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Propagation of plane waves in an orthotropic magneto-thermodiffusive rotating half-space

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Abstract. The present article is aimed at studying the reflection phenomena of plane waves in a homogeneous, orthotropic, initially stressed magneto-thermoelastic rotating medium with diffusion. The enuciation is applied to generalized thermoelasticity based on Lord-Shulman theory. There exist four coupled waves, namely, quasi-longitudinal P-wave (*qP*), quasi-longitudinal thermal wave (*qT*), quasi-longitudinal mass diffusive wave (*qMD*) and quasi-transverse wave (*qSV*) in the medium. The amplitude and energy ratios for these reflected waves are derived and the numerical computations have been carried out with the help of MATLAB programming. The effects of rotation, initial stress, magnetic and diffusion parameters on the amplitude ratios are depicted graphically. The expressions of energy ratios have also been obtained in explicit form and are shown graphically as functions of angle of incidence. It has been verified that during reflection phenomena, the sum of energy ratios is equal to unity at each angle of incidence. Effect of anisotropy is also depicted on velocities of various reflected waves.

Keywords: reflection; generalized thermoelasticity; rotation; magnetic; initial stress; diffusion

1. Introduction

The parabolic type of heat conduction equation, as was used initially in the study of thermoelastic behavior was found to yield some unrealistic situation in the sense that the velocity of heat signals was infinite. At present, various generalized thermoelasticity theories are being proposed to get rid of this physically inadmissible situation. Lord and Shulman (1967) postulated a generalized theory by incorporating thermal relaxation time with heat flux term in Fourier's law of heat conduction. Green and Lindsay (1972) also developed a beautiful theory of generalized thermoelasticity with two relaxation times. This theory does not violate classical Fourier's law in case of centrally symmetric bodies. But it includes temperature-rate in the constitutive relation and thus modifies all the equations of the coupled theory, not the heat conduction equation only. That's why, this theory is generally referred as temperature-rate-dependent thermoelasticity.

The problems of rotating bodies are more important than the corresponding problems of non-rotating bodies as most of the large bodies such as the earth, the moon and other planets have an angular velocity. Inspired by this idea, some researchers have investigated different problems of rotating media. Schoenberg and Censor (1973) concerned the effect of rotation on elastic waves. Sharma *et al*. (2008) analyzed the effect of rotation on different type of waves propagating in a thermoelastic medium. By employing the linear theory of thermoelasticity, Bayones and Abd-Alla (2018) studied the effect of time and rotation parameter on the stresses, displacement and temperature field in a thermoelastic half-space with heat source. Kalkal *et al*. (2018) employed normal mode analysis to analyze the effects of rotation and phase-lag parameters on the considered field variables in a micropolar generalized thermo-viscoelastic medium.

Recent years have seen an ever growing interest in investigation of the problems related to initially stressed elastic medium, due to its numerous applications in various fields, such as earthquake engineering, seismology and geophysics. The earth is assumed to be under high initial stresses. It is therefore of great interest to study the influence of these stresses on the propagation of stress waves. The elastodynamics of a body under initial stress is exposed in the treatise of Biot (1965). Montanaro (1999) developed the linear theory of thermoelasticity with initial stress for an isotropic medium. Yadav *et al*. (2017) studied the problem of reflection of plane waves from an initially stressed surface of the generalized electromicrostretch thermoelastic solid rotating with a uniform angular velocity in the context of Lord-Shulman (L-S) and Green-Lindsay (G-L) theories.

The study of dynamical problems of magneto-thermoelasticity has received much attention in literature during the past few decades. Knopoff (1955) and Chadwick (1957) introduced the theory of magneto-thermoelasticity. Paria (1962) discussed the development of magneto-thermoelasticity and also studied the propagation of plane magneto-thermoelastic waves in an isotropic unbounded medium. Nayfeh and Nemat-Nasser (1972) studied the

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propagation of plane waves in a solid under the influence of an electromagnetic field. Said and Othman (2016) investigated the wave propagation in a fiber-reinforced magneto-thermoelastic medium under two-temperature three-phase-lag Green-Naghdi theory without energy dissipation. Deswal *et al*. (2017) studied the magneto-thermo-viscoelastic interactions in a homogeneous, isotropic medium under generalized thermoelasticity theory without energy dissipation with fractional order strain. Biswas and Abo-Dahab (2018) studied the effects of initial stress and magnetic field on Rayleigh waves in a homogeneous magneto-thermoelastic orthotropic medium in the context of three-phase-lag model. Jain *et al*. (2018) discussed the propagation of plane waves in a fiber reinforced thermoelastic medium in the presence of moving internal heat source and gravity under the fractional order two-temperature theory.

Thermodiffusion, in an elastic solid, results from coupling of the fields of temperature, mass diffusion and strain. Nowacki (1974a,b,c) put forward the theory of thermoelastic diffusion by using a coupled thermoelastic model. Sherief *et al*. (2004) extended the theory of thermoelastic diffusion and derived the governing equations for the generalized thermoelastic diffusion problem in an elastic solid, which allows the finite speeds of propagation for thermoelastic and diffusive waves. Sharma *et al*. (2008) studied the dynamical behaviour in generalized thermoelastic diffusion medium under Green-Lindsay theory using Fourier transform method. Deswal and her co-workers (2009, 2011) examined some problems employing the theory of generalized thermoelastic diffusion under different kinds of loads. The fundamental solution for two-dimensional problem in an orthotropic magneto-thermoelastic diffusive medium was investigated by Kumar and Chawla (2013). By using Laplace and Hankel transforms, an axi-symmetric generalized thermoelastic diffusion problem with two-temperature and initial stress under fractional order heat conduction was discussed by Deswal *et al*. (2016). Under Green-Naghdi (G-N) theory of type II and III, Othman *et al*. (2017) studied the effect of gravitational field and temperature-dependent properties in an isotropic micropolar thermoelastic diffusive medium.

Keeping in view the above stated facts and applications of reflection phenomenon in a homogeneous, orthotropic, initially stressed, magneto-thermoelastic rotating medium with diffusion, the present paper is devoted to discuss the reflection phenomenon of plane waves at the boundary surface. The formulae for amplitude ratios and energy ratios corresponding to various reflected waves have been presented, when a set of coupled waves strikes obliquely at the boundary surface of the assumed model and their variations with angle of incidence are presented graphically. Effects of rotation, initial stress, magnetic and diffusion parameters on the reflection coefficients of thermoelastic waves are observed. It has been verified that at each angle of incidence, there is no dissipation of energy at the boundary surface during reflection. Moreover, the effect of anisotropy is also depicted on velocities of various reflected waves.

2. Basic governing equations

Following Sherief *et al*. (2004), the constitutive relations and field equations for a homogeneous, anisotropic, initially stressed, thermally and perfectly conducting elastic medium with rotation and diffusion in the context of L-S model, are given as

(i) the constitutive equations

$$\sigma_{ij} = C_{ijkl}e_{kl} - \beta_{ij}\theta - \gamma_{ij}c - P\left(\delta_{ij} + \omega_{ij}\right) \tag{1}$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \quad \omega_{ij} = \frac{1}{2}\left(u_{j,i} - u_{i,j}\right) \tag{2}$$

(ii) the equation of motion

$$\sigma_{ji,j} + F_i = \rho\left[\ddot{u} + \left(\vec{\Omega}\times\vec{\Omega}\times\vec{u}\right) + \left(2\vec{\Omega}\times\dot{\vec{u}}\right)\right]_i \tag{3}$$

(iii) heat conduction equation

$$K_{ij}\theta_{,ij} = \rho c_E\left(\dot{\theta} + \tau_0\ddot{\theta}\right) + T_0\beta_{ij}\left(\dot{u}_{i,j} + \tau_0\ddot{u}_{i,j}\right) + aT_0\left(\dot{c} + \tau_0\ddot{c}\right) \tag{4}$$

(iv) equation of mass diffusion

$$bD_{ij}c_{,ij} = D_{ij}\gamma_{km}e_{km,ij} + aD_{ij}\theta_{,ij} + \left(\dot{c} + \tau\ddot{c}\right) \tag{5}$$

where $C_{ijkl}(C_{ijkl}=C_{klij}=C_{jikl}=C_{ijlk})$ are the elastic parameters, $K_{ij}=K_i\delta_{ij}$ $(K_{ij}=K_{ji})$ are thermal conductivity, $\beta_{ij}=\beta_i\delta_{ij}(\beta_{ij}=\beta_{ji})$, $\gamma_{ij}=\gamma_i\delta_{ij}(\gamma_{ij}=\gamma_{ji})$ are the thermal and diffusion elastic coupling tensors, σ_{ij}'s are the components of stress, e_{ij}'s are the components of strain, u_i are components of displacement vector, δ_{ij} is the Kronecker delta, $\theta=T-T_0$, where T is absolute temperature, T_0 is temperature of the medium in its natural state assumed to be $\left|\frac{\theta}{T_0}\right| \ll 1$ and $c=C-C_0$, where C is non-equilibrium concentration, C_0 is mass concentration at natural state, a is measure of thermodiffusion effect, b is measure of diffusion effect, $D_{ij}=D_i\delta_{ij}(D_{ij}=D_{ji})$ are thermodiffusion constants, ρ is the mass density, c_E is the specific heat at constant strain, $\tau_0,$ τ are thermal and diffusion relaxation times respectively, P is initial stress, $\vec{\Omega}$ is angular velocity, F_i are the components of body force. In the above equations, a comma denotes material derivative and the summation convention is used.

Following Nayfeh and Nemat-Nasser (1972), the variation of the magnetic and electric fields for a perfectly conducting medium are given by linearized Maxwell's equations as

$$\text{curl}\ \vec{h} = \vec{J} + \varepsilon_0\frac{\partial\vec{E}}{\partial t} \tag{6}$$

$$\text{curl}\ \vec{E} = -\mu_0\frac{\partial\vec{h}}{\partial t} \tag{7}$$

$$\vec{E} = -\mu_0\left(\frac{\partial\vec{u}}{\partial t}\times\vec{H}\right) \tag{8}$$

$$\text{div}\ \vec{h} = 0 \tag{9}$$

The constitutive equations in an orthotropic, initially stressed thermoelastic medium can be written in matrix form as

$$\begin{bmatrix}\sigma_{xx}\\\sigma_{yy}\\\sigma_{zz}\\\sigma_{yz}\\\sigma_{xz}\\\sigma_{xy}\end{bmatrix}=\begin{bmatrix}C_{11}&C_{12}&C_{13}&0&0&0\\C_{12}&C_{22}&C_{23}&0&0&0\\C_{13}&C_{23}&C_{33}&0&0&0\\0&0&0&C_{44}&0&0\\0&0&0&0&C_{55}&0\\0&0&0&0&0&C_{66}\end{bmatrix}\begin{bmatrix}e_{xx}\\e_{yy}\\e_{zz}\\2e_{yz}\\2e_{xz}\\2e_{xy}\end{bmatrix}$$

$$-\begin{bmatrix}\beta_1\\\beta_2\\\beta_3\\0\\0\\0\end{bmatrix}\theta-\begin{bmatrix}\gamma_1\\\gamma_2\\\gamma_3\\0\\0\\0\end{bmatrix}c-\begin{bmatrix}1\\1\\1\\\omega_{yz}\\\omega_{xz}\\\omega_{xy}\end{bmatrix}P$$

where

$$\begin{aligned}\beta_1&=C_{11}\alpha_{1t}+C_{12}\alpha_{2t}+C_{13}\alpha_{3t},\\\beta_2&=C_{12}\alpha_{1t}+C_{22}\alpha_{2t}+C_{23}\alpha_{3t},\\\beta_3&=C_{13}\alpha_{1t}+C_{23}\alpha_{2t}+C_{33}\alpha_{3t},\\\gamma_1&=C_{11}\alpha_{1c}+C_{12}\alpha_{2c}+C_{13}\alpha_{3c},\\\gamma_2&=C_{12}\alpha_{1c}+C_{22}\alpha_{2c}+C_{23}\alpha_{3c},\\\gamma_3&=C_{13}\alpha_{1c}+C_{23}\alpha_{2c}+C_{33}\alpha_{3c},\end{aligned}\tag{10}$$

and $\alpha_{it},\alpha_{ic}\ (i=1,2,3)$ are coefficients of linear thermal and diffusion expansion respectively.

3. Problem formulation

Consider a homogeneous, orthotropic, initially stressed, thermally and perfectly conducting elastic medium with rotation and diffusion in the context of L-S model. We shall use the rectangular Cartesian co-ordinate system (*x,y,z*), having the surface of the half-space as the plane $x = 0$, with x-axis pointing vertically into the medium, so that the half-space occupies the region $x\ge 0$. The orientation of the primary magnetic field $\vec{H}=(0,0,H_0)$ is taken towards the positive direction of z-axis. Due to the application of this magnetic field, there arises in the medium an induced magnetic field $\vec{h}$ and an induced electric field $\vec{E}$. Further $\vec{h}$ and $\vec{E}$ are small in magnitude in accordance with the assumptions of the linear theory of thermoelasticity. We restrict our analysis to xy-plane. Thus all the quantities in the medium are independent of the variable z. So the displacement vector $\vec{u}$ and angular velocity $\vec{\Omega}$ will have the components respectively.

$$\vec{u}=(u,v,0),\qquad \vec{\Omega}=(0,0,\Omega)\tag{11}$$

The components of the initial magnetic field vector $\vec{H}$ are

$$H_x=0,\ H_y=0,\ H_z=H_0\tag{12}$$

The electric intensity vector is normal to both the magnetic intensity and the displacement vector. Also, the electric intensity vector $\vec{E}$ is parallel to the current density vector $\vec{J}$, thus

$$\begin{aligned}&E_x=E_1,\ E_y=E_2,\ E_z=0,\\&J_x=J_1,\ J_y=J_2,\ J_z=0\end{aligned}\tag{13}$$

From (6)-(9), one can obtain

$$E_1=-\mu_0H_0\frac{\partial v}{\partial t},\ E_2=\mu_0H_0\frac{\partial u}{\partial t},\ E_3=0\tag{14}$$

$$h_1=0,\ h_2=0,\ h_3=-H_0e\tag{15}$$

$$\begin{aligned}&J_1=-H_0\frac{\partial e}{\partial y}-\varepsilon_0\mu_0H_0\frac{\partial^2 v}{\partial t^2},\\&J_2=H_0\frac{\partial e}{dx}-\varepsilon_0\mu_0H_0\frac{\partial^2 u}{\partial t^2},\ J_3=0\end{aligned}\tag{16}$$

Lorentz's force $\vec{F}$ is given by the relation

$$\vec{F}=\mu_0\left(\vec{J}\times\vec{H}\right)\tag{17}$$

Inserting (12) and (16) in (17), we can obtain the components of the body force $\vec{F}$ as

$$\begin{aligned}&F_x=\mu_0H_0^2\left(\frac{\partial e}{\partial x}-\varepsilon_0\mu_0\frac{\partial^2 u}{\partial t^2}\right),\\&F_y=\mu_0H_0^2\left(\frac{\partial e}{\partial y}-\varepsilon_0\mu_0\frac{\partial^2 v}{\partial t^2}\right),\ F_z=0\end{aligned}\tag{18}$$

Taking into consideration (1), the requisite stress components are given as

$$\sigma_{xx}=C_{11}\frac{\partial u}{\partial x}+C_{12}\frac{\partial v}{\partial y}-\beta_1\theta-\gamma_1c-P\tag{19}$$

$$\sigma_{yy}=C_{12}\frac{\partial u}{\partial x}+C_{22}\frac{\partial v}{\partial y}-\beta_2\theta-\gamma_2c-P\tag{20}$$

$$\sigma_{xy}=\left(C_{66}+\frac{P}{2}\right)\frac{\partial u}{\partial y}+\left(C_{66}-\frac{P}{2}\right)\frac{\partial v}{\partial x}\tag{21}$$

$$\sigma_{yx}=\left(C_{66}-\frac{P}{2}\right)\frac{\partial u}{\partial y}+\left(C_{66}+\frac{P}{2}\right)\frac{\partial v}{\partial x}\tag{22}$$

By using summation convention and inserting the components of the body force and the stresses defined in (18)-(22) into (3) along with the consideration of two-dimensional problem, the equation of motion takes the form

$$\rho\left(\frac{\partial^2 u}{\partial t^2}-\Omega^2u-2\Omega\frac{\partial v}{\partial t}\right)=C_{11}\frac{\partial^2 u}{\partial x^2}\tag{23}$$

$$+\left(C_{12}+C_{66}+\frac{P}{2}\right)\frac{\partial^2 v}{\partial x\partial y}+\left(C_{66}-\frac{P}{2}\right)\frac{\partial^2 u}{\partial y^2}$$
$$-\beta_1\frac{\partial\theta}{\partial x}-\gamma_1\frac{\partial c}{\partial x}+\mu_0H_0^2\left(\frac{\partial e}{\partial x}-\varepsilon_0\mu_0\frac{\partial^2 u}{\partial t^2}\right),$$

$$\rho\left(\frac{\partial^2 v}{\partial t^2}-\Omega^2 v+2\Omega\frac{\partial u}{\partial t}\right)=\left(C_{66}-\frac{P}{2}\right)\frac{\partial^2 v}{\partial x^2}$$
$$+\left(C_{12}+C_{66}+\frac{P}{2}\right)\frac{\partial^2 u}{\partial x\partial y}+C_{22}\frac{\partial^2 v}{\partial y^2}-\beta_2\frac{\partial\theta}{\partial y} \tag{24}$$
$$-\gamma_2\frac{\partial c}{\partial y}+\mu_0H_0^2\left(\frac{\partial e}{\partial y}-\varepsilon_0\mu_0\frac{\partial^2 v}{\partial t^2}\right)$$

In xy-plane, equation of heat conduction (4) and equation of mass diffusion (5) are reduced to the forms

$$K_1\frac{\partial^2\theta}{\partial x^2}+K_2\frac{\partial^2\theta}{\partial y^2}$$
$$=\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\left(\rho c_E\theta+T_0\beta_1\frac{\partial u}{\partial x}+T_0\beta_2\frac{\partial v}{\partial y}+aT_0c\right) \tag{25}$$

$$b\left(D_1\frac{\partial^2 c}{\partial x^2}+D_2\frac{\partial^2 c}{\partial y^2}\right)$$
$$=a\left(D_1\frac{\partial^2\theta}{\partial x^2}+D_2\frac{\partial^2\theta}{\partial y^2}\right)+D_1\gamma_1\frac{\partial^3 u}{\partial x^3}+D_1\gamma_2\frac{\partial^3 v}{\partial x^2\partial y} \tag{26}$$
$$+D_2\gamma_1\frac{\partial^3 u}{\partial x\partial y^2}+D_2\gamma_2\frac{\partial^3 v}{\partial y^3}+\left(\frac{\partial}{\partial t}+\tau\frac{\partial^2}{\partial t^2}\right)c$$

To facilitate the solution, we introduce non-dimensional variables as follows

$$(x',y',u',v')=c_0\eta_0(x,y,u,v),$$
$$(t',\tau_0',\tau')=c_0^2\eta_0(t,\tau_0,\tau)$$
$$(\sigma_{ij}',P')=\frac{1}{\rho c_0^2}(\sigma_{ij},P),\quad \Omega'=\frac{\Omega}{c_0^2\eta_0}, \tag{27}$$
$$\theta'=\frac{\beta_1}{\rho c_0^2}\theta,\quad c'=\frac{\gamma_1}{\rho c_0^2}c$$

where

$$\eta_0=\frac{\rho c_E}{K_1},\quad c_0^2=\frac{C_{11}}{\rho}$$

Now, in terms of the non-dimensional quantities defined in (26) and (27), eqs. (19)-(26) along with some simplifications, provide the following relations

$$\sigma_{xx}=\frac{\partial u}{\partial x}+B_1\frac{\partial v}{\partial y}-\theta-c-P \tag{28}$$

$$\sigma_{yy}=B_1\frac{\partial u}{\partial x}+B_2\frac{\partial v}{\partial y}-B_3\theta-B_4c-P \tag{29}$$

$$\sigma_{xy}=B_6\frac{\partial u}{\partial y}+B_5\frac{\partial v}{\partial x} \tag{30}$$

$$\sigma_{yx}=B_5\frac{\partial u}{\partial y}+B_6\frac{\partial v}{\partial x} \tag{31}$$

$$\frac{\partial^2 u}{\partial t^2}-\Omega^2 u-2\Omega\frac{\partial v}{\partial t}=(1+B_7)\frac{\partial^2 u}{\partial x^2}+B_9\frac{\partial^2 v}{\partial x\partial y}$$
$$+B_5\frac{\partial^2 u}{\partial y^2}-\frac{\partial\theta}{\partial x}-\frac{\partial c}{\partial x}-B_8\frac{\partial^2 u}{\partial t^2} \tag{32}$$

$$\frac{\partial^2 v}{\partial t^2}-\Omega^2 v+2\Omega\frac{\partial u}{\partial t}=B_5\frac{\partial^2 v}{\partial x^2}+B_9\frac{\partial^2 u}{\partial x\partial y}+(B_2+B_7)\frac{\partial^2 v}{\partial y^2}$$
$$-B_3\frac{\partial\theta}{\partial y}-B_4\frac{\partial c}{\partial y}-B_8\frac{\partial^2 v}{\partial t^2} \tag{33}$$

$$\frac{\partial^2\theta}{\partial x^2}+\varepsilon_{11}\frac{\partial^2\theta}{\partial y^2}=\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\left(\theta+\varepsilon_{12}\frac{\partial u}{\partial x}+\varepsilon_{13}\frac{\partial v}{\partial y}+\varepsilon_{14}c\right) \tag{34}$$

$$\frac{\partial^2 c}{\partial x^2}+\varepsilon_{21}\frac{d^2 c}{\partial y^2}=\varepsilon_{22}\left(\frac{\partial^2\theta}{\partial x^2}+\varepsilon_{21}\frac{\partial^2\theta}{\partial y^2}\right)+\varepsilon_{23}\frac{\partial^3 u}{\partial x^3}$$
$$+\varepsilon_{24}\frac{\partial^3 v}{\partial x^2\partial y}+\varepsilon_{25}\frac{\partial^3 u}{\partial x\partial y^2}+\varepsilon_{26}\frac{\partial^3 v}{\partial y^3}+\varepsilon_{27}\left(\frac{\partial c}{\partial t}+\tau\frac{\partial^2 c}{\partial t^2}\right) \tag{35}$$

where

$$B_3=\frac{\beta_2}{\beta_1},\quad B_4=\frac{\gamma_2}{\gamma_1},\quad B_5=\frac{C_{66}}{C_{11}}-\frac{P}{2},,$$

$$B_9=B_1+B_6+B_7,\quad B_6=\frac{C_{66}}{C_{11}}+\frac{P}{2},$$

$$(B_1,B_2,B_7,B_8)=\frac{1}{C_{11}}\left(C_{12},C_{22},\mu_0H_0^2,\varepsilon_0\mu_0^2H_0^2c_0^2\right),$$

$$\varepsilon_{11}=\frac{K_2}{K_1},\quad (\varepsilon_{12},\varepsilon_{13},\varepsilon_{14})=\frac{T_0\beta_1}{C_{11}K_1\eta_0}\left(\beta_1,\beta_2,\frac{aC_{11}}{\gamma_1}\right),$$

$$\varepsilon_{21}=\frac{D_2}{D_1},\quad \varepsilon_{22}=\frac{a\gamma_1}{b\beta_1},$$

$$(\varepsilon_{23},\varepsilon_{24},\varepsilon_{25},\varepsilon_{26})=\frac{\gamma_1}{bC_{11}}(\gamma_1,\gamma_2,\gamma_1\varepsilon_{21},\gamma_2\varepsilon_{21}),$$

$$\varepsilon_{27}=\frac{1}{b\eta_0D_1}$$

4. Solution of the problem

For the analytic solution of eqs. (32)-(35) in the form of the harmonic traveling waves, we suppose the solution of the form

$$[u,v,\theta,c](x,y,t)=[u_1,v_1,\theta_1,c_1]\exp\left[\iota k\left(-x\cos\theta+y\sin\theta\right)-\iota\omega t\right] \quad (36)$$

where k is the wave number, ω is angular frequency having the definition $\omega=kV$, V being the phase velocity and ($\sin\theta$, $\cos\theta$) denotes the projection of wave normal onto the xy-plane.

Substituting from (36), into eqs. (32)-(35), we obtain the following set of equations

$$\left(F_{11}V^2+F_{12}\right)u_1+\left(F_{13}V^2-F_{14}\right)v_1-F_{15}V\theta_1-F_{16}Vc_1=0 \quad (37)$$

$$\left(F_{13}V^2+F_{14}\right)u_1-\left(F_{11}V^2+F_{16}\right)v_1-F_{17}V\theta_1-F_{18}Vc_1=0 \quad (38)$$

$$F_{22}Vu_1-F_{23}Vv-\left(F_{21}V^2-F_{24}\right)\theta_1-F_{25}V^2c_1=0 \quad (39)$$

$$F_{31}u_1-F_{32}v_1-F_{33}V\theta_1-V\left(F_{34}V^2-F_{35}\right)c_1=0 \quad (40)$$

where

$$F_{11}=-\left(\omega^2+\Omega^2+B_8\omega^2\right),$$

$$F_{12}=\left[(1+B_7)\cos^2\theta+B_5\sin^2\theta\right]\omega^2,\quad F_{13}=2\iota\Omega\omega.$$

$$F_{14}=\left(B_7+B_9\right)\cos\theta\sin\theta\omega^2,\quad F_{15}=\iota\omega\cos\theta,$$

$$F_{16}=\left[B_5\cos^2\theta+\left(B_2+B_7\right)\sin^2\theta\right]\omega^2,$$

$$F_{17}=\iota\omega B_3\sin\theta,\quad F_{18}=\iota\omega B_4\sin\theta,\quad F_{21}=\iota\omega+\tau_0\omega^2,$$

$$F_{22}=\iota\omega\varepsilon_{12}F_{21}\cos\theta,\quad F_{23}=\iota\omega\varepsilon_{13}F_{21}\sin\theta,$$

$$F_{24}=\omega^2\left(\cos^2\theta+\varepsilon_{11}\sin^2\theta\right),\quad F_{25}=\varepsilon_{14}F_{21},$$

$$F_{31}=\iota\omega^3\left(\varepsilon_{23}\cos^3\theta+\varepsilon_{25}\cos\theta\sin^2\theta\right),$$

$$F_{32}=\iota\omega^3\left(\varepsilon_{24}\cos^2\theta\sin\theta+\varepsilon_{26}\sin^3\theta\right),$$

$$F_{33}=\omega^2\varepsilon_{22}\left(\cos^2\theta+\varepsilon_{21}\sin^2\theta\right),$$

$$F_{34}=\varepsilon_{27}\left(\iota\omega+\tau\omega^2\right),\quad F_{35}=\omega^2\left(\cos^2\theta+\varepsilon_{21}\sin^2\theta\right)$$

It can be seen that eqs. (37)-(40) are coupled in u_1,v_1, θ_1 and c_1. The condition for existence of non-trivial solution of the homogeneous system of above four equations provides us the characteristic equation satisfied by V as

$$V^8+AV^6+BV^4+CV^2+D=0 \quad (41)$$

where

$$A=\frac{F_{71}F_{85}+F_{72}F_{84}-F_{81}F_{75}-F_{82}F_{74}}{F_{71}F_{84}-F_{81}F_{74}}$$

$$B=\frac{F_{71}F_{86}+F_{72}F_{85}+F_{81}F_{76}-F_{82}F_{75}+F_{83}F_{74}-F_{84}F_{73}}{F_{71}F_{84}-F_{81}F_{74}},$$

$$C=\frac{F_{72}F_{86}+F_{76}F_{82}+F_{83}F_{75}-F_{85}F_{73}}{F_{71}F_{84}-F_{81}F_{74}},$$

$$D=\frac{-\left(F_{76}F_{83}+F_{86}F_{73}\right)}{F_{71}F_{84}-F_{81}F_{74}},$$

$$F_{81}=F_{41}F_{67}+F_{45}F_{61},\quad F_{82}=F_{45}F_{62}-F_{41}F_{68}+F_{42}F_{67},$$

$$F_{83}=F_{42}F_{68}+F_{45}F_{63},\quad F_{84}=F_{43}F_{67}+F_{45}F_{64},$$

$$F_{85}=F_{44}F_{67}-F_{43}F_{68}-F_{45}F_{65},\quad F_{86}=F_{45}F_{66}-F_{44}F_{68},$$

$$F_{71}=F_{41}F_{55}-F_{45}F_{51},\quad F_{72}=F_{42}F_{55}-F_{41}F_{56}-F_{45}F_{52},$$

$$F_{73}=F_{42}F_{56},\quad F_{74}=F_{43}F_{55}-F_{45}F_{53},$$

$$F_{75}=F_{44}F_{55}-F_{43}F_{56}-F_{45}F_{54},\quad F_{76}=F_{44}F_{56},$$

$$F_{61}=F_{11}F_{34},\quad F_{62}=F_{12}F_{34}-F_{11}F_{35},\quad F_{63}=F_{12}F_{35}+F_{15}F_{31},$$

$$F_{64}=F_{13}F_{34},\quad F_{65}=F_{13}F_{35}+F_{14}F_{34},\quad F_{66}=F_{14}F_{35}+F_{15}F_{32},$$

$$F_{67}=F_{15}F_{34},\quad F_{68}=F_{15}\left(F_{33}+F_{35}\right),\quad F_{51}=F_{13}F_{25},$$

$$F_{52}=F_{14}F_{25}-F_{18}F_{22},\quad F_{53}=-F_{11}F_{25},$$

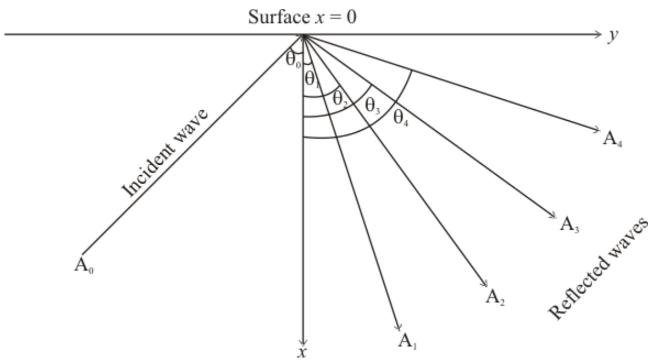
$$F_{54}=F_{18}F_{23}-F_{16}F_{25},\quad F_{55}=F_{18}F_{21}-F_{17}F_{25},\quad F_{56}=F_{18}F_{24},$$

$$F_{41}=F_{11}F_{18}-F_{13}F_{15},\quad F_{42}=F_{12}F_{18}-F_{14}F_{15},$$

$$F_{43}=F_{13}F_{18}+F_{11}F_{15},\quad F_{44}=F_{15}F_{16}-F_{14}F_{18},$$

$$F_{45}=F_{15}\left(F_{17}-F_{18}\right)$$

The roots of equation (41) give four values of V^2 which correspond to four coupled plane waves qP, qT, qMD and qSV propagating with velocities V_1,V_2,V_3 and V_4 respectively. Also, the velocity of these waves depends upon the frequency ω and wave number k. Hence, these waves are found to be dispersive and attenuated in nature.



5. Reflection at the boundary surface

Here we shall discuss the reflection phenomena, when a set of coupled waves becomes incident obliquely at the boundary of the half-space. We assume that a set of coupled plane qP wave propagating with velocity V_1 and making an angle θ_0 with the normal be made incident at the boundary surface x = 0. In order to satisfy the boundary conditions, we postulate that this incident qP wave gives rise to four reflected coupled plane waves qP, qT, qMD and qSV making an angle θ_1, θ_2, θ_3 and θ_4 respectively with the normal, as shown in Fig. A. Therefore, the full structure of the wave field consisting of the incident and reflected waves, can be written as

$$(u, v, \theta, c) = (1, \eta_1, \zeta_1, \xi_1) A_0 P_0^- + \sum_{i=1}^{4} (1, \eta_i, \zeta_i, \xi_i) A_i P_i^+ \quad (42)$$

where $P_0^- = \exp\left[\iota k_1 (-x\cos\theta_0 + y\sin\theta_0) - \iota\omega_1 t\right]$ is the phase factor of the incident wave at angle θ_0 with A_0 as amplitude constant, $P_i^+ = \exp\left[\iota k_i (x\cos\theta_i + y\sin\theta_i) - \iota\omega_i t\right]$ are the phase factors of the reflected waves corresponding to amplitude constants A_i at angles θ_i and η_i, ζ_i and ζ_i (i=1,2,3,4) are the coupling parameters between u_1,v_1, θ_1 and c_1. The expressions of these coupling parameters are given by

$$\eta_i = \frac{-\left(F_{71}V_i^4 + F_{72}V_i^2 - F_{73}\right)}{F_{74}V_i^4 + F_{75}V_i^2 - F_{76}},$$

$$\zeta_i = \frac{F_{61}V_i^4 + F_{62}V_i^2 - F_{63} + \eta_i\left(F_{64}V_i^4 - F_{65}V_i^2 + F_{66}\right)}{V_i\left(F_{67}V_i^2 - F_{68}\right)},$$

$$\xi_i = \frac{F_{31} - \eta_i F_{32} - F_{33}V_i\zeta_i}{V_i\left(F_{34}V_i^2 - F_{35}\right)}, (i = 1, 2, 3, 4)$$

6. Boundary conditions

The amplitudes A_0, A_1, A_2, A_3 and A_4 can be determined by imposing

The proper boundary conditions at the surface x = 0. Since, the boundary of the half-space is adjacent to vacuum, it is free from mechanical stresses, temperature and concentration. Mathematically, these conditions may be expressed as

$$\sigma_{xx} + \bar{\sigma}_{xx} + P = 0 \quad (43)$$

$$\sigma_{xy} + \vec{\sigma}_{xy} = 0 \quad (44)$$

$$\theta = 0 \quad (45)$$

$$c = 0 \quad (46)$$

where $\vec{\sigma}_{xj}\ (j = x, y, z)$ is the Maxwell stress given in the form

$$\vec{\sigma}_{xj} = \mu_0\left(H_x h_j + H_j h_x - H_k h_k \delta_{xj}\right) \quad (47)$$

The boundary conditions prescribed above in equations (43)-(46) are identically satisfied if and only if $\omega = \omega_1 = \omega_2 = \omega_3 = \omega_4$ and satisfy Snell's law, which gives the relation among angles of incident and reflected waves as

$$k_1\sin\theta_0 = k_1\sin\theta_1 = k_2\sin\theta_2 = k_3\sin\theta_3 = k_4\sin\theta_4 \quad (48)$$

which can also be expressed as (extended Snell's law)

$$\frac{\sin\theta_0}{V_1} = \frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{V_2} = \frac{\sin\theta_3}{V_3} = \frac{\sin\theta_4}{V_4} \quad (49)$$

From Snell's law (49), we observe that $\theta_0 = \theta_1$, the other angles of reflection depend upon the phase speeds V_1, V_2, V_3 and V_4 which are functions of material parameters.

Inserting the expressions of u,v, θ and c from equation (42) into expressions (28) and (30) and using relations given in (47) and (49), we obtain a system of four non-homogeneous equations in four unknowns by using the boundary conditions defined in (43)-(46). These four equations can be written in matrix form as

$$\sum_{j=1}^{4} b_{ij} Z_j = Y_i,\ i = 1, 2, 3, 4 \quad (50)$$

where

$$b_{1j} = k_j\left[(1+B_7)\cos\theta_j + (B_1+B_7)\eta_j\sin\theta_j - \frac{(\zeta_j+\xi_j)}{\iota k_j}\right],$$

$$b_{2j} = k_j\left(B_6\sin\theta_j + B_5\eta_j\cos\theta_j\right),\ b_{3j} = \zeta_j,\ b_{4j} = \xi_j,$$

$$Z_j = \frac{A_j}{A_0},\ (j = 1, 2, 3, 4), \text{ and}$$

$$Y_1 = k_1\left[(1+B_7)\cos\theta_1 - (B_1+B_7)\eta_1\sin\theta_1 + \frac{(\zeta_1+\xi_1)}{\iota k_1}\right],$$

$$Y_2 = -k_1\left(B_6\sin\theta - B_5\eta_1\cos\theta_1\right),\ Y_3 = -b_{31},\ Y_4 = -b_{41}$$

Here, Z_j(j=1,2,3,4) represent the reflection coefficients (ratio of the amplitudes of reflected waves to the amplitude of incident wave) of the reflected waves. It is clear that the various reflection coefficients depend on the angle of incidence, frequency of the incident wave and on the material properties of the medium.

7. Energy partition

Following Achenbach (1967), the instantaneous rate of work of surface traction is the scalar product of the surface traction and the particle velocity. This scalar product is called the power per unit area, denoted by P^* and represents the rate at which the energy is transmitted per unit area of the surface. The time average of P^* over a period, denoted by $<P^*>$, represents the average energy transmission per unit surface area per unit time. For the present case, the rate of energy transmission at the free plane surface x = 0 is given by

$$P^* = \left(\sigma_{xx} + \bar{\sigma}_{xx} + P\right)\dot{u} + \left(\sigma_{xy} + \bar{\sigma}_{xy}\right)\dot{v} \quad (51)$$

where superposed dot denotes temporal derivative. We shall now calculate P^* for the incident wave and for each of the reflected waves using the appropriate potentials. The energy ratios $E_i (i=1,2,3,4)$ of the various reflected waves are defined as the ratios of energy corresponding to the reflected waves to the energy of the incident wave. The expressions for these energy ratios $E_i (i=1,2,3,4)$ for reflected waves are defined as

$$E_i = \frac{< P_i^* >}{< P_0^* >} \tag{52}$$

where $<P_0^*>$ denotes the average energy carried along incident wave and $<P_0^*>$ (i = 1, 2, 3, 4) denote the average energy carried along reflected coupled waves. Thus, for an incident set of coupled wave, having phase speed V_1, the energy ratios of reflected waves by using expression (52), are given by

$$\begin{aligned} E_i &= R\Big[(1+B_7)\cos\theta_i + (B_1+B_7)\eta_i \sin\theta_i \\ &-\frac{(\zeta_i+\xi_i)}{\iota k_i} + \eta_i\left(B_6\sin\theta_i + B_5\eta_i\cos\theta_i\right)\Big]\frac{k_i}{k_1}Z_i^2, \\ R &= \Big[-(1+B_7)\cos\theta_1 + (B_1+B_7)\eta_1\sin\theta \\ &-\frac{(\zeta_1+\xi_1)}{\iota k_1} + \eta_1\left(B_6\sin\theta_1 - B_5\eta_1\cos\theta\right)\Big]^{-1} \end{aligned} \tag{53}$$

where i = 1, 2, 3, 4

8. Particular cases

8.1 Transversely isotropic medium

To discuss the problem of wave propagation and reflection phenomena for a transversely isotropic, initially stressed, thermally and perfectly conducting elastic rotating medium with axis of symmetry coinciding with x-axis under diffusion theory, it is sufficient to set the value of elastic parameters C_{ij} in equation (10) as:

$$C_{12} = C_{13},\quad C_{22} = C_{33},\quad C_{55} = C_{66},\quad C_{44} = \frac{1}{2}(C_{22} - C_{23}),$$
$$\beta_1 = C_{11}\alpha_{1t} + 2C_{12}\alpha_{2t},\quad \beta_2 = C_{12}\alpha_{1t} + (C_{22}+C_{23})\alpha_{2t},$$
$$\gamma_1 = C_{11}\alpha_{1c} + 2C_{12}\alpha_{2c},\quad \gamma_2 = C_{12}\alpha_{1c} + (C_{22}+C_{23})\alpha_{2c},$$
$$\alpha_{2t} = \alpha_{3t},\quad \gamma_{2c} = \gamma_{3c}.$$

In addition, if we also neglect magnetic, rotation and initial stress parameters in this particular case, then our results are concordant with Kumar and Kansal (2011), by making slight modifications in boundary conditions.

8.2 Isotropic medium

In order to study the problem of wave propagation and reflection phenomena for an isotropic, initially stressed, thermally and perfectly conducting elastic rotating medium with diffusion, it is sufficient to set the value of elastic parameters C_{ij} in equation (10) as

$$C_{11} = C_{22} = C_{33} = \lambda + 2\mu,\quad C_{12} = C_{13} = C_{23} = \lambda,$$
$$C_{44} = C_{55} = C_{66} = \mu,\quad \beta_1 = \beta_2 = \beta_3 = (3\lambda + 2\mu)\alpha_t,$$
$$\gamma_1 = \gamma_2 = \gamma_3 = (3\lambda + 2\mu)\alpha_c,\quad \alpha_{1t} = \alpha_{2t} = \alpha_{3t} = \alpha_t,$$
$$\gamma_{1c} = \gamma_{2c} = \gamma_{3c} = \gamma_c \text{ and } K_1 = K_2$$

Neglecting the diffusion and initial stress effect also and making suitable changes in boundary conditions, our results coincide with those of Othman and Song (2011).

8.3 Neglecting initial stress effect

In the absence of initial stress, we shall be left with the relevant problem of reflection phenomenon in an orthotropic, thermally and perfectly conducting elastic medium with rotation and diffusion in the context of Lord-Shulman model. In this case, it is sufficient to set the value of initial pressure as zero, i.e. P = 0. Taking into consideration the above mentioned modification, the corresponding reflection coefficients for the incidence of a set of coupled waves propagating with speed V_1 can be obtained from the system (50).

8.4 Neglecting magnetic field

By setting magnetic parameter H_0 = 0, in the equations of motion (23) and (24), we shall be dealing with a half space problem in an orthotropic initially stressed thermoelastic rotating medium with diffusion. Taking into consideration the above mentioned modification, system (50) will provide us the reflection coefficients for the corresponding problem. If we also remove initial stress and rotation effects and assimilate transversely isotropic medium instead of orthotropic medium in this particular case, then our results coincide with those of Bijarnia and Singh (2012) with appropriate changes in boundary conditions.

8.5 Without diffusion

If the diffusion effect is removed from the thermoelastic medium, then we shall be dealing a half-space problem in an orthotropic, initially stressed, thermally and perfectly conducting elastic medium with rotation. If we also assimilate Green-Lindsay model with isotropic medium instead of Lord-Shulman model with orthotropic medium and neglect rotation effect in this limiting case, then our results are in quite good agreement with those achieved by Abo-Dahab and Mohamed (2010), by making slight modifications in boundary conditions. In this particular case, the diffusive wave will disappear from the medium. So, by putting a = 0, b = 0, D_{ij} = 0 and c = 0 in governing equations, the equations (37)-(39), take the form

$$\left(F_{11}V^2 + F_{12}\right)u_1 + \left(F_{13}V^2 - F_{14}\right)v_1 - F_{15}V\theta_1 = 0 \tag{54}$$

$$\left(F_{13}V^2 + F_{14}\right)u_1 - \left(F_{11}V^2 + F_{16}\right)v_1 - F_{17}V\theta_1 = 0 \tag{55}$$

$$F_{22}Vu_1 - F_{23}Vv_1 - \left(F_{21}V^2 - F_{24}\right)\theta_1 = 0 \tag{56}$$

It can be seen that eqs. (54)-(56) are coupled in u_1, v_1, θ_1. The condition for existence of non-trivial solution of the system of above three equations, provides us

$$V^6 + A'V^4 + B'V^2 + C' = 0 \tag{57}$$

where

$$A' = \frac{\left(G_{11}G_{25} + G_{12}G_{24} - G_{13}G_{22} - G_{14}G_{21}\right)}{G_{11}G_{24} - G_{13}G_{21}},$$

$$B' = \frac{\left(G_{11}G_{26} + G_{12}G_{25} + G_{13}G_{23} - G_{14}G_{22}\right)}{G_{11}G_{24} - G_{13}G_{21}},$$

$$C' = \frac{\left(G_{12}G_{26} + G_{14}G_{23}\right)}{G_{11}G_{24} - G_{13}G_{21}}, \quad G_{11} = F_{11}F_{17} - F_{13}F_{15},$$

$$G_{12} = F_{12}F_{17} - F_{14}F_{15}, \quad G_{13} = F_{13}F_{17} + F_{11}F_{15},$$

$$G_{14} = F_{15}F_{16} - F_{14}F_{17}, \quad G_{21} = F_{11}F_{21},$$

$$G_{22} = F_{12}F_{21} - F_{11}F_{24} - F_{15}F_{22}, \quad G_{23} = F_{12}F_{24},$$

$$G_{24} = F_{13}F_{21}, \quad G_{25} = F_{15}F_{23} - F_{13}F_{24} - F_{14}F_{21},$$

$$G_{26} = F_{14}F_{24}.$$

The roots of equation (57) give three values of V^2 which correspond to three coupled plane waves qP, qT and qSV propagating with velocities V_1, V_2 and V_3 respectively.

Considering the above appropriate changes and following the similar steps as in general case, we obtain a system of three non-homogeneous equations in three unknowns by using the boundary conditions defined in (43)-(45). These three equations can be written in matrix form as

$$\sum_{j=1}^{3} b'_{ij} Z'_j = Y'_i, \quad i = 1, 2, 3 \tag{58}$$

where

$$b'_{1j} = k_j \left[\left(1 + B_7\right)\cos\theta + \left(B_1 + B_7\right)\eta'_j \sin\theta_j - \frac{\zeta'_j}{\iota k_j} \right],$$

$$b'_{2j} = k_j \left(B_6 \sin\theta_j + B_5 \eta'_j \cos\theta_j\right), \; b'_{3j} = \zeta'_j,$$

$$Y'_1 = k_1 \left[\left(1 + B_7\right)\cos\theta_1 - \left(B_1 + B_7\right)\eta'_1 \sin\theta_1 + \frac{\zeta'_1}{\iota k_1} \right],$$

$$Y'_2 = -k_1 \left(B_6 \sin\theta_1 - B_5 \eta'_1 \cos\theta_1\right), \; Y'_3 = -b'_{31},$$

$$\eta'_i = \frac{-\left(G_{11}V_i^2 + G_{12}\right)}{G_{13}V_i^2 + G_{14}}, \quad \zeta'_i = \frac{V_i\left(G_{22} - \eta'_i G_{23}\right)}{G_{21}V_i^2 - G_{24}},$$

$$Z'_j = \frac{A'_j}{A'_0}, \quad \left(i, j = 1, 2, 3\right)$$

Here, Z'_j represent the reflection coefficients (ratio of the amplitudes of reflected waves to the amplitude of incident wave) of the reflected waves.

9. Computational results and discussion

With an aim to illustrate the considered problem in greater detail, a numerical analysis is performed. Following Kumar and Kansal (2008) and Kumar and Singh (2009), we take the physical data of Cobalt material as:

$\rho = 8836\, kgm^{-3}$, $C_{11} = 3.071 \times 10^{11}\, kg\, m^{-1}s^{-2}$,
$C_{12} = 1.027 \times 10^{11}\, kgm^{-1}s^{-2}$, $C_{13} = 0.8 \times 10^{11}\, kgm^{-1}s^{-2}$,
$C_{22} = 3.581 \times 10^{11}\, kgm^{-1}s^{-2}$, $C_{23} = 1.650 \times 10^{11}\, kgm^{-1}s^{-2}$,
$C_{66} = 1.510 \times 10^{11}\, kgm^{-1}s^{-2}$, $K_1 = 0.690 \times 10^{2}\, Wm^{-1}K^{-1}$,
$K_2 = 0.695 \times 10^{2}\, Wm^{-1}K^{-1}$, $T_0 = 298K$,
$\beta_1 = 7.04 \times 10^{6}\, kgm^{-1}s^{-2}K^{-1}$, $\beta_2 = 6.90 \times 10^{6}\, kgm^{-1}s^{-2}K^{-1}$,
$c_E = 4.27 \times 10^{2}\, Jkg^{-1}K^{-1}$, $a = 1.2 \times 10^{4}\, m^2s^{-2}K^{-1}$,
$b = 0.9 \times 10^{6}\, m^5kg^{-1}s^{-2}$, $D_1 = 0.85 \times 10^{-8}\, kgsm^{-3}$,
$D_2 = 0.8 \times 10^{-8}\, kgsm^{-3}$, $\alpha_{1c} = 2.1 \times 10^{-4}\, kg^{-1}m^3$,
$\alpha_{2c} = 2.5 \times 10^{-4}\, kg^{-1}m^3$, $\tau_0 = 0.15\, s$, $\tau = 0.2\, s$,
$H_0 = 10\, Am^{-1}$, $\mu_0 = 0.3\, Hm^{-1}$, $\varepsilon_0 = 0.7\, Fm^{-1}$, $\omega = 0.01$,
$\Omega = 0.5$, $P = 1.0$.

Making use of above mentioned numerical values, we have computed the amplitude ratios and energy ratios corresponding to incident qP wave at different angles of incidence, varying from normal incidence to grazing incidence.

Figs. 1 (a-d) depict the effect of rotation (Ω= 0.3,0.5, 0.7) on the profile of reflection coefficients $|Z_i|(i=1,2,3,4)$ for a fixed value of initial stress P = 1.0 and magnetic parameter H_0 = 10.

In Fig. 1(a), we have illustrated the pattern of variation of reflection coefficient $|Z_1|$ versus angle of incidence for different values of angular velocity. From this figure, it can be noted that the reflection coefficient $|Z_1|$ corresponding to the reflected coupled qP wave dominates heavily over other reflection coefficients. Increase in the value of angular velocity results in decrease in numerical values of reflection coefficient Z_1, which illuminates the fact that the angular velocity is having a noticeable decreasing effect on the profile of reflection coefficient $|Z_1|$. It is evident from the plot that the reflection coefficient $|Z_1|$ has qualitatively similar behaviour for all the three values of angular velocity. The modulus values of Z_1 show continuously decreasing behaviour with increasing angle of incidence. A similar effect of rotation parameter is observed on the profile of reflection coefficients $|Z_2|$ and $|Z_3|$ in Figs. 1(b, c). However, $|Z_2|$ and $|Z_3|$ attain small numerical values in comparison to $|Z_1|$. Fig. 1(d) displays the effect of angular velocity on the profile of reflection coefficient $|Z_4|$. As can be seen from the plot that the absolute values of Z_4 for angular velocity (Ω=0.3) has large values in comparison to the values of angular velocity (Ω= 0.5, 0.7). It is also seen that all the curves show similar trend for the profile of reflection coefficient $|Z_4|$.

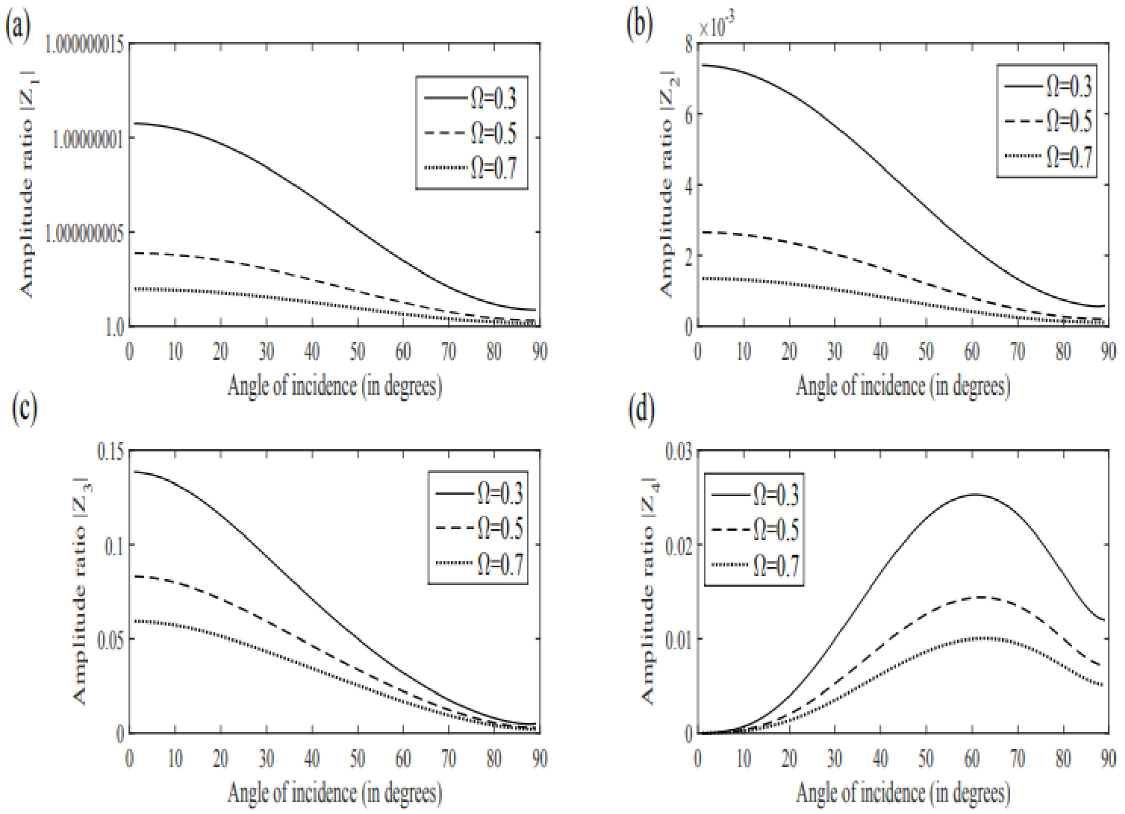


Fig. 1 (a-d) Variations of moduli of amplitude ratios for reflected waves to observe the effect of rotation parameter

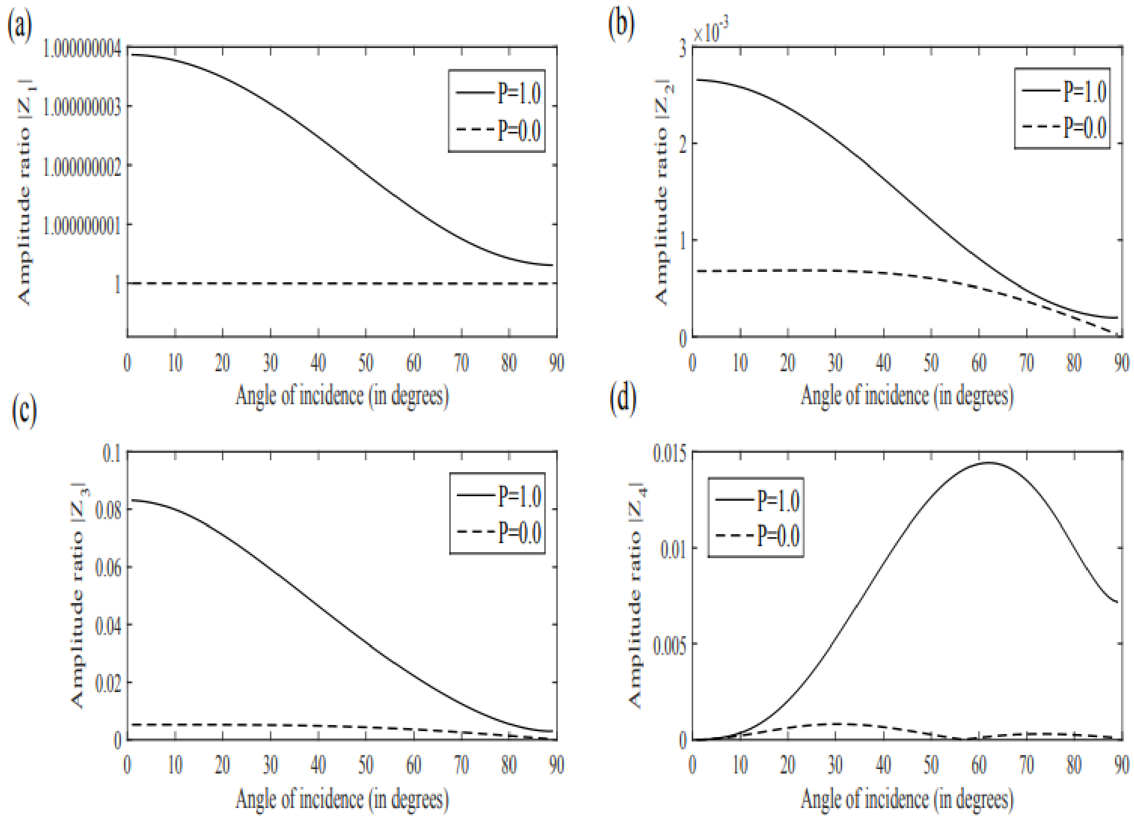


Fig. 2 (a-d) Variations of moduli of amplitude ratios for reflected waves to observe the effect of initial stress

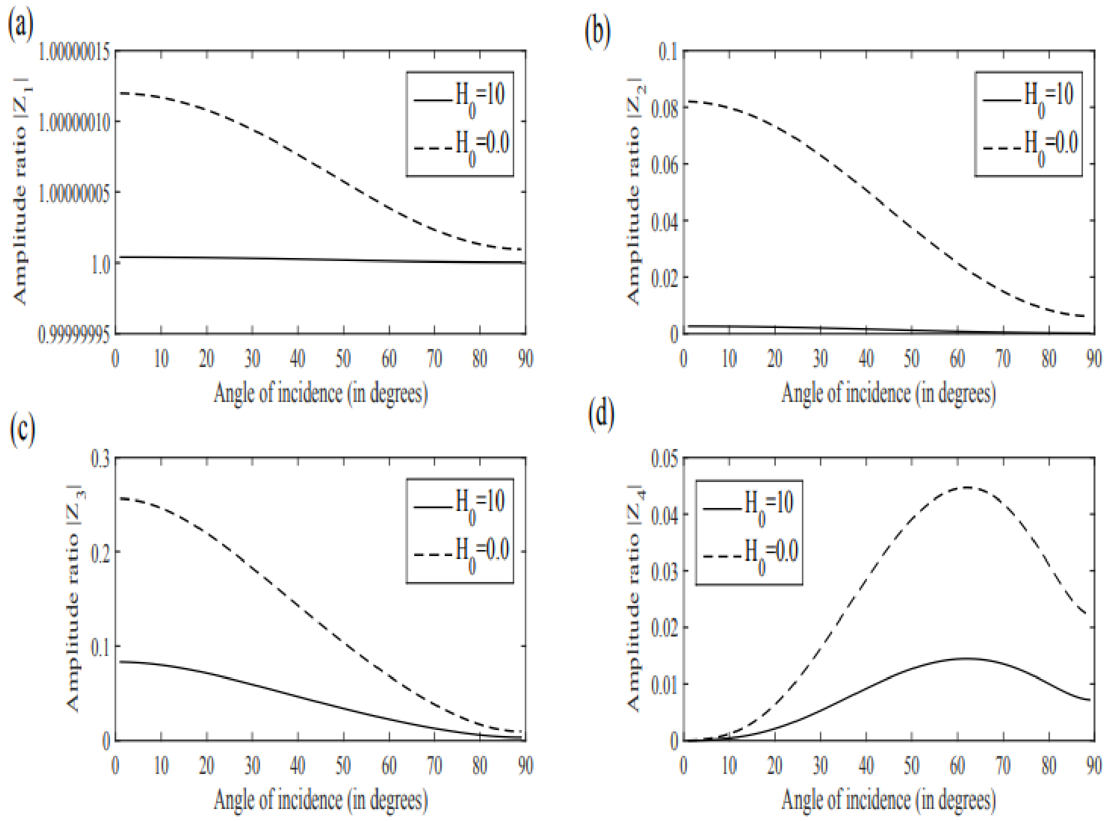


Fig. 3 (a-d) Variations of moduli of amplitude ratios for reflected waves to observe the effect of magnetic field

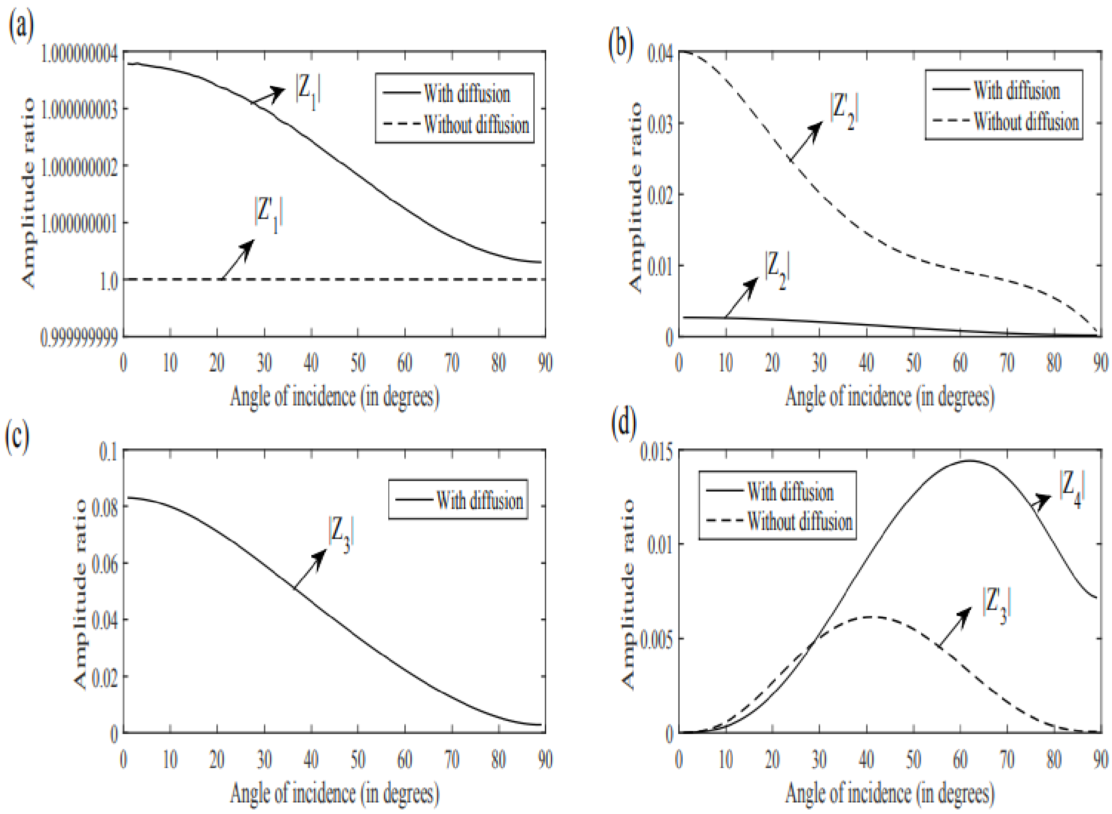


Fig. 4 (a-d) Variations of moduli of amplitude ratios for reflected waves in presence and absence of diffusion

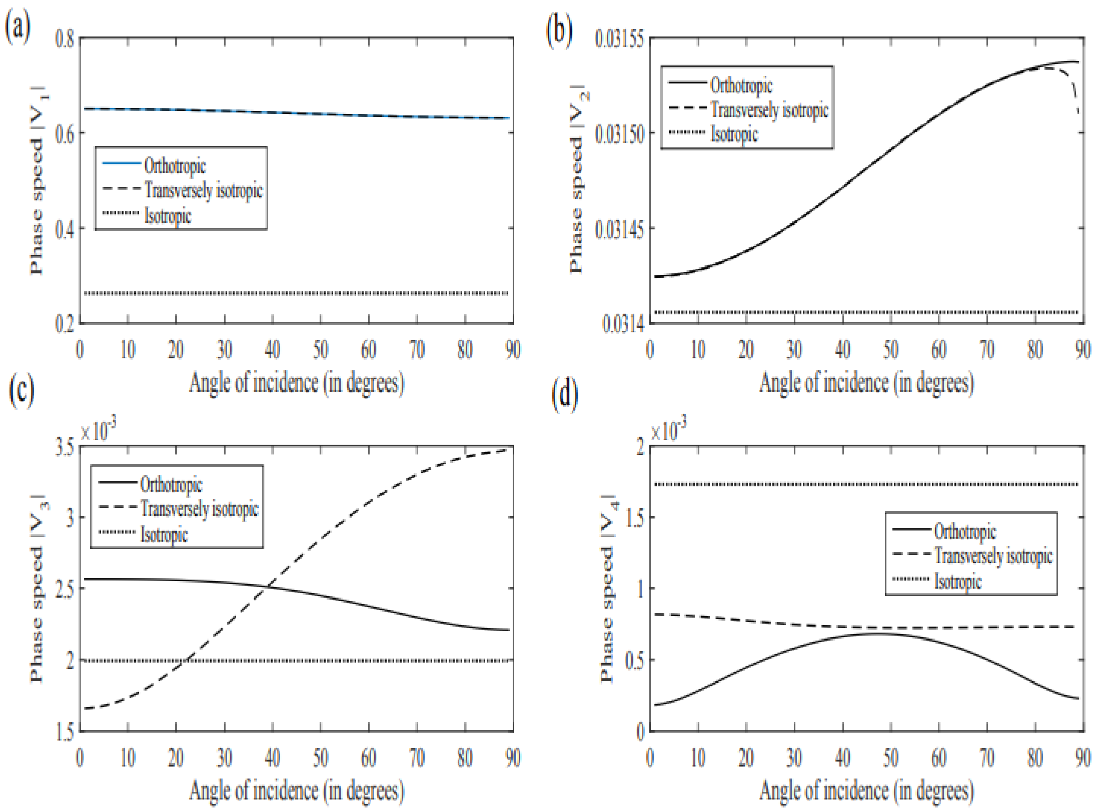


Fig. 5 (a-d) Variations of moduli of phase speeds against angle of incidence to observe the effect of anisotropy

In Figs. 2(a-d), the variations of amplitude ratios $|Z_i|(1 \le i \le 4)$, for incidence of coupled wave *qP*, have been shown graphically to observe the influence of initial stress for a fixed value of angular velocity $\Omega = 0.5$ and magnetic parameter $H_0 = 10$. In these figures, the solid line and dotted line refer to general case and without initial stress respectively. Fig. 2(a) manifests the effects of initial stress on the profile of reflection coefficient $|Z_1|$. Presence of initial stress increases the absolute values of reflection coefficient Z_1. Hence it has an increasing effect on the profile of $|Z_1|$. In Figs. 2(b, c), a similar trend is observed on the profile of reflection coefficients $|Z_2|$ and $|Z_3|$ as seen on the profile of reflection coefficient $|Z_1|$, in Fig. 2(a). The absolute values of Z_2 and Z_3 continuously decrease with the increase in angle of incidence, for both the cases (with and without initial stress). In Fig. 2(d), we have plotted the modulus values of the reflection coefficient Z_4 as a function of angle of incidence for both the cases. Presence of initial stress increases the values of reflection coefficient $|Z_4|$, in the whole range, hence indicating the increasing effect.

Figs. 3(a-d) display the absolute values of the amplitude ratios $|Z_i|(1 \le i \le 4)$ for two different cases, with and without magnetic field, for a fixed value of angular velocity $\Omega = 0.5$ and initial stress P = 1.0. The solid line and dashed line refer to general case and without magnetic field $(H_0 = 0.0)$ respectively. Fig. 3(a) shows the variations of reflection coefficient $|Z_1|$ versus angle of incidence for two different cases mentioned above. Presence of magnetic field decreases the absolute values of reflection coefficient, so it has a decreasing effect on the profile of this reflection coefficient, in the whole range. Figs. 3(b, c) reveal the variations of reflection coefficients $|Z_2|$ and $|Z_3|$ versus angle of incidence in the presence and absence of magnetic field. For both the cases, modulus values of Z_2 and Z_3 decrease in a similar fashion with increase in angle of incidence, but at different rates. Presence of magnetic field decreases the absolute values of reflection coefficients Z_2 and Z_3. Hence, it has a decreasing effect on the profile of both reflection coefficients. For the incidence of *qP* wave, the change in absolute values of reflection coefficient Z_4 with angle of incidence are represented in Fig. 3(d), in the presence and absence of magnetic field. It is evident from the plot that the reflection coefficient $|Z_4|$ has qualitatively similar behaviour in the presence and absence of magnetic field and the presence of magnetic field decreases the absolute values of Z_4.

Figs. 4(a-d), are plotted to demonstrate the variations of amplitude ratios $|Z_i|$ (i = 1, 2, 3, 4) under two different models, namely, with and without diffusion. Fig. 4(a) is drawn with the purpose to display a comparison of variation of reflection coefficient $|Z_1|$ versus angle of incidence for two different models. It is clear from the figure that the modulus values of Z_1 in the presence of diffusion are found to be larger as compared to without diffusion. Hence, it has an increasing effect. In Fig. 4(b), we have plotted the curves to exhibit the variations of reflection coefficient $|Z_2|$, in the presence and absence of diffusion model. It can be noticed

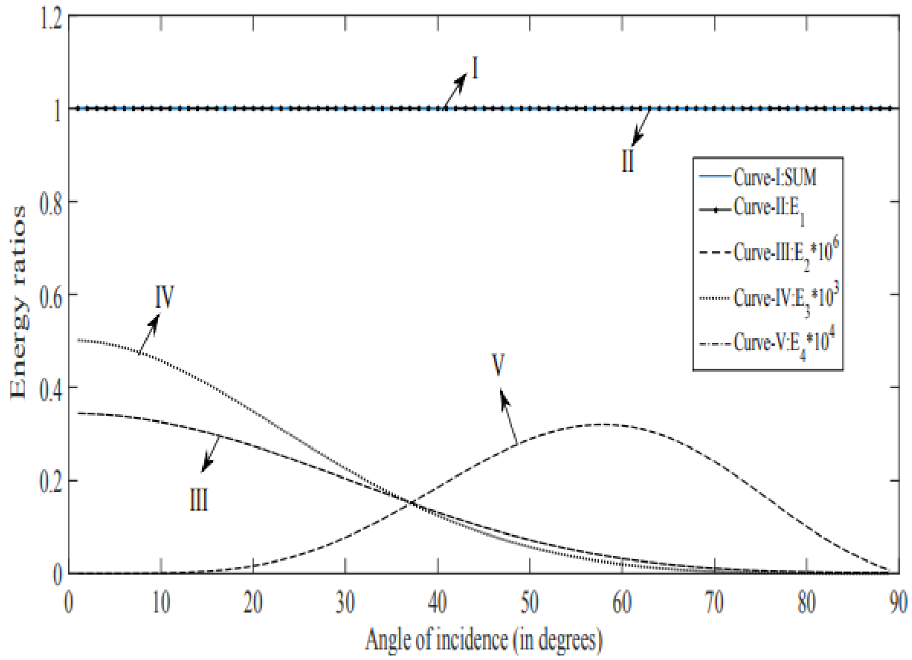


Fig. 6 Variations of moduli of energy ratios against angle of incidence of *qP* wave with speed V_1

from the figure that diffusion has a noticeable effect on the profile and its presence decreases the magnitude of reflection coefficient $|Z_2|$. Hence, it has a decreasing effect on the profile of reflection coefficient $|Z_2|$. In Fig. 4(c), we have plotted the modulus values of the reflection coefficient Z_3 as a function of angle of incidence. In this figure, we have only one curve of reflection coefficient Z_3 corresponding to the presence of diffusion, because in the absence of diffusion, the quasi-longitudinal mass diffusive wave (*qMD*) will disappear from the medium. The behavior of variation of reflection coefficient $|Z_4|$ against angle of incidence has been expressed in Fig. 4(d), for two different cases with and without diffusion. This reflection coefficient experiences a similar pattern of variations in both the models. It can also be noticed that reflection coefficient starts with zero value for both the cases at normal incidence. In this figure, we have compared the profile of reflection coefficients Z_3 and Z_3', because they appear corresponding to quasi-transverse wave (*qSV*) in thermoelastic diffusive medium and thermoelastic medium respectively.

In Figs. 5(a-d), the variations of the velocities of *qP*, *qT*, *qMD* and *qSV* waves have been shown graphically with the angle of propagation, when $\omega = 0.01$. The velocities of propagation of these plane waves are also compared with those for transversely isotropic and isotropic thermoelastic medium. We can see from these figures that the values of velocities vary at every angle of incidence in orthotropic and transversely isotropic media while in isotropic media, these values remain constant throughout the whole range. Thus, it is evident from the figures that the medium has an observable effect on the variations of the velocities. It is noticed that there exist four waves propagating in medium. The fastest among them is the quasi-longitudinal wave and the slowest of them is the quasi-transverse wave. The variations of modulus of energy ratios of reflected waves with the angle of incidence of coupled wave propagating with velocity V_1, are plotted in Fig. 6. In this figure, the comparisons of partition of energy between reflected *qP*, *qT*, *qMD* and *qSV* waves propagating with velocities V_1, V_2, V_3 and V_4 respectively for the incidence of *qP* wave are presented. The energy conversion in different ranges of angle of incidence is clearly noticed. We can see from the figure that the values of sum and E_1 are approximately same and equal to 1.0 and the values of E_2, E_3 and E_4 are very small. Since the reflection coefficients Z_2, Z_3 and Z_4 were found to be very small, therefore the corresponding energy ratios E_2, E_3 and E_4 are also very small. These energy ratios have been shown by curves III, IV and V in figure after multiplying their original values by the factors 10^6, 10^3 and 10^4 respectively. It can be seen from the figure that the energy carried by reflected coupled wave propagating with velocity V1 is maximum in comparison to energy carried by other reflected waves. Thus the major portion of energy is transported through *qP* wave while a very small amount of energy is carried by coupled *qT*, *qMD* and *qSV* waves. In the calculation of energy ratios, it has been verified that the sum of energy ratios is equal to unity for a fixed angle of incidence. This shows that there is no loss of energy during reflection of waves.

10. Concluding remarks

This article presents an in-depth analysis of plane wave propagation in an orthotropic, initially stressed, magneto-thermoelastic rotating medium with diffusion. From the analysis of the illustrations, we can arrive at the following conclusions:

- All the reflection coefficients Z_i (i = 1, 2, 3, 4) are highly influenced by angular velocity. Increase in the value of angular velocity acts to decrease the absolute values of all the reflection coefficients. Hence, it has a decreasing effect on the profile of reflection coefficients.
- Presence of initial stress parameter increases the absolute values of all the reflection coefficients. Thus, all the reflection coefficients are significantly sensitive towards the initial stress.
- The modulus values of all the reflection coefficients are highly influenced by the presence of magnetic field. It acts to decrease the magnitude of all the reflection coefficients Z_i (i = 1, 2, 3, 4).
- Significant impact of diffusion is observed on all the reflection coefficients Z_i (i = 1, 2, 3, 4). Presence of diffusion parameters increases the absolute values of reflection coefficient Z_1 and decreases the absolute values of Z_2 whereas on Z_4, its presence has both increasing and decreasing effects.
- In an anisotropic generalized thermoelastic medium, the velocities of propagation of reflected waves are found to depend upon the angle of incidence, whereas in an isotropic medium, velocities attain constant values.
- The numerical results reveal that the sum of the modulus values of energy ratios at the boundary surface is approximately unity at each angle of incidence. This shows that there is no dissipation of energy during reflection phenomena at the surface.

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Conflict of interest statement

The authors declare that they have no conflict of interest.

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