Structural Engineering and Mechanics, *Vol. 72, No. 1 (2019) 31-41*
DOI: https://doi.org/10.12989/sem.2019.72.1.031

Relevance vector based approach for the prediction of stress intensity factor for the pipe with circumferential crack under cyclic loading

A. Ramachandra Murthy*, S. Vishnuvardhan[a], M. Saravanan[b] and P. Gandhi[c]

CSIR-Structural Engineering Research Centre, Taramani, Chennai, India - 600113

(Received April 6, 2019, Revised May 1, 2019, Accepted May 2, 2019)

Abstract. Structural integrity assessment of piping components is of paramount important for remaining life prediction, residual strength evaluation and for in-service inspection planning. For accurate prediction of these, a reliable fracture parameter is essential. One of the fracture parameters is stress intensity factor (SIF), which is generally preferred for high strength materials, can be evaluated by using linear elastic fracture mechanics principles. To employ available analytical and numerical procedures for fracture analysis of piping components, it takes considerable amount of time and effort. In view of this, an alternative approach to analytical and finite element analysis, a model based on relevance vector machine (RVM) is developed to predict SIF of part through crack of a piping component under fatigue loading. RVM is based on probabilistic approach and regression and it is established based on Bayesian formulation of a linear model with an appropriate prior that results in a sparse representation. Model for SIF prediction is developed by using MATLAB software wherein 70% of the data has been used for the development of RVM model and rest of the data is used for validation. The predicted SIF is found to be in good agreement with the corresponding analytical solution, and can be used for damage tolerant analysis of structural components.

Keywords: Piping component; Cyclic loading; Stress intensity factor; Relevance vector machine

1. Introduction

The piping components of power plants, offshore oil and gas industry are generally subjected to cyclic loading and it is found that several failures are due to fatigue/cyclic loading. The failures are observed to occur well below the allowable stress limits under normal operating conditions and they are attributed to the presence of flaws. To evaluate structural integrity of piping components, a detailed analysis is required where in the reliable fracture parameter, stress intensity factor (SIF) is to be evaluated. Under linear elastic fracture mechanics domain, SIF governs design. Further, to design the structural component by leak before break (LBB) concept, evaluation of SIF is must. By using SIF, crack growth, remaining life and residual strength can be predicted which are very important for in-service inspection planning. In general, SIF can be determined by two ways, namely, numerical and analytical.

SIF can be determined by using a handbook (Tada *et al.* 1973; Sih, 1973; Rooke and Cartwright, 1976) or by employing numerical methods (Bergman and Brickstad, 1991, Zahoor, 1985, Zareei and Nabavi, 2016, Miyazaki and Mochizuki, 2011, and Kumar *et al.* 1985). Typical remaining life prediction process is explained in Fig. 1 (Yaguo Lei, 2017). From Fig. 1, it can be noted that Stage I indicates a normal operation stage and Stage II is an accelerated degradation stage throughout design period. During Stage I, the pipe is in healthy condition. After the damage or deterioration starts, the pipe condition goes to Stage II, where in remaining useful life (RUL) prediction is very important. The time to start prediction is defined as the first predicting time (FPT), which is denoted by t_{FPT} in Fig. 1. From Fig. 1, it can be observed that FPT is the transition between Stage I and Stage II. Fig. 1, (A) describes the RUL prediction process at a single time point t_k, and Fig. 1 (B) presents the real-time RUL prediction during Stage II. The real-time RUL prediction can be obtained by carrying out the RUL prediction from t_{FPT} to t_{EoL}.

Fatigue crack growth studies were also carried out on plate specimens with part through crack (Bhargava *et al.* 1998, Brennan *et al.* 2008) and on carbon steel pipes (Singh *et al.* 2003). In the present paper, an alternative approach based on statistical method for prediction of SIF is proposed. From the literature, it was observed that several advanced statistical models, namely, Gaussian regression process, extreme learning machine, least squares support vector machine, Artificial Neural Network, Relevance vector machine, support vector machine, multivariate adaptive regression splines are available to develop the models for the desired output (Yuvaraj *et al.*2013a, Yuvaraj *et al.* 2013b, Yuvaraj *et al.* 2014a, Yuvaraj *et al.* 2014b, Shantaram *et al.* 2014, Vishal *et al.* 2014, Susom Dutta *et al.* 2017, Jaideep Kaur *et al.* 2017, Erdem, 2017, Engin *et al.* 2015, Mansouri *et al.* 2016, Keprate, 2017). From the literature, it was observed that each model has its own merits and limitations.

*Corresponding author, Principal Scientist
E-mail: murthyarc@serc.res.in
[a] Senior Scientist
[b] Senior Scientist
[c] Chief Scientist

http://www.techno-press.com/journals/sem&subpage=7
ISSN: 1225-4568 (Print), 1598-6217 (Online)

Table 1 Specimen details

Pipe Material	SA 333 Gr.6	Diameter of pipe (mm)	219
Yield stress (MPa)	302	Thickness of pipe (mm)	15.13
Ultimate tensile stress (MPa)	450	Length of pipe (mm)	3003
Elongation %	36.7	Initial crack depth, a, mm	5.5
Young's modulus (GPa)	203	Initial notch length, 2c, mm	113.2
Fracture toughness (MPa√m)	55	Crack growth constants, C	8.346×10^{-11}
		m	2.3

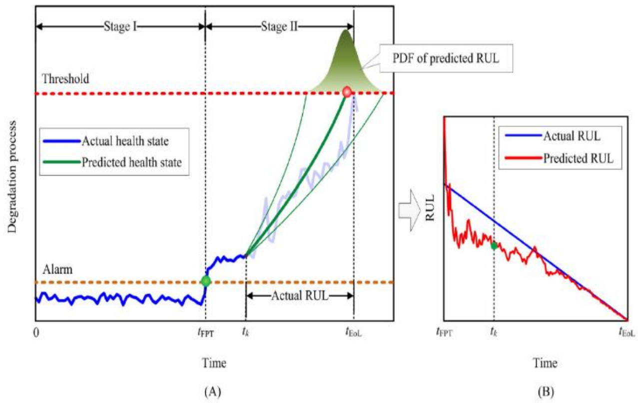


Fig. 1 Typical remaining useful life prediction process (Yaguo Lei, 2017)

In the present investigation, it is proposed to employ relevance vector machine (RVM) to predict SIF of a pipe with circumferential crack subjected to cyclic loading. RVM is an intelligent learning technique based on sparse Bayesian framework and the number of relevance vectors required for RVM is smaller compared to support vectors in support vector machine (Tipping, 2001, John *et al.* 2010). For the past several years, support vector machine (SVM) has been popular which is based on an alternative kernel technique for development of models. Although, SVM performance is significant, due to certain drawbacks, RVMs based approach has been proposed by improving the concepts of SVMs in a Bayesian context and can be effectively used for regression and classification problems. RVM offers several advantages that include the probabilistic predictions, automatic estimation of parameters, and the facility to utilise arbitrary basis functions. Further, there is no need to set the penalty parameter in RVM, which makes RVM more convenient to use than SVM. Sparsity can be achieved because in practice the posterior distributions of many of the weights are sharply peaked around zero. We call those training vectors associated with the remaining non-zero weights 'relevance' vectors, in deference to the principle of automatic relevance determination which motivates the approach (MacKay, 1994, Neal, 1996). The significant feature of RVM is that it uses lesser kernel functions. Further, it was mentioned that the run time of RVM is considerably faster than SVM.

Numerous applications of RVM were observed in the literature to predict the desired output (Wahyu *et al.* 2009, Dawei *et al.* 2002, Wei *et al.* 2005, Sarat Kumar Das and Pijush Samui 2008, Achmad *et al.* 2009, Xiaodong Wanga *et al.* 2009, Kefei Liu and Zhisheng Xu, 2011, Yuvaraj *et al.* 2014b). Enrico and Francesco (2012) predicted the crack growth in a structure by using RVM. Nurcihan (2014) predicted uniaxial compressive strength of volcanic rocks by using RVM and SVM. Sheng *et al.* (2015) presented the hybrid model of wavelet decomposition and artificial bee colony algorithm-based relevance vector machine (WABCRVM) for wind speed prediction. Recently, Prasanna *et al.* (2018) predicted compressive strength of various GGBS based concrete mixes for 28, 56, 90 and 180 days by using RVM.

The main objective of this manuscript is to present the mathematical background and the procedure for building, training and testing of the RVM model which will be used to predict SIF of pipeline.

2. Stress intensity factor for A Pipe with circumferential crack

Experiment was carried out on a piping component under fatigue loading. The nature of the crack is part-through in the circumferential direction. The details of the specimen are given below (Table 1) (Singh *et al.* 2003).

From the experiment, the out put such as number of cycles to failure, crack depth, crack length etc. were recorded under cyclic loading (Singh *et al.* 2003). SIF has been evaluated based on the crack profile obtained with respect to definite number of cycles. Although, several methods are available to evaluate SIF, RCC-MR approach has been followed in the present study. Brief details about RCC-MR are given below (Marie *et al.* 2007)). SIF for the case of piping component with part through external crack is given by (Fig. 2)

$K_I = \{\sigma_0 i_0 + \sigma_1 i_1(a/t) + \sigma_{gb} F_{gb}\}\sqrt{(\pi a)}$ for deepest point

$K_I = \{\sigma_0 i_0 + \sigma_1 i_1(a/t) + \sigma_{gb} F_{gb}\}\sqrt{(\pi c)}$ for surface point

Where,

In the present study, there is no internal pressure and thermal expansion is neglected. Only σ_{gb} is considered.

$\sigma_0 = [N_1/\{\pi(R_{out}^2 - R_{in}^2)] + [P\{R_{in}^2/(R_{out}^2 - R_{in}^2)\}] + \{(2E\alpha\theta_1 R_{out}/2(1-\mu)3t)*(1-[2R_{in}^2/R_{out}(R_{out+}R_{in})])\}$

$\sigma_1 = -E\alpha\theta_1/2(1-\mu)$

$\sigma_{gb} = M_2 R_{out}/[(\pi/4)(R_{out}^4 - R_{in}^4)]$

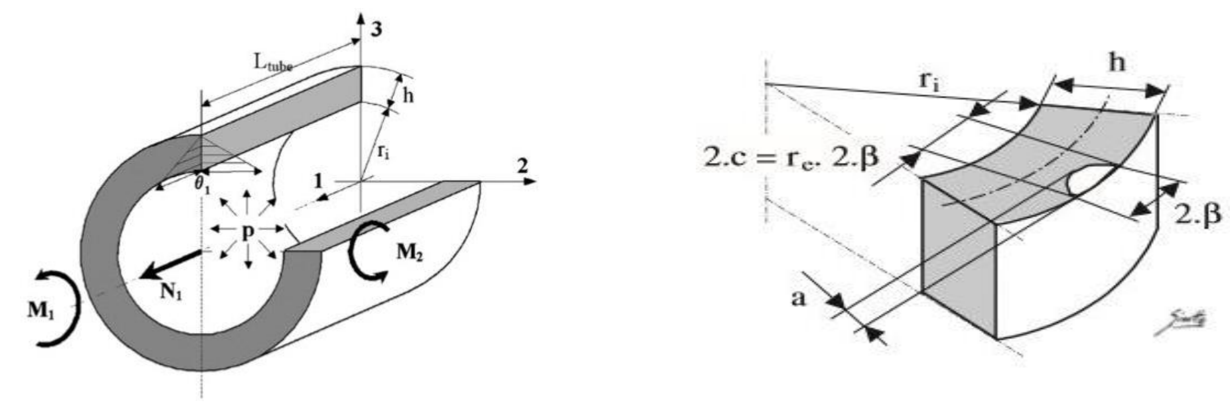
SIF at deepest point:

$$SIF\_Max = \{(F\_gb\_deep*\sigma\_gb\_max)*(\sqrt{(\pi*a)})\} \quad (1)$$

$$SIF\_Min = \{(F\_gb\_deep*\sigma\_gb\_min)*(\sqrt{(\pi*a)})\} \quad (2)$$

$$\sigma\_gb\_max = (M\_max * r_e)/\{(\pi/4)*(r_e^4 - r_i^4)\} \quad (3)$$

$$\sigma\_gb\_min = (M\_min * r_e)/\{(\pi/4)*(r_e^4 - r_i^4)\} \quad (4)$$



GEOMETRY OF THE TUBE:

r_i – internal radius
r_e – external radius
r_m – average radius
h – thickness
L_{tube} – length of tube
a: depth of defect
2c : length of the defect ($2c = 2\beta r_e$)
2β: angle of the defect (in radians)symmetrical position in relation to the bending plane

Loads:

P – Internal pressure
M_1 – Torsional moment along the axis 1
ϕ_1 – Rotation of section around axis 1
M_2 – Global bending moment along the axis 1
ϕ_1 – Rotation of section around axis 2
N_1 – Axial load (without pressure effect on the end closure)
u_1 – axial elongation
θ_1 – linear temperature gradient
θ_2 – quadratic temperature gradient

Fig. 2 Schematic diagram of piping component with part-through crack configuration

where,

F_gb_deep = Geometric factor at the deepest point of crack depth

σ_gb_max = Maximum global bending stress, σ_gb_min = Minimum global bending stress

a = Crack depth,

h = t = Thickness of pipe

In the present studies, SIF at deepest point is reported as it governs the design. SIF at deepest point has been evaluated from the Eq. 1 & 2 and the results are presented in Table 2.

P_Max =160 kN,

P_Min =16 kN, since the load ratio is 0.1

From the Eq. 3 & 4 the maximum and the minimum global bending stresses are computed corresponding to maximum and minimum load.

σ_gb_max = 140.1667 MPa,

σ_gb_min = 14.1667 MPa

From Eqns. 5 & 6, the number of cycles and the change in crack length at the surface direction for the corresponding change in crack depth is computed.

Paris Law

$$\frac{\mathbf{da}}{\mathbf{dN}} = \mathbf{C}\ (\Delta\mathbf{K})^{\mathbf{m}} \tag{5}$$

$$\frac{\mathbf{dc}}{\mathbf{dN}} = \mathbf{C}\ (\Delta\mathbf{K})^{\mathbf{m}} \tag{6}$$

Typical flow chart for computation of SIF at deepest and surface point is presented in Fig. 3.

3. Relevance vector machine

Relevance vector machine was initially proposed by Tipping (2001), is a special case of a sparse kernel model, adopts bayesian treatment of a generalized linear model of identical functional form as in the case of support vector machine (SVM). RVM produces simple models that have both a structure and a parameterization process together with respect to the data type. RVM is a probabilistic based approach, introduces a prior over the model weights governed by a set of hyperparameters associated with each weight, whose most probable values are iteratively determined from the data. RVM is based on a hierarchical prior, where an independent Gaussian prior is defined based on the weighted parameters in the first level, and an independent Gamma hyper prior is used for the variance parameters in die second level. The efficacy of the model will depend upon the calibration of related parameters and RVM uses the parameters (C, **Ɛ**, and r) and a kernel parameter (**σ**), respectively. In the present study, cross validation approach has been employed to determine the parameters of SVM (C and **Ɛ**) for various kernel parameter trials (**σ**). This section discusses a brief description about RVM. Full details about model are available in Tipping (2000, 2001). As already mentioned that RVM is an extension of SVM which employs Bayesian model and kernel function (Tipping 2001).

RVM starts with the base of linear models, i.e. the function of y(x) to be predicted at some arbitrary point x given a set of (typically noisy) measurements of the function t= (t_1, y, t_N) and with some training points x = (x_1,y, x_N)

$$t_i = y(x_i) + \varepsilon_i \tag{7}$$

Where, ε_i is the noise component of the measurement with mean zero Gaussian and variance σ^2. With a linear model assumption, the unknown function y(x) can be written as a linear combination of some known basis function i.e

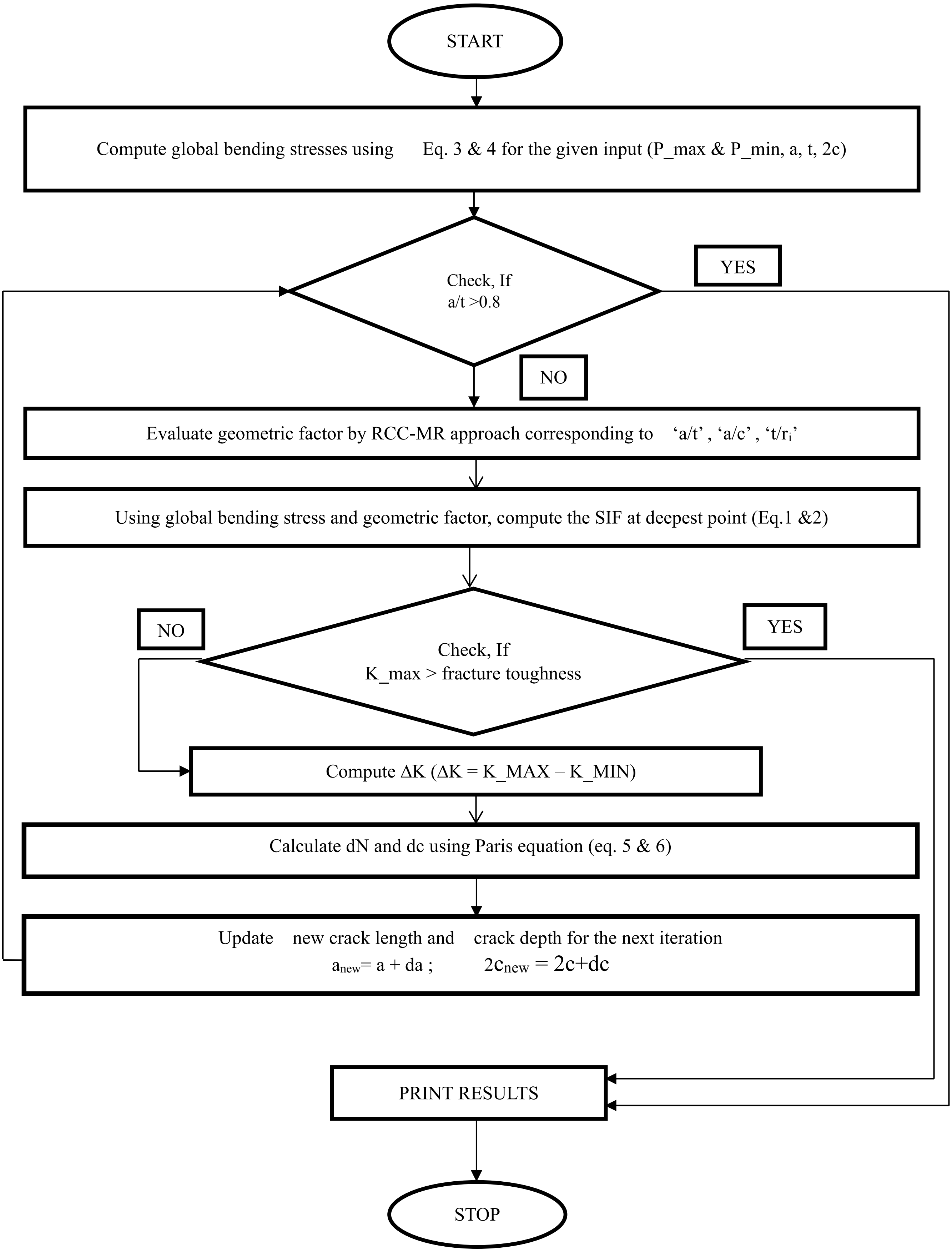


Fig. 3 Typical flow chart for determination of SIF

Table 2 No. of fatigue cycles vs Stress intensity factor

No. of fatigue Cycles, N (Experimental)	Crack depth, a mm (Experimental)	Geometric Factor (F_{gb})		SIF at deepest point,(MPa √m)
		at deepest point	at surface point	
2000	5.83	1.454	0.388	27.599
4000	6.03	1.467	0.400	28.319
8000	6.09	1.472	0.403	28.424
10000	6.13	1.476	0.405	28.743
12000	6.21	1.497	0.416	29.625
14000	6.33	1.498	0.416	29.892
15000	6.47	1.512	0.424	30.240
16000	6.53	1.518	0.428	30.503
18000	6.58	1.523	0.432	30.713
19000	6.65	1.530	0.436	31.027
20000	6.73	1.538	0.441	31.376
21000	6.91	1.556	0.454	32.158
22000	7.03	1.567	0.463	32.677
23500	7.42	1.604	0.482	34.348
25000	7.63	1.622	0.489	35.235
26000	7.81	1.638	0.496	35.988
27200	7.93	1.648	0.501	36.485
29040	8.53	1.695	0.526	38.920
30000	8.59	1.699	0.529	39.158
31000	8.67	1.705	0.533	39.474
32220	8.73	1.709	0.536	39.711
33000	8.86	1.720	0.547	39.918
33500	9.19	1.743	0.557	41.559
34000	9.43	1.765	0.568	42.608
34200	9.48	1.769	0.571	42.824
34520	9.53	1.773	0.573	43.039
35000	9.58	1.777	0.576	43.251
35500	9.60	1.781	0.578	43.464
36000	9.63	1.783	0.578	43.483
36000	9.64	1.786	0.579	43.506
36560	9.73	1.789	0.583	43.884
37000	9.81	1.793	0.588	44.094
37530	9.93	1.805	0.594	44.712
38010	10.01	1.810	0.599	45.038
38500	10.14	1.820	0.606	45.561
38800	10.23	1.826	0.611	45.918
39000	10.43	1.839	0.624	46.698
39250	10.68	1.844	0.640	47.645
39500	10.73	1.853	0.650	48.198
39730	10.79	1.863	0.651	48.293
39910	10.83	1.872	0.651	48.378
40000	10.88	1.875	0.651	48.505
41000	10.95	1.878	0.659	48.633
42000	11.2	1.882	0.677	49.514
42500	11.35	1.888	0.689	50.027
43000	11.5	1.895	0.700	50.530
43500	11.72	1.904	0.718	51.246
44000	11.8	1.907	0.725	51.500
44500	11.92	1.911	0.735	51.875
45000	12.1	1.917	0.750	52.425

$$y(x) = \sum_{i=1}^{M} w_i \varphi_i(x) \quad (8)$$

where, $w_i = (w_1,\ldots,w_M)$ = a vector consisting of the linear combination weights

y(x) = the output, a linearly-weighted sum of M, generally nonlinear and fixed basis functions

$\varphi_i(x) = (\varphi_1(x), \varphi_2(x), \ldots\ldots\ldots\ldots, \varphi_M(x))^T$.

For better predictions, the majority of parameters are default set to zero for the development of model (Tipping 2000, 2001).

$$t = \Phi w + \varepsilon \quad (9)$$

where, Φ= NxM design matrix, whose i[th] column is formed with the values of basis function $\Phi_i(x)$ at all the training points

$\varepsilon_i = (\varepsilon_1,\ldots, \varepsilon_N)$, the noise vector.

RVM starts with a set of data input $\{x_n\}_n^N = 1$ and their corresponding target vector $\{t_n\}_n^N = 1$.

The basic aim of the 'training' set is to learn a model of the dependency of the target vectors on the inputs to make accurate prediction of t for previously unseen value of x.

In SVM, the prediction is made by assuming the function of the form given below

$$y(x) = \sum_{i=1}^{N} w_i K(x, x_i) + w_0 \quad (10)$$

where, $w_i = w_1, w_2,\ldots, w_N$, weight vector

$K(x,x_i)$ = a kernel function and w_0 is the bias

In the present study, radial basis kernel function is employed and the related expression is given below

$$K(x_i, x) = \exp\left\{-\frac{(x_i - x)^T (x_i - x)}{2\sigma^2}\right\} \quad (11)$$

where, x_i and x = the training and test patterns, respectively.

d = a dimension of the input vector, σ = width of the basis function.

For a given input dataset, it is assumed as$\{x_n, t_n\}_n^N = 1$. Further, it is assumed that p (t|x) is Gaussian N (t|y(x), σ^2). The mean of this distribution for a given x is modelled by y(x) as mentioned in Eq. (10).

Due to the assumption of t_n independence, the likelihood of the total dataset can be written as

$$p(t|w,\sigma^2) = (2\pi\sigma^2)^{-N/2} \exp\left\{-\frac{1}{2\sigma^2}\|t - \Phi w\|^2\right\} \quad (12)$$

Where, $t_i = (t_1\ldots, t_N)^T$, $\omega_i = (\omega_0,\ldots,\omega_N)$ and

$$\Phi^T = \begin{bmatrix} 1 & K(x_1,x_1) & K(x_1,x_2) & \cdots & K(x_1,x_n) \\ 1 & K(x_2,x_1) & K(x_2,x_2) & \cdots & K(x_2,x_n) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & K(x_n,x_1) & K(x_n,x_2) & \cdots & K(x_n,x_n) \end{bmatrix}$$

Where, $K(x_i,x_n)$ is the kernel function

Simple parameters are generally preferred to constrain an explicit zero-mean Gaussian prior probability distribution over weights

$$p(w|\alpha) = \prod_{i=0}^{N} N(w_i|0, \alpha_i^{-1}) \quad (13a)$$

where α is a vector of N+1 hyperparameters, takes care the deviation of weight (Caesarendra 2010). By applying Bayes' rule, the posterior all unknowns can be computed, given the defined non-informative prior-distributions. To complete the specification of the hierarchical prior-distribution, hyperpriors over α are to be defined with noise variance σ^2. These quantities are examples of scale parameters and suitable priors are Gamma Distributions (Tipping 2000)

$$p(\alpha) = \prod_{i=0}^{N} \text{Gamma}(\alpha_i|a, b), \quad (13b)$$

$$p(\beta) = \prod_{i=0}^{N} \text{Gamma}(\beta|c, d) \quad (13c)$$

Where, $\beta = \sigma^{-2}$.

Hence, for α and σ, the distribution is "gamma" and for w, it is normal distribution and after the prior-distributions, Bayes rule is followed.

$$p(w,\alpha,\sigma^2|t) = \frac{p(t|w,\alpha,\sigma^2)p(w,\alpha,\sigma^2)}{p(t)} \quad (14a)$$

For a new test point (X_*) corresponding to target (t_*), the predictive distribution is as follows

$$p(t_*|t) = \int p(t_*|w,\alpha,\sigma^2)p(w,\alpha,\sigma^2|t)dw\, d\alpha\, d\sigma^2 \quad (14b)$$

Since the above equation cannot be solved directly, it can be solved by decomposition of posterior (equation 15) by assigning appropriate weights due to the property of normalising integral, which is convolution of Gaussians

$$p(w,\alpha,\sigma^2|t) = p(w|t,\alpha,\sigma^2)p(\alpha,\sigma^2|t) \quad (15)$$

Equation (15) can be modified as

$$p(w|t,\alpha,\sigma^2) = \frac{p(t|w,\sigma^2)p(w,\alpha)}{p(t|\alpha,\sigma^2)} \quad (16)$$

By employing the Bayes rule, the equation (16) can be modified as

$$p(w|t,\alpha,\sigma^2) = (2\pi)^{-(N+1)/2} |\Sigma|^{-1/2} \exp\left\{-\frac{1}{2}(w-\mu)^T \Sigma^{-1}(w-\mu)\right\} \quad (17)$$

The posterior covariance,

$$\Sigma = (\sigma^{-2}\Phi^T\Phi + A)^{-1} \quad (18)$$

Mean, $\mu = \sigma^{-2} \Sigma \Phi^T t$ (19)

A = diag(α_0, $\alpha_1 \ldots \alpha_N$).

Maximization of $p(\alpha, \alpha_{\epsilon_n}^2 | y) \alpha p(y|\alpha, \alpha_{\epsilon_n}^2) p(\alpha) p(\alpha_{\epsilon_n}^2)$ with respect to a α and σ^2, results in a search for the hyper-parameters posterior.

For the general case of uniform hyperpriors, maximization is need to be done for the term $p(y|\alpha, \alpha_{\epsilon_n}^2)$, which can be computed as follows

$$p(y|\alpha, \alpha_{\epsilon_n}^2) = \int p(y|w, \alpha_{\epsilon_n}^2) p(w|\alpha) dw$$
$$= (2\pi)^{-1/2} \left| \alpha_{\epsilon_n}^2 I + \Phi A^{-1} \Phi^T \right|^{1/2} \quad (20)$$
$$\times \exp\left\{ -\frac{1}{2} y^T \left(\alpha_{\epsilon_n}^2 I + \Phi A^{-1} \Phi^T \right)^{-1} y \right\}$$

At convergence of the hyperparameter determination process, predictions can be based on the posterior distribution on the weights, conditioned on the maximized most probable values of α and $\sigma_{\epsilon_n}^2, \alpha_{MP}$ and $\sigma^2{}_{MP}$ respectively.

$$p(y_* | y, \alpha_{MP}, \sigma_{MP}^2) = \int p(y_* | w, \sigma_{MP}^2) \; p(w | y, \alpha_{MP}, | \sigma_{MP}^2) \, dw \quad (21)$$

From the equation (21), it can be noted that the both the terms in the integrand are Gaussian, It can be re-written as

$$p(y_* | y, \alpha_{MP}, \sigma_{MP}^2) = N(y_* | t_*, \sigma_*^2) \quad (22)$$

$$t_* = \mu^T \Phi(x_*) \quad (23)$$

With $\sigma_*^2 = \sigma_{MP}^2 + \Phi(x_*)^T \Sigma \Phi(x_*)$ (24)

The predictive variance is the sum of two variance components, namely, the estimated noice on the data and uncertainity in the prediction of the weights.

3.1 RVM based analysis

For prediction of the stress intensity factor for the piping component having part-through crack in the circumferential direction under cyclic loading, RVM model has been developed. From Table 2, it can be noted that SIF is a function of applied load, geometric parameter and crack configuration. Normalization of the data has been carried out before presenting the input to statistical machine learning algorithm. MALAB software has been used for development of RVM model. Thus, equation 25 has been used for the linear normalization of the data to the data values between 0 and 1.

$$x_i^n = \frac{x_i^a - x_i^{\min}}{x_i^{\max} - x_i^{\min}} \quad (25)$$

where, x_i^a and x_i^n are i^{th} components of the input vector before and after normalization, respectively, x_i^{max} and x_i^{min} are the maximum and minimum values of all the components of the input vector before the normalization.

3.1.1 Development of RVM model

SIF data of about 50 are tabulated for development of model. About 70 % of data set has been used for the development of RVM model and remaining 30% of the data set has been used for testing and verification of the developed model. Testing and verification of the model is done by comparing the predicted values obtained by using RVM model with the computed values. The important aspect of development of RVM model is that the selection of kernel width which was obtained by using post modelling analysis (Wahyu *et al.*2010). Post-modelling analysis of the training and testing R values is associated with the number of relevance vectors (NRV) involved in the model and their corresponding weights & variation in the kernel width (σ). The value of σ is initially assumed as 0.13 and for the assumed value of σ, the model has been developed. Fig. 4 shows the typical process of development of RVM model. The developed model provides the NRVs used and their corresponding weights (w_i). The quality of the developed model is evaluated based on the coefficient of correlation (R).

$$R = \frac{\sum_{i=1}^{n} (E_{ai} - \overline{E}_a)(E_{pi} - \overline{E}_p)}{\sqrt{\sum_{i=1}^{n} (E_{ai} - \overline{E}_a)} \sqrt{\sum_{i=1}^{n} (E_{pi} - \overline{E}_p)}} \quad (26)$$

where, E_{ai} and E_{pi} are the actual and predicted values, respectively.

$\bar{E}_a$ and $\bar{E}_p$ are mean of actual and predicted E values corresponding to n patterns. In each iteration, R value is computed and the model is finalized when the R value is closer to one.

It is observed that the testing R value achieved its maximum at kernel widths shown in Table 3 for the corresponding models, involving minimum number of relevance vectors. The training and testing R values obtained for models are presented in Table 3.

Table 4 shows the weights for RVM model for the prediction of SIF. By using equations 20,21 and Table 4 with w_o as zero, the following equation has been obtained from the developed RVM model.

$$y = Stress \text{ intensity factor}$$
$$= \sum_{i=1}^{35} w_i \exp\left\{ -\frac{(x_i - x)^T (x_i - x)}{0.034} \right\} \quad (27)$$

Variance for training and testing data set for the developed model is plotted and shown in Figs. 5 and 6.

Table 3 Performance of developed RVM model

Parameters	Coefficient of correlation (R)		width (σ)	No. of RVs used out of total 35 dataset
	Training	Testing		
RVM model for SIF	0.994	0.992	0.13	24

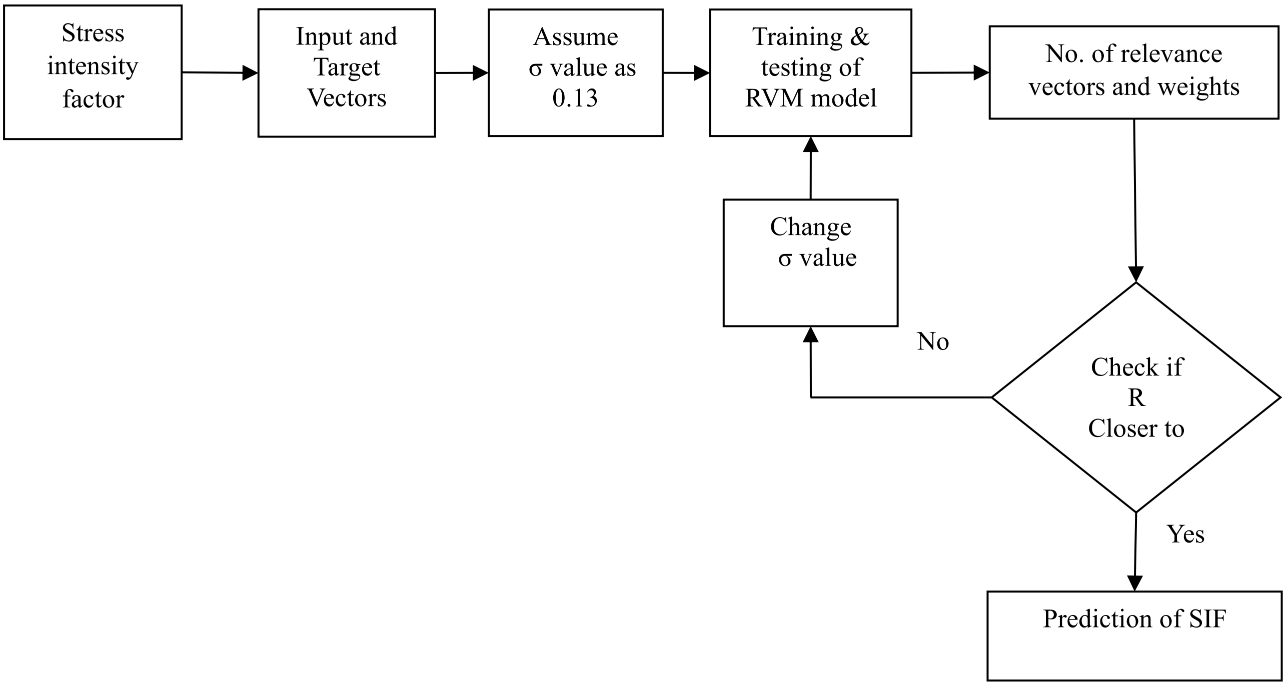


Fig. 4 Schematic diagram- development of RVM model

Table 4 Values of weights (w_i) for prediction of SIF

i =1,2…35	w_i	i =1,2…35	w_i
1	0.1	19	0.01
2	0.022	20	0.041
3	0.02	21	0.01
4	0	22	0.05
5	0.12	23	0.062
6	0.03	24	0.045
7	0.14	25	0.085
8	0.12	26	0.01
9	0.04	27	0.03
10	0.120	28	0.072
11	0.11	29	0.053
12	0.05	30	0.01
13	0.043	31	0.024
14	0.092	32	0.04
15	0.11	33	0.08
16	0.11	34	0.021
17	0.102	35	0.02
18	0.1		

The normalised output vector obtained from the RVM model is converted back to original value by using the equation below.

$$x_i^a = x_i^n\left(x_i^{\max} - x_i^{\min}\right) + x_i^{\min} \tag{28}$$

where, x_i^n is the normalized result obtained after the test for the i[th] component.

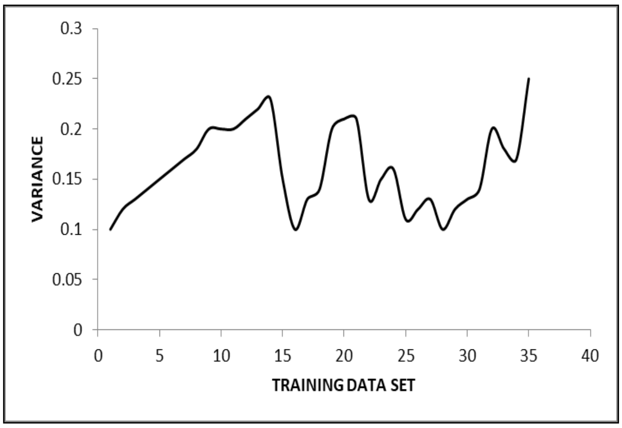


Fig. 5 Variance of training data set for stress intensity factor

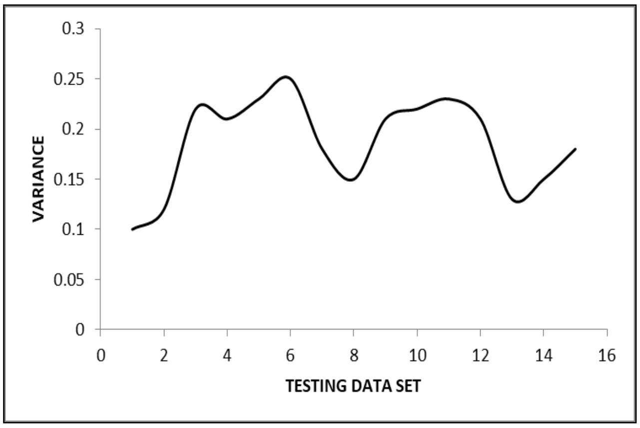


Fig. 6 Variance of testing data set for stress intensity factor

Table 5 Predicted and analytical SIF values

Crack depth, a mm (Experimental)	SIF at deepest point, (Mpa √m)	
	Analytical	Predicted
6.13	28.743	27.45
6.33	29.892	27.9
6.47	30.240	28.34
6.58	30.713	30.87
6.91	32.158	31.93
7.42	34.348	32.86
7.93	36.485	34.87
8.67	39.474	40.21
9.19	41.559	42.78
9.58	43.251	41.98
10.01	45.038	42.87
10.68	47.645	48.21
10.95	48.633	48.11
11.92	51.875	50.12
12.1	52.425	49.52

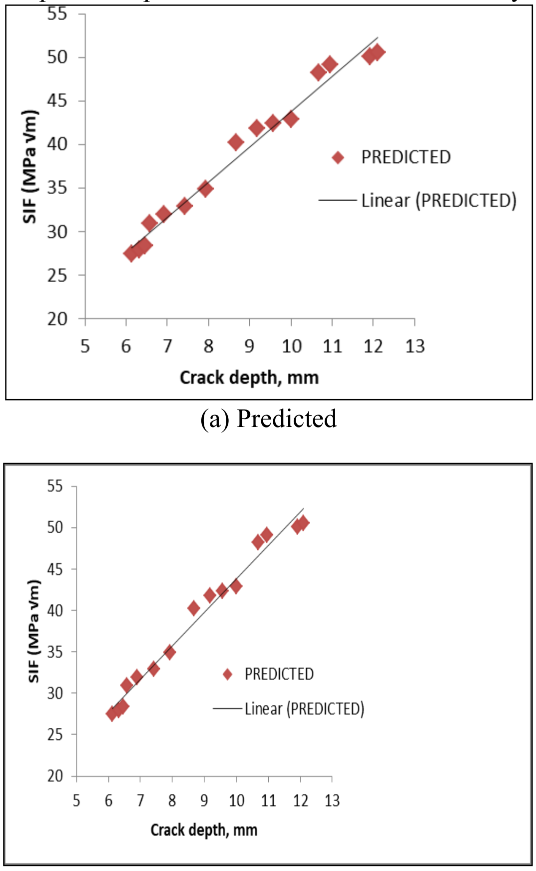
x_i^a is the actual result obtained for i[th] componenet, and x_i^{max} and x_i^{min} are the maximum and minimum values of all the components of the corresponding input vector before the normalization.

The developed RVM model has been verified with the remaining 15 data sets and the results are presented in Table 5.

From Table 5, it can be observed that the predicted SIF is in very good agreement with the corresponding analytical values. The maximum % of difference between predicted and Analytical is ±5%. Fig. 6 shows predicted and analytical SIF w.r.t crack depth and the comparison plot of predicted and the corresponding analytical SIF is shown in Fig. 7. From Table 5 and Figs. 6 and 7, it can be concluded that the developed model is robust and reliable.

4. Summary and Conclusions

An advanced statistical model based on relevance vector machine has been developed to predict stress intensity factor for the piping component having part through crack in the circumferential direction. To develop model, the experimental data such as number of cycles, crack depth, crack length, and the applied load has been taken. Stress intensity factor has been analytically computed by using RCC-MR approach. MATLAB software has been used for training and development of RVM based model. About 70% of the data (35 dataset) has been used for development of model and the remaining 30% of the data (15 dataset) is used validation. It was observed that the developed RVM model can produce a sparse solution, indicating that a with that of analytically obtained values and the proposed equation can be used to compute SIF of a pipe with part significant number of the weights are minimum, which produces compact, computationally capable models that are also simple. The predicted SIF is found to be very close



(a) Predicted

(b) Analytical

Fig. 7 Crack depth vs. SIF

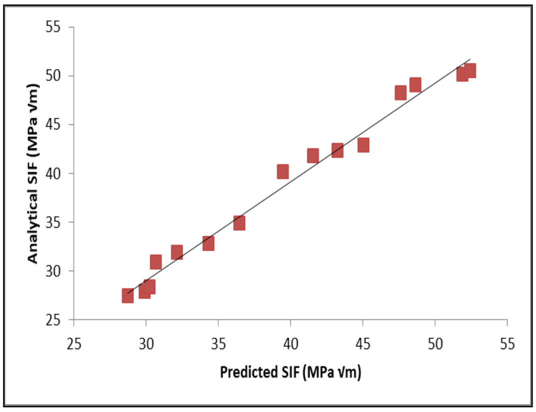


Fig. 8 Predicted vs. analytical SIF

through crack in the circumferential direction. The R value for the developed model is found to be closer to 1 indicating better predictability of the model. The predicted SIF is useful for crack growth studies, remaining life prediction and residual strength evaluation of piping component which in turn useful for in-service inspection scheduling and repair.

Acknowledgements

The authors thank the staff of the Fatigue & Fracture

Laboratory for the co-operation and suggestions provided during the investigations. This paper is being published with the kind permission of the Director, CSIR-SERC.

References

Bergman, M. and Brickstad, B. (1991), "Stress intensity factors for circumferential cracks in pipes analyzed by FEM using line spring elements", *Int J Fract,* **47**(1), R17-R19. https://doi.org/10.1002/mawe.19870180710.

Bhargava, R.Y., Bhasin, V. and Kushwaha, H.S. (1998), "Assuring It is safe", International conference of Integrating Structural Integrity, Inspection, Monitoring, Safety and Risk Assessment, Institution of Mechanical Engineers, United Kingdom., May.

Brennan, F.P., Ngiam, S.S. and Lee, C.W. (2008), "An experimental and analytical study of fatigue crack shape control by cold working", *Eng Fract Mech*, **75**(3-4), 355-363. https://doi.org/10.1016/j.engfracmech.2007.03.033.

Dawei, H., Ian, C. and Weiping, K. (2012), "Flow modelling using Relevance Vector Machine (RVM)", *Proceedings of the Fifth International Conference on Hydroinformatics*, 1429-1435, Cardiff, United Kingdom.

Engin, S., Ozturk, O. and Okay, F. (2015), "Estimation of ultimate torque capacity of the SFRC beams using ANN", *Struct Eng Mech*, **53**(5), 939-956. http://dx.doi.org/10.12989/sem.2015.53.5.939.

Erdem, H. (2017). "Predicting the moment capacity of RC slabs with insulation materials exposed to fire by ANN", *Struct Eng Mech*, **64**(3), 339-346. http://dx.doi.org/10.12989/sem.2017.64.3.339.

Jaideep, K. and Kamaljit, K. (2017), "A Fuzzy Approach for an IoT-based Automated Employee Performance Appraisal", *Comput Mater Con*, **53**(1), 23-36. https://doi.org/10.3970/cmc.2017.053.024.

John, F., Todd, K.M., Mac, M. and Gunther, J.H. (2010), "Application of the relevance vector machine to canal flow prediction in the Sevier River Basin", *Agric Water Manage,* **97**(2), 208-214. https://doi.org/10.1016/j.agwat.2009.09.010.

Kefei, L. and Zhisheng, X. (2011), "Traffic Flow Prediction of Highway Based on Wavelet Relevance Vector Machine", *J Inf Comput Sci*. **8**(9)**,** 1641-1647.

Keprate, A. and Ratnayake, R.M.C. (2017), "Enhancing offshore process safety by selecting fatigue critical pipeline locations for inspection using Fuzzy-AHP based approach", *Process Saf Environ,* **106,** 34-51. https://doi.org/10.1016/j.psep.2016.02.013.

Kumar, V., German, M.D. and Schumacher, B.I. (1985), "Analysis of elastic surface cracks in cylinders using the line spring model and shell finite element method", *ASME J Press Vessel Technol*, **107**(4), 403-411. https://doi.org/10.1115/1.3264474.

MacKay, D. J. C. (1994), *Bayesian Methods for Back Propagation Networks", Models of Neural Networks III. Physics of Neural Networks*. Springer, New York, USA.

Mansouri, I., Safa, M., Ibrahim, Z., Kisi, O., ahir, M.M., Baharom, S. and Azimi. (2016), "Strength prediction of rotary brace damper using MLR and MARS", *Struct Eng Mech*, **60**(3)**,** 471-488. https://doi.org/10.12989/sem.2016.60.3.471.

Marie, S., Chapuliot, S., Kayser, Y., Lacier, M.H., Drubay, B., Barthelet, B., Le, P.D., Rougier, V., Naudin, C., Gilles, P. and Triay, M. (2007), "French RSE-M and RCC-MR code appendices for flaw analysis: Presentation of the fracture calculation-Part III: cracked pipes", *Int J Pressure Vessel Piping*, **84**(10-11), 614-658. https://doi.org/10.1016/j.ijpvp.2007.05.005.

Miyazaki, M. and Mochizuki, M. (2011). "The effects of residual stress distribution and component geometry on the stress intensity factor of surface cracks". *ASME J Press Vessel Technol,***133**(1). 011701-7. https://doi.org/10.1115/PVP2005-71462.

Neal, R. M. (1996), *Bayesian Learning for Neural Networks*, Springer, New York., USA.

Nurcihan, C. (2014), "Application of support vector machines and relevance vector machines in predicting uniaxial compressive strength of volcanic rocks", *J Afr Earth Sci*, **100**, 634-644. https://doi.org/10.1016/j.jafrearsci.2014.08.006.

Prasanna, P.K., Murthy, A.R. and Srinivasu, K. (2018), "prediction of compressive strength of GGBS based concrete using RVM", *Struct Eng Mech*, **68**(6), 691-700. http://dx.doi.org/10.12989/sem.2018.68.6.691.

Rooke, D.P. (1976), *Cartwright DJ. Compendium of stress intensity factors*. London, UK.

Sarat, K.D. and Pijush, S. (2008), "Prediction of Liquefaction Potential Based on CPT Data: A Relevance Vector Machine Approach", *12th International Conference of International Association for Computer Methods and Advances in Geomechanics (IACMAG),* Goa, India.

Shantaram, P., Shreya, S., Pijush, S., Murthy, A.R. (2014), "Prediction of fracture parameters of high strength and ultra high strength concrete beams using Gaussian process regression and Least squares support vector machine", *Comp Model Eng*, **101**(2), 139-158. https://doi.org/10.3970/cmes.2014.101.139.

Sheng-wei, F. and Yong, H. (2015), "Wind speed prediction using the hybrid model of wavelet decomposition and artificial bee colony algorithm-based relevance vector machine", *Int J Elec Power*, **73**, 625-631. https://doi.org/10.1016/j.ijepes.2015.04.019.

Sih, G.C. (1973), *Handbook of stress intensity factors", Institute of Fracture and Solid Mechanics*, Leigh University, USA.

Singh, P.K., Vaze, K.K., Bhasin, V., Kushwaha, H.S., Gandhi, P. and Ramachandra Murthy, D.S. (2003), "Crack initiation and growth behavior of circumferentially cracked pipes under cyclic and monotonic loading", *Int J Pressure Vessel Piping*, **80**, 629-640. https://doi.org/10.1016/S0308-0161(03)00132-7.

Susom, D., Murthy, A.R., Dookie, K. and Pijush, S. (2017), "Prediction of Compressive Strength of Self-Compacting Concrete Using Intelligent Computational Modelling", *Comput Mater Con*, **53**(2), 157-174. https://doi.org/10.3970/cmc.2017.053.167.

Tada, H.P., Paris, P.C. and Irwin, G.R. (1973), *The stress analysis of cracks handbook*, Del Research Corporation, Pennsylvania, USA.

Tipping, M.E. (2001), "Sparse Bayesian learning and the relevance vector machine", *J Mach Learn Res*, **1,** 211–244. https://doi.org/10.1162/15324430152748236.

Vishal, S.S., Henyl, R.S., Pijush, S. and Murthy, A.R. (2014), "Prediction of Fracture Parameters of High Strength and Ultra-High Strength Concrete Beams using Minimax Probability Machine Regression and Extreme Learning Machine", *Comput Mater Con*, **44** (2), pp. 73-84. https://doi.org/10.3970/cmc.2014.044.073.

Wahyu, C., Achmad, W. and Bo-Suk, Y. (2009), "Application of relevance vector machine and logistic regression for machine degradation assessment", *Mech Sys Signal Pr*, **24**, 1161–1171. https://doi.org/10.1016/j.ymssp.2009.10.011.

Widodo, A., Kim, E. Y., Son, J. D., Yang, B. S., Tan, A. C., Gu, D. S., Choi, B.K. and Joseph, M. (2009), "Fault diagnosis of low speed bearing based on relevance vector machine and support vector machine", *Expert Syst Appl*, **36**, 7252-7261. https://doi.org/10.1016/j.eswa.2008.09.033.

Xiaodong, W., Meiying, Y. and Duanmu, C.J. (2009), "Classification of data from electronic nose using relevance vector machines", *Sens Actuators B Chem*, **140**, 143-148. https://doi.org/10.1016/j.snb.2009.04.030.

Yaguo, L. (2017), *Intelligent Fault Diagnosis and Remaining Useful Life Prediction of Rotating Machinery*, Butterworth-Heinemann, Elsevier Inc., Oxford, United Kingdom.

Yuvaraj, P., Murthy, A.R., Nagesh, R.I., Pijush, S. and Sekar, S.K. (2013a), "Multivariate adaptive regression splines model to predict fracture characteristics of high strength and ultra high strength concrete beams", *Comput Mater Con*, **36**(1), 73-97. https://doi.org/10.3970/cmc.2013.036.073.

Yuvaraj, P., Murthy, A.R., Nagesh, R.I., Pijush, S. and Sekar, S.K. (2014a), "ANN model to predict fracture characteristics of high strength and ultra high strength concrete beams", *Comput Mater Con*, **41**(3), 193-213. https://doi.org/10.3970/cmc.2014.041.193.

Yuvaraj, P., Murthy, A.R., Nagesh, R.I., Pijush, S. and Sekar, S.K. (2014b), "Prediction of fracture characteristics of high strength and ultra high strength concrete beams based on relevance vector machine", *Int J Damage Mech*, **23**(7), 979–1004. https://doi.org/10.1177/1056789514520796.

Yuvaraj, P., Murthy, A.R., Nagesh, R.I., Pijush, S. and Sekar, S.K. (2013a), "Support vector regression based models to predict fracture characteristics of high strength and ultra high strength concrete beams", *Eng Fract Mech*, **98**, 29-43. https://doi.org/10.1016/j.engfracmech.2012.11.014.

Zahoor, A. (1985). "Closed-form expressions for fracture mechanics analysis of cracked pipes". *ASME J Press Vessel Technol*, **107**(2), 203-205. https://doi.org/10.1115/1.3264435.

Zareei, A. and Nabavi, S.M. (2016), "Calculation of stress intensity factors for circumferential semi-elliptical cracks with high aspect ratio in pipes", *Int J Press Vessel Pip*, **146**, 32-38. https://doi.org/10.1016/j.ijpvp.2016.05.008.

PL