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Investigation on thermal buckling of porous FG plate resting on elastic foundation via quasi 3D solution

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Abstract. The present article deals with thermal buckling of functionally graded plates with porosity and resting on elastic foundation. The basic formulation is based on quasi 3D theory. The present theory contains only four unknowns and also accommodates the thickness stretching effect. Porosity-dependent material coefficients of the plate are compositionally graded throughout the thickness according to a modified micromechanical model. Different patterns of porosity distributions are considered. The thermal loads are assumed to be uniform, linear and non-linear temperature rises through the thickness direction. The plate is assumed to be simply supported on all edges. Various numerical examples are given to check the accuracy and reliability of the present solution, in which both the present results and those reported in the literature are provided. In addition, several numerous new results for thick FG plates with porosity are also presented.

Keywords: functionally graded plate; thermal buckling; quasi 3D theory; porosity; elastic foundation; simply supported plate

1. Introduction

Recently, intelligent structures and materials have received the attention of researchers because of their enormous application potentials. Functionally graded materials (FGMs) are among these materials. These materials have been widely used in several fields, with a focus on thermal engineering applications (thermal barrier structures subjected to severe thermal gradients).

The notable development in the application of these materials as FG plates and FG beams with a micro or nanometric length scale due to their exceptional thermal, mechanical and electrical properties has led to a provocation in modeling of micro/nano scale structures. Buckling is a mode of failure that a structure can experiment in certain situations.

Therefore, many scientific articles have been recently published on the thermal behavior of FG plates.

Amiri Rad *et al.* (2014) have studied the buckling behavior of functionally graded plates under un-iaxial and bi-axial tension loads and containing a crack using classical plate theory (CPT) in framework of finite elements. Using the same theory, Ma and Wang (2003) investigated the axisymmetric nonlinear bending and thermal post-buckling behavior of a functionally graded circular plate with simply and clamped boundary conditions. The plate has been subjected to mechanical, thermal and combining thermal–mechanical loading.

Avcar (2015) studied the combined effects of rotary inertia, shear deformation and material non-homogeneity on the values of natural frequencies of the simply supported beam.

Using Euler-Bernoulli beam theory, Avcar and his co-author (2016 b, 2018) have examined the free vibration of FG beams resting on elastic foundation with and without axial force.

Zhao *et al.* (2009) studied the buckling behavior of FG plates under mechanical and thermal loading using a formulation based on the first-order shear deformation plate theory (FSDT) by employing the element-free kp-Ritz method. Based on the FSDT, Saidi and Hasani Baferani (2010) presented an analytical solution for thermal buckling of FG annular sector plates with simply supported radial edges. Using the FSDT and the Finite Element Method, Abolghasemi *et al.* (2014) studied the buckling of FG Plates with an elliptical cutout under combined thermal and mechanical loads.

Trabelsi *et al.* (2018) investigate the thermal post-buckling responses of FG shell structures. They use a geometrically nonlinear analysis based on a modified

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FSDT. Ghiasian *et al.* (2014) analyzed the bifurcation behaviour of moderately thick heated annular FG plates using the FSDT. Bouazza *et al.* (2010) studied the thermo-elastic buckling of plate using the FSDT. The plate was assumed to be subjected to two types of thermal loadings.

Using the Timoshenko beam theory as well as the Euler–Bernoulli beam theory, Avcar (2016 a) studied the effect of material non-homogeneity and two-parameter elastic foundation on the fundamental frequency parameters of the simply supported beams.

It should be noted that the FSDT violates the equilibrium conditions on the top and bottom surfaces of the plate, a shear correction factor is required to compensate for the error due to a constant shear strain hypothesis in the thickness (Huu-Tai and Dong-Ho 2011). Therefore, the FSDT depends on a shear correction factor that is difficult to estimate for composite materials. To overcome this handicap, various higher-order shear deformation theories (HSDT) have been developed and proposed by researchers.

The theory of third order shear deformation (TSDT) does not require a shear correction factor and satisfies zero transverse shear stresses on the top and bottom surfaces of the plate attracted considerable research interest (Baferani *et al.* 2011, Akbarzadeh *et al.* 2011, Shariat and Eslami 2007, Bodaghi and Saidi 2010, Thai HT and Kim 2013). Using Reddy's third-order shear deformation plate theory, Cong *et al.* (2017) investigated the non-linear dynamic response of stiffened FGM plate in thermal medium subjected to mechanical and thermal loads considering the temperature-dependent materials properties. Also, Duc *et al.* (2016) studied the thermal stability of eccentrically stiffened FGM plate using the same theory.

Tran *et al.* (2013) presented an iso-geometric finite element approach (IGA) in combination with the third-order deformation plate theory for thermal buckling analysis of FGM plates. Using a local Kriging meshless method, Zhang *et al.* (2014) studied the mechanical and thermal buckling behaviors of ceramic–metal functionally grade plates (FGPs). Lezgy-Nazargah, & Meshkani (2018) presented a finite element model based on a parametrized mixed variational principle for the static and free vibration analysis of functionally graded material (FGM) plates rested on Winkler-Pasternak elastic foundations. Applying Galerkin's method, Sofiyev and Avcar (2010) investigated the stability of cylindrical shells subjected to axial load and resting on Winkler-Pasternak foundations.

Tounsi *et al.* (2013) used a refined trigonometric shear deformation theory (RTSDT) for studying the thermo-elastic bending response of FG sandwich plates. Using the HSDT, Zhang *et al.* (2017) studied the thermal post-buckling of FG elliptical plates in different thermal environments. They use the Ritz method to determine the central deflection-temperature curves. Duc and Tung (2011) have used the HSDT theory to investigate the buckling and postbuckling behaviors of thick FG plates resting on elastic foundations and subjected to in-plane compressive, thermal and thermo-mechanical loads. Also, Duc and Quan (2013) presented an analytical solution for the non-linear post-buckling of imperfect eccentrically stiffened P-FGM double-curved thin shallow shells in thermal environments. Using the HSDT in conjunction with the Green-Lagrange kinematics, Mahapatra *et al.* (2015a, 2015b, 2016a, 2016b, 2016c) have studied different behavior of shells and laminated composite plates under hydro-thermal environment.

Bachir Bouiadjra *et al.* (2013) investigated the non-linear behavior of FG plates under thermal loads using an efficient sinusoidal shear deformation theory. Abazid *et al.* (2018) have presented a theory taking into account both shear and normal strains effects to study the static bending of various types of FGM sandwich plates resting on Pasternak foundations in hygrothermal environment.

Bousahla *et al.* (2016) used a four-variable refined plate theory for thermal buckling analysis of plates with FG coefficient of thermal expansion and subjected to different temperature rises across the thickness direction. Beldjelili *et al.* (2016) studied the hygro-thermo-mechanical bending behavior of sigmoid FG plate resting on elastic foundations. Sobhy (2017) studied the hygro-thermo-mechanical vibration and buckling of exponentially graded nanoplates resting on Pasternak foundations. By using the same four-variable refined plate theory and by considering the concept of the neutral surface position, El Hassar *et al.* (2016) studied the thermal stability of solar FG plates subjected to uniform, linear and non-linear temperature rises across the thickness direction and resting on elastic foundation.

Using a new displacement field that containing undetermined integral terms, Chikh *et al.* (2017) studied the thermal buckling of cross-ply laminated plates. Menasria *et al.* (2017) analyzed the thermal buckling response of functionally graded (FG) sandwich plate. Sekkal *et al.* (2017) used the same theory for studying the buckling and free vibration of FG sandwich plate. Mahmoudi *et al.* (2018) investigated the influence of the micromechanical models on the free vibration of simply supported FG plate resting on elastic foundation using the same displacement field.

Based on the nonlocal strain gradient elasticity theory, Arani *et al.* (2019) studied the thermo-electro-mechanical buckling of a sandwich nano-beams with face-sheets made of FG carbon nano-tubes reinforcement composite.

Zghal and his co-autors (Zghal *et al.* 2017, Zghal *et al.* 2018a, Zghal *et al*. 2018b, Frikha *et al.* 2018a, Zghal *et al.* 2018c, Frikha *et al.* 2018b, Trabelsi *et al.* 2019) have studied the mechanical, vibrational and buckling of FG plate, shells and its derivatives like FG-CNTRC.

Porosities occurring inside the material structure during the production progress have an important effect on the mechanical behaviors of inhomogeneous structures (Barati 2017).

As for porous stuctures, Ait Atmane *et al.* (2017) have used a quasi 3D beam theory to analyze bending, buckling and free vibration characteristics of porous FG beams on elastic foundations.

Shojaeefard *et al.* (2017) studied the free vibration and thermal buckling of micro temperature dependent FG porous circular plate subjected to a nonlinear thermal load using both CPT and the FSDT theories in conjunction with the modified couple stress theory. Using the HSDT, Cong *et*

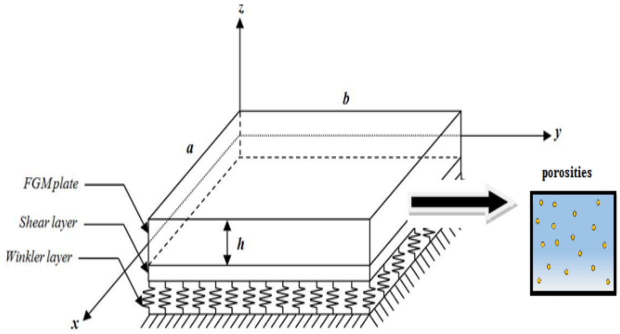


Fig. 1 Coordinate system and geometry for rectangular FG porous plates resting on elastic foundation

al. (2018) have presented a study for buckling and post-buckling behavior of FG plate with porosities resting on elastic foundations and subjected to mechanical, thermal and thermo-mechanical loads.

Gupta and Talha (2018) studied the flexural and vibration response of FG plates with porosity based on non-polynomial higher-order shear and normal deformation theory.

In this article, our aim is to extend the work of Bachir Bouiadjra *et al.* (2018) to study the effect of porosity on the thermal stability of FG plates on elastic foundation. For this purpose, the displacement field is expressed in the same manner as that reported in Bachir Bouiadjra *et al.* (2018). The present theory takes into account the stretching effect due to its quasi three dimensional nature. Moreover, the model satisfied exactly stress boundary conditions on the top and the bottom of the plate. The proposed quasi 3D solution contains only four unknowns. Different porosity distributions are considered. The thermal loads are assumed to be uniform, linear and non-linear temperature rises through the thickness direction.

To check the validity of the present quasi 3D theory, the obtained results are compared with the available data in the literature. Parametric study is performed to show the influences of the porosity distributions and the thermal loads on the buckling of the FG plate.

2. Effective material properties of the FG porous plate

Consider a FG thick rectangular porous plate of uniform thickness h, length a and width b as shown in Figure 1. The plate is assumed to rest on a Winkler-Pasternak elastic foundation. The mechanical characteristics of the porous plate are assumed to be vary according to a well-defined law as a function of the distribution of the porosity. In the following, five distributions of the porosity in the direction of the thickness are considered

Porosity distribution 1 (Mouaici *et al.* 2016, Wattanasakulpong and Chaikittiratana 2015)

$$E(z)=(E_c-E_m)V(z)+E_m-\frac{\alpha}{2}(E_c+E_m) \quad (1)$$

Porosity distribution 2 (Wattanasakulpong and Chaikittiratana 2015)

$$E(z)=(E_c-E_m)V(z)+E_m-\frac{\alpha}{2}(E_c+E_m)\left(1-\frac{2|z|}{h}\right) \quad (2)$$

Porosity distribution 3 (Gupta and Talha 2018)

$$\begin{aligned} E(z)=&(E_c-E_m)V(z)\ +E_m \\ &-log\left(1+\frac{\alpha}{2}\right)(E_c+E_m)\left(1-\frac{2|z|}{h}\right) \end{aligned} \quad (3)$$

Porosity distribution 4 (Chen *et al.* 2015)

$$\begin{aligned} E(z)&=E_1\left[1-e_0\,cos(\pi\zeta)\right] \\ G(z)&=G_1\left[1-e_0\,cos(\pi\zeta)\right] \\ \rho(z)&=\rho_1\left[1-e_m\,cos(\pi\zeta)\right] \end{aligned} \quad (4)$$

Porosity distribution 5 (Chen *et al.* 2015)

$$\begin{aligned} E(z)&=E_1\left[1-e_0\,cos\left(\frac{\pi}{2}\zeta+\frac{\pi}{4}\right)\right] \\ G(z)&=G_1\left[1-e_0\,cos\left(\frac{\pi}{2}\zeta+\frac{\pi}{4}\right)\right] \\ \rho(z)&=\rho_1\left[1-e_m\,cos\left(\frac{\pi}{2}\zeta+\frac{\pi}{4}\right)\right] \end{aligned} \quad (5)$$

Where $\zeta= z/h$, the porosity coefficient $e_0=1-E_0/E_1=1-G_0/G_1 \quad (0<e_0<1)$, the minimum and maximum values of Young's modulus E_0 and E_1 are related to the minimum and maximum values of shear modulus G_0 and G_1 by

$$G_i=E_i/\left[2(1+\nu)\right](i=0,1) \quad (6)$$

The porosity coefficient for mass density is defined as

$$e_m=1-\rho_0/\rho_1,\ (0<e_m<1) \quad (7)$$

ρ_0 *and* ρ_1 are respectively the minimum and maximum values of mass density.

The relationship between density and Young's modulus for an open-cell metal form is given by Gibson and Ashby (1982) and Choi and Lakes (1995)

$$\frac{E_0}{E_1}=\left(\frac{\rho_0}{\rho_1}\right)^2 \quad (8)$$

This can be used to obtain the relationship between e_0 and e_m as below

$$e_m=1-\sqrt{1-e_0} \quad (9)$$

$V(z)$ is the volume fraction and it is assumed to follow a simple power law as follow Bourada *et al*. (2018), Attia *et al*. (2018), Bousahla *et al*. (2016), Achouri *et al*. (2019)

$$V(z)=\left(\frac{z}{h}+\frac{1}{2}\right)^p \quad (10)$$

where "p" is the power law index. Note that, when $p=0$, one obtains a fully homogeneous ceramic plate. Whereas, if $p \approx \infty$, a fully metal plate is obtained. E_m and E_c are the Young modulus of metal and ceramic, respectively.

α is termed as porosity volume fraction (α<1). α=0 indicates the perfect FG plate.

3. Kinematics

The displacement field satisfying the conditions of transverse shear stresses (and hence strains) vanishing at (x, y, ±*h/2*) on the outer (top) and inner (bottom) surfaces of the plate, is given as follows

$$\begin{aligned} u\ (x,y,z,t) &= u_0 - z\frac{\partial w_b}{\partial x} + f\ (z)\frac{\partial w_s}{\partial x} \\ v\ (x,y,z,t) &= v_0 - z\frac{\partial w_b}{\partial y} + f\ (z)\frac{\partial w_s}{\partial y} \\ w(x,y,z,t) &= w_b + g(z)w_s \end{aligned} \tag{11}$$

With

$$f(z) = -z\left(1-\frac{4z^2}{3h^2}\right) \quad , \quad g(z) = \frac{1}{5}\frac{\partial f(z)}{\partial z} \tag{12}$$

Where $u_0(x,y)$, $v_0(x,y)$, $w_b(x,y)$ *and* $w_s(x,y)$ are the four unknown displacement functions of the middle surface of the plate.

The kinematic relations can be obtained as follows

$$\begin{aligned} &\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} + f\ (z)\begin{Bmatrix} \eta_x \\ \eta_y \\ \eta_{xy} \end{Bmatrix}, \varepsilon_z = g'(z)\varepsilon_z^0, \\ &\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = f'\ (z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} + g\ (z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \end{aligned} \tag{13}$$

Where

$$\begin{aligned} &\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_b}{\partial x^2} \\ -\dfrac{\partial^2 w_b}{\partial y^2} \\ -2\dfrac{\partial^2 w_b}{\partial x \partial y} \end{Bmatrix}, \\ &\begin{Bmatrix} \eta_x \\ \eta_y \\ \eta_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial^2 w_s}{\partial x^2} \\ \dfrac{\partial^2 w_s}{\partial y^2} \\ 2\dfrac{\partial^2 w_s}{\partial x \partial y} \end{Bmatrix}, \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial w_s}{\partial y} \\ \dfrac{\partial w_s}{\partial x} \end{Bmatrix}, \varepsilon_z^0 = w_s \end{aligned} \tag{14}$$

4. Constitutive relations

The linear constitutive relations are

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{pmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{pmatrix} \varepsilon_x - \alpha_t T \\ \varepsilon_y - \alpha_t T \\ \varepsilon_z - \alpha_t T \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{pmatrix} \tag{15}$$

where (σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, τ_{yx}) and (ε_x, ε_y, ε_z, γ_{xy}, γ_{yz}, γ_{yx}) are the stress and strain components, respectively. α_t is the coefficient of thermal expansion, and T is the distribution of the temperature load. Using the material properties defined above, stiffness coefficients, Q_{ij} can be expressed as

$$\begin{aligned} Q_{11} &= Q_{22} = Q_{33} = \frac{E(z)}{1-\nu^2}, \\ Q_{12} &= Q_{13} = Q_{23} = \frac{\nu E(z)}{1-\nu^2}, \\ Q_{44} &= Q_{55} = Q_{66} = \frac{E(z)}{2(1+\nu)}, \end{aligned} \tag{16}$$

5. Equations of motion

Considering the static version of the principle of virtual work, the following expressions can be obtained

$$\begin{aligned} \int_\Omega\int (&\sigma_x\,\delta\varepsilon_x + \sigma_y\,\delta\varepsilon_y + \sigma_z\,\delta\varepsilon_z + \tau_{xy}\,\delta\gamma_{xy} \\ &+ \tau_{xz}\,\delta\gamma_{xz} + \tau_{yz}\,\delta\gamma_{yz})d\Omega\,dz + \int_\Omega f_e \delta w d\Omega = 0 \end{aligned} \tag{17}$$

Where Ω is the top surface.

And f_e is the density of reaction force of foundation, and for the Pasternak fundation model can be written as

$$f_e = k_w\left(w_b + g(z)w_s\right) - k_p \nabla^2\left(w_b + g(z)w_s\right) \tag{18a}$$

Where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{18b}$$

Substituting Eqs. (13) and (18) into Eq. (17) and integrating through the thickness of the plate, we can obtain

$$\begin{aligned} \int_\Omega \Big\{ &N_1\delta\varepsilon_x^0 + M_1\,\delta k_x + P_1\delta\eta_x + N_2\delta\varepsilon_y^0 + M_2\delta k_y + P_2\delta\eta_y + R_3\delta\varepsilon_z^0 \\ &+ Q_4\delta\gamma_{yz}^0 + K_4\delta\gamma_{yz}^0 + Q_5\delta\gamma_{xz}^0 + K_5\delta\gamma_{xz}^0 + N_6\delta\gamma_{xy}^0 + M_6\delta k_{xy} \\ &+ P_6\delta\eta_{xy} + \left[k_w\left(w_b + g(z)w_s\right) - k_p\nabla^2\left(w_b + g(z)w_s\right)\right]\delta w\Big\} d\Omega = 0 \end{aligned} \tag{19}$$

The stress resultants N, M, P, Q and R are defined by

$$\left(N_i, M_i, P_i\right) = \int_{-h/2}^{h/2} \sigma_i \left(1, z, f(z)\right) dz, \qquad \left(i = 1, 2, 6\right)$$

$$\left(K_i, Q_i\right) = \int_{-h/2}^{h/2} \sigma_i \left(f'(z), g(z)\right) dz, \qquad \left(i = 4, 5\right) \tag{20}$$

$$R_i = \int_{-h/2}^{h/2} \sigma_i g'(z) dz, \qquad \left(i = 3\right)$$

The governing equations of equilibrium can be derived from eq. (19) by integrating the displacement gradients by parts and setting the coefficients where δu_0, δv_0, δw_b, δw_s zero.

Thus one can obtain the equilibrium equations associated with the present shear deformation theory

$$\begin{aligned}
&\frac{\partial N_1}{\partial x} + \frac{\partial N_6}{\partial y} = 0 \\
&\frac{\partial N_6}{\partial x} + \frac{\partial N_2}{\partial y} = 0 \\
&\frac{\partial^2 M_1}{\partial x^2} + 2\frac{\partial^2 M_6}{\partial x \partial y} + \frac{\partial^2 M_2}{\partial y^2} + \bar{N} - f_e = 0 \\
&-\frac{\partial^2 P_1}{\partial x^2} - \frac{\partial^2 P_2}{\partial y^2} - 2\frac{\partial^2 P_6}{\partial x \partial y} + \frac{\partial Q_4}{\partial y} + \frac{\partial K_4}{\partial y} + \frac{\partial Q_5}{\partial x} \\
&\quad + \frac{\partial K_5}{\partial x} - R_3 + g(z)\bar{N} - g(z) f_e = 0
\end{aligned} \tag{21}$$

With

$$\bar{N} = \left[\begin{array}{c} N_x \dfrac{\partial^2 (w_b + g(z) w_s)}{\partial x^2} + 2N_{xy} \dfrac{\partial^2 (w_b + g(z) w_s)}{\partial x \partial y} \\ + N_y \dfrac{\partial^2 (w_b + g(z) w_s)}{\partial y^2} \end{array}\right] \tag{22}$$

The stability equations for FG plates may be obtained by means of the adjacent-equilibrium criterion. Let us assume that the state of equilibrium of plate under thermal loads is defined in terms of the displacement components u_0^0, v_0^0, w_b^0 and w_s^0. The displacement components of a neighboring state of the stable equilibrium differ by u_0^1, v_0^1, w_b^1 and w_s^1 with respect to the equilibrium position. Thus, the total displacements of a neighboring state are

$$\begin{aligned} u_0 = u_0^0 + u_0^1,\ v_0 = v_0^0 + v_0^1,\ w_b = w_b^0 + w_b^1, \\ w_s = w_s^0 + w_s^1 \end{aligned} \tag{23}$$

Accordingly, the stress resultants are divided into two terms representing the stable equilibrium and the neighboring state. The stress resultants with superscript 1 are linear functions of displacement with superscript 1. Considering all these mentioned above and using Eqs. (21) and (23), the stability equations becomes

$$\begin{aligned}
&\frac{\partial N_1^1}{\partial x} + \frac{\partial N_6^1}{\partial y} = 0 \\
&\frac{\partial N_6^1}{\partial x} + \frac{\partial N_2^1}{\partial y} = 0 \\
&\frac{\partial^2 M_1^1}{\partial x^2} + 2\frac{\partial^2 M_6^1}{\partial x \partial y} + \frac{\partial^2 M_2^1}{\partial y^2} + \bar{N}^1 - f_e^1 = 0 \\
&-\frac{\partial^2 P_1^1}{\partial x^2} - \frac{\partial^2 P_2^1}{\partial y^2} - 2\frac{\partial^2 P_6^1}{\partial x \partial y} + \frac{\partial Q_4^1}{\partial y} + \frac{\partial K_4^1}{\partial y} + \frac{\partial Q_5^1}{\partial x} \\
&\quad + \frac{\partial K_5^1}{\partial x} - R_3^1 + g(z)\bar{N}^1 - g(z) f_e^1 = 0
\end{aligned} \tag{24}$$

With

$$\bar{N}^1 = \left[\begin{array}{c} N_x^0 \dfrac{\partial^2 (w_b^1 + g(z) w_s^1)}{\partial x^2} + 2N_{xy}^0 \dfrac{\partial^2 (w_b^1 + g(z) w_s^1)}{\partial x \partial y} \\ + N_y^0 \dfrac{\partial^2 (w_b^1 + g(z) w_s^1)}{\partial y^2} \end{array}\right] \tag{25}$$

The terms N_x^0, N_y^0 and N_{xy}^0 are the pre-buckling force resultants obtained as

$$N_x^0 = N_y^0 = -\int_{-h/2}^{h/2} \frac{\alpha_t(z) E(z) T}{1-\nu} dz, \quad N_{xy}^0 = 0 \tag{26}$$

Using Eq. (15) in Eq. (20), the stress resultants of the plate can be related to the total strains by

$$\begin{aligned}
&N_i^1 = A_{ij}\varepsilon_j^0 + B_{ij}k_j + C_{ij}\eta_j + F_{ij}\varepsilon_z^0 - N_i^T, && (i = 1, 2, 6) \\
&M_i^1 = B_{ij}\varepsilon_j^0 + G_{ij}k_j + H_{ij}\eta_j + K_{ij}'\varepsilon_z^0 - M_i^T, && (i = 1, 2, 6) \\
&P_i^1 = C_{ij}\varepsilon_j^0 + H_{ij}k_j + L_{ij}\eta_j + O_{ij}\varepsilon_z^0, - P_i^T && (i = 1, 2, 6) \\
&Q_i^1 = Q_{ij}'\gamma_j^0 + P_{ij}'\gamma_j^0, && (i = 4, 5) \\
&K_i^1 = Q_{ij}'\gamma_j^0 + S_{ij}\gamma_j^0, && (i = 4, 5) \\
&R_i^1 = F_{ij}\varepsilon_j^0 + K_{ij}'k_j + O_{ij}\eta_j + U_{ij}\varepsilon_z^0 - R_i^T, && (i = 3)
\end{aligned} \tag{27}$$

where

$$\begin{aligned}
&\left(A_{ij}, B_{ij}, C_{ij}, D_{ij}, E_{ij}, F_{ij}\right) = \int_{-h/2}^{h/2} Q_{ij}\left(1, z, f(z), g(z), f'(z), g'(z)\right) dz \\
&\left(G_{ij}, H_{ij}, K_{ij}', L_{ij}\right) = \int_{-h/2}^{h/2} Q_{ij}\left(z^2, zf(z), zg'(z), f^2(z)\right) dz \\
&\left(O_{ij}, P_{ij}', Q_{ij}', S_{ij}\right) = \int_{-h/2}^{h/2} Q_{ij}\left(f(z)g'(z), g^2(z), g(z)f'(z), f'^2(z)\right) dz \\
&U_{ij} = \int_{-h/2}^{h/2} Q_{ij}\left(g'^2(z)\right) dz
\end{aligned} \tag{28}$$

The stress and moment resultants, $N_1^T = N_2^T$, $M_1^T = M_2^T$, $P_1^T = P_2^T$ *and* R_3^T due to thermal loading are defined by

$$\begin{Bmatrix} N_1^T \\ M_1^T \\ P_1^T \\ R_3^T \end{Bmatrix} = \int_{-h/2}^{h/2} \frac{E(z)}{1-\nu^2}(1+2\nu)\alpha_t(z)T \begin{Bmatrix} 1 \\ z \\ f(z) \\ g'(z) \end{Bmatrix} dz, \tag{29}$$

6. Solution procedure

For the analytical solution of Eqs. (24), the Navier method is used under the specified boundary conditions. The displacement functions are selected as the following Fourier series

$$\begin{Bmatrix} u_0^1(x,y) \\ v_0^1(x,y) \\ w_b^1(x,y) \\ w_s^1(x,y) \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U^1 \cos(\alpha x)\sin(\beta y) \\ V^1 \sin(\alpha x)\cos(\beta y) \\ W_b^1 \sin(\alpha x)\sin(\beta y) \\ W_s^1 \sin(\alpha x)\sin(\beta y) \end{Bmatrix} \tag{30}$$

Where U^1, V^1, W_b^1 and W_s^1 are arbitrary parameters to be determined and $\alpha=m\pi/a$, $\beta=n\pi/b$.

Substituting Eq. (30) into Eq. (24), one obtains

$$[K]\{\Delta\} = 0 \tag{31}$$

Where $\{\Delta\}=\{U^1, V^1, W_b^1, W_s^1\}^t$ and *[K]* is the symmetric matrix given by

$$[K] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{12} & a_{22} & a_{23} & a_{24} \\ a_{13} & a_{23} & a_{33} & a_{34} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{bmatrix} \tag{32}$$

In which

$$\begin{aligned}
a_{11} &= \alpha^2 A_{11} + \beta^2 A_{66} \\
a_{12} &= \alpha\beta\left(A_{12} + A_{66}\right) \\
a_{13} &= -\left(\alpha^3 B_{11} + \alpha\beta^2\left(B_{12} + 2B_{66}\right)\right) \\
a_{14} &= \alpha^3 C_{11} + \alpha\beta^2\left(C_{12} + 2C_{66}\right) - \alpha F_{13} \\
a_{22} &= \beta^2 A_{22} + \alpha^2 A_{66} \\
a_{23} &= -\left(\beta^3 B_{22} + \alpha^2\beta\left(B_{12} + 2B_{66}\right)\right) \\
a_{24} &= \beta^3 C_{22} + \alpha^2\beta\left(C_{12} + 2C_{66}\right) - \beta F_{23} \\
a_{33} &= \alpha^4 G_{11} + \beta^4 G_{22} + 2\alpha^2\beta^2\left(G_{12} + 2G_{66}\right) + k_w \\
&\quad + k_p(\alpha^2 + \beta^2) + N_x^0\alpha^2 + N_y^0\beta^2 \\
a_{34} &= -\alpha^4 H_{11} - \beta^4 H_{22} - 2\alpha^2\beta^2\left(H_{12} + 2H_{66}\right) \\
&\quad + \alpha^2 K_{13}' + \beta^2 K_{23}' + k_w g(-h/2) \\
&\quad + k_p g(-h/2)(\alpha^2 + \beta^2) + \left(N_x^0\alpha^2 + N_y^0\beta^2\right) \\
a_{44} &= \alpha^4 L_{11} + \beta^4 L_{22} + 2\alpha^2\beta^2\left(L_{12} + 2L_{66}\right) + \beta^2\left(Q_{44}' + P_{44}'\right) \\
&\quad + \alpha^2\left(Q_{55}' + P_{55}'\right) + \beta^2\left(S_{44} + Q_{44}'\right) + \alpha^2\left(S_{55} + Q_{55}'\right) + U_{33} \\
&\quad - 2\alpha^2 O_{13} - 2\beta^2 O_{23} + k_w g(-h/2)^2 + k_p g(-h/2)^2(\alpha^2 + \beta^2) \\
&\quad + \left(N_x^0\alpha^2 + N_y^0\beta^2\right)
\end{aligned} \tag{33}$$

6.1 Thermal buckling solution

In the following, the solution of the equation $|K| = 0$ for different types of thermal loading conditions is presented. The temperature change varies only through the thickness.

6.1.1 Buckling of FG plates under uniform temperature rise

The plate initial temperature is assumed to be T_i. The temperature is uniformly raised to a final value T_f in which the plate buckles. The temperature change is $\Delta T = T_f - T_i$.

6.1.2 Buckling of FG plates subjected to a graded temperature change across the thickness

For FG plates, the temperature change is not uniform in this case. The temperature is assumed to be varied according to the power law variation through-the-thickness as follows

$$T(z) = \Delta T\left(\frac{z}{h} + \frac{1}{2}\right)^{\eta} + T_m \tag{34}$$

where the buckling temperature difference $\Delta T = T_c - T_m$ where T_c and T_m are the temperature of the top surface which is ceramic-rich and the bottom surface which is metal-rich, respectively. η is the temperature exponent ($0 < \eta < \infty$). Note that the value of η equal to unity represents a linear temperature change across the thickness. While the value of η excluding unity represents a non-linear temperature change through-the-thickness.

The following dimensionless expressions of Winkler's and Pasternak's elastic foundation parameters, as well as the critical buckling temperature difference are used in the present analysis

$$k_w = \frac{a^4}{D}K_W,\ k_p = \frac{a^2}{D}K_P,\ T_{cr} = 10^{-3}\Delta T_{cr}$$

Where $D = E_c h^3 / \left[12\left(1-\nu^2\right)\right]$

7. Numerical results and discussion

In this section, thermal buckling of porous FG plate resting on elastic foundation is investigated based on a quasi 3D solution.

The FGM plate is taken to be made of Aluminium and Alumina with the following material properties

- Metal (Aluminium): $E_M = 70\times10^9$ N/m^2; $\nu = 0.3$; $\alpha_M = 23{,}40\,.10^{-6}\ /\ K$.
- Ceramic (Alumina): $E_C = 380\times10^9$; N/m^2; $\nu = 0.3$; $\alpha_c = 7{,}410\,.10^{-6}\ /\ K$.

For First, numerical tests are performed to confirm the

Table 1 Critical buckling temperature of a square FG plate without elastic foundation (a/h = 10, a=b) under different temperature loads (η=3)

p	Theory	Uniform	Linear	Non linear
0	Tebboun *et al.* (2015) 2D	1,6188	3,2276	6,4553
	Van do and Chin-Hyung (2018) 3D	1,6091	3,2082	/
	Present 3D	1,6049	3,1999	6,3997
1	Tebboun *et al.* (2015) 2D	0,7585	1,4131	2,8270
	Van do and Chin-Hyung (2018) 3D	0,7548	1,4062	/
	Present 3D	0,7529	1,4027	2,8062
5	Tebboun *et al.* (2015) 2D	0,6790	1,1601	2,0152
	Van do and Chin-Hyung (2018) 3D	0,6743	1,1520	/
	Present 3D	0,6885	1,1765	2,0438
10	Tebboun *et al.* (2015) 2D	0,6925	1,2184	2,0972
	Van do and Chin-Hyung (2018) 3D	0,6871	1,2088	/
	Present 3D	0,7039	1,2385	2,1319

accuracy of the proposed model. For verification purpose, the obtained results are compared with those of the 2D solution of Tebboun *et al*. (2015) and the 3D model of Van do and Chin-Hyung (2018).

Tables 1-3 present comparison of the critical buckling temperature for different temperature load and different case of elastic foundation.

In Table 1, a comparative study is carried out between the results obtained by the present quasi 3D solution and those reported by Tebboun *et al.* (2015) and Van do and Chin-Hyung (2018). Results are presented for perfect square FG plate under three cases of temperature rise across the thickness.

The results of the present theory show very good agreement with the two mentioned solutions for both uniform and linear temperature load. For the case of the non linear temperature load, there is a slight difference. This is because the Tebboun *et al.* (2015) solution uses a 2D model that neglects the stretching effect. The latter has a significant effect in the case of thick plates.

Another comparison between the results of this method and those of Tebboun *et al.* (2015) is shown in Table 2. The results are presented for the two cases of Winkler and Pasternak elastic foundations.

It is found that the results are in very good agreement for both cases of thermal loading uniform and linear. The difference between the results increases slightly for the case of non-linear loading especially with the presence of Pasternak foundation. This is due to the quasi-3D solution elaborated in the present study, contrary to Tebboun *et al.* (2015) where the thickness stretching effect is neglected.

To illustrate the accuracy of present theory for the case of non linear thermal loading, critical buckling temperature of FG perfected plate on elastic foundation with different values of the power law index are presented in Table3. The comparison is made with the results of Van do and Chin-Hyung (2018). It can be seen that the results are almost identical.

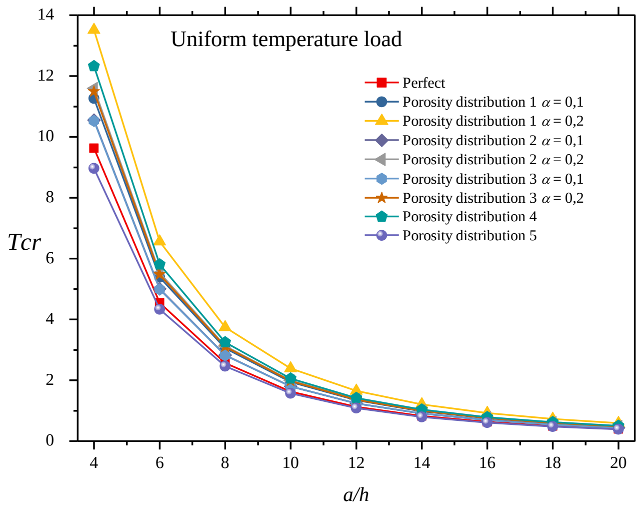
As can be seen, our results are in good agreement with the published ones, and it can be concluded that the present theory is accurate for the prediction of critical thermal buckling loads.

Figure 2 shows the effects of the ratio *a/h* on the critical buckling temperature T_{cr} of a porous FG plates under a uniform, linear and non-linear temperature loads. Five different cases of porosity distribution are studied.

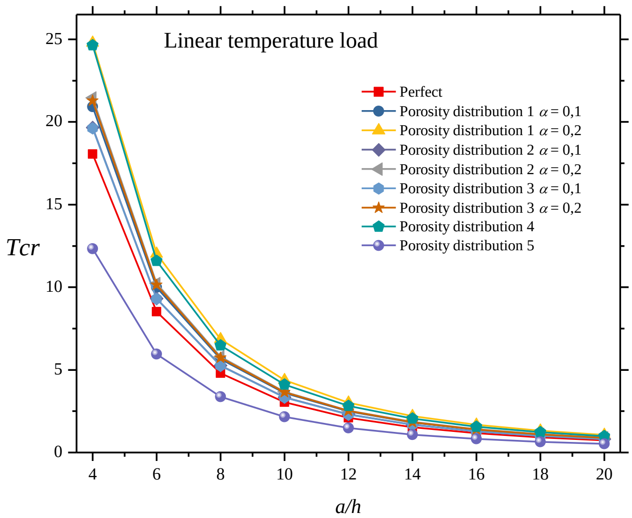
It is observed that, with increasing the plate ratio *a/h*, the critical buckling temperature decreases regardless of the distribution of porosity and the thermal loading. From these figures, it is found that the critical buckling temperature max are obtained with distribution 1 (α = 0.2) and those min with distribution 5. Also, we can see that there is not a big difference between the values of the critical loads obtained by the different distributions and for the different modes of loading (uniform-linear-nonlinear). The exception is made for distributions 1 (α = 0.2) and 5 where the difference increases with the variation of thermal loading.

In Figure 3, the influence of aspect ratio (*b/a*) and on the critical buckling temperature of a porous FGM plate is shown for different porosity distribution. The plate is subjected to three thermal loading through its thickness, uniform, linear and non-linear. It is observed from this figure that the critical buckling temperature decreases with increases in aspect ratio and this whatever the thermal loading and the porosity distribution. The same ascertainment noted above remains valid. Namely that all the distributions give more or less close values with the exception of the distribution 1 (α = 0.2) and 5 where a consequent difference is observed.

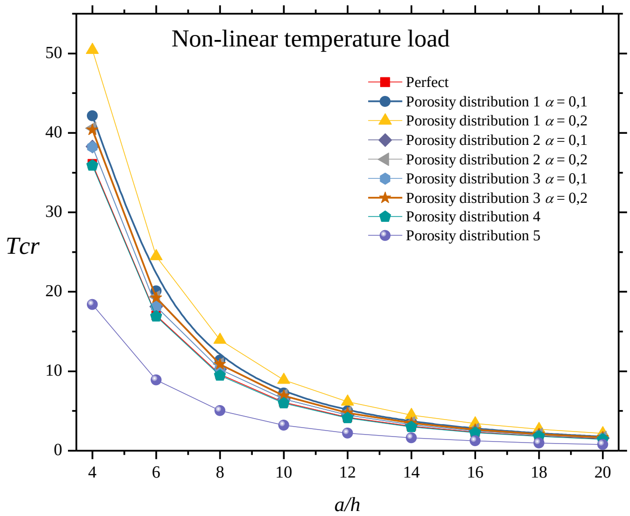
Figure 4 represent the effect of porosity distribution on the critical buckling temperature versus the power law index for a porous FG plate resting on Pasternak foundation. It is observed that the critical buckling temperature increases as the power law index increase for all the porosity distribution considered here.



(a)

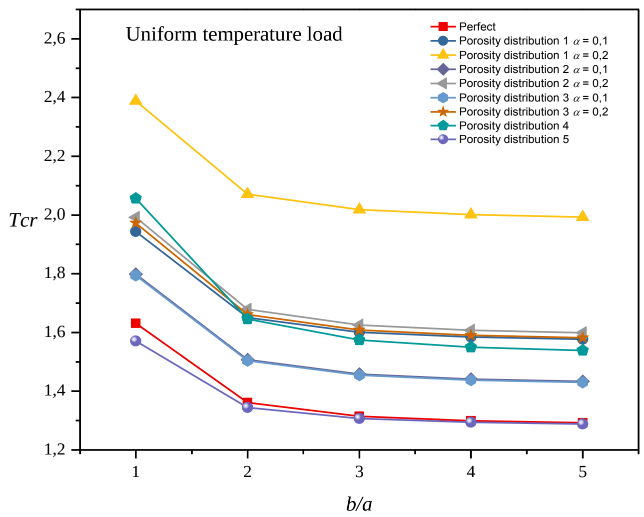


(b)

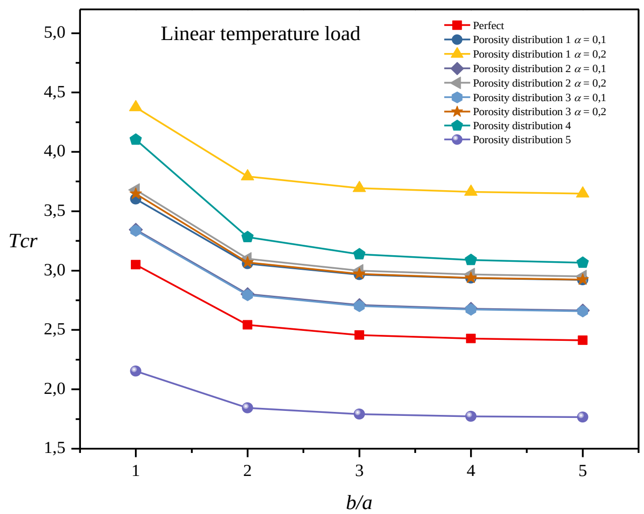


(c)

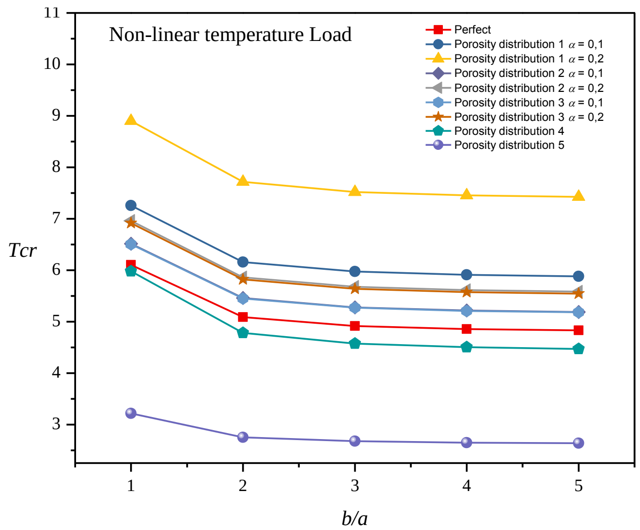
Fig. 2 Critical buckling temperature T_{cr} versus the thickness ratio a/h (a) Uniform temperature; (b) Linear temperature; (c) Non-linear temperature (η=3) (p = 1, a = b, k_w = k_p = 10)



(a)



(b)



(c)

Fig. 3 Critical buckling temperature T_{cr} versus the ratio b/a (a) Uniform temperature; (b) Linear temperature; (c) Non-linear temperature (η=3) (p = 1, a/h = 10, k_w = k_p = 10)

Table 2 Critical buckling temperature of a square FG plate (a/h = 10, a=b) on elastic foundation under different temperature loads

p	Theory	K_w=10 K_p=0			K_w=10 K_p=10		
		Uniform	Linear	Non linear	Uniform	Linear	Non linear
0	Tebboun *et al.* (2015) 2D	1,6627	3,3154	6,6308	2,5290	5,0479	10,0958
	Present 3D	1,6512	3,2925	6,6585	2,5596	5,1091	10,2183
1	Tebboun *et al.* (2015) 2D	0,7994	1,4898	2,9804	1,6067	3,0040	6,0098
	Present 3D	0,7957	1,4830	2,9668	1,6315	3,0505	6,1027
5	Tebboun *et al.* (2015) 2D	0,7356	1,2577	2,1847	1,8547	3,1839	5,5309
	Present 3D	0,7479	1,2787	2,2213	1,8997	3,2613	5,6653
10	Tebboun *et al.* (2015) 2D	0,7565	1,3318	2,2924	2,0196	3,5699	6,1449
	Present 3D	0,7712	1,3578	2,3372	2,0723	3,6635	6,3059

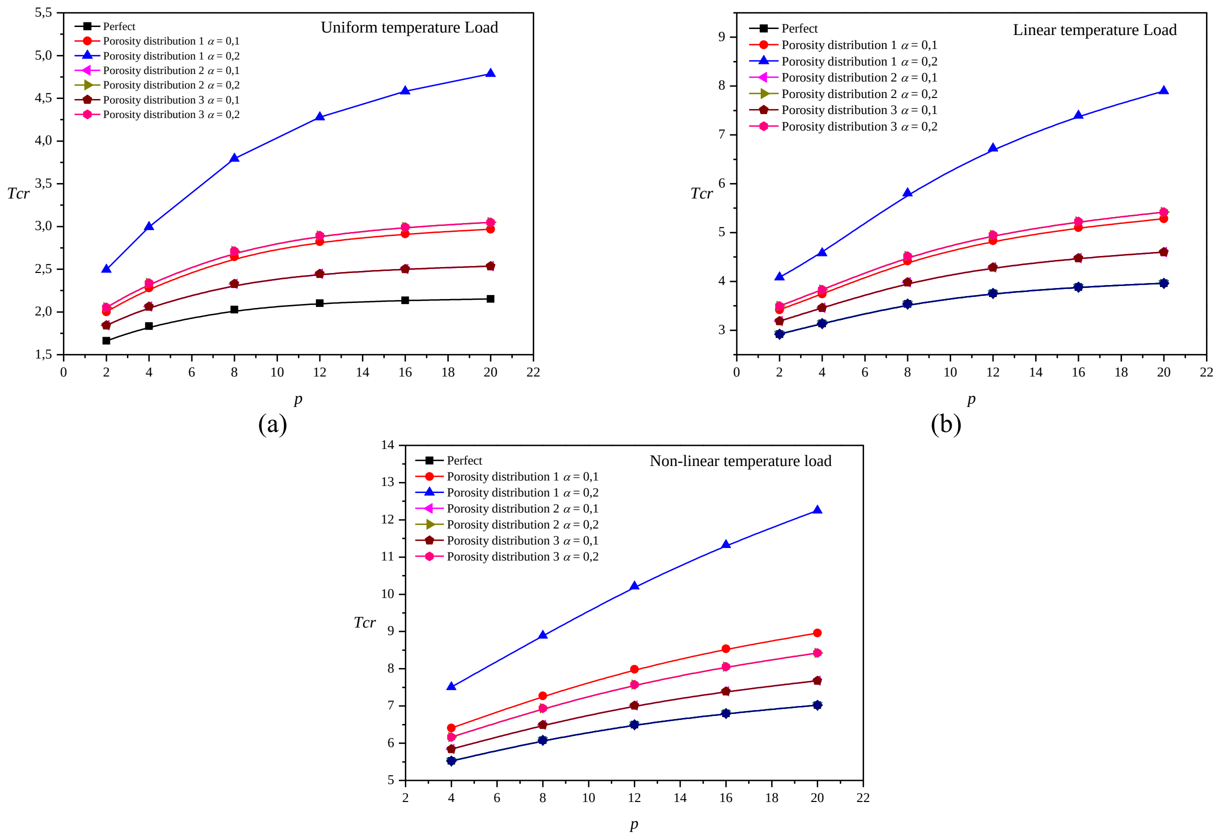


Fig. 4 Critical buckling temperature T_{cr} versus the power law index "p" (a) Uniform temperature; (b) Linear temperature; (c) Non-linear temperature (η=3) (a/h = 10, a=b, k_w = k_p = 10)

For the three figures above, we can say that the presence of an elastic foundation including that of Pasternak modified the overall behavior of the plate. If the plate wasn't supported by a foundation or even supported by an elastic foundation but of Winkler type, the results would have been reversed. That is, the perfect plate gives the high values. Indeed, when the plate rests on a Pasternak foundation the stiffness of the system increases result of the incorporation of a shear layer leading to an increase in flexural stiffness. This same case has been observed in the work of Shahsavari *et al.* (2018) in particular in Table 5 where the fundamental frequency for imperfect plates is greater than that of the perfect plates for the case of the Pasternak foundation.

Concerning the difference between the values of the critical temperature between the imperfect plates, this can be related to the way of microvoids concentration within FG plate.

8. Conclusion

In this paper, we have developed a new refined quasi-three-dimensional (3D) shear deformation theory for the solutions of static bending of FG plate.

Thermal buckling of simply supported FG porous plates under different types of thermal loading (uniform, linear and non linear distribution through the thickness) and

Table 3 Critical buckling temperature T_{cr} of a square FG plate under the nonlinear temperature rise (a/h=10, a=b).

p	Theory	η=2	η=5	η=10
0	Van do and Chin-Hyung (2018) 3D	4,812	9,625	17,645
	Present 3D	4,800	9,600	17,599
1	Van do and Chin-Hyung (2018) 3D	2,096	4,297	8,151
	Present 3D	2,091	4,287	8,130
5	Van do and Chin-Hyung (2018) 3D	1,584	2,829	4,964
	Present 3D	1,618	2,889	5,070
10	Van do and Chin-Hyung (2018) 3D	1,663	2,862	4,734
	Present 3D	1,704	2,932	4,851

resting on elastic foundation has been analyzed by using a quasi 3D shear deformation theory. All comparison studies show that the critical buckling temperature obtained by the proposed quasi 3D solution with four unknowns is in perfect agreement with other shear deformation theories.

From the results and comparisons between different porosity distributions, it has been found that the different distributions give values more or less close to with the exception of model 1($\alpha = 0.2$) and 5 where a difference is found.

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