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Reliability analysis of laminated composite shells by response surface method based on HSDT

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Abstract. Reliability analysis of composite structures considering random variation of involved parameters is quite important as composite materials revealed large statistical variations in their mechanical properties. The reliability analysis of such structures by the first order reliability method (FORM) and Monte Carlo Simulation (MCS) based approach involves repetitive evaluations of performance function. The response surface method (RSM) based metamodeling technique has emerged as an effective solution to such problems. In the application of metamodeling for uncertainty quantification and reliability analysis of composite structures; the finite element model is usually formulated by either classical laminate theory or first order shear deformation theory. But such theories show significant error in calculating the structural responses of composite structures. The present study attempted to apply the RSM based MCS for reliability analysis of composite shell structures where the surrogate model is constructed using higher order shear deformation theory (HSDT) of composite structures considering the uncertainties in the material properties, load, ply thickness and radius of curvature of the shell structure. The sensitivity of responses of the shell is also obtained by RSM and finite element method based direct approach to elucidate the advantages of RSM for response sensitivity analysis. The reliability results obtained by the proposed RSM based MCS and FORM are compared with the accurate reliability analysis results obtained by the direct MCS by considering two numerical examples.

Keywords: reliability; response surface method; laminated shell; higher order shear deformation theory; finite element analysis

1. Introduction

Laminated composite materials are extensively used for construction of various important engineering structures e.g. aircraft structures, submarine, wind turbine blade, bridge deck, commercial vehicles and ship structures etc. The composite materials offer numerous advantages like lightweight, high stiffness, improved chemical and environmental resistance, high fatigue resistance and above all ability to tailor the properties. However, the experimental and analytical studies on composite materials revealed large statistical variations in their mechanical properties. Thereby, the response of laminated composite structure is largely influenced by various uncertain parameters. Thus, safety assessment of composite structures i.e. reliability analysis of composite structures considering random variation of involved parameters is quite important. The present study deals with reliability analysis of laminated composite shell structures.

The reliability assessment of structure requires the computation of probability of failure (p_f). Several methods of reliability analysis have been developed which can be classified into two groups: analytic techniques and simulation methods. In the first group, one can see the well-known First Order Reliability Method (FORM) and Second Order Reliability Method (SORM). Such methods aim at building a local approximation of the limit state function from geometric considerations such as the gradients and curvatures. In the second groups, the methods based on Monte Carlo Simulations (MCS) can be found. These are now well established and documented in numerous texts (Thoft-Christensen and Barker 1982, Ditlevsen and Madsen 1996, Melchers 1999, Halder and Mahadevan 2000).

The applications of reliability analysis methods to composite structures are also notable in recent past. Reliability of laminate composite structure using first order second moment method was introduced by Cederbaum *et al*. (1990) considering stress failure criteria. Boyer *et al*. (1997) presented the application of reliability methods i.e. FORM and SORM of laminated composite structures considering stress-strain failure criteria. An analytical probabilistic modeling for stochastic initial failure and reliability of a laminated thin-walled composite structure was proposed by Yushanov *et al*. (1998). A comparison between MCS technique and FORM based reliability assessment of composite structure was presented by Sciuva and Lomario (2003) considering loads, geometries and material properties as stochastic variables. Chen *et al*. (2005) showed fuzzy reliability methods for laminated

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composites where loads were considered as random design variables and strengths as fuzzy variables. Finite element based methodology for statistical analysis of angle-ply composite structures considering material properties as random variables was proposed by Antonio and Hoffbauer (2007).

Using Spectral Stochastic finite element method (SFEM) to obtain response of composite structure considering material properties as random variables was presented by Ngah and Young (2007). Biagi and Medico (2008) presented MCS based reliability analysis of composite cylindrical shells under axial compression with stacking sequence of fiber as a random variable. Papadopoulos and Lagaros (2009) presented stochastic vulnerability based robust design of laminated shell structures considering random geometric and material properties. Chiachio *et al*. (2012) presented a survey on reliability analysis of composites encompassing different types of reliability methods, random variables and failure criteria for laminated composite plate and shell. Gomes *et al*. (2011) presented reliability based optimization of laminated composite using artificial neural networks and genetic algorithms considering loading, fiber orientation angle and ply thickness as random variables. Reliability analysis and sensitivity analysis of composite structures was presented by Gosling *et al*. (2014). Basic input variables were taken as fiber orientation angle and ply thickness. The FORM was proposed for reliability analysis and compared with MCS based results. Stochastic free vibration analysis of composite shell structure based on Kriging metamodeling approach was presented by Dey *et al*. (2015). Sasikumar *et al*. (2015) proposed a polynomial chaos based SFEM for reliability analysis of composite structures with individual lamina properties as random variables. Haeri and Fadaee (2016) presented an advanced Kriging model to approximate the mechanical model of a laminated composite structure for estimation the probability of failure considering stress criterion to define the performance function for reliability analysis. A bottom up surrogated based approach is employed by Dey *et al*. (2016) for reliability assessment of laminated composites considering uncertainty in ply orientation angle, elastic modulus and mass density. FSDT was used to construct the surrogated model of the shell. Several parametric studies were presented to determine the stochastic natural frequencies and mode shapes of the shell. Mukhopadhyay *et al*. (2016) presented the effect of noise on surrogated based stochastic frequency analysis of spherical composite shallow shells considering material properties, ply orientation angle and mass density of the laminate as input random parameters. The Kriging based surrogated model was used to calculate the frequency of the shell whereas the surrogated model was developed using finite element formulation of laminated composite shell based on FSDT. Mukhopadhyay *et al*. (2017) presented a critical comparative assessment in terms of accuracy and computational efficiency of Kriging model variants for uncertainty quantification of natural frequencies of composite doubly curved shells. Five types of Kriging model variants were studied and all the models were constructed using FSDT of laminated shell. Direct MCS was used to check the effectiveness of each model with uncertainty in ply orientation angle, elastic modulus and mass density of the shell. Dey *et al*. (2017) investigated different types of surrogate models which are used in reliability analysis of composites for comparative assessment of uncertainty in natural frequencies. Both computational efficiency and accuracy were compared for each model. The results obtained by the MCS method were compared with the results of the different metamodels.

The reliability analysis of composite structures as discussed above used the FORM and MCS based approach which involves repetitive evaluations of performance function. The second moment based FORM algorithms require computation of gradients and hessians of performance function. For implicit performance function, finite difference methods are usually adopted for approximating the gradients of the performance functions. This requires a large number of numerical computations. Furthermore, the second moment methods cannot always provide desired accuracy, particularly when the levels of uncertainty in the parameters are relatively large. Whereas, in direct MCS approach, repeated evaluation of performance function involves large number of executions of the FE model of a structure. Thus, assessing the reliability of a complex composite structure requires a deal between the reliability algorithms and numerical methods used to model the mechanical behavior of the system and development of approach requiring fairly low computational time becomes important for safety assessments of composite structures. The response surface method (RSM) based metamodeling technique has emerged as an effective solution in this regard allowing a convenient way to achieve a balance between the number of execution of the FE model and the accuracy of computed reliability.

The early application of RSM in structural reliability was made by Faravelli (1989) and the subsequent works (Bucher and Bourgund 1990, Liu and Moses 1994, Rajashekhar and Ellingwood 1993, Basaga *et al*. 2012) for reliability analysis of large and complex structural system are well known. The position of the sample points, the type of polynomial response and its performance is the subject of investigated by several researchers (Rajashekhar and Ellingwood 1993, Allaix and Carbone 2011, Goswami *et al*. 2016). The optimal number of sampling points required to construct response surface is another important issue which is well studied in Haldar *et al*. (2012), Huh and Haldar (2011).

In the application of metamodeling for uncertainty quantification and reliability analysis of composite structures (Dey *et al*. 2015, Haeri and Fadaee 2016, Dey *et al*. 2016, Mukhopadhyay *et al*. 2016, Mukhopadhyay *et al*. 2017, Dey *et al*. 2017) the finite element model was formulated either by classical laminate theory or first order shear deformation theory (FSDT). But such theories show significant error in calculating the responses of composite structures. Thus, it is felt important to use accurate and efficient method of response evaluation of composite structures for constructing response surface. The present study attempted to apply the RSM for reliability analysis of composite shell structures where the surrogate model is constructed using HSDT of laminated shell. For this, the finite element based HSDT model proposed by Thakur and Ray (2015a,b) is adopted to obtain much accurate response of composite shell structures. A second order polynomial function based RSM is used to approximate the composite shell responses considering the uncertainties in the material properties (E_1, E_2, G_{12}, G_{13}, G_{23}, γ_{12}), external load (P_0), ply thickness (h) and radius of curvature (R) of shell structure. The sensitivity of responses of the shell is also obtained by RSM and finite element method based direct approach to illustrate the advantages of RSM for response sensitivity analysis. The reliability results obtained by the proposed RSM based MCS and FORM are compared with the most accurate reliability analysis results obtained by the direct MCS. The RSM based reliability analysis algorithm is elucidated with the help of two numerical examples i.e. reliability analysis of four layered laminated composite cylindrical symmetric (0°/90°/90°/0°) and anti-symmetric (0°/90°/0°/90°) shell structures.

2. Response analysis of laminated composite shell

The evaluation of response of a structure plays the pivotal role for successful evaluation of its reliability. The theoretical model of laminated composite shell structure is modified day to day to achieve an accurate and improved realistic result. Classical laminated shell theory based on love-Kirchhoff assumption (Love 1888) was first introduced neglecting the transverse shear deformation of the laminates. Considering the constant shear deformation and using a linear displacement field through the thickness, FSDT (Reddy 1984) was introduced, which is also unable to predict the actual behavior of composite laminates. The HSDT (Reddy and Liu 1985, Kant and Menon 1989, Sayyad and Ghugal 2014, Thakur and Ray 2015a,b, Thakur, Ray and Chakraborty 2016) involves non-linear distribution of displacement field across the thickness of the laminate gives more accurate results as compared to FSDT. Thus, for reliability analysis use of more accurate laminated shell theory is important for proper quantification of responses considering uncertain parameters. The more realistic solution of laminated composite shell structure of any shapes obtained by the finite element based HSDT models proposed by Thakur and Ray (2015a,b) which considers the effect of thickness coordinate to radius ratio (z/R) in strain components as well as normal and shear stress resultants is adopted in the present study for obtaining accurate responses of the shell structures. The model is briefly discussed in the following.

A doubly curved laminated shell with s, r, z co-ordinate system is shown in Fig. 1. R_s and R_r are the radius of curvature in s and r directions respectively, h is the total thickness of the shell along z direction and n is the number of lamina.

The stress-strain relationship for a typical k^{th} lamina in a laminated composite shell shown in Fig. 1 is given by

$$\begin{Bmatrix}\sigma_s\\ \sigma_r\\ \tau_{sr}\\ \tau_{sz}\\ \tau_{rz}\end{Bmatrix}=\begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0\\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45}\\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55}\end{bmatrix}\begin{Bmatrix}\varepsilon_s\\ \varepsilon_r\\ \varepsilon_{sr}\\ \varepsilon_{sz}\\ \varepsilon_{rz}\end{Bmatrix} \tag{1}$$

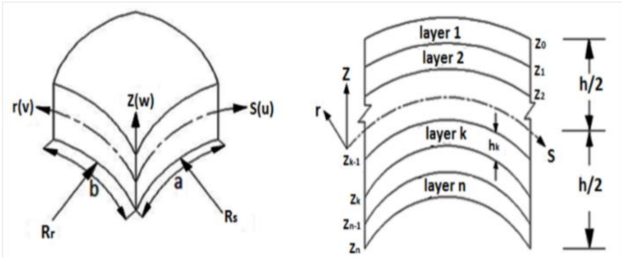

Fig. 1 Geometry of laminated shell

The displacement field considering Taylor series expansion is expressed by,

$$\begin{aligned}
&U(s,r,z)=u(s,r)+z\theta_s(s,r)+z^2u^*(s,r)\\
&+z^3\theta_s^*(s,r)\\
&V(s,r,z)=v(s,r)+z\theta_r(s,r)+z^2v^*(s,r)\\
&+z^3\theta_r^*(s,r)\\
&W(s,r,z)=w(s,r)
\end{aligned} \tag{2}$$

Where, u, v and w are the displacement components at the middle surface of the shell and

$$\theta_s(s,r)=\left(\frac{\partial U}{\partial z}\right)_{z=0},u^*(s,r)=\frac{1}{2}\left(\frac{\partial^2 U}{\partial z^2}\right)_{z=0},\theta_s^*(s,r)=\frac{1}{6}\left(\frac{\partial^3 U}{\partial z^3}\right)_{z=0}$$

$$\theta_r(s,r)=\left(\frac{\partial V}{\partial z}\right)_{z=0},v^*(s,r)=\frac{1}{2}\left(\frac{\partial^2 V}{\partial z^2}\right)_{z=0},\theta_r^*(s,r)=\frac{1}{6}\left(\frac{\partial^3 V}{\partial z^3}\right)_{z=0}$$

Considering $\frac{z}{R_r}$ and $\frac{z}{R_s}$ terms in strain-displacement relationship, the strain components at the mid-plane can be expressed as (Thakur and Ray 2015a,b),

$$\begin{aligned}
&\varepsilon_{so}=\frac{\partial u}{\partial s}+\frac{w}{R_s},\varepsilon_{ro}=\frac{\partial v}{\partial r}+\frac{w}{R_r},\varepsilon_{sro}=\frac{\partial v}{\partial s},\\
&\varepsilon_{rso}=\frac{\partial u}{\partial r}\\
&\kappa_s=\frac{\partial\theta_s}{\partial s},\kappa_r=\frac{\partial\theta_r}{\partial r},\kappa_{sr}=\frac{\partial\theta_r}{\partial s},\kappa_{rs}=\frac{\partial\theta_s}{\partial r}\\
&\varepsilon_{so}^*=\frac{\partial u^*}{\partial s},\varepsilon_{ro}^*=\frac{\partial v^*}{\partial r},\varepsilon_{sro}^*=\frac{\partial v^*}{\partial s},\varepsilon_{rso}^*=\frac{\partial u^*}{\partial r}\\
&\kappa_s^*=\frac{\partial\theta_s^*}{\partial s},\kappa_r^*=\frac{\partial\theta_r^*}{\partial r},\kappa_{sr}^*=\frac{\partial\theta_r^*}{\partial s},\kappa_{rs}^*=\frac{\partial\theta_s^*}{\partial r}
\end{aligned} \tag{3}$$

And the transverse shear strain components at the mid-plane are

$$\begin{aligned}
&\varepsilon_{szo}=\theta_s+\frac{\partial w}{\partial s}-\frac{u}{R_s},\quad \varepsilon_{rzo}=\theta_r+\frac{\partial w}{\partial r}-\frac{v}{R_r}\\
&\kappa_{sz}=2u^*,\kappa_{rz}=2v^*\\
&\varepsilon_{szo}^*=3\theta_s^*+\frac{u^*}{R_s},\quad \varepsilon_{rzo}^*=3\theta_r^*+\frac{v^*}{R_r}
\end{aligned} \tag{4}$$

$$\kappa_{sz}^* = \frac{2\theta_s^*}{R_s}, \kappa_{rz}^* = \frac{2\theta_r^*}{R_r}$$

Using the relationship between stress resultants and mid-plane strain components, the rigidity matrix of laminated shell can be obtained as (Thakur and Ray 2015a,b)

$$[\mathrm{D}] = \begin{bmatrix} [\mathrm{A}] & [\mathrm{B}] & [\mathrm{C}] & [\mathrm{D}] & [0] & [0] & [0] & [0] \\ [\mathrm{B}] & [\mathrm{C}] & [\mathrm{D}] & [\mathrm{E}] & [0] & [0] & [0] & [0] \\ [\mathrm{C}] & [\mathrm{D}] & [\mathrm{E}] & [\mathrm{F}] & [0] & [0] & [0] & [0] \\ [\mathrm{D}] & [\mathrm{E}] & [\mathrm{F}] & [\mathrm{G}] & [0] & [0] & [0] & [0] \\ [0] & [0] & [0] & [0] & [\mathrm{H}] & [\mathrm{I}] & [\mathrm{J}] & [\mathrm{K}] \\ [0] & [0] & [0] & [0] & [\mathrm{I}] & [\mathrm{J}] & [\mathrm{K}] & [\mathrm{L}] \\ [0] & [0] & [0] & [0] & [\mathrm{J}] & [\mathrm{K}] & [\mathrm{L}] & [\mathrm{M}] \\ [0] & [0] & [0] & [0] & [\mathrm{K}] & [\mathrm{L}] & [\mathrm{M}] & [\mathrm{N}] \end{bmatrix} \quad (5)$$

Where, [0] is a null matrix. The elements of the rigidity matrix are computed and presented in Appendix A.

The finite element analysis has been carried out by using an eight noded C^0 isoparametric shell element with nine degrees of freedom at each node (u, v, w ,θ_s, θ_r, u^*, v^*, θ_s^*, θ_r^*). The displacement field can be expressed as

$$\{u\} = \sum_{i=1}^{8} [N_i]\{u_i\} \quad (6)$$

Where, $[N_i]$ is the shape function of the associated node of isoparametric quadrilateral element and u_i is the nodal displacement. The strain-displacement relationship at the mid-plane of the laminated shell can be expressed in the following matrix form as,

$$\{\varepsilon\} = [B]\{u\} \quad (7)$$

Where, [B] is the differential operator matrix of interpolation function which can be derived from strain-displacement equations (Eq. 3 and Eq. 4). The computations of non-zero terms of [B] matrix are shown in Appendix B. The element stiffness matrix [K_e] and the mass matrix [M_e] for an element can be obtained as

$$[K_e] = \iint [B]^T[D][B]dsdr = \int_{-1}^{1}\int_{-1}^{1} [B]^T[D][B] \,|\, J \,|\, d\xi d\eta$$

$$[M_e] = \iint [N]^T[I][N]dsdr = \int_{-1}^{1}\int_{-1}^{1} [N]^T[I][N] \,|\, J \,|\, d\xi d\eta \quad (8)$$

Where [I] is the inertia matrix and $|J|$ is the determinant of Jacobian matrix. The finite element static equilibrium equation can be written as,

$$[K]\{u\} = \{P\} \quad (9)$$

Where [K] is the overall stiffness matrix and u is the deflection.

3. Response surface method

The RSM, first proposed by Box and Wilson (1954), is a statistical technique designed for better understanding about the overall response of structural system by design of experiment (DOE) (Das and Zheng 2000, Roussouly *et al*. 2013, Khuri and Mukhopadhyaya 2010) and subsequent analysis of experimental data. The RSM is a simple function (polynomial type in most of the cases) which is fitted by a set of carefully selected data points referred as DOE. Thus, it is basically a system identification procedure where the output parameters (i.e. displacement, natural frequency, stresses) are directly obtained by substituting the value of input parameters (i.e. loading, structural geometry and materials properties).

If there are n response values y_i corresponding to n numbers of observed data, x_{ij}(denotes the i-th observation of the j-th input variable x_j in a DOE), the relationship between the response and the input variables can be expressed as

$$\mathbf{y} = \mathbf{X\beta} + \varepsilon_y \quad (10)$$

In the above multiple non-linear regression model, **X, y, β** and $\boldsymbol{\varepsilon}_\mathbf{y}$ are the design matrix containing the input data from the DOE, response vector, unknown co-efficient vector and error vector, respectively. Typically, the quadratic polynomial form used in the RSM is as following:

$$\mathbf{y} = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k}\sum_{j=1}^{k} \beta_{ij} x_i x_j \quad (11)$$

The Least Square Method (LSM) is the most widely adopted technique to construct polynomial response surface. The best estimate of the polynomial coefficients are obtained in the sense of least squares by minimizing the sum of squares of the distance between the original data points and the points in the fitted curve. Suppose the data points are (x_1,y_1), (x_2,y_2),…, (x_n,y_n), where x is the independent variable and y is the dependent variable and the fitting curve is $f(x) = a_1 x + a_2$. Then the deviations or errors d of $f(x)$ from each data points are $d_1 = y_1 - f(x_1), d_2 = y_2 - f(x_2), \ldots d_n = y_n - f(x_n)$. According to the least squares method, the sum of squares of those errors is minimum i.e.

$$L = d_1^{\,2} + d_2^{\,2} + \ldots + d_n^{\,2} = \sum_{i=1}^{n} d_i^{\,2} = \sum_{i=1}^{n} [y_i - f(x_i)]^2$$

$$= \sum_{i=1}^{n} [y_i - f(a_1 x_i + a_2)]^2 = \text{minimum}$$

The coefficient of the fitted curve $f(x)$ can be obtained by setting the gradient of the error (L) with respect to a_1 and a_2 is equal to zero i.e.

$$\frac{\partial L}{\partial a_1} = 0 \text{ and } \frac{\partial L}{\partial a_2} = 0$$

In the LSM of estimation technique, the unknown polynomial coefficients of Eq. (11) are obtained by minimizing the error norm defined as:

$$L = \sum_{i=1}^{n} \left(y_i - \beta_0 - \sum_{i=1}^{k} \beta_i x_i - \sum_{i=1}^{k}\sum_{j=1}^{k} \beta_{ij} x_i x_j \,) \right)^2 \quad (12)$$

$$= (\mathbf{y} - \mathbf{X\beta})^T (\mathbf{y} - \mathbf{X\beta})$$

And the least squares estimate of $\boldsymbol{\beta}$ is obtained as,

$$\boldsymbol{\beta} = \left[\mathbf{X}^T\mathbf{X}\right]^{-1}\left\{\mathbf{X}^T\mathbf{y}\right\} \tag{13}$$

Once the polynomial coefficients $\boldsymbol{\beta}$ are obtained from the above Eq. (13), the response y can be readily evaluated for any set of input parameters. To fit an accurate model within reasonable time, it is required that the initial input data (**X** and **y**) are selected judiciously. The CCD is adopted in the present study to generate the input data sets where 2^n+2n+1 number of functions evolutions are required.

The capability of response surface model is usually judged by the statistical indices i.e. Root Mean Square Error (*RMSE*), the coefficient of determination (R^2) and the average prediction error (ε_m), given by (Goswami 2010)

$$\begin{gathered} RMSE = \sqrt{\sum_{i=1}^{p}\frac{\left(\hat{y}_i - y_i\right)^2}{p}} \\ R^2 = \frac{\sum_{i=1}^{p}\left(\hat{y}_i - \bar{y}_i\right)^2}{\sum_{i=1}^{p}\left(y_i - \bar{y}_i\right)^2} \\ \varepsilon_m = \frac{\sqrt{\sum_{i=1}^{p}100\frac{\left|\hat{y}_i - y_i\right|}{y_i}}}{p} \end{gathered} \tag{14}$$

Where p is the total number of samples, y_i is the actual response obtained by the FEM-MCS for the ith sample point, $\bar{y}_i$ is the mean value of actual responses and $\hat{y}_i$ is the predicted response obtained by the RSM-MCS for the ith sample point. It has been noted from the convergence study of direct MCS results presented in Numerical Study that approximately after 20,000 simulations the probability of failure with respect to deflection criteria does not change significantly. Thus, conservatively, sample size p is taken as 30,000 in evaluating Eq. (14). A larger value of R^2 (i.e. a value closer to 1) and smaller values of RMSE and ε_m indicates a better fit of the RSM model.

4. Sensitivity analysis

Sensitivity analysis of laminated shell i.e. changes in deflection, stress, frequency, buckling etc due to the change in design variables is of great importance to study the effect of variation in the design variables on the performance of the shell for proper design of such structures. There exists some design variable showing significant effect on the structural response sensitivity whereas some other variables may have negligible effect. Thus, the sensitivity gradient provides essential information for choosing a search direction to obtain improved and feasible new design points. Also, the sensitivity analysis is significant for optimization, re-analysis and damage assessment and reliability analysis of such composite structure effectively.

The sensitivity of responses can be obtained directly using the FE equations as presented through Eqs. (6) to (9) in section 2. The design variables considered for deflection sensitivity are: material parameters (E_1, E_2, γ_{12}, G_{12}, G_{13}, G_{23}), fiber orientation angle (θ), radius of curvature (R_r and R_s) and external load (P_0). Now, taking the differentiation of the FE equilibrium Eq. (9) with respect to any design variable (say d) can be expressed as

$$\begin{gathered} \left[K\right]\frac{\partial\{u\}}{\partial d} + \{u\}\frac{\partial\left[K\right]}{\partial d} = \frac{\partial\{P\}}{\partial d} \\ i.e.\ \frac{\partial\{u\}}{\partial d} = \left[K\right]^{-1}\left(\frac{\partial\{P\}}{\partial d} - \{u\}\frac{\partial\left[K\right]}{\partial d}\right) \end{gathered} \tag{15}$$

It may be noted that one needs to evaluate the above gradient of the response repeatedly for reliability analysis by FORM, optimum design and model updating and it will be a computationally challenging task for such analysis of real complex structures. However, the response sensitivity can be obtained easily using the explicit response surface obtained by RSM. The sensitivity of any desired output response approximated by response surface obtained by RSM (Eq. 11) with respect to design variable x_l can be simply obtained as following

$$\frac{\partial y}{\partial x_l} = \beta_l + 2\beta_{ll}x_l + \sum_{i\neq l}^{k}\beta_{il}x_i \tag{16}$$

Where $\beta_{il} = \beta_{li}$

However, the numerical values of the sensitivity derivatives obtained with respect to each design variable will be of different order, because the order i.e. range of design variables are different. Thus, it is not possible to make a comparative assessment of importance of each variable on the overall sensitivity of response of the shell to identify the most significant design parameter. Thus, the sensitivity parameters are normalized and the normalized sensitivity coefficient for *l-th* design parameter x_l is obtained as

$$\left(\frac{dy}{dx_l}\right)_{scaled} = \frac{\left(\frac{\partial y}{\partial x_l}\right)}{\left(\frac{y}{x_l}\right)} \tag{17}$$

The sensitivity information obtained from the above provides the nature of variation of the responses of a laminated shell with respect to *l-th* input parameter x_l. This information indicates that the changes in performance in a design associated with enhancement or reduction of respective variables. Now, it is important to make a comparative assessment of importance of each variable to identify the most decisive input parameters that predominantly affect the responses of composite shell structures. For this, the importance factor can be obtained to rank the variables in order of their relative significance. The importance factor is defined as

$$S_l = \frac{\left(\frac{dy}{dx_l}\right)^2_{scaled}}{\sum_{l=1}^{m}\left(\frac{dy}{dx_l}\right)^2_{scaled}} \tag{18}$$

Table 1 Non-dimensional central deflection ($\overline{w} = wE_2 / P_0$) of laminated cross-ply shell for uniformly distributed sinusoidal load. (a=b; a/h=10; R_s=R_r=R)

Lamination scheme	R/a	Laminated theory			
		3D Elasticity (Bhimaraddi 1993)	CST (Bhimaraddi 1993)	PSD (Bhimaraddi 1993)	Present HSDT
0°/90°	1	4.6920	3.5718 (23.87%)	3.7686 (19.68%)	4.0823 (12.99%)
	2	8.8092	7.1163 (19.21%)	7.8119 (11.32%)	8.1438 (7.55%)
	3	10.512	8.7192 (17.05%)	9.7489 (7.25%)	9.9928 (4.93%)
	4	11.263	9.4655 (15.95%)	10.675 (5.22%)	10.855 (3.62%)
	5	11.639	9.8559 (15.32%)	11.167 (4.05%)	11.306 (2.86%)
	10	12.150	10.429 (14.16%)	11.896 (2.09%)	11.970 (1.48%)
	20	12.258	10.583 (13.66%)	12.094 (1.33%)	12.148 (0.89%)
	Plate	12.257	10.636 (13.22%)	12.161 (0.78%)	12.209 (0.39%)
0°/90°/0°	1	4.0811	2.4008 (41.17%)	3.0770 (24.60%)	3.4036 (16.60%)
	2	6.3134	3.5965 (43.03%)	5.3616 (15.07%)	5.5874 (11.49%)
	3	6.9888	3.9619 (43.31%)	6.2163 (11.05%)	6.6647 (4.63%)
	4	7.7476	4.1080 (46.97%)	6.5836 (15.02%)	6.8226 (11.93%)
	5	7.3674	4.1794 (43.27%)	6.7688 (8.12%)	7.0452 (4.37%)
	10	7.5123	4.2784 (43.04%)	7.0325 (6.38%)	7.1031 (5.44%)
	20	7.5328	4.3039 (42.86%)	7.1016 (5.72%)	7.1226 (5.44%)
	plate	7.5169	4.3125 (42.62%)	7.1250 (5.21%)	7.1256 (5.20%)

Where, S_l is the importance factor of sensitivity of y due to the input variable x_l and m is the total number of input variables. It may be noted that the value of S_l as obtained from Eq. (18) is normalized between 0 and 1. Thus, this value can serve as an indicator for identifying the relative importance of various input parameters influencing the responses of structure. This is highly useful at early design stage to identify the important input parameters; thereby reducing the number of significant variables for further optimum design and reliability analysis.

5. Reliability analysis

The assurance of performance is referred to as reliability. This performance function is described as,

$$\mathrm{Z} = \mathrm{g}(\mathbf{X}) \tag{19}$$

Where, $\mathbf{X}$ is the vector consists of the random structural parameters. The failure occurs when Z<0 and the probability of failure is mathematically expressed by the following multi-dimensional integral,

$$\mathrm{P_f}(\mathrm{Z}<0) = \int .. \int_{g(\mathbf{X})<0} f_X(\mathbf{X}) d\mathbf{X} \tag{20}$$

Where, $f_{\mathbf{X}}(\mathbf{X})$ is the n-dimensional joint probability density function (PDF) of the basic random variables. In general, the joint PDF of the random variables is seldom available. Moreover, evaluating the multiple integral is a formidable task. The MCS technique or various second moment based approximate methods i.e. the FORM and SORM is usually performed to evaluate the probability of failure. In the present study RSM based MCS and FORM is used for reliability analysis of laminated shells.

6. Numerical study

The effectiveness of sensitivity analysis and reliability evaluation of laminated composite shell by RSM is numerically demonstrated by considering cross-ply cylindrical symmetric and anti-symmetric shells. The boundary conditions for both the numerical examples are taken as simply supported at all edges of the laminates, i.e. $v = w = \theta_r = v^* = \theta_r^* = 0$ at s = 0, a and $u = w = \theta_s = u^* = \theta_s^* = 0$ at r = 0, b. The loading condition used in each example is sinusoidally distributed load, i.e $P = P_0 \sin(\pi s/a)\sin(\pi r/b)$, where P_0 is the constant load.

Simply supported cross-ply anti-symmetric (0°/90°) and symmetric (0°/90°/0°) laminated doubly curved shells are considered first to study the accuracy of HSDT (Thakur and Ray 2015a,b) model which is used for constructing the response surface for reliability analysis. The aspect ratio is taken as 1 and a/h ratio is considered as 10. The term *R/a* ratio varies from 1 to infinity. The results in terms of non-dimensional central deflections are computed by HSDT model (Thakur and Ray 2015a,b) and have been presented in Table 1 and compared with Bhimaraddi (1993). Bhimaraddi (1993) has used the three-dimensional theory of elasticity to obtain the exact solution of central deflection of

Table 2 The statistical properties of various random parameters of laminated shell

Variable	Distribution	Unit	Mean	COV
E_1	Normal	GPa	135	0.1
E_2	Normal	GPa	10.97	0.1
G_{12}	Normal	GPa	5.77	0.1
G_{13}	Normal	Gpa	5.77	0.1
G_{23}	Normal	GPa	3.45	0.1
γ_{12}	Normal	Dimensionless	0.24	0.1
P_0	Normal	kPa	1	0.1
h	Normal	m	0.002/ply	0.05
R	Normal	m	50	0.1

laminated shell. Bhimaraddi (1993) also used classical shell theory (CST) and a higher order theory i.e. parabolic shear deformation theory (PSD) neglecting the effect of *z/R* ratio in the analysis. The present laminated shell theory is based on HSDT considering the effect of *z/R* ratio. It can be observed from Table 1 that the present HSDT model shows consistent results when compared to the exact 3D solution.

The RSM based reliability analysis of composite shell structures are elucidate numerically by considering four layered symmetric (0°/90°/90°/0°) and anti-symmetric (0°/90°/0°/90°) laminated composite cylindrical shells. The dimensions are considered as a=1m and b=3m and radius of curvature R=R_s =50m and R_r =inf. The uncertain parameters considered are: material properties (E_1, E_2, G_{12}, G_{13}, G_{23}, γ_{12}), external load (P_0), ply thickness (h) and radius of curvature (R). The mean values of design variables with other statistical properties are shown in Table 2.

6.1 Example 1: Symmetric (0°/90°/90°/0°) cross-ply laminated cylindrical shell

In this example four layered cross-ply symmetric (0°/90°/90°/0°) laminated cylindrical shell is considered to check the accuracy of present LSM based RSM. The maximum deflection of the shell is approximated by RSM considering second order polynomial with cross term as an explicit function of the nine random variables. The central composite design (CCD) is adopted as DOE scheme. For nine input variables, total numbers of design points required are 2^9+2×9+1=531 to generate the response surface by CCD. A Matlab programme is used to generate these data points for developing the RSM using finite element method based on HSDT of laminated shell structure. It may be noted that response surface for any other response quantity of interest i.e. stress strain etc. at any location can be easily obtained once the DOE data are generated.

To study the capability of RSM to approximate the response of the composite shell, various statistical indices i.e. Root Mean Square Error (RMSE), co-efficient of determination (R^2) and direct error (ε_m) are computed first. The values of RMSE, R^2 and ε_m as obtained by the RSM for central deflection of symmetric (0°/90°/90°/0°) cross-ply laminated cylindrical shell are 0.00133, 0.999999 and 0.5196 respectively. It can be noted that the value of R^2 is nearest to one and the value of RMSE and direct error (ε_m) are very small which indicates that the RSM can approximate the shell response with sufficient accuracy.

Table 3 Non-dimensional sensitivity of central deflection with respect to different design variables for symmetric cross-ply (0°/90°/90°/0°) laminated cylindrical shell

Sensitivities	RSM	Direct	Error (%)
$\frac{\partial U}{\partial E_1}$	-1.0034	-0.9548	4.84
$\frac{\partial U}{\partial E_2}$	-2.2537×10^{-2}	-2.1472×10^{-2}	4.72
$\frac{\partial U}{\partial G_{12}}$	-2.2898×10^{-2}	2.1819×10^{-2}	4.71
$\frac{\partial U}{\partial G_{13}}$	-5.5922×10^{-4}	-5.4039×10^{-4}	3.36
$\frac{\partial U}{\partial G_{23}}$	-1.4028×10^{-3}	-1.3612×10^{-3}	2.96
$\frac{\partial U}{\partial \nu_{12}}$	-1.4589×10^{-2}	-1.3929×10^{-2}	4.52
$\frac{\partial U}{\partial P_0}$	1.0295	0.9999	2.87
$\frac{\partial U}{\partial h}$	-3.1131	-2.9915	3.90
$\frac{\partial U}{\partial R}$	5.0817×10^{-3}	4.4899×10^{-3}	11.64

Now, the sensitivity analysis is performed on the developed RSM to identity the significant random variables. Table 3 presents the results of non-dimensional sensitivity of central deflection with respect to different design variables obtained by RSM and FEM (direct method). It is clearly observed that the RSM based approach provides almost same result in terms of sensitivity of central deflection compare to the direct method. The error in RSM based sensitivity with respect to radius of curvature is noted to be much higher with respect to other parameters. This is arising because the sensitivity or first derivative of the central deflection of shell with respect to radius of curvature by direct method is carried out neglecting the higher order terms of thickness coordinate to radius of curvature (*z/R*) ratio e.g. $(z/R)^2$, $(z/R)^3$.

The sensitivity-based importance factor of central deflection with respect to all design variables obtained by the RSM depicted in Fig.2. It may be readily noted from the figure that the most influential parameters are E_1, P_0 and h.

The reliability analysis is now taken up for the following performance function

$$g(x)=U-U_0 \tag{21}$$

where, U is the central deflection of the laminated shell and U_0 is the maximum allowable deflection. The reliability is now estimated using the response surface already approximated to predict the central deflection. Fig. 3 shows the probability of failure with respect to number of simulation by RSM based MCS and direct MCS method. For direct MCS, the complete FE analysis is performed for each simulation to obtain the deflection and it is repeated for total number of simulation considered. The allowable

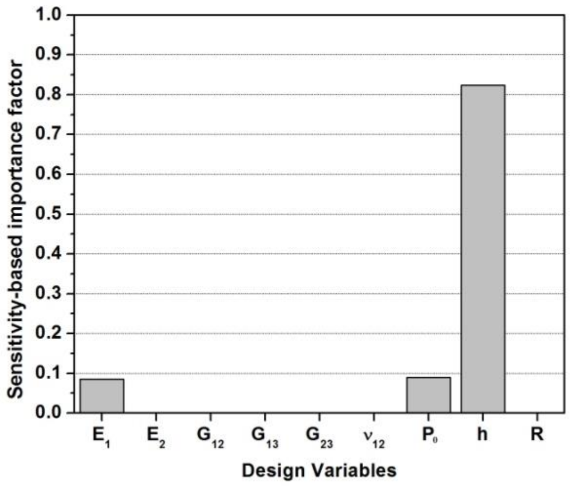

Fig. 2 Sensitivity-based importance factor of central deflection for symmetric (0°/90°/90°/0°) cross-ply laminated shell with respect to different design variables

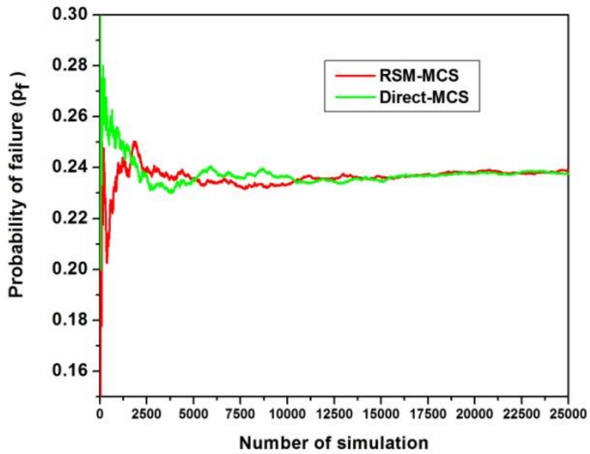

Fig. 3 The probability of failure of symmetric (0°/90°/90°/0°) cross-ply laminated shell with respect to deflection criteria

central deflection is taken as 2.2 mm in this case. It is clearly observed that the reliability results converge and merge with nearly at 20,000 simulations. Thus, for further parametric study 20,000 simulations is taken for reliability estimation. In this regard, by comparing the results, one can easily notice the capability of the RSM based MCS to estimate the reliability of the composite shell.

The effectiveness of the RSM in terms of processing time for estimating reliability with respect to deflection criteria is studied first. Table 4 shows the comparison of results of processing time for such analysis as required by Direct-MCS, RSM-MCS, FORM-Direct and FORM-RSM for cross-ply laminated Symmetric (0°/90°/90°/0°) shell. The allowable deflection is considered as 2.2 mm and 20000 simulations are taken in Direct-MCS and RSM-MCS approach. It can be observed from Table 4 that for 20000 FEM runs, the Direct-MCS required 14480 sec i.e. about 4 hours to obtain the probability of failure with respect to deflection criteria. Whereas, the same result obtained from RSM-MCS required 0.134 sec only. Also, it can be studied that FORM-Direct and FORM-MCS show significant error as compared to Direct-MCS. Thus, the RSM approach can

Table 4 Comparison of processing times for probability of failure of cross-ply laminated symmetric (0°/90°/90°/0°) shell with respect to deflection criteria

Method	Relative processing time (sec)	Probability of failure (p_f)
Direct-MCS	14480	0.2341
RSM-MCS	0.134	0.2377
FORM-Direct	12.35	0.2173
FORM-RSM	0.030	0.2148

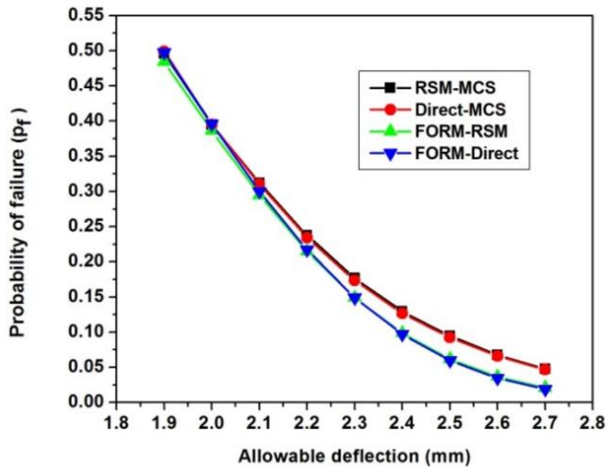

Fig. 4 The probability of failure of symmetric (0°/90°/90°/0°) cross-ply laminated shell with respect to deflection criteria for different allowable deflection

be effectively used in reliability assessment of laminated shell structures to reduce the computational cost of analysis.

Fig. 4 presents the probability of failure of the shell for different allowable deflection. The reliability is estimated by RSM based MCS and by RSM based FORM and compare with the results of direct MCS and direct FEM based FORM. It can be observed that results obtained by RSM based MCS is totally merged with FEM based MCS results. As expected, RSM and direct FEM based FORM show significant error when compared with to direct MCS based results.

As noted from Fig. 2 that the most influential parameters affecting the shell deflection are E_1, P_0 and h. thus, further parametric studies are considered with respect to these three parameters to study the nature of variation and performance of RSM or reliability analysis. However, it is also well known that radius of curvature(R) is an important factor for response analysis of shell structure. Thus, it is also taken for parametric study of reliability analysis. Fig. 5 to Fig. 8 shows the results of probability of failure of central deflection with respect to coefficient of variation (COV) of design parameters E_1, P_0, h and R, respectively. The allowable central deflection in each case is taken as 2.2 mm. As expected, the probability of failure of central deflection increases with the increase value of COV of each design parameters. It may be noted that the present RSM based MCS shows quite good estimate of probability of failures and are quite close to the most accurate direct based MCS estimate. Whereas, as expected the FORM results show significant error.

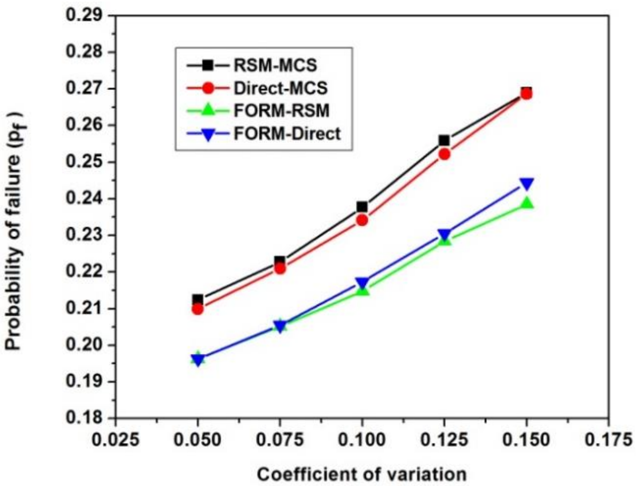

Fig. 5 The probability of failure of symmetric (0°/90°/90°/0°) cross-ply laminated shell with respect to deflection criteria for different COV of E_1

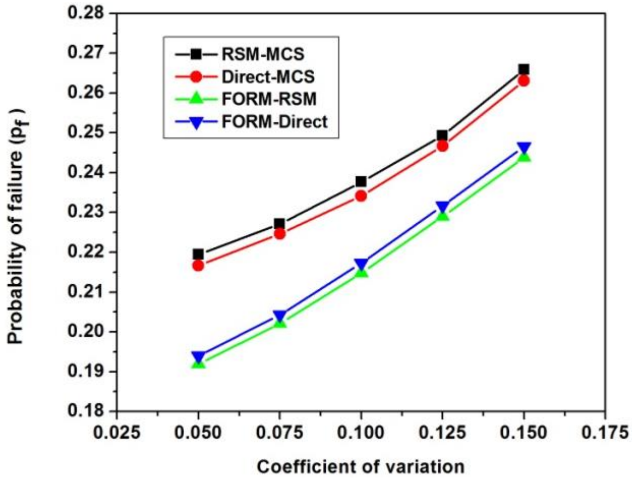

Fig. 6 The probability of failure of symmetric (0°/90°/90°/0°) cross-ply laminated shell with respect to deflection criteria for different COV of P_0

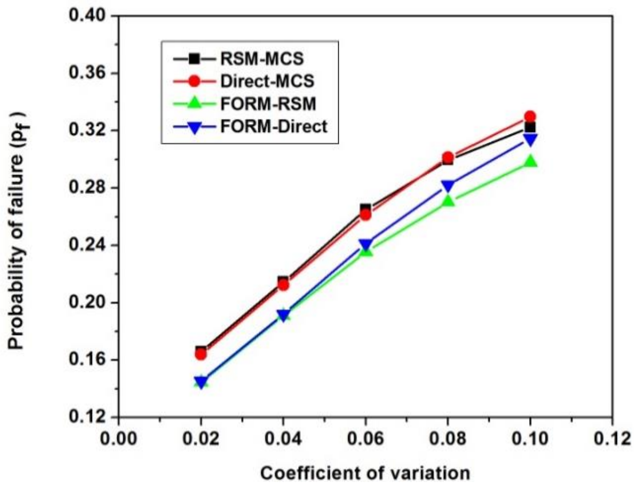

Fig. 7 The probability of failure of symmetric (0°/90°/90°/0°) cross-ply laminated shell with respect to deflection criteria for different COV of h

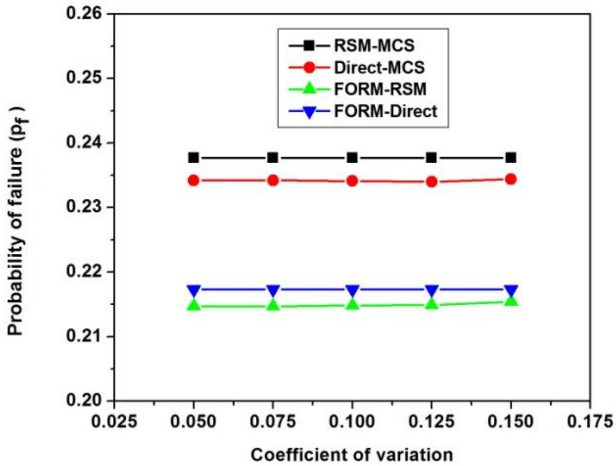

Fig. 8 The probability of failure of symmetric (0°/90°/90°/0°) cross-ply laminated shell with respect to deflection criteria for different COV of R_x

6.2 Example 2: Anti-symmetric (0°/90°/0°/90°) cross-ply laminated shell

Four layered cross-ply anti-symmetric (0°/90°/0°/90°) laminated cylindrical shell is now taken up to further study the capability of RSM based reliability analysis of composite shell structure. As earlier, the central deflection of the shell is approximated by RSM. The nature of polynomial function, number of random variables and DOE scheme remains same as earlier.

To study the capability of RSM to approximate the response of the composite shell, the various statistical indices i.e. RMSE, R^2 and ε_m are computed and the corresponding values are 0.00207, 0.999173 and 0.4289. It can be noted that the value of R^2 is nearest to one and the value of RMSE and direct error (ε_m) are very small which indicates that the RSM can approximate the shell response with sufficient accuracy.

The non-dimensional sensitivity of central deflection with respect to different design variables are shown in Table 5 and the result of sensitivity of deflection by present RSM compares well with that of FEM results for this case also. Fig. 9 shows the variation of sensitivity-based importance factor for central deflection for anti-symmetric (0°/90°/0°/90°) laminated cylindrical shell with respect to all design variables obtained by the RSM. The observations on results are as earlier i.e. the most effective parameters on deflection are E_1, P_0 and h.

Taking allowable deflection 4.4 mm, the variations of probability of failure of central deflection with respect to number of simulations are shown in Fig. 10. Based on which number of simulation for further study is fixed at 20,000 for this example problem also.

Table 6 shows the results in terms of processing times for probability of failure of cross-ply laminated anti-symmetric (0°/90°/0°/90°) shell with respect to deflection criteria. The allowable deflection is taken as 4.4 mm. It can be observed that RSM-MCS can properly estimate the probability of failure as compared to Direct-MCS with less

Table 5 Non-dimensional sensitivity of central deflection with respect to different design variables for anti-symmetric (0°/90°/0°/90°) cross-ply laminated cylindrical shell

Sensitivities	RSM	FEM	Error (%)
$\frac{\partial U}{\partial E_1}$	-0.8623	-0.8278	3.00
$\frac{\partial U}{\partial E_2}$	-0.1367	-0.1316	3.73
$\frac{\partial U}{\partial G_{12}}$	-4.0956×10^{-2}	-3.9326×10^{-2}	3.97
$\frac{\partial U}{\partial G_{13}}$	-7.0261×10^{-4}	-6.8411×10^{-4}	2.63
$\frac{\partial U}{\partial G_{23}}$	-4.3075×10^{-4}	-4.1951×10^{-2}	2.60
$\frac{\partial U}{\partial \nu_{12}}$	-1.5751×10^{-2}	-1.5167×10^{-2}	3.70
$\frac{\partial U}{\partial P_0}$	1.0258	1.0000	2.51
$\frac{\partial U}{\partial h}$	-3.1004	-2.9903	3.55
$\frac{\partial U}{\partial R}$	8.0930×10^{-3}	7.1575×10^{-3}	11.55

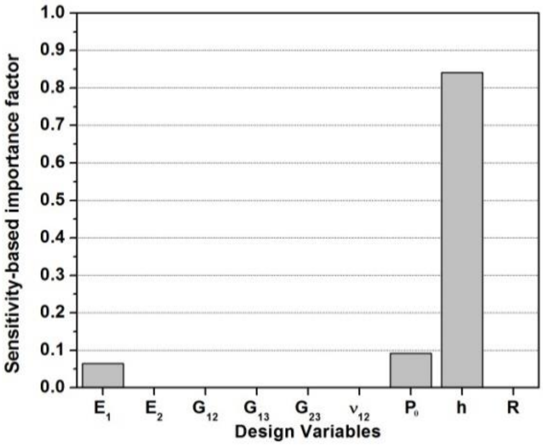

Fig. 9 Sensitivity-based importance factor of central deflection for anti-symmetric (0°/90°/0°/90°) cross-ply laminated shell with respect to different design variables

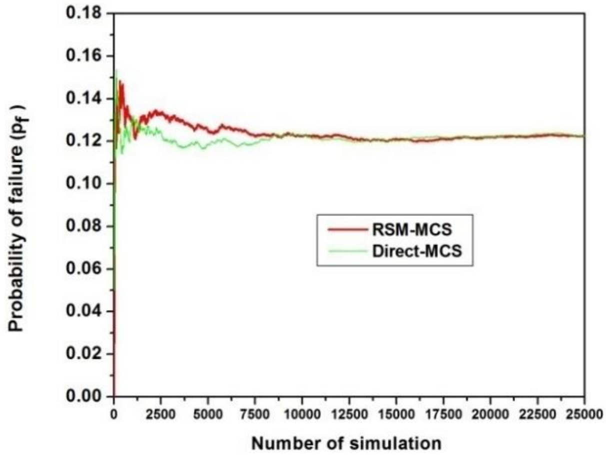

Fig. 10 The probability of failure of anti-symmetric (0°/90°/0°/90°) cross-ply laminated shell with respect to deflection criteria

Table 6 Comparison of processing times for probability of failure of cross-ply laminated anti-symmetric (0°/90°/0°/90°) shell with respect to deflection criteria

Method	Relative processing time (sec)	Probability of failure (p_f)
Direct-MCS	14500	0.1212
RSM-MCS	0.153	0.1235
FORM-Direct	12.46	0.0919
FORM-RSM	0.029	0.0939

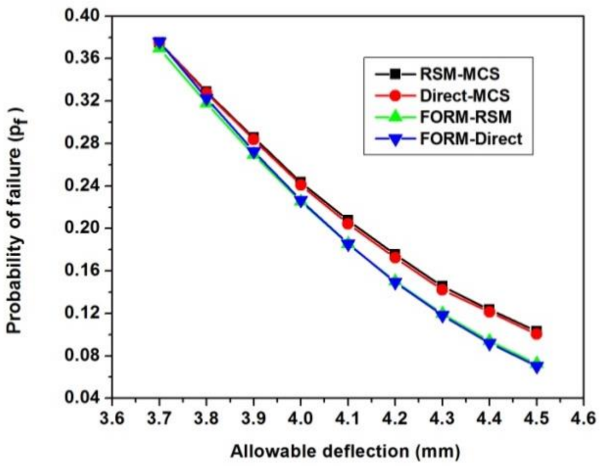

Fig. 11 The probability of failure of anti-symmetric (0°/90°/0°/90°) cross-ply laminated shell with respect to deflection criteria for different allowable deflection

Fig. 11 presents the probability of failure for different allowable deflection for anti-symmetric (0°/90°/0°/90°) laminated cylindrical shell. The observations on the results are same as earlier and capability of RSM based MCS for reliability analysis of composite shell is confirmed for this problem also.

Fig. 12 to Fig. 15 further show the results of parametric study i.e. the variation of the probability of failure of the shell with respect to COV of design parameters E_1, P_0, h and R respectively. The allowable central deflection in each case is taken as 4.4 mm. The same trend is observed i.e. the probability of failure of central deflection is increases with the increase COV values of each design parameters and the RSM based MCS estimate the probability of failure quite accurately whereas FORM based results show considerable error.

7. Summary and conclusions

The reliability assessment of laminated composite shell structure is presented in the framework of RSM to replace the repeated response analysis of complex finite element model of the composite shell. In doing so, useful sensitivity information and importance factor estimation capability of the RSM based approach is also demonstrated. The result of

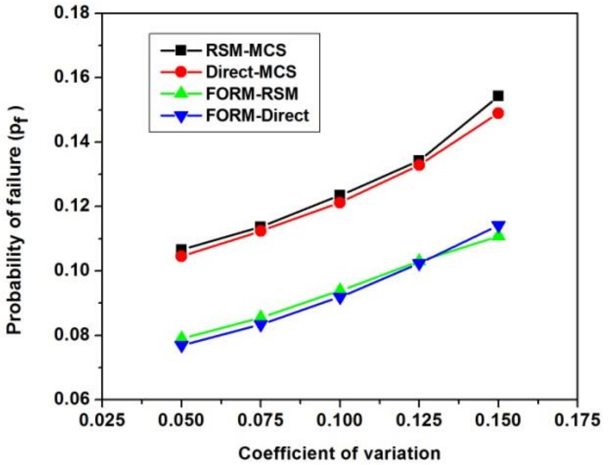

Fig. 12 The probability of failure of anti-symmetric (0°/90°/0°/90°) cross-ply laminated shell with respect to deflection criteria for different COV of E_1

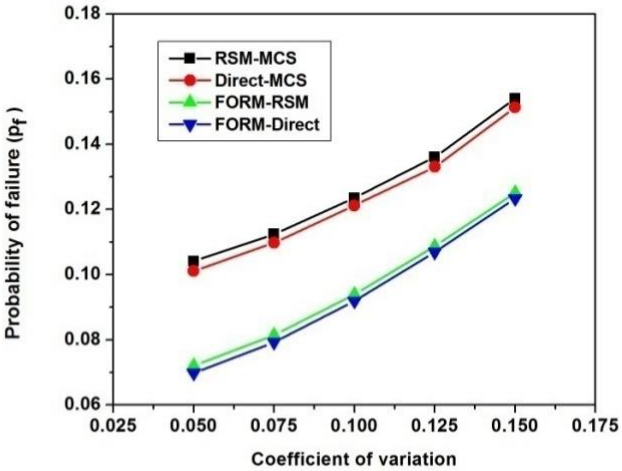

Fig. 13 The probability of failure of anti-symmetric (0°/90°/0°/90°) cross-ply laminated shell with respect to deflection criteria for different COV of P

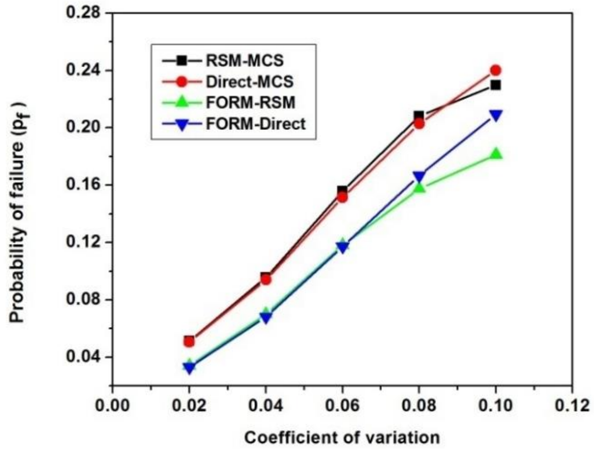

Fig. 14 The probability of failure of anti-symmetric (0°/90°/0°/90°) cross-ply laminated shell with respect to deflection criteria for different COV of h

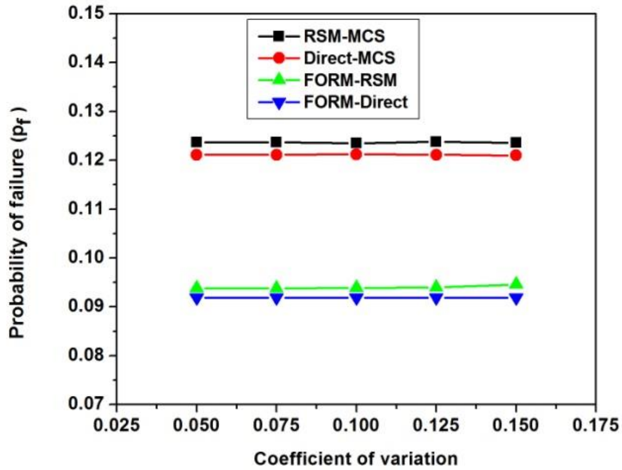

Fig. 15 The probability of failure of anti-symmetric (0°/90°/0°/90°) cross-ply laminated shell with respect to deflection criteria for different COV of R_x

reliability quantities by the RSM based approach is compared with the most accurate direct MCS to study the accuracy of the RSM based approach. It has been generally noted from the numerical study that RSM based MCS can estimate the reliability of the composite shells considered in the presents study for wide range of variations of probability of failures ranges. As the FEM analysis of laminated shell takes much computational time, the RSM can be used efficiently in reliability analysis to balance the accuracy and cost of computational time. In this regards it may be noted that the FEM analysis for deflection of laminated composite shell structures takes 0.72 sec in Matlab programming for single run. Thus, for constructing of RSM for nine input variables by CCD requires 531 runs i.e. about 6.5 minutes whereas for direct MCS with 20,000 runs required about 4 hrs. Thus, RSM based approach can be successfully used as an efficient alternative for reliability estimate of complex composite shell structures.

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APPENDIX A

Rigidity matrix of laminates

$$
\begin{Bmatrix} N_s \\ N_r \\ N_{sr} \\ N_{rs} \\ M_s \\ M_r \\ M_{sr} \\ M_{rs} \\ N_s^* \\ N_r^* \\ N_{sr}^* \\ N_{rs}^* \\ M_s^* \\ M_r^* \\ M_{sr}^* \\ M_{rs}^* \end{Bmatrix} =
\begin{bmatrix}
\overline{A_{11}} & A_{12} & \overline{A_{16}} & A_{16} & \overline{B_{11}} & B_{12} & \overline{B_{16}} & B_{16} & \overline{C_{11}} & C_{12} & \overline{C_{16}} & C_{16} & \overline{D_{11}} & D_{12} & \overline{D_{16}} & D_{16} \\
A_{12} & A_{22} & A_{26} & A_{26} & B_{12} & B_{22} & B_{26} & B_{26} & C_{12} & C_{22} & C_{26} & C_{26} & D_{12} & D_{22} & D_{26} & D_{26} \\
\overline{A_{16}} & A_{26} & \overline{A_{66}} & A_{66} & \overline{B_{16}} & B_{26} & \overline{B_{66}} & B_{66} & \overline{C_{16}} & C_{26} & \overline{C_{66}} & C_{66} & \overline{D_{16}} & D_{26} & \overline{D_{66}} & D_{66} \\
A_{16} & A_{26} & A_{66} & A_{66} & B_{16} & B_{26} & B_{66} & B_{66} & C_{16} & C_{26} & C_{66} & C_{66} & D_{16} & D_{26} & D_{66} & D_{66} \\
\overline{B_{11}} & B_{12} & \overline{B_{16}} & B_{16} & \overline{C_{11}} & C_{12} & \overline{C_{16}} & C_{16} & \overline{D_{11}} & D_{12} & \overline{D_{16}} & D_{16} & \overline{E_{11}} & E_{12} & \overline{E_{16}} & E_{16} \\
B_{12} & B_{22} & B_{26} & B_{26} & C_{12} & C_{22} & C_{26} & C_{26} & D_{12} & D_{22} & D_{26} & D_{26} & E_{12} & E_{22} & E_{26} & E_{26} \\
\overline{B_{16}} & B_{26} & \overline{B_{66}} & B_{66} & \overline{C_{16}} & C_{26} & \overline{C_{66}} & C_{66} & \overline{D_{16}} & D_{26} & \overline{D_{66}} & D_{66} & \overline{E_{16}} & E_{26} & \overline{E_{66}} & E_{66} \\
B_{16} & B_{26} & B_{66} & B_{66} & C_{16} & C_{26} & C_{66} & C_{66} & D_{16} & D_{26} & D_{66} & D_{66} & E_{16} & E_{26} & E_{66} & E_{66} \\
\overline{C_{11}} & C_{12} & \overline{C_{16}} & C_{16} & \overline{D_{11}} & D_{12} & \overline{D_{16}} & D_{16} & \overline{E_{11}} & E_{12} & \overline{E_{16}} & E_{16} & \overline{F_{11}} & F_{12} & \overline{F_{16}} & F_{16} \\
C_{12} & C_{22} & C_{26} & C_{26} & D_{12} & D_{22} & D_{26} & D_{26} & E_{12} & E_{22} & E_{26} & E_{26} & F_{12} & F_{22} & F_{26} & F_{26} \\
\overline{C_{16}} & C_{26} & \overline{C_{66}} & C_{66} & \overline{D_{16}} & D_{26} & \overline{D_{66}} & D_{66} & \overline{E_{16}} & E_{26} & \overline{E_{66}} & E_{66} & \overline{F_{16}} & F_{26} & \overline{F_{66}} & F_{66} \\
C_{16} & C_{26} & C_{66} & C_{66} & D_{16} & D_{26} & D_{66} & D_{66} & E_{16} & E_{26} & E_{66} & E_{66} & F_{16} & F_{26} & F_{66} & F_{66} \\
\overline{D_{11}} & D_{12} & \overline{D_{16}} & D_{16} & \overline{E_{11}} & E_{12} & \overline{E_{16}} & E_{16} & \overline{F_{11}} & F_{12} & \overline{F_{16}} & F_{16} & \overline{G_{11}} & G_{12} & \overline{G_{16}} & G_{16} \\
D_{12} & D_{22} & D_{26} & D_{26} & E_{12} & E_{22} & E_{26} & E_{26} & F_{12} & F_{22} & F_{26} & F_{26} & G_{12} & G_{22} & G_{26} & G_{26} \\
\overline{D_{16}} & D_{26} & \overline{D_{66}} & D_{66} & \overline{E_{16}} & E_{26} & \overline{E_{66}} & E_{66} & \overline{F_{16}} & F_{26} & \overline{F_{66}} & F_{66} & \overline{G_{16}} & G_{26} & \overline{G_{66}} & G_{66} \\
D_{16} & D_{26} & D_{66} & D_{66} & E_{16} & E_{26} & E_{66} & E_{66} & F_{16} & F_{26} & F_{66} & F_{66} & G_{16} & G_{26} & G_{66} & G_{66}
\end{bmatrix}
\begin{Bmatrix} \varepsilon_{so} \\ \varepsilon_{ro} \\ \varepsilon_{sro} \\ \varepsilon_{rso} \\ \kappa_s \\ \kappa_r \\ \kappa_{sr} \\ \kappa_{rs} \\ \varepsilon_{so}^* \\ \varepsilon_{ro}^* \\ \varepsilon_{sro}^* \\ \varepsilon_{rso}^* \\ \kappa_s^* \\ \kappa_r^* \\ \kappa_{sr}^* \\ \kappa_{rs}^* \end{Bmatrix}
$$

$$
\begin{Bmatrix} Q_s \\ Q_r \\ S_s \\ S_r \\ Q_s^* \\ Q_r^* \\ S_s^* \\ S_r^* \end{Bmatrix} =
\begin{bmatrix}
\overline{A_{44}} & A_{45} & \overline{B_{44}} & B_{45} & \overline{C_{44}} & C_{45} & \overline{D_{44}} & D_{45} \\
A_{45} & A_{55} & B_{45} & B_{55} & C_{45} & C_{55} & D_{45} & D_{55} \\
\overline{B_{44}} & B_{45} & \overline{C_{44}} & C_{45} & \overline{D_{44}} & D_{45} & \overline{E_{44}} & E_{45} \\
B_{45} & B_{55} & C_{45} & C_{55} & D_{45} & D_{55} & E_{45} & E_{55} \\
\overline{C_{44}} & C_{45} & \overline{D_{44}} & D_{45} & \overline{E_{44}} & E_{45} & \overline{F_{44}} & F_{45} \\
C_{45} & C_{55} & D_{45} & D_{55} & E_{45} & E_{55} & F_{45} & F_{55} \\
\overline{D_{44}} & D_{45} & \overline{E_{44}} & E_{45} & \overline{F_{44}} & F_{45} & \overline{G_{44}} & G_{45} \\
D_{45} & D_{55} & E_{45} & E_{55} & F_{45} & F_{55} & G_{45} & G_{55}
\end{bmatrix}
\begin{Bmatrix} \varepsilon_{szo} \\ \varepsilon_{rzo} \\ \kappa_{sz} \\ \kappa_{rz} \\ \varepsilon_{szo}^* \\ \varepsilon_{rzo}^* \\ \kappa_{sz}^* \\ \kappa_{rz}^* \end{Bmatrix}
$$

Where, the values of above matrix coefficients are

$$
\left(A_{ij}, B_{ij}, C_{ij}, D_{ij}, E_{ij}, F_{ij}, G_{ij}, H_{ij}\right) = \sum_{k}^{k+1} \overline{Q}_{ij}^{(k)} \int_{z_k}^{z_{k+1}} \left(1, z, z^2, z^3, z^4, z^5, z^6, z^7\right) dz
$$

and

$$
\begin{aligned}
\overline{A_{ij}} &= A_{ij} - C_0 B_{ij}, A_{ij} = A_{ij} + C_0 B_{ij} \\
\overline{B_{ij}} &= B_{ij} - C_0 C_{ij}, B_{ij} = B_{ij} + C_0 C_{ij} \\
\overline{C_{ij}} &= C_{ij} - C_0 D_{ij}, C_{ij} = C_{ij} + C_0 D_{ij} \\
\overline{D_{ij}} &= D_{ij} - C_0 E_{ij}, D_{ij} = D_{ij} + C_0 E_{ij} \\
\overline{E_{ij}} &= E_{ij} - C_0 F_{ij}, E_{ij} = E_{ij} + C_0 F_{ij} \\
\overline{F_{ij}} &= F_{ij} - C_0 G_{ij}, F_{ij} = F_{ij} + C_0 G_{ij} \\
\overline{G_{ij}} &= G_{ij} - C_0 H_{ij}, G_{ij} = G_{ij} + C_0 H_{ij}
\end{aligned}
$$

Where, i, j = 1, 2, 4, 5, 6
And

$$
C_0 = \left(\frac{1}{R_s} - \frac{1}{R_r}\right)
$$

APPENDIX B

The non-zero terms of strain-displacement matrix [B] are as follows

$$B_{11} = B_{42} = B_{54} = B_{85} = B_{96} = B_{12,7} = B_{13,8} = B_{16,9} = B_{17,3} = \frac{\partial N_i}{\partial s}$$

$$B_{13} = -B_{17,1} = \frac{N_i}{R_s}$$

$$B_{22} = B_{31} = B_{65} = B_{74} = B_{10,7} = B_{11,7} = B_{14,9} = B_{15,8} = B_{18,3} = \frac{\partial N_i}{\partial r}$$

$$B_{23} = -B_{18,2} = \frac{N_i}{R_r}$$

$$B_{17,4} = B_{18,5} = N_i$$

$$B_{19,6} = B_{20,7} = 2N_i$$

$$B_{21,6} = \frac{N_i}{R_s}$$

$$B_{22,7} = \frac{N_i}{R_r}$$

$$B_{21,8} = B_{22,9} = 3N_i$$

$$B_{23,8} = \frac{2N_i}{R_s}$$

$$B_{24,9} = \frac{2N_i}{R_r}$$