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Damage assessment based on static and dynamic responses applied to foundation beams

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Abstract. Foundations are a vital part of structures. Over time, the foundations can deteriorate due to unforeseen overloads and/or settlements, resulting in the appearance of cracks in the concrete. These cracks produce changes in the static and dynamic behavior of the affected foundation, which alter its load carrying capacity. In this work, non-destructive techniques of relative simplicity of application are presented for the detection, location, and quantification of damage, using numerical models, solved with the finite element method and Power Series. For this, two types of parameters are used: static (displacement and elastic curvature) and dynamics (natural frequencies). In the static analysis, the damage detection is done by means of a finite elements model representing a beam supported on an elastic foundation with a discrete crack that varies in length and location. With regard to dynamic analysis, the governing equations of the model are presented and a method based on Power Series is used to obtain the solution for a data set, which could be the Winkler coefficient, the location of the crack or the frequency. In order to validate the proposed methodologies, these techniques are applied to data obtained from laboratory tests.

Keywords: foundation beam; failure detection; reinforced concrete; structural dynamics

1. Introduction

A structure can be damaged by different reasons, as by accidental overloads in excess of its designed strength, or by deterioration of its mechanical properties over time, or by environmental influence, reducing the structural stiffness at the damaged zone thus changing its static and dynamic behaviour, being the magnitude of the change a function of the severity and the location of the damage. Moreover, it is required that structures must in safety work during its service life, but damages instigate an earlier breakdown period on the structures. Therefore, it required regular costly inspections. So, the capability of identifying both location and severity of damages of faulted elements in a structural system is greatly needed under the present demands of constantly maintaining the safety of engineering structures.

In most structures, in order to make an overall estimate of the damage, visual inspection and nondestructive evaluation (NDE) is relatively easy to perform, but in the case of foundations the main limitation of visual inspection and some of the traditional approaches is that they require access to potentially damaged regions, which may be dangerous or impossible. Probably for this reason and despite the importance of foundations, the specific literature on damage to foundations is limited (Karatzetzou and Pitilakis 2018).

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For these reasons, it is important to develop methods based on structural parameters, by static or dynamic measurements, such as mass, stiffness, modulus of elasticity and in modal parameters, such as natural frequencies, modal forms and modal damping. Structural damage can be determined by methods that consider the changes in the static response of the structures, i.e. displacement variations and/or strain, (Caddemi and Morassi 2007), (Robles and Ortega 2011), (Abdo 2012), (Xiang *et al.* 2013), (Bernal 2014), (Lozano-Galant *et al.* 2015), (Zhang *et al.* 2015), (Boumechra 2017), (Ercolani *et al.* 2017), (Erdenebat *et al.* 2018), (Ercolani *et al.* 2018 a), or by dynamic methods, where modal parameters are analysed, such as frequencies and modal forms (Gounaris and Dimarogonas,1988), (Salawu 1997), (Doebling 1998), (Patil and Maiti 2005), (Ciambella *et al.* 2011), (Yang *et al.* 2013), (Homaei *et al.* 2014), (Zhao *et al.* 2017), (Khalili and Vosoughi 2018), (Pedram *et al.* 2018), (Faye *et al.* 2018), (Ercolani *et al.* 2018b). Depending on the analysed structure and the equipment availability to make the measurements in situ, one method, dynamic or static, could be more convenient than another, and in some cases, complementary (Jiang *et al.* 2004), (Schommer *et al.* 2017). Rytter (1993) classified damage detection in four levels: level 1 takes into account the existence of structural damage; level 2 is level 1 plus damage localization; in level 3 the quantification of damage severity is added, and level 4 is level 3 plus prediction of the structure's remaining life.

This paper deals with techniques for detection, localization and quantification of damage, using numerical models, solved with the finite elements method and Powers Series, using different parameters, static and dynamic, as

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the damage index. In the case of the static approach, two techniques are introduced. The first one is related to the variation of the flexibility of the damaged structure, relative to the undamaged one. The second technique is based on the elastic curvature variation, analysed using the finite differences method.

It should be noted that, in order to perform on-site measurements, the concrete surface of the affected foundation must be exposed. In the particular case of the foundation beam that is analysed in this work, access is more direct if the columns and the foundation are part of subsoil, for example garages or deposit, simplifying the measurements (static or dynamic).

Tests are performed on elastic foundation beams, adopting a ground reaction which varies linearly according to ground deformation. The techniques presented here allow for the assessment of foundation structures by means of non-destructive methods, in order to evaluate the Residual Load Capacity.

In the case of the dynamic test, the assessment of location and depth of cracks by evaluation of natural frequencies in the damaged structural component has been widely used. One of the reasons for this tendency is the fact that frequencies are dynamic parameters which can be rather simply obtained through measurements. Their experimental determination for a given cracked element is relatively direct, this is due to the fact that the experimental determination of the natural frequencies can be from one single location, thus offering scope for the development of a fast and global NDE technique. Nevertheless, the inverse problem for determining crack parameters (location and depth) for a given set of measured frequencies is not as simple. To achieve significant results a suitable model and an efficient numerical technique must be used.

A simple and widely accepted model is the representation of the decreased rigidity at the site of the crack through a rotational spring. In the direct problem, if the spring constant and location are known, the resolution of the algorithm results in the natural frequencies of the structural system. Within the discipline of Fracture Mechanics, equivalence can be determined for a certain type of problems, between the spring constant and the depth of the crack. In this way, if the crack depth is known the value for the equivalent spring constant can be obtained and vice versa.

To locate and quantify cracks by a dynamic analysis, an indirect method is proposed, i.e. once the response is known the data of the crack are obtained. Firstly, the algorithm for solving the direct problem is stated and then, is used to formulate the inverse problem.

Using dynamic measurements, the values of the first three natural frequencies are obtained. These values are entered as data in the algorithm developed to solve the inverse problem, that is, to predict the spring constant and the location of the crack. In the resolution of the inverse problem, the crack is always considered open and is modelled by a rotational spring, whose constant is an invariant. Then, using the equivalence from the Fracture Mechanics theory, the depth of damage can be obtained (Filipich and Rosales 2002).

A simply supported beam of reinforced concrete, previously damaged, was tested in the laboratory. Through the comparison of the experimental results and those obtained by the proposed numerical model, the quality of the results was assessed. The resulting parameters values attained satisfactory precision.

2. Detection, localization and quantification of damage through static methods

As mentioned in the Introduction, a damaged structure has a lower relative rigidity than the undamaged structure. This is evidenced in the static response, by the vertical displacements and curvature of the elastica produced by the action of the acting loads.

The vertical displacements and the curvature of the elastica of a structure are functions of the structural geometry, of the properties of the material and of the loads acting on it. In a numerical model, the damage can be simulated as a change in the properties of the material or as an alteration in the geometry. In this work, the damage is modelled as a discrete crack, which alters the geometry and, more specifically, the second moment of area of the affected cross-section.

Models representative of the structure, consisting in a foundation beam supported by a Winkler-type elastic medium, were built with and without damage. They were analysed through software based on finite elements methods (Algor 23 2010). Values for vertical displacement and elastic curvature were obtained for different points. It must be noted that the values of vertical displacement may be obtained experimentally, either by in-situ measurement on the actual structure, or in a scaled physical model.

A linear Winkler model is adopted in virtue of the type of soil adopted in this work, dense sand. This type of soil has low shear effect. If the soil had shear capacity, as in the case of rocks or gravels then other models should be adopted, for example, a Pasternak model. Moreover, the length to height ratio of the studied foundation is between 3 and 6, range in which not considering the shear effects generates on flexion behaviour errors up to 10 % (Teodoru *et al*. 2006). These errors are considered acceptable in comparison to other simplifications that are usually assumed in a model, such as the case of considering reinforced concrete as a homogenous and isotropic material.

2.1 Numerical modelling

Fig. 1 shows the model used in this study, with its dimensions. It involves a foundation beam with a crack placed at distance x from the left end, and the plate with a crack at x1 from the left end. In these models, rectangular Plate - Shell elements were used with nodes at their vertices. with a density, in areas far from the crack, of 400 elements/m2. In the nearby crack area, were used triangular elements with a mesh density of 12870 elements/m2. The change in stiffness due to damage was modelled by incorporating a discrete crack.

Reinforced concrete was considered homogeneous, continuous and isotropic. The adopted properties correspond

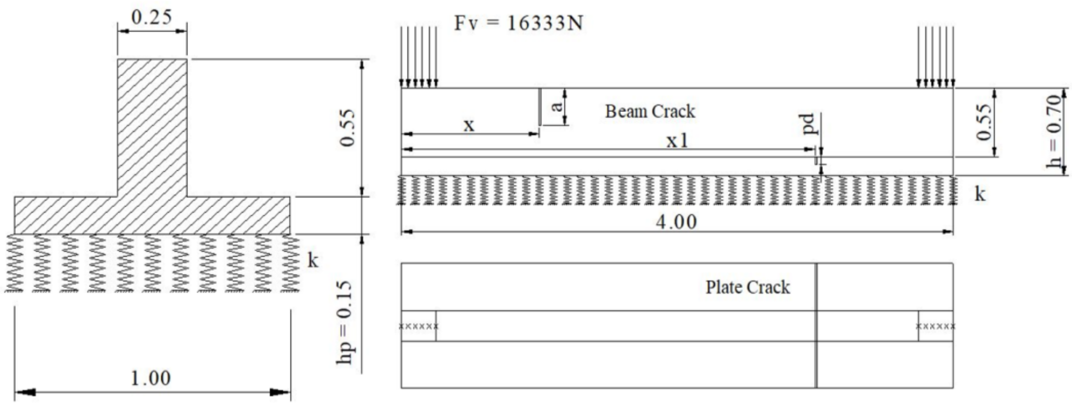

Fig. 1 Model of a foundation beam with cracks on the beam and on the plate (length in meters)

to a high-strength concrete, with a characteristic compression strength of 40 MPa, E = 3.10 10^{10} N/m^2; ν = 0.15 and ρ = 2351 Kg/m^3. The value of the total load applied in each column, located at the ends of the beam is *Fv* = 16333 N. The value for Ground Coefficient Reaction (*k*) is 9.8 10^7 N/m^3, corresponding to a dense uniform sand (Maheshwari 2011).

2.2 Assessment of the presence of damage by static methods

To assess the presence of damage, a comparison was made between the values of the vertical displacements, corresponding to the structure with and without damage, using this variable as an index of the existence of damage.

It is worth mentioning that no longitudinal truss was included in the crack, in order to decrease the software run time, which would have been increased five-fold. Nevertheless, with the purpose of verifying the magnitude of the error implied by this simplification, a model was built with a truss element included in the crack. The maximum difference detected in vertical displacement between models with and without truss was 0.02 %.

2.2.1 Model with damage on the beam

Fig. 2 shows the vertical displacements of a foundation beam, with damage at a fixed position, x = 1.5 m, where x is the coordinate on the longitudinal axis measured from the centre of the left support, as a function of the relative location of each point, for different relations a/h, where a is the depth of the damage and h is the total height of the beam. The measurement of the vertical displacements was made on the beam. In addition, the curve corresponding to the vertical displacements of the undamaged structure has been included in this graph. It should be mentioned that as the crack location is closer to the centre of the beam, the change of curvature in the graphics, that represent the vertical displacements versus the position, is more prominent. If the determination of the vertical displacements of a beam that has a crack is done on the plate, the corresponding graph is similar to that of Fig. 2, with smoother changes of slopes of the graphs, that is, the method loses sensitivity.

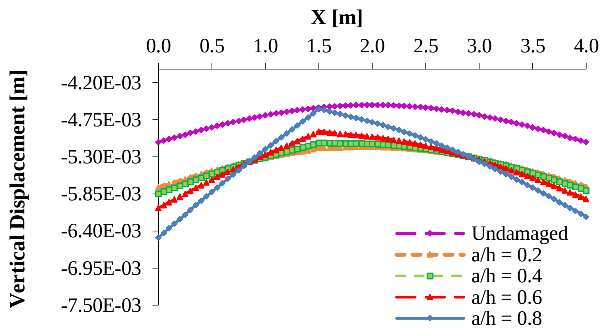

Fig. 2 Vertical displacements with damage of varying severity, measured on the beam

2.2.2 Model with damage on the plate

When the crack is in the plate, the generated rigidity change is lower than if the crack is in the beam. Therefore, the changes in the curvatures are not very noticeable, making the detection of damage rather difficult in these cases.

2.2.3 Model with damage on the beam and on the plate

When cracks appear simultaneously on the beam and on the plate, this methodology does not allow the detection of cracks on the plate, regardless of whether the measurement is made on the beam or on the plate. It should be noted that when the measurement is made on the plate the technique loses sensitivity compared to the measurements made on the beam.

2.3 Damage location

Location of damage is performed after assessing its presence in the structure. Two methods are hereby presented:

- Vertical displacement variation method, and
- Elastic curvature method.

Notice that both methods are fit for the analysis of static as well as dynamic behaviour.

2.3.1 Vertical displacement variation method

This method is based in the determination of the

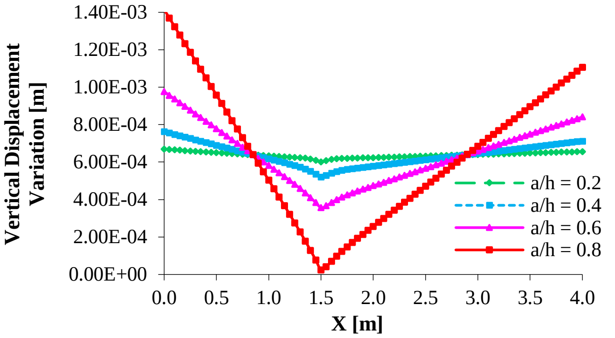

Fig. 3 Vertical displacement variation with damage on the beam, measured on the beam

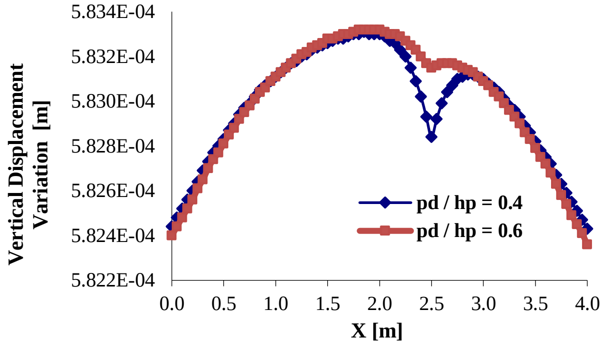

Fig. 4 Vertical displacement variation with damage on the plate, measured on the plate

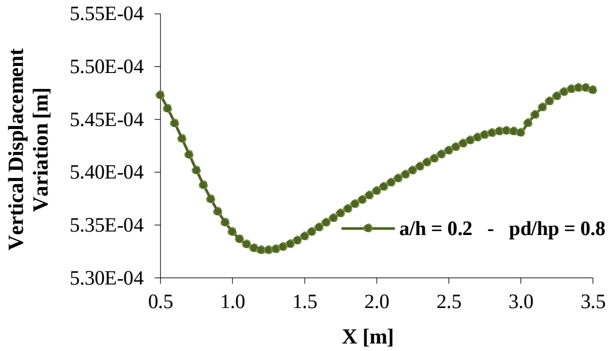

Fig. 5 Vertical displacements variation with damage on the beam and on the plate, measured on the plate

difference between the vertical displacement of the intact structure and that of the damaged structure. This parameter, called vertical displacement variation (VDV_i) is an index of variations in the structural rigidity, and is defined as:

$$VDV = |w_{id} - w_{iu}| \tag{1}$$

where w_{id} is the vertical displacement of point i in the damaged structure and w_{iu} is the displacement of the same point in the undamaged structure. If these data are available for several points of the structure, the generated plotting allows for an estimate of the location of the damage. If data are not available on vertical displacements of the undamaged structure, which is a frequent situation, an estimate may be made analytically or by means of numerical models.

2.3.1.1 Model with damage on the beam

Fig. 3 shows the variation of the vertical displacements, for each point of the mesh, for a foundation beam with damage on the beam, at x = 1.5 m. Four different crack depths are displayed, a/h = 0.2, a/h = 0.4, a/h = 0.6, and a/h = 0.8. It can be observed that the slope change of the vertical displacement occurs in coincidence with the damage and then, the sign of the curvature also varies.

It is interesting to note that, as the depth of the damage increases, the change in the slope of these curves, coinciding with the location of the crack, is more noticeable. When vertical displacements are measured on the plate, the variations in the slopes are smaller.

2.3.1.2 Model with damage on the plate

Fig. 4 displays the variation of vertical displacements along the foundation beam, with damage on the plate at x = 2.5 m. Two crack depths are plotted, pd/hp = 0.4 and pd/hp = 0.6. It can be appreciated that as before, the slope changes are coincident with the damage location. Notice that vertical displacements were measured on the plate. It should be noted that as the damage is closer to the columns, the slope change is lower and the method loses sensitivity. Also, if displacements measurements are made on the beam, the method is not accurate.

2.3.1.3 Model with damage on the beam and on the plate

Fig. 5 shows the variation of the vertical displacement of a foundation beam, with damage in a fixed position in the beam and in the plate, at x = 1 m and x = 3 m, respectively, as a function of the longitudinal axis of the foundation, for a/h = 0.2 and pd/hp = 0.8. The measurements were made on the plate. It can be noted that the curvature varies smoothly when the damage is on the beam, but it shows a sharp indentation when the damage is located on the plate.

When measurements are made on the beam, only cracks on the beam can be located, whereas measuring on the plate allows for locating cracks on the beam and on the plate.

2.3.2 Elastic curvature method

The values of vertical displacements obtained for a certain point of the structure, either by experimental measurements or, as in this case, by developing a numerical simulation, can be used to obtain the curvature of the deformed structure through an approximation with central finite differences (Lu *et al*. 2002). In this way, the curvature is given by:

$$\frac{d^2w}{dx^2} = \frac{w_{i+1} - 2w + w_{i-1}}{S^2} \qquad i = 2, \ldots, n-1 \tag{2}$$

where s is the distance between two adjacent points (step), w is the displacement at a given point and n is the number of grid points.

Of note, this method is independent of any information about the undamaged structure, an important advantage when such data is unavailable.

For an undamaged structure, the plotting of the curve renders a smooth shape. The emergence of a peak or a disruption of shape indicates an abnormal variation in the

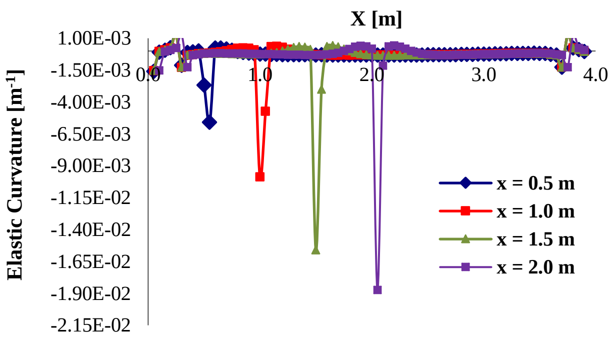

Fig. 6 Elastic curvature with damage on the beam, measured on the beam

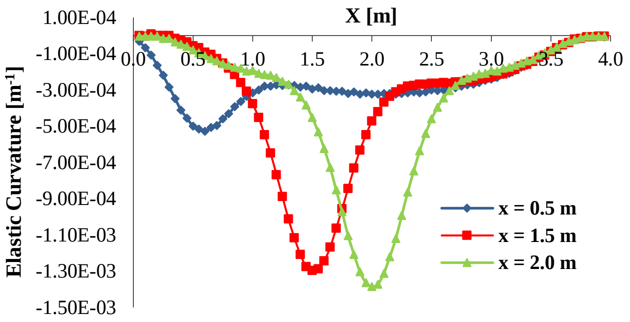

Fig. 7 Elastic curvature with damage on the beam, measured on the plate

rigidity or flexibility at that precise position, induced by the damage. Thus, one or more damaged areas can be located using such measurements.

Fig. 6 shows the elastic curvature as calculated according to Equ. (2), as a function of the longitudinal axis, for a foundation beam with damage at x = 0.5 m, x = 1 m, x = 1.5 m, x = 2.0 m and a crack depth of a/h = 0.6.

Fig.7 shows the variation in the elastic curvature calculated by Equ. (2), with the vertical displacements measured on the plate, as a function of the longitudinal axis, for a foundation beam damaged at x = 0.5 m, x = 1.5 m, x = 2.0 m, with a crack depth of a/h = 0.6.

From the observation of Figs. 6 and 7, it can be said that if the displacements are measured on the beam, the shifts in the curvature is more pronounced.

2.4. Damage quantification

Once the damage has been located through the techniques previously described here, the following step is to establish its magnitude. Although the damaged area could be visually detected, there is uncertainty as regards its magnitude in the interior of the structure. In order to address with this, it is necessary to make a numerical model of the structure with the damage simulated in the location determined visually, or located using the methodologies previously presented, or by the use of an experimental technique, for example, ultrasound techniques.

In the analysis performed by this method, the selected mechanical parameter is the volumetric displacements (*VD*). This parameter is defined as the sum of the products

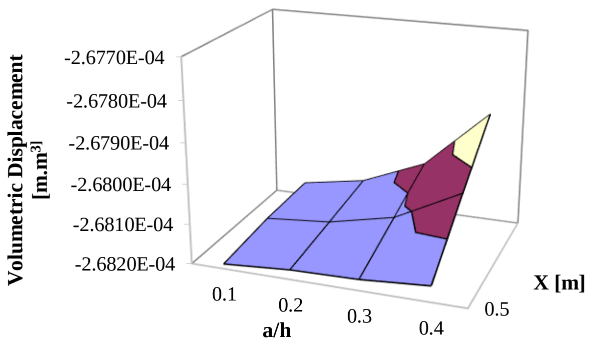

Fig. 8 Volumetric displacements of a foundation beam as a function of location (x) and damage severity (a/h)

of the vertical displacements at each point of the mesh d_i, times their influence volume, V_i:

$$VD = \sum_{i=1}^{n} |\delta_i V_i| \tag{3}$$

where the Volume of Influence (*Vi*) is the volume whose gravity centre is the point at which displacement is measured. The length of the sides of this volume is half the distance between the two adjacent nodes, in each of the three directions.

This parameter has the advantage of accounting for a greater volume of information, so if an error exists in any of the vertical displacement measurements, it would have less influence on the volumetric displacement.

In order to quantify the damage, models with different locations and damage severities are numerically analysed, resulting in a three dimensional graph of the parameters: location (x), severity (a/h) and a third, volumetric displacement, linked to the mechanical behaviour of the damaged structure (Ortega *et al.* 1998; Robles *et al.* 2001). Then, using Fig. 8, with the location of damage (x), and with the calculated volumetric displacement, the value of (a/h), corresponding to the severity of damage is obtained.

2.5. Experimental verification

In order to validate the methodology of detection and location of damage presented, reinforced concrete beams, simply supported of the following dimensions: 0.08 m wide, 0.16 m high and 2.20 m total length, were tested in the Laboratory of Structural Models of the Department of Engineering, of the Universidad Nacional del Sur, Argentina.

The reinforced concrete beams were built with 2 steel rods of 4.2 mm in diameter at the top and 2 with a diameter of 10 mm at the bottom and stirrups of 6 mm each 12 cm. A load was applied in the centre of the beam, by means of a hydraulic system capable of incrementing the load in stages of 1280 kg each, up to a total of 7070 kg.

Vertical displacement was measured at every load stage, by means of 10 fleximeters, spaced every 15 cm along the beam. Measurements were made starting from the undamaged beam up to a fully cracked beam, including the different stages of damage, as displayed in Fig. 9 a and b. It

(a) General view

(b) Cracked zone

Fig. 9 Views of a beam test

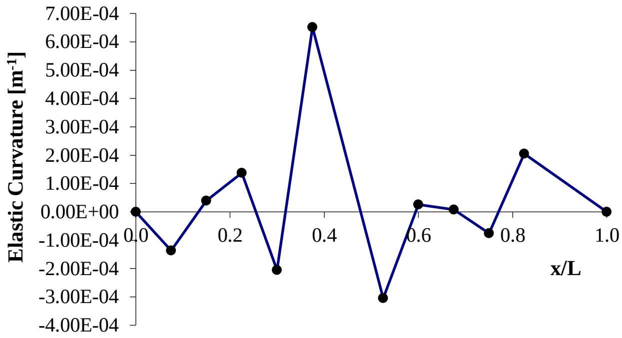

Fig. 10 Elastic curve of a beam with a 7070 kg load at its centre

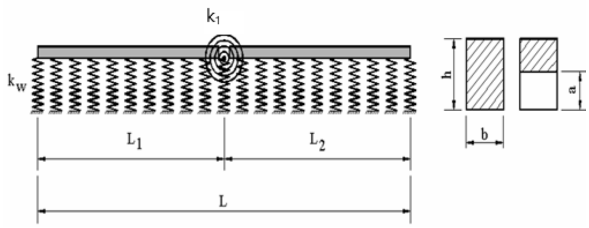

Fig. 11 Winkler foundation beam with an intermediate rotational spring, representing a section with a crack depth a

should be noted that the width and length of the cracks was measured at every load stage, using a scale graded with a precision of 0.5 mm.

After measuring vertical displacements, the elastic curvature was calculated and plotted in Fig. 10. Comparing Fig. 10, with that existing in another work (Robles *et al.* 2008), it can be seen that more peaks appear in the experimental graph than in the resulting graphs of the numerical model. This is due to the errors that are presented in the experimental determinations. On the other hand, the number of points measured is lower than the determinations made in the numerical model. However, the location of the crack is easily detectable, which indicates that this methodology is reliable and of practical application.

It is to be noted that, in the first loading steps, the location of the crack cannot be clearly identified. Only after approximately half of the maximum load, the location of the crack can be identified. The trend of the graphs is similar to that shown in Fig. 10.

The measurement of displacements in the working environment can be done by means of mechanical fleximeters, with a precision of a hundredth of a millimetre, or with electronic instrumentation such as LVDT, with a precision equal or exceeding that of the fleximeters. It is interesting to note that the accuracy of fleximeters is enough since, with the tenth of a millimetre, the variations of the displacements caused by the failures can already be detected.

3. Detection, localization and quantification of damage through dynamic methods

For different reasons, for example, space to apply loads or inaccessibility of the studied part, cost and test time, dynamic methods are the most appropriate. For this reason, this section presents the detection, location and quantification of damage using the dynamic method, in particular, a solution to the inverse problem using a power series algorithm.

3.1 Elastic foundation beam: Statement of the vibration problem

The model shown in Fig. 11 was adopted, representing a beam resting on an elastic ground, with a Winkler type response, with a rigidity modulus k_w kg/m^3. To simulate the mechanical effect of the crack in the beam, a rotational spring is used, which represents the change in the flexibility in the damaged section. The spring is characterized, with an elastic constant k_1, and the material of the structural element, by means of its density (ρ) and elasticity modulus (E).

To generalize the problem, it can be assumed that the beam has two zones with different sections (F_1 and F_2), inertias (J_1 and J_2) and lengths (L_1 and L_2). In the case that is analysed, the sections and inertias remain constant.

The equations that allow analyzing this problem of natural vibrations of the beam-spring system used here are:

$$\begin{aligned} EIv_1'''' + \bar{w}v_1 + \rho A\ddot{v}_1 = 0 \\ EIv_2'''' + \bar{w}v_2 + \rho A\ddot{v}_2 = 0 \end{aligned} \tag{4}$$

where the point indicates the derivative with respect to the temporal variable.

The non-dimensional equations that govern the natural vibrations of the beam spring system are:

$$v_1^{''''}-\alpha_1^4\lambda^2 v_1=0; \quad v_2^{''''}-(1-\alpha_1)^4\lambda^2 v_2=0 \tag{5}$$

where

$a_1=L_1/L$; x is the spatial variable adimensionalized with L, x_1 with L_1; x_2 with L_2,

$$v_1=v(x_1); \quad v_2=v_2(x_2); \quad \lambda_2 = \Omega^2 - \omega;$$

$$\Omega^2 = \frac{\rho A \omega^2 L^y}{EI}; \qquad \omega = \frac{\bar{\omega} L^y}{EI}$$

The boundary conditions and continuity conditions at $x = L_1$; (α =1 and x_2 =0) are:

$$\begin{gathered} \left[v_1''(0)\delta v_1'(0)\right] = 0 \ \left[v_1'''(0)\delta v_1(0)\right] = 0 \\ \left[v_2''(0)\delta v_2'(0)\right] = 0 \ \left[v_2'''(0)\delta v_2(0)\right] = 0 \end{gathered} \tag{6}$$

$$\begin{aligned} v_2(0) &= v_1(1) \\ v_2'(0) &= \frac{(1-\alpha_1)}{k\alpha_1^2}\left[v_1''(1) + k\alpha_1 v_1''(1)\right] \\ v_2''(0) &= \frac{(1-\alpha_1)^2}{\alpha_1^2} v_1''(1) \\ v_2'''(0) &= \frac{(1-\alpha_1)^3}{\alpha_1^2} v_1'''(1) \end{aligned} \tag{7}$$

3.2 Solution of the direct problem using a power series algorithm

Power Series are a widely known and used tool, in structural mechanics. Filipich and Rosales have used this tool in the solution of highly non-linear problems (Filipich and Rosales 2002, Filipich *et al.* 2004, Rosales and Filipich 2003, Rosales *et al*. 2003).

In the problem that is being treated in this research, the equations are linear and the advantage of this approach is its algebraic simplicity, with a fast convergence. Therefore, in the solution of these problems, it takes less computational time, an advantage that is relevant in the resolution of the inverse problem.

The unknowns of the problem are the functions $v_1(x_1)$ and $v_2(x_2)$ which represent the mode shape in each zone of the beam. These functions are expanded in Power Series, as follows:

$$v_1(x) = \sum_{i=0}^{M} A_{1i} x_1^i \qquad v_2(x) = \sum_{i=0}^{M} A_{2i} x_2^i \tag{8}$$

Once the Equ. (8) is replaced in the governing differential system Eqs. (5), the following recurrences equations are obtained:

$$A_{1(i+4)} = \frac{\alpha^4\lambda^2 A_{1i}}{\varphi_{4i}} \qquad A_{2(i+4)} = \frac{(1-\alpha)^4\lambda^2 A_{2i}}{\varphi_{4i}} \tag{9}$$

where $\varphi_{lk} = (k+l)!\,/\,k!$, being k, l integers. The conditions of continuity lead to

$$\begin{aligned} A_{20} &= \sum_{i=0} A_{1i} \\ A_{21} &= \frac{(1-\alpha)}{k\alpha^2}\left[\sum_{i=0} \varphi_{2i} A_{(i+2)+k\alpha \sum \varphi_{li} A_{1(i+1)}}\right] \\ A_{22} &= \frac{(1-\alpha)^2}{k\alpha^2}\left[\sum_{i=0} \varphi_{2i} A_{1(i+2)}\right] \\ A_{23} &= \frac{(1-\alpha)^3}{6\alpha^3}\left[\sum_{i=0} \varphi_{3i} A_{1(i+3)}\right] \end{aligned} \tag{10}$$

In Eqs. (9) and (10), the unknowns are A_{ij} with i =1, 2 and j = 0, 1, 2, 3 and the eigenvalues λ, and they are obtained by stating the external boundary corresponding to the particular case under study. In summary, having the position of the spring α and its stiffness constant $k_1 = k_r L_1/EI_1$ as input data, the eigenvalues λ of the problem can be obtained. This is the solution to the direct problem. Now, with (λ and α) or (λ and k) known, it is possible to obtain the parameters k or α, respectively. In this way, the inverse problem is solved.

3.3 Inverse problem

As stated above, the natural frequencies methods for crack detection have been widely used in the last decades. Crack detection can be considered as an inverse problem. The procedure is as follows:

a) The measurement of the first three natural frequencies of the cracked, or presumably cracked, structural element is made.

b) Each one of these frequencies is introduced as data in the algorithm.

c) A k_1 *vs.* α_1 curve is obtained and subsequently plotted, for every frequency.

d) The crossing point of all three curves gives the sought values. Due to the inevitable numerical errors and dispersion of the data, the intersection of the curves does not determine a point but a triangular area. The barycenter of this triangle is representative the position of the spring (equivalent to the crack) α_1 and the stiffness k_1 (directly related with the crack depth a).

e) Once the barycenter is found, and consequently the values of α_1 and k_1, the size (depth) of the crack can be derived from a relation between the crack and the stiffness of the equivalent rotational spring (Ostachowicz and Krawczuk, 1991). It should be noted that the relationship was derived for the case of a clamped beam and here, it will be used approximately for the beam under study.

$$k_1 = \frac{Ebh^2}{72\pi f(r^*)} \tag{11}$$

being: $r^*=a/h$ and $f(r^*) = 0.6384r^{*2}-1.035\, r^{*3}+3.7201\, r^{*4}-5.1773\, r^{*5}+7.5531\, r^{*6}-7.3324\, r^{*7}+2.4909\, r^{*8}$

3.4 Damage detection using natural frequencies obtained with a numerical model

In order to verify the methodology, natural frequencies obtained using a numerical method were used. In this section, an undamaged foundation beam was modeled, with the finite elements method, by means of ALGOR software (Algor 23 2010), resting on an elastic medium, measuring 0.20 m in width; 0.30 m in height and 4.00 m in length. The adopted material was reinforced concrete with E = 3.10×10^{10} [N/m^2]; $\nu = 0.15$ and $\rho = 2351$ [Kg/m^3].

In order to take into account the discrepancy between the 2D model, analyzed by the finite element method, and the Bernoulli beam model of the detection algorithm, the adjustment procedure was used (Nandwana and Maiti 1997). The theoretical frequencies were calculated, for the undamaged foundation beam resting on an elastic medium (Blevins 1979), and a dimensionless factor was obtained (ratio of the theoretical frequencies and the frequencies calculated by means of the finite elements method) as shown below:

$$Z_i = \frac{\lambda_i^T}{\lambda_i^M} \qquad (12)$$

In this case, the theoretical dimensionless frequencies for a foundation beam resting on an elastic medium are: $\lambda_1^T=23.16250$, $\lambda_2^T=61.96350$, $\lambda_3^T=121.05192$. On the other hand, the dimensionless frequencies of the undamaged beam, as calculated with the method of the finite elements, are: $\lambda_1^M = 22.73380$, $\lambda_2^M = 58.34810$, $\lambda_3^M = 109.08310$.

Hence, the values of the factor for the "zero setting" are in this case: $Z_1 = 1.01885$, $Z_2 = 1.06196$, $Z_3 = 1.10972$ for each mode, respectively.

Then, the same model with a discrete crack was built in finite elements, measuring 1 mm in width, with a sequentially altered length, for a crack positioned at a distance x of 1m from the supporting point, as a function of the crack depth. In each case, the first three natural frequencies ($f^{\,M}$) were calculated. The dimensionless frequencies were calculated for each model (λ_1^M, λ_2^M, λ_3^M) multiplying the natural frequencies resulting from the finite elements method, times the value $2\pi L^2\sqrt{\frac{\rho F}{EJ}} = 0.316348$, such as $\lambda_i^M = 2\pi L^2\sqrt{\frac{\rho F}{EJ}}\, f = 0.316348 f$

The corrected dimensionless frequencies were introduced to a program built in a Matlab environment, taking into account the solution algorithm for the Inverse Problem, thus obtaining a curve k_I (spring stiffness) which is directly related with the crack depth as a function of α_I (spring position, L_I/L).

3.4.1 Application cases

To validate the methodology four models of the beam study in section 3.4, with a crack positioned at a distance x = 1.0 m from the supporting point, and crack's depth of 0.035 m (a/h=0.05), 0.07 m (a/h=0.1), 0.105 m (a/h=0.15) and 0.14 m (a/h=0.20) were analyzed. Fig. 12 displays the curve k_I as a function of α_1 corresponding to the three frequencies input into the algorithm. In reference to the intersection of these curves, it should be noted that the three curves do not exactly intersect at a point, defining a small

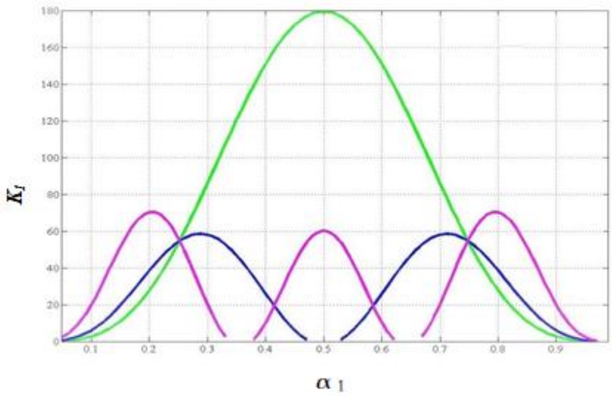

Fig. 12 Curves of k_I as a function of α_I for Model I

Table 1 Location, crack depth and errors of analyzed beams

Location*	$\hat{a}$**	%Error ε_l	Crack depth [m]	%Error ε_a
1.008	0.252	0.2	0.046	1.33
1.083	0.270	2.0	0.098	0.66
0.987	0.247	0.3	0.149	0.33
0.960	0.240	1.1	0.223	7.66

* Location: Distance from the right support in [m]
** Location of the crack corresponding to the triangle barycenter

area. The central point of this area gives the crack location. It is interesting to note that these curves are similar for the four models, changing only the coordinates of the k_I axis. For this reason, this figure is considered representative of all four models.

From Fig. 12, it is determined the location of the crack at $\alpha_I = 0.252$, indicating the crack is at a distance of 1.008 m from the right support, with an error:

$$\varepsilon_l = \left|\frac{\widehat{L_1} - L_1}{L}\right| = \left|\frac{1.008 - 1.00}{4}\right|.100 = 0.2\% \qquad (13)$$

The hat on a variable indicates the detected value.

After locating the crack and with the values of α and k_I, the crack size (depth a_I) is estimated by a relationship between it and the equivalent rotational spring stiffness as shown in Eq. (11). Solving this equation, a crack depth value of 0.046 m is obtained, with an error:

$$\varepsilon_a = \left|\frac{\hat{a} - a}{h}\right| = \left|\frac{0.046 - 0.050}{0.3}\right|.100 = 1.3\% \qquad (14)$$

The values of the four analyzed cases are displayed in Table 1.

3.5 Experimental detection by means of resolution of the inverse problem

In order to validate the dynamic methodology of detection and location of damage presented above, a reinforced concrete beam, simply supported, was tested in the Structural Model Laboratory of the UNS.

To simulate the crack by means of a rotational spring and calculate its theoretical natural frequencies, the structural scheme indicated in Fig. 13 was adopted.

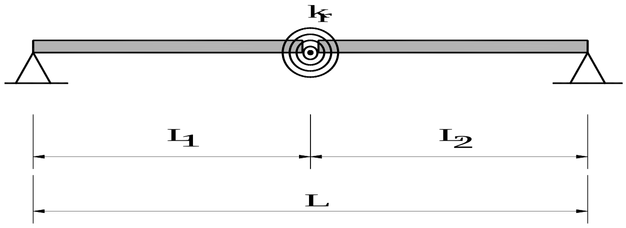

Fig. 13 Beam with a rotational spring of constant k_l in the midpoint of the spam

Fig. 14 Instrumentation used to measure natural frequencies

For calculating the theoretical frequencies, the homogenized section of the reinforced-concrete beam was found. This section measures 16.3 cm wide by 8 cm high and 199,5 cm length (L); the upper steel section is 0.6411 cm^2 (1 Ø 8 + 1 Ø 4.2), the same as the lower section. The parameters of the equivalent section are: b_{eqh}=1.9404 cm, A_{eq}=17.1941 cm^2, I_{eq}= 99.2121 cm^4.

$$\Omega^T = \omega^T L^2 \sqrt{\frac{\rho A_{eq}}{E I_{eq}}} = 2\pi f^T L^2 \sqrt{\frac{\rho A_{eq}}{E I_{eq}}} \qquad (15)$$

Replacing the data from the homogenized beam, the frequencies are calculated, with the following results:

$$f_1^T = \pi^2 \Omega_1^T = 42.206 \text{ Hz}, \; f_2^T = 4\pi^2 \Omega_2^T = 168.824 \text{ Hz},$$
$$f_3^T = 9\pi^2 \Omega_3^T = 379.856 \text{ Hz}$$

For the measurement of the frequencies, the interfase Vernier LabQuest TM was employed. The accelerometer “3-Axis”, 5g for an accelerations range of ±49 m/seg2 in three orthogonal axes, with a frequency response between 0 and 100 Hz, was placed at a distance 0.25 L, measured from the support. The beam was excited dynamically with an impact hammer (Modal Hammer Meggitt, Model 2302) at a distance of 0.25 L, measured from the other support. The accelerometer is connected to a data acquisition system (Labquest Interface, 2008). The LOGGER PRO program (LOGGER PRO, 2008) was used to process the natural frequencies of vibration. Fig. 14 shows the instruments used for the test.

With the gathered data a statistical analysis was performed, and the first three frequencies of the undamaged beam were found, with the following results:

$$f_{1s} = 19.530 \text{ Hz}, \; f_{2s} = 121.090 \text{ Hz}, \; f_{3s} = 359.380 \text{ Hz}$$

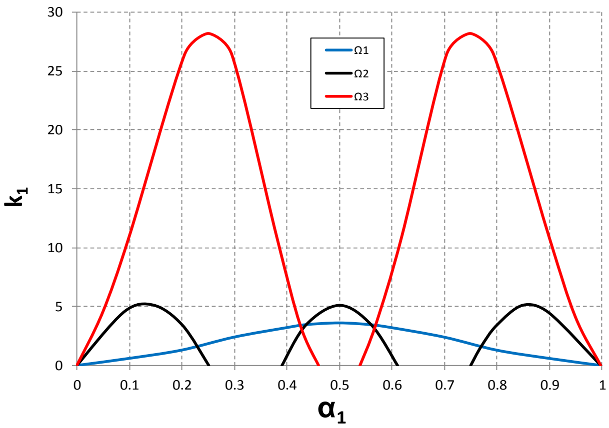

Fig. 15 Curves k_l as a function of α_l, for the experimentally tested beam

Once the theoretical frequencies were calculated and the experimental frequencies, measured, the “zero setting” factors were calculated, which values in this case are: $Z_1 = 2.16$, $Z_2 = 1.39$ and $Z_3 = 1.06$.

In order to crack the beam, a concentrated load was gradually applied onto its centre, and the natural frequencies were measured with the procedure described above. The first three frequencies for the simply supported cracked beam were calculated, with the following results:

$$f_{1fc} = 15.60 \text{ Hz}, f_{2fc} = 117.10 \text{ Hz}, \; f_{3fc} = 316.40 \text{ Hz}$$

Once the frequencies were experimentally found, the previously calculated “zero setting” factors were applied, in order to obtain the corrected frequencies for the cracked beam. The values in this case are:

$$f_{1fc} = 33.78 \, Hz, \; f_{2fc} = 163.39 \, Hz, \; f_{3fc} = 334.44 \, Hz.$$

The corrected dimensionless frequencies were then obtained, with values of:

$$\Omega_1 = 7.8987, \Omega_2 = 38.2068, \; \Omega_3 = 78.2055.$$

These frequencies were fed to a program in a Matlab environment, using the algorithm for solving the inverse problem in Matlab environment. A curve displaying the variation of k_l - spring stiffness was obtained, directly related to the crack depth as a function of α1 (spring location, L_l/L).

Fig. 15 depicts the curves for each frequency. It should be remembered that the three curves do not exactly intersect at a point, defining a small area. The central point of this area gives the crack location. For example, for the location of the crack at α_l = 0.561, reveals the crack is locate at 1.00± 0.12 m from the supports, with an error:

$$\varepsilon_l = \left| \frac{\widehat{L_1} - L_1}{L} \right| = \left| \frac{1.12 - 1.00}{1.995} \right| .100 = 0.06\% \qquad (16)$$

Once the midpoint was located, and together with it, the values for α_l y k_l, the depth of the crack was obtained by means of Equ. (11), at 2.53 cm.

It should be noted that, due to the symmetry of the problem, two points are detected (0.439 and 0.561) with the same error. However, the present approach does not allow distinguishing which one is the appropriate. Thus, an additional means should be employed to select the proper detected point.

4. Conclusions

With the aim of quantifying structural pathologies in reinforced concrete beam supported on an elastic Winkler type foundation, in this work two techniques were developed that used static parameters. We conclude that when vertical displacements measurements are made on the beam, only cracks on the beam can be located, whereas measuring on the plate allows for locating cracks on the beam and on the plate. It should be noted that, of the two studied static methods, the one based on the curvature of the elastic is more efficient in detecting damage than the approach using the vertical displacements.

An application that used dynamic parameters for the detection, location and quantification of damage was presented, by means of the solution of the inverse problem. The results show a high level of accuracy, taking into account that reinforced-concrete elements do not exhibit as a rule, precisely located cracks, having a cracked area instead.

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