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Reliability analysis of simply supported beam using GRNN, ELM and GPR

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Abstract. This article deals with the application of reliability analysis for determining the safety of simply supported beam under the uniformly distributed load. The uncertainties of the existing methods were taken into account and hence reliability analysis has been adopted. To accomplish this aim, Generalized Regression Neural Network (GRNN), Extreme Learning Machine (ELM) and Gaussian Process Regression (GPR) models are developed. Reliability analysis is the probabilistic style to determine the possibility of failure free operation of a structure. The application of probabilistic mathematics into the quantitative aspects of a structure and improve the qualitative aspects of a structure. In order to construct the GRNN, ELM and GPR models, the dataset contains Modulus of Elasticity (E), Load intensity (w) and performance function (δ) in which E and w are inputs and δ is the output. The achievement of the developed models was weighed by various statistical parameters; one among the most primitive parameter is Coefficient of Determination (R^2) which has 0.998 for training and 0.989 for testing. The GRNN outperforms the other ELM and GPR models. Other different statistical computations have been carried out, which speaks out the errors and prediction performance in order to justify the capability of the developed models.

Keywords: beam; deflection; ELM; GPR; GRNN; prediction

1. Introduction

Over Building structures are constructed wherever the normal landscape should be modified to empower the fulfillment of framework ventures. Additional requesting basic prerequisites, generally fluctuating landscape and dynamic urban improvement make building structures of different sorts important. Structures are presented to exceptional dimensions of worry from traffic and various natural variables. Generally, every civil engineering designs and structures has its own uncertainties, extensively in loading, material strength and in the adopted analyzing techniques. The concept factor of safety was employed in order to assess the uncertainty in characteristic load and strength which has the criteria that load should be lesser than the material strength. In regarding to the designing and analyzing techniques appropriate methods should be adopted as it has to yield not only safety but also economic designs.

Beam is an inevitable element of any structure that resists loads applied laterally along its axis. The load applied on the beam generates shear forces and bending moments, which in turn create stress, strain and deflections. Beams are characterized by their support, length, size and material. The application of soft computing techniques has widely scattered on the issues on beams, columns, also beams and column joints. The demeanor of the joint shear beam-column is eminently problematic and non-linear, especially when disparate variables influenced the joint shear strength. Yaseen *et al.* (2018) utilized the deep learning neural network (DLNN) for forecasting the joint shear strength of RC beam-column connections. The optimal input combination attributes were accomplished in their model 6 and model 14 with better regression values 0.88 and 0.92. The new hybrid artificial intelligence technique which merged the regression and optimization, Support Vector Regression with Firefly Optimization Algorithm (SVR-FFA) had benefitted the prediction of shear strength of Steel Fiber Reinforced Concrete Beam (SFRCB) (Abeer *et al.* 2018). The data was collected from different researchers which comprises of various concrete beam dimensional properties and the characteristics of concrete. This practiced technique exposed the best possible outcome in terms of forecasting accuracy with R^2=0.96. This article considered the simply supported reinforced concrete beams with various grades. The upcoming figure 1 provides the details of the beam.

Theoretically, the performance function (δ) can be computed by the following equation

$$\delta = \frac{L}{325} - \frac{5wL^4}{384EI} \tag{1}$$

where L is the length of the beam; w is the load applied per unit length; E is the modulus of elasticity and I is the moment of inertia. Also EI is called flexural rigidity and it is inversely proportional to the deflection. When the value of $\delta \leq 0$ then it will be considered as unsafe and when $\delta > 0$ then it is safe.

In general the term 'reliability' is utilized to raise the

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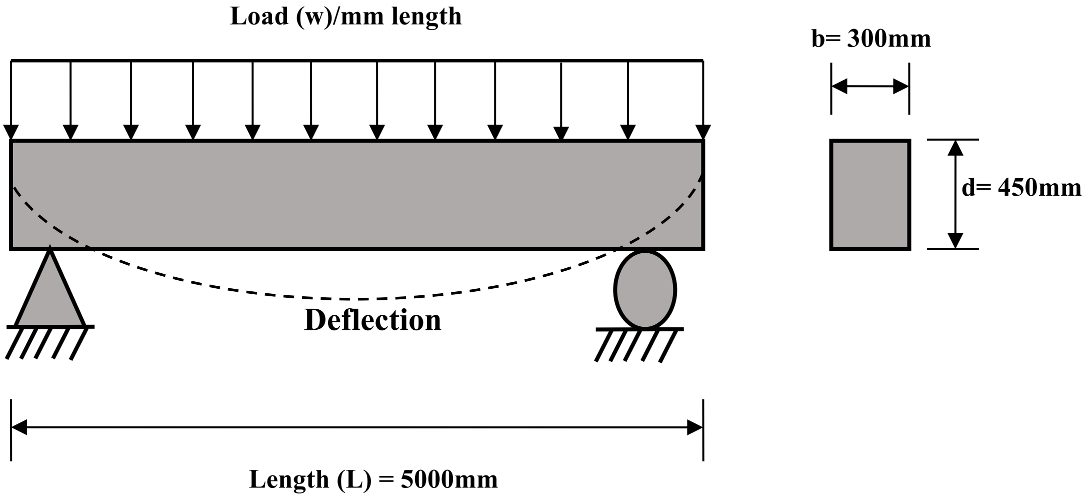

Fig. 1 Schematic diagram of simply supported beam.

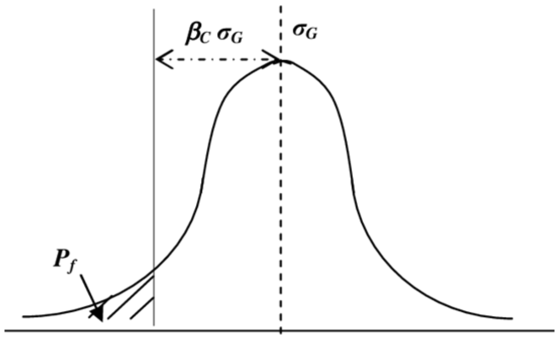

Fig. 2 Cornell reliability index, probability of failure

limit of a framework to continue playing out its proposed capacity. The application of reliability theory to geometric design can allow designers to determine the probability of non-compliance (P_{nc}) as well as providing a measure of safety, called reliability index (β) (Navin 1992). Mathematically, reliability can be expressed as

$$Reliability = 1 - P(f) \tag{2}$$

where P(f) is the probability of failure.

Cornell (1969) utilized typical couple of variables approach (mean (μ) and variance (σ)), in which he presumed the resulting probability of G is a normal dissemination. Then he delineated the reliability index (β_c) is the absolute value of the ordinate of the point corresponding to G =0 on the standard normalized probability plots. This was depicted in the upcoming figure (2) (Selmi *et al.* 2010). The equation (3) provides the formula for determining the reliability index

$$\beta_C = \frac{\mu_G}{\sigma_G} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 - \sigma_S^2}} \tag{3}$$

When the value of β_c is more, the probability of failure is less.

Reliability analysis forecasts the deterioration rate of elements and completes system reliability. These predictions are utilized to

- assess the design possibility
- compare the design alternatives
- identify and diagnose the promising failure
- compromise system design aspects and
- track reliability enhancement.

Reliability analyses of deteriorating reinforced concrete structures affected by reinforcement corrosion due to crack width, strength of rebar, etc. Pipelines are one of the most secured and cost effective structures for transporting the crude oil and natural gas. The deformity due to corrosion is one of the major menaces to burst out the oil and gas. The deterministic mechanisms declined to estimate the failure probability, hence reliability analysis, CHL-RF algorithm was adopted to compute the burst pressure of intact pipes and the strength left out due to corrosion. The outcome conveyed that the adopted probabilistic model provided promising outcome with minor errors (Behrooz and Miri, 2014). Hua and Nan (2015) provided the numerical sample by utilizing the Weibull model for demonstrating the performance of the concrete structures with respect to reinforcement corrosion. Chaotic Conjugate Stability Transformation Method (CCSTM) was the technique utilized to revamp the efficiency and robustness of eight distinct structural or mechanical issues (Keshtegar, 2016). One among the issue was the cantilever beam with external forces, moments and loads had two different limit state functions. CCSTM provided the reliability index of 3.468414 and 3.048462 within 6 and 7 iterations which is least when compared with other reliability methods. Likewise, CCSTM had outperformed other reliability models on other instances based on the speed convergence and efficiency. Jesna and Anjaneyulu (2016) has developed safety evaluation criterion for the horizontal curves on two lane highways. They implied that the curves with higher reliability indices are secured than those with inferior values. Gunner (2016) utilized ANN and predicted the shear strength of RCC deep beams with high precision. The adopted model forecasted the ultimate shear strength with more accuracy of R^2=0.97. Wei *et al.* (2017) made a quantitative study in paratransit service reliability under different zoning strategies with the real time data from Houston, Los Angeles and Boston. Olmati *et al.* (2017) has utilized reliability analysis of punching in the reinforced concrete flat slabs. They also derived the safety factors for computing the probability of punching under accidental loads. They concluded that falling of slab is more pernicious than removal of column. Longitudinal reinforced concrete beams were designed according to the Eurocode 2 and the shear failure was identified and diagnosed with the help of reliability analysis (Slowik *et al.* 2017). The shear strength prediction of the steel fibre reinforced concrete (SFRC) was determined in an optimum manner for in order to use in vast applications. Riza (2017) adopted ANN for predicting the shear strength and ductility of RC beams. The flexural behavior of RCC beams and ultra-high performance fiber-reinforced concrete (UHPFRC) was predicted by using various analytical models. The models predicted the strength of the RC beams after pre-loadings and also subjected to fatigue loading. It was found that the forecasted values are in the vicinity of the measured values (Ramachandra *et al.* 2018). Many researchers and scientists have successfully utilized this reliability analysis in disparate fields (Wei *et al.* 2011, Ignacio *et al.* 2012, Zhang and Jiang 2012, Yinghe *et al.* 2013, Wang *et al.* 2014, Jeffrey *et al.* 2015, Abdelouafi *et al.* 2015, Arvydas and John 2016, Zhou *et al.* 2016, Jahani *et al.* 2016, Baoyu *et al.* 2017, Yaseen and Keshtegar 2018).

Generalized Regression Neural Network (GRNN) was originally proposed by Specht (1991) It is a variation of the radial basis neural networks which is based on kernel regression networks (Kim *et al.* 2004, Cigizoglu and Alp 2005, Celikoglu and Cigizoglu 2007) frequently used for function approximation. GRNN recommended a new calibration procedure for travel mode choice analysis in a transportation modeling framework (Celikoglu, 2006). GRNN outperforms other models like RBFNN, FFBPNN, MVLR by consuming least time and root mean square error (1154) with more coefficient of determination (0.975). In order to iron out the dynamic network loading at an unsignalized highway node Celikoglu (2007) adopted three different Neural Networks and GRNN was one among that. After calibrating the Neural Network components with conical delay function formulation, the delays forming were computed this was the outcome of capacity, capability and flow competition. GRNN provided better result compared with other models when it underwent statistical evaluation (R^2=0.998 and RMSE=0.298). Hilmi and Hitmet (2007) took advantage of GRNN in the area of transportation engineering, prediction of daily trip flows. GRNN accomplished by yielding positive values and adjacent values. This would be supportive for the advanced traffic management systems. Gaurav and Hasmat (2016) had predicted the wind speed in the western region of India by the GRNN model. In order to elevate the efficiency and accuracy of the classroom teaching, GRNN was utilized and yield the output with minimal error percentage (Cheng and Xiong, 2017). Application of GRNN has outstanding records in diverse fields (Shaikh *et al.* 2010, Schaffer *et al.* 2012, Walker *et al.* 2012, Campbell *et al.* 2013, Harish *et al.* 2014, Chen *et al.* 2015, Divya *et al.* 2016).

Extreme Learning Machine (ELM) was framed by Huang *et al.* (2006). Kindie *et al.* (2016) has utilized ELM in classifying the heart diseases and its severity namely low, average, high and serious with an encouraging accuracy level. ELM has been effectively used by Cheng and Xiong (2017) for the prediction of dam displacement. By depicting the minimal error percentages, they justified the efficiency of ELM. Fatigue is the foundational procedure that outcomes in the crack of the material when exposed to fluctuating stress with a tensile strength less than that of the material itself. Hence, the fatigue performance has to be under consideration before implementing the material under loads. Yaseen *et al.* (2019) had foreseen the fatigue failure with the help of modern data-intelligence model ELM by considering the input variables such as the geometry dimension, the stress, and the orientations of the fiber glass reinforced material. ELM model showed its capability with promising accuracy in prediction and satisfactory range of residual errors. Thus ELM has proved its capability not only in prediction but also in classifying the data. Various applications used ELM in different fields (Lahouari*et al.* 2013, Peng *et al.* 2014, Lei *et al.* 2015, Zuo *et al.* 2015, Yara and Awad, 2015, Bartosz 2016).

Gaussian Process Regression (GPR), a supervised learning algorithm framed to figure out the regression and classification obstacles (Rasmussen and Williams, 1996). For prediction in smart grid, Mori and Nakona (2014) proposed the new method for Locational Marginal Pricing (LMP). LMP is vital for maintaining the economic efficiency in power market where they do maximize the profit with lesser risks. They adopted GPR for forecasting the LMP and succeeded with encouraging accuracy comparatively. Fei *et al.* (2015) utilized GPR for evaluating the stability of soil slopes. Latin hypercube adopted for generating the samples and the dataset was then incorporated to GPR for determining the failure probability of slopes. The application of GPR was in numerous fields with wide range (Ma and Yan 2014, Douglas and Kwang 2016, Robert *et al.* 2017, Milica *et al.* 2017).

This article determines the performance function of simply supported beam by utilizing the GRNN, ELM and GPR models. The details of the beam are depicted in the fig 1. MATLAB is used develop these models. The following flowchart represents the algorithm of the proposed models.

2. Details of GRNN

GRNN, a variation to radial basis neural networks typically utilized for functional approximation. It can be applicable for the classification, regression and prediction. The essential objective of this GRNN is to attain the flawless mapping between the input and the target vector with least errors. GRNN has two different layers: radial basis layer and the linear layer. The radial basis layers encompass the concealed neuron which was proportionate to number of inputs. The net input to each neuron is the product of the weighted input with its bias. The concealed

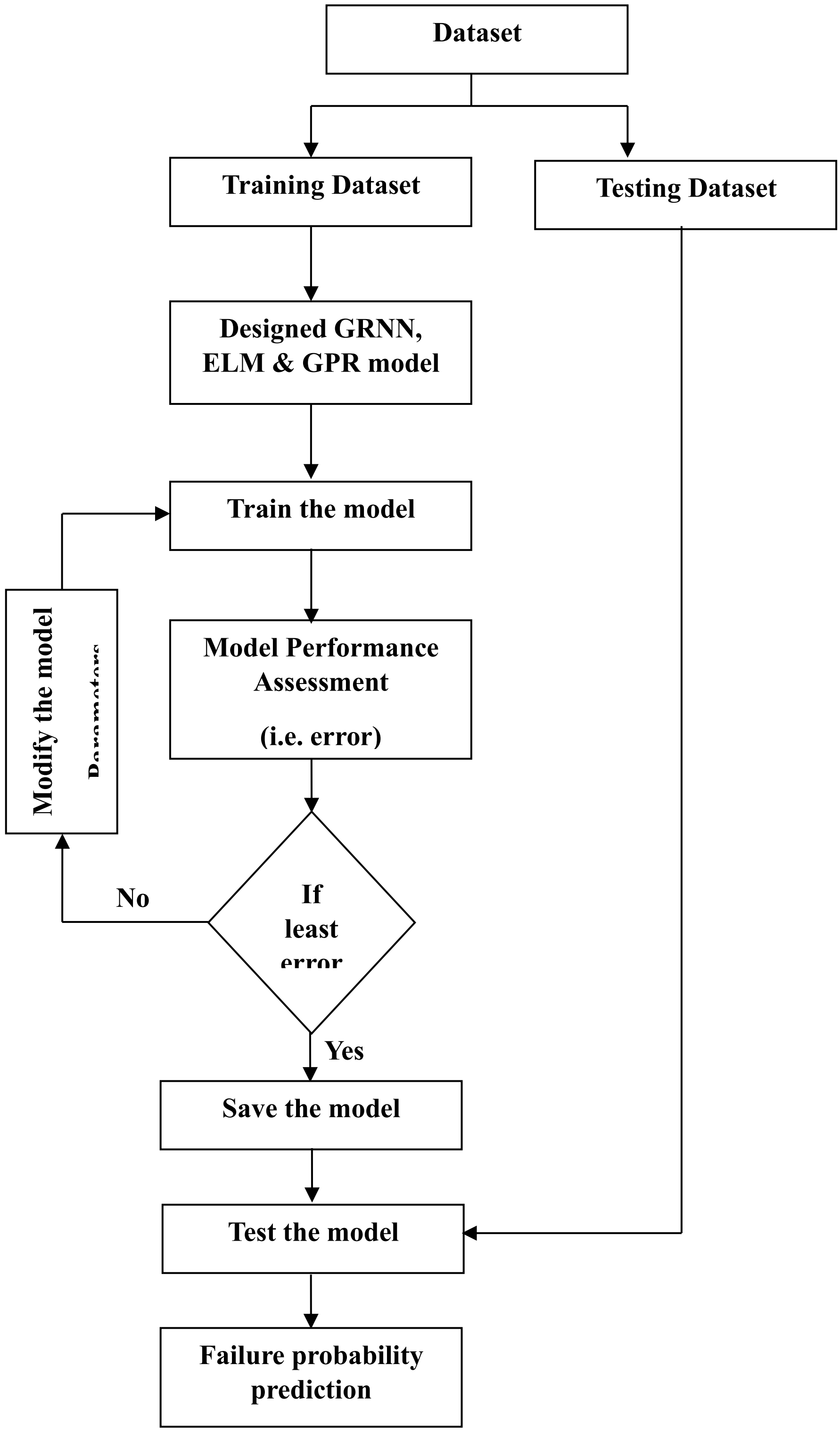

Fig. 3 Proposed Algorithm of the adopted model

neurons compute the Euclidean distance from midpoint of neuron to the test case. This is linked up with the linear layer which is also called summation layer. GRNN is not in need of any repetitive training procedure (Gaurav and Hasmat 2016).

Normal distribution has been utilized in GRNN as the probability density function. The general equation for GRNN is as follows (Specht 1991)

$$Y(x) = \frac{\sum_{k=1}^{n} Y_k \exp\left(\frac{-d_k^2}{2\sigma^2}\right)}{\sum_{k=1}^{n} \exp\left(\frac{-d_k^2}{2\sigma^2}\right)} \tag{4}$$

where Y(x) is the output; σ is the smoothness parameter or standard deviation

d_k is the length between the training sample and the point of prediction and it can be calculated as

$$d_k^2 = \left(x - x_k\right)^T \left(x - x_k\right) \quad (5)$$

x_k is the training sample; T is transpose.

The equation (4) can predict the performance of systems based on training samples; forecast the smooth multi-dimensional curves and interpolate between training samples. GRNN has various pros like no need of back propagation, high veracity and adjustment of noises in the inputs. The preeminent benefit of GRNN are expeditious learning and merging to the ideal relapse surface as the number of tests gets to be exceptionally high (Specht 1991). Also GRNN provides specific convenience with sparse data in the real time environment, because the surface of the regression is directly exemplified everywhere. If one predicted value is similar to the observed value, then the second value will bisect hyperspace into maximum and minimum halves with glossy progression between them, so that the surface becomes more complex when there are more data has been added. Also, this method grasps from the data and generalizes from the instances as soon as they are stocked. The leading con of this technique specifies the gob of reckoning required for the trained system to forecast the new output. But this disadvantage can be visibly moved by the cluster version of GRNN (Specht 1991).

GRNN adopts 80 datasets which consists of Modulus of Elasticity (E) and Load intensity (w) as the input and yield the failure probability of the reinforced concrete simply supported beam. In order to develop the GRNN model, the dataset has been cleaved into two: training dataset and testing dataset.

Training dataset is helps to construct the model whereas the testing dataset is used to evaluate the developed model. For the data segregation there is no thumb rule as it was the trial and error approach. Sitharam *et al.*, (2008) had utilized 90% of the available data for the training dataset and the remaining for the testing dataset. Therefore, 70% of data say 56 datasets were considered as training dataset and the remaining 30% say 24 as testing dataset. As mentioned previously, GRNN was constructed by the MATLAB. The data has been normalized between 0 and 1 by using the formula (6)

$$x_{normalized} = \frac{x - x_{\min}}{x_{\max} - x_{\min}} \quad (6)$$

where x is the value, x_{min} is the minimum value and x_{max} is the maximum value.

3. Details of ELM

The conventional learning algorithm based on ANN which adopts the empirical risk minimization principle may exact the non-linear function of the input to output data. ELM is an adequate learning algorithm for single hidden-layer feedforward neural networks (SLFN) projected by Huang *et al.* (2004, 2006). ELM arbitrarily chooses the input weight matrix and hidden layer biases and the output weights of SLFN's can be resolved analytically through uncomplicated generalized contrary action of the hidden-layer output matrices (Lei *et al.* 2015).

Let us consider N random distinct samples $D = \left\{\left(x_i, y_i\right)_{i=1}^{N}\right\}$ where $x_i = \left(x_{i1}, x_{i2}, \ldots x_{im}\right)^T \in R^m$ and $y_i = \left(y_{i1}, y_{i2}, \ldots y_{in}\right)^T \in R^n$. Also with P hidden nodes and activation function k(x) can approximate these N samples with no error, there exist β_i, w_i and b_i such that

$$f_P\left(x_j\right) = \sum_{i=1}^{P} \alpha_i K\left(w_i, b_i, x_j\right) = y_j \quad (7)$$

where j=1,2,3,…N.

The above equation can be rewritten as the following equation (8)

$$H\alpha = Y \quad (8)$$

Where

$$H\begin{pmatrix} w_1, w_2, \ldots w_P, b_1, \\ b_2, \ldots b_P, x_1, x_2, \ldots x_P \end{pmatrix} = \begin{bmatrix} k\left(w_1.x_1 + b_1\right) \ldots k\left(w_P.x_1 + b_P\right) \\ . \quad \ldots.. \quad . \\ . \quad \ldots.. \quad . \\ . \quad \ldots.. \quad . \\ k\left(w_1.x_N + b_1\right) \ldots k\left(w_P.x_N + b_P\right) \end{bmatrix}$$

$$\alpha = \begin{bmatrix} \alpha_1^T \\ . \\ . \\ \alpha_P^T \end{bmatrix}_{Pxm} \text{ and } \quad Y = \begin{bmatrix} y_1^T \\ . \\ . \\ y_P^T \end{bmatrix}_{Nxm} \quad (9)$$

where $w_i = \left(w_{i1}, w_{i2}, \ldots w_{im}\right)^T \in R^m$ is the weight connects the i[th] hidden node to the input nodes, $\alpha_i = \left(\alpha_{i1}, \alpha_{i2}, \ldots \alpha_{im}\right)^T \in R^m$ is the weight links the i[th] hidden node to the output nodes, b_i is the threshold, H is the hidden layer output matrix. Therefore the output weights can be computed by determining the minimum least square solution for the given linear system and it is given by the upcoming equation (10)

$$\hat{\alpha} = H^{+}Y \quad (10)$$

where $\hat{\alpha}$ is the predicted value of α and H^{+} is the Mooree Penrose generalized inverse of matrix H.

This technique elevates the training speed and the generalization performance which is considered as the pros of ELM (Kheon *et al.* 2015). Also it dwindles the computation burden without immolating the prediction veracity by figuring out the befitting activation function. The main disadvantage is that it can classify linearly separable problems.

ELM adopts the same training and testing dataset as utilized by GRNN. Training dataset used for developing the model and the testing dataset is utilized for verifying the developed model. MATLAB was used to construct the ELM model

4. Details of GPR

Gaussian Process is a Bayesian technique which indicates earlier dissemination over the uncharted function and then given some information forecasts the posterior (Rasmussen and Williams 2005). GPR contains mean function in which is what we anticipate the uncharted function to glimpse like previously we have seen any information. Also it has the kernel function which point out the prior information of the correlation of function values for divergent parts of input space. For every input point the kernel function define where the function value lie and therefore defines the correlation between the known and the unknown function values. By this process the known function values give great impact on the places where the data was missing. Also, for every input point the Gaussian process defines a Gaussian distribution over possible function values with mean and variance (Milica *et al.* 2016). Let us consider the stochastic variable Y was disseminated as

$$Y \sim N\left(\mu, \sigma^2\right) \tag{11}$$

where μ is mean and σ is covariance. This means the observation Y we would forecast the value is not too far off μ and if the series of observation we would hope around 68% of the values would fall in the range ($\mu-\sigma$, $\mu+\sigma$).

A Gaussian process extends the concept of a stochastic variable to a stochastic function $f(x)$. Therefore the stochastic function is distributed as

$$f(x) \sim GP\left(m(x), k\left(x, x'\right)\right) \tag{12}$$

where $m(x)$ is the mean function and $k(x,x')$ is the covariance function.

In order to comprehend the Gaussian processes let us consider p dimensional stochastic variable spread as

$$Y \sim N_p(\mu, \Sigma) \tag{13}$$

where p x 1 is the mean vector μ whereas p x $p$is the covariance matrix Σ as per the character of multivariate normal distribution.

For instance if we take one sample from Y we could predict that sample is close enough to μ and if we take more samples we could strongly believe that it would lie on the contours produced by Σ as its role is to explain how variable the distinct samples of Y are. In order to determine the f(x) where f is dependent variable and x is the independent variable and any unobserved pair (x^*, f^*) as (Jesper and Stamatios 2015)

$$\begin{bmatrix} f \\ f^* \end{bmatrix} \sim N_{n+1}\left(0, \begin{bmatrix} K(X,X) & k\left(X,x^*\right) \\ k\left(x^*,X\right) & k\left(x^*,x^*\right) \end{bmatrix}\right) \tag{14}$$

where $K(X,X)$ is an n x n matrix of covariance between all the points in the training data, $k(X, x^*)$ is an n x 1 vector of covariance between the unobserved point x^* and training data, $k(x^*, x^*)$ is the variance. In the typical regression the mean ($\bar{f}$) from f and then it integrates to f^*

$$p\left(f^* \mid x^*, X, f\right) = N\begin{pmatrix} k\left(x^*, X\right)K(X,X)^{-1} \\ f, k\left(x^*, x^*\right) - k\left(x^*, X\right) \\ K(X,X)^{-1}k\left(X, x^*\right) \end{pmatrix} \tag{15}$$

The above equation (15) expressed X and f by maximizing the joint probability of f^*conditional on x^* in order to determine the f^*.

Thus the forecasting equation (15) assumes the training data is flawless (no uncertainty in the dependent variable f). The covariance function $k(x,x')$ does not model any fault in the data, but the variability of the function itself. For all values of x^*, the output plots passed exactly all of the training points (x_i, f_i) when we calculate $k(x^*,X)K(X,X)^{-1}$. Hence for utilizing our noisy data the model has to be accompanied by measurement error. Therefore the equation (14) was converted into (Jesper and Stamatios 2015)

$$\begin{bmatrix} f \\ f^* \end{bmatrix} \sim N_{n+1}\left(0, \begin{bmatrix} K(X,X) + \sigma^2 I & k\left(X,x^*\right) \\ k\left(x^*,X\right) & k\left(x^*,x^*\right) \end{bmatrix}\right) \tag{16}$$

where the conditional likelihood and the variance change to

$$\hat{f}\left(x^*\right) = k\left(x^*, X\right)\left(K(X,X) + \sigma^2 I\right)^{-1} f \tag{17}$$

and

$$Cov\left(\hat{f}\left(x^*\right)\right) = k\left(x^*, x^*\right) - k\left(x^*, X\right) \left(K(X,X) + \sigma^2 I\right)^{-1} k\left(x, x^*\right) \tag{18}$$

where σ^2 is the variance of the observed error and I is the identity matrix.

The equation (17) gives an approach to make forecasts for both observed and imperceptibly points x^* given few scrutiny x and f, provided we know the function $k(x,x')$. Genton (2001) explained that for a function $k(x,x')$ to work as a covariance function it requires to deliver a positive definite matrix $K(X,X)$ for any set of X of points in the domain $f(x)$.

The memory requirement and problem complexity of GPR has greatly developed, which are considered as the pro of this method. It learns the regularization parameter and the kernel function quickly. The issue of this method includes the efficiency loss due to high dimensional spaces; not dealing with the discontinuous data. GPR chose the same training and testing dataset as utilized by GRNN and ELM. Training dataset used to build the model and the testing dataset was utilized for scrutinizing the built model. MATLAB was used to evolve the GPR model.

The performance of the developed models can be assessed by Coefficient of Determination (R^2) value as it has the capacity to discover the probability of future

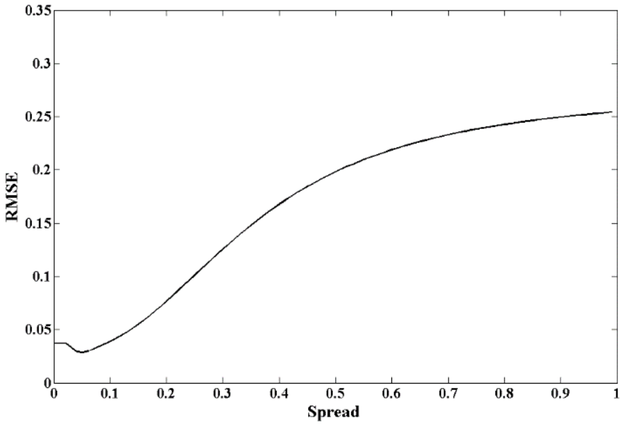

Fig. 4 Plot between Spread and RMSE.

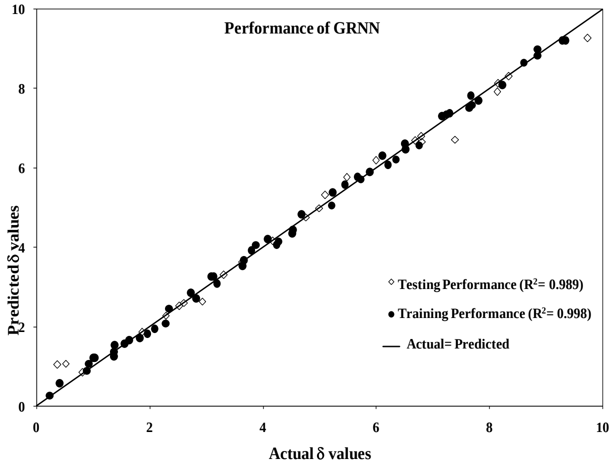

Fig. 5 Performance of GRNN model.

occasions falling inside the anticipated results. If the R^2 value is near to 1, then it was said to be the good model. The formula for determining this R^2 value is as follows

$$R^2 = \frac{\sum_{t=1}^{n}\left(d_t - d_{mean}\right)^2 - \sum_{t=1}^{n}\left(d_t - y_t\right)}{\sum_{t=1}^{n}\left(d_t - d_{mean}\right)^2} \quad (19)$$

where n is the number of training and testing samples

d_t is the measured value; d_{mean} is the mean of actual value; y_t is the predicted value

There are other statistical parameters for justifying the performance of the developed model (Yagiz *et al.* 2012, Nurichan 2014, Chandwani *et al.* 2015, Yaseen *et al.* 2018, Khestegar *et al.* 2019). The parameter comprised of Mean Absolute Percentage Error (MAPE) correlate the enduring error for each data point with respect to the measured or predicted value. Minimum values of MAPE express the exceptional performance of the model and maximum value depicts the incapability of the model. Coefficient of Efficiency (E) is the ratio of residual error variance to measured variance in observed data, in which the value closes to unity; point out the veracity of the model. Root Mean Square Error (RMSE) analyzes the measured values to the forecasted values and reckons the square root of the average residual error. The least values of RMSE justifies positively about the prediction performance of the model. Normalized Mean Biased Error (NMBE) weighs the potential of the model to forecast a value which is situated away from the average value. The positive values of NMBE point out over-prediction whereas the negative value considered as under-prediction of the model. Weighted Mean Absolute Percentage Error (WMAPE) computes the weighted mean absolute percentage error of the prediction. Root mean square error to observation's standard deviation ratio (RSR) joins the advantages of mistake list measurements and incorporates a scaling/standardization factor, with the goal that the subsequent measurement revealed qualities can apply to different constituents. Variance Account Factor (VAF) describes the ratio of error variance to the observed variance. Performance Index (PI) was used to scrutinize the veracity of the statistical techniques; however RMSE, VAF and R^2 are not high caliber. Instead of R^2, Adjusted Determination of Coefficient (Adj R^2) was used to determine the PI. The formula for determining those statistical parameters were listed below

$$WMAPE = \frac{\sum_{t=1}^{n}\left|\frac{d_t - y_t}{d_t}\right| \text{x}\, d_t}{\sum_{1}^{n} d_t} \quad (20)$$

$$E = 1 - \left[\frac{\sum_{1}^{n}\left(d_t - y_t\right)^2}{\sum_{1}^{n}\left(d_t - d_{mean}\right)^2}\right] \quad (21)$$

$$RMSE = \sqrt{\frac{\sum_{t=1}^{n}\left(d_t - y_t\right)^2}{n}} \quad (22)$$

$$Adj\, R^2 = 1 - \left(1 - R^2\right)\frac{(n-1)}{(n-p-1)} \quad (23)$$

$$MAPE = \frac{1}{N}\sum_{1}^{N}\frac{\left|d_t - y_t\right|}{d_t} \text{x}\, 100 \quad (24)$$

$$RSR = \frac{RMSE}{\sqrt{\sum_{1}^{N}\frac{\left(d_i - d_{mean}\right)^2}{N}}} \quad (25)$$

$$NMBE = \frac{\frac{1}{N}\sum_{1}^{N}\left(y_t - d_t\right)}{\frac{1}{N}\sum_{1}^{N} d_t} \text{x}\, 100 \quad (26)$$

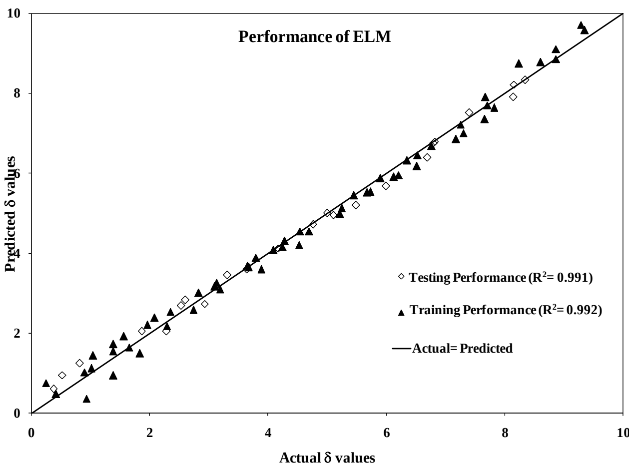

Fig. 6 Performance of ELM model.

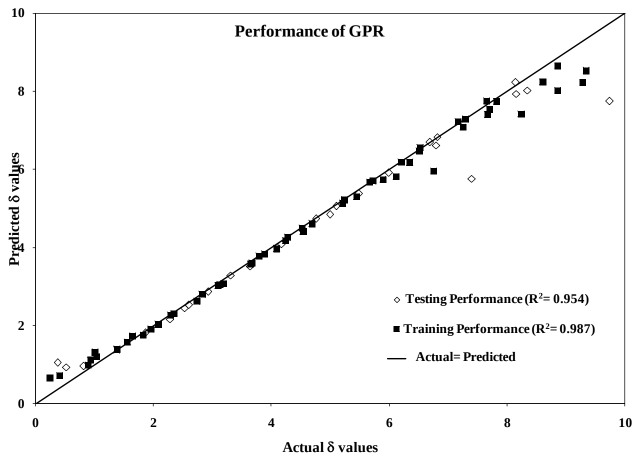

Fig. 7 Performance of GPR model

$$VAF = \left(1 - \frac{\mathrm{var}\left(d_t - y_t\right)}{\mathrm{var}\left(d_t\right)}\right) \mathrm{x}100 \tag{27}$$

$$PI = Adj\ R^2 + 0.01VAF - RMSE \tag{28}$$

These statistical parameters will justify the capability and prediction accuracy of the developed models and the values are tabulated in the upcoming section.

5. Results and discussion

GRNN generates a plot between the spread value (σ) and Root Mean Square Error (RMSE) in which the value of σ should be the one with minimum RMSE.

The figure (4) depicted the optimum spread value as 0.0449 which has the least RMSE. Thus the tuning parameter has been determined and the performance of the GRNN model was exposed in the upcoming figure (5)

The above figure 5 depicts the training and testing performance of the GRNN model with the value of R^2 nearby one and hence GRNN proved its efficiency in determining the performance function (δ). For the ELM model, the number of hidden nodes was found out by the trail an error approach and for this data ELM provides the best performance with 7 numbers of hidden nodes. The following figure (6) establishes the performance of ELM model with the value of R^2 close to one.

Thus the capability of ELM model was depicted clearly in the above figure 6. Even though the R^2 value of ELM is comparatively less than the GRNN model, ELM also has its better figure in determining the performance function (δ). GPR has two different designing parameters ε and width (σ) of radial basis function and the values are purely based on trial and error approach. GPR provides the best result when ε =0.001 and σ = 0.3. The performance of GPR was assessed by the coefficient of correlation. The following figure (7) explores the capability of GPR. Thus the developed GPR also has shown its efficiency in determining δ.

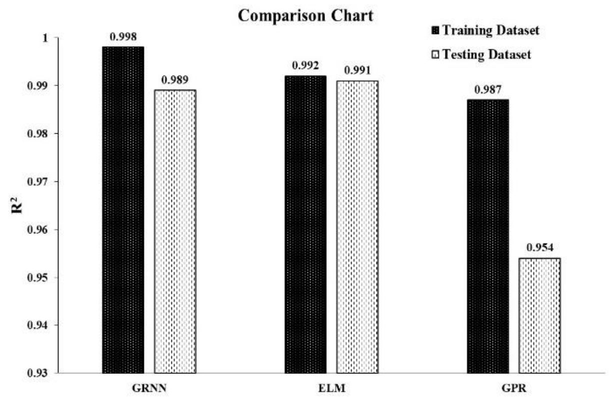

Fig. 8 Comparison of models based on R^2 value

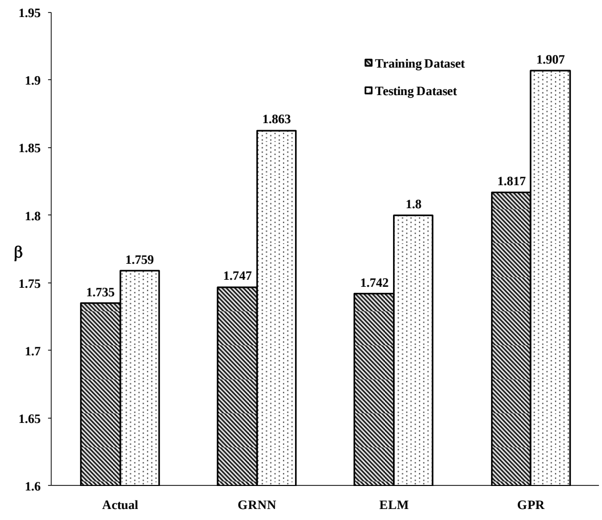

Fig. 9 Comparison chart of the developed models based on β values

The training and the testing performance were clearly depicted that the other two developed model outperforms the GPR model. The upcoming figure (8) delivers the efficiency of the developed GRNN, ELM and GPR model based on the R^2 value of training and testing dataset. In the figure 8 the value of R^2 of GRNN model is preeminent than the other two models. This proves that the developed GRNN model has superlative capability in determining δ.

Table 1 Statistical parameters of the developed models

	GRNN		ELM		GPR	
	Training	Testing	Training	Testing	Training	Testing
WMAPE	0.024	0.067	0.041	0.041	0.038	0.059
NS	0.999	0.997	0.997	0.997	0.996	0.988
RMSE	0.125	0.271	0.238	0.251	0.297	0.562
R^2	0.998	0.989	0.992	0.991	0.987	0.954
Adj R^2	0.998	0.998	0.991	0.991	0.987	0.95
MAPE(%)	0.329	14.148	11.107	11.872	8.247	15.288
RSR	0.048	0.103	0.091	0.095	0.114	0.214
NMBE(%)	0.005	0.115	-0.004	0.745	-2.455	-3.501
VAF(%)	99.681	89.24	99.299	99.516	97.1	91.161
PI	1.869	1.707	1.746	1.734	1.661	1.3

Now the reliability index was computed using the equation (3) and the comparison of the measured and the predicted models has plotted and depicted in the upcoming figure (9).

The reliability index values were compared in which the values computed by the models GRNN and ELM were close to the measured reliability indices. The following table 1 describes the various statistical parameters and its values.

The above table 1 expose that developed models performed well in predicting the performance functions. The values of NMBE clarifies that the GRNN model was neither over nor under predicted the performance function. Comparing with the developed models, the values of GRNN in each statistical parameter have depicted the superior capability and justified its potential.

6. Conclusions

This article expected to build up a specialist framework to anticipate the performance function of the simply supported beam by considering the modulus of elasticity and load. It also interprets how the adopted GRNN, ELM and GPR utilized the 80 data from experiments in order to expose its capability of determining the performance function of simply supported beam. The flow chart explains the technical procedure for developing and determining the performance function. The developed models have been evaluated by the reliability analysis and other statistical investigations. In this, GRNN has only one tuning parameter whereas the GPR and ELM has 2 and 3 tuning parameters. Furthermore, GRNN is consistent and the time consumption was also relatively lesser. GRNN approximates any discretionary function among input and yield vectors, drawing the function gauge straightforwardly from the training data. Three different methods came with better outcome; however GRNN exhibits very acceptable giving close or once in a while even prevalent performance when looked at other built ELM and GPR models. The other statistical criteria's also justifies the capacity of GRNN for this specific problem. Thus GRNN is considered as an authoritative and reliable tool for determining the performance function of the simply supported beam for the futuristic purposes. Users can use the

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