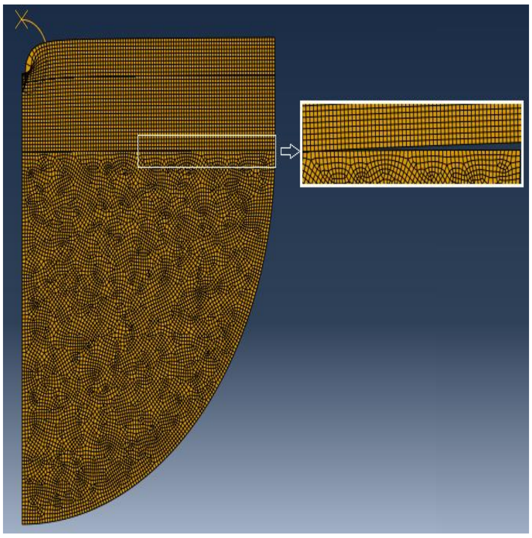

Fig. 6 Deformed shape of model in ABAQUS

Table 1 Comparison of the contact widths for various values of the ratio (h_1/h_2)

(h_1/h_2)	Source	(a/h_1)	(b/h_1)
	Adıbelli *et al.* (2013)	0.699	4.0612
2	Present(ANSYS)	0.69	4.00
	Present(ABAQUS)	0.68	4.00
	Adıbelli *et al.* (2013)	0.7083	2.7457
1	Present(ANSYS)	0.71	2.70
	Present(ABAQUS)	0.70	2.75
	Adıbelli *et al.* (2013)	0.7195	2.1071
0.5	Present(ANSYS)	0.73	2.00
	Present(ABAQUS)	0.71	2.10
	Adıbelli *et al.* (2013)	0.7365	1.6203
0.1	Present(ANSYS)	0.75	1.60
	Present(ABAQUS)	0.73	1.65

punch modeling for ABAQUS software. Element type CAX4R, which is 4-node bilinear axisymmetric quadrilateral, reduced integration, hourglass control, is selected for all other elements. Surface to surface contact type is used for contact and interaction modeling. The interaction between the rigid punch and the layer is modeled by defining the master surface and slave surface. The interaction between the elastic half-plane and the layer is similarly modeled. Tie is used as the constrain type between layers. Surface to surface is used as discretization method.

The finite element models of prior to analysis of the problem created in the ANSYS and ABAQUS software are shown in Fig. 3 and Fig. 4. The deformed shapes that occur after analysis of these models are shown in Fig.5 and Fig.6.

3. Results and discussion

In this section, normal and shear stresses are calculated for various dimensionless quantities, such as shear modulus factor-1, $(m = G_2/G_1)$, shear modulus factor-2, $(n = G_3/G_2)$, radius factor (R/h_1), load factor $(G_1/(P/h_1))$and height factor (h_2/h_1). Normal stresses (σ_x, σ_y) along the axis of symmetry *y (x=0)* and shear stress (τ_{xy}) near this axis *(x=0.05)*. Material constants in numerical applications are taken to be $(\kappa_1 = \kappa_2 = \kappa_3 = 2)$

Example 1: In the first example the present numerical results are compared with the analytical results of Adıbelli *et al.* (2013) to verify the accuracy of the present model. Here the following dimensionless quantities are used: $(\mu_1/\mu_2 = 1, \mu_3/\mu_2 = 1, R/h_2 = 250, \mu_2/(P/h_2) = 250)$ As seen from Table 1, the present results are in good agreement with the previously published ones which shows the adopted solution appears to be a good approximation method to solve actual contact problems.

Example 2: In the second example, the variations of dimensionless the normal stress (x-component), $\sigma_x/(P/h_1)$, along the axis of the symmetry for load factor, $G_1/(P/h_1)$, and shear modulus factor-2, *n*, are obtained and illustrated in Figs. 7 and 8, respectively. Here the following dimensionless quantities are used: $(h_2/h_1 = 2, m = n = 2, R/h_1 = 500)$ for Fig. 7 and $(h_2/h_1 = 2, m = 2, R/h_1 = 500, G_1/(P/h_1) = 100)$ for Fig. 8.

As seen from Fig. 7, the compressive stresses occur on the whole of the first layer while the compressive stresses occur in a small area of the upper part of the second layer. The value of dimensionless normal stress decreases and finally becomes zero at a point and then it changes sign and acts tensile stress to half a plane. Besides, only compressive stresses act on the half-plane due to the variation of the load. Furthermore, as the load increases, the value of the load factor decreases and so the value of dimensionless normal stress increases.

It is clear from Fig. 8 that the values of dimensionless normal stress are so close to the each other in the lower regions. Compressive stresses occur in the first layer and half-plane as well as the second layer includes tensile and compressive stress zones and two curves intersect at a value of tensile stress zone. Furthermore, it is found that the values of dimensionless normal stress increase with the increasing shear modulus factor-2.

Example 3: In the third example, the variations of the dimensionless normal stress(y-component),σ_y, along axis of symmetry are shown in Figs. 9-12.

Fig. 9 presents the variation of dimensionless normal stress along the axis of symmetry versus various values of load factor, $G_1/(P/h_1)$. Here the following dimensionless quantities are used: $(h_2/h_1 = 2, m = n = 2, R/h_1 = 500)$. It is found that dimensionless normal stress increases with the increasing load factor.

Fig. 10 illustrates the variation of the dimensionless normal stress along the axis of symmetry versus various values of the radius factor, *R/h₁*. Here the following dimensionless quantities are employed: $(h_2/h_1 = 2, m = n = 2, G/(P/h_1 = 100))$ It is observed that the values of dimensionless stress decreases with the increasing radius factor.

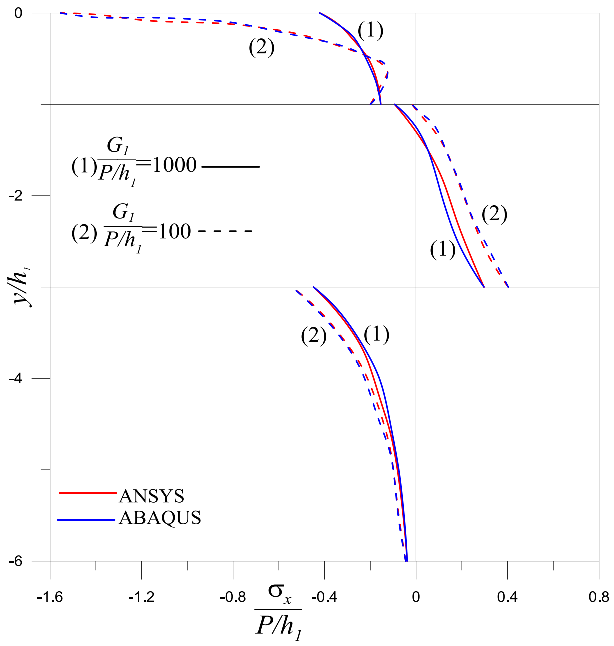

Fig. 7 The variation of the dimensionless normal stress (x-component) along the axis of symmetry versus load factor

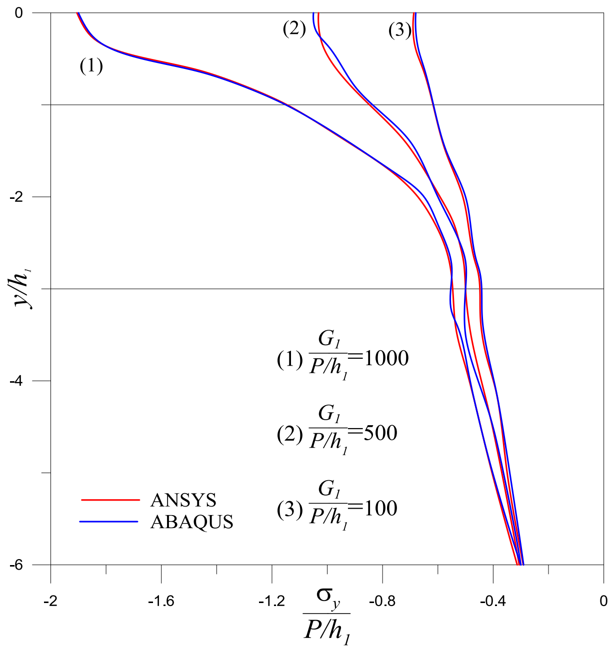

Fig. 9 The variation of dimensionless normal stress(y-component), along the axis of symmetry versus load factor

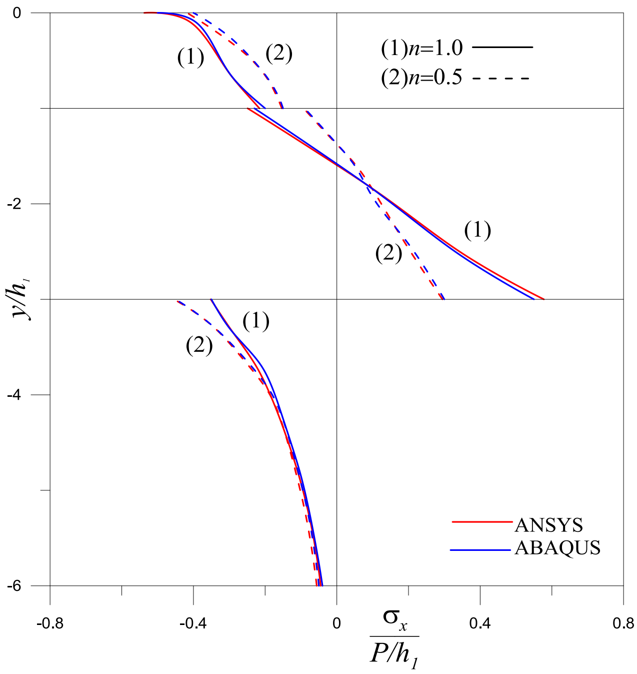

Fig. 8 The variation of the dimensionless normal stress (x-component), along the axis of symmetry versus shear modulus factor-2

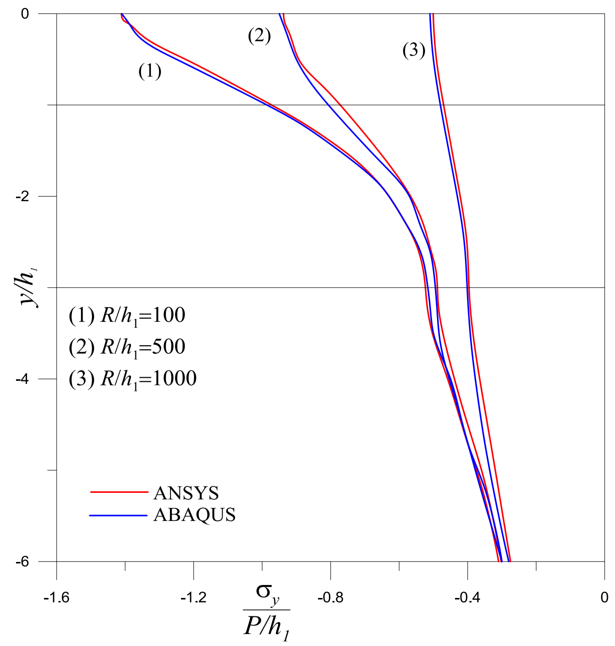

Fig. 10 The variation of the dimensionless normal stress (y-component), along the axis of symmetry versus radius factor

Fig. 11 plots the variation of dimensionless normal stress along the axis of symmetry for various values of shear modulus factor-1, *m*. Here the following dimensionless quantities are adopted: $(h_2/h_1 = 2, n = 2, R/h_1 = 500, G/(P/h_1 = 100))$. It is seen that the values of normal stress increase with the increasing shear modulus factor-1.

Fig. 12 shows the variation of the dimensionless normal stress along the axis of symmetry for various values of shear modulus factor-2, *n*. Here the following dimensionless quantities are taken into account: $(h_2/h_1 = 2, m = 2, R/h_1 = 500, G/(P/h_1 = 100))$. It is concluded that the values of normal stress decrease with the increasing shear modulus factor-2.

Example 4: Due the shear stress, (τ_{xy}), along the symmetry plane (x=0) equals to zero, the variations of the dimensionless shear stress, $\tau_{xy}(0.5, y)/(P/h_1)$, are investigated for the value of *x=0.5* in Figs. 13-16.

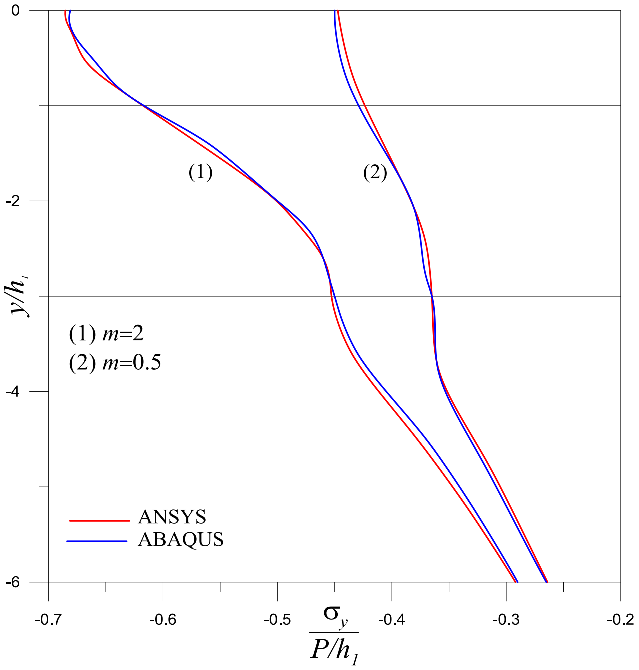

Fig. 11 The variation of the dimensionless normal stress (y-component) along the axis of symmetry versus shear modulus factor-1

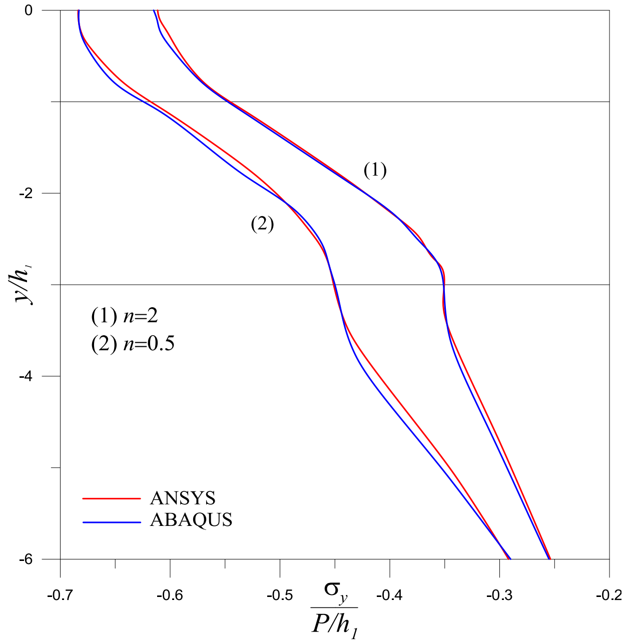

Fig. 12 The variation of the dimensionless normal stress (y-component) along the axis of symmetry versus shear modulus factor-2

Fig. 13 presents the variation of the dimensionless shear stress for various values of load factor, $G_1/(P/h_1)$. Here the following dimensionless quantities are employed: $(h_2/h_1 = 2, m = n = 2, R/h_1 = 500)$. It is found that the values of dimensionless shear stress decrease with the decreasing load factor.

Fig. 14 shows the variation of the dimensionless shear stress for various value of radius factor, R/h_1. Here the

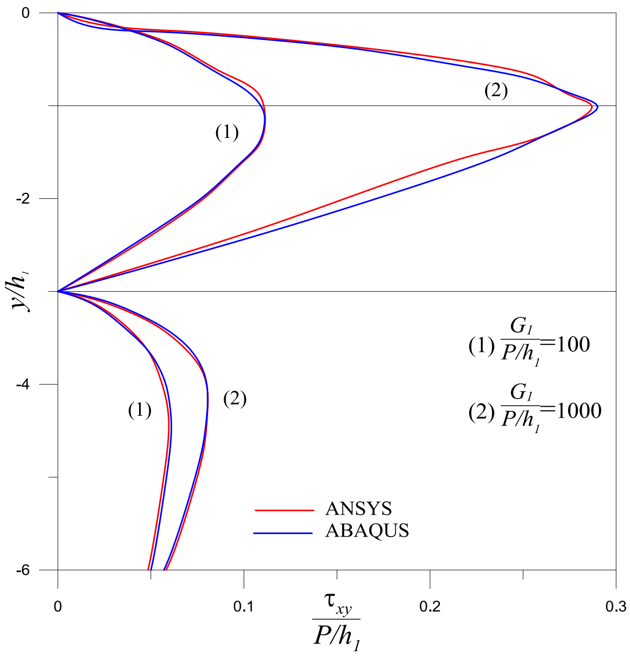

Fig. 13 The variation of the dimensionless shear stress versus load factor

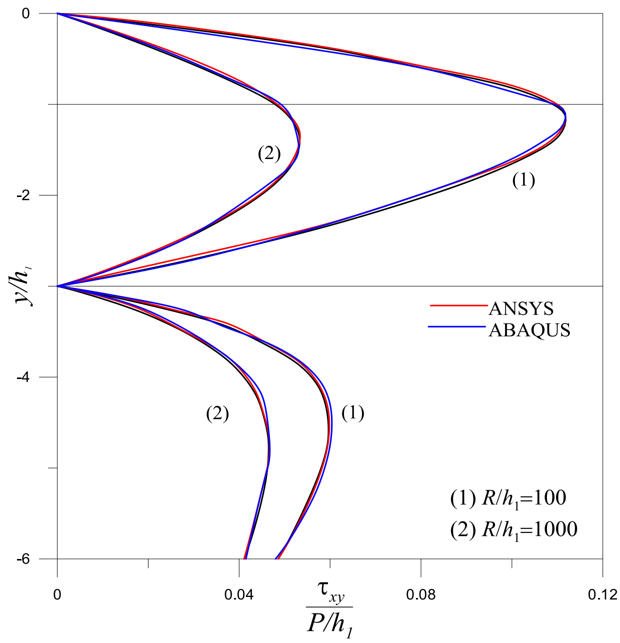

Fig. 14 The variation of the dimensionless shear stress versus radius factor

following dimensionless quantities are used: $(h_2/h_1 = 2, m = n = 2, G/(P/h_1 = 100))$. It is seen that the values of dimensionless shear stress decrease with the increasing radius factor.

Figs. 15 and 16 illustrate the variation of the dimensionless shear stress for various values of shear modulus factor 1 and 2. Here the following dimensionless quantities are used:
$(h_2/h_1 = 2, n = 2, R/h_1 = 500, G/(P/h_1 = 100))$ .It is observed that dimensionless shear stress increases with the increasing shear modulus factors.

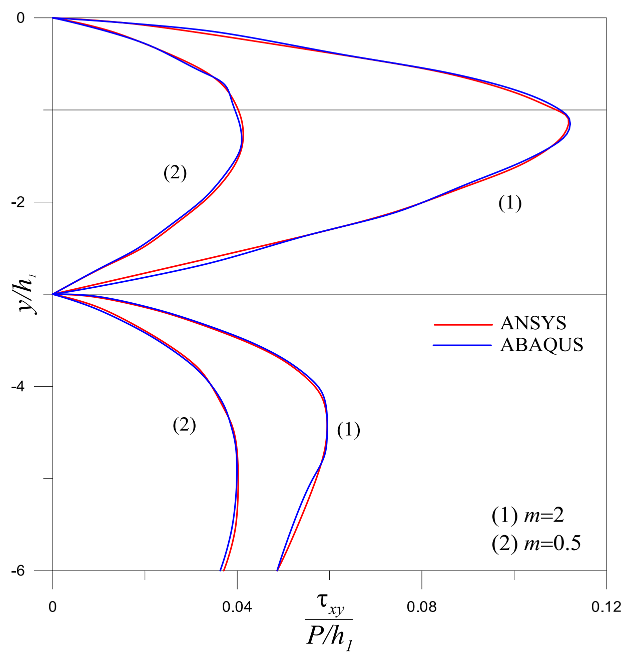

Fig. 15 The variation of the dimensionless shear stress versus shear modulus factor-1

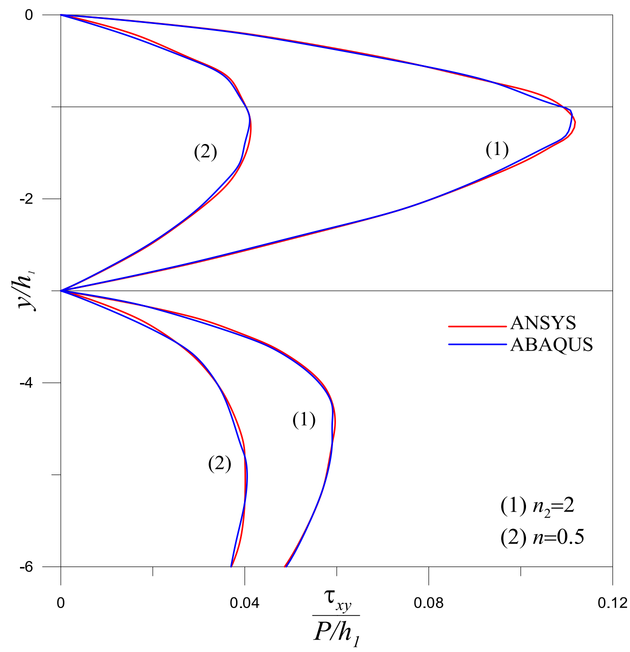

Fig. 16 The variation of the dimensionless shear stress versus shear modulus factor-2

4. Conclusion

A numerical analysis method for the symmetric contact problem of two bonded layers resting on an elastic half plane compressed with a rigid punch using FEM based ANSYS and ABAQUS software have been given in the present paper. For this aim, the plane symmetric double receding contact problem of a rigid stamp and two infinite elastic layers with different elastic constants and heights is investigated, in detail.

Briefly, the following results are obtained:

- Dimensionless normal stress (x-component), decreases with the increasing load factor while it increases with the increasing shear modulus factor-1.
- Dimensionless normal stress (y-component) increases with the increasing load factor while it decreases with the increasing radius factor.
- Dimensionless normal stress (y-component) increases with the increasing shear modulus factor-1 while it decreases with the increasing shear modulus factor-1.
- Dimensionless shear stress decrease with the decreasing load and increasing radius factors.
- Dimensionless shear stress increases with the increasing shear modulus factors.

Finally, it is concluded that the considered dimensionless quantities have significant influence on the normal and shear stress distributions.

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