Structural Engineering and Mechanics, *Vol. 72, No. 5 (2019) 653-673*
DOI: https://doi.org/10.12989/sem.2019.72.5.653

A new higher-order shear and normal deformation theory for the buckling analysis of new type of FGM sandwich plates

Sara Chelahi Chikr[1,2], Abdelhakim Kaci*[1,3], Redha Yeghnem[1, 3] and Abdelouahed Tounsi[3,4]

[1]Université Dr Tahar Moulay, Faculté de Technologie, Département de Génie Civil et Hydraulique, BP 138 Cité En-Nasr 20000 Saida, Algérie
[2]Laboratoire des Ressources Hydriques et Environnement, Université Dr Tahar Moulay, BP 138 Cité En-Nasr20000 Saida, Algérie
[3]Material and Hydrology Laboratory, University of Sidi Bel Abbes, Faculty of Technology, Civil Engineering Department, Algeria
[4] Department of Civil and Environmental Engineering, King Fahd University of Petroleum & Minerals, 31261 Dhahran, Eastern Province, Saudi Arabia

(Received March 26, 2019, Revised July 24, 2019, Accepted July 30, 2019)

Abstract. This work investigates a novel quasi-3D hyperbolic shear deformation theory is presented to discuss the buckling of new type of sandwich plates. This theory accounts for both shear deformation and thickness stretching effects by a hyperbolic variation of all displacements through the thickness. The enhancement of this formulation is due to the use of only five unknowns by including undetermined integral terms, contrary to other theories where we find six or more unknowns. It does not require shear correction factors and transverse shear stresses vary parabolically across the thickness. A new type of FGM sandwich plates, namely, both FGM face sheets and FGM hard core are considered. The governing equations and boundary conditions are derived using the principle of virtual displacements. Analytical solutions are obtained for a simply supported plate. The accuracy of the present theory is verified by comparing the obtained results with quasi-3D solutions and those predicted by higher-order shear deformation theories. The comparison studies show that the obtained results are not only more accurate than those obtained by higher-order shear deformation theories, but also comparable with those predicted by quasi-3D theories with a greater number of unknowns.

Keywords: buckling analysis; functionally graded plate; new type of sandwich plate; refined plate theory; novel quasi-3D theory

1. Introduction

In many applications such as aeronautics, automotive, railway or building, the ratio between mechanical rigidity in flexion and mass is paramount. When it comes to flat components, the sandwich structure is a very relevant solution because of their high rigidity and low weight as well as their energy absorption capacity. Sandwich panels, because of their high stiffness compared to their lightness, are more and more used in high performance products. Sandwich plates can be utilized to fabricate light structures with high strength / stiffness / weight ratios, noise, vibration and "*hardness insulation*", "*thermal insulation*", or construction of structures with discrete functional layers (Vinson 2001, Vinson 2005, Lindström and Hallström 2010, Dean *et al*. 2011). There are several types of sandwich structures. The most used structure is the sandwich structure with homogeneous faces and a homogeneous core (Librescu and Hause 2000). With the development of advanced materials, functionally graded materials (FGM) are materials in the family of engineering composites fabricated from a mixture of metal and ceramic (Meziane *et al*. 2014, Zidi *et al*. 2014, Hamidi *et al*. 2015, El-Haina *et al*. 2017, Sekkal *et al*. 2017a, Menasria *et al*. 2017, Mahmoudi *et al*. 2017, Hellal *et al*. 2019, Meksi *et al*. 2019).

The idea of FGM was initiated in 1984 by some materials investigators, in Sendai, Japan, for thermal barriers (Koizumi 1997). Currently, materials specialists are developing novel processing techniques to include compositional gradients into various material systems (Kirigulige *et al*. 2005, Pollien *et al*. 2005, Gupta 2007, Yaghoobi *et al*. 2014, Houari *et al*. 2016, Bouderba *et al*. 2016, Bennoun *et al*. 2016, Zidi *et al*. 2017, Bellifa *et al*. 2017a, Hachemi *et al*. 2017, Ait Atmane *et al*. 2017, Sekkal *et al*. 2017b, Mohammadimehr *et al*. 2018, Benchohra *et al*. 2018, Belabed *et al*. 2018, Chaabane *et al*. 2019, Zarga *et al*. 2019). FGMs are currently exploring in the design of sandwich plates. Two novel kinds of sandwich structures with "*FGM face plates*" and a "*homogeneous core*" (Zenkour 2005, Shen and Li 2008, Zenkour and Sobhy 2010, Sobhy 2013, Fazzolari 2015) or with homogeneous facial leaves and an FGM core (Fazzolari 2015, Kashtalyan and Menshykova 2009, Alibeigloo and Liew 2014, Liu *et al*. 2016) have been proposed and studied. In a typical FGM plate, the properties of the material vary continuously and smoothly in thickness by mixing two different materials, depending on the volume fraction of the constituent materials (Miyamoto *et al*. 1999) in order to eliminate mechanically and thermally induced stresses by the

*Corresponding author, Ph.D.
E-mail: kaci_abdelhakim@yahoo.fr

http://www.techno-press.com/journals/sem&subpage=7
ISSN: 1225-4568 (Print), 1598-6217 (Online)

disagreement of the properties of the materials at the interfaces, which is an important problem in "*conventional sandwich structures*" (Swaminathan *et al*. 2015, Thai and Kim 2015, Dai *et al*. 2016). As a result, many researchers have paid close attention to the mechanical, thermal and thermomechanical behavior of functional graded sandwich structures (FG). In addition, the effect of material composition on mechanical behaviors of FG structures has been studied by many researchers (Alshorbagy *et al*. 2011, Eltaher *et al*. 2013 and 2014, Attia *et al*. 2015, Hamed *et al*. 2016, Abdelaziz *et al*. 2017, Eltaher *et al*. 2018ab, Attia *et al*. 2018ab).

Therefore, the increase in FGM applications requires the development of theoretical and numerical formulations for the investigation of this type of FGM shells and plates (Avcar and Mohammed 2018, Soliman *et al*. 2018, Selmi and Bisharat 2018, Aldousari 2017, Kolahchi *et al*. 2016, Kar and Panda 2015, Ahmed 2014, Li and Batra 2013, Chih-Ping and Hao-Yuan 2010, Ying *et al*. 2009, Lü *et al*. 2009 a,b, Wu and Huang 2009, Vel and Batra 2004).

In the "*classical plate theory*" (CPT), which is based on the Kirchhoff hypothesis, gives only acceptable results for thin plate analysis. This is the result of neglecting the transverse shear strains. However, the accuracy of the First Order Shear Deformation (FSDT) theory (Reissner 1945, Mindlin 1951, Reissner 1944) explains the effect of shear deformation by means of the linear variation of plane displacements across the thickness and depends on the shear correction factor. Higher order shear deformation (HSDT) theories provide better accuracy for transverse shear stresses without the need for a shear correction factor, they have been proposed for plate analysis by Levinson (1980), Ambartsumian (1958), Touratier (1991), Soldatos (1992), Karama *et al*. (2003) and Reddy (2004). It should be indicated that the 2-D plate theories mentioned above (i.e. CPT, FSDT and HSDT) do not consider the impact of thickness stretching assuming a constant transverse displacement through thickness, leading to shear deformation theories that neglect stretching $\varepsilon_{zz} = 0$. This assumption is appropriate for thin or medium-thick functional dimpled plates (FG), but is inadequate for thick FG plates (Qian *et al*. 2004). The importance of stretching effect in FG plates has been emphasized in the work of Carrera *et al*. (2011), using finite element approximations. Since the quasi-3D theory takes into account a higher order variation of plane and transverse displacements across the thickness, the shear deformation effect and the stretching effect of thickness are taken into account $\varepsilon_{zz} \neq 0$. Many quasi-3D theories have been proposed in the literature (Matsunaga 2009, Chen *et al*. 2009, Talha and Singh 2010, Ferreira *et al*. 2011, Reddy 2011, Neves *et al*. 2012a, Neves *et al*. 2012b, Neves *et al*. 2013). These theories are cumbersome and expensive because of many unknowns such as the theories of Talha and Singh (2010) with thirteen unknowns; Chen *et al*. (2009) and Reddy (2011) with eleven unknowns and Ferreira *et al*. (2011) and Neves *et al*. (Neves *et al*. 2012a, Neves *et al*. 2012b, Neves *et al*. 2013) with nine unknowns. Although some well-known quasi-3D theories developed by Zenkour (2007) and recently by Mantari and Guedes Soares (2012, 2013) have six unknowns, they are even more complicated than the FSDT. It is therefore necessary to develop a simple quasi-3D theory.

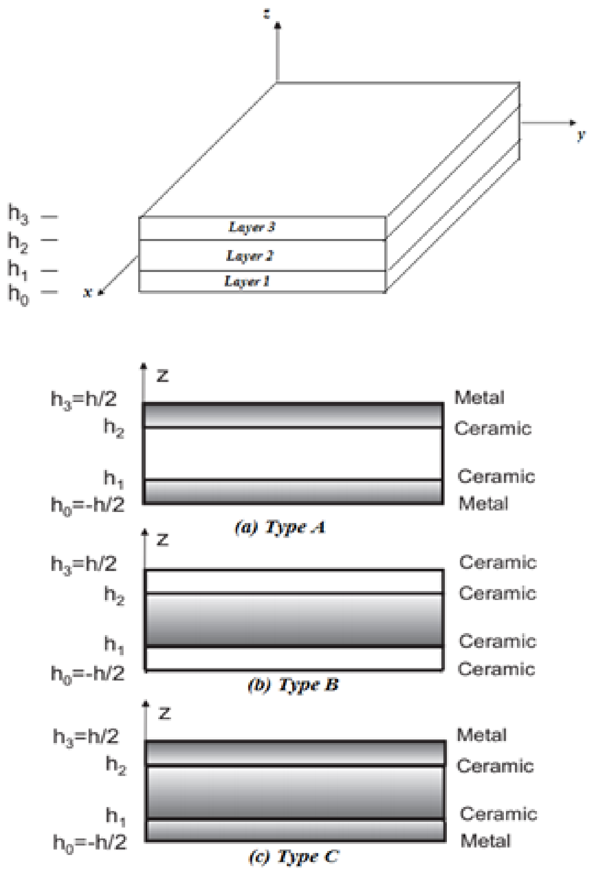

Fig. 1 Geometry of the FGM sandwich plate (Dongdong *et al*. 2017)

The phenomenon of buckling and postbuckling of structures is a considerable topic for structural engineering and many authors examined these problems in the open literature such as (Emam and Eltaher 2016, Emam *et al*. 2018, Eltaher *et al*. 2019ab, Mohamed *et al*. 2019). Thus, it is more interesting to consider this topic in new researches.

Using these theories, although much work has been done for FG plates (Carrera *et al*. 2011, Jha *et al*. 2012, Bessaim *et al*. 2013, Belabed *et al*. 2014, Hebali *et al*. 2014, Akavci 2016, Benahmed *et al*. 2017). More recently, Tounsi and his colleagues (Hebali *et al*. 2016, Meftah *et al*. 2017, Krenich *et al*. 2017, Besseghier *et al*. 2017) have developed a new theory of refined plates for the mechanical behavior of simply supported plates with only four unknowns. These theories have a new field of displacement that introduces indeterminate integral variables. As opposed to five or more numbers, in the case of other shear deformation theories higher. It should be noted that the thickness stretching effect is ignored in these new four-variable theories and that the transverse displacement is considered constant in the thickness direction, as in the thin-plate theories of type Kirchhoff-Love. This seems quite insufficient because the

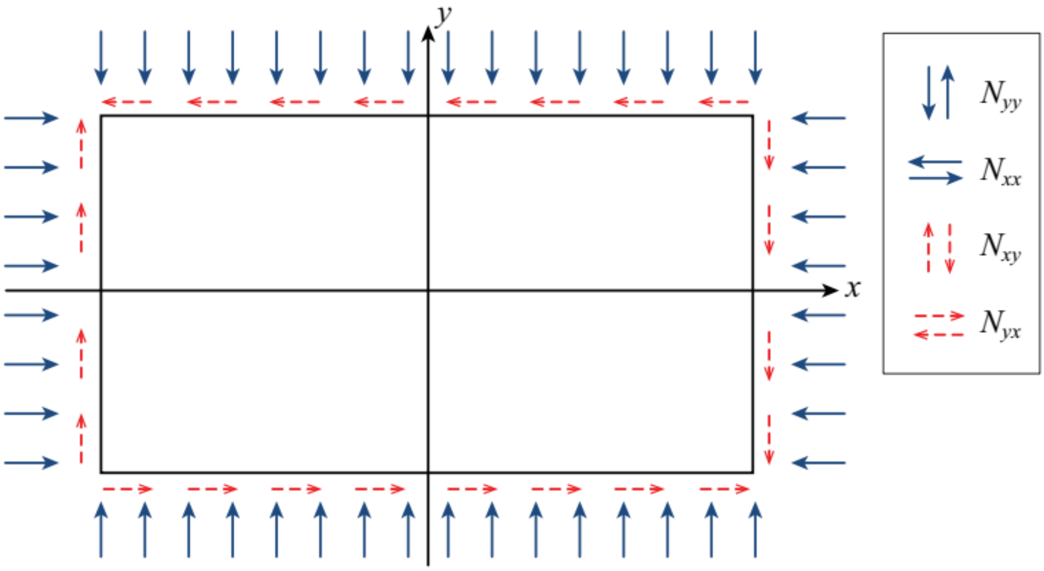

Fig. 2 Rectangular plate subjected to in-plane forces (Neves *et al*. 2012)

FGM plates are characterized by a strong variation of the properties of the material in the direction of the thickness.

In this paper, the goal is to improve the new theory of normal shear and normal deformation of higher order with only five unknowns developed by Tounsi and his colleagues (Akavci 2016, Benahmed *et al*. 2017, Hebali *et al*. 2016, Meftah *et al*. 2017). The highlight of this work is that, in addition to including the effect of thickness stretching ($\varepsilon_z \neq 0$), new type of FGM sandwich plates are considered as a practical design. Using this theory, we studied the buckling analysis of a new type of FGM sandwich plate composed of FGM faceplates and a FGM core. The governing equations are deduced from the principle of virtual work and solved by the Navier method. Comparative studies are conducted to verify the accuracy and efficiency of the current theory.

2. Theoretical formulation

Three types of sandwich plate as shown in Fig. 1 are considered in this study: (1) Type A: sandwich plate with FGM face sheets and homogeneous core and, (2) Type B: sandwich plate with homogeneous face sheets and FGM core and (3) Type C: sandwich plate with FGM faces sheets and FGM core. Rectangular Cartesian coordinates (x, y, z) are used to describe infinitesimal deformations of a three-layer sandwich elastic plate occupying the region [$0, a$]×[$0, b$] ×[$-h/2$, $h/2$] in the unstressed reference configuration. The mid-plane is defined by z=0 and its external bounding planes being defined by $z=\pm h/2$. The vertical positions of the bottom surface, the two interfaces between the core and faces layers, and the top surface are denoted, respectively, by $h_0 = -h/2$, h_1, $h_2$and $h_3=h/2$.

In the present study, the sandwich plate is subjected to compressive in-plane forces acting on the mid-plane of the plate. N_x^0 and N_y^0 denote the in-plane loads perpendicular to the edges x=0and y=0 respectively, and N_{xy}^0 denotes the distributed shear force parallel to the edges x=0 and y=0respectively (see Fig. 2).

2.1 Material properties of the face sheets

The bottom face sheet varies from a metal-rich surface ($z= h_0 = -h/2$)) to a ceramic-rich surface while the top face sheet varies from a ceramic-rich surface to a metal-rich surface ($z=h_3=h/2$). The volume fraction of the face sheets follows a power-law function through the thickness

$$V^{(1)} = \left(\frac{z-h_0}{h_1-h_0}\right)^p, \qquad z \in [h_0, h_1]$$
$$V^{(3)} = \left(\frac{z-h_3}{h_2-h_3}\right)^p, \qquad z \in [h_2, h_3] \tag{1}$$

where $V^{(n)}$, (n=1,3), denotes the volume fraction function of layer n; p is the volume fraction index ($0 \leq p \leq +\infty$), which dictates the material variation profile through the thickness.

Note that any isotropic material can be obtained as a particular case of the power-law function by setting p=0. The volume fraction for the metal phase is given as $V_m=1-V_c$.

The effective material properties for nth layer, like the Young's modulus $E^{(n)}$ and the Poisson's ratio $\nu^{(n)}$ can be determined by the linear rule of mixture as

$$P^{(n)}(z) = P_m + (P_c - P_m)V^{(n)} \tag{2}$$

where subscripts m and c represent metal and ceramic respectively

2.2 Material properties of the sandwich core

The volume fraction of the sandwich core is given as

$$V^{(2)} = \left(\frac{2|z|}{h_2-h_1}\right)^k, \qquad z \in [h_1, h_2] \tag{3}$$

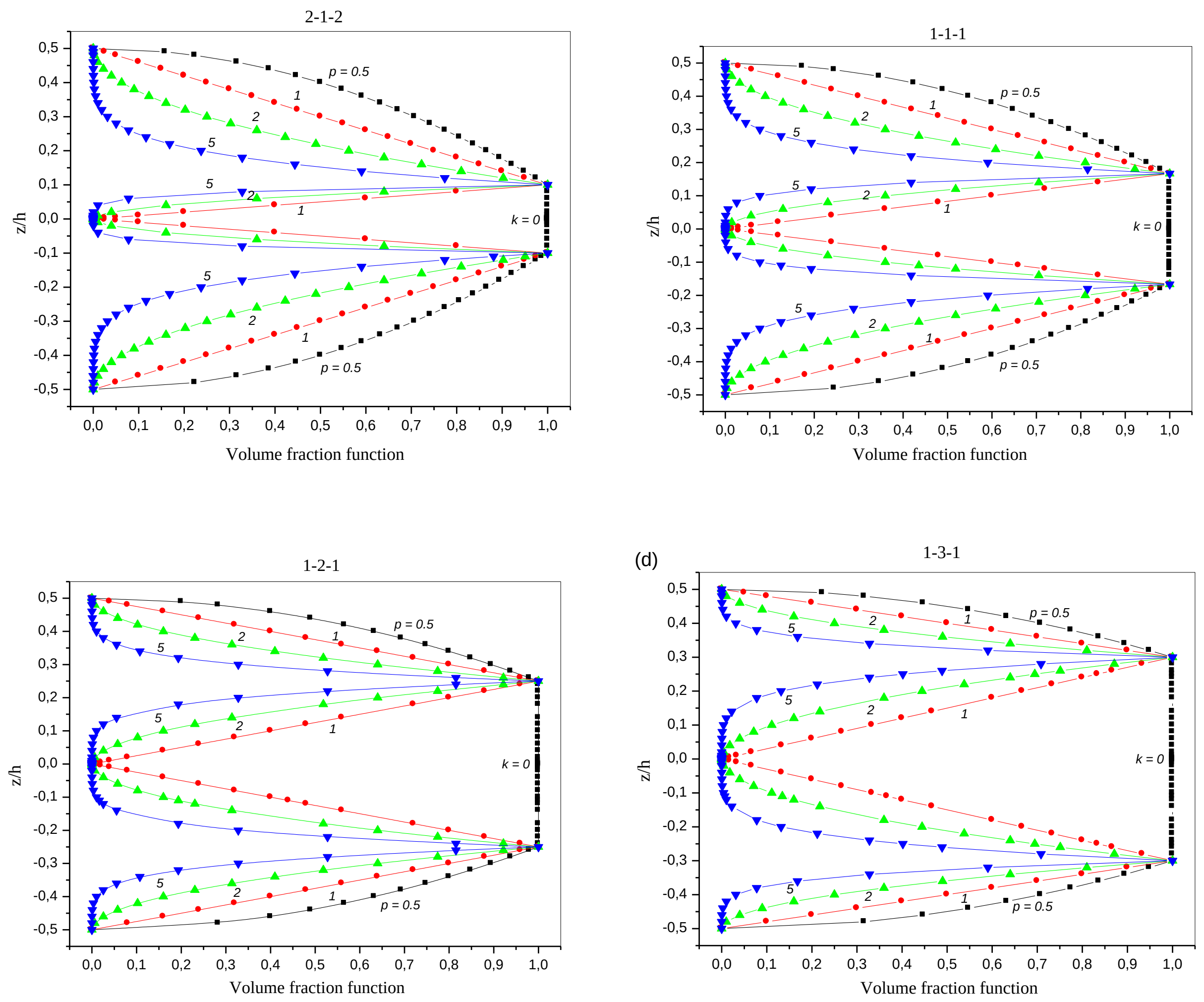

Fig. 3 Variation of volume fraction function through plate thickness for various values of $\boldsymbol{p}=\{\mathbf{0.5, 1, 2, 5}\}$ and $\boldsymbol{k}=\{\mathbf{0, 1, 2, 5}\}$ and different types of FGM sandwich plates: (a) 2-1-2, (b)1-1-1, (c)1-2-1,(d) 1-3-1

where k is the inhomogeneity parameter which takes values greater than or equal to zero.

The effective material properties of the FGM core are assumed to vary exponentially along the thickness direction

$$P^{(2)} = P_m \exp\left(\beta V^{(2)}\right), \qquad \beta = \ln \frac{P_c}{P_m} \tag{4}$$

where $P^{(2)}$ represents the effective material properties of the FGM core. P_m and P_c refer to the effective material properties of metal and ceramic, respectively

2.3 Different types of f symmetric sandwich plates

According to various layer thickness ratios, sandwich plates can be classified into (2-1-2), (1-1-1), (1-2-1), (1-3-1) sandwich plates and so on.

Several kinds of sandwich plates are presented in Fig.3 showing the through-the-thickness variation of the volume fraction of the above four kinds of symmetric FGM sandwich plates for various values of p={0.5, 1, 2, 5} and k={0, 1, 2, 5}.

2.3.1 The (2-1-2) sandwich plate

In this state the core of the plate is half the face thickness (see Fig. 3a). Thus,

$$h_1 = -\frac{h}{10}, \qquad h_2 = \frac{h}{10} \tag{5}$$

2.3.2 The (1-1-1) FGM sandwich plate

As shown in Fig. 3b the plate is made of three equal-thickness layers. So, one gets

$$h_1 = -\frac{h}{6}, \qquad h_2 = \frac{h}{6} \tag{6}$$

2.3.3. The (1-2-1) FGM sandwich plate

Here the core thickness is twice the face thickness (see Fig. 3c). Then

$$h_1 = -\frac{h}{4}, \qquad h_2 = \frac{h}{4} \tag{7}$$

2.3.4 The (1-3-1) EGM sandwich plate

In this state, as shown in Fig. 3d, the face layers are one third the core thickness layer. So, one gets

$$h_1 = -\frac{3h}{10}, \qquad h_2 = \frac{3h}{10} \tag{8}$$

2.4 Kinematics and constitutive equations

The aim of this paper is to develop a simple quasi-3D theory in which only five unknowns compared to other high order theory that uses six variables. The displacement field satisfying the conditions of transverse shear stresses (and hence strains) vanishing at a point (x, y, $\pm h/2$) on the outer (top) and inner (bottom) surfaces of the plate, is given as follows (Boukhari *et al*. 2016, Younsi *et al*. 2018, Abualnour *et al*. 2018, Bouhadra *et al*. 2018, Tounsi *et al*. 2019, Boukhlif *et al*. 2019, Zaoui *et al*. 2019)

$$\begin{aligned}
u(x,y,z) &= u_0(x,y) - z\frac{\partial w_0}{\partial x} + k_1\, f(z)\int \theta(x,y)\,dx \\
v(x,y,z) &= v_0(x,y) - z\frac{\partial w_0}{\partial y} + k_2\, f(z)\int \theta(x,y)\,dy \\
w(x,y,z) &= w_0(x,y) + g(z)\varphi_z
\end{aligned} \tag{9}$$

The coefficients k_1 and k_2 depend on the geometry and the $f(z)$ is hyperbolic shape function in the form

$$f(z) = \frac{(h/\pi)\sinh\left(\frac{\pi}{h}z\right) - z\cosh(\pi/2)}{[\cosh(\pi/2) - 1]} \tag{10}$$

and $g(z)$ is given as follows

$$g(z) = f'(z) \tag{11}$$

where (u,v,w) are the displacement components of a general point (x,y,z) in the FG plate, $(u_0, v_0, w_0, \theta, \varphi_z)$ are only five unknown displacements of the mid-plane of the plate and h is the plate thickness. Using the displacement field in Eq. (9), the linear strains ε_{ij} are given as:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z)\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} \tag{12a}$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix}, \qquad \varepsilon_{zz} = g'(z)\varepsilon_z^0 \tag{12b}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \qquad \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \tag{13a}$$

$$\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} k_1\theta \\ k_2\theta \\ k_1\dfrac{\partial}{\partial y}\int \theta\,dx + k_2\dfrac{\partial}{\partial x}\int \theta\,dy \end{Bmatrix},$$

$$\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_2\int \theta\,dy + \dfrac{\partial \varphi_z}{\partial y} \\ k_1\int \theta\,dx + \dfrac{\partial \varphi_z}{\partial x} \end{Bmatrix}, \qquad \varepsilon_z^0 = \varphi_z \tag{13b}$$

and

$$g'(z) = \frac{dg(z)}{dz} \tag{14}$$

The integrals used in the above equations shall be resolved by a Navier type method and can be written as follows:

$$\frac{\partial}{\partial y}\int \theta\,dx = A'\frac{\partial^2\theta}{\partial x \partial y}, \quad \frac{\partial}{\partial x}\int \theta\,dy = B'\frac{\partial^2\theta}{\partial x \partial y}, \quad \int \theta\,dx = A'\frac{\partial\theta}{\partial x}, \quad \int \theta\,dy = B'\frac{\partial\theta}{\partial y} \tag{15}$$

where the coefficients A' and B' are expressed according to the type of solution employed, in this case by using Navier. Therefore, A' and B' are expressed as follows:

$$A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2}, \quad k_1 = \alpha^2, \quad k_2 = \beta^2 \tag{16}$$

where α and β are defined in expression (31).

The constitutive relations of a FG plate can be expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} \tag{17}$$

where (σ_x, σ_y, σ_z, τ_{yz}, τ_{xz}, τ_{xy}) and (ε_x, ε_y, ε_z, γ_{yz}, γ_{xz}, γ_{xy}) are stress and strain components, respectively.

The computation of the elastic constants C_{ij} depends on which assumption of ε_z are considered. If $\varepsilon_z = 0$, then C_{ij} are the plane stress-reduced elastic constants

$$C_{11} = C_{22} = \frac{E(z)}{1-\nu^2}, \quad C_{12} = \nu\, C_{11} \tag{18a}$$

$$C_{44} = C_{55} = C_{66} = G(z) = \frac{E(z)}{2(1+\nu)} \tag{18b}$$

If $\varepsilon_z \neq 0$ (thickness stretching), then C_{ij} are 3D elastic constants, given by

$$C_{11}=C_{22}=C_{33}=\frac{(1-\nu)}{\nu}\lambda(z),\qquad C_{12}=C_{13}=C_{23}=\lambda(z) \tag{19a}$$

$$C_{44}=C_{55}=C_{66}=G(z)=\mu(z)=\frac{E(z)}{2(1+\nu)} \tag{19b}$$

Where $\lambda(z)=[\nu E(z)]/[(1-2\nu)(1+\nu)]$ and $\mu(z)=G(z)=E(z)/[2(1+\nu)]$ = Lame's coefficients.

The module E and G and the elastic coefficients C_{ij} vary through the thickness, according to Eqs.(2) and (4).

2.5 Governing equations

The principle of virtual displacements is used herein to derive the governing equations (Tounsi *et al.* 2013, Yahia *et al.* 2015).

The principle can be stated in an analytical form as (Bouderba *et al.* 2013, Bousahla *et al.* 2014, Bourada *et al.* 2015, Beldjelili *et al.* 2016, Bellifa *et al.* 2016, Fahsi *et al.* 2017, Benadouda *et al.* 2017, Zine *et al.* 2018, Fourn *et al.* 2018)

$$\delta U+\delta V=0 \tag{20}$$

Where δU is the virtual strain energy and δV is the virtual work done by external loads. The virtual strain energy is expressed by

$$\begin{aligned}\delta U&=\iint_A\int_{-h/2}^{h/2}\Big[\sigma_x\delta\varepsilon_x+\sigma_y\delta\varepsilon_y+\sigma_z\delta\varepsilon_z+\sigma_{xy}\delta\gamma_{xy}\\&\qquad+\sigma_{xz}\delta\gamma_{xz}+\sigma_{yz}\delta\gamma_{yz}\Big]dAdz\\&=\int_A\Big[N_x\delta\varepsilon_x^0+M_x\delta k_x^b+S_x\delta k_x^s+N_y\delta\varepsilon_y^0\\&\qquad+M_y\delta k_y^b+S_y\delta k_y^s+N_z\delta\varepsilon_z^0+N_{xy}\delta\gamma_{xy}^0\\&\qquad+M_{xy}\delta k_{xy}^b+S_{xy}\delta k_{xy}^s+Q_{xz}\delta\gamma_{xz}^0+Q_{yz}\delta\gamma_{yz}^0\Big]dA\end{aligned} \tag{21}$$

where A is the top surface and the stress resultants N, M, S and Q are expressed by

$$(N_i,M_i,S_i)=\int_{-h/2}^{h/2}(1,z,f)\sigma_i dz,\qquad i=x,y,xy \tag{22a}$$

$$Q_i=\int_{-h/2}^{h/2}g(z)\sigma_i dz,\qquad i=xy,yz \tag{22b}$$

$$N_z=\int_{-h/2}^{h/2}\sigma_z\ g'(z)dz \tag{22c}$$

The external virtual work due to in-plane forces and shear forces applied to the plate is given as:

$$\begin{aligned}\delta V=-\int_A\Bigg[&N_x^0\frac{\partial w}{\partial x}\frac{\partial\delta w}{\partial x}+N_{xy}^0\frac{\partial w}{\partial y}\frac{\partial\delta w}{\partial x}\\&+N_{yx}^0\frac{\partial w}{\partial x}\frac{\partial\delta w}{\partial y}+N_y^0\frac{\partial w}{\partial y}\frac{\partial\delta w}{\partial y}\Bigg]dA\end{aligned} \tag{23}$$

N_x^0 and N_y^0 are in- plane loads perpendicular to the edges x=0 and y=0 respectively, and N_{xy}^0 and N_{yx}^0 are the distributed shear forces parallel to the edges x=0 and y=0 respectively.

Substituting the expressions for δU and δV from Eqs. (21) and (23) into Eq. (20) and integrating by parts and collecting the coefficients of $\delta u_0,\delta v_0,\delta w_0,\delta\theta$ and $\delta\varphi_z$, the following equations of motion of the plate are obtained as

$$\begin{aligned}&\delta u_0:\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0\\&\delta v_0:\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=0\\&\delta w_0:\frac{\partial^2 M_x}{\partial x^2}+\frac{\partial^2 M_y}{\partial y^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}-\overline{N}=0\\&\delta\theta:-k_1S_x-k_2S_y-(k_1A'+k_2B')\frac{\partial^2 S_{xy}}{\partial x\partial y}+k_1A'\frac{\partial Q_{xz}}{\partial x}+k_2B'\frac{\partial Q_{yz}}{\partial y}=0\\&\delta\varphi_z:\frac{\partial Q_{xz}}{\partial x}+\frac{\partial Q_{yz}}{\partial y}-N_z-\overline{N}=0\end{aligned} \tag{24}$$

with

$$\overline{N}=N_x^0\frac{\partial^2 w_0}{\partial x^2}+N_{xy}^0\frac{\partial^2 w_0}{\partial x\partial y}+N_y^0\frac{\partial^2 w_0}{\partial y^2} \tag{25}$$

By substituting Eq. (9) into Eq. (12) and the subsequent results into Eq. (22), the stress resultants are obtained as

$$\begin{Bmatrix}N_x\\N_y\\N_{xy}\\M_x\\M_y\\M_{xy}\\S_x\\S_y\\S_{xy}\\N_z\end{Bmatrix}=\begin{bmatrix}A_{11}&A_{12}&0&B_{11}&B_{12}&0&B_{11}^s&B_{12}^s&0&L\\A_{12}&A_{22}&0&B_{12}&B_{22}&0&B_{12}^s&B_{22}^s&0&L\\0&0&A_{66}&0&0&B_{66}&0&0&B_{66}^s&0\\B_{11}&B_{12}&0&D_{11}&D_{12}&0&D_{11}^s&D_{12}^s&0&L^a\\B_{12}&B_{22}&0&D_{12}&D_{22}&0&D_{12}^s&D_{22}^s&0&L^a\\0&0&B_{66}&0&0&D_{66}&0&0&D_{66}^s&0\\B_{11}^s&B_{12}^s&0&D_{11}^s&D_{12}^s&0&H_{11}^s&H_{12}^s&0&R\\B_{12}^s&B_{22}^s&0&D_{12}^s&D_{22}^s&0&H_{12}^s&H_{22}^s&0&R\\0&0&B_{66}^s&0&0&D_{66}^s&0&0&H_{66}^s&0\\L&L&0&L^a&L^a&0&R&R&0&R^a\end{bmatrix}\begin{Bmatrix}\varepsilon_x^0\\\varepsilon_y^0\\\gamma_{xy}^0\\k_x^b\\k_y^b\\k_{xy}^b\\k_x^s\\k_y^s\\k_{xy}^s\\\varepsilon_z^0\end{Bmatrix} \tag{26a}$$

$$\begin{Bmatrix}Q_{yz}\\Q_{xz}\end{Bmatrix}=\begin{bmatrix}A_{44}^s&0\\0&A_{55}^s\end{bmatrix}\begin{Bmatrix}\gamma_{yz}^0\\\gamma_{xz}^0\end{Bmatrix} \tag{26b}$$

where

$$\begin{Bmatrix}A_{11}&B_{11}&D_{11}&B_{11}^s&D_{11}^s&H_{11}^s\\A_{12}&B_{12}&D_{12}&B_{12}^s&D_{12}^s&H_{12}^s\\A_{66}&B_{66}&D_{66}&B_{66}^s&D_{66}^s&H_{66}^s\end{Bmatrix}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\lambda(z)\left[1,z,z^2,f,z\,f,f^2\right]\begin{Bmatrix}\frac{1-\nu}{\nu}\\1\\\frac{1-2\nu}{2\nu}\end{Bmatrix}dz \tag{27a}$$

$$\begin{Bmatrix}L\\L^a\\R\\R^a\end{Bmatrix}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\lambda(z)\begin{Bmatrix}1\\z\\f\\g'\frac{1-\nu}{\nu}\end{Bmatrix}g'dz \tag{27b}$$

By substituting Eq. (26) into Eq. (24), the equations of motion can be expressed in terms of displacements $(\delta u_0,\delta v_0,\delta w_0,\delta\theta,\delta\varphi_z)$ as where $d_{ij}.d_{ijl}$ and d_{ijlm} are the following differential operators

$$d_{ij}=\frac{\partial^2}{\partial x_i \partial x_j},\qquad d_{ijl}=\frac{\partial^3}{\partial x_i \partial x_j \partial x_l},\qquad d_i=\frac{\partial}{\partial x_i},$$
$$d_{ijlm}=\frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m},\qquad (i,j,l,m=1,2). \tag{29}$$

3. Analytical solution of simply supported FG sandwich plate

Consider a simply supported rectangular sandwich plate with length a and width b and subjected to an in-plane loading in two directions $\left(N_x^0=-N_{cr}, N_y^0=-\gamma N_{cr}, \gamma=N_y^0/N_x^0 \;\; and \;\; N_{xy}^0=0\right)$ as shown in Fig. 2. The simply supported boundary conditions are given as follows

$$N_x=M_x=S_x=v_0=w_0=\frac{\partial\theta}{\partial y}=\varphi_z=0 \qquad at\ x=0,a$$
$$N_y=M_y=S_y=u_0=w_0=\frac{\partial\theta}{\partial x}=\varphi_z=0 \qquad at\ x=0,b \tag{30}$$

In this section, an analytical solution for buckling is presented for a simply supported rectangular sandwich plate. Based on the Navier approach, the solutions are assumed as

$$\begin{Bmatrix} u_0(x,y)\\ v_0(x,y)\\ w_0(x,y)\\ \theta(x,y)\\ \varphi_z(x,y)\end{Bmatrix}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix} U_{mn}\cos(\alpha x)\sin(\beta y)\\ V_{mn}\sin(\alpha x)\cos(\beta y)\\ W_{mn}\sin(\alpha x)\sin(\beta y)\\ \Theta_{mn}\sin(\alpha x)\sin(\beta y)\\ \Phi_{mn}\sin(\alpha x)\sin(\beta y)\end{Bmatrix} \tag{31}$$

where $U_{mn}, V_{mn}, W_{mn}, \Theta_{mn}, \Phi_{mn}$ are Fourier coefficients to be determined for each pair of m and n.

with

$$\alpha=m\pi/a\ ,\quad \beta=n\pi/b\ . \tag{32}$$

Substituting Eqs. (31) into Eq. (28), the analytical solutions can be obtained from the following equations

$$\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} & s_{15}\\ s_{12} & s_{22} & s_{23} & s_{24} & s_{25}\\ s_{13} & s_{23} & s_{33} & s_{34} & s_{35}\\ s_{14} & s_{24} & s_{34} & s_{44} & s_{45}\\ s_{15} & s_{25} & s_{35} & s_{45} & s_{55}\end{bmatrix}\begin{Bmatrix} U_{mn}\\ V_{mn}\\ W_{mn}\\ \Theta_{mn}\\ \Phi_{mn}\end{Bmatrix}=\begin{Bmatrix}0\\0\\0\\0\\0\end{Bmatrix} \tag{33}$$

Where

$$\begin{aligned}
s_{11}&=-\left(A_{11}\alpha^2+A_{66}\beta^2\right),\\
s_{12}&=-\alpha\beta\left(A_{12}+A_{66}\right),\\
s_{13}&=\left(B_{12}+2B_{66}\right)\alpha\beta^2+B_{11}\alpha^3\\
s_{14}&=\left(B_{11}^s k_1+B_{12}^s k_2\right)\alpha-B_{66}^s\left(k_1A'+k_2B'\right)\alpha\beta^2,\\
s_{15}&=L\alpha\\
s_{22}&=-\left(A_{22}\beta^2+A_{66}\alpha^2\right),\\
s_{23}&=B_{22}\beta^3+\left(2B_{66}+B_{12}\right)\alpha^2\beta\\
s_{24}&=\left(B_{12}^s k_1+B_{22}^s k_2\right)\beta-B_{66}^s\left(k_1A'+k_2B'\right)\alpha^2\beta,\\
s_{25}&=L\beta\\
s_{33}&=-\left(D_{11}\alpha^4+D_{22}\beta^4+2\left(2D_{66}+D_{12}\right)\alpha^2\beta^2\right.\\
&\quad\left.+N_x^0\alpha^2+N_y^0\beta^2\right)\\
s_{34}&=2D_{66}^s\left(k_1A'+k_2B'\right)\alpha^2\beta^2-k_2\left(D_{12}^s\alpha^2+D_{22}^s\beta^2\right)\\
&\quad-k_1\left(D_{11}^s\alpha^2+D_{12}^s\beta^2\right)\\
s_{35}&=-L^a\left(\alpha^2+\beta^2\right)-N_x^0\alpha^2-N_y^0\beta^2\\
s_{44}&=-H_{11}^s k_1^2-H_{22}^s k_2^2-2H_{12}^s k_1k_2\\
&\quad-H_{66}^s\left(k_1A'+k_2B'\right)^2\alpha^2\beta^2\\
&\quad-A_{55}^s k_1^2A'^2\alpha^2-A_{44}^s k_2^2B'^2\beta^2\\
s_{45}&=-\left(k_2B'A_{44}^s\beta^2+k_1R+k_1A'A_{55}^s\alpha^2+k_2R\right)\\
s_{55}&=-\left(A_{55}^s\alpha^2+A_{44}^s\beta^2+R^a+N_x^0\alpha^2+N_y^0\beta^2\right)
\end{aligned} \tag{34}$$

4. Numerical results and discussions

To illustrate the proposed approach, a new type of FGM sandwich plates, namely, both FGM face sheets and FGM hard core are used for the buckling analysis of simply supported functionally graded sandwich plates. The combination of materials consists of alumina and aluminum. The Young's modulus and the Poisson's ratio were selected to be E_m=70E_0 and ν_m = 0.3 for aluminum; and E_c=380E_0 and ν_c = 0.3 for alumina being E_0=1GPa, respectively.

For convenience, following non-dimensional critical buckling load is used in presenting the numerical results in graphical and tabular forms

$$\overline{N}_{cr}=\frac{N_{cr}a^2}{100h^3E_0} \tag{35}$$

4.1 Critical buckling load of sandwich plate with FGM face sheets and homogeneous core (Type A)

In this example, a simply supported square sandwich plate with FGM face sheets and homogeneous core is subjected to compressive in-plane forces acting on the mid-plane of the plate. It can be considered as a special case of the present study, where k=0.

Numerical results are given in Tables 1 and 2 using the present refined five-variable plate theory. The critical buckling loads obtained from the present theory with $\varepsilon_{zz} \neq 0$ and ε_{zz} = 0 with aspect ratio of a/h=10 are tabulated and compared with quasi-3D hyperbolic sine shear deformation theory (Neves *et al.* 2012) for various power-law exponents p and thickness ratios. Both tables include results obtained from classical plate theory (CLPT), first-order shear

Table 1 Comparison of nondimensional critical buckling load of square FG sandwich plates with FGM face sheets and homogeneous core subjected to uniaxial compressive load (γ=0, a/h=10)

p	Theory	$\overline{N}_{cr}$ 1-0-1	2-1-2	1-1-1	1-2-1
0	CLPT	13.73791	13.73791	13.73791	13.73791
	FSDT	13.00449	13.00449	13.00449	13.00449
	Zenkour (2005) (TSDT)	13.00495	13.00495	13.00495	13.00495
	Zenkour (2005) (SSDT)	13.00606	13.00606	13.00606	13.00606
	Neves *et al*. (2012c) $\varepsilon_{zz}=0$	13.00532	13.00532	13.00532	13.00532
	Neves *et al*. (2012c) $\varepsilon_{zz}\neq 0$	12.95304	12.95304	12.95304	12.95304
	Present $\varepsilon_{zz}=0$	13.00552	13.00552	13.00552	13.00552
	Present $\varepsilon_{zz}\neq 0$	12.98429	12.98429	12.98429	12.98429
0.5	CLPT	7.65398	8.25597	8.78063	9.61525
	FSDT	7.33732	7.91320	8.41034	9.19517
	Zenkour (2005) (TSDT)	7.36437	7.94084	8.43645	9.21681
	Zenkour (2005) (SSDT)	7.36568	7.94195	8.43712	9.21670
	Neves *et al*. (2012c) $\varepsilon_{zz}=0$	7.18707	7.74315	8.22141	8.97351
	Neves *et al*. (2012c) $\varepsilon_{zz}\neq 0$	7.16191	7.71617	8.19283	8.94221
	Present $\varepsilon_{zz}=0$	7.36380	7.94046	8.43647	9.21757
	Present $\varepsilon_{zz}\neq 0$	7.35541	7.93147	8.42681	9.20640
1	CLPT	5.33248	6.02733	6.68150	7.78406
	FSDT	5.14236	5.81379	6.43892	7.48365
	Zenkour (2005) (TSDT)	5.16713	5.84006	6.46474	7.50656
	Zenkour (2005) (SSDT)	5.16846	5.84119	6.46539	7.50629
	Neves *et al*. (2012c) $\varepsilon_{zz}=0$	5.07825	5.73007	6.33558	7.34408
	Neves *et al*. (2012c) $\varepsilon_{zz}\neq 0$	5.06123	5.71125	6.31501	7.32025
	Present $\varepsilon_{zz}=0$	5.16629	5.83941	6.46450	7.50719
	Present $\varepsilon_{zz}\neq 0$	5.16191	5.83465	6.45911	7.49996
5	CLPT	2.73080	3.10704	3.65732	4.85717
	FSDT	2.63842	3.02252	3.55958	4.71475
	Zenkour (2005) (TSDT)	2.65821	3.04257	3.57956	4.73469
	Zenkour (2005) (SSDT)	2.66006	3.04406	3.58063	4.73488
	Neves *et al*. (2012c) $\varepsilon_{zz}=0$	2.64662	3.01870	3.54145	4.66071
	Neves *et al*. (2012c) $\varepsilon_{zz}\neq 0$	2.63658	3.00819	3.53014	4.64707
	Present $\varepsilon_{zz}=0$	2.65679	3.04141	3.57874	4.73463
	Present $\varepsilon_{zz}\neq 0$	2.65398	3.04023	3.57873	4.73404
10	CLPT	2.56985	2.80340	3.25924	4.38221
	FSDT	2.46904	2.72626	3.17521	4.26040
	Zenkour (2005) (TSDT)	2.48727	2.74632	3.19471	4.27991
	Zenkour (2005) (SSDT)	2.48928	2.74844	3.19456	4.38175
	Neves *et al*. (2012c) $\varepsilon_{zz}=0$	2.48179	2.73094	3.16842	4.21795
	Neves *et al*. (2012c) $\varepsilon_{zz}\neq 0$	2.47199	2.72089	3.15785	4.20550
	Present $\varepsilon_{zz}=0$	2.48574	2.74498	3.19373	4.27964
	Present $\varepsilon_{zz}\neq 0$	2.48217	2.74301	3.19359	4.28002

deformation plate theory (FSDT, K=5/6 as shear correction factor), Reddy's third-order shear deformation plate theory (TSDPT) and sinusoidal shear deformation plate theory (SSDPT) (Zenkour 2005). Table 1 refers to the uniaxial buckling load and Table 2 refers to the bi-axial buckling load. However, the TSDPT and the SSDPT (Zenkour 2005), which omit the thickness-stretching effect, slightly overestimate the critical buckling loads. It is worth noting

Table 2 Comparison of non-dimensional critical buckling load of square FG sandwich plates with FGM face sheets and homogeneous core subjected to biaxial compressive load (γ=1, a/h=10)

p	Theory	$\overline{N}_{cr}$			
		1-0-1	2-1-2	1-1-1	1-2-1
0	CLPT	6.86896	6.86896	6.86896	6.86896
	FSDT	6.50224	6.50224	6.50224	6.50224
	Zenkour (2005) (TSDT)	6.50248	6.50248	6.50248	6.50248
	Zenkour (2005) (SSDT)	6.50303	6.50303	6.50303	6.50303
	Neves *et al*. (2012c) $\varepsilon_{zz} = 0$	6.50266	6.50266	6.50266	6.50266
	Neves *et al*. (2012c) $\varepsilon_{zz} \neq 0$	6.47652	6.47652	6.47652	6.47652
	Present $\varepsilon_{zz} = 0$	6.50276	6.50276	6.50276	6.50276
	Present $\varepsilon_{zz} \neq 0$	6.49215	6.49215	6.49215	6.49215
0.5	CLPT	3.82699	4.12798	4.39032	4.80762
	FSDT	3.66866	3.95660	4.20517	4.59758
	Zenkour (2005) (TSDT)	3.68219	3.97042	4.21823	4.60841
	Zenkour (2005) (SSDT)	3.68284	3.97097	4.21856	4.60835
	Neves *et al*. (2012c)$\varepsilon_{zz} = 0$	3.59354	3.87157	4.11071	4.48676
	Neves *et al*. (2012c)$\varepsilon_{zz} \neq 0$	3.58096	3.85809	4.09641	4.47110
	Present $\varepsilon_{zz} = 0$	3.68190	3.97023	4.21823	4.60878
	Present $\varepsilon_{zz} \neq 0$	3.67770	3.96573	4.21340	4.60320
1	CLPT	2.66624	3.01366	3.34075	3.89203
	FSDT	2.57118	2.90690	3.21946	3.74182
	TSDPT (a)	2.58357	2.92003	3.23237	3.75328
	SSDPT (a)	2.58423	2.92060	3.23270	3.75314
	Neves *et al*. (2012c)$\varepsilon_{zz} = 0$	2.53913	2.86503	3.16779	3.67204
	Neves *et al*. (2012c)$\varepsilon_{zz} \neq 0$	2.53062	2.85563	3.15750	3.66013
	Present $\varepsilon_{zz} = 0$	2.58315	2.91970	3.23225	3.75359
	Present $\varepsilon_{zz} \neq 0$	2.58096	2.91732	3.22956	3.74998
5	CLPT	1.36540	1.55352	1.82866	2.42859
	FSDT	1.31921	1.51126	1.77979	2.35737
	Zenkour (2005) (TSDT)	1.32910	1.52129	1.78978	2.36734
	Zenkour (2005) (SSDT)	1.33003	1.52203	1.79032	2.36744
	Neves *et al*. (2012c) $\varepsilon_{zz} = 0$	1.32331	1.50935	1.77072	2.33036
	Neves *et al*. (2012c) $\varepsilon_{zz} \neq 0$	1.31829	1.50409	1.76507	2.32354
	Present $\varepsilon_{zz} = 0$	1.32839	1.52071	1.78937	2.36731
	Present $\varepsilon_{zz} \neq 0$	1.32699	1.52012	1.78936	2.36702
10	CLPT	1.28493	1.40170	1.62962	2.19111
	FSDT	1.23452	1.36313	1.58760	2.13020
	Zenkour (2005) (TSDT)	1.24363	1.37316	1.59736	2.13995
	Zenkour (2005) (SSDT)	1.24475	1.37422	1.59728	2.19087
	Neves *et al*. (2012c) $\varepsilon_{zz} = 0$	1.24090	1.36547	1.58421	2.10897
	Neves *et al*. (2012c) $\varepsilon_{zz} \neq 0$	1.23599	1.36044	1.57893	2.10275
	Present $\varepsilon_{zz} = 0$	1.24287	1.37249	1.59687	2.13982
	Present $\varepsilon_{zz} \neq 0$	1.24109	1.37150	1.59680	2.14001

that the developed theory consists of five unknowns, while the number of unknowns in the TSDPT (Zenkour 2005), SSDPT (Zenkour 2005) and quasi-3D theory(Neves *et al*. 2012) is five and six, respectively. Consequently, it may be concluded that the presentquasi-3D theory is not only more accurate than the higher order shear deformation theory (TSDPT and SSDPT) having the same five unknowns, but also comparable with the quasi-3D theory having more number of unknowns.

Table 3 Effect of aspect ratio a/b on the critical buckling load of sandwich plate with FGM face sheets and homogeneous core subjected to uniaxial compressive load (b/h=10, p=2)

Scheme	Theory	$\overline{N}_{cr}$ $b/a=0.5$	$b/a=1$	$b/a=2$
1-2-1	CLPT	14.69209[a]	6.17302	2.41134
	FSDT	23.86154[a]	5.96539	2.21831
	Zenkour (2005) (TSDT)	23.94786[a]	5.98697	2.23758
	Zenkour (2005) (SSDT)	23.94721[a]	5.98680	2.23745
	Present $\varepsilon_{zz}=0$	23.94924[a]	5.98731	2.23787
	Present $\varepsilon_{zz}\neq 0$	23.93448[a]	5.98362	2.22756

Table 4 Effect of aspect ratio b/a on the critical buckling load of sandwich plate with FGM face sheets and homogeneous core subjected to biaxial compressive load (b/h=10, p=2)

Scheme	Theory	$\overline{N}_{cr}$ $b/a=0.5$	$b/a=1$	$b/a=2$
1-2-1	CLPT	7.71628	3.08651	1.92907
	FSDT	7.55199	2.98269	1.77464
	Zenkour (2005) (TSDT)	7.56924	2.99348	1.79006
	Zenkour (2005) (SSDT)	7.56911	2.99340	1.78996
	Present $\varepsilon_{zz}=0$	7.56952	2.99365	1.79030
	Present $\varepsilon_{zz}\neq 0$	7.57280	2.99181	1.78205

Comparisons are given in Table 3 and 4on the basis of the symmetric (1-2-1) type of FG sandwich plates with FGM face sheets and homogeneous core. The critical buckling loads decrease with increasing aspect ratio (b/a) and γ. In general, the critical buckling loads obtained by the CLPT are much higher than those computed from the shear deformation theories. This implies the well-known fact that the results estimated by the CLPT are grossly in error for a thick plate.

4.2 Critical buckling load of sandwich plate with homogeneous face sheets and FGM core (Type B)

As a special case of the present study, where p=0, a simply supported square sandwich plate with homogeneous face sheets and FGM core is considered. The buckling analysis is conducted.

Table 5 refers to the uniaxial buckling load and Table 6 refers to the bi-axial buckling load for sandwich plates with homogeneous face sheets and FGM core. The critical buckling load is considered for k=0,1,2,3,4 and 5 and various layer thickness ratios of 2-1-2, 1-1-1, 1-2-1 and 1-3-

Table 5 Comparison of nondimensional critical buckling load of square sandwich plate with homogeneous face sheets and FGM core subjected to uniaxial compressive load (γ=0, b/h=10)

k	Theory	$\overline{N}_{cr}$ 2-1-2	1-1-1	1-2-1	1-3-1
0	CLPT	13.73791	13.73791	13.73791	13.73791
	FSDT	13.00449	13.00449	13.00449	13.00449
	Present $\varepsilon_{zz}=0$	13.00552	13.00552	13.00552	13.00552
	Present $\varepsilon_{zz}\neq 0$	12.98429	12.98429	12.98429	12.98429
1	CLPT	6.02733	6.68150	7.78406	8.62740
	FSDT	5.81379	6.43892	7.48365	8.27710
	Present $\varepsilon_{zz}=0$	5.83941	6.46450	7.50719	8.29787
	Present $\varepsilon_{zz}\neq 0$	5.83465	6.45911	7.49996	8.28871
2	CLPT	4.27003	4.938150	6.17302	7.174950
	FSDT	4.14152	4.78594	5.96539	6.91465
	Present $\varepsilon_{zz}=0$	4.16303	4.80807	5.98731	6.93519
	Present $\varepsilon_{zz}\neq 0$	4.16125	4.80614	5.98362	6.92921
3	CLPT	3.60656	4.23251	5.472570	6.52045
	FSDT	3.50492	4.11184	5.30098	6.29726
	Present $\varepsilon_{zz}=0$	3.52477	4.13228	5.32182	6.31736
	Present $\varepsilon_{zz}\neq 0$	3.52370	4.13160	5.31978	6.31296
4	CLPT	3.28636	3.87079	5.09232	6.15541
	FSDT	3.19613	3.76489	4.93906	5.95188
	Present $\varepsilon_{zz}=0$	3.21530	3.78449	4.95930	5.97172
	Present $\varepsilon_{zz}\neq 0$	3.21428	3.78431	4.95816	5.96825
5	CLPT	3.10704	3.65732	4.85717	5.92486
	FSDT	3.02252	3.55958	4.71475	5.73332
	Present $\varepsilon_{zz}=0$	3.04141	3.57874	4.73463	5.75301
	Present $\varepsilon_{zz}\neq 0$	3.04023	3.57873	4.73404	5.75015

1. We can see from this table that with the increase of the inhomogeneity parameter k, the dimensionless critical buckling load decreases slightly. The dimensionless critical buckling load obtained by the present quasi-3D theory is smaller than those computed by FSDT and present theory with ε_{zz}=0.

Tables 7 and 8 compares the dimensionless the critical buckling loads of sandwich plates with homogeneous face sheets and FGM core with b/h=10, k=1 for different values of aspect ratios and two types of FG sandwich plates. As the aspect ratio (b/a) increases, the dimensionless critical buckling loads of the sandwich plate decreases.

Fig. 4 exhibit plots of the critical buckling loads versus the side to-thickness ratio b/h for k=2 and b=2a. Once again, it is well known that the CLPT critical buckling loads

Table 6 Comparison of nondimensional critical buckling load of square sandwich plate with homogeneous face sheets and FGM core subjected to biaxial compressive load (γ=1, b/h=10)

k	Theory	$\overline{N}_{cr}$ 2-1-2	1-1-1	1-2-1	1-3-1
0	CLPT	6.86896	6.86896	6.86896	6.86896
	FSDT	6.50224	6.50224	6.50224	6.50224
	Present ε_{zz}=0	6.50276	6.50276	6.50276	6.50276
	Present $\varepsilon_{zz}\neq$0	6.49215	6.49215	6.49215	6.49215
1	CLPT	3.01366	3.34075	3.89203	4.31370
	FSDT	2.90690	3.21946	3.74182	4.13854
	Present ε_{zz}=0	2.91970	3.23225	3.75359	4.14894
	Present $\varepsilon_{zz}\neq$0	2.91732	3.22956	3.74998	4.14436
2	CLPT	2.13502	2.46907	3.08651	3.58747
	FSDT	2.07076	2.39297	2.98269	3.45733
	Present ε_{zz}=0	2.08152	2.40404	2.99365	3.46760
	Present $\varepsilon_{zz}\neq$0	2.08063	2.40307	2.99181	3.46461
3	CLPT	1.80328	2.11625	2.73628	3.26022
	FSDT	1.75246	2.05592	2.65049	3.14863
	Present ε_{zz}=0	1.76238	2.06614	2.66091	3.15868
	Present $\varepsilon_{zz}\neq$0	1.76185	2.06580	2.65989	3.15648
4	CLPT	1.64318	1.93540	2.54616	3.07771
	FSDT	1.59806	1.88244	2.46953	2.97594
	Present ε_{zz}=0	1.60765	1.89225	2.47965	2.98586
	Present $\varepsilon_{zz}\neq$0	1.60714	1.89215	2.47908	2.98412
5	CLPT	1.55352	1.82866	2.42859	2.96243
	FSDT	1.51126	1.77979	2.35737	2.86666
	Present ε_{zz}=0	1.52071	1.78937	2.36731	2.87651
	Present $\varepsilon_{zz}\neq$0	1.52012	1.78936	2.36702	2.87507

Table 7 Effect of aspect ratio a/b on dimensionless critical buckling load of sandwich plate with homogeneous face sheets and FGM core subjected to uniaxialcompressive load (b/h=10, k=1)

Scheme	Theory	$\overline{N}_{cr}$ b/a=1	b/a=2	b/a=3	b/a=4	b/a=5
2-1-2	Present ε_{zz}=0	12.82442136	4.570440555	3.151983704	2.448120964	1.966724410
	Present $\varepsilon_{zz}\neq$0	12.80279002	4.540517500	3.110700226	2.398852630	1.913711950
1-1-1	Present ε_{zz}=0	12.58918067	4.433738040	3.010884797	2.302268731	1.823994414
	Present $\varepsilon_{zz}\neq$0	12.56669963	4.404547034	2.972117875	2.257543296	1.777242289
1-2-1	Present ε_{zz}=0	12.08507206	4.190150055	2.789869262	2.092502608	1.630469776
	Present $\varepsilon_{zz}\neq$0	12.06100174	4.161995616	2.754442612	2.053396868	1.591044352
1-3-1	Present ε_{zz}=0	11.64341795	4.002021930	2.636276317	1.957338592	1.512222546
	Present $\varepsilon_{zz}\neq$0	11.61849764	3.974650134	2.602887757	1.921362234	1.476641033

are independent of the b/h ratio. Critical buckling loads decrease as γ increases. The results of the sandwich plate (1-3-1) are superior to those of (2-1-2), (1-1-1) and (1-2-1) and this depends on the thickness of the core layer.

Table 8 Effect of aspect ratio a/b on dimensionless critical buckling load of sandwich plate with homogeneous face sheets and FGM core subjected to biaxial compressive load (b/h=10, k=1)

Scheme	Theory	$\overline{N}_{cr}$ b/a=1	b/a=2	b/a=3	b/a=4	b/a=5
2-1-2	Present ε_{zz}=0	6.412210682	3.656352442	2.836785332	2.304113852	1.891081164
	Present $\varepsilon_{zz}\neq$0	6.401395008	3.632413998	2.799630206	2.257743652	1.840107646
1-1-1	Present ε_{zz}=0	6.294590336	3.546990429	2.709796318	2.166841158	1.753840782
	Present $\varepsilon_{zz}\neq$0	6.283349815	3.523637632	2.674906091	2.124746633	1.708886817
1-2-1	Present ε_{zz}=0	6.042536034	3.352120048	2.510882336	1.969414221	1.567759403
	Present $\varepsilon_{zz}\neq$0	6.030500868	3.329596495	2.478998351	1.932608818	1.529850339
1-3-1	Present ε_{zz}=0	5.821708975	3.201617542	2.372648684	1.842201027	1.454060140
	Present $\varepsilon_{zz}\neq$0	5.809248819	3.179720106	2.342598983	1.808340926	1.419847147

Fig. 5 show the variation of the critical buckling loads as a function of side-to-thickness ratio (b/h) of (2-1-2) FGM sandwich plates with FGM face sheets and homogeneous core using various plate theories (b=2a) using the present new simple quasi-3D hyperbolic shear deformation theory. It can be seen that the critical buckling loads become maximum for the ceramic plates and minimum for the metal plates. It is seen that the results increase smoothly as the amount of ceramic in the sandwich plate increases. Also, the buckling load of plate under uniaxial compression is almost the twice of that of the case of the plate under biaxial compression.

Fig. 6 show the critical buckling loads of sandwich plates with respect to the power-law index. It can be seen from these figures that with the increase of the power-law index, they decrease for sandwich plates with homogeneous face sheets and FGM core.

4.3 Critical buckling load of sandwich plate with FGM face sheets and FGM core (Type C)

In this example, a simply supported square sandwich plate with FGM face sheets and FGM core under mechanical loads is considered.

Table 9 and 10 performs the dimensionless critical buckling load for a sandwich plate with FGM face sheets and FGM core for various values of p = {0, 1, 2, 3, 4, 5} and various layer thickness ratios {2-1-2, 1-1-1, 1-2-1, 1-3-1}. The plate is subjected to uniaxial and biaxial compressive load. The inhomogeneity parameter k is set to be 1. It can be seen from this table that as the index p increases, the dimensionless critical buckling load decreases. The critical buckling loadobtained by the present theory are smaller than those computed by using the CLPT, FSDT and Present with $\varepsilon_{zz} = 0$.

Tables 11 and 12 reveals the dimensionless critical buckling load of the sandwich plate under mechanical loads

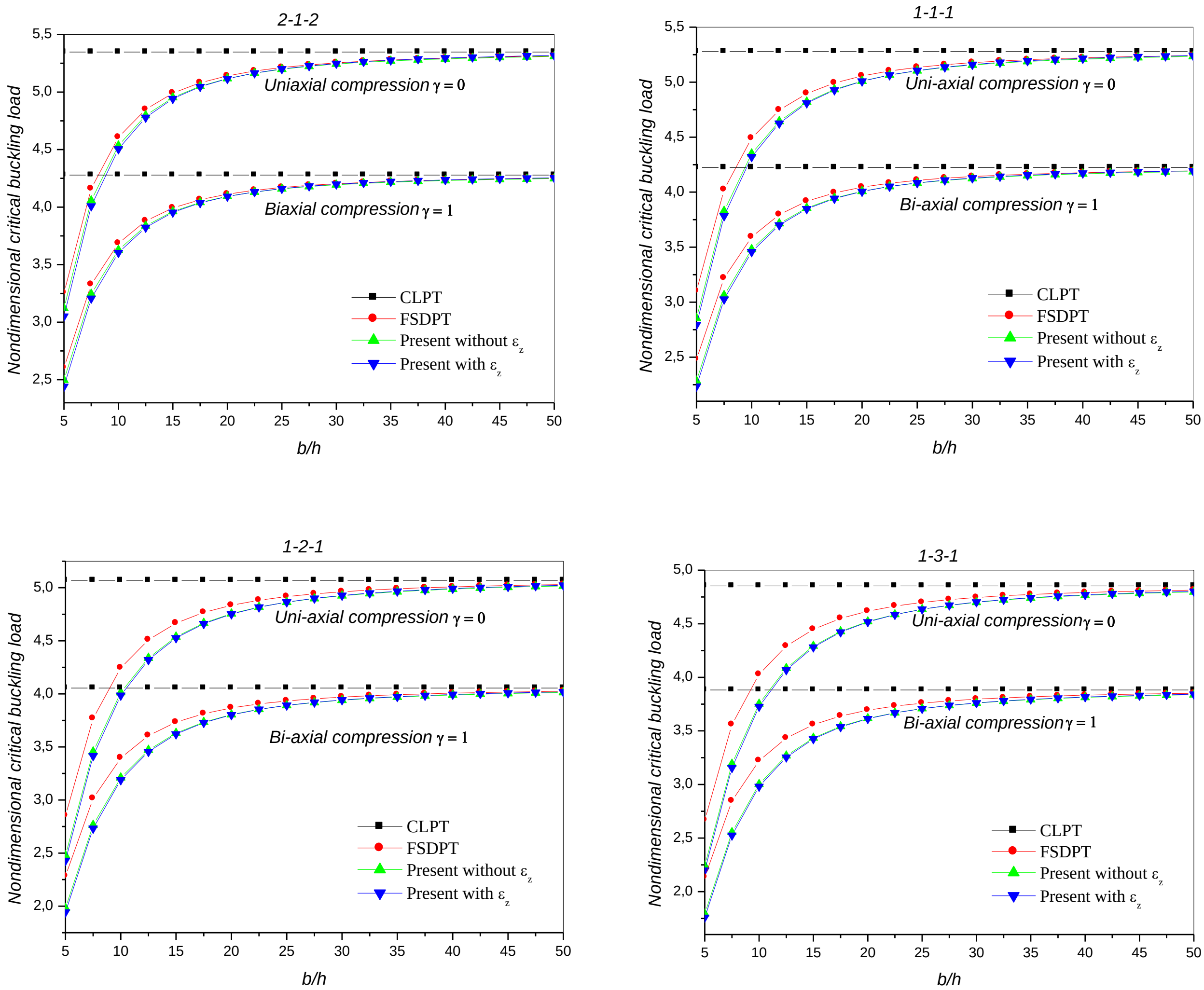

Fig. 4 Critical buckling load as a function of side-to-thickness ratio *(b/h*) of FGM sandwich plates with homogeneous face sheets and FGM core using various plate theories (*b*=2*a, k*=2)

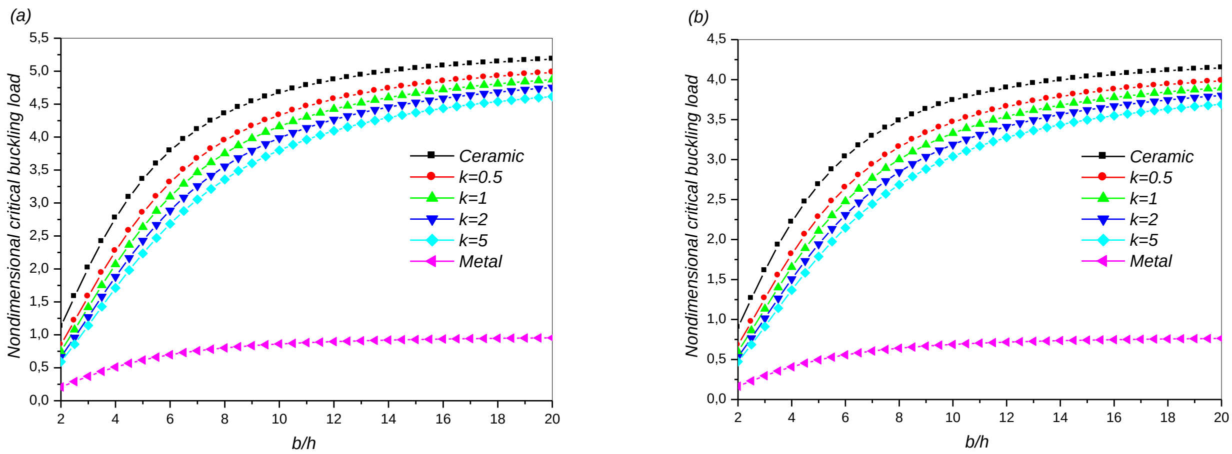

Fig. 5 Nondimensional critical buckling load as a function of side-to-thickness ratio (b/h) of (1-2-1) FGM sandwich plates with homogeneous face sheets and FGM core for various values of *k*. (a) Plate subjected to uniaxial compressive load and (b) Plate subjected to biaxial compressive load

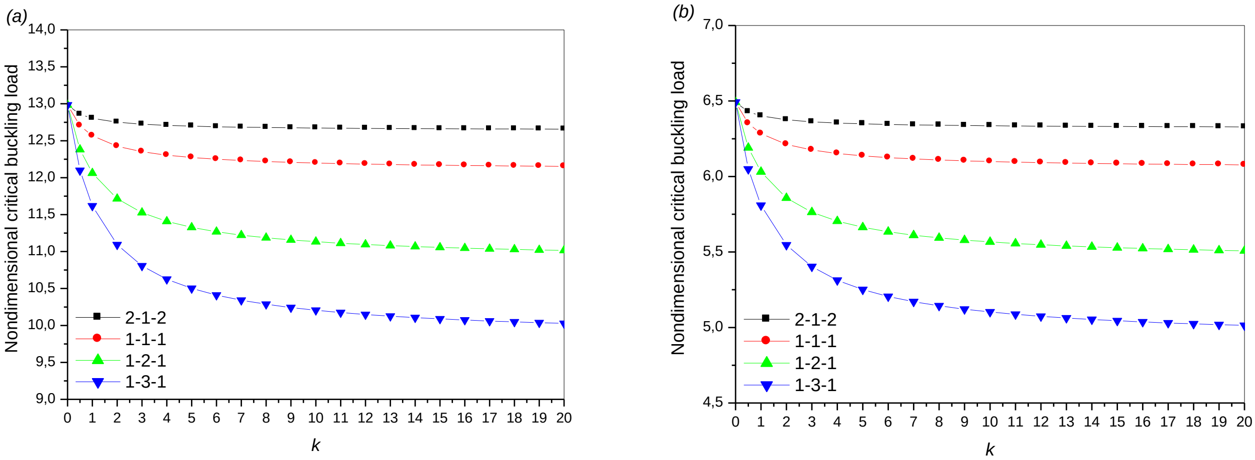

Fig. 6 Effect of the power-law index k on the nondimensional critical buckling loads of Al/Al_2O_3 sandwich square plates with homogeneous face sheets and FGM core(b/h=10) (a) Plate subjected to uniaxial compressive load ($\gamma = 0$) and (b) Plate subjected to biaxial compressive load ($\gamma = 1$)

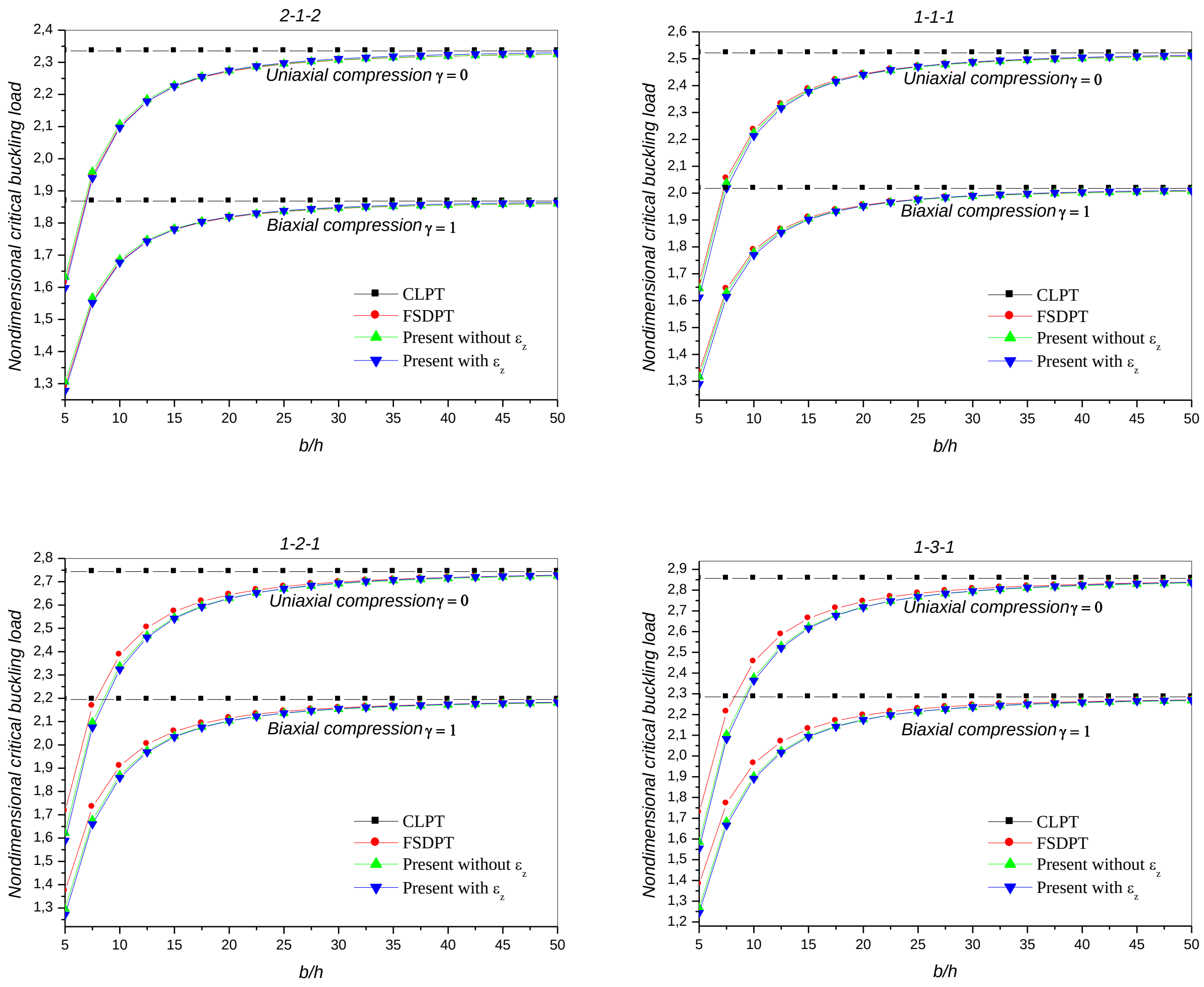

Fig. 7 Critical buckling load as a function of side-to-thickness ratio (b/h) of FGM sandwich plates plate with FGM face sheets and FGM core using various plate theories (b=2a, $p=1$, $k=2$).

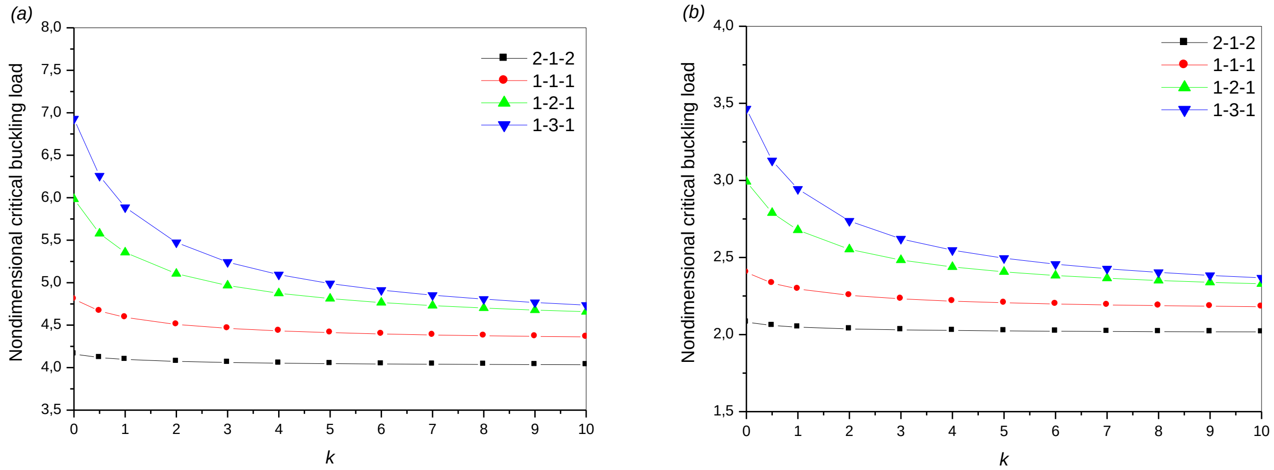

Fig. 8 Effect of the power-law index k on the nondimensional critical buckling loads of Al/Al_2O_3 sandwich square plates with FGM face sheets and FGM core (b/h=10, p=2) (a) Plate subjected to uniaxial compressive load (γ = 0) and (b) Plate subjected to biaxial compressive load (γ = 1)

Table 9 Comparison of nondimensional critical buckling load of square sandwich plate with FGM face sheets and FGM core subjected to uniaxial compressive load ($\gamma = 0, a/h = 10, k = 1$)

Scheme	Theory	$\overline{N}_{cr}$ $b/a = 1$	$b/a = 2$	$b/a = 3$	$b/a = 4$	$b/a = 5$
2-1-2	CLPT	3.572256269	1.395412606	1.102548231	1.008185608	0.9659380956
	FSDT	3.448009362	1.280094086	0.9342267776	0.7717917380	0.6577956584
	Present $\varepsilon_{zz} = 0$	3.464658434	1.294926794	0.9545325160	0.7980593381	0.6889375175
	Present $\varepsilon_{zz} \neq 0$	3.463246171	1.288667312	0.9435587644	0.7823968675	0.6692740888
1-1-1	CLPT	4.073680342	1.591281383	1.257308746	1.149700800	1.101523164
	FSDT	3.917513742	1.447067697	1.048352660	0.8587281094	0.7255320520
	Present $\varepsilon_{zz} = 0$	3.929381180	1.457542915	1.062508331	0.8767844736	0.7466646737
	Present $\varepsilon_{zz} \neq 0$	3.927719665	1.450763339	1.050923389	0.8606835962	0.7269823555
1-2-1	CLPT	4.936524369	1.928329831	1.523618633	1.393218303	1.334836189
	FSDT	4.716874931	1.727248991	1.235867170	0.9981380800	0.8314832594
	Present $\varepsilon_{zz} = 0$	4.714156914	1.725026666	1.233223682	0.9953305240	0.8289359990
	Present $\varepsilon_{zz} \neq 0$	4.710309338	1.716591897	1.220003355	0.9780034489	0.8087928015
1-3-1	CLPT	5.594161026	2.185219151	1.726592909	1.578820836	1.512661141
	FSDT	5.319394855	1.935305200	1.372197780	1.097122555	0.9049736365
	Present $\varepsilon_{zz} = 0$	5.299628136	1.918784039	1.351611880	1.073319192	0.8799632784
	Present $\varepsilon_{zz} \neq 0$	5.293300430	1.908706959	1.336858530	1.054834742	0.8592527295

with p = 3 and k = 1 for different values of aspect ratios and layer thickness ratios. As the aspect ratio b/a increases, the dimensionless critical buckling load of the sandwich plate decreases.

Fig. 7 exhibit plots of the critical buckling loads versus the side to-thickness ratio b/h for k=2, p=1, b=2a. Once again, it is well known that the CLPT critical buckling loads are independent of the b/h ratio. Critical buckling loads decrease as γ increases. The results of the sandwich plate (1-3-1) are superior to those of (2-1-2), (1-1-1) and (1-2-1) and this depends on the thickness of the core layer.

Fig.8 show the effect of the power-law index k on the nondimensional critical buckling loads of Al/Al_2O_3 sandwich square plates with FGM face sheets and FGM core (b/h=10, p=2). It can be seen from these figures that with the increase of the power-law index, they decrease for sandwich plates.

5. Conclusions

A novel quasi-3D hyperbolic shear deformation theory for buckling analysis of FG sandwich plates with both FGM face sheets and FGM core is proposed in this paper. It contains only five unknowns, accounts for a hyperbolic distribution of transverse shear stress and satisfies the traction free boundary conditions. The governing equations are obtained through the principle of virtual work. The Navier-type solutions are derived and compared with the existing solutions to verify the validity of the developed theory. Benchmark comparisons of the solutions obtained for a degradation model (functionally graded face sheets and homogeneous core) with ones in literature are conducted to verify the accuracy and efficiency of the present theory. The critical buckling loads of sandwich plates with homogeneous face sheets and FGM core or with FGM face sheets and FGM core are presented. Parameter studies are carried out to analyze the influences of power index ***p***, inhomogeneity parameter ***k***, geometrical parameters (b/a, b/h ratios). The main findings can be summarized as follows:

(1) Comparative studies reveal that the present theory is not only accurate, but also more efficient since the number of unknown functions involved in the present theory is only five, compared with other theories containing six or more unknown functions.

(2) Critical buckling loads are presented for sandwich plate with homogeneous face sheets and symmetric FGM core as well as sandwich plate with FGM face sheets and FGM core.

(3) Parameter studies show that: (a) power index ***p*** and inhomogeneity parameter ***k*** play an important role in eliminating interface problems of sandwich plates; (b) the dimensionless critical buckling loads decreases as the aspect ratio b/a increases; (c) Once again, it is well known that the CLPT critical buckling loads are independent of the b/h ratio; (d) Critical buckling loads decrease as γ increases; (e) The results of the sandwich plate (1-3-1) are superior to those of (2-1-2), (1-1-1) and (1-2-1) and this depends on the thickness of the core layer.

An improvement of the present study will be considered in the future work to consider other types of materials and structures (Al-Basyouni *et al*. 2015, Mahi *et al*. 2015, Larbi Chaht *et al*. 2015, Zemri *et al*. 2015, Belkorissat *et al*. 2015, Ahouel *et al*. 2016, Bousahla *et al*. 2016, Draiche *et al*. 2016, Bounouara *et al*. 2016, Bellifa *et al*. 2017b, Yeghnem *et al*. 2017, Klouche *et al*. 2017, Chikh *et al*. 2017, Khetir *et al*. 2017, Mouffoki *et al*. 2017, Karami *et al*. 2017, Bouafia *et al*. 2017, Bakhadda *et al*. 2018, Bouadi *et al*. 2018, Kaci *et al*. 2018, Shahsavari *et al*. 2018, Behera and Kumari 2018, Kadari *et al*. 2018, Yazid *et al*. 2018, Mokhtar *et al*. 2018, Ayat *et al*. 2018, Youcef *et al*. 2018, Cherif *et al*. 2018, Karami *et al*. 2018abcd, Bourada *et al*. 2018 and 2019, Adda Bedia *et al*. 2019, Berghouti *et al*. 2019, Khiloun *et al*. 2019, Bendaho *et al*. 2019, Bouanati *et al*. 2019, Boulefrakh *et al*. 2019, Karami *et al*. 2019abc, Boutaleb *et al*. 2019, Bensattalah *et al*. 2019, Semmah *et al*. 2019, Draoui *et al*. 2019).

References

Aaleti, S. and Sritharan, S. (2009), "A simplified analysis method for characterizing unbonded post-tensioned precast wall systems", *Eng. Strcut.*, **31**(12), 2966-2975. https://doi.org/10.1016/j.engstruct.2009.07.024.

Abdelaziz, H.H., Meziane, M.A.A, Bousahla, A.A., Tounsi, A., Mahmoud, S.R. and Alwabli, A.S. (2017), "An efficient hyperbolic shear deformation theory for bending, buckling and free vibration of FGM sandwich plates with various boundary conditions", *Steel Compos. Struct.,* **25**(6), 693-704. https://doi.org/10.12989/scs.2017.25.6.693.

Abualnour, M., Houari, M.S.A., Tounsi, A., Adda Bedia, E.A., Mahmoud, S.R. (2018), "A novel quasi 3D trigonometric plate theory for free vibration analysis of advanced composite plates", *Compos. Struct.*, 184, 688-697. https://doi.org/10.1016/j.compstruct.2017.10.047.

Adda Bedia, W., Houari, M.S.A., Bessaim, A., Bousahla, A.A., Tounsi, A., Saeed, T., Alhodaly, M.S. (2019), "A new hyperbolic two-unknown beam model for bending and buckling analysis of a nonlocal strain gradient nanobeams", *J. Nano Res.,* **57**, 175-191. https://doi.org/10.4028/www.scientific.net/JNanoR.57.175.

Ahmed, A. (2014), "Post buckling analysis of sandwich beams with functionally graded faces using a consistent higher order theory", *Int. J. Civil Struct. Environ*., **4**(2), 59-64.

Ahouel, M., Houari, M.S.A., Adda Bedia, E.A. and Tounsi, A. (2016), "Size-dependent mechanical behavior of functionally graded trigonometric shear deformable nanobeams including neutral surface position concept", *Steel Compos. Struct*., **20**(5), 963-981. https://doi.org/10.12989/scs.2016.20.5.963.

Ait Atmane, H, Tounsi, A., Bernard, F. (2017), "Effect of thickness stretching and porosity on mechanical response of a functionally graded beams resting on elastic foundations", *International J. Mech. Mater. Design*, **13**(1), 71-84. https://doi.org/10.1007/s10999-015-9318-x.

Akavci, S.S. (2016), "Mechanical behavior of functionally graded sandwich plates on elastic foundation", *Compos. Part B Eng.,* **96**,136-152. https://doi.org/10.1016/j.compositesb.2016.04.035.

Al-Basyouni, K.S., Tounsi, A. and Mahmoud, S.R. (2015), "Size dependent bending and vibration analysis of functionally graded micro beams based on modified couple stress theory and neutral surface position", *Compos. Struct*., **125**, 621-630. https://doi.org/10.1016/j.compstruct.2014.12.070.

Aldousari, S.M. (2017), "Bending analysis of different material distributions of functionally graded beam", *Appl. Phys. A,* **123**, 296. https://doi.org/10.1007/s00339-017-0854-0

Alibeigloo, A., Liew, K.M. (2014), "Free vibration analysis of sandwich cylindrical panel with functionally graded core using three-dimensional theory of elasticity*", Compos Struct,* **113**, 23-30. https://doi.org/10.1016/j.compstruct.2014.03.004.

Alshorbagy, A.E., Eltaher, M.A., Mahmoud, F.F. (2011), "Free vibration characteristics of a functionally graded beam by finite element method", *Appl. Math. Model.*, **35**(1), 412-425. https://doi.org/10.1016/j.apm.2010.07.006.

Ambartsumian, S.A. (1958), "On the theory of bending plates", *Izv Otd Tech Nauk AN SSSR*, **5**, 69-77. https://doi.org/10.1002/sapm1944231184.

Attia, A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "Free vibration analysis of functionally graded plates with temperature-dependent properties using various four variable refined plate theories", *Steel Compos. Struct*., **18**(1),187-212. https://doi.org/10.12989/scs.2015.18.1.187.

Attia, A., Bousahla, A.A., Tounsi, A., Mahmoud, S.R., Alwabli, A.S. (2018a), "A refined four variable plate theory for thermoelastic analysis of FGM plates resting on variable elastic foundations", *Struct. Eng. Mech*., **65**(4), 453-464. https://doi.org/10.12989/sem.2018.65.4.453.

Attia, M.A., Eltaher, M.A., Soliman, A., Abdelrahman, A.A., Alshorbagy, A.E. (2018b), "Thermoelastic crack analysis in functionally graded pipelines conveying natural gas by an FEM", *J. Appl. Mech.*, **10**(4), https://doi.org/10.1142/S1758825118500369.

Avcar, M and Mohammed, W.K.M. (2018), "Free vibration of functionally graded beams resting on Winkler-Pasternak foundation", *Arab. J. Geosci.,* **11**, 232. https://doi.org/10.1007/s12517-018-3579-2.

Ayat, H., Kellouche, Y., Ghrici, M., Boukhatem, B. (2018), "Compressive strength prediction of limestone filler concrete using artificial neural networks", *Adv. Comput. Design*, **3**(3), 289-302. https://doi.org/10.12989/acd.2018.3.3.289.

Bakhadda, B., Bachir Bouiadjra, M., Bourada, F., Bousahla, A.A., Tounsi, A., Mahmoud, S.R. (2018), "Dynamic and bending analysis of carbon nanotube-reinforced composite plates with elastic foundation", *Wind Struct.*, **27**(5), 311-324. https://doi.org/10.12989/was.2018.27.5.311.

Behera, S., Kumari, P. (2018), "Free vibration of Levy-type rectangular laminated plates using efficient zig-zag theory", *Adv. Comput. Design*, **3**(3), 213-232. https://doi.org/10.12989/acd.2018.3.3.213.

Belabed, Z., Bousahla, A.A., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. (2018), "A new 3-unknown hyperbolic shear deformation theory for vibration of functionally graded sandwich plate", *Earthq. Struct.*, **14**(2), 103-115. https://doi.org/10.12989/eas.2018.14.2.103.

Belabed, Z, Houari , MSA, Tounsi, A, Mahmoud, SR, Anwar Bég, O. (2014), "An efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates", *Compos. B Eng.,* **60**, 274-283. https://doi.org/10.1016/j.compositesb.2013.12.057.

Beldjelili, Y., Tounsi, A. and Mahmoud, S.R. (2016), "Hygro-thermo mechanical bending of S-FGM plates resting on variable elastic foundations using a four-variable trigonometric plate theory", *Smart Struct. Syst.,* **18**(4), 755-786. https://doi.org/10.12989/sss.2016.18.4.755.

Belkorissat, I., Houari, M.S.A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "On vibration properties of functionally graded nano-plate using a new nonlocal refined four variable model", *Steel Compos. Struct*., **18**(4), 1063-1081. http://dx.doi.org/10.12989/scs.2015.18.4.1063.

Bellifa, H., Bakora, A., Tounsi, A., Bousahla, A.A. and Mahmoud, S.R. (2017a), "An efficient and simple four variable refined plate theory for buckling analysis of functionally graded plates", *Steel Compos. Struct*., **25**(3), 257-270. https://doi.org/10.12989/scs.2017.25.3.257.

Bellifa, H., Benrahou, K.H., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2017b), "A nonlocal zeroth-order shear deformation theory for nonlinear postbuckling of nanobeams", *Struct. Eng. Mech*., **62**(6), 695-702. https://doi.org/10.12989/sem.2017.62.6.695.

Bellifa, H., Benrahou, K.H., Hadji, L., Houari, M.S.A. and Tounsi, A. (2016), "Bending and free vibration analysis of functionally graded plates using a simple shear deformation theory and the concept the neutral surface position", *J Braz. Soc. Mech. Sci.Eng.*, **38**, 265-275. https://doi.org/10.1007/s40430-015-0354-0.

Benadouda, M., Ait Atmane, H., Tounsi, A., Bernard, F., Mahmoud, S.R. (2017), "An efficient shear deformation theory for wave propagation in functionally graded material beams with porosities", *Earthq. Struct.*, **13**(3), 255-265. https://doi.org/10.12989/eas.2017.13.3.255.

Benahmed, A, Houari, MSA, Benyoucef, S, Belakhdar, K, Tounsi, A. (2017), "A novel quasi-3D hyperbolic shear deformation theory for functionally graded thick rectangular plates on elastic foundation", *Geomechan Eng.,* **12**(1), 9-34. https://doi.org/10.12989/gae.2017.12.1.009.

Benchohra, M., Driz, H., Bakora, A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2018), "A new quasi-3D sinusoidal shear deformation theory for functionally graded plates", *Struct. Eng. Mech*., **65**(1), 19-31. https://doi.org/10.12989/sem.2018.65.1.019.

Bendaho, B., Belabed, Z., Bourada, M., Benatta, M.A., Bourada, F. and Tounsi, A. (2019), "Assessment of new 2D and quasi-3D Nonlocal theories for free vibration analysis of size-dependent functionally graded (FG) nanoplates", *Adv. Nano Res.,* **7**(4), 279-294. https://doi.org/10.12989/anr.2019.7.4.277.

Bennoun, M., Houari, M.S.A. and Tounsi, A. (2016), "A novel five variable refined plate theory for vibration analysis of functionally graded sandwich plates", *Mech. Adv. Mater. Struct.*, **23**(4), 423 -431. https://doi.org/10.1080/15376494.2014.984088.

Berghouti, H., Adda Bedia, E.A. Benkhedda, A. and Tounsi, A. (2019), "Vibration analysis of nonlocal porous nanobeams made of functionally graded material", *Adv. Nano Res.*, **7**(5), 351-364. https://doi.org/10.12989/anr.2019.7.5.351.

Bensattalah, T., Zidour, M., Daouadji, T.S. (2019), "A new nonlocal beam model for free vibration analysis of chiral single-walled carbon nanotubes", *Compos. Mater. Eng.*, **1**(1), 21-31. https://doi.org/10.12989/cme.2019.1.1.021.

Bessaim, A., Houari, M.S.A., Tounsi, A., Mahmoud, S.R, Adda Bedia, E.A. (2013), "A new higher order shear and normal deformation theory for the static and free vibration analysis of sandwich plates with functionally graded isotropic face sheets", *J. Sandwich Struct. Mater.*, **15**(6), 671-703. https://doi.org/10.1177/1099636213498888.

Besseghier, A, Houari, MSA, Tounsi, A, Mahmoud, SR. (2017), "Free vibration analysis of embedded nanosize FG plates using a new nonlocal trigonometric shear deformation theory", *Smart Struct. Syst*., **19**(6), 601-614. https://doi.org/10.12989/sss.2017.19.6.601.

Bouadi, A., Bousahla, A.A., Houari, M.S.A., Heireche, H., Tounsi, A. (2018), "A new nonlocal HSDT for analysis of stability of single layer graphene sheet", *Adv. Nano Res.*, **6**(2), 147-162. https://doi.org/10.12989/anr.2018.6.2.147.

Bouafia, K., Kaci, A., Houari, M. S. A., Benzair, A., Tounsi, A. (2017), "A nonlocal quasi-3D theory for bending and free flexural vibration behaviors of functionally graded nanobeams", *Smart Struct. Syst.*., **19**(2), 115-126. https://doi.org/10.12989/sss.2017.19.2.115.

Bouanati, S., Benrahou, K.H., Ait Atmane, H., Ait Yahia, S., Bernard, F., Tounsi, A., Adda Bedia, E.A. (2019), "Investigation of wave propagation in anisotropic plates via quasi 3D HSDT", *Geomech. Eng.*, **18**(1), 85-96. https://doi.org/10.12989/gae.2019.18.1.085.

Bouderba, B., Houari, M. S. A., Tounsi, A. and Mahmoud, S.R. (2016), "Thermal stability of functionally graded sandwich plates using a simple shear deformation theory", *Struct. Eng. Mech.*, **58**(3), 397-422. https://doi.org/10.12989/sem.2016.58.3.397.

Bouderba, B., Houari, M.S.A. and Tounsi, A. (2013), "Thermomechanical bending response of FGM thick plates resting on Winkler-Pasternak elastic foundations", *Steel Comp. Struct*., **14**(1), 85-104. https://doi.org/10.12989/scs.2013.14.1.085.

Bouhadra, A., Tounsi, A., Bousahla, A.A., Benyoucef, S., Mahmoud, S.R. (2018), "Improved HSDT accounting for effect of thickness stretching in advanced composite plates", *Struct. Eng. Mech.*, **66**(1), 61-73. https://doi.org/10.12989/sem.2018.66.1.061.

Boukhari, A., Ait Atmane, H., Houari, M.S.A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2016), "An efficient shear deformation theory for wave propagation of functionally graded material plates", *Struct. Eng. Mech*., **57**(5), 837-859. https://doi.org/10.12989/sem.2016.57.5.837.

Boukhlif, Z., Bouremana, M., Bourada, F., Bousahla, A.A., Bourada, M., Tounsi, A., Al-Osta, M.A. (2019), "A simple quasi-3D HSDT for the dynamics analysis of FG thick plate on elastic foundation", *Steel Compos. Struct.*, **31**(5), 503-516. https://doi.org/10.12989/scs.2019.31.5.503.

Boulefrakh, L., Hebali, H., Chikh, A., Bousahla, A.A., Tounsi, A., Mahmoud, S.R. (2019), "The effect of parameters of visco-Pasternak foundation on the bending and vibration properties of a thick FG plate", *Geomech. Eng.*, **18**(2), 161-178. https://doi.org/10.12989/gae.2019.18.2.161.

Bounouara, F., Benrahou, K.H., Belkorissat, I. and Tounsi, A. (2016), "A nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates resting on elastic foundation", *Steel Compos. Struct*., **20**(2), 227-249. https://doi.org/10.12989/scs.2016.20.2.227.

Bourada, F., Bousahla, A.A., Bourada, M., Azzaz, A., Zinata, A., Tounsi, A. (2019), "Dynamic investigation of porous functionally graded beam using a sinusoidal shear deformation theory", *Wind Struct.*, **28**(1), 19-30. https://doi.org/10.12989/was.2019.28.1.019.

Bourada, F., Amara, K., Bousahla, A.A., Tounsi, A., Mahmoud, S.R. (2018), "A novel refined plate theory for stability analysis of hybrid and symmetric S-FGM plates", *Struct. Eng. Mech.*, **68**(6), 661-675. https://doi.org/10.12989/sem.2018.68.6.661.

Bourada, M., Kaci, A., Houari, M.S.A. and Tounsi, A. (2015), "A new simple shear and normal deformations theory for functionally graded beams", *Steel Compos. Struct*., **18**(2), 409-423. https://doi.org/10.12989/scs.2015.18.2.409.

Bousahla, A.A., Benyoucef, S., Tounsi, A. and Mahmoud, S.R. (2016), "On thermal stability of plates with functionally graded coefficient of thermal expansion", *Struct. Eng. Mech*., **60**(2), 313-335. https://doi.org/10.12989/sem.2016.60.2.313.

Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Adda Bedia, E.A. (2014), "A novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates", *Int. J. Computat. Methods*, **11**(6), 1350082. https://doi.org/10.1142/S0219876213500825.

Boutaleb, S., Benrahou, K.H., Bakora, A., Algarni, A., Bousahla, A.A., Tounsi, A., Mahmoud, S.R., Tounsi, A. (2019), "Dynamic Analysis of nanosize FG rectangular plates based on simple nonlocal quasi 3D HSDT", *Adv. Nano Res.*, **7**(3), 189-206. https://doi.org/10.12989/anr.2019.7.3.191.

Carrera, E, Brischetto, S, Cinefra, M, Soave, M. (2011), "Effects of thickness stretching in functionally graded plates and shells", *Compos. B Eng*, **42**(2), 123-133. https://doi.org/10.1016/j.compositesb.2010.10.005.

Chaabane, L.A., Bourada, F., Sekkal, M., Zerouati, S., Zaoui, F.Z., Tounsi, A., Derras, A., Bousahla, A.A., Tounsi, A. (2019), "Analytical study of bending and free vibration responses of functionally graded beams resting on elastic foundation", *Struct. Eng. Mech.*, **71**(2), 185-196. https://doi.org/10.12989/sem.2019.71.2.185.

Chen, CS, Hsu, CY, Tzou, G.J. (2009), "Vibration and stability of functionally graded plates based on a higher-order deformation theory", *J. Reinforced Plastics Compos.*, **28**(10), 1215-1234. https://doi.org/10.1177%2F0731684408088884.

Cherif, R.H., Meradjah, M., Zidour, M., Tounsi, A., Belmahi, H., Bensattalah, T. (2018), "Vibration analysis of nano beam using differential transform method including thermal effect", *J. Nano Res.*,**54**, 1-14. https://doi.org/10.4028/www.scientific.net/JNanoR.54.1

Chikh, A., Tounsi, A., Hebali, H. and Mahmoud, S.R. (2017), "Thermal buckling analysis of cross-ply laminated plates using a simplified HSDT", *Smart Struct. Syst*., **19**(3), 289-297. https://doi.org/10.12989/sss.2017.19.3.289.

Chih-Ping, W, Hao-Yuan, L. (2010), "An RMVT-based third-order shear deformation theory of multilayered functionally graded material plates", *Compos. Struct.,* **92**(10), 2591-2605. https://doi.org/10.1016/j.compstruct.2010.01.022.

Dai, HL, Rao, YN, Dai, T. (2016), "A review of recent researches on FGM cylindrical structures under coupled physical interactions, 2000-2015", *Compos. Struct.,* **152**, 199-225. https://doi.org/10.1016/j.compstruct.2016.05.042.

Dean, J., Fallah, A.S., Brown, P.M., Louca, L.A., Clyne, T.W. (2011), "Energy absorption during projectile perforation of lightweight sandwich panels with metallic fibre cores", *Compos. Struct*., **93**(3), 1089-1095. https://doi.org/10.1016/j.compstruct.2010.09.019.

Draiche, K., Tounsi, A. and Mahmoud, S.R. (2016), "A refined theory with stretching effect for the flexure analysis of laminated composite plates", *Geomech. Eng*., **11**(5), 671-690. https://doi.org/10.12989/gae.2016.11.5.671.

Draoui, A., Zidour, M., Tounsi, A., Adim, B. (2019), "Static and dynamic behavior of nanotubes-reinforced sandwich plates using (FSDT)", *J. Nano Res.*, **57**, 117-135. https://doi.org/10.4028/www.scientific.net/JNanoR.57.117.

El-Haina, F., Bakora, A., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2017), "A simple analytical approach for thermal buckling of thick functionally graded sandwich plates", *Struct. Eng. Mech*., **63**(5), 585-595. https://doi.org/10.12989/sem.2017.63.5.585.

Eltaher, M.A., Mohamed, N., Mohamed, S.A., Seddek, L.F. (2019a), "Periodic and nonperiodic modes of postbuckling and nonlinear vibration of beams attached to nonlinear foundations", *Appl. Math. Model.*, **75**, 414-445. https://doi.org/10.1016/j.apm.2019.05.026.

Eltaher, M.A., Mohamed, N., Mohamed, S., Seddek, L.F. (2019b), "Postbuckling of curved carbon nanotubes using energy equivalent model", *J. Nano Res.*, **57**, 136-157. https://doi.org/10.4028/www.scientific.net/JNanoR.57.136.

Eltaher, M. A., Attia, M. A., Soliman, A. E. and Alshorbagy, A. E. (2018a), "Analysis of crack occurs under unsteady pressure and temperature in a natural gas facility by applying FGM", *Struct. Eng. Mech.*, **66**(1), 97-111. https://doi.org/10.12989/sem.2018.66.1.097.

Eltaher, M.A., Fouda, N., El-midany, T., Sadoun, A.M. (2018b), "Modified porosity model in analysis of functionally graded porous nanobeams", *J. Brazil. Soc. MEch. Sci. Eng.*, **40**(3), 141. https://doi.org/10.1007/s40430-018-1065-0.

Eltaher, M.A., Khairy, A., Sadoun, A.M., Omar, F.A. (2014), "Static and buckling analysis of functionally graded Timoshenko nanobeams", *Appl. Math. Comput.*, **229**, 283-295. https://doi.org/10.1016/j.amc.2013.12.072.

Eltaher, M.A., Alshorbagy, A.E., Mahmoud, F.F. (2013), "Determination of neutral axis position and its effect on natural frequencies of functionally graded macro/nanobeams", *Compos. Struct.*, **99**, 193-201. https://doi.org/10.1016/j.compstruct.2012.11.039.

Emam, S., Eltaher, M., Khater, M., Abdalla, W. (2018), "Postbuckling and free vibration of multilayer imperfect nanobeams under a pre-stress load", *Appl. Sci.*, **8**(11), 2238. https://doi.org/10.3390/app8112238.

Emam, S., Eltaher, M.A. (2016), "Buckling and postbuckling of composite beams in hygrothermal environments", *Compos. Struct.*, **152**, 665-675. https://doi.org/10.1016/j.compstruct.2016.05.029.

Fahsi, A., Tounsi, A., Hebali, H., Chikh, A., Adda Bedia, E.A. and Mahmoud, S.R. (2017), "A four variable refined nth-order shear deformation theory for mechanical and thermal buckling analysis of functionally graded plates", *Geomech. Eng*., **13**(3), 385-410. https://doi.org/10.12989/gae.2017.13.3.385.

Fazzolari, F.A. (2015), "Natural frequencies and critical temperatures of functionally graded sandwich plates subjected to uniform and non-uniform temperature distributions", *Compos Struct*, **121**,197-210. https://doi.org/10.1016/j.compstruct.2014.10.039.

Ferreira, A.J.M., Carrera, E., Cinefra, M., Roque, C.M.C., Polit, O. (2011), "Analysis of laminated shells by a sinusoidal shear deformation theory and radial basis functions collocation, accounting for through-the-thickness deformations", *Compos.*

Part B Eng., **42**(5), 1276-1284. https://doi.org/10.1016/j.compositesb.2011.01.031,

Fourn, H., Ait Atmane, H., Bourada, M., Bousahla, A.A., Tounsi, A., Mahmoud, S.R. (2018), "A novel four variable refined plate theory for wave propagation in functionally graded material plates", *Steel Compos. Struct.*, **27**(1), 109-122. https://doi.org/10.12989/scs.2018.27.1.109.

Gupta, N. (2007), "A functionally graded syntactic foam material for high energy absorption under compression", *Mater Lett.,* **61**(4-5), 979-982. https://doi.org/10.1016/j.matlet.2006.06.033.

Hachemi, H., Kaci, A., Houari, M.S.A., Bourada, A., Tounsi, A., Mahmoud, S.R. (2017), "A new simple three-unknown shear deformation theory for bending analysis of FG plates resting on elastic foundations", *Steel Compos. Struct.*, **25**(6), 717-726. https://doi.org/10.12989/scs.2017.25.6.717.

Hamed, M.A., Eltaher, M.A., Sadoun, A.M. and Almitani, K.H. (2016), "Free vibration of symmetric and sigmoid functionally graded nanobeams", *Appl. Phys. A*, **122**(9), 829. https://doi.org/10.1007/s00339-016-0324-0.

Hamidi, A., Houari, M.S.A..Mahmoud, S.R. and Tounsi, A. (2015), "A sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates", *Steel Comp. Struct.*, **18**(1), 235-253. https://doi.org/10.12989/scs.2015.18.1.235.

Hebali, H., Bakora, A., Tounsi, A., Kaci, A. (2016), "A novel four variable refined plate theory for bending, buckling, and vibration of functionally graded plates", *Steel Compos Struct.*, **22**(3), 473-495. https://doi.org/10.12989/scs.2016.22.3.473.

Hebali, H., Tounsi, A., Houari, M.S.A., Bessaim, A., Adda Bedia, E.A. (2014), "New quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of functionally graded plates", *J. Eng. Mech. (ASCE),* **140**(2), 374-383. https://doi.org/10.1061/(ASCE)EM.1943-7889.0000665.

Hellal, H., Bourada, M., Hebali, H., Bourada, F., Tounsi, A., Bousahla, A.A., Mahmoud, S.R. (2019), "Dynamic and stability analysis of functionally graded material sandwich plates in hygro-thermal environment using a simple higher shear deformation theory", *J. Sandwich Struct. Mater.*, https://doi.org/10.1177/1099636219845841.

Houari, M.S.A., Tounsi, A., Bessaim, A. and Mahmoud, S.R. (2016), "A new simple three -unknown sinusoidal shear deformation theory for functionally graded plates", *Steel Compos. Struct.*, **22**(2), 257-276. https://doi.org/10.12989/scs.2016.22.2.257.

Jha, D.K., Kant, T., Singh, R.K. (2012), "Higher order shear and normal deformation theory for natural frequency of functionally graded rectangular plates", *Nucl Eng Des*, **250**, 8-13. https://doi.org/10.1016/j.nucengdes.2012.05.001.

Kaci, A., Houari, M.S.A., Bousahla, A.A., Tounsi, A., Mahmoud, S.R. (2018), "Post-buckling analysis of shear-deformable composite beams using a novel simple two-unknown beam theory", *Struct. Eng. Mech.*, **65**(5), 621-631. https://doi.org/10.12989/sem.2018.65.5.621.

Kadari, B., Bessaim, A., Tounsi, A., Heireche, H., Bousahla, A.A. and Houari, M.S.A. (2018), "Buckling analysis of orthotropic nanoscale plates resting on elastic foundations", *J. Nano Res.*, **55**, 42-56. https://doi.org/10.4028/www.scientific.net/JNanoR.55.42.

Kar, V.R. and Panda, S.K. (2015), "Nonlinear flexural vibration of shear deformable functionally graded spherical shell panel", *Steel Compos. Struct.,* **18**(3), 693-709. https://doi.org/10.12989/scs.2015.18.3.693.

Karama, M., Afaq, K.S. and Mistou, S. (2003), "Mechanical behaviour of laminated composite beam by new multi-layered laminated composite structures model with transverse shear stress continuity", *Int. J. Solids Struct*., **40**, 1525-1546. https://doi.org/10.1016/S0020-7683(02)00647-9.

Karami, B., Janghorban, M., Tounsi, A. (2019a), "On exact wave propagation analysis of triclinic material using three dimensional bi-Helmholtz gradient plate model", *Struct. Eng. Mech.*, **69**(5), 487-497. https://doi.org/10.12989/sem.2019.69.5.487.

Karami, B., Janghorban, M., Tounsi, A. (2019b), "Wave propagation of functionally graded anisotropic nanoplates resting on Winkler-Pasternak foundation", *Struct. Eng. Mech.*, **7**(1), 55-66. https://doi.org/10.12989/sem.2019.70.1.055.

Karami, B., Shahsavari, D., Janghorban, M., Tounsi, A. (2019c), "Resonance behavior of functionally graded polymer composite nanoplates reinforced with grapheme nanoplatelets", *J. Mech. Sci.*, **156**, 94-105. https://doi.org/10.1016/j.ijmecsci.2019.03.036.

Karami, B., Janghorban, M. and Tounsi, A. (2018a), "Galerkin's approach for buckling analysis of functionally graded anisotropic nanoplates/different boundary conditions", *Eng. Comput.*. https://doi.org/10.1007/s00366-018-0664-9.

Karami, B., Janghorban, M., Shahsavari, D., Tounsi, A. (2018b), "A size-dependent quasi-3D model for wave dispersion analysis of FG nanoplates", *Steel Compos. Struct.*, **28**(1), 99-110. https://doi.org/10.12989/scs.2018.28.1.099.

Karami, B., Janghorban, M., Tounsi, A. (2018c), "Nonlocal strain gradient 3D elasticity theory for anisotropic spherical nanoparticles", *Steel Compos. Struct.*, **27**(2), 201-216. https://doi.org/10.12989/scs.2018.27.2.201.

Karami, B., Janghorban, M., Tounsi, A. (2018d), "Variational approach for wave dispersion in anisotropic doubly-curved nanoshells based on a new nonlocal strain gradient higher order shell theory", *Thin-Walled Struct.*, **129**, 251-264. https://doi.org/10.1016/j.tws.2018.02.025.

Karami, B., Janghorban, M., Tounsi, A. (2017), "Effects of triaxial magnetic field on the anisotropic nanoplates", *Steel Compos. Struct.*, **25**(3), 361-374. https://doi.org/10.12989/scs.2017.25.3.361.

Kashtalyan, M., Menshykova, M. (2009), "Three-dimensional elasticity solution for sandwich panels with a functionally graded core", *Compos Struct,* **87** (1), 36-43. https://doi.org/10.1016/j.compstruct.2007.12.003.

Khetir, H., Bachir Bouiadjra, M., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), "A new nonlocal trigonometric shear deformation theory for thermal buckling analysis of embedded nanosize FG plates", *Struct. Eng. Mech., Int. J.*, **64**(4), 391-402. https://doi.org/10.12989/sem.2017.64.4.391.

Khiloun, M., Bousahla, A.A., Kaci, A., Bessaim, A., Touns, A., Mahmoud, S.R. (2019), "Analytical modeling of bending and vibration of thick advanced composite plates using a four-variable quasi 3D HSDT", *Eng. Comput.*., https://doi.org/10.1007/s00366-019-00732-1.

Kirigulige, MS, Kitey, R, Tippur, HV. (2005), "Dynamic fracture behaviour of model sandwich structures with functionally graded core: A feasibility study", *Compos Sci Technol,* **65**, 1052-1068. https://doi.org/10.1016/j.compscitech.2004.10.029.

Klouche, F., Darcherif, L., Sekkal, M., Tounsi, A., and Mahmoud, S.R. (2017), "An original single variable shear deformation theory for buckling analysis of thick isotropic plates", *Struct. Eng. Mech.*, **63**(4), 439-446. https://doi.org/10.12989/sem.2017.63.4.439.

Koizumi, M. (1997), "FGM activities in Japan", *Composites Part B: Engineering*, **28**(1-2), 1-4. https://doi.org/10.1016/S1359-8368(96)00016-9.

Kolahchi, R., Safari, M. and Esmailpour, M. (2016), "Dynamic stability analysis of temperature-dependent functionally graded CNT-reinforced visco-plates resting on orthotropic elastomeric medium", *Compos. Struct.*, **150**, 255-265. https://doi.org/10.1016/j.compstruct.2016.05.023.

Krenich, F., Heireche, H., Houari, M.S.A., Tounsi, A. (2017), "A novel nonlocal four variable platetheory for thermal stability of single-layered graphene sheets embedded in anelastic substrate medium", *Curr. Nanomater.*, **1**, 215-222.

Larbi Chaht, F., Kaci, A., Houari, M.S.A., Tounsi, A., Anwar Bég,

O. and Mahmoud, S.R. (2015), "Bending and buckling analyses of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect", *Steel. Compos. Struct*., **18**(2), 425-442. https://doi.org/10.12989/scs.2015.18.2.425.

Levinson, M. (1980), "An accurate simple theory of static and dynamics of elastic plates", *Mech. Res. Commun.,* **7**, 343-350.

Levy, M. (1877), "Mémoire sur la théorie des plaques élastique planes", *J Math Appl,* **30**, 219-306.

Li, R.R. and Batra, R. (2013), "Relations between buckling loads of functionally graded Timoshenko and homogeneous Euler-Bernoulli beams", *Compos. Struct.*, **95**, 5-9. https://doi.org/10.1016/j.compstruct.2012.07.027.

Librescu, L., Hause, T. (2000), "Recent developments in the modeling and behavior of advanced sandwich constructions: a survey", *Compos. Struct*., **48**(1-3), 1-17. https://doi.org/10.1016/S0263-8223(99)00068-9.

Lindström, A., Hallström, S. (2010), "Energy absorption of SMC/balsa sandwich panels with geometrical triggering features", *Compos Struct*, **92**(11), 2676-2684. https://doi.org/10.1016/j.compstruct.2010.03.018.

Liu, B., Ferreira, A.J.M., Xing, Y.F., Neves, A.M.A. (2016), "Analysis of functionally graded sandwich and laminated shells using a layerwise theory and a differential quadrature finite element method", *Compos Struct*, **136**, 546-553. https://doi.org/10.1016/j.compstruct.2015.10.044.

Lü, C.F., Lim, C.W., Chen, W.Q. (2009), "Exact solutions for free vibrations of functionally graded thick plates on elastic foundations", *Mech. Adv. Mater. Struct*., **16**, 576-584. https://doi.org/10.1080/15376490903138888.

Lü, C.F., Lim, C.W., Chen, W.Q. (2009), "Semi-analytical analysis for multi-directional functionally graded plates: 3-D elasticity solutions", *Int. J. Numer. Meth. Eng*., **79**, 25-44. https://doi.org/10.1002/nme.2555.

Mahi, A., Adda Bedia, E.A., Tounsi, A. (2015), "A new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates", *Appl. Math. Model.*, **39**, 2489-2508. https://doi.org/10.1016/j.apm.2014.10.045.

Mahmoudi, A., Benyoucef, A., Tounsi, A., Benachour, A., Adda Bedia, E.A., Mahmoud, S.R. (2017), "A refined quasi-3D shear deformation theory for thermo-mechanical behavior of functionally graded sandwich plates on elastic foundations", *J. Sandwich Struct. Mater.*, https://doi.org/10.1177/1099636217727577.

Mantari, J.L., Guedes Soares, C. (2012), "Generalized hybrid quasi-3D shear deformation theory for the static analysis of advanced composite plates", *Compos. Struct.*, **94**(8), 2561-2575. https://doi.org/10.1016/j.compstruct.2012.02.019.

Mantari, J.L., Guedes Soares, C. (2013), "A novel higher-order shear deformation theory with stretching effect for functionally graded plates", *Compos. Part B Eng.*, **45**(1), 268-281. https://doi.org/10.1016/j.compositesb.2012.05.036.

Matsunaga, H. (2009), "Stress analysis of functionally graded plates subjected to thermal and mechanical loadings", *Compos. Struct.,* **87**(4), 344-357. https://doi.org/10.1016/j.compstruct.2008.02.002.

Meftah, A., Bakora, A., Zaoui, F.Z., Tounsi, A., Adda Bedia, E.A. (2017), "A non-polynomial fourvariable refined plate theory for free vibration of functionally graded thick rectangular plates on elastic foundation", *Steel Compos Struct.*, **23**(3), 317-330. https://doi.org/10.12989/scs.2017.23.3.317

Meksi, R, Benyoucef, S., Mahmoudi, A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, SR. (2019), "An analytical solution for bending, buckling and vibration responses of FGM sandwich plates", *J. Sandw. Struct.Mater*., **21**(2), 727-757. https://doi.org/10.1177/1099636217698443.

Menasria, A., Bouhadra, A., Tounsi, A., Bousahla, A.A. and Mahmoud, S.R. (2017), "A new and simple HSDT for thermal stability analysis of FG sandwich plates", *Steel Comp. Struct*., **25**(2), 157-175. https://doi.org/10.12989/scs.2017.25.2.157.

Meziane, M.A.A., Abdelaziz, H.H. and Tounsi, A. (2014), "An efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions", *J. Sandw. Struct. Mater*.,**16**(3), 293-318. https://doi.org/10.1177/1099636214526852.

Mindlin, RD. (1951), "Influence of rotatory inertia and shear on flexural motions of isotropic elastic plates", *J. Appl. Mech*., **18**, 31-38.

Miyamoto, Y., Kaysser, W.A., Rabin, B.H., Kawasaki, A., Ford, R.G. (1999), "Functionally graded materials: design, processing and applications", *Kluwer Academic Publishers*.

Mohamed, N., Eltaher, M.A., Mohamed, S., and Seddek, L.F., (2019), "Energy equivalent model in analysis of postbuckling of imperfect carbon nanotubes resting on nonlinear elastic foundation", *Struct. Eng. Mech.*, **70**(6), 737-750. https://doi.org/10.12989/sem.2019.70.6.737.

Mohammadimehr, M., Atifeh, S.J., Navi, B.R. (2018), "Stress and free vibration analysis of piezoelectric hollow circular FG-SWBNNTs reinforced nanocomposite plate based on modified couple stress theory subjected to thermo-mechanical loadings", *J. Vib. Control*, **24**(15), 3471-3486. https://doi.org/10.1177%2F1077546317706887.

Mokhtar, Y., Heireche, H., Bousahla, A.A., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. (2018), "A novel shear deformation theory for buckling analysis of single layer graphene sheet based on nonlocal elasticity theory", *Smart Struct. Syst.*, **21**(4), 397-405. https://doi.org/10.12989/sss.2018.21.4.397.

Mouffoki, A., Adda Bedia, E.A., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. (2017), "Vibration analysis of nonlocal advanced nanobeams in hygro-thermal environment using a new two-unknown trigonometric shear deformation beam theory", *Smart Struct. Syst.*, **20**(3), 369-383. https://doi.org/10.12989/sss.2017.20.3.369.

Neves, A.M.A., Ferreira, A.J.M., Carrera, E., Roque, C.M.C., Cinefra, M., Jorge, R.M.N. and Soares, C.M.M. (2012a), "A quasi-3D sinusoidal shear deformation theory for the static and free vibration analysis of functionally graded plates", *Compos. Part B Eng.*, **43**(2), 711-725. https://doi.org/10.1016/j.compositesb.2011.08.009.

Neves, A.M.A., Ferreira, A.J.M., Carrera, E., Cinefra, M., Roque, C.M.C., Jorge, R.M.N. and Soares, C.M.M. (2012b), "A quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of functionally graded plates", *Compos. Struct.*, **94**(5), 1814-1825. https://doi.org/10.1016/j.compstruct.2011.12.005.

Neves, A.M. A., Ferreira, A.J.M., Carrera, E., Cinefra, M., Jorge, R.M.N. and Soares, C.M.M. (2012c), "Buckling analysis of sandwich plates with functionally graded skins using a new quasi-3D hyperbolic sine shear deformation theory and collocation with radial basis functions", *ZAMM - J. Appl. Math. Mech.* **92** (9), 749-766. https://doi.org/10.1002/zamm.201100186.

Neves, A.M.A., Ferreira, A.J.M., Carrera, E., Cinefra, M., Roque, C.M.C., Jorge, R.M.N. and Soares, C.M.M. (2013), "Static, free vibration and buckling analysis of isotropic and sandwich functionally graded plates using a quasi-3D higher-order shear deformation theory and a meshless technique", *Compos. Part B Eng.*, **44**(1), 657-674. https://doi.org/10.1016/j.compositesb.2012.01.089.

Pollien, A., Conde, Y., Pambaguian, L. and Mortensen, A. (2005), "Graded open-cell aluminum foam core sandwich beams", *Mater. Sci. Eng. A*, **404**, 9-18. https://doi.org/10.1016/j.msea.2005.05.096.

Qian, L.F., Batra, R.C. and Chen, L.M. (2004), "Static and dynamic deformations of thick functionally graded elastic plates by using higher-order shear and normal deformable plate theory and meshless local Petrov-Galerkin method", *Compos. Part B Eng.*,

35(6-8), 685-697. https://doi.org/10.1016/j.compositesb.2004.02.004.

Reddy, J.N. (2004), *Mechanics of Laminated Composite Plates and Shells: Theory and Analysis*, *2nd ed.,* CRC Press, Florida, USA.

Reddy, J.N. (2011), "A general nonlinear third-order theory of functionally graded plates", *J. Aerosp. Lightweight Struct.*, **1**(1), 1-21. http://dx.doi.org/10.3850/S201042861100002X.

Reissner, E. (1944), "On the theory of bending of elastic plates", *J. Math. Phys*., **23**, 184-191. https://doi.org/10.1002/sapm1944231184.

Reissner, E. (1945), "The effect of transverse shear deformation on the bending of elastic plates", *J. Appl. Mech*., **12**, 69-77. https://doi.org/10.1177/002199836900300316.

Sekkal, M., Fahsi, B., Tounsi, A., Mahmoud, S.R. (2017a), "A novel and simple higher order shear deformation theory for stability and vibration of functionally graded sandwich plate", *Steel Compos. Struct.*, **25**(4), 389-401. https://doi.org/10.12989/scs.2017.25.4.389.

Sekkal, M., Fahsi, B., Tounsi, A., Mahmoud, S.R. (2017b), "A new quasi-3D HSDT for buckling and vibration of FG plate", *Struct. Eng. Mech.*, **64**(6), 737-749. https://doi.org/10.12989/sem.2017.64.6.737.

Selmi, A., Bisharat, A. (2018), "Free vibration of functionally graded SWNT reinforced aluminum alloy beam", *J. Vibroengineering,* **20**(5), 2151-2164. https://doi.org/10.21595/jve.2018.19445.

Semmah, A., Heireche, H., Bousahla, A.A., Tounsi, A. (2019), "Thermal buckling analysis of SWBNNT on Winkler foundation by non local FSDT", *Adv. Nano Res.*, **7**(2), 89-98. https://doi.org/10.12989/anr.2019.7.2.089.

Shahsavari, D., Karami, B., Li, L. (2018), "A high-order gradient model for wave propagation analysis of porous FG nanoplates", *Steel Compos. Struct.*, **29**(1), 53-66. https://doi.org/10.12989/scs.2018.29.1.053.

Shen, H.S., Li, S.R. (2008), "Postbuckling of sandwich plates with FGM face sheets and temperature-dependent properties", *Compos Pt B-Eng*, **39**(2), 332-344. https://doi.org/10.1016/j.compositesb.2007.01.004.

Sobhy, M. (2013), "Buckling and free vibration of exponentially graded sandwich plates resting on elastic foundations under various boundary conditions", *Compos Struct*, **99**, 76-87. https://doi.org/10.1016/j.compstruct.2012.11.018.

Soldatos, K.P. (1992), "A transverse shear deformation theory for homogeneous monoclinic plates", *Acta Mech*, **94**, 195-200. https://doi.org/10.1007/BF01176650.

Soliman, A.E., Eltaher, M.A., Attia, M.A, Alshorbagy, A.E. (2018), "Nonlinear transient analysis of FG pipe subjected to internal pressure and unsteady temperature in a natural gas facility", *Struct. Eng. Mech.*, **66**(1), 85-96. https://doi.org/10.12989/sem.2018.66.1.085.

Swaminathan, K., Naveenkumar, D.T., Zenkour, A.M., Carrera, E. (2015), "Stress, vibration and buckling analyses of FGM plates—A state-of-the-art review", *Compos. Struct.*, **120**, 10-31. https://doi.org/10.1016/j.compstruct.2014.09.070.

Talha, M., Singh, B.N. (2010), "Static response and free vibration analysis of FGM plates using higher order shear deformation theory", *Appl. Math. Model.*, **34**(12), 3991-4011. https://doi.org/10.1016/j.apm.2010.03.034.

Thai, H.T., Kim, S.E. (2015), "A review of theories for the modeling and analysis of functionally graded plates and shells", *Compos. Struct.*, **128**, 70-86. https://doi.org/10.1016/j.compstruct.2015.03.010.

Tounsi, A, Houari, MSA, Benyoucef, S., Adda Bedia, E.A. (2013), "A refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates", *Aerosp. Sci. Technol,* **24**(1), 209-220. https://doi.org/10.1016/j.ast.2011.11.009.

Tounsi, A., Ait Atmane, H., Khiloun, M., Sekkal, M., Taleb, O., Bousahla, A.A. (2019), "On buckling behavior of thick advanced composite sandwich plates", *Compos. Mater. Eng.*, **1**(1), 1-19. https://doi.org/10.12989/cme.2019.1.1.001.

Touratier, M. (1991), "An efficient standard plate theory", *Int. J. Eng. Sci*., **29**(8), 901-916. https://doi.org/10.1016/0020-7225(91)90165-Y.

Vel, S.S., Batra, R.C. (2004), "Three-dimensional exact solution for the vibration of functionally graded rectangular plates", *J. Sound Vib*., **272**, 703-730. https://doi.org/10.1016/S0022-460X(03)00412-7.

Vinson, J.R. (2001), "Sandwich structures", *Appl. Mech. Rev.*, **54**(3), 201-214. https://doi.org/10.1115/1.3097295.

Vinson, J.R. (2005), "Sandwich structures: past, present, and future", *Sandwich Structures 7: Advancing with Sandwich Structures and Materials. Netherlands*, Springer, Germany. 3-12.

Wu, C.P., Huang, S.E. (2004), "Three-dimensional solutions of functionally graded piezothermo-elastic shells and plates using a modified Pagano method", *Comput. Mater. Continua*, **12**, 251-282.

Yaghoobi, H., Valipour, M.S., Fereidoon, A., Khoshnevisrad, P. (2014), "Analytical study on post-buckling and nonlinear free vibration analysis of FG beams resting on nonlinear elastic foundation under thermo-mechanical loadings using VIM", *Steel Compos. Struct.*, **17**(5), 753-776. http://dx.doi.org/10.12989/scs.2014.17.5.753.

Yahia, S.A., Ait Atmane, H., Houari, M.S.A. and Tounsi, A. (2015), "Wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories", *Struct. Eng. Mech*., **53**(6), 1143-1165. http://dx.doi.org/10.12989/sem.2015.53.6.1143.

Yazid, M., Heireche, H., Tounsi, A., Bousahla, A.A., Houari, M.S.A. (2018), "A novel nonlocal refined plate theory for stability response of orthotropic single-layer graphene sheet resting on elastic medium", *Smart Struct. Syst..*, **21**(1), 15-25. https://doi.org/10.12989/sss.2018.21.1.015.

Yeghnem, R., Guerroudj, H.Z., Amar, LHH., Meftah, S.A., Benyoucef, S., Tounsi, A., Adda Bedia, E.A. (2017), "Numerical modeling of the aging effects of RC shear walls strengthened by CFRP plates: A comparison of results from different "code type" models", *Comput. Concrete*, 19(5), 579-588. https://doi.org/10.12989/cac.2017.19.5.579.

Ying, J., Lü, C.F., Lim, C.W. (2009), "3D thermoelasticity solutions for functionally graded thick plates", *J. Zhejiang Univ. Sci, A*, **10**(3), 327-336. https://doi.org/10.1631/jzus.A0820406.

Youcef, D.O., Kaci, A., Benzair, A., Bousahla, A.A., Tounsi, A. (2018), "Dynamic analysis of nanoscale beams including surface stress effects", *Smart Struct. Syst..*, **21**(1), 65-74. https://doi.org/10.12989/sss.2018.21.1.065.

Younsi, A., Tounsi, A, Zaoui, F.Z., Bousahla, A.A., Mahmoud, S.R. (2018), "Novel quasi-3D and 2D shear deformation theories for bending and free vibration analysis of FGM plates", *Geomech. Eng.*, **14**(6), 519-532. https://doi.org/10.12989/gae.2018.14.6.519.

Zaoui, F.Z., Ouinas, D., Tounsi, A. (2019), "New 2D and quasi-3D shear deformation theories for free vibration of functionally graded plates on elastic foundations", *Compos. Part B*, **159**, 231-247. https://doi.org/10.1016/j.compositesb.2018.09.051.

Zarga, D., Tounsi, A., Bousahla, A.A., Bourada, F., Mahmoud, S.R. (2019), "Thermomechanical bending study for functionally graded sandwich plates using a simple quasi-3D shear deformation theory", *Steel Compos. Struct.*, **32**(3), 389-410. https://doi.org/10.12989/scs.2019.32.3.389.

Zemri, A., Houari, M.S.A., Bousahla, A.A., Tounsi, A. (2015), "A mechanical response of functionally graded nanoscale beam: an assessment of a refined nonlocal shear deformation theory beam theory", *Struct. Eng. Mech.*, **54**(4), 693-710. https://doi.org/10.12989/sem.2015.54.4.693.

Zenkour, A. (2007), "Benchmark trigonometric and 3-D elasticity solutions for an exponentially graded thick rectangular plate", *Archive Appl. Mech.*, **77**(4), 197-214. https://doi.org/10.1007/s00419-006-0084-y.

Zenkour, A., Sobhy, M. (2010), "Thermal buckling of various types of FGM sandwich plates", *Compos Struct*, **93**(1), 93-102. https://doi.org/10.1016/j.compstruct.2010.06.012.

Zenkour, A. (2005), "A comprehensive analysis of functionally graded sandwich plates: Part 2 - buckling and free vibration", *Int. J. Solids Struct*., **42**(18-19), 5243-5258. https://doi.org/10.1016/j.ijsolstr.2005.02.016.

Zidi, M., Houari, M.S.A., Tounsi, A., Bessaim, A., Mahmoud, S.R. (2017), "A novel simple two-unknown hyperbolic shear deformation theory for functionally graded beams", *Struct. Eng. Mech.*, **64**(2), 145-153. https://doi.org/10.12989/sem.2017.64.2.145.

Zidi, M., Tounsi, A., Houari, M.S.A..Adda Bedia, E.A. and Anwar Bég, O. (2014), "Bending analysis of FGM plates under hygro-thermo-mechanical loading using a four variable refined plate theory", *Aerosp. Sci. Technol*. ,**34**, 24-34. https://doi.org/10.1016/j.ast.2014.02.001.

Zine, A., Tounsi, A., Draiche, K., Sekkal, M., Mahmoud, S.R. (2018), "A novel higher-order shear deformation theory for bending and free vibration analysis of isotropic and multilayered plates and shells", *Steel Compos. Struct.*, **26**(2), 125-137. https://doi.org/10.12989/scs.2018.26.2.125.

CC