Structural Engineering and Mechanics, *Vol. 72, No. 1 (2019) 131-139*
DOI: https://doi.org/10.12989/sem.2019.72.1.131

Static analysis of monoclinic plates via a three-dimensional model using differential quadrature method

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(Received March 12, 2019, Revised April 21, 2019, Accepted May 17, 2019)

Abstract. According to the properties of monoclinic materials, the normal and shear stresses are depending on both normal and shear strains. In the current investigation, the static analysis of monoclinic plates based on three dimensional elasticity theory is investigated. New governing equations and boundary conditions are derived for monoclinic plates and the Differential Quadrature Method (DQM) is used to solve the static problem. In our method of solution, no approximation is used and the DQM is adopted in all directions. By showing the differences between our results and the results for especially orthotropic plates, one can find that it is worth to investigate the monoclinic plates to have more accurate results.

Keywords: monoclinic materials; static analysis; three-dimensional elasticity theory; differential quadrature method; rectangular plates

1. Introduction

Although mechanics of isotropic and functionally graded structures have been studied for several times (Amar *et al.*, 2017, Attia *et al.*, 2018, Barati, 2017, Bellifa *et al.*, 2017, Bouderba *et al.*, 2016, Dash *et al.*, 2018, Ebrahimi and Barati, 2018, Guessas *et al.*, 2018, Kaghazian *et al.*, 2017, Khetir *et al.*, 2017, She *et al.*, 2018, Sobhy, 2017, Yousfi *et al.*, 2018, Zemri *et al.*, 2015, She *et al.*, 2019b, She *et al.*, 2019a, Karami and Shahsavari, 2019, Karami *et al.*, 2019d, Karami and Karami, 2019, Karami *et al.*, 2019c, Lurie and Solyaev, 2018, Rajasekaran and Khaniki, 2017, Ghayesh *et al.*, 2017, Lu *et al.*, 2017, Ghayesh *et al.*, 2019, Ghayesh, 2019, Shahsavari *et al.*, 2018, Karami *et al.*, 2019e), but a few researches have been conducted on static and dynamic characteristics of anisotropic ones because of the complexity in the modeling of such structures (Ferreira and Batra, 2005, Ferreira *et al.*, 2009, Chaudhuri, 2012, Soldatos, 2004, Batra *et al.*, 2004, Kumar and Tomar, 2006, Singhal and Bindal, 2012, Karami *et al.*, 2018b, Karami *et al.*, 2018c, Karami *et al.*, 2018a, Karami *et al.*, 2019a, Karami *et al.*, 2019b, Karami and Janghorban, 2019a). (Ferreira and Batra, 2005) studied the collocation method with multiquadrics basis functions and a first-order shear deformation theory are used to find natural flexural frequencies of a square plate made of orthotropic, monoclinic and hexagonal materials subjected to different boundary conditions. They found that computational effort required with this approach is considerably less than that needed with the analysis of the three-dimensional elasticity equations by the finite element method. (Ferreira *et al.*, 2009) used the first-order shear deformation theory and a meshless method based on radial basis functions in a pseudo-spectral framework for predicting the free vibration behavior of thick orthotropic, monoclinic and hexagonal plates. (Chaudhuri, 2012) derived three-dimensional asymptotic stress field in the vicinity of the front of a semi-infinite through-thickness crack/anti-crack weakening/reinforcing an infinite monoclinic plate, of finite thickness and subjected to far-field anti-plane shear loading. (Soldatos, 2004) developed complex potential formalisms for the solution of the bending problem of inhomogeneous anisotropic plates, on the basis of the most commonly used refined plate theories. (Batra *et al.*, 2004) presented the natural frequencies of thick square plates made of orthotropic, trigonal, monoclinic, hexagonal and triclinic materials using finite element method. (Kumar and Tomar, 2006) proposed the free transverse vibration of monoclinic rectangular plates with continuously varying thickness and density. In 2012, on the basis of classical plate theory, vibration characteristics of monoclinic rectangular plate of exponentially varying thickness resting on elastic foundation were studied by (Singhal and Bindal, 2012). (Karami *et al.*, 2018b) studied the ramen frequency and radial wave propagation of anisotropic nanoparticles via a three-dimensional model. In another work, the authors investigated the wave characteristics of monoclinic, triclinic, trigonal and hexagonal plates under the triaxial magnetic field effects (Karami *et al.*, 2017). Further, wave propagation of doubly-curved shells made of different anisotropic materials is investigated by the same authors (Karami *et al.*, 2018c). (Karami *et al.*, 2018a) investigated the buckling response of functionally graded nanoplates

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http://www.techno-press.com/journals/sem&subpage=7
ISSN: 1225-4568 (Print), 1598-6217 (Online)

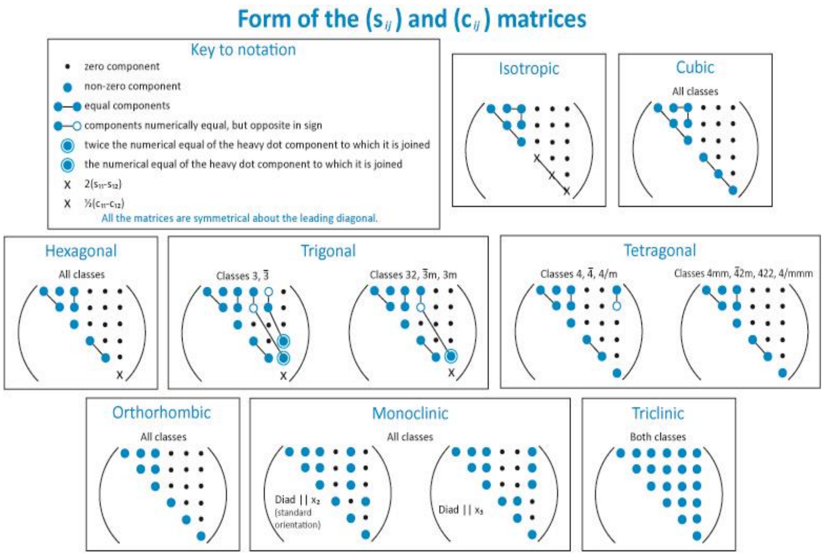


Fig. 1 The most general stress-strain relationship.

made of anisotropic materials where the problem were solved via the Galerkin method.

As noted above, there are limited studies that examine the behavior of anisotropic materials, because the modeling of these materials is more difficult than isotropic ones (due to the more components of the stiffness matrix compared to isotropic materials). Hence, in this paper, after the modeling of the plate made of anisotropic materials, we will examine the possibility of replacing anisotropic models with a simple model in which the elastic components are less than anisotropic one. The fundamental question of this study is the possibility of damage to the anisotropic model at the time of its replacement with less elastic components.

For the first time, using three dimensional elasticity theory, the static analysis of simply supported and fully clamped rectangular plates made of monoclinic materials is studied via the Differential Quadrature Method (DQM). Our results are verified with the results available in the literature and good agreement is achieved. The effects of different parameters such as geometry and boundary conditions on the displacements and stresses are also presented. According to the low effort on studying monoclinic materials, the present work can be used as bench mark for future works.

2. Monoclinic Materials

The most general stress-strain relationship (generalized Hooke's law) within the theory of linear elasticity is that of the materials without any plane of symmetry, i.e., anisotropic materials (see Fig. 1). If there is a plane of symmetry, the material is termed monoclinic. Itis important to keep in mind that a material which is anisotropic on one length scale may be isotropic on another (usually larger) length scale. In present work, although the formulation is derived for monoclinic materials but the investigation is take place on especially orthotropic plates, too (see in Table 1).

Table 1 Elastic constants for different kind of materials (Triclinic, Monoclinic, Orthotropic, Transversely isotropic, Isotropic)

	Independent Constants	Nonzero On-axis	Nonzero Off-axis	Nonzero General
Triclinic	21	36	36	36
Monoclinic	13	20	36	36
Orthotropic	9	12	20	36
Transversely-isotropic	5	12	20	36
Isotropic	2	12	12	12

3. Governing equations

According to the properties of monoclinic materials, the normal stresses are depending on both normal and shear strains. Shear stresses are also depending on both normal and shear strains. In the following equation, the general stress-strain relation with linear behavior is presented. The stress-strain relations for monoclinic materials can be written as follow

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & C_{15} & 0 \\ C_{12} & C_{22} & C_{23} & 0 & C_{25} & 0 \\ C_{13} & C_{23} & C_{33} & 0 & C_{35} & 0 \\ 0 & 0 & 0 & C_{44} & 0 & C_{46} \\ C_{15} & C_{25} & C_{35} & 0 & C_{55} & 0 \\ 0 & 0 & 0 & C_{46} & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} \tag{1}$$

where C_{11}=10.89GPa, C_{22}=11.47GPa, C_{33}=11.32GPa, C_{12}=4.57GPa, C_{13}=6.45GPa, C_{15}=0.78GPa, C_{23}=1.51GPa, C_{25}=0.23GPa, C_{35}=0.04GPa, C_{44}=3.67GPa, C_{46}=-0.08GPa, C_{55}=0.85GPa, C_{66}=2.89GPa.

The equilibrium equations for investigating the static analysis of monoclinic plates in Cartesian coordinates are defined as follow,

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = 0 \tag{2}$$

$$\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\tau_{yz}}{\partial z}+\frac{\partial\tau_{yx}}{\partial x}=0$$

$$\frac{\partial\sigma_{zz}}{\partial z}+\frac{\partial\tau_{xz}}{\partial x}+\frac{\partial\tau_{yz}}{\partial y}=0$$

The linear strain-displacements relations can be written as follow,

$$\begin{aligned}
\varepsilon_{xx} &= \frac{\partial U}{\partial x}\\
\varepsilon_{yy} &= \frac{\partial V}{\partial y}\\
\varepsilon_{zz} &= \frac{\partial W}{\partial z}\\
\gamma_{xy} &= \frac{\partial U}{\partial y}+\frac{\partial V}{\partial x}\\
\gamma_{xz} &= \frac{\partial U}{\partial z}+\frac{\partial W}{\partial x}\\
\gamma_{yz} &= \frac{\partial W}{\partial y}+\frac{\partial V}{\partial z}
\end{aligned} \tag{3}$$

Now by substituting equation (3) in equation (1) and insert the result relations in equilibrium equation (2), the governing equations for monoclinic rectangular plates can be expressed as below,

$$\begin{aligned}
&C_{11}\frac{\partial^2 U}{\partial x^2}+C_{44}\frac{\partial^2 U}{\partial y^2}+C_{55}\frac{\partial^2 U}{\partial z^2}+C_{15}\frac{\partial^2 W}{\partial x^2}\\
&\quad+C_{46}\frac{\partial^2 W}{\partial y^2}+C_{35}\frac{\partial^2 W}{\partial z^2}\\
&\quad+\frac{2C_{15}\,\partial^2 U}{\partial x\,\partial z}
\end{aligned} \tag{4}$$

$$\begin{aligned}
&+(C_{12}+C_{44})\frac{\partial^2 V}{\partial x\,\partial y}+(C_{46}+C_{25})\frac{\partial^2 V}{\partial y\,\partial z}+(C_{13}\\
&\quad+C_{55})\frac{\partial^2 W}{\partial x\,\partial z}=0
\end{aligned}$$

$$\begin{aligned}
&C_{44}\frac{\partial^2 V}{\partial x^2}+C_{22}\frac{\partial^2 V}{\partial y^2}+C_{66}\frac{\partial^2 V}{\partial z^2}+(C_{12}+C_{44})\frac{\partial^2 U}{\partial x\,\partial y}\\
&\quad+(C_{46}+C_{25})\frac{\partial^2 U}{\partial y\,\partial z}\\
&+2C_{46}\frac{\partial^2 V}{\partial x\,\partial z}+(C_{25}+C_{46})\frac{\partial^2 W}{\partial x\,\partial y}+(C_{23}\\
&\quad+C_{66})\frac{\partial^2 W}{\partial y\,\partial z}=0
\end{aligned} \tag{5}$$

$$\begin{aligned}
&C_{15}\frac{\partial^2 U}{\partial x^2}+C_{46}\frac{\partial^2 U}{\partial y^2}+C_{35}\frac{\partial^2 U}{\partial z^2}+C_{55}\frac{\partial^2 W}{\partial x^2}\\
&\quad+C_{66}\frac{\partial^2 W}{\partial y^2}+C_{33}\frac{\partial^2 W}{\partial z^2}\\
&+(C_{13}+C_{55})\frac{\partial^2 U}{\partial x\,\partial z}+(C_{46}+C_{25})\frac{\partial^2 V}{\partial x\,\partial y}\\
&\quad+(C_{23}+C_{66})\frac{\partial^2 V}{\partial y\,\partial z}\\
&\quad+2C_{35}\frac{\partial^2 W}{\partial x\,\partial z}=0
\end{aligned} \tag{6}$$

The boundary conditions for simply support and fully clamped rectangular plates are defined as,

Fully clamped:

$$U=0\,,V=0\,,W=0\text{at }x=0\,,x=a\,,y=0\,,y=b$$

Simply support: (7)

$$\sigma_{xx}=0\,,V=W=0\quad\text{at }x=0\,,x=a$$

$$\sigma_{yy}=0\,,U=W=0\quad\text{at }y=0\,,y=b$$

For both simply supported and Fully clamped boundary condition, one can have,

$$\begin{aligned}
&\tau_{xz}=0\,,\tau_{yz}=0\,,\sigma_{zz}=0\ \text{ at } z=0\\
&\tau_{xz}=0\,,\tau_{yz}=0\,,\sigma_{zz}=q\ \text{ at } z=h
\end{aligned} \tag{8}$$

With substituting the appropriate elastic constant for monoclinic materials and using the above equations, the static analysis of monoclinic plates under mechanical loading can be studied. If it is not impossible to solve these equations analytically, it seems that it is difficult to obtain such a solution. Hence, here the differential quadrature method as an efficient and accurate numerical tool is employed to solve these systems of equations.

4. Differential Quadrature method

The basic idea of the differential quadrature method (Janghorban, 2011, Malekzadeh, 2007, Bacciocchi *et al.*, 2016, Tornabene *et al.*, 2014, Karami and Janghorban, 2019b) is that the derivative of a function, with respect to a space variable at a given sampling point, is approximated as the linear weighted sums of its values at all of the sampling points in the domain of that variable. In order to illustrate the DQ approximation, consider a function $f(\xi,\eta)$ having its field on a rectangular domain $0\le\xi\le a$ and $0\le\eta\le b$. Let, in the given domain, the function values be known or desired on a grid of sampling points. According to DQ method, the r^{th} derivative of a function $f(\xi,\eta)$ can be approximated as

$$\left.\frac{\partial^r f(\xi,\eta)}{\partial\xi^r}\right|_{(\xi,\eta)=(\xi_i,\eta_j)}=\sum_{m=1}^{N_\xi}A_{im}^{\xi(r)}f(\xi_m,\eta_j)=\sum_{m=1}^{N_\xi}A_{im}^{\xi(r)}f_{mj} \tag{9}$$

for i=1,2,…, N_ξ , j=1,2,…, N_η and r=1,2,…, $N_\xi-1$

From this equation one can deduce that the important components of DQ approximations are weighting coefficients and the choice of sampling points. In order to determine the weighting coefficients a set of test functions should be used in Eq. (9). For polynomial basis functions DQ, a set of Lagrange polynomials are employed as the test functions. The weighting coefficients for the first-order derivatives in ξ -direction are thus determined as specially

$$A_{ij}^{\xi}=\begin{cases}\dfrac{1}{L_{\xi}}\dfrac{M(\xi_i)}{(\xi_i-\xi_j)M(\xi_j)} & \text{for} \quad i\neq j\\ -\sum\limits_{\substack{j=1\\ i\neq j}}^{N_{\xi}}A_{ij}^{\xi} & \text{for} \quad i=j\end{cases} \tag{10}$$

i, j=1,2…, N_{ξ}

where L_{ξ} is the length of domain along the $\xi-$direction and

$$M(\xi_i)=\prod_{k=1,i\neq k}^{N_{\xi}}(\xi_i-\xi_k) \tag{11}$$

The weighting coefficients of second order derivative can be obtained as [16],

$$[B_{ij}^{\xi}]=[A_{ij}^{\xi}][A_{ij}^{\xi}]=[A_{ij}^{\xi}]^2 \tag{12}$$

In a similar manner, the weighting coefficients for η-direction can be obtained. In numerical computations, Chebyshev-Gauss-Lobatto quadrature points are used, that is,

$$\frac{\xi_i}{a}=\frac{1}{2}\{1-\cos[\frac{(i-1)\pi}{(N_{\xi}-1)}]\}$$
$$\frac{\eta_j}{b}=\frac{1}{2}\{1-\cos[\frac{(j-1)\pi}{(N_{\eta}-1)}]\} \tag{13}$$

for i =1,2,? N_{ξ} and j =1,2,? N_{η}

It can be mentioned that in some cases although using equal spacing between nodes may cause results with less accurate results, but it needs less nods than above spacing. It is a good idea that other researchers investigate other branches of differential quadrature method and compare the results to find out which of them is the most accurate numerical tool.

Now by using the DQ method, the governing Eqs. (4-6) can be discretized as follow,

$$\begin{aligned}&C_{11}\sum_{m=1}^{N_x}B_{(i,m)}^{x}U_{mjk}+C_{44}\sum_{n=1}^{N_y}B_{(j,n)}^{y}U_{ink}\\&\quad+C_{55}\sum_{p=1}^{N_z}B_{(k,p)}^{z}U_{ijp}\\&+C_{15}\sum_{m=1}^{N_x}B_{(i,m)}^{x}W_{mjk}+C_{46}\sum_{n=1}^{N_y}B_{(j,n)}^{y}W_{ink}\\&\quad+C_{35}\sum_{p=1}^{N_z}B_{(k,p)}^{z}W_{ijp}\\&+2C_{15}\sum_{m=1}^{N_x}\sum_{p=1}^{N_z}A_{(i,m)}^{x}A_{(k,p)}^{z}U_{mjp}\\&+(C_{12}+C_{44})\sum_{m=1}^{N_x}\sum_{n=1}^{N_y}A_{(i,m)}^{x}A_{(j,n)}^{y}V_{mnk}\\&+(C_{46}+C_{25})\sum_{n=1}^{N_y}\sum_{p=1}^{N_z}A_{(j,n)}^{y}A_{(k,p)}^{z}V_{inp}\\&+(C_{13}+C_{55})\sum_{m=1}^{N_x}\sum_{p=1}^{N_z}A_{(i,m)}^{x}A_{(k,p)}^{z}W_{mjp}=0\end{aligned} \tag{14}$$

$$\begin{aligned}&C_{44}\sum_{m=1}^{N_x}B_{(i,m)}^{x}V_{mjk}+C_{22}\sum_{n=1}^{N_y}B_{(j,n)}^{y}V_{ink}\\&\quad+C_{66}\sum_{p=1}^{N_z}B_{(k,p)}^{z}V_{ijp}\\&(C_{12}+C_{44})\sum_{m=1}^{N_x}\sum_{n=1}^{N_y}A_{(i,m)}^{x}A_{(j,n)}^{y}U_{mnk}\\&+(C_{25}+C_{46})\sum_{n=1}^{N_y}\sum_{p=1}^{N_z}A_{(j,n)}^{y}A_{(k,p)}^{z}U_{inp}\\&+2C_{46}\sum_{m=1}^{N_x}\sum_{p=1}^{N_z}A_{(i,m)}^{x}A_{(k,p)}^{z}V_{mjp}+(C_{46}\\&+C_{25})\sum_{m=1}^{N_x}\sum_{n=1}^{N_y}A_{(i,m)}^{x}A_{(j,n)}^{y}W_{mnk}\\&+(C_{23}+C_{66})\sum_{n=1}^{N_y}\sum_{p=1}^{N_z}A_{(j,n)}^{y}A_{(k,p)}^{z}W_{inp}=0\end{aligned} \tag{15}$$

$$\begin{aligned}&C_{15}\sum_{m=1}^{N_x}B_{(i,m)}^{x}U_{mjk}+C_{46}\sum_{n=1}^{N_y}B_{(j,n)}^{y}U_{ink}\\&\quad+C_{35}\sum_{p=1}^{N_z}B_{(k,p)}^{z}U_{ijp}\\&\quad+C_{55}\sum_{m=1}^{N_x}B_{(i,m)}^{x}W_{mjk}\\&+C_{66}\sum_{n=1}^{N_y}B_{(j,n)}^{y}W_{ink}+C_{33}\sum_{p=1}^{N_z}B_{(k,p)}^{z}W_{ijp}\\&+(C_{13}+C_{55})\sum_{m=1}^{N_x}\sum_{p=1}^{N_z}A_{(i,m)}^{x}A_{(k,p)}^{z}U_{mjp}\\&+(C_{25}+C_{46})\sum_{m=1}^{N_x}\sum_{n=1}^{N_y}A_{(i,m)}^{x}A_{(j,n)}^{y}V_{mnk}\\&+(C_{23}+C_{66})\sum_{n=1}^{N_y}\sum_{p=1}^{N_z}A_{(j,n)}^{y}A_{(k,p)}^{z}V_{inp}\\&+2C_{35}\sum_{m=1}^{N_x}\sum_{p=1}^{N_z}A_{(i,m)}^{x}A_{(k,p)}^{z}W_{mjp}=0\end{aligned} \tag{16}$$

Table 2 comparison of deflection $\overline{W}$, normal stresses($\bar{\sigma}_x$, $\bar{\sigma}_y$) and shear stresses $\bar{\tau}_{xy}$ in rectangular isotropic plate subjected to uniformly distributed load (*a/b*=1, *h/a*=0.1, $\overline{W}=\frac{100*E*W(\max)}{q*h*S^4}$,$(\bar{\sigma}_x,\bar{\sigma}_y,\bar{\tau}_{xy})=(\frac{\sigma_x,\sigma_y,\tau_{xy}(\max)}{q*S^2})$

Method	$\overline{W}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$
(Ghugal and Sayyad, 2010)	11.34	0.638	0.245	0.277
(Krishna Murty, 1986)	11.31	0.613	0.31	0.278
(Reddy, 1984)	11.42	0.612	0.278	0.28
(Mindlin, 1951)	11.42	0.61	0.277	0.276
(Kirchoff, 1850)	11.06	0.61	0.278	0.277
Present	11.378	0.612	0.282	0.278

The boundary conditions can be discretized in a similar way by using DQ method.

5. Numerical results

In this work, static analysis is reported to study the mechanical characteristics of monoclinic plates by several numerical examples for the first time. After the validation of the developed differential quadrature method, it is used to present static behavior of the monoclinic plates.

To examine the accuracy and convergence of the developed differential quadrature method, our results are compared with the results of different mathematical formulations in Table 2. One can easily find that our numerical results are in a good agreement with other results especially with the results of exact solution.

Fig. 2 depicts the effects of length to thickness ratio on the normal stresses for fully clamped rectangular plate subjected to uniform load. It can be seen that for the normal stress in the *z*- direction, the results for thin plates are almost the same for monoclinic and especially orthotropic but for thick plates, the differences between the results cannot be ignored.

The influences of length to thickness ratio on the displacements for fully clamped rectangular plate subjected to uniform load are demonstrated in Fig. 3. From this figure, one can find that for square plates (*b/a*=1), the displacements for monoclinic and especially orthotropic plates are almost the same but for rectangular plates *b/a*≠1, the differences are considerable. From this figure it is obtained that the aspect ratio plays an important role in studying bending of monoclinic plates especially for thick and moderately thick plates.

In Fig. 4, the effects of aspect ratio on the normal stresses for fully clamped rectangular plate under uniform load is proposed. It is shown that with the increase of aspect ratio, the normal stresses in the *x* and *z* directions are increase but the stresses in the *y* direction are almost decrease. By studying the differences between our results for monoclinic plates and the results for especially orthotropic plates, one can easily find that it is worth to investigate the monoclinic plates to have more accurate results.

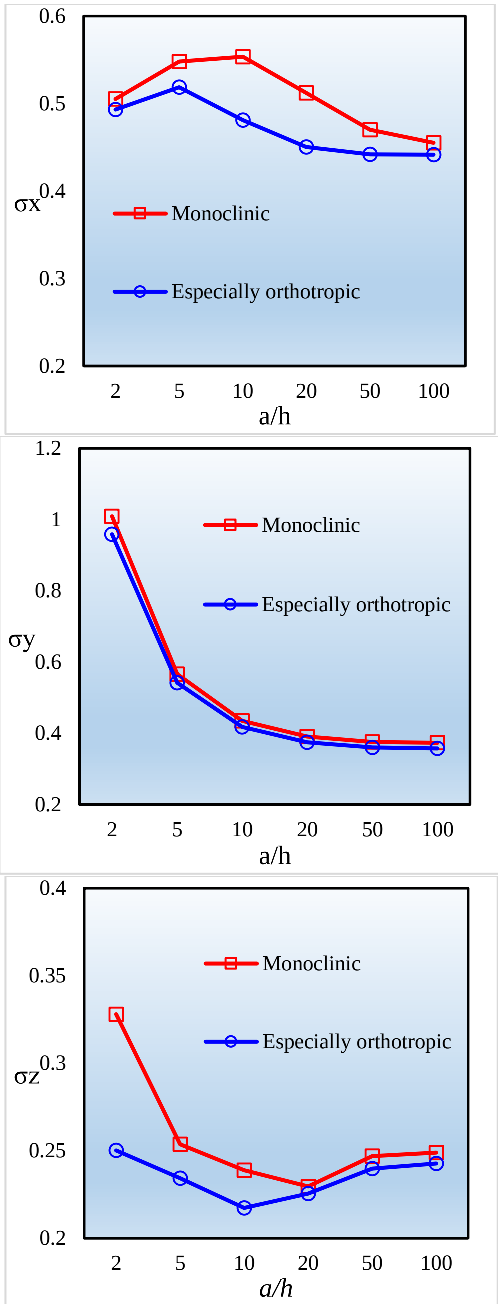
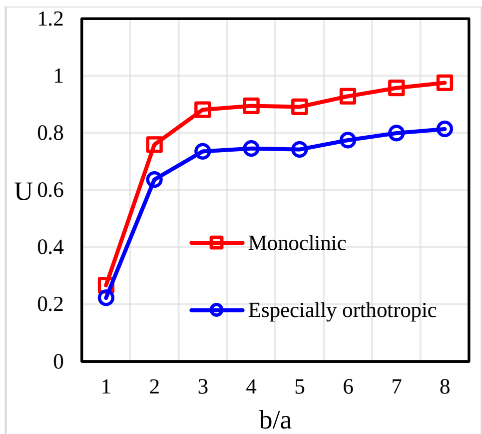


Fig. 2 Normal stresses for fully clamped rectangular plate subjected to uniform load (*b/a*=2)



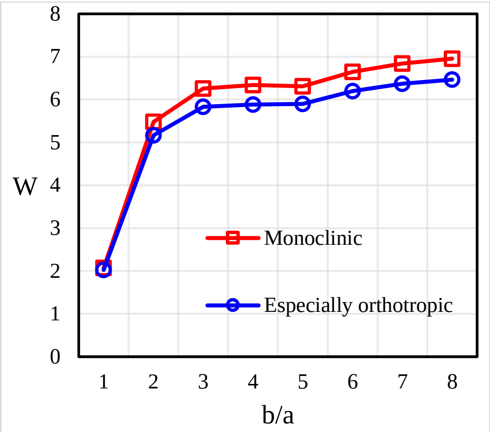


Fig. 3 Displacements for fully clamped rectangular plate subjected to uniform load (h/a=0.1).

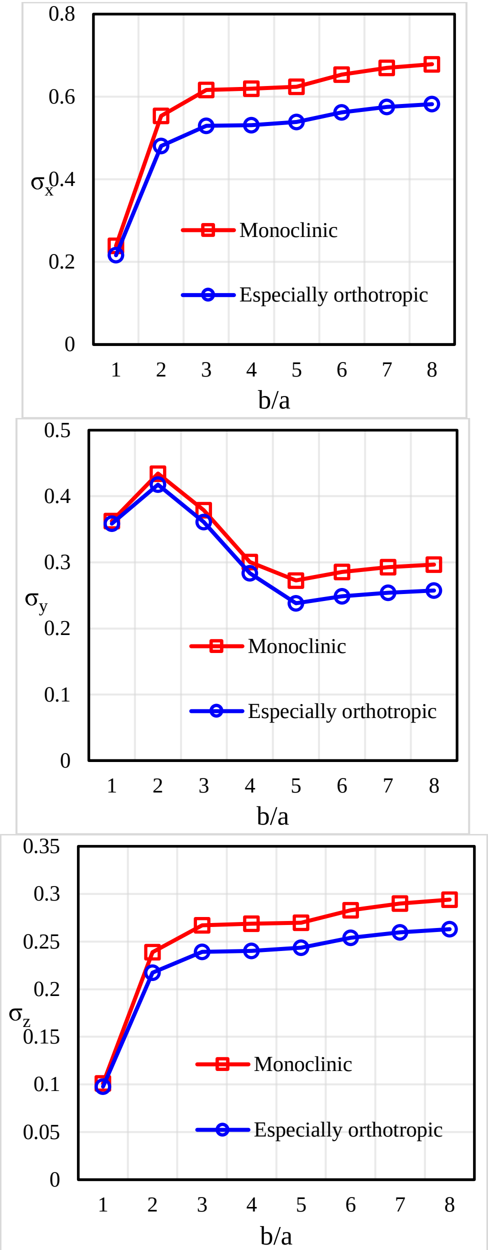


Fig. 4 Normal stresses for fully clamped rectangular plate subjected to uniform load (h/a=0.1).

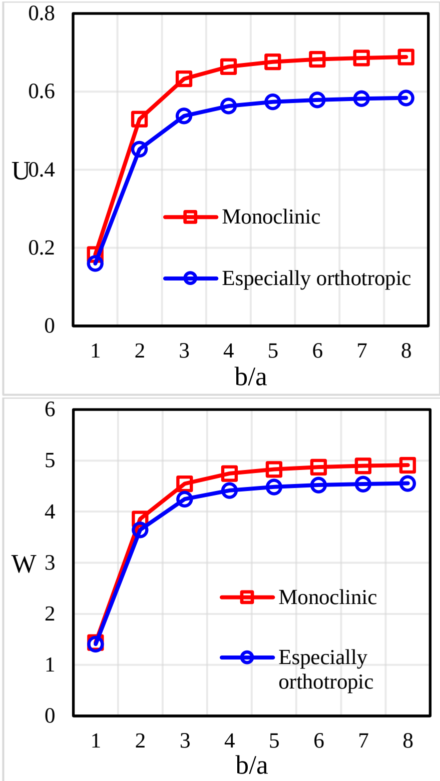


Fig. 5 Displacements for fully clamped rectangular plate subjected to sinusoidal loading (h/a=0.1).

The displacements for fully clamped rectangular plate subjected to sinusoidal loading are illustrated in Fig. 5. By comparing this figure with Fig. 3, it is found that the general trend of displacements is the same although the displacements in the x-direction for plate under sinusoidal loading seem to be more than those for plate under uniform load.

Fig. 6 shows the influences of aspect ratio on the displacements of simply supported rectangular plate subjected to uniform load. With considering this figure and figure 3 for fully clamped boundary condition, it may be concluded that the differences between the results of monoclinic and especially orthotropic plates are more significant for simply supported boundary condition. From this figure, it is also seen that in the most cases, increasing the aspect ratio will cause increasing the displacements in all directions.

The effects of aspect ratios on the shear stresses of simply supported rectangular plate subjected to uniform load are investigated in Fig.7. In almost all the cases, with the increase of aspect ratio, the shear stresses decrease except for the τ_{xz}. It is also shown that for the all aspect ratios presented in these three figures, the shear stresses for monoclinic rectangular plate are more than those for monoclinic square plate.

In Fig. 8, the influences of length to thickness ratio on the shear stresses of simply supported rectangular plate subjected to sinusoidal loading are studied. From this figure, it is found that the results are almost the same for specially orthotropic and monoclinic plates except for τ_{xy}. One can also find an unexpected behavior for shear stress in the xy plane for monoclinic plate which can be explained more in future works

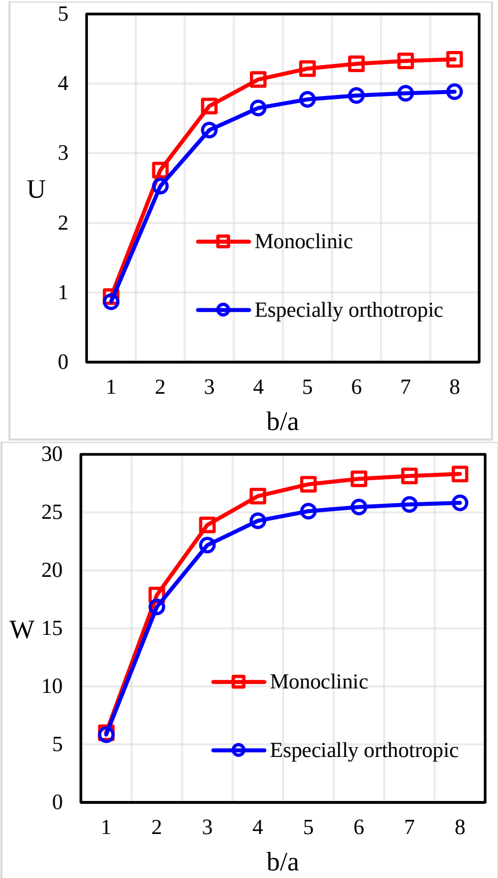
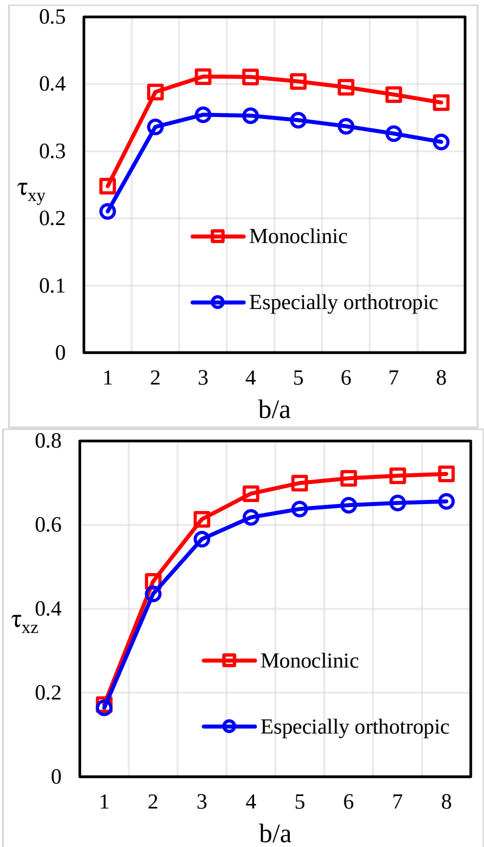


Fig. 6 Displacements for simply supported rectangular plate subjected to uniform load (h/a=0.1).



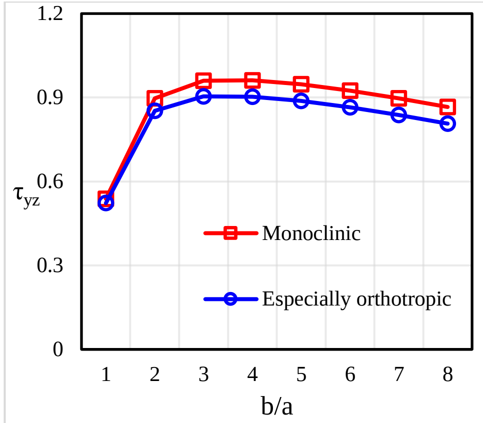


Fig. 7 Shear stresses for simply supported rectangular plate subjected to uniform load (h/a=0.1).

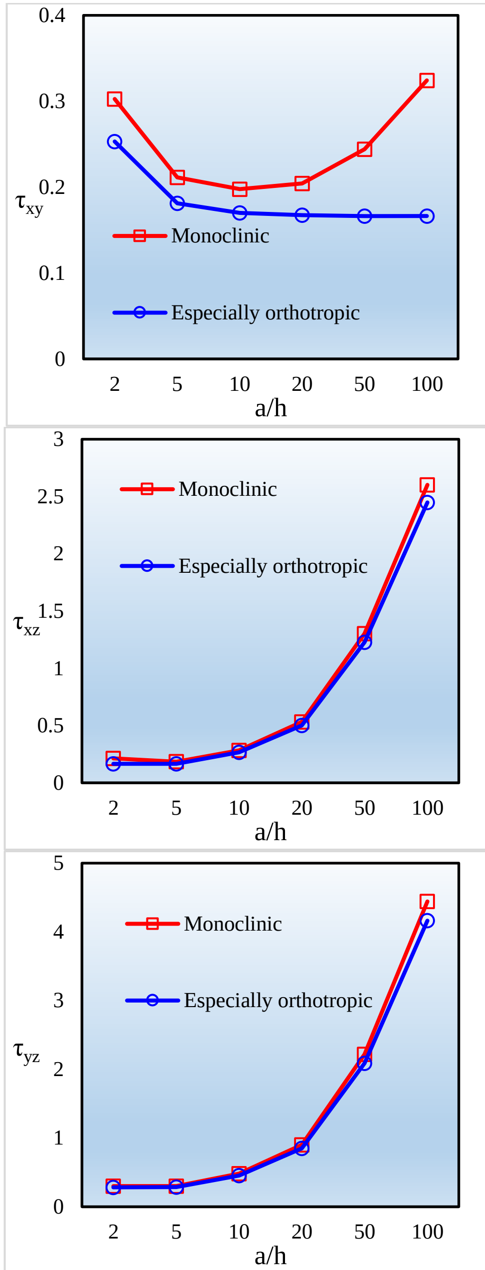


Fig. 8 Shear stresses for simply supported rectangular plate subjected to sinusoidal loading (b/a=2)

6. Conclusions

For the first time, a numerical study for static analysis of monoclinic plates was performed using three-dimensional elasticity theory without any approximation in the modeling. Differential quadrature method was utilized to solve the problem for different boundary conditions. It was also the first time that the monoclinic plate response for static problem were compared with their simple form (especially orthotropic plate).

The influences of length-to-thickness ratio, width-to-thickness ratio and boundary conditions on static characteristics of the monoclinic plate were studied. As a result, the following conclusions are notable.

- Excellent accuracy and convergence were obtained by the developed differential quadrature method on the static response of rectangular plates.
- For the same loading conditions, displacements obtained for simply supported monoclinic plate is higher than the fully clamped once.
- By studying the differences between our results for monoclinic plates and the results for especially orthotropic plates, one could find that it is worth to investigate the monoclinic plates to have more accurate results.

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