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Flexural behaviour of steel beams reinforced by carbon fibre reinforced polymer: Experimental and numerical study

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Abstract. The paper presents the results of an experimental and numerical programme to characterize the behaviour of steel beams reinforcement by composite plates. Important failure mode of such plated beams is the debonding of the composite plates from the steel beam due to high level of stress concentration in the adhesive at the ends of the composite plate. In this new research, an experimental and numerical finite element study is presented to calculate the stresses in the sika carbodur and sika wrap reinforced steel beam under mechanical loading. The main objective of the experimental program was the evaluation of the force transfer mechanism, the increase of the load capacity of the steel beam and the flexural stiffness. It also validated different analytical and numerical models for the analysis of sika carbodur and sika wrap reinforced steel beams. In particular, a finite element model validated with respect to the experimental data and in relation to the analytical approach is presented. Experimental and numerical results from the present analysis are presented in order to show the advantages of the present solution over existing ones and to reconcile debonding stresses with strengthening quality.

Keywords: numerical analysis; experimental study; steel beam; interfacial stresses; strengthening; composite plate

1. Introduction

Many countries around the world have acquired over the years a considerable built heritage. Whether steel structures, bridges, marine platforms, silos and tanks, these structures age and become damaged. This heritage, whether in the world or in Algeria represents one of the most expensive assets of a country and these works require to be rehabilitated. In Algeria, rehabilitation in most cases only concerned buildings damaged during earthquakes. Nevertheless, some initiatives to rehabilitate existing undamaged structures have been reinforced by the sika carbodur and sika wrap process. In addition, the lifetime of a structure is determined by the maximum permissible reduction of the chosen performance. Below this value, stability, safety or functional performance are no longer sufficient and therefore action is urgently needed. Exceptional solicitations can lead to immediate ruin of the structure or shorten its life. The service life can be increased by repairing the construction after a certain period of use. Indeed, these works will reach their useful life in the near future, hence the need to find economic solutions to rehabilitate them effectively. Reinforcement can be advantageous in the face of a new build for a comparable level of performance. Strengthening steel structures is an advantageous way of extending their life. Several reinforcement techniques are available on the market, including the one of external reinforcement by composite materials. Several researchers (Tounsi 2006, Tahar 2016d, Benachour 2008, Smith 2001, Shen 2001, Roberts 1989, Yuan 2019, Rabahi 2016, Adim 2016, Ameur 2008, Hassaine Daouadji 2016b, Panjehpour 2016, Ashraful 2018, Attia 2018, Sallai 2015, Rabia 2016b, Mazari 2015, Tlidji 2014, Al saidy 2004, Benyoucef 2006, Zidour 2014, Benachour 2001, Lazreg 2016a, Hadji 2016b, Hassaine Daouadji 2016a, Benhenni 2018b, Benferhat 2016, Tayeb 2019, Belkacem 2016c, Bellifa 2017, Tahar 2016c, Panjehpour 2014a, Benhenni 2018a, Adim 2016a, and Bouhadra 2018) have indeed been interested in these new materials because they have interesting characteristics to address the growing problems of structural deficiency and environmental degradation of national and global infrastructure.

The premature damage of the steel elements of structures requires a strengthening or a repairing. Among the existing techniques of reinforcement, the composite materials recently developed and are being increasingly used as alternatives for conventional materials primarily because of their high strength, specific stiffness, light weight and adjustable properties (Hassaine Daouadji 2016c, Rabahi 2018, Bouakaz 2014, Bensatallah 2018, Rabia 2016a, Panjehpour 2011, Tahar 2016a, Chaded 2018, El Mahi 2014, Tayeb 2018a, Lazreg 2016b, Adim 2016c, Belkacem 2016a, Hadji 2016a, Abdelhak 2016, Benhenni 2019b, Hassaine Daouadji 2013, Jones 1988, Benferhat

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2018 and Khalifa 2018). However, a significant problem is associated in the flexural strengthening of the steel beams: debonding between the fiber reinforced polymer FRP plate and steel beam. As matter of fact, since the reinforcement by composite materials was implemented, many tests laboratory allowed to conclude that the delamination of the reinforcement plate is the most frequent failure mode due to a high stresses concentration at the end of the reinforcement plate (Tounsi 2007, Panjehpour 2014c, Belkacem 2018, Zidani 2015, Hassaine Daouadji 2017, Adim 2016b, Tayeb 2018b, Bensatallah 2016, Panjehpour 2014b, Benhenni 2019a, Tahar 20016b, Rabahi2019, Benferhat 2019, Zohra 2016, Abderezak 2018, Tounsi 2009, Belkacem 2016b, Bouakaz 2014 and Chaded 2018).

This work is part of the logical continuation of our research work. It aims to analyze the mechanical behavior of a steel beam - FRP plate (sika carbodur or sika wrap) assembled by the bonding technique. We thus study the analytical and numerical modeling of the interface stresses (delamination) in the steel beam simply supported under mechanical loading in bending, reinforced by a FRP plate. The present method is based on the improved theoretical solution to predict the interfacial stresses developed by Tounsi (2006), by taking of account the adherend shear deformations and by assuming a parabolic shear stress through the thickness of both the steel beam and bonded plate. The results of our present model were validated by comparison with the results of interlaminar stresses resulting from the literature.

2. Analytical approach

2.1 Assumptions

The present analysis takes into consideration the transverse shear stress and strain in the beam and the plate but ignores the transverse normal stress in them. One of the analytical approach proposed by Hassaine Daouadji (2016d) for steel beam strengthened with a bonded composite plate (Fig 1 and 2) was used in order to compare it with a finite element analysis.

The analytical approach is based on the following assumptions (Hassaine Daouadji 2016d, Tahar 2018)

- Elastic stress strain relationship for steel beam, composite plate and adhesive;
- There is a perfect bond between the composite plate and the steel beam;
- The adhesive is assumed to only play a role in transferring the stresses from the steel beam to the composite plate reinforcement;
- The stresses in the adhesive layer do not change through the direction of the thickness;

Since the composite laminate is an orthotropic material, its material properties vary from layer to layer. In theoretical study (Tahar 2017, Hassaine Daouadji 2016d), the laminate theory is used to determine the stress and strain behaviours of the externally bonded composite plate in order to investigate the whole mechanical performance of the composite – strengthened structure. The laminate theory is used to estimate the strain of the symmetrical composite plate.

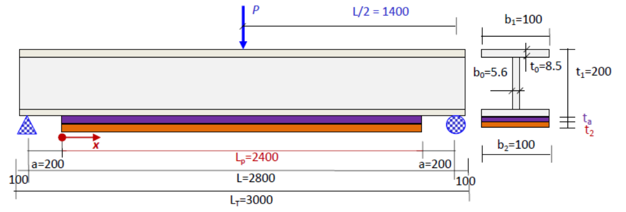

Fig. 1 steel beam strengthened with a bonded composite plate loaded with a concentrated load P at mid-span

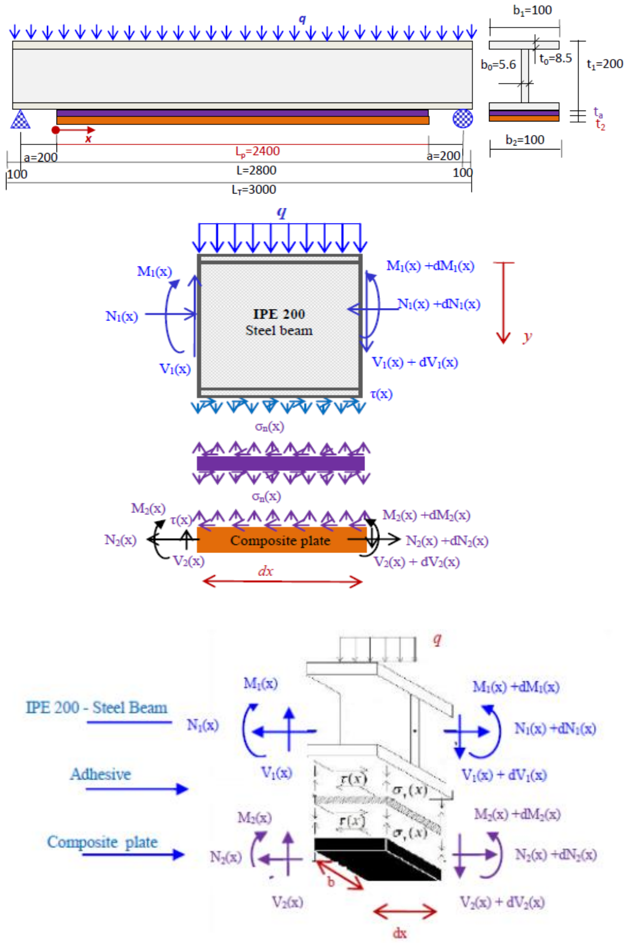

Fig. 2 Steel beam simply supported reinforced with composite plate bonded and loaded by a uniformly distributed load

2.2 Shear stress distribution along the FRP – Steel interface

The governing differential equation for the interfacial shear stress is expressed as (Hassaine Daouadji 2016d)

$$\frac{d^2\tau(x)}{dx^2} - \frac{\left(A'_{11} + \frac{b_2}{E_1A_1} + \frac{\left(\frac{t_1+t_2}{2}\right)\left(\frac{t_1+t_2}{2}+t_a\right)}{E_1I_1D'_{11}+b_2}b_2D'_{11}\right)}{\frac{t_a}{G_a}+\frac{t_1}{3G_1}\eta}\tau(x) + \frac{\left(\frac{t_1+t_2}{E_1I_1D'_{11}+b_2}D'_{11}\right)}{2\left(\frac{t_a}{G_a}+\frac{t_1}{3G_1}\eta\right)}V_T(x)=0 \qquad (1)$$

Where η is a geometrical coefficient which is given as

$$\eta=\frac{1}{2A_1t_1^2}\left[b_1\left(-t_0^3+6t_0t_1^2-t_1^3+(t_1-t_0)^3\right)+b_0\left(3t_1^2(t_1-2t_0)-(t_1-t_0)^3+t_0^3\right)\right] \quad (2)$$

For a rectangular section ($b_1= b_0$), η =1 which corresponds to the same expression given by Hassaine Daouadji (2016d). However, for I-beam section we have η <1. For simplicity, the general solutions presented below are limited to loading which is either concentrated or uniformly distributed over part or the whole span of the beam, or both (Fig 2). For such loading, $d^2V_T(x)/dx^2 = 0$, and the general solution to Eq.1 is given by

$$\tau(x)=\Delta_1\cosh(\delta x)+\Delta_2\sinh(\delta x)+\frac{1}{2\delta^2\left(\frac{t_a}{G_a}+\frac{t_1}{3G_1}\eta\right)}\left(\frac{t_1+\frac{t_2}{2}}{E_1I_1D'_{11}+b_2}D'_{11}\right)V_T(x) \quad (3)$$

were δ is given as:

$$\delta=\sqrt{\frac{1}{\frac{t_a}{G_a}+\frac{t_1}{3G_1}\eta}\left(A'_{11}+\frac{b_2}{E_1A_1}+\frac{\left(\frac{t_1+t_2}{2}\right)\left(\frac{t_1+t_2}{2}+t_a\right)}{E_1I_1D'_{11}+b_2}b_2D'_{11}\right)} \quad (4)$$

and Δ_1 and Δ_2 are constant coefficients determined from the boundary conditions. In the present study, a simply supported beam has been investigated which is subjected to a uniformly distributed load (Fig. 1). The interfacial shear stress for this uniformly distributed load at any point is written as (Hassaine Daouadji 2016d)

$$\tau(x)=\left[\frac{t_1a}{2E_1I_1\left(\frac{t_a}{G_a}+\frac{t_1}{3G_1}\eta\right)}(L-a)-\frac{\left(\frac{t_1+t_2}{E_1I_1D'_{11}+b_2}D'_{11}\right)}{2\delta^2}\right]\frac{qe^{-\delta x}}{\delta}+\frac{t_1+t_2}{2\delta^2(E_1I_1D'_{11}+b_2)}D'_{11}q\left(\frac{L}{2}-a-x\right) \quad (5)$$

$$0\le x\le L_P$$

Where q is the uniformly distributed load and x; a; L and L_p are defined in Fig. 2.

A simply supported beam was investigated which is subjected to a concentrated load P at mid-span as shown in Fig.2. The interfacial shear stress for this load case at any point is written as:

The concentrated load P is in the reinforced zone

$$\tau(x)=\frac{\frac{t_1}{\frac{t_a}{G_a}+\eta\frac{t_1}{3G_1}}}{2E_1I_1\delta}Pae^{-\delta x}+\frac{\frac{1}{\frac{t_a}{G_a}+\eta\frac{t_1}{3G_1}}}{2\delta^2}\left(\frac{t_1+t_2}{E_1I_1D'_{11}+b_2}D'_{11}\right)P\cosh(\delta x)e^{-\delta(\frac{l}{2}-a)} \quad (6)$$

The concentrated load P is positioned in the not reinforced zone

$$\tau(x)=\frac{\frac{t_1}{\frac{t_a}{G_a}+\eta\frac{t_1}{3G_1}}}{2E_1I_1\delta}Pbe^{-\delta x} \quad (7)$$

2.3 Normal stress distribution along the FRP – Steel interface

<u>Case 1:</u> *Steel beam strengthened with a bonded composite plate under evenly distributed load*

The following governing differential equation for the interfacial normal stress for this uniformly distributed load (Fig 2) (Hassaine Daouadji 2016d)

$$\frac{d^4\sigma_n(x)}{dx^4}+K_n\left(D'_{11}+\frac{b_2}{E_1I_1}\right)\sigma_n(x)-K_n\left(D'_{11}\frac{t_2}{2}-\frac{y_1b_2}{E_1I_1}\right)\frac{d\tau(x)}{dx}+\frac{qK_n}{E_1I_1}=0 \quad (8)$$

The general solution to this fourth–order differential equation is

$$\sigma_n(x)=e^{-\varphi x}\left[\Delta_3\cos(\varphi x)+\Delta_4\sin(\varphi x)\right]+e^{\varphi x}\left[\Delta_5\cos(\varphi x)+\Delta_6\sin(\varphi x)\right]-\left(\frac{y_1b_2-\frac{D'_{11}E_1I_1t_2}{2}}{D'_{11}E_1I_1+b_2}\right)\frac{d\tau(x)}{dx}-\frac{1}{D'_{11}E_1I_1+b_2}q \quad (9)$$

For large values of x it is assumed that the normal stress approaches zero and, as a result, $\Delta_5= \Delta_6= 0$. The general solution therefore becomes

$$\sigma_n(x)=e^{-\phi x}\left[\Delta_3\cos(\phi x)+\Delta_4\sin(\phi x)\right]-\left(\frac{y_1b_2-\frac{D'_{11}E_1I_1t_2}{2}}{D'_{11}E_1I_1+b_2}\right)\frac{d\tau(x)}{dx}-\frac{1}{D'_{11}E_1I_1+b_2}q \quad (10)$$

Where

$$\phi=\sqrt[4]{\frac{K_n}{4}\left(D'_{11}+\frac{b_2}{E_1I_1}\right)} \quad (11)$$

As is described by Hassaine Daouadji (2016d), the constants Δ_3 and Δ_4 in Eq.9 are determined using the appropriate boundary conditions and they are written as follows

$$\Delta_3=\frac{K_n\left[V_T(0)+\sqrt[4]{\frac{K_n}{4}(D'_{11}+\frac{b_2}{E_1I_1})}M_T(0)\right]}{2\left[\sqrt[4]{\frac{K_n}{4}(D'_{11}+\frac{b_2}{E_1I_1})}\right]^3E_1I_1}-\frac{b_2K_n\left(\frac{y_1}{E_1I_1}-\frac{D'_{11}t_2}{2b_2}\right)}{2\left[\sqrt[4]{\frac{K_n}{4}(D'_{11}+\frac{b_2}{E_1I_1})}\right]^3}\tau(0)+\frac{r_1\left[\frac{d^4\tau(0)}{dx^4}+\sqrt[4]{\frac{K_n}{4}(D'_{11}+\frac{b_2}{E_1I_1})}\frac{d^3\tau(0)}{dx^3}\right]}{2\left[\sqrt[4]{\frac{K_n}{4}(D'_{11}+\frac{b_2}{E_1I_1})}\right]^3} \quad (12)$$

$$\Delta_4=-\frac{K_n}{2\sqrt{\frac{K_n}{4}(D'_{11}+\frac{b_2}{E_1I_1})}.E_1I_1}M_T(0)-\frac{\frac{y_1b_2-\frac{D'_{11}E_1I_1t_2}{2}}{D'_{11}E_1I_1+b_2}}{2\sqrt{\frac{K_n}{4}(D'_{11}+\frac{b_2}{E_1I_1})}}\frac{d^3\tau(0)}{dx^3} \quad (13)$$

The above expressions for the constants Δ_3 and Δ_4 has been left in terms of the bending moment $M_T(0)$ and shear force $V_T(0)$ at the end of the soffit plate. With the constants Δ_3 and Δ_4 determined, the interfacial normal stress can then be found using Eq.8.

<u>Case 2:</u> *Steel beam strengthened with a bonded composite plate loaded with a concentrated load P at mid-span*:

The following governing differential equation for the interfacial normal stress loaded with a concentrated load P at mid-span (Fig 1) (Hassaine Daouadji 2016d)

$$\frac{d^4\sigma_n(x)}{dx^4}+\frac{E_a}{t_a}\left(D_{11}^{'}+\frac{b_2}{E_1I_1}\right)\sigma_n(x)$$
$$-\frac{E_a}{t_a}\left(D_{11}^{'}y_2-\frac{t_1b_2}{2E_1I_1}\right)\frac{d\tau(x)}{dx}+\frac{2P}{L}\frac{E_a}{t_aE_1I_1}=0 \quad (14)$$

The general solution to this fourth – order differential equation is

$$\sigma_n(x)=e^{-\varphi x}\left[\Delta_7\cos(\varphi x)+\Delta_8\sin(\varphi x)\right]$$
$$+e^{\varphi x}\left[\Delta_9\cos(\varphi x)+\Delta_{10}\sin(\varphi x)\right] \quad (15)$$
$$-n_1\frac{d\tau(x)}{dx}-n_2\frac{2P}{L}$$

For large values of x it is assumed that the normal stress approaches zero, and as a result $\Delta_9=\Delta_{10}=0$. The general solution therefore becomes

$$\sigma_n(x)=e^{-\varphi x}\left[\Delta_7\cos(\varphi x)+\Delta_8\sin(\varphi x)\right]$$
$$-\left(\frac{t_1b_2-D_{11}^{'}E_1I_1t_2}{2(D_{11}^{'}E_1I_1+b_2)}\right)\frac{d\tau(x)}{dx}-\frac{2P}{L(D_{11}^{'}E_1I_1+b_2)} \quad (16)$$

The constants Δ_7 and Δ_8 in Eq. (16) are written as follow

$$\Delta_7=\frac{E_a}{2\phi^3t_aE_1I_1}\left[V_T(0)+\phi M_T(0)\right]-\frac{E_ab_2}{2t_a\phi^3}\left(\frac{t_1}{2E_1I_1}-\frac{D_{11}^{'}t_2}{2b_2}\right)\tau(0)$$
$$+\frac{t_1b_2-D_{11}^{'}E_1I_1t_2}{4\phi^3(D_{11}^{'}E_1I_1+b_2)}\left(\frac{d^4\tau(0)}{dx^4}+\phi\frac{d^3\tau(0)}{dx^3}\right) \quad (17)$$

$$\Delta_8=-\frac{E_a}{2\phi^2t_aE_1I_1}M_T(0)-\frac{\frac{t_1b_2-D_{11}^{'}E_1I_1t_2}{2(D_{11}^{'}E_1I_1+b_2)}}{2\phi^2}\frac{d^3\tau(0)}{dx^3} \quad (18)$$

The above expressions for the constants Δ_7 and Δ_8 have been left in terms of the bending moment $M_T(0)$ and shear force $V_T(0)$ at the end of the composite plate.

3. Numerical modeling

The general purpose finite element program Abaqus (2007) was used. A three dimensional finite element (3D FE) model was developed to account for the geometric and the nonlinear material behavior of steel composite beams investigated in the present work. Von Misses yield criterion was adopted in the nonlinear analysis. Eight-node brick element S4R was employed to model the steel beam, adhesive layer and composite plate as recommended previously by several researchers in the literature (Tounsi 2006, Hassaine Daouadji 2016). Different mesh sizes were used to test the convergence and to get the appropriate accuracy of the numerical solution as verified by the experimental results in the tolerable processing time. In comparison with laboratory tests which are highly time and cost demanding, the numerical simulation is cheaper, time-saving, not so dangerous and more information. As the computational power has intensely increased, numerical methods, in particular the finite element method (FEM), have also been resorted for analysis of many practical engineering problems. One of the advantages of FE calculation is that detailed distribution of the normal and shear stress along the interface can be produced. In these cases numerical methods must be adopted. In this work a finite element model was developed using the commercial code Abaqus to evaluate the stresses in the reinforced beams and in the adhesives. In order to reduce the computational effort, the beam was modeled (Fig. 3).

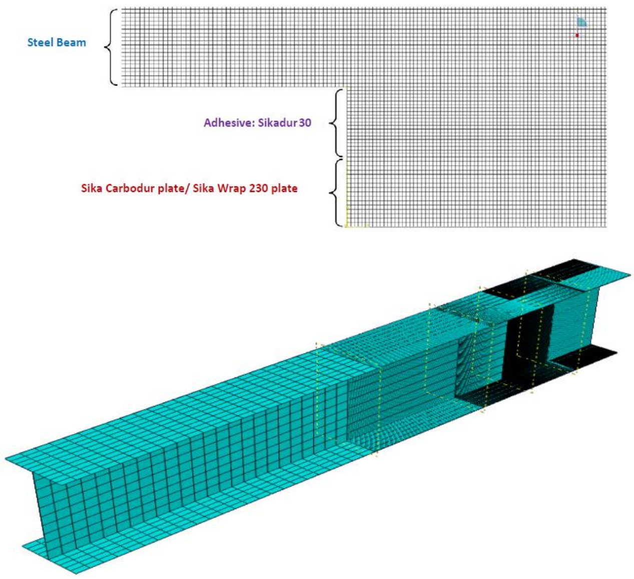

Fig. 3 FE mesh of Mesh of an "IPE" steel beam

The modeling process in Abaqus consists of defining the various components of the model individually i.e. the steel I-beam, CFRP plate and adhesive layer were defined as parts, each compatible with the other so as to provide a complete analysis. The modeling itself is an iterative process, in that it takes several analyses to be able to simulate a particular set of characteristics effectively. A 4-node linear quadrilateral, type S4R was established, in which only one half of the beam was considered because of symmetry geometry and loading of the beam (Fig. 3). All nodes at mid-span were restrained to produce the required symmetry, and nodes at the end of the steel I-beam were restrained to represent simply roll-supported conditions.

The finite element mesh was refined in correspondence of the reinforcement ends in order to capture the relevant stress concentration with a total of 27289 elements and 28234 nodes for steel I-beam and a total number of 5646 elements and 6223 nodes for each FRP plate and adhesive. The number of elements used depends largely on the geometric parameters such as the length and the cross-sectional perimeter. In order to obtain accurate stress results at the ends of the plate, a fine mesh was deployed in these areas, as shown in Fig. 3. The relevant geometrical and mechanical properties used in the finite element analysis were the same as that used in the analytical method. To simulate correctly the interaction behavior between the various components of the composite beams, a surface-to-surface contact interaction describes contact between two deformable surfaces. Element types and material properties were then specified and assigned to each corresponding part. In this work, the stresses have been obtained from the

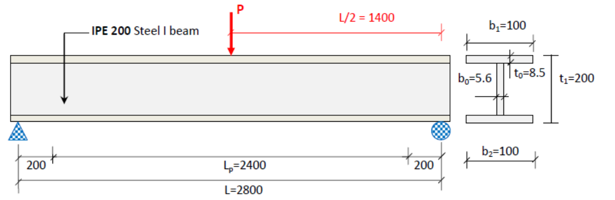

Fig. 4a Steel beam without reinforcement "Reference Beam"

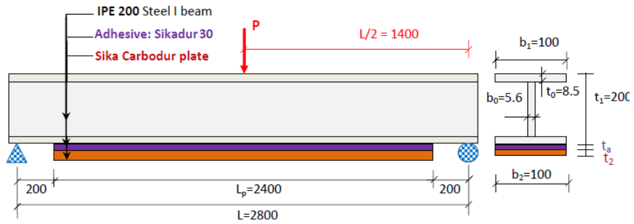

Fig. 4b Steel beam reinforced with a Sika carbodur plate

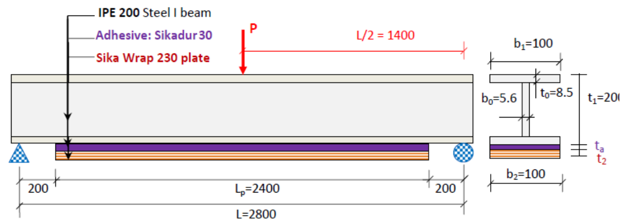

Fig. 4c Steel beam reinforced with four layers of Sika Wrap

Table 1 Property of the reinforcement material

Material property	Sika Carbodur	Sika wrap	Adhesive Sikadur 30
Young's modulus - (MPa)	≥ 165000	230000	12800
Tensile strength - (MPa)	≥ 2800	≥ 3650	-
Width - (mm)	100	100	100
Thickness - (mm)	1,2	0,48	2

average values of the stress in the bottom elements of the adhesive layer.

4. Experimental study

The effectiveness of the epoxy bonding of FRP plates on the reinforcement of steel girders was examined by testing two steel beams IPE 200 reinforced with composite plates. In addition, an identical beam without reinforcements was tested as a reference case (witness beam). The geometry of the beams and reinforcements is shown in Figs 4a, 4b and 4c. Four identical steel beams were reinforced; among them two beams were reinforced by a layer of Sika carbodur and the other two by four layers of Sika wrap using the same epoxy adhesives in this case Sikadur 30, whose characteristics of the materials mentioned are shown in Table 1.

The reinforced steel girders were instrumented using 3 mm strain gauges, which were mounted on the surface of the CFRP plates and steel girders. After a twenty days, the specimens were instrumented using 3 mm strain gauges with resistance 120 U that were mounted on the surface of

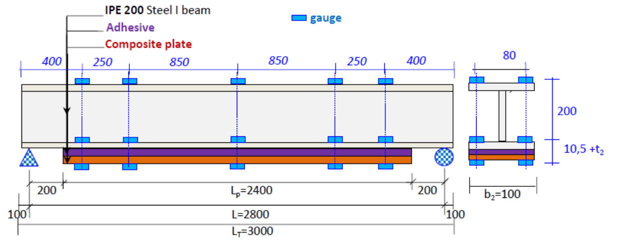

Fig. 5 Strains gages locations for the steel beams reinforced with composite plate.

the composite sheets and steel beams, the locations of strain gauges for samples are shown in Figs. 5.

The experimental setup is the three-point bending tests that were performed using a laboratory-level test frame, the loading was applied using a block of spherical seat support. The beams involved in the study were simply supported at both ends by a cylindrical rolling bearing. The monotonic load was applied using a hydraulic actuator, the load, the mid-range deviation and the deformations at different points were recorded with a data acquisition system.

5. Results and analysis of this study

5.1 Experimental and numerical comparison of the results

To validate the present model, a steel section IPE 200 is used here. One of the tested for the steel beams reinforced with composite plate (sika carbodur and sika wrap), is analysed here using the present improved solution. The beam is simply supported and subjected to three-point bending. The geometry and materials properties of the specimen are summarized in the table 1 and 2. As it can be seen from Fig 6, the predicted numerical results are in reasonable agreement with the experimental results. We can say that, the use of composite materials associated with glues on stretched surfaces is a very effective way to reinforce the structural beams, especially for undersized beams.

The bonding of the composite on tensioned surfaces increases the ultimate strength of the reinforced beams and by decreasing the deflection of the structures (Fig. 6), it also increases their stiffness. This phenomenon helps to reduce corrosion and improve the durability of reinforced structures.

5.2 Validation by comparison of the present analytical and numerical model with the results from the literature

One of advantages of FEM simulation is that the detailed distributions of the normal and shear stresses along the interfaces can be produced. A comparison of the interfacial shear and normal stresses from the different existing closed – form solutions and the present solution is undertaken in this section. A steel beams bonded with sika carbodur and sika wrap plate soffit plate is considered. The beam is simply supported and subjected to a uniformly

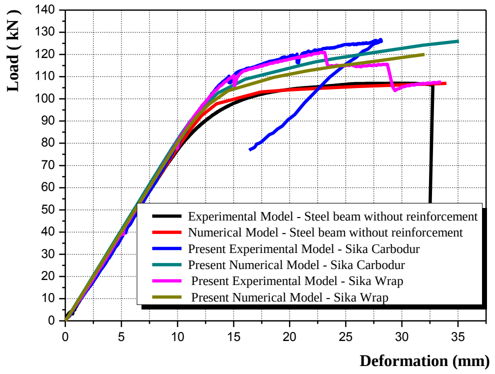

Fig. 6 Experimental and numerical comparison for the steel beams reinforced with composite plate

Table 2 Comparison with analytical approach of interfacial shear and normal stresses (MPa)

Steel Beam bonded with a (prestressed) thin plate subjected to a uniformly distributed load

Model	Prestressing load	Steel beam with Sika carbodur plate τ(x)	Steel beam with Sika carbodur plate σ(x)	Steel beam with four layers of Sika Wrap τ(x)	Steel beam with four layers of Sika Wrap σ(x)
Bouakaz 2014	P=0	7,5046	5,2206	5,6336	2,8924
Hassaine Daouadji 2016	P=0	5,6307	3,95702	4,2256	2,16936
Present Analytical Model	P=0	5,9363	4,09119	4,4609	2,27601
	P=10kN	-1,9301	-1,225	-6,259	-3,13696
	P=20kN	-9,7979	-6,5431	-16,98	-8,54996

Steel Beam bonded with a (prestressed) thin plate subjected to a Single Point Distributed Load

Model	Prestressing load	Steel beam with Sika carbodur plate τ(x)	Steel beam with Sika carbodur plate σ(x)	Steel beam with four layers of Sika Wrap τ(x)	Steel beam with four layers of Sika Wrap σ(x)
Bouakaz 2014	P=0	8,0783	5,6224	6,0592	3,1164
Hassaine Daouadji 2016	P=0	6,0630	4,2172	4,5444	2,3437
Present Analytical Model	P=0	6,3885	4,4037	4,7951	2,4473
	P=10kN	-1,4785	-0,9130	-5,924	-2,9664
	P=20kN	-9,3449	-6,2296	-16,64	-8,3798

Steel Beam bonded with a (prestressed) thin plate subjected to a Two Symmetric Point Load

Model	Prestressing load	Steel beam with Sika carbodur plate τ(x)	Steel beam with Sika carbodur plate σ(x)	Steel beam with four layers of Sika Wrap τ(x)	Steel beam with four layers of Sika Wrap σ(x)
Bouakaz 2014	P=0	9,1043	6,2573	6,2927	3,5344
Hassaine Daouadji 2016	P=0	6,8284	4,6927	5,1952	2,6506
Present Analytical Model	P=0	7,2403	4,9341	5,5085	2,794
	P=10kN	-0,6267	-0,3822	-5,211	-2,6197
	P=20kN	-8,4931	-5,6992	-15,93	-8,0336

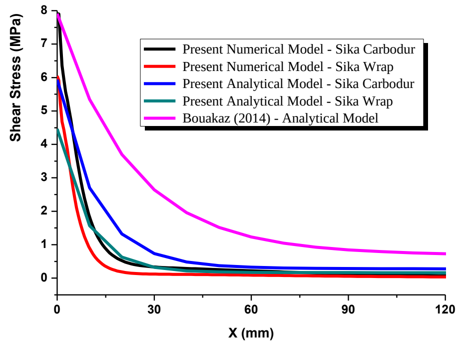

Fig. 7 Comparison of interfacial shear stress for the steel beams reinforced with composite plate

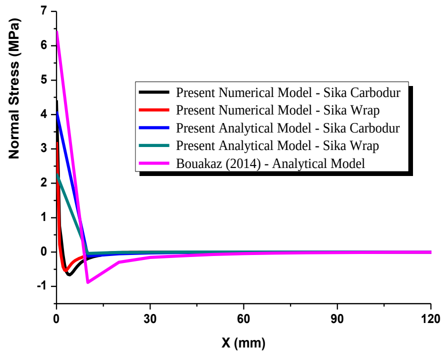

Fig. 8 Comparison of interfacial normal stress for the steel beams reinforced with composite plate

distributed load. A summary of the geometric and material properties is given in figure 1 and table 1.

The FEM solutions are compared with the analytical solution (Hassaine Daouadji 2016d and Bouakaz 2014) (table 2) and the interfacial shear and normal stress distributions near the end of composite plate (sika carbodur and sika wrap plate) are shown in The FEM results are in reasonable agreement with the analytical results (Table 2). The interfacial normal stresses change sign at a short distance away from the plate end. In the region of the negative interfacial normal stresses, the additional bending deformations in the composite due to interfacial shear stresses are considered in some theoretical solutions, such as Bouakaz (2014) and Hassaine Daouadji (2016d). The results of the peak interfacial shear and normal stresses are given in figure 7 and 8 for the beams strengthened by bonding with sika carbodur and sika wrap plate. As it can be seen from the results, the peak interfacial stresses assessed by the present theory are smaller compared to those given by Hassaine Daouadji (2016) and Bouakaz (2014) solutions.

Fig 7 and 8 plots the interfacial shear and normal stresses near the plate end for the example RC beam bonded with sika carbodur and sika wrap plate for the uniformly distributed load case. Overall, the predictions of the different solutions agree closely with each other. The interfacial normal stress is seen to change sign at a short distance away from the plate end. The present analysis gives lower maximum interfacial shear and normal stresses

Table 3 Comparison of interfacial shear and normal stresses (MPa): Uniformly Distributed Load

Steel Beam bonded with a (prestressed) thin plate subjected to a uniformly distributed load										
Present Model	Steel beam with Sika carbodur plate		Steel beam with four layers of Sika Wrap		Steel beam with perfect FGM plate α=0		Steel beam with perfect FGM plate α=0,2		Steel beam with honeycomb sandwich plate	
	τ(x)	σ(x)	τ(x)	σ(x)	τ(x)	σ(x)	τ(x)	σ(x)	τ(x)	σ(x)
P=0	5,9363	4,09119	4,4609	2,27601	6,0767	4,02429	5,52318	3,80394	5.06953	5.84642
P=10 kN	-1,9301	-1,225	-6,259	-3,13696	-1,587	-0,9442	-2,99813	-1,94522	-2.18584	-1.59043
P=20 kN	-9,7979	-6,5431	-16,98	-8,54996	-9,250	-5,91269	-11,5195	-7,69439	-9.44118	-9.02726
P=40 kN	-25,532	-17,177	-38,42	-19,376	-24,57	-15,8496	-28,5621	-19,1927	-23.9519	-23.9010

Table 4 Comparison of interfacial shear and normal stresses (MPa): Single Point Distributed Load

Steel Beam bonded with a (prestressed) thin plate subjected to a Single Point Distributed Load										
Present Model	Steel beam with Sika carbodur plate		Steel beam with four layers of Sika Wrap		Steel beam with perfect FGM plateα=0		Steel beam with perfect FGM plateα=0,2		Steel beam with honeycomb sandwich plate	
	τ(x)	σ(x)	τ(x)	σ(x)	τ(x)	σ(x)	τ(x)	σ(x)	τ(x)	σ(x)
P=0	6,3885	4,4037	4,7951	2,4473	6,5406	4,3324	5,942	4,0931	5,4591	6,3023
P=10 kN	-1,478	-0,9130	-5,924	-2,966	-1,1234	-0,636	-2,579	-1,6555	-1,7964	-1,1339
P=20 kN	-9,344	-6,2296	-16,64	-8,379	-8,7873	-5,604	-11,10	-7,4044	-9,0524	-8,5708
P=40 kN	-25,08	-16,864	-38,08	-19,20	-24,116	-15,54	-28,14	-18,901	-23,563	-23,442

Table 5 Comparison of interfacial shear and normal stresses (MPa): Two Symmetric Point Load

Steel Beam bonded with a (prestressed) thin plate subjected to a Two Symmetric Point Load										
Present Model	Steel beam with Sika carbodur plate		Steel beam with four layers of Sika Wrap		Steel beam with perfect FGM plateα=0		Steel beam with perfect FGM plateα=0,2		Steel beam with honeycomb sandwich plate	
	τ(x)	σ(x)	τ(x)	σ(x)	τ(x)	σ(x)	τ(x)	σ(x)	τ(x)	σ(x)
P=0	7,2403	4,9341	5,5085	2,794	7,4033	4,8437	6,7592	4,6006	6,1424	6,6499
P=10 kN	-0,626	-0,3822	-5,211	-2,619	-0,260	-0,124	-1,7619	-1,148	-1,1131	-0,78605
P=20 kN	-8,493	-5,6992	-15,93	-8,033	-7,924	-5,093	-10,284	-6,8973	-8,3691	-8,2225
P=40 kN	-24,22	-16,333	-37,37	-18,86	-23,25	-15,02	-27,325	-18,393	-22,88	-23,095

than those predicted by Bouakaz (2014), indicating that the inclusion of adherend shear de formation effect in the beam and soffit plate leads to lower values of σ_{max} and τ_{max}. Therefore, it appears that the adherent shear deformation reduces the concentration of interfacial stresses and thus makes the adhesive shear distribution more uniform, this is the object of the present analytical analysis, finally, we can say that the objective of reducing the concentration of interface stresses is achieved.

5.3 Effect of the prestressing force (P0) on adhesive stress

The numerical results of the present solution are presented to study the effect of the prestressing force P_0 on the distribution of interfacial stress in a steel beam strengthened with bonded prestressed composite plate (Sika carbodur plate, Sika Wrap, perfect and FGM plate, honeycomb sandwich plate). Three value of P_0 are considered in this study (P_0=0; P_0=10, P_0=20, and P_0=40 kN).

Table 3, 4 and 5 shows that the interfacial shear and normal stress for the steel beam strengthened with bonded prestressed composite plate for the mid-point load case, From these results, one can observe:

- Maximum stress occur at the ends of adhesively bonded plates, and the normal, or peeling, stress disappears at around 200 mm from the end of the plates.
- It is seen that increasing the value of prestressing force P_0 leads to high stress concentrations.

5.4 Effect of the composite stiffness and thickness

Table 3, 4 and 5 gives interfacial shear and normal stress for the steel beam bonded with the prestressed composite plate, which demonstrate the effect of plate material properties on interfacial stresses. In contrary to adhesive stiffness, interfacial stress increases as the plate material becomes softer (from Sika carbodur plate, Sika Wrap perfect and FGM plate, honeycomb sandwich plate). This is due to the fact that the initial deformations needed to

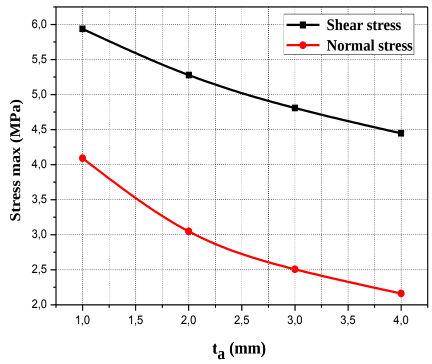

Fig. 9 Effect of adhesive layer thickness on interfacial stress at the ends of bonded sika carbodur plate

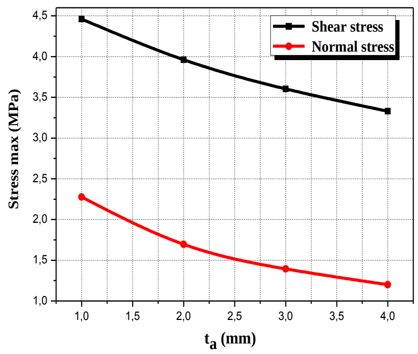

Fig. 10 Effect of adhesive layer thickness on interfacial stress at the ends of bonded sika wrap plate

accomplish a specific prestressing force was however higher for stiffer laminates. The instantaneous loss in prestressing force was however higher for stiffer laminates. The thickness of the composite plate (Sika carbodur plate, Sika Wrap, perfect and FGM plate, honeycomb sandwich plate) is an important design variable in practice.

The results show that the rigidity and thickness of the composite considerably reduces edge peeling and shear stress. This is because the initial stress will be lower for thicker composite.

5.5 Effect of adhesive layer thickness

Figures 9 and 10 shows the effect of the thickness of the adhesive layer on interfacial stress. It is seen that increasing the thickness of the adhesive layer leads to significant reduction in peak interfacial stress. Thus using thick adhesive layer, especially in the vicinity of the edge, is recommended.

6. Conclusions

The object of the present research was the reinforcement of steel beams by composite materials. This research has just confirmed and completed some points previously made on the subject of rehabilitation of structures, after an analysis and modeling of the interface stresses of metal beams reinforced by bending by plates made of composite materials. In conclusion, it is very clear that a suitable repair and / or reinforcement design taking into account all key parameters of the steel IPE beam, the CFRP plate repair and the interface adhesives is very important. To better understand the behavior of glued beam repairs, which will help engineers optimize their design parameters, we have carefully studied and analyzed the effects of a few parameters, in this case the geometric and material characteristics that influence the rigidity and stability of the reinforced beam. The results of the tests presented on steel beams reinforced with composite materials (sika carbodur and sika wrap) are very promising. For all the two types of reinforcement considered in this study, the reinforcement technique improved the load capacity. The effect of the composite strips on rigidity was also remarkable, while a significant increase in elastic rigidity is obtained when the reinforcement is achieved by a sika carbodur plate.

In any point at a distance from the support greater than the development length, the analytical models provide an accurate estimation of the stresses for the one layer configuration and a slightly conservative estimation for the four layers configuration. In the latter case, the model could be improved accounting for the adhesive flexibility. Finally, the proposed finite element model produces estimation of the stresses in the adhesive layer in good agreement with the result obtained by the analytical approach. Moreover, the stress in the composite strips for both the analyzed reinforcement configurations agrees with the experimental results. By comparing with experimental results, the present closed – solution provides satisfactory predictions to the interfacial shear stress in the plated beams.

Acknowledgments

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