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Multi-objective BESO topology optimization for stiffness and frequency of continuum structures

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Abstract. Topology optimization of structures seeking the best distribution of mass in a design space to improve the structural performance and reduce the weight of a structure is one of the most comprehensive issues in the field of structural optimization. In addition to structures stiffness as the most common objective function, frequency optimization is of great importance in variety of applications too. In this paper, an efficient multi-objective Bi-directional Evolutionary Structural Optimization (BESO) method is developed for topology optimization of frequency and stiffness in continuum structures simultaneously. A software package including a Matlab code and Abaqus FE solver has been created for the numerical implementation of multi-objective BESO utilizing the weighted function method. At the same time, by considering the weaknesses of the optimized structure in single-objective optimizations for stiffness or frequency problems, slight modifications have been done on the numerical algorithm of developed multi-objective BESO in order to overcome challenges due to artificial localized modes, checker boarding and geometrical symmetry constraint during the progressive iterations of optimization. Numerical results show that the proposed Multiobjective BESO method is efficient and optimal solutions can be obtained for continuum structures based on an existent finite element model of the structures.

Keywords: topology optimization; stiffness problem; frequency problem; BESO; Multiobjective

1. Introduction

Reducing cost and designing an economic structure which is mostly possible by reducing the structural mass is the main goal of structural optimization. Therefore, a structure meeting the goals and expectations of engineering with the least weight is considered as an ideal structure. In the field of structural optimization, optimization occurs in three branches including size, shape, and topology, depending on design variable selection. The goal of topology optimization as the most comprehensive type of structural optimization is determining the mass distribution of structural material in the design space leading to structural performance improvement and weight loss. In this branch of optimization, the design area and boundary conditions are specified first and then the general form of structure, location of members, and geometry of design space are subjected to be changed until the design criteria are achieved.

Topology optimization of structures has been studied extensively in past decades on beams, plates and 2D structures to reduce weight in design area alongside their mechanical performance improvement. It's worth mentioning that many studies have already been made on 2D structures in order to evaluate their mechanical performance. It should be noted that these studies have evaluated mechanical behavior concentrating on the full design area. For example, Hadji, L., Khelifa, Z. and Adda Bedia, E.A. (2016), represented a new higher-order shear deformation model for functionally graded beams. Buckling analysis of isotropic and orthotropic plates using a novel four variable refined plate theory was studied by Bourada, F., Amara, and K., Tounsi, A. (2016). Bennoun, M., et al. (2016), represented a novel five variable refined plate theory for vibration analysis of functionally graded sandwich plates. Wave propagation in functionally graded beams using various higher-order shear deformation beams theories have been studied by Hadji, L., Zouatnia, N., and Kassoul, A. (2017).

Appearance of fast computing methods by computers alongside with several structural optimization methods invention like homogenization (Bendsoe and Kikuchi 1998), solid isotropic material with penalization method (Zhou and Rozvany, 1991, Sigmund and Petersson, 1998, Rozvany *et al.* 1992, Ritz, 2001, Bendsoe and Sigmund, 2003), evolutionary structural optimization method (Xie and Steven, 1993 and 1997), and level-set method (Sethian Wiegmann, 2000, Wan *et al.* 2003) have drawn researchers attention to optimize already known engineering structures. the evolutionary structural optimization method is performed for discrete design variables. It can be said that the optimization procedure is to find the best topology of a structure by determining for every point in the design domain whether there should be material (solid element) or not (void element). In Bi-directional Evolutionary Structural Optimization (BESO) approach which was introduced for the first time by Yang et al. (1999) and then its extended version was presented by Huang and Xie

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(2007), unlike initial methods which gradually omitted unnecessary elements from finite element model of the structure, addition of elements simultaneous with their omission from the design area is possible too. The criterion of addition and omission of an element is determined based on the influence of that element on the changes of objective function. This criterion is called the element sensitivity number. Increasing the structural stiffness against a volume constraint has been noticeably taken into consideration on the most structural optimization problems which have been studied previously. Besides structural stiffness as objective function, frequency optimization is of great importance in many engineering fields like aerospace and automotive. A frequent goal of designing the vibrating structures is to avoid the resonance of the structure for external excitation frequencies achieved by maximizing the fundamental frequency.

Fewer researches have been done on frequency optimization problems compared with wide-ranging articles which have been issued for topology optimization with stiffness as the objective function. Homogenization (Tenek and Hagiwara, 1994, Ma, 1995) and solid isotropic material with penalization approach (Kosaka and Swan, 1999, Du and Olhoff, 2007) are of used methods in researches done on frequency optimization problems. Nevertheless, solid isotropic material with penalization approach does not function properly in low-density regions due to the occurrence of artificial local vibration modes (Pedersen, 2000). Bi-directional Evolutionary Structural Optimization (BESO) algorithm which had previously been implemented efficiently in stiffness problems was applied by Yang et al. (1999) and then its amended version by Zuo and Xie (2010) for solving frequency problems.

Generally, the high ability of the BESO method in solving stiffness problems has been welcomed by researchers of topology optimization area in recent years. Stiffness optimization of elastoplastic structures (Xia *et al.* 2017) and composite materials (Sun *et al.* 2011), tension-based topology optimization (Xia *et al.* 2018), topology optimization of continuous structures with uniform boundaries (Da *et al.* 2017), simultaneous topology optimization of large-scale structures and microstructure for natural frequency (Liu *et al.* 2016), and structural topology optimization under frequency and displacement constraints (Zuo *et al.* 2012) are the newest studies in this field, which used BESO approach.

In most of the topology optimization studies made until now, only one objective function has been considered, but there may be several objective functions in many real case problems. It should be noted that in a single-objective problem, structural performance in other areas might be subjected to changes that are not often favorable in the design process. As a result, optimizing different criteria together should be taken into consideration. There have been few studies on Multiobjective topology optimization compared with single-objective topology optimization. In this regard, Sujin Bureerat and Tawatchai Kunakote have worked on this field using classic optimization evolutionary algorithms (Kunakote and Bureerat, 2011). David Munk et.al (2018) have also worked on this field using an updated smart normal constraint method which is combined with a Bi-directional Evolutionary Structural Optimization (SNC-BESO) algorithm in a multi-physics problem. In this regard, different applications for topology optimization have been also considered by researchers (Zhiyi *et al.* 2018, Zhou, 2016, Nguyen and Lee, 2015, Banh *et al.* 2018).

Stiffness and frequency behavior of structures as two important factors in real-design problems were studied as separate objective functions in past researches (Yang, X., *et al.* (1999) and Zuo, Z.H., *et al.* (2012)). Therefore, establishing a balance between these two features can have high importance in the process of designing and analyzing structures related to automotive, aviation and construction industries. The phenomenon of multiple frequencies, mesh dependency of topology responses, checker-boarding, geometric symmetry constraint, and occurrence of artificial localized vibration modes in low-density regions are the most important challenges faced by the designer in frequency optimization problems which influence the manufacturability of the optimized design too.

In this study, the development of the bi-directional evolutionary algorithm is considered for topology optimization of continuum structures with frequency and stiffness objective functions simultaneously as a dual-purpose optimization. In this regard, the method of weight functions is used for creating one general criterion and so linear combination of these two weighted criteria constitutes a general criterion of the structure. Also, there is no limitation in types and numbers of objective functions within the developed procedure but for maintaining coherence with recent works, stiffness and frequency objective functions are used in this paper. To the best of authors' knowledge Multi-objective BESO method that has been rarely used in previous works, is introduced in this paper for objective functions of compliance and fundamental natural frequency. In fact, it is the sequences of the BESO method in which different objective functions can be combined with weight functions method while there are some tricky computational issues and constraints implementation that should be taken into account.

Topology optimization problem with stiffness objective function which is the base for the development of topology optimization methods and modified Bi-directional Evolutionary Structural Optimization algorithm for frequency problems alongside its relevant challenges have been represented elaborately in previous work of the authors (Teimouri and Asgari, 2018) in a problem of two-dimensional elastic beam. The fundamentals and relations of the multi-objective BESO method has been represented for a general objective function composed of stiffness and first natural frequency criteria. Then the developed algorithm has been implemented in a problem of two-dimensional elastic beam and the best mass distribution is presented in the design area. At the same time, considering the weaknesses of the optimized structure in single-objective optimizations for both stiffness and frequency problems, slight modifications have been made on the numerical algorithm of developed multi-objective BESO in order to overcome challenges due to artificial localized modes, checker-boarding and geometrical symmetry constraint during the progressive iterations of optimization. MATLAB software has been used for implementing the developed algorithm for solving optimization problems while ABAQUS finite element solver has been used for static and modal analysis of the structure. Algorithm stages have been implemented completely through creating an interaction between these two powerful computer tools. The decision for elements addition and deletion is intellectually made in all optimization iterations by calling up the output information of finite element solver by MATLAB and performing sensitivity analysis on all elements. Consequently, an efficient multi-objective Bi-directional Evolutionary Structural Optimization (MBESO) method is developed for topology optimization of frequency and stiffness in continuum structures simultaneously. Numerical results show that the proposed Multi-objective BESO method is efficient and optimal solutions can be obtained for continuum structures based on an existent finite element model of the structures.

2. Multi-objective topology optimization of continuum structures

2.1 The weighed function method for BESO

The Weight Function Method has been implemented in order to create a multi-objective problem based on separated functions. In this method which is regarded as the most popular method for multi-objective optimization problems, a Multi-Objective (MO) problem is changed into a Single-Objective (SO) problem as follows where *F* is multi-criteria objective function, W_i is weight function of criteria *i*, and f_i (x) is criteria *i*.

Minimize or Maximize: $F = \sum_{i=1}^{k} w_i f_i(x)$ (1-a)

Inequality Constraint: $g_j \leq 0$ (1-b)

Equality Constraint: $h_i = 0$ (1-c)

The weight function of each objective function is selected due to its importance by a designer. Using the Weight Function Method:

- Importance of different functions can be specified and,
- A balance between two different criteria with different physical natures can be maintained.

Also, the following equation should be maintained for each weight function:

$$\sum_{i=1}^{k} w_i = 1 \qquad (2)$$

It should be noted that normalizing objective functions with different dimensions or physical natures is very important. One method for this purpose is as follows which should be implemented on all objective functions

$$f_i^{norm} = \frac{f_i(X) - f_i^0}{f_i^{max} - f_i^0}, \quad f_i^0 \equiv \text{utopia point} \qquad (3)$$

Where f_i^0 is the optimum value of the objective function f_i, in single-objective optimization problem and f_i^{max} is its maximum value. The following points are important in weight functions method:

- Obtained solution through this method is definitely an optimum Pareto solution.
- If some of the weight functions are selected zero, a poor Pareto solution is obtained.
- the Collection of Pareto solutions can be obtained by changing weight functions.
- All possible Pareto solutions are not obtained in this method.

Based on the above-mentioned method, the BESO method has been developed for multi-objective optimization. Multi-objective BESO method which will be introduced in this paper for the first time in fact is the sequence of the conventional BESO method in which different objective functions can be combined through weight functions method.

In the Multiobjective BESO approach, at first elements sensitivity numbers are calculated separately for each objective function like a single-objective (SO) problem. Then elemental sensitivity numbers with respect to Multiobjective function (general criterion) are obtained using a linear combination of these SO sensitivity numbers. So, the process of deletion and addition of an element is done based on the sensitivity number. Fundamentals and relations of the Multiobjective BESO method will be represented for a general objective function composed of stiffness and frequency criteria.

2.2 Problem statement

Stiffness is one of the key factors that must be taken into account in the design of a large variety of structures. Commonly the mean compliance *C*, the inverse measure of the overall stiffness of a structure, is considered as the first objective function; hence minimizing structural compliance is equivalent to maximizing its stiffness. Considering structural stiffness as the first criterion of multi-objective topology optimization, the single-objective stiffness problem under a volume constraint is represented as follows

Minimize: $C = \frac{1}{2} f^{T} u$ (4-a)

With volume constraint of: $V^* - \sum_{e=1}^{N} V_i x_i = 0$ (4-b)

Design variable as: $x_i = \{0 \text{ or } x_{min}, 1\}$ (4-c)

On the other hand to improve structural frequency performance, j^{th} natural frequency is considered as the second criterion in Multiobjective topology optimization problem

Maximize: ω_j (5-a)

With volume constraint of: $V^* - \sum_{i=1}^{N} V_i x_i = 0$ (5-b)

Design variable as: $x_i = x_{min}$ or 1 (5-c)

Due to the fact that all objective functions should be simultaneously minimized or maximized in a multiobjective optimization problem, frequency objective function can be represented as follows without changing single-objective frequency problem

$$\text{minimize:} \quad -\omega_j \tag{6}$$

As mentioned before, according to the dimensional difference between the two objective functions, they should be normalized. Using equation (3), the normalized forms of both objective functions are represented in equations 7-a and 7-b which respectively show dimensionless mean compliance and dimensionless j^{th} natural frequency. C^0 and ω_j^0 are numerical values corresponding to the single-objective topology optimization problem

$$C^{norm} = \frac{C - C_i^0}{C^{max} - C^0}, \quad C^0 \equiv \text{utopia point} \tag{7-a}$$

$$(-\omega_j)^{norm} = \frac{-\omega_j + \omega_j^0}{(-\omega_j)^{max} + \omega_j^0}, \tag{7-b}$$

$$(-\omega_j)^0 \equiv \text{utopia point}$$

Finally using the weight functions method, the Multiobjective topology optimization problem with a general objective function (stiffness- frequency) is represented as follows

$$\text{Minimize:} \quad F = w_C C^{norm} + w_\omega (-\omega_j)^{norm} \tag{8-a}$$

$$\text{Volume constraint of:} \quad V^* - \sum_{i=1}^{N} V_i x_i = 0 \tag{8-b}$$

$$\text{Design variable:} \quad x_i = \{x_{\min}, 1\} \tag{8-c}$$

2.3 Implementing Weighted Multiobjective BESO

2.3.1 Interpolation Relation of Material Distribution

The relation between elemental design variable and material properties (Young's modulus and Density) should be specified in a soft-kill approach. This relation has been presented and discussed separately for both stiffness and frequency problems in reference (Teimouri and Asgari, 2018). A unique relation for material distribution should be specified in a Multiobjective topology optimization problem, which satisfies challenges related to each objective function. Presented interpolation relation for stiffness problem is not usable in frequency problem due to the occurrence of artificial vibration modes in low-density regions (Huang, 2010). Therefore, it can be said that the occurrence of artificial modes is still an imitating factor in a Multiobjective stiffness and frequency optimization problem. In this research, the interpolation scheme for frequency problem (Huang, 2010) will be used in the Multiobjective problem as the material distribution relation as follows so that E^1 and ρ^1 are respectively density and Young's modulus of applied material in the structure. In most studies with the mentioned approach for stiffness problems, penalty values of p and q are considered respectively 1 and 3.

$$\rho(x_i) = x_i \rho^1 \tag{9a}$$

$$E(x_i) = \left[\frac{1 - x_{min}}{1 - x_{min}^p} x_i^p - \frac{x_{min}^p - x_{min}}{1 - x_{min}^p}\right] E^1 \tag{9b}$$

2.3.2 Sensitivity Analysis and Sensitivity Number

By calculating the variation of general objective function (Equation of 8-a) with respect to changes of the elemental design variable, the sensitivity analysis is performed as follows

$$\frac{dF}{dx} = w_C \frac{dC^{norm}}{dx} + w_\omega \frac{d(-\omega_j)^{norm}}{dx} = w_C \alpha_C^i + w_\omega \alpha_\omega^i \tag{10}$$

Performing a sensitivity analysis based on the above-mentioned material distribution scheme over each dimensionless objective function of compliance and frequency, the elemental sensitivity numbers corresponding to each one of the objective functions are calculated as follows:

In the stiffness problem

$$\alpha_C^i = \frac{-1}{p} \frac{\mathrm{d}C^{norm}}{dx_i} = \left(\frac{1}{k_C}\right) \begin{cases} \frac{1}{2} u_i^T K_i^1 u_i & x_i = 1 \quad ? \\ \frac{1}{2} u_i^T \frac{1 - x_{\min}}{1 - x_{\min}^p} x_{min}^{p-1} K_i^1 u_i & x_i = x_{min} \end{cases} \tag{11}$$

And in the frequency problem

$$\alpha_\omega^i = \frac{-1}{p} \frac{\mathrm{d}\omega_j^{norm}}{\mathrm{d}x_i} = \left(\frac{1}{k_\omega}\right) \begin{cases} \frac{1}{2\omega_j} u_j^T \left(\frac{1 - x_{\min}}{1 - x_{\min}^p} K_i^1 - \frac{\omega_j^2}{P} M_i^1\right) u_j & x_i = 1 \\ \frac{1}{2\omega_j} u_j^T \left(\frac{x_{\min}^{p-1} - x_{\min}^p}{1 - x_{\min}^p} K_i^1 - \frac{\omega_j^2}{P} M_i^1\right) u_j ? & x_i = x_{min} \end{cases} \tag{12}$$

Wherein, p is the penalty parameter related to the soft-kill approach and $k_c$and k_w are the numerical constant values which are resulted from dimensionless objective function derivatives.

If x_{min} descends to zero (a number such as 10^{-6}), the elemental sensitivity number will be as follows:

In the stiffness problem

$$\alpha_C^i = \frac{-1}{p} \frac{d\,C^{norm}}{dx_i} = \left(\frac{1}{k_C}\right) \begin{cases} \frac{1}{2} u_i^T K_i^1 u_i & x_i = 1 \quad ? \\ \frac{1}{2} u_i^T x_{min}^{p-1} K_i^1 u_i & x_i = x_{min} \end{cases} \tag{13}$$

And in the frequency problem

$$\alpha_\omega^i = \frac{-1}{p} \frac{\mathrm{d}\omega_j}{\mathrm{d}x_i} = \left(\frac{1}{k_\omega}\right) \begin{cases} \frac{1}{2\omega_j} u_j^T \left(K_i^1 - \frac{\omega_j^2}{P} M_i^1\right) u_j & x_i = 1 \\ \frac{1}{2\omega_j} u_j^T \left(-\frac{\omega_j^2}{P} M_i^1\right) u_j ? & x_i = x_{min} \end{cases} \tag{14}$$

When calculating frequency sensitivity numbers in problems with symmetry in geometry and loading, the symmetry of frequency sensitivity numbers must be considered.

Finally, the total sensitivity number of elements related to the general objective function is defined as follows

$$\alpha^i = \frac{-1}{p}\frac{dF}{dx} = (\frac{-1}{p})(w_C \frac{dC^{norm}}{dx} + w_\omega \frac{d(-\omega_j)^{norm}}{dx}) \\ = w_C \alpha_C^i + w_\omega \alpha_\omega^i \quad (15)$$

2.3.3 Sensitivity numbers improvements:

To solve checker-boarding and mesh dependency problem in the BESO method, which are the most important numerical problems in a topology optimization problem, a method has been used to improve the elemental sensitivity numbers in which elemental sensitivities are converted to nodal sensitivity numbers (α_i^n) and then again the sensitivity number related to each element is obtained from the numbers assigned to the nodes being present in a specific radius of that element called the filter radius (Yang *et al.* 1999)

$$\widehat{\alpha_i} = \frac{\sum_{j=1}^{M} w(r_{ij})\alpha_j^n}{\sum_{j=1}^{M} w(r_{ij})} \quad (16)$$

In which r_{ij} is the distance between the center of the ith element and jth node. M is the total number of nodes in the structure and $w(r_{ij})$ is the weight factor assigned to the related node which is calculated from the following equation

$$w(r_{ij}) = \begin{Bmatrix} r_{min} - r_{ij} & \text{for } r_{ij} < r_{min} \\ 0 & \text{for } r_{ij} \geq r_{min} \end{Bmatrix} \quad (17)$$

Based on gained experience in computer computations of evolutionary algorithms, averaging from each general improved elemental sensitivity number reduces numerical instabilities in the convergence of objective function considering the history of a sensitivity number in the optimization process.

$$\alpha_i = \frac{\alpha_i^k + \alpha_i^{k-1}}{2} \quad (18)$$

In this equation, k is the optimization iteration number. In each optimization iteration $\alpha_i^k = \alpha_i$ will be used for the next iteration.

Ideal optimality criteria in the BESO approach is such that the sensitivity numbers of all elements remain the same if the elemental design variables are not changed. Therefore, the elemental design variable x_i of elements with high sensitivity numbers should be increased and the elemental design variable x_i of elements with low sensitivity numbers should be decreased. Considering that in the BESO approach, the design variable is a discrete quantity and only values of 1 or x_{min} is allowed to be allocated, the optimality criteria is described in a way that solid elements sensitivity numbers are always more than soft elements sensitivity numbers, therefore, for elements with low sensitivity numbers, design variables are changed

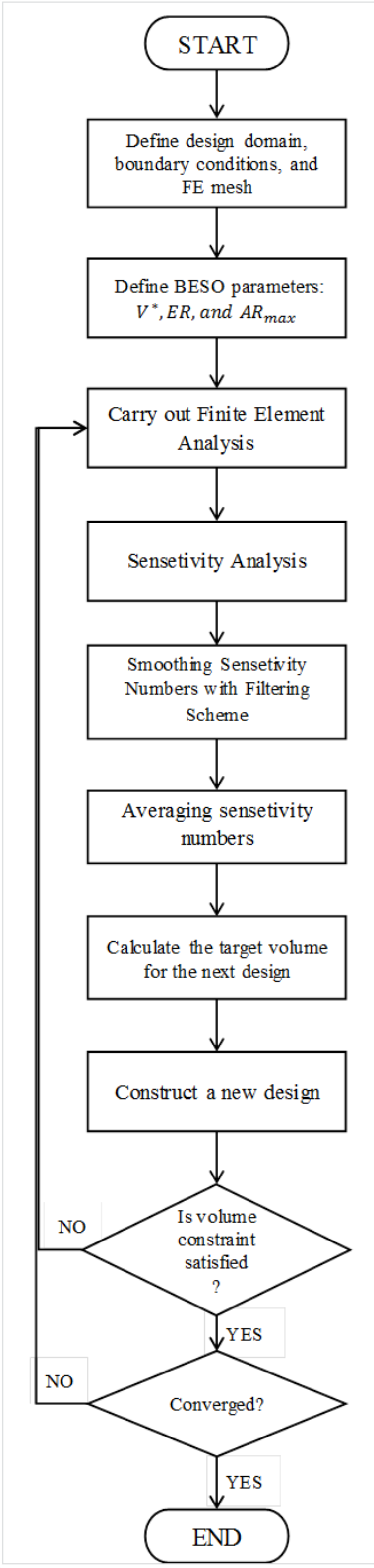

Fig. 1 Flowchart of the BESO method

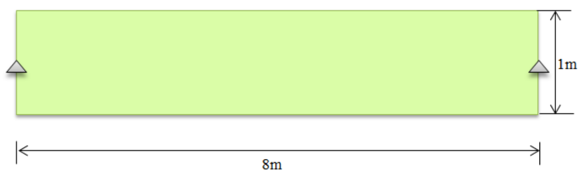

Fig. 2 Design domain of a simply supported 2D beam

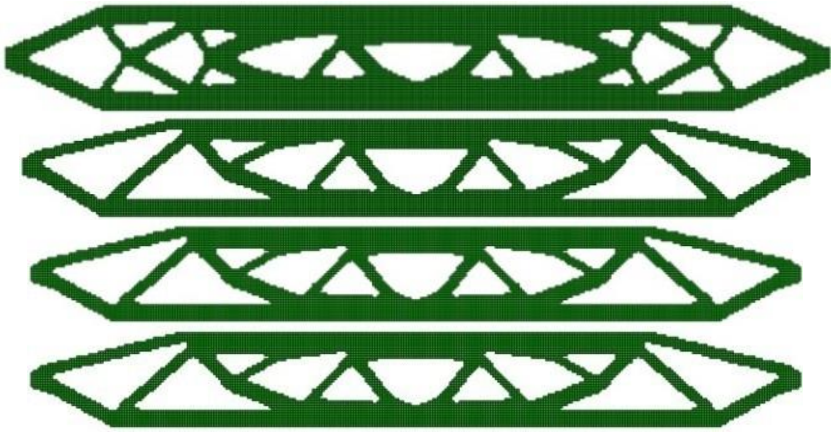

Fig. 3 Topology evolution histories of the structure for maximum stiffness and volume constraint of 50% at iteration: a) 10, b) 25, c) 50, and d) 43

from 1 to x_{min} and for elements with high sensitivity numbers, design variables are changed from x_{min} to 1.

Decrease and increase of the volume fraction (the number of deleted / added elements to the total number of elements of the design area in each iteration) is determined by two parameters of AR_{max} and ER. The volume of the structure in each step is calculated according to equation (19), and as soon as reaching the design objective volume (satisfied optimization constraint), its value is kept constant. Then the objective function convergence is evaluated.

$$V_{k+1} = V_k(1 \pm ER) \qquad (k = 1,2,3 \ldots) \tag{19}$$

The repeatable process of the BESO algorithm is shown in Fig. 1 which explains the necessary steps for reaching a reasonable topology through meeting the objective function and satisfying the design constraint.

2.3.4 Geometrical symmetry

In most structural optimization problems, for problems with symmetry in loading and fundamental geometry, geometrical symmetry of the resulted topology is considered as a logical requirement in the optimization process. In the first step, what is geometrically needed to achieve a symmetric topology, is having the same elemental sensitivity numbers for elements geometrically symmetric in order to have an equal chance of deletion/addition. Then, the designer should consider elements deletion/addition procedure to satisfy the geometrical symmetry. In this research, the symmetry constraint in the multi-objective BESO approach is considered in the deletion /addition stage of the main optimization process.

3. Numerical implementation and results

Multiobjective topology optimization for objective functions of stiffness and fundamental natural frequency has been considered in this section. In this example, the goal of the optimization is to maximize the fundamental frequency and stiffness of a 2D structure (Fig. 2). The objective volume is 50% of the structure's initial volume and dimensions of the beam are considered as 8m×1m. Four-node plane stress elements are used for discretization of the design area. The Young's modulus and the Poisson's

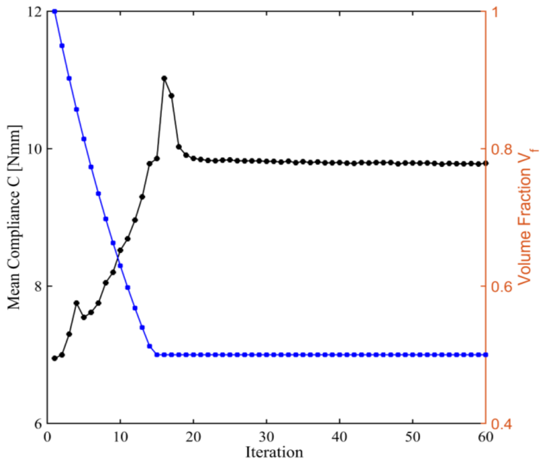

Fig. 4 Evolution histories of the mean compliance and volume fraction for stiffness objective function and volume constraint of 50%

ratio of the structure are respectively E=10MPa and v=0.3, and the structural density is considered ρ=1kg/m^3. The number of the elements is proportional to the dimensions of the beam (40×320). The required parameters for the BESO procedure are selected as follows:

ER=0.02, AR_{max}=0.02, x_{min}=10^{-6}, r_{min}= 0.075m and penalty parameter is p=3.

A 1N force is considered for stiffness optimization of the structure in the middle of the beam lower length. The two-objective BESO procedure starts to reduce the volume of a structure by deletion/addition of elements from the design area in a gradual process which is determined by ARmax and ER. By allocating numerical quantities for the above-mentioned parameters in the MATLAB optimization code, and through using ABAQUS software for modal and static analysis, the elements that should be omitted or added to the structure will be specified in optimization iterations. Finally, the iterations will be continued up to convergence of the general objective function. In the following sections, the optimization results for both objective functions will be represented separately. Then the two-objective problem will be solved considering the general objective function which is a linear combination of stiffness and fundamental natural frequency.

Table 1 comparing the results of optimized structure with initial structure

	Mean compliance [Nmm]	Fundamental natural frequency [rad/sec]	Volume fraction
Initial structure	6.949	135.67	1
Optimized structure with BESO method	9.8	147	0.5

Fig. 5 First mode shape of the structure having an Eigen value of 147 [rad/sec]

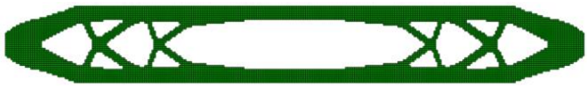

Fig. 6 Optimum topology of the structure for maximum fundamental frequency and volume constraint of 50%

3.1 Stiffness optimization

A single objective topology optimization for stiffness has been comprehensively discussed in the reference (Teimouri and Asgari 2018). In Figs. 3 and 4, the structure topology evolution as well as the historical evolution of the objective function and volume fraction is illustrated. Satisfying the volume constraint after 15 iterations, the compliance of the structure is converged to a magnitude of 9.73 [N.mm]. In the present research, the convergence is considered for 0.01%.

In table 1, the topology optimization results of the 2D beam with maximized stiffness as objective function and volume constraint of 50% is compared with the initial structure. To evaluate the frequency performance of the structure, the frequency response of the final topology is shown in the Fig. 5.

3.2 Fundamental natural frequency optimization

The topology optimization of this structure considering the fundamental natural frequency as the objective function is also offered in the reference (Teimouri and Asgari, 2018) and (Yang *et al.* 1999). Implementing the same approach, the final topology of the structure is shown in Fig. 6. In table 2, topology optimization results of the beam structure for fundamental natural frequency and volume constraint of 50% is compared with those of the initial structure. Using the modified BESO presented by authors (Teimouri and Asgari, 2018), the fundamental natural frequency of the structure is increased for 25% and at the same time, its mass (volume) is halved.

As it's understood from table 2, unlike the stiffness objective function which resulted in improvement of the structure frequency behavior, the compliance of the

Table 2 Comparing the results of optimized structure with initial structure

	Mean Compliance [Nmm]	1^{st}Natural frequency [rad/sec]	2^{nd}Natural frequency [rad/sec]	Volume fraction
Initial structure	6.949	135.67	493.36	1
Optimized structure with BESO method	35	170	172.73	0.5

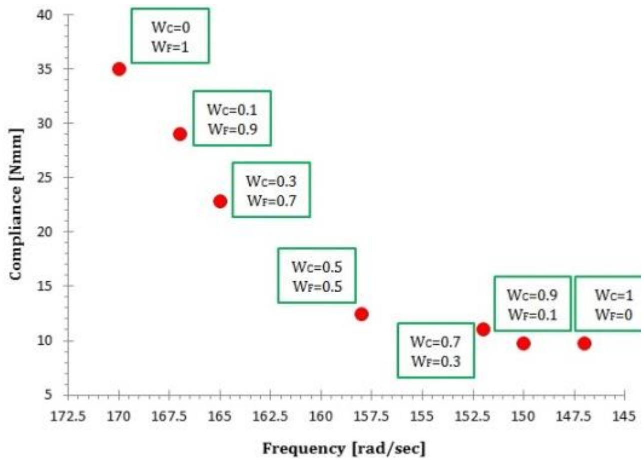

Fig. 7 The Pareto-front for different weighed coefficients

structure is noticeably increased during frequency optimization which is considered as a great disadvantage in the single objective topology optimization for frequency in bridge-like structures.

3.3 The Multiobjective BESO for optimization of stiffness and natural frequency

In this section, two-objective topology optimization results for a 2D beam problem (Fig. 2) are represented using the weighted BESO method (combination of BESO and weighed function methods).

According to the problem statement in equation (8), the obtained results for seven different coefficients (weighed functions of w_c and w_ω) are presented in table 3 as some of the Pareto solutions.

In Fig. 7 the Pareto-front of optimized objective functions of stiffness and fundamental natural frequency is illustrated for different weighted coefficients. Fig. 8 shows the evolution of compliance objective function for different weighted coefficients. According to Fig. 8, increasing the compliance weighed coefficient leads to an increase in its importance in general design criteria directing the beam topology design toward a stiffer structure with weaker frequency performance. Finally, by changing the weighed compliance coefficient to "1", the Multiobjective problem is practically converted to a single objective problem with minimum mean compliance (stiffest structure).

In Fig. 9 the evolution of fundamental natural frequency is shown for different weighted coefficients. According to the figure, increasing the frequency weighted coefficient leads to an increase in its importance in the general design

Table 3 Topology optimization results of two-objective problem (stiffness and fundamental frequency) using weighed BESO method

w_C	w_ω	C [Nmm]	ω_j [rad / s ec]	C^{norm}	$(-\omega_j)^{norm}$	F
0	1	35	170	0.46	-0.06	-0.0636
0.1	0.9	29	167	0.48	0.12	0.1543
0.3	0.7	22.8	165	0.32	0.13	0.1897
0.5	0.5	12.4	158	0.059	0.32	0.1891
0.7	0.3	11	152	0.024	0.49	0.1638
0.9	0.1	9.8	150	-0.004	0.56	0.0524
1	0	9.8	147	-0.005	0.66	-0.005

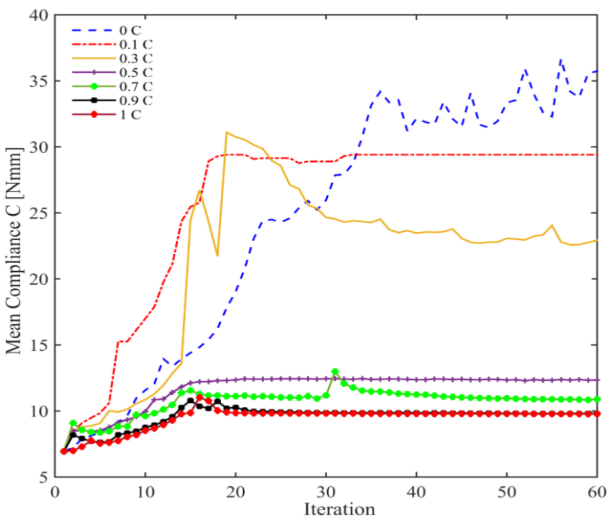

Fig. 8 Evolution histories of the mean compliance for different weighed coefficients

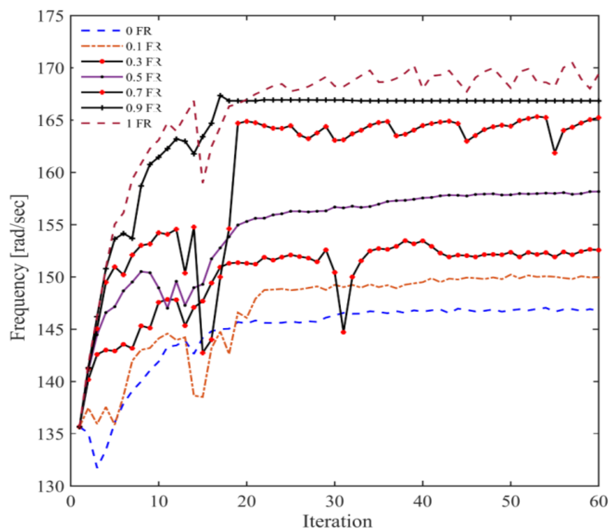

Fig. 9 Evolution histories of the fundamental natural frequency for different weighed coefficients

criteria and as a result of that the beam topology design will result in a softer structure with better frequency performance. Finally, by changing the frequency weighted coefficient to “1”, the two-objective problem is practically converted to a single objective problem with maximum

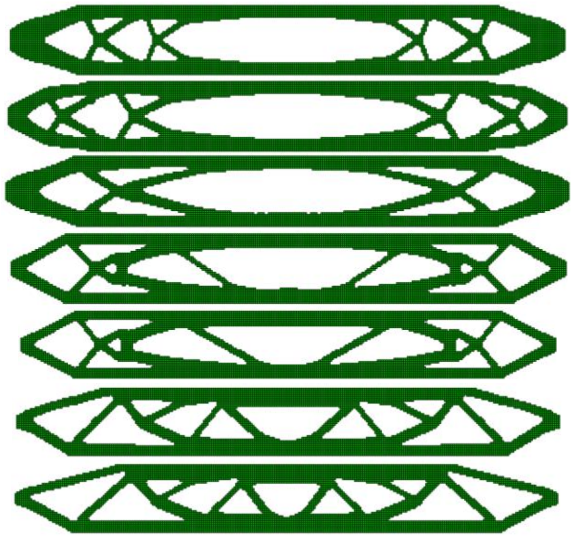

Fig. 10 Final topologies of the two-objective optimization problem for different weighed coefficients of: a) 0, b) 0.1, c) 0.3, d) 0.5, e) 0.7, f) 0.9, and g) 1

fundamental natural frequency (best frequency performance).

In Fig. 10, the topologies for different weighted coefficients are shown. According to the figure, the middle parts of the structure are more important in its stiffness. In addition, these parts have greater impact on the structural mass and their omission will result in the fundamental natural frequency increase. Therefore selecting of a middle topology, in which both objective functions have suitable performance, is of great importance.

4. Conclusion

Considering the importance of different objective functions in structural problems and the necessity of Multiobjective optimization, in this paper, an efficient Multiobjective BESO method was developed for topology optimization of continuum structures simultaneously. The stiffness and frequency problem is solved as a two-objective topology optimization problem in a 2D continuum structure (rectangular beam). For this purpose, at first, both objective functions are optimized on mentioned geometry as single objective functions. According to the results, unlike stiffness single objective problem that leads to improvement of the structure frequency performance, the frequency single objective problem has no positive role in structure stiffness improvement and the structure compliance increases noticeably. Therefore by implementing the Multiobjective BESO based on the weighed coefficient method, a balance between both objective functions is made in the design. This type of topology optimization procedure can mostly be used in designing bridge-like structures or structures related to vehicle and aerospace industries where the stiffness and frequency performance are simultaneously important. During the progressive iterations of optimization, some modifications have been made on the developed Multiobjective BESO algorithm in order to overcome numerical challenges raised from artificial localized modes, checkerboarding and geometrical symmetry constraint. By implementing the Multiobjective BESO method in MATLAB-ABAQUS software package, the topologies resulted from different weighted coefficients are stiffer in comparison with a single objective frequency problem and have better frequency performance compared to the single objective stiffness problem. The Multiobjective BESO method in MATLAB-ABAQUS software package has been practically implemented and is applicable for 2D structures with different geometrical and boundary conditions and other objective functions. Numerical results show that the proposed Multiobjective BESO method is efficient and its optimal solutions can be obtained for continuum structures based on an existent finite element model of the structures.

Symbols

English symbols

ER,AR	evolutionary ratio, incremental ratio
E	Elasticity Modulus, N/m^2
KE, ME Element	Stiffness matrix (N/m), element mass matrix (kg)
u_i	elemental displacement vector
x	elemental design variable

Greek symptoms

P	density, kg/m^3
ω_j	j-fundamental frequency (rad/sec)
a	modified element sensitivity number

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