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Analytical methods for determining the cable configuration and construction parameters of a suspension bridge

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Abstract. Main cable configurations under final dead load and in the unloaded state and critical construction parameters (e.g. unstrained cable length, unstrained hanger lengths, and pre-offsets for tower saddles and splay saddles) are the core considerations in the design and construction control of a suspension bridge. For the purpose of accurate calculations, it is necessary to take into account the effects of cable strands over the anchor spans, arc-shaped saddle top, and tower top pre-uplift. In this paper, a method for calculating the cable configuration under final dead load over a main span, two side spans, and two anchor spans, coordinates of tangent points, and unstrained cable length are firstly developed using conditions for mechanical equilibrium and geometric relationships. Hanger tensile forces and unstrained hanger lengths are calculated by iteratively solving the equations governing hanger tensile forces and the cable configuration, which gives careful consideration to the effect of hanger weight. Next, equations for calculating the cable configuration in the unloaded state and pre-offsets of saddles are derived from the cable configuration under final dead load and the conditions for unstrained cable length to be conserved. The equations for the main span, two side spans and two anchor spans are then solved simultaneously. In the proposed methods, coupled nonlinear equations are solved by turning them into an unconstrained optimization problem, making the procedure simplified. The feasibility and validity of the proposed methods are demonstrated through a numerical example.

Keywords: suspension bridge; cable shape; unstrained length; hanger tension; saddle; pre-offset

1. Introduction

A suspension bridge consists of main cables, hangers, a deck, towers, anchorages, tower saddles, splay saddles, cable clamps and other components. The ultimate aim of structural design and construction calculations is to achieve the desired bridge configuration with high accuracy. The first step in this process is to determine a target cable configuration under final dead load, which then serves as a data source for subsequent construction calculations. As post-installation adjustment of cable configuration is very difficult, accurately determining cable configuration in the unloaded state and construction parameters is a key to achieving target configurations of the completed bridge. Critical construction parameters include the unstrained cable length, unstrained hanger lengths, and pre-offsets for tower saddles and splay saddles. Accurate calculations of these parameters require full consideration of effects of cable strands over the anchor spans and the arc-shaped tops of saddles. Moreover, it is also necessary to take into account the effect of tower top pre-uplift when calculating the pre-offsets of saddles.

Cable saddles are used to turn main cables and thus they can directly constrain the deformation of main cables. Main cables are tangent to the arc-shaped tops of saddles in all conditions. A tower saddle normally has only one circular arc, while a splay saddle features multiple circular arcs of different radii. This increases the difficulties of calculating cable configurations over side spans and anchor spans.

Offsets are preset for both tower saddles and splay saddles during installation. Under final dead load, the horizontal components of tension in the cable segments at the two sides of each tower saddle are equal, and the sum of torques by cable tensions about the each splay saddle's center of rotation is zero. However, the external loads applied by different spans to a main cable differ. For example, a long main span can exert a relatively great load on the main cable, while a side span can exert only a low or even zero load on the main cable. In the unloaded state, these external loads do not arise and the internal force in a cable segment is equal to the internal force under final dead load minus the internal force arising from external load on it. Compared to external load from the side span, the external load from the main span can produce a greater internal force. If the saddles are installed at the positions for the completed bridge, there will inevitably be great unbalanced forces acting on the saddles in the unloaded state. These unbalanced forces may cause displacements of towers and sliding of cable strands in the saddles. To eliminate such unbalanced forces, tower saddles and splay saddles need to be offset by proper distances and angles, respectively, during installation. The resulting changes in span lengths between saddles will lead to significant

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changes in the sags of the main cable over different spans, thereby changing the tension in the main cable. Then the main cable segments at the two sides of a saddle will be in equilibrium with each other. The proper offset distance or angle for a saddle is the pre-offset. During the installation of deck, the saddles move towards their target positions. Therefore, the pre-offsets of saddles vanish in the bridge's completed state.

The elevation of a bare tower top is higher than the tower top's elevation under final dead load. The elevation difference is referred to as pre-uplift. Under final dead load, a main cable can exert a huge downward force on each tower, generating compression in the tower. If the bare tower is pre-uplifted to offset the amount of compression, the tower top's elevation under final dead load will reach the target level.

Configuration calculations for suspension bridges mostly use finite element methods based on the finite displacement theory (Irvine 1981, Jayaraman and Knudson 1981, Kim and Lee 2002, Thai and Kim 2011, Wang and Yang, Karoumi 2012, Sun *et al*. 2014) or analytical methods based on suspended-cable mechanics (O'Brien 1964, O'Brien and Francis 1964, Chen *et al*. 2013, 2015, Wang *et al* 2015, Jung *et al*. 2015). Given the structural characteristics of a suspension bridge, finite element methods usually involve simulating a construction process based on construction characteristics and requirements regarding cables' mechanical behavior and configuration under final dead load and then calculating cable configuration in the unloaded state via multiple loop iterations. However, finite element methods are inefficient in local detail processing and require an immense and complex computing system and a complicated computational process. In analytical methods, the first step is to calculate unstrained cable length on the basis of main cables' designed state under final dead load. Then cable configuration in the unloaded state is calculated based on the principle that the unstrained length of any cable segment remains constant during structural construction and after completion. In comparison, analytical methods have more explicit computational process and better capability of detail processing such as cable length correction, and are thus more widely applied to cable configuration calculation for suspension bridges. However, analytical shape-finding methods considering the effects of cable strands over the anchor spans, arc-shaped saddle top, and tower top pre-uplift are seldom reported. In addition, approaches for determining the pre-offsets of tower saddles and splay saddles are rare.

This paper proposes analytical methods for calculating cable configurations under final dead load and in the unloaded state and relevant construction parameters, which takes the effects of cable strands over the anchor spans, arc-shaped saddle top and tower top pre-uplift into account. They are later applied to the calculations involved in construction control for a suspension bridge with a 730m main span. The results demonstrate the feasibility and validity of the proposed methods.

Cable configuration calculation requires solving a number of coupled non-linear transcendental equations. For example, in the calculation of cable configuration in the unloaded state, the number of coupled nonlinear transcendental equations reaches up to 17. To avoid complicated iterations for equations, the problem of solving a system of equations can be transformed to an unconstrained optimization problem. Then the system of equations can be solved by the generalized reduced gradient (GRG) method (Wilde and Beightler 1967, Lasdon *et al*. 1974, 1978) for nonlinear programming.

2. Calculation of cable configuration under final dead load

The suspension bridge under study has one main span, two side spans and two anchor spans, as shown in Fig. 1. Calculation of cable configuration can only start from the configuration under final dead load. Unstrained cable length is an important parameter that relates the cable configuration under final dead load to that in the unloaded state, because the unstrained length of any cable segment remains constant in different stages of construction. In calculation of cable configuration under final dead load, the main span is first considered, followed by the side spans and then anchor spans. Specifically, the first step is to calculate the horizontal component of the tension in main cable over the main span. Next, cable configuration over a side span is calculated based on the principle that the horizontal components of the tension in the two cable segments at the two sides of each tower top are equal. Then cable configuration calculation is performed for an anchor span based on the conditions for the equilibrium at each splay saddle. Given the three-dimensional geometry of the cable strands over the anchor spans, the cable strands are usually treated as a whole when calculating their configuration and the anchor point is located at the center of the front anchor plane.

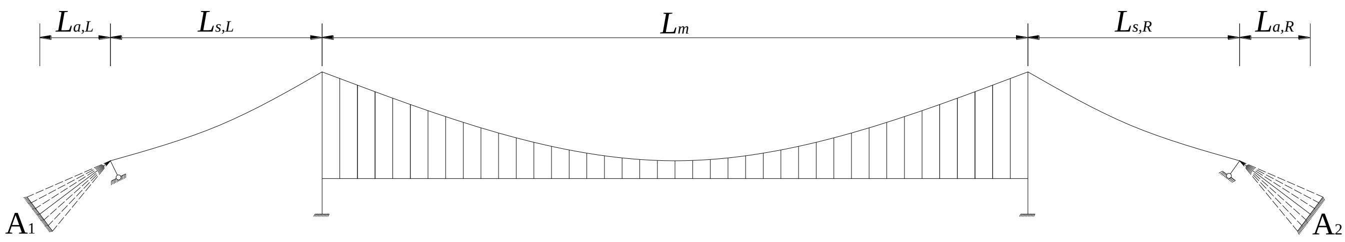

Fig. 1 Schematic of the whole bridge

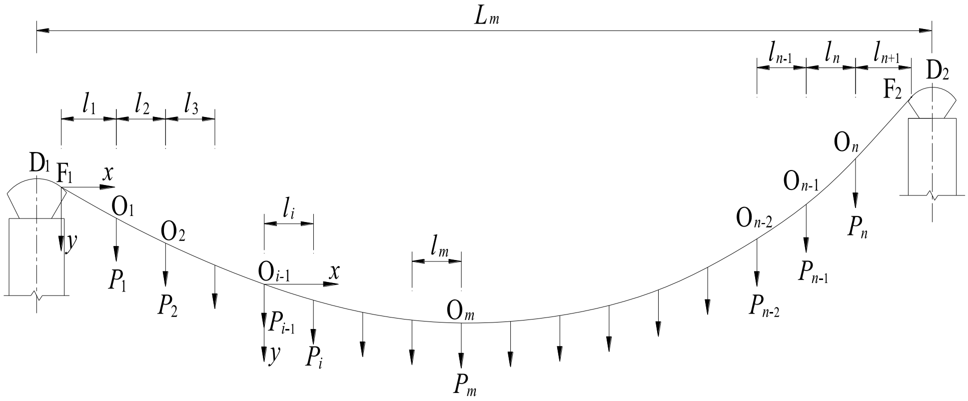

Fig. 2 Cable configuration under final dead load

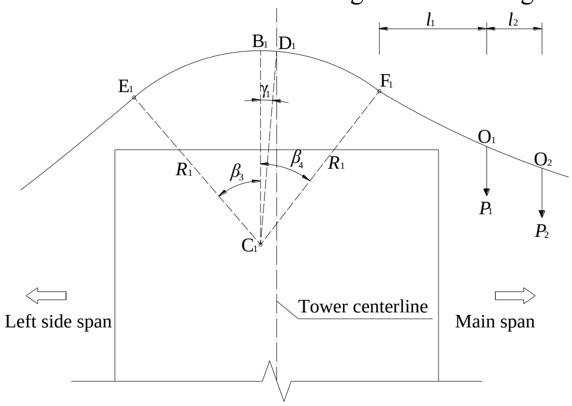

Fig. 3 Left tower saddle

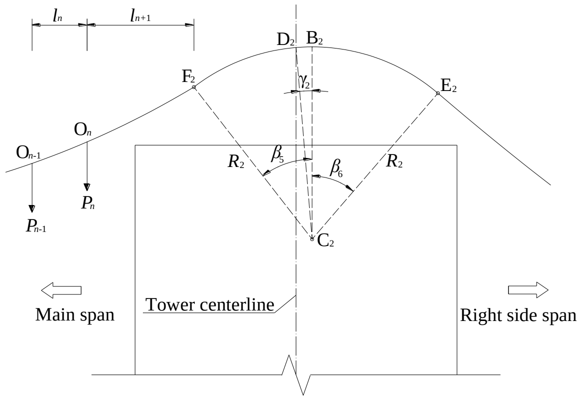

Fig. 4 Right tower saddle

2.1 Main span

2.1.1 Cable configuration

Under the action of concentrated forces from hangers, the cable in the bridge's completed state consists of many catenary segments hanging between adjacent hangers. As shown in Fig. 2, a number of coordinate systems are established, with the origin being at the left tangent point, F_1, and the suspension points, O_1~O_n, along the cable, the positive *x*-axis pointing right, and the positive *y*-axis pointing downward. Then the shape of an arbitrary cable segment is governed by the following catenary equation:

$$y = c\cosh\left(\frac{x}{c} + a_i\right) + b_i \tag{1}$$

where $c=-H/q$, in which H is the horizontal component of the cable tension in the bridge's completed state (kN) and q is the cable weight per unit length (kN/m); a_i and b_i are parameters of the catenary equation.

From the boundary condition $y(0)=c\cosh a_i+b_i=0$, we obtain $b_i=-c\cosh a_i$. Plugging this into the equation above, the catenary equation can then be rewritten as:

$$y = c\left[\cosh\left(\frac{x}{c} + a_i\right) - \cosh a_i\right] \tag{2}$$

Three constraint conditions are introduced: (1) the elevation difference between the left tangent point and the mid-span point is closed; (2) the elevation difference between the two tangent points on the left and right saddles is closed; and (3) the length of the orthogonal projection of the rightmost catenary segment on the horizontal plane, l_{n+1}, meets the relevant design requirement. The corresponding equations are as follows:

$$\sum_{i=1}^{m} \Delta h_i = \Delta h_{F_1O_m} \tag{3-1}$$

$$\sum_{i=1}^{n} \Delta h_i = \Delta h_{F_1F_2} \tag{3-2}$$

$$l_{n+1} = l_{O_nD_2} - l_{F_2D_2} \tag{3-3}$$

where m is the number of the cable segments between the left tangent point, F_1, and the mid-span point, O_m; n is the number of suspension points along the cable; Δh_i denotes the elevation difference between the two endpoints of an arbitrary catenary segment of the cable; $\Delta h_{F_1O_m}$ is the elevation difference between the left tangent point, F_1, and the mid-span point, O_m; $\Delta h_{F_1F_2}$ is the elevation difference between the left tangent point, F_1, and the right tangent point, F_2; $l_{O_nD_2}$ is the designed horizontal distance between the rightmost hanger and the right tower's centerline; $l_{F_2D_2}$ is the horizontal distance between the right tangent point, F_2, and the right tower's centerline.

There are three unknown quantities hidden in the three equations: H – the horizontal cable tension in the bridge's completed state; a_1 – the parameter in the catenary equation for the first catenary segment; and l_{n+1} – the length of the orthogonal projection of the rightmost catenary segment on the horizontal plane. The next step is expressing the other parameters in the equations as functions of the three quantities.

The Δh_i can be expressed as follows:

$$\Delta h_i = y(l_i) - y(0) = c\left[\cosh\left(\frac{l_i}{c} + a_i\right) - \cosh a_i\right] \tag{4}$$

where l_i is the length of the orthogonal projection of an arbitrary catenary segment on the horizontal plane, as shown in Fig. 2.

In the left tower saddle (Fig. 3), for a given elevation of the circle center, C_1, in the geodetic system, denoted as h_{C_1}, the elevation of the left tangent point, F_1, in the geodetic system, h_{F_1}, can be written as

$$h_{F_1} = h_{C_1} + R_1 \cos\beta_4 = h_{C_1} + R_1 \operatorname{sech} a_1 \tag{5}$$

where R_1 is the radius of the left tower saddle's arc-shaped top, as shown in Fig. 3; β_4 is the angle between the vertical segment B_1C_1 and the segment connecting point F_1 and the circle center, C_1. Since $\tan\beta_4 = \frac{dy}{dx}\big|_{x=0} = \sinh a_1$, then $\cos\beta_4 = \operatorname{sech} a_1$, $\sin\beta_4 = \tanh a_1$.

For a given elevation of point O_m in the geodetic system, denoted as h_{O_m}, the elevation difference between points F_1 and O_m, denoted as $\Delta h_{F_1O_m}$, can be expressed by

$$\Delta h_{F_1O_m} = h_{F_1} - h_{O_m} = h_{C_1} + R_1 \operatorname{sech} a_1 - h_{O_m} \tag{6}$$

In the right tower saddle (Fig. 4), for a given elevation of the circle center, C_2, in the geodetic system, denoted as h_{C_2}, the elevation of the right tangent point, F_2, in the geodetic system, h_{F_2}, can be expressed as

$$h_{F_2} = h_{C_2} + R_2 \cos\beta_5 = h_{C_2} + R_2 \operatorname{sech}\left(\frac{l_{n+1}}{c} + a_{n+1}\right) \tag{7}$$

where R_2 is the radius of the right tower saddle's arc-shaped top, as shown in Fig. 4; β_5 is the angle between the vertical segment B_2C_2 and the segment connecting point F_2 and the circle center, C_2. Since $\tan\beta_5 = -\frac{dy}{dx}\big|_{x=l_{n+1}} = -\sinh\left(\frac{l_{n+1}}{c} + a_{n+1}\right)$ then $\cos\beta_5 = \operatorname{sech}\left(\frac{l_{n+1}}{c} + a_{n+1}\right)$, $\sin\beta_5 = -\tanh\left(\frac{l_{n+1}}{c} + a_{n+1}\right)$.

Then the elevation difference between tangent points F_1 and F_2, denoted as $\Delta h_{F_1F_2}$, can be expressed by

$$\begin{aligned}\Delta h_{F_1F_2} &= h_{F_1} - h_{F_2} = h_{C_1} - h_{C_2} + \\ &R_1 \operatorname{sech} a_1 - R_2 \operatorname{sech}\left(\frac{l_{n+1}}{c} + a_{n+1}\right)\end{aligned} \tag{8}$$

The horizontal distance between the right tangent point, F_2, and the right tower's centerline, denoted as $l_{F_2D_2}$, can be expressed by

$$\begin{aligned} l_{F_2D_2} &= l_{F_2B_2} - l_{D_2B_2} = R_2 \sin\beta_5 - R_2 \sin\gamma_2 \\ &= -R_2\left[\tanh\left(\frac{l_{n+1}}{c} + a_{n+1}\right) + \sin\gamma_2\right]\end{aligned} \tag{9}$$

where $l_{F_2B_2}$ represents the horizontal distance between points F_2 and B_2; $l_{D_2B_2}$ is the horizontal distance between points D_2 and B_2; γ_2 is the angle between the vertical segment B_2C_2 and the segment connecting point D_2 and the circle center, C_2, as shown in Fig. 4.

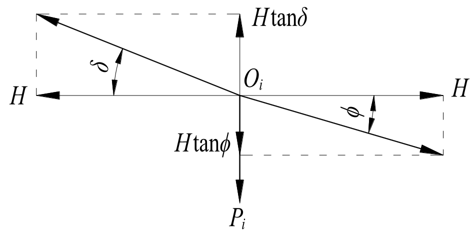

Fig. 5 Equilibrium between forces at a suspension point

At an arbitrary suspension point on the main cable, the axial tensile force can be decomposed into a horizontal component and a vertical one, as shown in Fig. 5. Through the force equilibrium in the vertical direction, we can obtain (Zhang *et al*. 2018).

$$H\tan\delta = H\tan\phi + P_i \tag{10}$$

where P_i is the hanger tensile force; δ and ϕ are the inclination angles for the cable segments at the left and right of the suspension point, O_i, respectively.

Substituting $\tan\delta = \sinh(l_i/c + a_i)$ and $\tan\phi = \sinh a_{i+1}$ into the equation above leads to

$$H\sinh(l_i/c + a_i) = H\sinh a_{i+1} + P_i \tag{11}$$

Then

$$a_{i+1} = \operatorname{asinh}\left[\sinh\left(\frac{l_i}{c} + a_i\right) - \frac{P_i}{H}\right] \tag{12}$$

In the equation above, the expression of l_1 is needed. As shown in Fig. 3, the horizontal distance between the tangent point on the left tower saddle and the first hanger, l_1, can be expressed as

$$\begin{aligned} l_1 &= l_{D_1O_1} - R_1(\sin\beta_4 - \sin\gamma_1) \\ &= l_{D_1O_1} - R_1(\tanh a_1 - \sin\gamma_1)\end{aligned} \tag{13}$$

where $l_{D_1O_1}$ is the horizontal distance between the left tower's centerline and the first hanger; γ_1 is the angle between the vertical segment B_1C_1 and the segment from the left tower saddle's circle center, C_1, to its actual apex, D_1, as shown in Fig.3.

Plugging Eqs. (4), (6), (8), (9), and (12) into Eq. (3) yields three nonlinear governing equations that are coupled to each other. The equations can be rewritten as three functions.

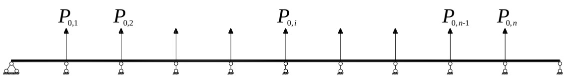

Fig. 6 Rigidly supported continuous beam

$$f_1(H, a_1, l_{n+1}) = 0 \tag{14-1}$$

$$f_2(H, a_1, l_{n+1}) = 0 \tag{14-2}$$

$$f_3(H, a_1, l_{n+1}) = 0 \tag{14-3}$$

In order to solve the resultant system of these equations more conveniently, it can be transformed into an unconstrained optimization problem.

$$\min\left[\sum_{i=1}^{3} f_i^2\left(H, a_1, l_{n+1}\right)\right] \tag{15}$$

This equation can be solved by using the generalized reduced gradient (GRG) algorithm, which is now available in the Microsoft Excel. Solving this equation gives the expressions for the aforementioned unknown quantities: H, a_1 and l_{n+1}. Then the cable shape over the main span, which includes the position of each tangent point, the elevation of each suspending point, and so on, can be determined.

2.1.2 Hanger tensile force and unstrained hanger length

In the calculation above, the hanger tensile force, P_i, is the axial tensile force at a hanger's upper end. It can be resolved into two components: axial tensile force at the hanger's lower end, denoted as $P_{0,i}$, and hanger weight. $P_{0,i}$ can be calculated using the rigidly supported continuous beam method (Kim and Lee 2001, Jung et al. 2013, Thai and Choi 2013, Cao *et al*. 2017), namely by replacing the hangers for the stiffening girder with rigid supports (Fig. 6) so that $P_{0,i}$ equals the reaction force applied by each rigid support. However, hanger weight needs to be iteratively calculated. This is because calculating hanger lengths requires determining the elevations of suspension points along each main cable, which in term requires the exact values of hanger tensile forces. The iterative process involves the following steps:

(1) Assume the axial tensile force at a hanger's upper end, P_i, equals the value of $P_{0,i}$.

(2) Calculate the elevations of suspension points along the cable from P_i using the method described in the previous subsection.

(3) Calculate the unstrained length of each hanger using the equation below:

$$S_{i,\mathrm{h}} = \frac{L_{i,\mathrm{h}}}{1 + \dfrac{P_i - 0.5 w_i L_{i,\mathrm{h}}}{E_\mathrm{h} A_i}} \tag{16}$$

where $L_{i,\mathrm{h}}$ is the strained length of the ith hanger, $L_{i,\mathrm{h}}=h_{i,\mathrm{c}}-h_{i,\mathrm{d}}$; $h_{i,\mathrm{c}}$ is the elevation of the ith suspension point on the main cable in the geodetic system; $h_{i,\mathrm{d}}$ is the elevation of the corresponding anchor point on the deck in the geodetic system; w_i is the hanger weight per unit length (kN/m); E_h is the elastic modulus of the steel wires used to produce the hangers; and A_i is the cross-sectional area of the ith hanger.

(4) Calculate the axial tensile force at the upper end using the equation below:

$$P_i^{'} = P_{0,i} + S_{i,\mathrm{hanger}} w_i \tag{17}$$

(5) Decide whether the iterative process converges according to the criterion shown in the following equation:

$$\max_{1 \le i \le n}\left(\frac{\left|P_i^{'} - P_i\right|}{P_i}\right) \le \varepsilon \tag{18}$$

where ε is the threshold and can be set at 0.035%. The iterative processes to solve for P_i and cable configuration terminate when this formula holds. Otherwise, assign the value of $P_i^{'}$ to P_i and go back to step (2) and repeat.

2.1.3 Unstrained cable length

The unstrained cable length over the main span can be divided into three components: multiple catenary segments, the arc segment D_1F_1 on the left tower saddle, and the arc segment F_2D_2 on the right tower saddle.

The total unstrained length of catenary segments in the main span, denoted as $S_{c,m}$, can be expressed as

$$S_{c,m} = \sum_{i=1}^{n+1} S_{c,i} \tag{19}$$

where $S_{c,i}$ is the unstrained length of the ith catenary segment. It can be formulated as follows:

$$\begin{aligned} S_{c,i} &= c\left[\sinh\left(\frac{l_i}{c} + a_i\right) - \sinh a_i\right] \\ &- \frac{H}{2EA}\left\{l_i + \frac{c}{2}\left[\sinh 2\left(\frac{l_i}{c} + a_i\right) - \sinh 2a_i\right]\right\} \end{aligned} \tag{20}$$

The unstrained length of the arc segment D_1F_1 on the left tower saddle can be written as

$$S_{\mathrm{D_1F_1}} = \frac{R_1(\beta_4 - \gamma_1)}{1 + \dfrac{H \cosh a_1}{EA}} \tag{21}$$

The unstrained length of the arc segment on the right tower saddle, F_2D_2, can be expressed as

$$S_{\mathrm{F_2D_2}} = \frac{R_2(\beta_5 - \gamma_2)}{1 + \dfrac{H \cosh(l_{n+1}/c + a_{n+1})}{EA}} \tag{22}$$

Then the unstrained cable length over the main span can be calculated using the equation below:

$$S_m = S_{c,m} + S_{\mathrm{D_1F_1}} + S_{\mathrm{F_2D_2}} \tag{23}$$

2.2 Side spans

2.2.1 Cable configuration

After the calculation for the main span is completed, the

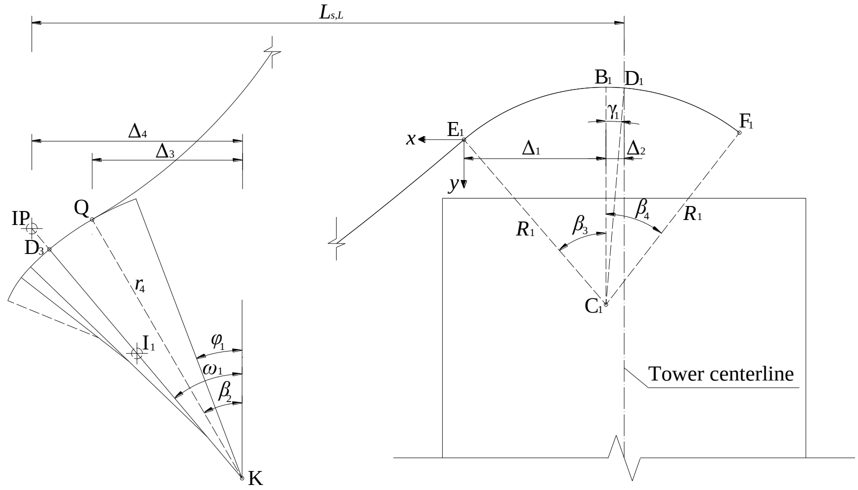

Fig. 7 Cable configuration under final dead load over the left side span

cable configurations over the side spans are calculated. The calculation method is basically the same as that used for the main span, except that the known quantities are slightly different. The sag to span ratio and elevation of the mid-span point are known in the calculation for the main span, but are unknown in the calculation for the side spans. However, the horizontal component of the tension in main cable over a side span can be inferred from the equilibrium conditions for the saddle mounted on each tower top. It is generally assumed that towers are not subject to any horizontal force from main cables. Therefore, the horizontal component of the tension in main cable over a side span is considered equal to that over the main span. This means that H is a known quantity in the cable configuration calculation for the side spans and it takes the value calculated for the main span.

Due to the absence of hanger tensile force, the main cable over a side span can be viewed as a complete catenary when calculating its configuration. Take for example the left side span. As shown in Fig. 7, a coordinate system is established, with the origin being at the left tangent point, E_1, the positive x-axis pointing left, and the positive y-axis pointing downward. Then the catenary equation for main cable over the left side span can be written as

$$y = c\cosh\left(\frac{x}{c}+a_s\right)+b_s \tag{24}$$

where a_s and b_s are parameters of the catenary equation, and the subscript s denotes a side span.

From the boundary condition $y(0)=c\cosh a_s+b_s=0$, we obtain $b_s=-c\cosh a_s$. Substituting this into the equation above, the catenary equation can then be rewritten as

$$y = c\left[\cosh\left(\frac{x}{c}+a_s\right)-\cosh a_s\right] \tag{25}$$

Since both the elevation difference and horizontal distance between the tangent points on the tower and splay saddles are closed, we have

$$\Delta h_s = \Delta h_{E_1Q} \tag{26-1}$$

$$l_s = \Delta l_{E_1Q} \tag{26-2}$$

where Δh_s is the elevation difference between the two endpoints of the catenary segment over the left side span; Δh_{E_1Q} and Δl_{E_1Q} represent the elevation difference and horizontal distance, respectively, between tangent points E_1 and Q; and l_s is the length of the orthogonal projection of the catenary segment over the left side span on the horizontal plane.

There are two unknown quantities hidden in this equation system: a_s and l_s. Next, other parameters in the equations are to be expressed as functions of the two unknown quantities.

The elevation difference between the two endpoints of the catenary segment is given by

$$\Delta h_s = y(l_s)-y(0) = c\left[\cosh\left(\frac{l_s}{c}+a_s\right)-\cosh a_s\right] \tag{27}$$

In the geodetic system, the elevation of tangent point E_1 on the left tower saddle, denoted as h_{E_1}, can be calculated from the elevation of circle center C_1, denoted as h_{C_1}, using the equation below:

$$h_{E_1} = h_{C_1} + R_1\cos\beta_3 = h_{C_1} + R_1\operatorname{sech} a_s \tag{28}$$

where β_3 is the angle between the vertical segment B_1C_1 and the segment connecting tangent point E_1 to the circle center C_1. Since $\tan\beta_3 = \frac{dy}{dx}\Big|_{x=0} = \sinh a_s$, then $\cos\beta_3 = \operatorname{sech} a_s$ and $\sin\beta_3 = \tanh a_s$.

In the geodetic system, the elevation of tangent point Q on the splay saddle, h_Q, can be calculated from the elevation of the splay saddle's circle center, h_K, using

$$h_Q = h_K + r_4\cos\beta_2 = h_K + r_4\operatorname{sech}\left(\frac{l_s}{c}+a_s\right) \tag{29}$$

where r_4 is the radius of the arc at tangent point Q; and β_2 is the angle between the vertical line and the segment connecting points Q and K. Since

$\tan\beta_2 = \frac{\mathrm{d}y}{\mathrm{d}x}\Big|_{x=l_s} = \sinh\left(\frac{l_s}{c}+a_s\right)$, then $\cos\beta_2 = \mathrm{sech}\left(\frac{l_s}{c}+a_s\right)$ $\sin\beta_2 = \tanh\left(\frac{l_s}{c}+a_s\right)$.

So the elevation difference between tangent points Q and E_1 can be expressed as

$$\Delta h_{E_1Q} = h_{E_1} - h_Q = h_{C_1} - h_K + R_1\mathrm{sech}a_s - r_4\,\mathrm{sech}\left(\frac{l_s}{c}+a_s\right) \tag{30}$$

Their horizontal distance, denoted as Δl_{E_1Q}, can be expressed as

$$\Delta l_{E_1Q} = L_{s,L} - \Delta_1 - \Delta_2 + \Delta_3 - \Delta_4 \tag{31}$$

where $L_{s,L}$ is the length of a side span, which is equivalent to the horizontal distance between the IP points of the left tower saddle and left splay saddle; Δ_1 is the horizontal distance between the tangent point E_1, and point B_1, $\Delta_1=R_1\sin\beta_3$; Δ_2 is the horizontal distance between points B_1 and D_1, $\Delta_2=R_1\sin\gamma_1$; Δ_3 is the horizontal distance between the tangent point on the left splay saddle, Q, and circle center K, $\Delta_3=r_4\sin\beta_2$; Δ_4 is the horizontal distance between the IP point and circle center of the left splay saddle, $\Delta_4=l_K\sin\omega_1$, in which l_K is the distance between the splay saddle's IP point and circle center K, and ω_1 is the angle between the vertical line and the segment connecting the IP point and K.

Plugging Eqs. (27), (30) and (31) into Eq. (26) gives two coupled equations, which have the following functional forms:

$$f_1(a_s, l_s) = 0 \tag{32-1}$$

$$f_2(a_s, l_s) = 0 \tag{32-2}$$

An objective function is constructed by using a_s and l_s as the variables

$$\min\left[\sum_{i=1}^{2} f_i^2(a_s, l_s)\right] \tag{33}$$

By solving the equations, we can get the values of a_s and l_s, and then the main cable's configuration and internal forces under final dead load. The method described above can also be utilized to calculate the configuration and internal forces for the main cable over the right side span.

2.2.2 Unstrained cable length

The unstrained cable length over the left side span consists of three parts: the catenary segment QE_1, the arc segment D_3Q on the left splay saddle, and the arc segment E_1D_1 on the left tower saddle.

The unstrained length of the catenary segment QE_1 can be expressed as

$$S_{c,s} = c\left[\sinh\left(\frac{l_s}{c}+a_s\right) - \sinh a_s\right] - \frac{H}{2EA}\left\{l_s + \frac{c}{2}\left[\sinh 2\left(\frac{l_s}{c}+a_s\right) - \sinh 2a_s\right]\right\} \tag{34}$$

The unstrained length of the arc segment D_3Q can be expressed as

$$S_{D_3Q} = \frac{r_3(\omega_1-\varphi_1-\theta_4) + r_4[(\varphi_1+\theta_4)-\beta_2]}{1+\dfrac{H\cosh(l_s/c+a_s)}{EA}} \tag{35}$$

where θ_4 is the central angle of the rightmost circular arc on the left splay saddle (Fig. 8).

The unstrained length of the arc segment E_1D_1 has the following form

$$S_{E_1D_1} = \frac{R_1(\beta_3+\gamma_1)}{1+\dfrac{H\cosh a_s}{EA}} \tag{36}$$

The total unstrained cable length over the left side span can be calculated using

$$S_{s,L} = S_{c,s} + S_{D_3Q} + S_{E_1D_1} \tag{37}$$

The unstrained cable length over the right side span can be obtained through the same method.

2.3 Anchor spans

2.3.1 Cable configuration

The cable configuration under final dead load over the anchor spans is calculated usually after the calculation for the side spans is completed. The calculation method is roughly the same as that used for the side spans except for some differences. Over the anchor spans, the splay saddles are tilted, making it necessary to consider the balance of torques about the splay saddle's center of rotation. Therefore, the horizontal component of cable tension over an anchor span is unknown.

This subsection only presents the calculation for the left anchor span. As shown in Fig. 8, the splay saddle consists of 4 circular arcs. The radii of these arcs, from the anchor span to the side span, are r_1, r_2, r_3, and r_4, respectively, and the corresponding central angles are θ_1, θ_2, θ_3, and θ_4, respectively.

A coordinate system is established, with the origin being at the tangent point on the left splay saddle, J, the positive x-axis pointing left, and the positive y-axis pointing downward. Then the catenary equation for main cable over the left anchor span can be written as

$$y = c_a\cosh\left(\frac{x}{c_a}+a_a\right) + b_a \tag{38}$$

where $c_a=-H_a/q$, in which H_a represents the horizontal component of cable tension over the anchor span and is unknown; a_a and b_a are parameters of the catenary equation for main cable over the anchor span; and the subscript a denotes an anchor span.

From the boundary condition $y(0)=c_a\cosh a_a+b_a=0$, we obtain $b_a=-c_a\cosh a_a$. Putting this into the equation above, the catenary equation can then be rewritten as follows:

$$y = c_a\left[\cosh\left(\frac{x}{c_a}+a_a\right) - \cosh a_a\right] \tag{39}$$

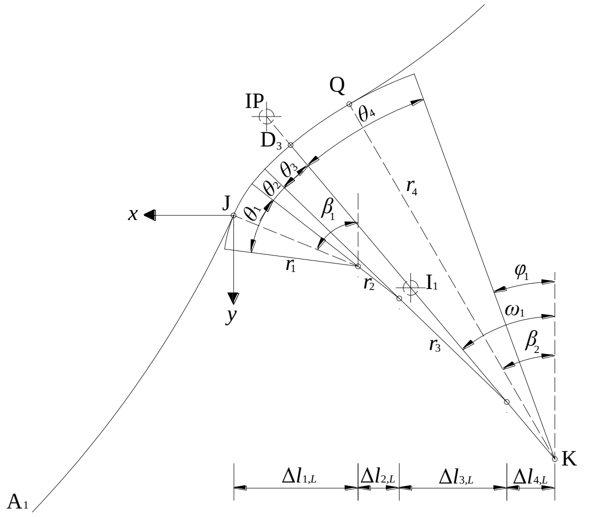

Fig. 8 Cable configuration under final dead load over the left anchor span

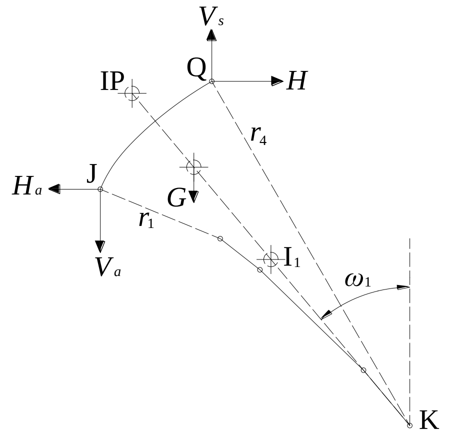

Fig. 9 Balance of torques acting on the left splay saddle under final dead load

Since both the elevation difference and horizontal distance between the left anchor point A_1 and tangent point J are closed, and the sum of torques about the left splay saddle's center of rotation, I_1, is zero, the following equation system can be constructed:

$$\Delta h_a = \Delta h_{JA_1} \tag{40-1}$$

$$l_a = \Delta l_{JA_1} \tag{40-2}$$

$$\sum M_{I_1} = 0 \tag{40-3}$$

where Δh_a is the elevation difference between the two endpoints of the catenary segment over the left anchor span; Δh_{JA_1} and Δl_{JA_1} are the elevation difference and horizontal distance, respectively, between tangent point J and the left anchor point, A_1; l_a is the length of the orthogonal projection of the catenary segment over the left anchor span on the horizontal plane; and $\sum M_{I_1}$ represents the sum of torques about point I_1 due to cable tension and the splay saddle's weight.

There are three unknown quantities hidden in the equation system: a_a, l_a and H_a. Next, other parameters in the equations are to be expressed as functions of the three quantities.

The elevation difference between the two endpoints of the catenary segment, denoted as Δh_a, can be described by

$$\begin{aligned}\Delta h_a &= y(l_a) - y(0) \\ &= c_a\left[\cosh\left(\frac{l_a}{c_a} + a_a\right) - \cosh a_a\right]\end{aligned} \tag{41}$$

In the geodetic system, the elevation of tangent point J on the left splay saddle, denoted as h_J, can be inferred from the elevation of the splay saddle's circle center K, denoted as h_K, using

$$h_J = h_K + \Delta h_{1,L} + \Delta h_{2,L} + \Delta h_{3,L} + \Delta h_{4,L} \tag{42}$$

where $\Delta h_{1,L}$ represents the elevation difference between tangent point J and the center of the first circular arc of the splay saddle, $\Delta h_{1,L} = r_1\cos\beta_1$, in which β_1 is the angle between the vertical line and the segment connecting tangent point J and the center of the first circular arc. Since $\tan\beta_1 = \frac{\mathrm{d}y}{\mathrm{d}x}\big|_{x=0} = \sinh a_a$, then $\cos\beta_1 = \mathrm{sech}\, a_a$, $\sin\beta_1 = \tanh a_a$; $\Delta h_{2,L}$ is the elevation difference between the centers of the first and second circular arcs of the splay saddle, $\Delta h_{2,L} = (r_2 - r_1)\cos(\theta_2 + \theta_3 + \theta_4 + \varphi_1)$, in which φ_1 is the angle between the vertical line and the segment connecting the outmost endpoint of the fourth circular arc and its circle center K; $\Delta h_{3,L}$ is the elevation difference between the centers of the second and third arcs of the splay saddle, $\Delta h_{3,L} = (r_3 - r_2)\cos(\theta_3 + \theta_4 + \varphi_1)$; and $\Delta h_{4,L}$ is the elevation difference between the centers of the third and fourth arcs of the splay saddle, $\Delta h_{4,L} = (r_4 - r_3)\cos(\theta_4 + \varphi_1)$.

So the elevation difference between points J and A_1, denoted as Δh_{JA_1}, can be written as

$$\begin{aligned}\Delta h_{JA_1} &= h_J - h_{A_1} \\ &= (h_K + \Delta h_{1,L} + \Delta h_{2,L} + \Delta h_{3,L} + \Delta h_{4,L}) - h_{A_1}\end{aligned} \tag{43}$$

where h_{A_1} is the elevation of point A_1 in the geodetic system.

The horizontal distance between points J and A_1, denoted Δl_{JA_1}, can be written as

$$\Delta l_{JA_1} = L_{a,L} - (\Delta l_{1,L} + \Delta l_{2,L} + \Delta l_{3,L} + \Delta l_{4,L} - \Delta_4) \tag{44}$$

where $L_{a,L}$ is the length of the left anchor span, i.e. the horizontal distance between the left splay saddle's IP point and left anchor point A_1; $\Delta l_{1,L}$ is the horizontal distance between the tangent point on the left splay saddle, J, and the center of the first circular arc of the splay saddle, $\Delta l_{1,L} = r_1\sin\beta_1$; $\Delta l_{2,L}$ is the horizontal distance between the centers of the first and second circular arcs of the splay saddle, $\Delta l_{2,L} = (r_2 - r_1)\sin(\theta_2 + \theta_3 + \theta_4 + \varphi_1)$; $\Delta l_{3,L}$ is the horizontal distance between the centers of the second and third circular arcs of the splay saddle,

$\Delta l_{3,L} = (r_3 - r_2)\sin(\theta_3 + \theta_4 + \varphi_1)$; and $\Delta l_{4,L}$ is the horizontal distance between the third circular arc's center and the fourth circular arc's center, K, $\Delta l_{4,L} = (r_4 - r_3)\sin(\theta_4 + \varphi_1)$.

As demonstrated in Fig. 9, the sum of torques about point I_1 due to the tension in the cable segments at the two sides of the left splay saddle and the splay saddle's self-weight can be written as

$$\sum M_{I_1} = H \cdot e_{s1} + V_s \cdot e_{s2} - (H_a \cdot e_{a1} + V_a \cdot e_{a2} + G \cdot e_g) \tag{45}$$

where e_{s1} denotes the eccentricity of the horizontal component, H, of cable tension over the side span, and is defined by $e_{s1} = r_4 \cos\beta_2 - (l_K - l_I)\cos\omega_1$, in which l_I is the distance between the splay saddle's IP point and center of rotation, I_1; V_s denotes the vertical component of cable tension over the side span at tangent point Q, $V_s = H\tan\beta_2$; e_{s2} is the eccentricity of V_s, $e_{s2} = r_4 \sin\beta_2 - (l_K - l_I)\sin\omega_1$; e_{a1} is the eccentricity of the horizontal component, H_a, of cable tension over the anchor span, $e_{a1} = \Delta h_{1,L} + \Delta h_{2,L} + \Delta h_{3,L} + \Delta h_{4,L} - (l_K - l_I)\cos\omega_1$; V_a is the vertical component of cable tension over the left anchor span at tangent point J, $V_a = H_a \tan\beta_1$; e_{a2} is the eccentricity of V_a, $e_{a2} = \Delta l_{1,L} + \Delta l_{2,L} + \Delta l_{3,L} + \Delta l_{4,L} - (l_K - l_I)\sin\omega_1$; G is the force of gravity acting on the splay saddle; and e_g is the eccentricity of G, $e_g = l_g \sin\omega_1$, in which l_g denotes the distance between the splay saddle's center of gravity and center of rotation I_1.

Substituting Eqs. (41), (43), (44), and (45) into Eq. (40) yields three coupled equations, which take the following functional forms:

$$f_1(H_a, a_a, l_a) = 0 \tag{46-1}$$

$$f_2(H_a, a_a, l_a) = 0 \tag{46-2}$$

$$f_3(H_a, a_a, l_a) = 0 \tag{46-3}$$

An objective function is constructed by using a_a, l_a, and H_a as the variables:

$$\min\left[\sum_{i=1}^{3} f_i^2\left(H_a, a_a, l_a\right)\right] \tag{47}$$

By solving these equations, we can get the values of a_a, l_a, H_a, and then the cable configuration under final dead load over the left anchor span. This method described above can also be used to calculate the cable configuration over the right anchor span.

2.3.2 Unstrained cable length

The unstrained cable length over the left anchor span is split into two parts: the catenary segment A_1J and the arc segment JD_3 on the left splay saddle.

The unstrained length of the catenary segment A_1J can be expressed as

$$S_{c,a} = c_a\left[\sinh\left(\frac{l_a}{c_a} + a_a\right) - \sinh a_a\right] - \frac{H_a}{2EA}\left\{l_a + \frac{c_a}{2}\left[\sinh 2\left(\frac{l_a}{c_a} + a_a\right) - \sinh 2a_a\right]\right\} \tag{48}$$

The unstrained length of the arc segment JD_3 is given by

$$S_{JD_3} = \frac{r_1[\beta_1 - (\theta_2 + \theta_3 + \theta_4 + \varphi_1)] + r_2\theta_2 + r_3[\theta_3 - (\omega_1 - \varphi_1 - \theta_4)]}{1 + \dfrac{H_a \cosh a_a}{EA}} \tag{49}$$

Then the total unstrained cable length over the left anchor span can be calculated using the equation below:

$$S_{a,L} = S_{c,a} + S_{JD_3} \tag{50}$$

This calculation method is also applicable to the right anchor span.

2.4 Tower top pre-uplift

Under final dead load, a tower tends to undergo axial compression due to the downward compressive forces from the main cables. The amount of compression can be calculated as follows:

$$\Delta h_t = \frac{h_t}{1 - \dfrac{H(\tan\beta_3 + \tan\beta_4)}{E_t A_t}} - h_t \tag{51}$$

where h_t is the tower's target height of the completed bridge; E_t is the tower's elastic modulus; and A_t is the cross-sectional area of a tower column.

During construction, the tower top pre-uplift, Δh_t, is normally set as the calculated amount of compression.

3. Calculation of cable configuration in the unloaded state

Unlike in calculation of cable configuration under final dead load, we need to consider different spans simultaneously when calculating cable configuration in the unloaded state and pre-offsets for cable saddles. This is because cable configuration over any span and relevant parameters are affected by adjacent span(s). Coupled equations are established using the conditions for geometrical compatibility at the nodes between adjacent spans, mechanical equilibrium conditions, and geometric boundary conditions. Then parameters in the system of equations are expressed as functions of unknown quantities. The last step is to solve for the unknown quantities by nonlinear programming.

3.1 Unknown quantities

The horizontal component of cable tension over each span, parameters of corresponding catenary equation, and positions of saddles and tangent points in the unloaded state differ from those under final dead load. For this reason, unknown quantities in the unloaded state fall into the following five groups:

(1) The horizontal components of cable tension over different spans: $H'_{a,L}, H'_{s,L}, H'_m, H'_{s,R}, H'_{a,R}$, where the subscripts a, s and m represent an anchor span, side span and main span, respectively; the subscript L and R represent the left and right spans, respectively; and the superscript ' indicates a parameter in the unloaded state. As $H'_{s,L}=H'_m=H'_{s,R}$, the three can be regarded as one unknown quantity.

(2) Parameters of the catenary equations for different spans: $a'_{a,L}, a'_{s,L}, a'_m, a'_{s,R}, a'_{a,R}$.

(3) Lengths of the orthogonal projection of catenary segments over different spans on the horizontal plan: $l'_{a,L}, l'_{s,L}, l'_m, l'_{s,R}, l'_{a,R}$.

(4) Pre-offset distances for the tower saddles: $\Delta_{m,L}$, $\Delta_{m,R}$.

(5) Pre-offset angles for the splay saddles: $\alpha_{s,L}$, $\alpha_{s,R}$.

There are a total of 17 unknown quantities, and thus we need 17 equations to calculate them.

3.2 Coupled equations

Coupled equations can be derived from the conditions necessary for closed elevation difference and horizontal distance, conserved unstrained cable length, and balance of torques acting on each splay saddle.

(1) For the elevation difference between the two endpoints of cable segment over each span to be closed, the following conditions must be satisfied:

$$\begin{aligned}\Delta h_m &= \Delta h'_m\\ \Delta h_{s1} &= \Delta h'_{s,L}\\ \Delta h_{s2} &= \Delta h'_{s,R}\\ \Delta h_{a1} &= \Delta h'_{a,L}\\ \Delta h_{a2} &= \Delta h'_{a,R}\end{aligned} \tag{52-1}$$

where Δh_m is the elevation difference between the circle centers of the left and right tower saddles, C_1 and C_2 (Figs. 3 and 4), under final dead load ; $\Delta h'_m$ is Δh_m expressed in terms of unknown parameters in the unloaded state; Δh_{s1} is the elevation difference between points I_1 and C_1 (Fig. 7) under final dead load, which is a known parameter; $\Delta h'_{s,L}$ is Δh_{s1} expressed in terms of unknown parameters in the unloaded state; Δh_{a1} is the elevation difference between points A_1 and I_1 (Fig. 8) under final dead load, a known quantity; $\Delta h'_{a,L}$ is the elevation difference between points A_1 and I_1 (Fig. 14) in the unloaded state and can be expressed as a function of unknown parameters in the unloaded state; and the subscripts "1" and "2" indicate a left span and a right span, respectively.

(2) For the horizontal distance between the ends of each span to be closed, the following conditions must be satisfied:

$$\begin{aligned}L_m &= L'_m\\ L_{s1} &= L'_{s,L}\\ L_{s2} &= L'_{s,R}\\ L_{a1} &= L'_{a,L}\\ L_{a2} &= L'_{a,R}\end{aligned} \tag{52-2}$$

where L_m is the horizontal distance between the centerlines of the left and right towers under final dead load (Fig. 1), a known quantity; L'_m is the horizontal distance between the two towers' centerlines in the unloaded state and can be expressed as a function of the aforementioned unknown parameters in the unloaded state; L_{s1} and $L'_{s,L}$ are the horizontal distances between point I_1 (Figs. 7 and 12) and the left tower's centerline under final dead load and in the unloaded state, respectively; and L_{a1} and $L'_{a,L}$ denote the horizontal distances between points A_1 and I_1 (Figs. 8 and 14) under final dead load and in the unloaded state, respectively.

(3) For the unstrained cable length over each span to be conserved, the following conditions must be satisfied:

$$\begin{aligned}S_m &= S'_m\\ S_{s,L} &= S'_{s,L}\\ S_{s,R} &= S'_{s,R}\\ S_{a,L} &= S'_{a,L}\\ S_{a,R} &= S'_{a,R}\end{aligned} \tag{52-3}$$

where S and S' represent the unstrained cable lengths over a span under final dead load and in the unloaded state, respectively.

(4) The conditions necessary for the sum of torques about each splay saddle's center of rotation to be zero are as follows:

$$\begin{aligned}\sum M'_{I_1} &= 0\\ \sum M'_{I_2} &= 0\end{aligned} \tag{52-4}$$

where I_2 is the right splay saddle's center of rotation.

Next, the parameters in the equations will be expressed in terms of the aforementioned 17 unknown quantities.

3.2.1 Main span

Expressions are built from relevant unknown quantities to describe the following three parameters: $\Delta h'_m$, the elevation difference between the circle centers of the left and right tower saddles, C_1 and C_2, for the completed bridge; L'_m, the distance between the left and right towers' centerlines; and S'_m, the unstrained cable length over the main span.

(1) $\Delta h'_m$ can be expressed as:

$$\begin{aligned}\Delta h'_m &= \Delta h'_{C_1C_2} = h'_{C_1} - h'_{C_2}\\ &= -\Delta h_{t,L} - R_1\cos\beta'_4 + \Delta h'_{c,m} + R_2\cos\beta'_5 + \Delta h_{t,R}\end{aligned} \tag{53}$$

where $\Delta h_{t,L}$ and $\Delta h_{t,R}$ are the pre-uplifts for the left and right tower tops, respectively; and $\Delta h'_{c,m}$ denotes the elevation difference between the catenary segment's two endpoints, F'_1 and F'_2, in the unloaded state (Figs. 10 and 11), which is given by

$$\Delta h'_{c,m} = y(l'_m) - y(0) = c'_m\left[\cosh\left(\frac{l'_m}{c'_m} + a'_m\right) - \cosh a'_m\right] \tag{54}$$

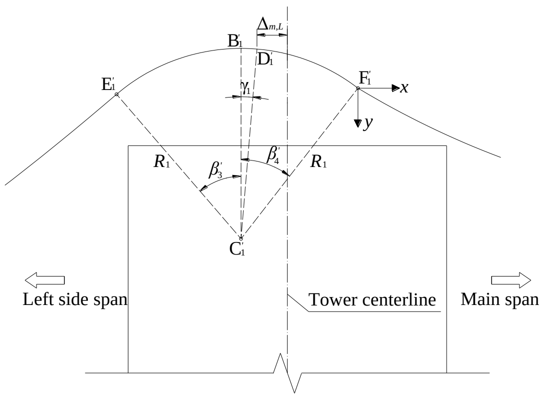

Fig. 10 Left tower saddle in the unloaded state

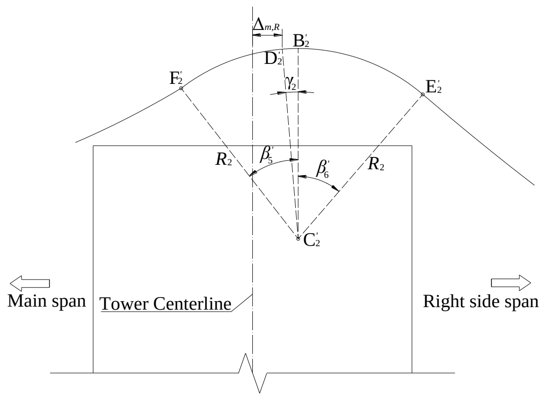

Fig. 11 Right tower saddle in the unloaded state

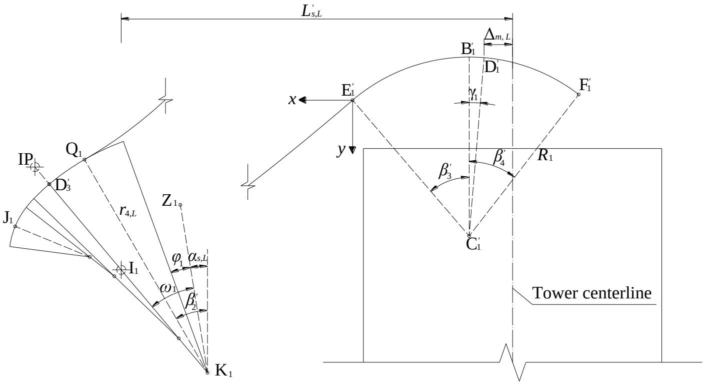

Fig. 12 Cable configuration in the unloaded state over the left side span

where l'_m is the length of the orthogonal projection of the catenary segment $F'_1F'_2$ on the horizontal plane in the unloaded state; $c'_m = -H'_m / q'$, in which H'_m denotes the horizontal component of cable tension in the unloaded state (kN) and q' denotes the free cable's weight per unit length (kN/m).

(2) L'_m can be described by

$$\begin{aligned} L'_m = & -\Delta_{m,L} - R_1 \sin\gamma_1 + R_1 \sin\beta'_4 + l'_m \\ & + R_2 \sin\beta'_5 - \Delta_{m,R} - R_2 \sin\gamma_2 \end{aligned} \tag{55}$$

where $\Delta_{m,L}$ and $\Delta_{m,R}$ are the pre-offsets for the left and right tower saddles, respectively (Figs. 10 and 11).

(3) S'_m can be described by

$$S'_m = S'_{c,m} + S'_{D'_1F'_1} + S'_{F'_2D'_2} \tag{56}$$

where $S'_{c,m}$, $S'_{D'_1F'_1}$, and $S'_{F'_2D'_2}$ are the unstrained lengths of the catenary segment $F'_1F'_2$, the arc segment $D'_1F'_1$ (Fig. 10), and the arc segment $F'_2D'_2$ (Fig. 11), respectively. They are given by the following equations

$$\begin{aligned} S'_{c,m} = & c'_m\left[\sinh\left(\frac{l'_m}{c'_m} + a'_m\right) - \sinh a'_m\right] \\ & - \frac{H'_m}{2EA}\left\{l'_m + \frac{c'_m}{2}\left[\sinh 2\left(\frac{l'_m}{c'_m} + a'_m\right) - \sinh 2a'_m\right]\right\} \end{aligned} \tag{57-1}$$

$$S'_{D'_1F'_1} = \frac{R_1(\beta'_4 - \gamma_1)}{1 + \dfrac{H'_m \cosh a'_m}{EA}} \tag{57-2}$$

$$S'_{F'_2D'_2} = \frac{R_2(\beta'_5 - \gamma_2)}{1 + \dfrac{H'_m \cosh a'_m}{EA}} \tag{57-3}$$

3.2.2 Left side span

In this subsection, three expressions are created from relevant unknown quantities to describe the following three parameters: $\Delta h'_{s,L}$, the elevation difference between points I_1 and C_1; $L'_{s,L}$, the horizontal distance between point I_1 and the left tower's centerline (Fig. 12); and $S'_{s,L}$, the unstrained cable length over the left side span.

(1) $\Delta h'_{s,L}$ can be expressed by

$$\begin{aligned} \Delta h'_{s,L} = \Delta h'_{C_1I_1} = h_{C_1} - h'_{I_1} = & -(l_{K,L} - l_{I,L})\cos(\omega_1 + \alpha_{s,L}) \\ & + r_{4,L}\cos\beta'_2 + \Delta h'_{c,s,L} - \Delta h_{t,L} - R_1\cos\beta'_3 \end{aligned} \tag{58}$$

where $l_{K,L}$ and $l_{I,L}$ are the distances from the left splay saddle's IP point to its circle center, K_1, and center of rotation, I_1, respectively; $\alpha_{s,L}$ is the preset offset angle for the left splay saddle, i.e. the angle between the vertical line and segment Z_1K_1, which was a vertical segment before the left splay saddle rotates; and $\Delta h'_{c,s,L}$ is the elevation difference between the two endpoints of the catenary

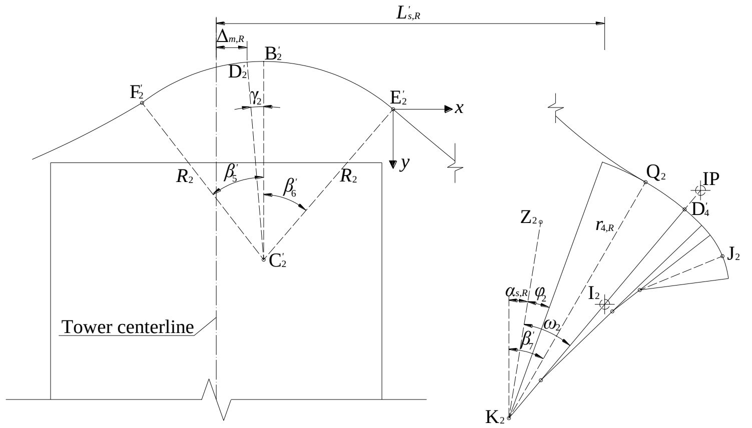

Fig. 13 Cable configuration in the unloaded state over the right side span

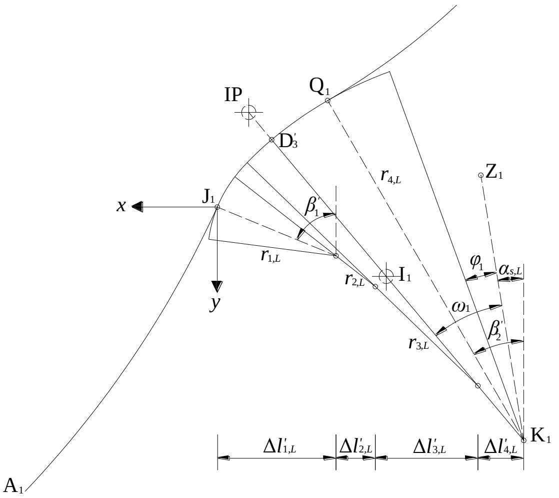

Fig. 14 Cable configuration in the unloaded state over the left anchor span

segment over the left side span, defined by

$$\Delta h'_{c,s,L} = y(l'_{s,L}) - y(0) = c'_{s,L}\left[\cosh\left(\frac{l'_{s,L}}{c'_{s,L}} + a'_{s,L}\right) - \cosh a'_{s,L}\right].$$

(2) $L'_{s,L}$ can be formulated as

$$\begin{aligned} L'_{s,L} &= (l_{\mathrm{K},L} - l_{\mathrm{I},L})\sin(\omega_1 + \alpha_{s,L}) - r_{4,L}\sin\beta'_2 \\ &+ l'_{s,L} + R_1(\sin\beta'_3 + \sin\gamma_1) + \Delta_{m,L} \end{aligned} \tag{59}$$

where $l'_{s,L}$ is the length of the orthogonal projection of the catenary segment over the left side span on the horizontal plane.

(3) $S'_{s,L}$ can be expressed as

$$S'_{s,L} = S'_{c,s,L} + S'_{\mathrm{D}'_3\mathrm{Q}_1} + S'_{\mathrm{E}'_1\mathrm{D}'_1} \tag{60}$$

where $S'_{c,s,L}$, $S'_{\mathrm{D}'_3\mathrm{Q}_1}$, and $S'_{\mathrm{E}'_1\mathrm{D}'_1}$ are the unstrained cable lengths of $\mathrm{Q}_1\mathrm{E}'_1$ (the catenary segment over the left side span, as shown in Fig. 12), $\mathrm{D}'_3\mathrm{Q}_1$ (the arc segment on the splay saddle), and $\mathrm{E}'_1\mathrm{D}'_1$ (the arc segment on the tower saddle), respectively. They can be expressed in the following forms:

$$\begin{aligned} S'_{c,s,L} &= c'_{s,L}\left[\sinh\left(\frac{l'_{s,L}}{c'_{s,L}} + a'_{s,L}\right) - \sinh a'_{s,L}\right] \\ &- \frac{H'_{s,L}}{2EA}\left\{l'_{s,L} + \frac{c'_{s,L}}{2}\left[\sinh 2\left(\frac{l'_{s,L}}{c'_{s,L}} + a'_{s,L}\right) - \sinh 2a'_{s,L}\right]\right\} \end{aligned} \tag{61-1}$$

$$S'_{\mathrm{D}'_3\mathrm{Q}_1} = \frac{r_{3,L}(\omega_1 - \varphi_1 - \theta_{4,L}) + r_{4,L}[(\varphi_1 + \theta_{4,L} + \alpha_{s,L}) - \beta'_2]}{1 + \dfrac{H'_{s,L}\cosh(l'_{s,L}/c'_{s,L} + a'_{s,L})}{EA}} \tag{61-2}$$

$$S'_{\mathrm{E}'_1\mathrm{D}'_1} = \frac{R_1(\beta'_3 + \gamma_1)}{1 + \dfrac{H'_{s,L}\cosh a'_{s,L}}{EA}} \tag{61-3}$$

3.2.3 Right side span

Three expressions are created from relevant unknown quantities to describe the following three parameters: $\Delta h'_{s,R}$, the elevation difference between points C_2 and I_2; $L'_{l,R}$, the horizontal distance between point I_2 and the right tower's centerline (Fig. 13); and $S'_{s,R}$, the unstrained cable length over the right side span.

(1) $\Delta h'_{s,R}$ can be expressed as

$$\begin{aligned}\Delta h'_{s,R} &= \Delta h'_{C_2 I_2} = h_{C_2} - h'_{I_2} = -(l_{K,R} - l_{I,R})\cos(\omega_2 + \alpha_{s,R}) \\ &+ r_{4,R}\cos\beta'_7 + \Delta h'_{c,s,R} - R_2\cos\beta'_6 - \Delta h_{t,R}\end{aligned} \tag{62}$$

where $\alpha_{s,R}$ is the preset offset angle for the right splay saddle, i.e. the angle between the vertical line and segment Z_2K_2, which was a vertical segment before the right splay saddle rotates; and $\Delta h'_{c,s,R}$ is the elevation difference between the two endpoints of the catenary segment over the right side span, $\Delta h'_{c,s,R} = c'_{s,R}\left[\cosh\left(\frac{l'_{s,R}}{c'_{s,R}} + a'_{s,R}\right) - \cosh a'_{s,R}\right]$.

(2) $L'_{l,R}$ can be formulated as

$$\begin{aligned}L'_{l,R} &= (l_{K,R} - l_{I,R})\sin(\omega_2 + \alpha_{s,R}) - r_{4,R}\sin\beta'_7 \\ &+ l'_{l,R} + R_2(\sin\beta'_6 + \sin\gamma_2) + \Delta_{m,R}\end{aligned} \tag{63}$$

(3) $S'_{sl,R}$ can be expressed as

$$S'_{sl,R} = S'_{c,s,R} + S'_{D'_2E'_2} + S'_{Q_2D'_4} \tag{64}$$

where $S'_{c,s,R}$, $S'_{D'_2E'_2}$, and $S'_{Q_2D'_4}$ are the unstrained lengths of catenary segment E'_2Q_2, the arc segment on the tower saddle, $D'_2E'_2$, and the arc segment on the splay saddle, $Q_2D'_4$, respectively. They are given by

$$\begin{aligned}S'_{c,s,R} &= c'_{s,R}\left[\sinh\left(\frac{l'_{s,R}}{c'_{s,R}} + a'_{s,R}\right) - \sinh a'_{s,R}\right] \\ &- \frac{H'_{s,R}}{2EA}\left\{l'_{s,R} + \frac{c'_{s,R}}{2}\left[\sinh 2\left(\frac{l'_{s,R}}{c'_{s,R}} + a'_{s,R}\right) - \sinh 2a'_{s,R}\right]\right\}\end{aligned} \tag{65-1}$$

$$S'_{D'_2E'_2} = \frac{R_2(\beta'_6 + \gamma_2)}{1 + \dfrac{H'_{s,R}\cosh a'_{s,R}}{EA}} \tag{65-2}$$

$$S'_{Q_2D'_4} = \frac{r_{3,R}(\omega_2 - \varphi_2 - \theta_{4,R}) + r_{4,R}[(\varphi_2 + \theta_{4,R} + \alpha_{s,R}) - \beta'_7]}{1 + \dfrac{H'_{s,R}\cosh(l'_{s,R}/c'_{s,R} + a'_{s,R})}{EA}} \tag{65-3}$$

3.2.4 Left anchor span

Here we provide four expressions created from relevant unknown quantities to describe the following parameters: $\Delta h'_{a,L}$, the elevation difference between anchor point A_1 and the splay saddle's center of rotation, I_1; $L'_{a,L}$, the horizontal distance between points A_1 and I_1; $S'_{a,L}$, the unstrained cable length over the anchor span; and $\sum M'_{I_1}$, the sum of torques about the splay saddle's center of rotation.

(1) $\Delta h'_{a,L}$ can be described by the equation below:

$$\begin{aligned}\Delta h'_{a,L} &= \Delta h'_{I_1A_1} = h'_{I_1} - h_{A_1} = \Delta h'_{c,a,L} - \\ &(\Delta h'_{1,L} + \Delta h'_{2,L} + \Delta h'_{3,L} + \Delta h'_{4,L}) + (l_{K,L} - l_{I,L})\cos(\omega_1 + \alpha_{s,L})\end{aligned} \tag{66}$$

where $\Delta h'_{c,a,L}$ represents the elevation difference between the two endpoints of the catenary segment over the left anchor span in the unloaded state, $\Delta h'_{c,a,L} = c'_{a,L}\left[\cosh\left(\frac{l'_{a,L}}{c'_{a,L}} + a'_{a,L}\right) - \cosh a'_{a,L}\right]$, in which $l'_{a,L}$ is the length of the orthogonal projection of this catenary segment on the horizontal plane; $\Delta h'_{1,L}$ is the elevation difference between tangent point J_1 and the center of the splay saddle's first circular arc (Fig. 14); $\Delta h'_{2,L}$ is the elevation difference between the centers of the first and second circular arcs of the splay saddle; $\Delta h'_{3,L}$ is the elevation difference between the centers of the second and third circular arcs of the splay saddle; and $\Delta h'_{4,L}$ is the elevation difference between the center of the third circular arc and the fourth arc's center, K_1. They are given by

$$\Delta h'_{1,L} = r_{1,L}\cos\beta'_1 \tag{67-1}$$

$$\Delta h'_{2,L} = (r_{2,L} - r_{1,L})\cos(\theta_{2,L} + \theta_{3,L} + \theta_{4,L} + \varphi_1 + \alpha_{s,L}) \tag{67-2}$$

$$\Delta h'_{3,L} = (r_{3,L} - r_{2,L})\cos(\theta_{3,L} + \theta_{4,L} + \varphi_1 + \alpha_{s,L}) \tag{67-3}$$

$$\Delta h'_{4,L} = (r_{4,L} - r_{3,L})\cos(\theta_{4,L} + \varphi_1 + \alpha_{s,L}) \tag{67-4}$$

(2) $L'_{a,L}$ can be written as

$$\begin{aligned}L'_{a,L} &= l'_{a,L} + (\Delta l'_{1,L} + \Delta l'_{2,L} + \Delta l'_{3,L} + \Delta l'_{4,L}) \\ &- (l_{K,L} - l_{I,L})\sin(\omega_1 + \alpha_{s,L})\end{aligned} \tag{68}$$

where $\Delta l'_{1,L}$ is the horizontal distance between tangent point and the center of the splay saddle's first circular arc; $\Delta l'_{2,L}$ is the horizontal distance between the centers of the first and second circular arcs of the splay saddle; $\Delta l'_{3,L}$ is the horizontal distance between the centers of the second and third circular arcs; and $\Delta l'_{4,L}$ is the horizontal distance between the third circular arc's center and the fourth circular arc's center K_1. They can be expressed in the following forms:

$$\Delta l'_{1,L} = r_{1,L}\sin\beta'_1 \tag{69-1}$$

$$\Delta l'_{2,L} = (r_{2,L} - r_{1,L})\sin(\theta_{2,L} + \theta_{3,L} + \theta_{4,L} + \varphi_1 + \alpha_{s,L}) \tag{69-2}$$

$$\Delta l'_{3,L} = (r_{3,L} - r_{2,L})\sin(\theta_{3,L} + \theta_{4,L} + \varphi_1 + \alpha_{s,L}) \tag{69-3}$$

$$\Delta l'_{4,L} = (r_{4,L} - r_{3,L})\sin(\theta_{4,L} + \varphi_1 + \alpha_{s,L}) \tag{69-4}$$

(3) $S'_{a,L}$ can be expressed as

$$S'_{a,L} = S'_{c,a,L} + S'_{J_1D'_3} \tag{70}$$

where $S'_{c,a,L}$ and $S'_{J_1D'_3}$ are the unstrained cable lengths of

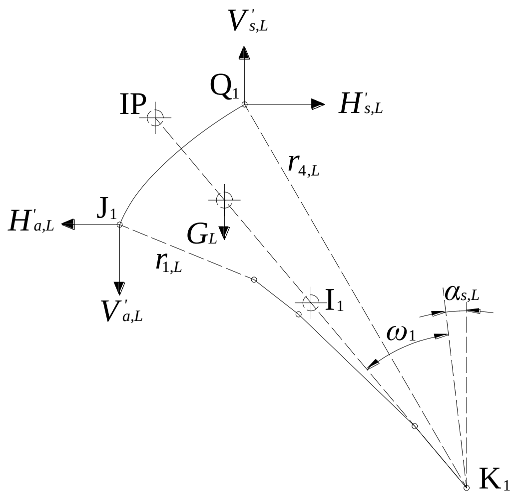

Fig. 15 Balance of torques acting on the left splay saddle in the unloaded state

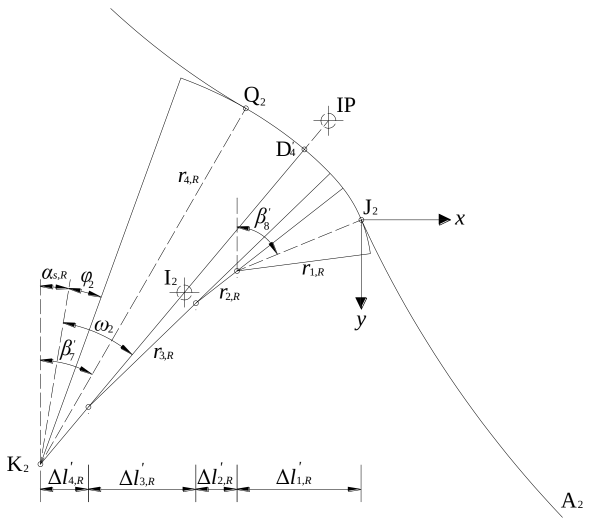

Fig. 16 Cable configuration in the unloaded state over the right anchor span

the catenary segment A_1J_1 and the arc segment $J_1D'_3$. They are given by

$$S'_{c,a,L} = c'_{a,L}\left[\sinh\left(\frac{l'_{a,L}}{c'_{a,L}} + a'_{al,L}\right) - \sinh a'_{a,L}\right] - \frac{H'_{a,L}}{2EA}\left\{l'_{a,L} + \frac{c'_{a,L}}{2}\left[\sinh 2\left(\frac{l'_{a,L}}{c'_{a,L}} + a'_{a,L}\right) - \sinh 2a'_{a,L}\right]\right\} \tag{71-1}$$

$$S'_{J_1D'_3} = \frac{r_{1,L}[\beta'_1 - (\theta_{2,L} + \theta_{3,L} + \theta_{4,L} + \varphi_1 + \alpha_{s,L})] + r_{2,L}\theta_{2,L} + r_{3,L}[\theta_{3,L} - (\omega_1 - \varphi_1 - \theta_{4,L})]}{1 + \frac{H'_{a,L}\cosh a'_{a,L}}{EA}} \tag{71-2}$$

(4) In the unloaded state, the splay saddle over the left anchor span deviates by a certain angle, and the horizontal components of cable tension at the two sides of the splay saddle tend to change. Therefore, the sum of torques acting on the splay saddle needs to be recalculated (Fig. 15):

$$\sum M'_{I_1} = H'_{s,L}\cdot e'_{s1,L} + V'_{s,L}\cdot e'_{s2,L} - (H'_{a,L}\cdot e'_{a1,L} + V'_{a,L}\cdot e'_{a2,L} + G_L\cdot e'_{g,L}) \tag{72}$$

where $e'_{s1,L}$ denotes the eccentricity of the horizontal component of cable tension over the left side span, $e'_{s1,L} = r_{4,L}\cos\beta'_2 - (l_{K,L} - l_{I,L})\cos(\omega_1 + \alpha_{s,L})$; $V'_{s,L}$ represents the vertical component of cable tension over the left side span at tangent point Q_1, $V'_{s,L} = H'_{s,L}\tan\beta'_2$; $e'_{s2,L}$ is the eccentricity of $V'_{s,L}$, $e'_{s2,L} = r_{4,L}\sin\beta'_2 - (l_{K,L} - l_{I,L})\sin(\omega_1 + \alpha_{s,L})$; $e'_{a1,L}$ is the eccentricity of the horizontal component of cable tension over the left anchor span, $e'_{a1,L} = \Delta h'_{1,L} + \Delta h'_{2,L} + \Delta h'_{3,L} + \Delta h'_{4,L} - (l_{K,L} - l_{I,L})\cos(\omega_1 + \alpha_{s,L})$; $V'_{a,L}$ is the vertical component of cable tension over the left anchor span at tangent point J_1, $V'_{a,L} = H'_{a,L}\tan\beta'_1$; $e'_{a2,L}$ is the eccentricity of $V'_{a,L}$, $e'_{a2,L} = \Delta l'_{1,L} + \Delta l'_{2,L} + \Delta l'_{3,L} + \Delta l'_{4,L} - (l_{K,L} - l_{I,L})\sin(\omega_1 + \alpha_{s,L})$; G_L is the force of gravity exerted on the left splay saddle; and $e'_{g,L}$ is the eccentricity of G_L, $e'_{g,L} = l_{g,L}\sin(\omega_1 + \alpha_{s,L})$.

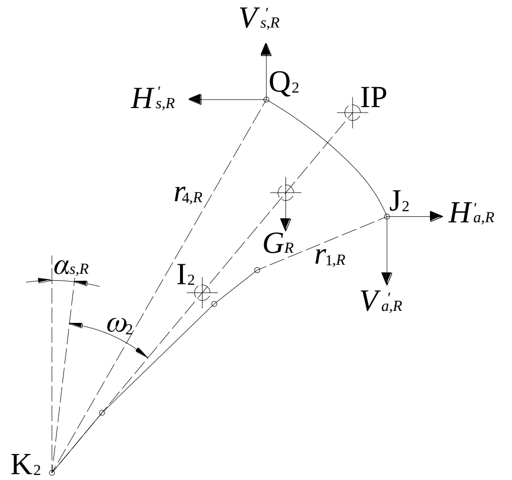

Fig. 17 Balance of torques acting on the right splay saddle in the unloaded state

3.2.5 Right anchor span

Expressions are constructed from relevant unknown quantities to describe the following four parameters: $\Delta h'_{a,R}$, the elevation difference between the right splay saddle's center of rotation, I_2, and the right anchor point, A_2; $L'_{a,R}$, the horizontal distance between points I_2 and A_2; $S'_{a,R}$, the unstrained cable length over right anchor span; and $\sum M'_{I_2}$, the sum of torques about the right splay saddle's center of rotation.

(1) $\Delta h'_{a,R}$ can be expressed in the following form:

$$\Delta h'_{a,R} = \Delta h'_{I_2A_2} = h_{I_2} - h_{A_2} = \Delta h'_{c,a,R} - (\Delta h'_{1,R} + \Delta h'_{2,R} + \Delta h'_{3,R} + \Delta h'_{4,R}) + (l_{K,R} - l_{I,R})\cos(\omega_2 + \alpha_{s,R}) \tag{73}$$

where $\Delta h'_{c,a,L}$ is the elevation difference between the two endpoints of the catenary segment over the right anchor span in the unloaded state, $\Delta h'_{c,a,R} = c'_{a,R}\left[\cosh\left(\frac{l'_{a,R}}{c'_{a,R}} + a'_{a,R}\right) - \cosh a'_{a,R}\right]$; $l_{I,R}$ is the

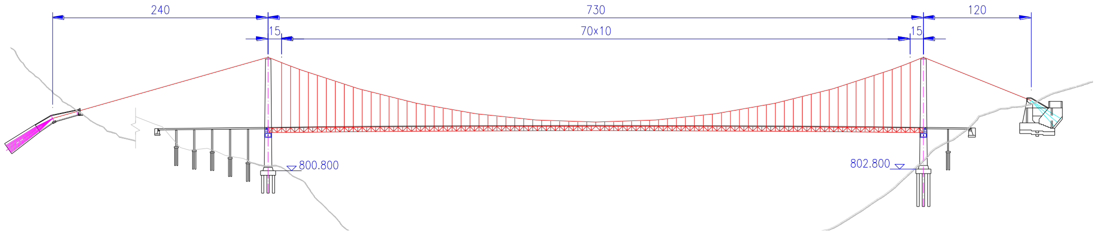

Fig. 18 Overall layout of the Jindong Bridge (Unit: m)

Fig. 19 The completed Jindong bridge under final dead load

Fig. 20 The unloaded state of the Jindong bridge

Fig. 21 A splay saddle of the Jindong bridge

distance from the right splay saddle's IP point to its center of rotation, I_2; $\Delta h^{'}_{1,R}$ is the elevation difference between tangent point J_2 and the center of the first circular arc of the right splay saddle; $\Delta h^{'}_{2,R}$ is the elevation difference between the centers of the first and second circular arcs of the splay saddle; $\Delta h^{'}_{3,R}$ is the elevation difference between the centers of the second and third circular arcs; and $\Delta h^{'}_{4,R}$ is the elevation difference between the center of the third circular arc and the fourth circular arc's center, K_2. They can be written in the following forms:

$$\Delta h^{'}_{1,R} = r_{1,R} \cos \beta^{'}_{8} \tag{74-1}$$

$$\Delta h^{'}_{2,R} = (r_{2,R} - r_{1,R}) \cos(\theta_{2,R} + \theta_{3,R} + \theta_{4,R} + \varphi_2 + \alpha_{s,R}) \tag{74-2}$$

$$\Delta h^{'}_{3,R} = (r_{3,R} - r_{2,R}) \cos(\theta_{3,R} + \theta_{4,R} + \varphi_2 + \alpha_{s,R}) \tag{74-3}$$

$$\Delta h^{'}_{4,R} = (r_{4,R} - r_{3,R}) \cos(\theta_{4,R} + \varphi_2 + \alpha_{s,R}) \tag{74-4}$$

(2) $L^{'}_{a,R}$ can be described by

$$\begin{aligned} L^{'}_{l,R} &= l^{'}_{l,R} + (\Delta l^{'}_{1,R} + \Delta l^{'}_{2,R} + \Delta l^{'}_{3,R} + \Delta l^{'}_{4,R}) \\ &\quad - (l_{\mathrm{K},R} - l_{\mathrm{I},R}) \sin(\omega_2 + \alpha_{s,R}) \end{aligned} \tag{75}$$

where $\Delta l^{'}_{1,R}$ is the horizontal distance between tangent point J_2 and the center of the splay saddle's first circular arc (Fig. 16); $\Delta l^{'}_{2,R}$ is the horizontal distance between the centers of the first and second circular arcs of the splay saddle; $\Delta l^{'}_{3,R}$ is the horizontal distance between the centers of the second and third circular arcs; and $\Delta l^{'}_{4,R}$ denotes the horizontal distance between center of the third circular arc and the fourth circular arc's center, K_2. They are given by

$$\Delta l^{'}_{1,R} = r_{1,R} \sin \beta^{'}_{8} \tag{76-1}$$

$$\Delta l^{'}_{2,R} = (r_{2,R} - r_{1,R}) \sin(\theta_{2,R} + \theta_{3,R} + \theta_{4,R} + \varphi_2 + \alpha_{s,R}) \tag{76-2}$$

$$\Delta l^{'}_{3,R} = (r_{3,R} - r_{2,R}) \sin(\theta_{3,R} + \theta_{4,R} + \varphi_R + \alpha_{s,R}) \tag{76-3}$$

$$\Delta l^{'}_{4,R} = (r_{4,R} - r_{3,R}) \sin(\theta_{4,R} + \varphi_2 + \alpha_{s,R}) \tag{76-4}$$

(3) $S^{'}_{a,R}$ can be expressed as

$$S^{'}_{a,R} = S^{'}_{c,a,R} + S^{'}_{\mathrm{D}^{'}_{4}\mathrm{J}_2} \tag{77}$$

where $S'_{c,a,R}$ and $S'_{\mathrm{D}'_4\mathrm{J}_2}$ are the unstrained cable lengths of the catenary segment over the right anchor span, J_2A_2, and the arc segment on the right splay saddle, $\mathrm{D}'_4\mathrm{J}_2$, respectively. They are given by

$$S'_{c,a,R} = c'_{a,R}\left[\sinh\left(\frac{l'_{a,R}}{c'_{a,R}} + a'_{a,R}\right) - \sinh a'_{a,R}\right] - \frac{H'_{a,R}}{2EA}\left\{l'_{a,R} + \frac{c'_{a,R}}{2}\left[\sinh 2\left(\frac{l'_{a,R}}{c'_{a,R}} + a'_{a,R}\right) - \sinh 2a'_{a,R}\right]\right\} \tag{78-1}$$

$$S'_{\mathrm{D}'_4\mathrm{J}_2} = \frac{r_{1,R}[\beta'_8 - (\theta_{2,R} + \theta_{3,R} + \theta_{4,R} + \varphi_2 + \alpha_{s,R})] + r_{2,R}\theta_{2,R} + r_{3,R}[\theta_{3,R} - (\omega_2 - \varphi_2 - \theta_{4,R})]}{1 + \dfrac{H'_{a,R}\cosh a'_{a,R}}{EA}} \tag{78-2}$$

(4) According to Fig. 17, $\sum M'_{\mathrm{I}_2}$ can be described by the equation below

$$\sum M'_{\mathrm{I}_2} = H'_{s,R}\cdot e'_{s1,R} + V'_{s,R}\cdot e'_{s2,R} - (H'_{a,R}\cdot e'_{a1,R} + V'_{a,R}\cdot e'_{a2,R} + G_R\cdot e'_{g,R}) \tag{79}$$

where $e'_{s1,R}$ denotes the eccentricity of the horizontal component of cable tension over the left side span, $e'_{s1,R} = r_{4,R}\cos\beta'_7 - (l_{\mathrm{K},R} - l_{\mathrm{I},R})\cos(\omega_2 + \alpha_{s,R})$; $V'_{s,R}$ denotes the vertical component of cable tension over the left side span at tangent point Q_2, $V'_{s,R} = H'_{s,R}\tan\beta'_7$; $e'_{s2,R}$ represents the eccentricity of $V'_{s,R}$, $e'_{s2,R} = r_{4,R}\sin\beta'_7 - (l_{\mathrm{K},R} - l_{\mathrm{I},R})\sin(\omega_2 + \alpha_{s,R})$; $e'_{a1,R}$ is the eccentricity of the horizontal component of cable tension over the left anchor span, $e'_{a1,R} = \Delta h'_{1,R} + \Delta h'_{2,R} + \Delta h'_{3,R} + \Delta h'_{4,R} - (l_{\mathrm{K},R} - l_{\mathrm{I},R})\cos(\omega_2 + \alpha_{s,R})$; $V'_{a,R}$ is the vertical component of cable tension over the left anchor span at tangent point J_2, $V'_{a,R} = H'_{a,R}\tan\beta'_8$; $e'_{a2,R}$ is the eccentricity of $V'_{a,R}$, $e'_{a2,R} = \Delta l'_{1,R} + \Delta l'_{2,R} + \Delta l'_{3,R} + \Delta l'_{4,R} - (l_{\mathrm{K},R} - l_{\mathrm{I},R})\sin(\omega_2 + \alpha_{s,R})$; G_R is the force of gravity acting on the right splay saddle; and $e'_{g,R}$ is the eccentricity of G_R, $e'_{g,R} = l_{g,R}\cdot\sin(\omega_2 + \alpha_{s,R})$.

3.3 Equation solving

Substituting above expressions for the parameters built from the unknown quantities into Eq. (52) gives 17 coupled equations. Rearranging each equation to the left-hand side, we can get 17 functions of the form $f_i(\ )=0$. The following objective function is then constructed by using the aforementioned 17 unknown quantities as the variables:

$$\min\left(\sum_{i=1}^{17} f_i^2\right) \tag{80}$$

Solving this equation will give the values of the 17 known quantities in the unloaded state. Then the cable configuration in the unloaded state and pre-offsets for the saddles can be calculated from the values obtained.

4. Example analysis

4.1 Profile of the Jindong bridge

Located in the Kunming City, Yunnan Province, China, the Jindong Bridge is currently the suspension bridge with the longest span over the Jinsha River. As shown in Fig. 18, the bridge is spanned as 240m+730m+120m. The sag to span ratio of the main cable is 1/10, and the deck width is 20m. The center-to-center spacing between the upstream and downstream main cables is 17.5m. The left and right ends of the cable are anchored by gravity-type and tunnel-type anchorages, respectively. The left tower is 126m high, and the right is 124m high, both with a portal frame structure.

Prefabricated parallel wire strands (PPWS) are adopted for the two cables. Each strand consists of 91*Φ*5.2 high-strength galvanized steel wires and is regular hexagon in cross section. Each cable is made of 91 strands and thus has 8281 wires in total. Each main cable is connected with 71 hangers. Each hanger is composed of 109*Φ*5.0 high-strength galvanized steel wires.

The steel truss girder consists of the main truss, top and bottom bracings, and transverse trusses. The main truss is a Warren truss with 5m height, 17.5m width, and 5.0m panel length, and it has a transverse truss in each panel. The bridge floor system combines longitudinal I-beams and concrete slab decks. They are simply supported by the top chords of the cross members in the main truss. The slab decks and longitudinal beams are fastened by shear pins.

The bridge's completed and unloaded states are shown in Figs. 19 and 20, respectively. A splay saddle is shown in Fig. 21.

4.2 Cable configuration under final dead load

Tables 1 and 2 summarize the known parameters needed for calculating the cable geometry of the completed bridge. Elevations of hanger anchor points on the deck, $h_{i,\mathrm{deck}}$, and axial tensile forces at the hangers' lower ends, $P_{0,i}$, are also known parameters, as listed in Table 5.

4.2.1 Main span

The values of the unknown quantities, H, a_1, and l_{n+1} were determined by solving Eq. (15). Then other unknown parameters can be determined. Results of the main span for the completed bridge are listed in Table 3. Geometric parameters of all catenary cable segments are shown in Table 4. Calculated hanger parameters are listed in Table 5. Equation numbers, based on which the unknown parameters are calculated, are also listed in these tables.

4.2.2 Side spans

The values of the unknown quantities, a_s and l_s were determined by solving Eq. (33). Then other unknown parameters can be determined, as listed in Table 6.

4.2.3 Anchor spans

The values of the unknown quantities, a_a , l_a, and H_a were determined by solving Eq. (47). Then other unknown parameters can be determined, as listed in Table 7.

4.2.4 Tower top pre-uplift

Based on Eq. (51), the pre-uplifts, Δh_t, of the left and right tower tops were determined as 0.031m and 0.023m, respectively.

Table 1 Known parameters of the main span

Basic parameters of the completed bridge		Symbol	Unit	Value
Horizontal distance between the centerlines of the left and right towers, i.e. the main span length		L_m	m	730
Main cable	Weight per unit length	q	kN/m	14.268
	Elastic modulus	E	GPa	197.03
	Cross-sectional area	A	m^2	0.1759
	Number of the cable segments between the left tangent point, F_1, and the mid-span point, O_m	m	/	36
	Elevation of point O_m in the geodetic system	h_{O_m}	m	856
Hanger	Number	n	/	71
	Spacing	$l_2 \sim l_{71}$	m	10
	Horizontal distance between left tower's centerline and the first hanger	$l_{D_1O_1}$	m	15
	Horizontal distance between the rightmost hanger and right tower's centerline	$l_{O_{71}D_2}$	m	15
	Elastic modulus	E_h	GPa	199
	Cross-sectional area	A_i	m^2	0.00214
	Weight per unit length	w_i	kN/m	0.1835
Left tower saddle	Elevation of the circle center, C_1, in the geodetic system	h_{C_1}	m	923.21
	Angle between vertical segment B_1C_1 and segment C_1D_1	γ_1	°	2.365
	Radius of the arc-shaped top	R_1	m	5.5
Right tower saddle	Elevation of the circle center, C_2, in the geodetic system	h_{C_2}	m	923.09
	Angle between vertical segment B_2C_2 and segment C_2D_2	γ_2	°	-0.635*
	Radius of the arc-shaped top	R_2	m	5.5

*Note: the minus sign indicates that the line segment from point C_2 to point D_2 is bankward rather than riverward.

Table 2 Known parameters of the side and anchor spans

Basic parameters of the completed bridge			Symbol	Unit	Value	
					Left	Right
Side span length			L_s	m	240	120
Anchor span length			L_a	m	16.38	15.32
Elevation of the anchor point A_i in the geodetic system (i=1 and i=2 denote left and right, respectively)			h_{A_i}	m	848.53	867.14
Splay saddle	Elevation of the circle center, K, in the geodetic system		h_K	m	854.68	874.96
	Radius of the arc-shaped top	First circular arc	r_1	m	1.781	1.781
		Second circular arc	r_2	m	3.081	3.081
		Third circular arc	r_3	m	4.781	4.781
		Fourth circular arc	r_4	m	5.781	5.781
	Central angle	First circular arc	θ_1	°	19.78	15.6
		Second circular arc	θ_2	°	6	6
		Third circular arc	θ_3	°	6	6
		Fourth circular arc	θ_4	°	10	10
	Angle between the vertical line and the segment connecting the IP point, center of gravity, center of rotation, I_1, and the fourth circular arc's circle center, K		ω_1	°	25.01	30.86
	Angle between the vertical line and the segment connecting the outmost endpoint of the fourth circular arc and its circle center, K		φ_1	°	15.01	20.86
	Distance between the IP point and the center of rotation, I_1		l_I	m	3.05	3.05
	Distance from the IP point to the circle center, K, of the fourth circular arc		l_K	m	5.875	5.866
	Self-weight		G	kN	406.262	384.792
	Distance between center of gravity and center of rotation, I_1		l_g	m	1.879	1.879
Tower	Target height of the completed bridge		h_t	m	126	124
	Elastic modulus		E_t	GPa	32.5	32.5
	Cross-sectional area of a tower column		A_t	m^2	8.260	12.707

Table 3 Results of the main span for the completed bridge

Parameter	Symbol	Unit	Value	Eq.
Horizontal component of the cable tension	H	kN	94239.75	(15)
A parameter of the catenary equation	c	m	-6604.97	$c=-H/q$
Elevation of the left tangent point, F_1, in the geodetic system	h_{F_1}	m	928.327	(5)
Elevation of the right tangent point, F_2, in the geodetic system	h_{F_2}	m	928.207	(7)
Elevation difference between points F_1 and O_m	$\Delta h_{F_1O_m}$	m	72.327	(6)
Horizontal distance between the right tangent point, F_2, and the right tower's centerline	$l_{F_2D_2}$	m	2.077	(9)
Angle between the vertical segment B_1C_1 and the segment connecting point F_1 and the circle center, C_1	β_4	°	21.506	$\tan\beta_4 = \sinh a_1$
Angle between the vertical segment B_2C_2 and the segment connecting point F_2 and the circle center, C_2	β_5	°	21.503	$\tan\beta_5 = -\sinh\left(\frac{l_{n+1}}{c}+a_{n+1}\right)$
Unstrained cable length of the arc segment D_1F_1 on the left tower saddle	$S_{D_1F_1}$	m	1.832	(21)
Total unstrained cable length of catenary segments	$S_{c,m}$	m	742.828	(19)
Unstrained cable length of the arc segment F_2D_2 on the right tower saddle	$S_{F_2D_2}$	m	2.119	(22)
Unstrained cable length over the main span	S_m	m	746.779	(23)

Table 4 Geometric parameters of all catenary cable segments in the main span

No.	a_i	b_i (m)	l_i (m)	Δh_i (m)	$S_{c,i}$ (m)	No.	a_i	b_i (m)	l_i (m)	Δh_i (m)	$S_{c,i}$ (m)
	a_1:Eq.(15) a_2-a_{72}: Eq.(12)	b_i=-ccosha_i	l_1: Eq(13)	Eq.(4)	Eq.(20)		Eq.(12)	b_i=-ccosha_i	l_{72}: Eq(15)	Eq.(4)	Eq.(20)
1	0.38449	6122.742	13.211	5.191	14.153	37	-0.00469	6604.900	10	-0.055	9.973
2	0.37172	6153.875	10	3.795	10.665	38	-0.01561	6604.168	10	-0.164	9.974
3	0.36113	6178.940	10	3.682	10.626	39	-0.02653	6602.648	10	-0.273	9.977
4	0.35070	6202.937	10	3.571	10.588	40	-0.03745	6600.342	10	-0.382	9.980
5	0.34024	6226.345	10	3.460	10.551	41	-0.04836	6597.251	10	-0.491	9.985
6	0.32973	6249.154	10	3.349	10.516	42	-0.05927	6593.375	10	-0.601	9.991
7	0.31920	6271.342	10	3.238	10.481	43	-0.07017	6588.717	10	-0.710	9.998
8	0.30863	6292.885	10	3.128	10.448	44	-0.08107	6583.279	10	-0.819	10.006
9	0.29805	6313.773	10	3.017	10.416	45	-0.09196	6577.063	10	-0.929	10.016
10	0.28744	6333.993	10	2.906	10.384	46	-0.10285	6570.072	10	-1.038	10.026
11	0.27681	6353.543	10	2.796	10.354	47	-0.11372	6562.309	10	-1.147	10.038
12	0.26616	6372.407	10	2.685	10.325	48	-0.12459	6553.778	10	-1.257	10.051
13	0.25548	6390.584	10	2.575	10.297	49	-0.13544	6544.482	10	-1.366	10.065
14	0.24479	6408.062	10	2.465	10.270	50	-0.14629	6534.425	10	-1.476	10.081
15	0.23408	6424.838	10	2.354	10.245	51	-0.15712	6523.612	10	-1.585	10.097
16	0.22336	6440.903	10	2.244	10.220	52	-0.16794	6512.047	10	-1.695	10.115
17	0.21261	6456.251	10	2.134	10.197	53	-0.17875	6499.737	10	-1.805	10.133
18	0.20185	6470.875	10	2.025	10.175	54	-0.18954	6486.684	10	-1.914	10.153
19	0.19107	6484.771	10	1.915	10.153	55	-0.20032	6472.897	10	-2.024	10.175
20	0.18028	6497.930	10	1.805	10.133	56	-0.21108	6458.378	10	-2.134	10.197
21	0.16947	6510.349	10	1.695	10.115	57	-0.22182	6443.137	10	-2.244	10.220
22	0.15865	6522.021	10	1.586	10.097	58	-0.23255	6427.177	10	-2.354	10.245
23	0.14782	6532.943	10	1.476	10.081	59	-0.24326	6410.507	10	-2.464	10.270
24	0.13698	6543.108	10	1.366	10.065	60	-0.25395	6393.132	10	-2.575	10.297
25	0.12612	6552.513	10	1.257	10.051	61	-0.26462	6375.061	10	-2.685	10.325
26	0.11525	6561.153	10	1.147	10.038	62	-0.27528	6356.298	10	-2.796	10.354
27	0.10438	6569.026	10	1.038	10.026	63	-0.28591	6336.855	10	-2.906	10.384
28	0.09349	6576.126	10	0.929	10.016	64	-0.29652	6316.736	10	-3.017	10.415
29	0.08260	6582.452	10	0.819	10.006	65	-0.30710	6295.951	10	-3.127	10.448
30	0.07171	6588.000	10	0.710	9.998	66	-0.31767	6274.508	10	-3.238	10.481
31	0.06080	6592.768	10	0.601	9.991	67	-0.32820	6252.423	10	-3.349	10.516
32	0.04989	6596.754	10	0.492	9.985	68	-0.33871	6229.709	10	-3.460	10.551
33	0.03898	6599.956	10	0.382	9.980	69	-0.34917	6206.403	10	-3.571	10.588
34	0.02806	6602.372	10	0.273	9.977	70	-0.35960	6182.503	10	-3.682	10.625
35	0.01715	6604.002	10	0.164	9.974	71	-0.37019	6157.541	10	-3.795	10.665
36	0.00623	6604.845	10	0.055	9.973	72	-0.38247	6127.724	12.923	-5.078	13.844

Table 5 Known and calculated hanger parameters

No.	$h_{i,\mathrm{d}}$ (m)	$L_{i,\mathrm{h}}$ (m) $=h_{i,\mathrm{c}}-h_{i,\mathrm{d}}$	$S_{i,\mathrm{h}}$ (m) Eq.(16)	$P_{0,i}$ (kN)	P_i (kN) Eq.(18)	No.	$h_{i,\mathrm{d}}$ (m)	$L_{i,\mathrm{h}}$ (m) $=h_{i,\mathrm{c}}-h_{i,\mathrm{d}}$	$S_{i,\mathrm{h}}$ (m) Eq.(16)	$P_{0,i}$ (kN)	P_i (kN) Eq.(18)
1	849.685	73.450	72.606	1074.5	1087.8	37	851.699	4.356	4.315	885.7	886.5
2	849.799	69.542	68.870	900.7	913.3	38	851.694	4.525	4.482	885.6	886.4
3	849.909	65.749	65.128	881.5	893.5	39	851.685	4.806	4.760	885.7	886.6
4	850.016	62.071	61.483	882.9	894.2	40	851.674	5.199	5.150	885.6	886.5
5	850.120	58.507	57.952	884.4	895.0	41	851.659	5.706	5.652	885.7	886.7
6	850.220	55.057	54.535	885.3	895.3	42	851.641	6.324	6.265	885.6	886.7
7	850.317	51.722	51.231	885.5	894.9	43	851.620	7.056	6.989	885.7	887.0
8	850.411	48.500	48.040	885.7	894.5	44	851.595	7.899	7.825	885.6	887.0
9	850.501	45.393	44.963	885.6	893.9	45	851.567	8.856	8.772	885.7	887.3
10	850.588	42.399	41.998	885.8	893.5	46	851.536	9.925	9.831	885.6	887.4
11	850.672	39.520	39.146	885.6	892.8	47	851.501	11.107	11.002	885.7	887.7
12	850.753	36.754	36.406	885.7	892.4	48	851.463	12.401	12.284	885.6	887.9
13	850.830	34.102	33.779	885.6	891.8	49	851.422	13.809	13.678	885.7	888.2
14	850.904	31.563	31.265	885.7	891.4	50	851.378	15.329	15.184	885.6	888.4
15	850.975	29.138	28.862	885.6	890.9	51	851.330	16.962	16.802	885.7	888.8
16	851.042	26.826	26.572	885.7	890.6	52	851.279	18.708	18.531	885.6	889.0
17	851.107	24.628	24.395	885.6	890.1	53	851.225	20.567	20.372	885.7	889.4
18	851.167	22.542	22.329	885.7	889.8	54	851.167	22.539	22.326	885.6	889.7
19	851.225	20.570	20.376	885.6	889.3	55	851.107	24.624	24.391	885.7	890.2
20	851.279	18.711	18.534	885.7	889.1	56	851.042	26.823	26.569	885.6	890.5
21	851.330	16.965	16.804	885.6	888.7	57	850.975	29.134	28.859	885.7	891.0
22	851.378	15.332	15.187	885.7	888.5	58	850.904	31.559	31.261	885.6	891.3
23	851.422	13.811	13.681	885.6	888.1	59	850.830	34.098	33.775	885.7	891.9
24	851.463	12.404	12.286	885.7	888.0	60	850.753	36.750	36.402	885.6	892.3
25	851.501	11.109	11.004	885.6	887.6	61	850.672	39.515	39.141	885.8	893.0
26	851.536	9.927	9.833	885.7	887.5	62	850.588	42.395	41.993	885.6	893.3
27	851.567	8.858	8.774	885.6	887.2	63	850.501	45.388	44.958	885.7	893.9
28	851.595	7.901	7.826	885.7	887.1	64	850.411	48.495	48.036	885.6	894.4
29	851.620	7.057	6.990	885.6	886.9	65	850.317	51.716	51.226	885.6	895.0
30	851.641	6.325	6.266	885.7	886.8	66	850.220	55.052	54.530	885.2	895.2
31	851.659	5.706	5.653	885.6	886.6	67	850.120	58.501	57.947	884.6	895.2
32	851.674	5.200	5.151	885.7	886.6	68	850.016	62.065	61.478	882.8	894.1
33	851.685	4.806	4.761	885.6	886.5	69	849.909	65.743	65.122	881.6	893.5
34	851.694	4.525	4.482	885.7	886.5	70	849.799	69.535	68.864	900.6	913.2
35	851.699	4.356	4.315	885.7	886.5	71	849.685	73.444	72.600	1074.6	1087.9
36	851.700	4.300	4.259	885.5	886.3						

Table 6 Results of side spans for the completed bridge

Parameter	Symbol	Unit	Value		Eq.
			Left	Right	
Horizontal component of the cable tension	H	kN	94239.75		Table 3
Parameters of the catenary equation	a_s	/	0.30170	0.40643	(33)
	b_s	m	6907.88	7146.12	b_s=-$c$cosha_s
	c	m	-6604.97		Table 3
Length of the orthogonal projection of the catenary segment on the horizontal plane	l_s	m	237.180	117.072	(33)
Elevation of tangent point E_1 on the tower saddle	h_{E_1}	m	928.469	928.165	(28)
Elevation of tangent point Q on the splay saddle	h_Q	m	860.258	880.387	(29)
Angle between vertical line and the segment QK	β_2	°	15.052	22.670	$\tan\beta_2 = \sinh\left(\frac{l_s}{c}+a_s\right)$
Angle between the vertical segment B_1C_1 and the segment connecting tangent point E_1 to circle center C_1	β_3	°	17.031	21.730	$\tan\beta_3 = \sinh a_s$
Horizontal distance between the tangent point E_1 and point B_1	Δ_1	m	1.611	2.120	$\Delta_1=R_1\sin\beta_3$
Horizontal distance between points B_1 and D_1	Δ_2	m	0.227	-0.061	$\Delta_2=R_1\sin\gamma_1$
Horizontal distance between the tangent point on the splay saddle, Q, and circle center K	Δ_3	m	1.501	2.140	$\Delta_3=r_4\sin\beta_2$
Horizontal distance between the IP point and circle center, K, of the splay saddle	Δ_4	m	2.484	3.009	$\Delta_4=l_K\sin\omega_1$
Unstrained cable length of the arc segment D_3Q on the splay saddle	S_{D_3Q}	m	1.002	0.918	(35)
Unstrained cable length of the catenary segment QE_1	$S_{c,s}$	m	246.107	126.076	(34)
Unstrained cable length of the arc segment E_1D_1 on the tower saddle	$S_{E_1D_1}$	m	1.857	2.109	(36)
Total unstrained cable length over the side span	S_s	m	248.965	129.104	(37)

Table 7 Results of anchor spans for the completed bridge

Parameter	Symbol	Unit	Value		Eq.
			Left	Right	
Horizontal component of the cable tension	H_a	kN	80394.20	78101.52	(47)
Parameters of the catenary equation	a_a	/	0.65259	0.76218	(47)
	b_a	m	6877.59	7142.33	b_a=-$c_a$cosha_a
	c_a	m	-5634.58	-5473.89	c_a=-H_a /q
Length of the orthogonal projection of the catenary segment on the horizontal plane	l_a	m	15.765	14.769	(47)
Elevation difference between the tangent point J and the anchor point A_i (i=1 and i=2 denote left and right, respectively)	Δh_{JA_i}	m	11.007	12.353	(43)
Angle between the vertical line and the segment connecting tangent point J and the center of the first circular arc	β_1	°	34.989	39.968	$\tan\beta_1 = \sinh a_a$
Elevation of tangent point J on the splay saddle	h_J	m	859.536	879.500	(42)
Unstrained cable length of the catenary segment	$S_{c,a}$	m	19.173	19.198	(48)
Unstrained cable length of the arc segment JD_3 on the splay saddle	S_{JD_3}	m	0.758	0.731	(49)
Total unstrained cable length over the anchor span	S_a	m	19.931	19.929	(50)

Table 8 Known parameters from the completed bridge

Elevation difference between points	Symbol	Value (m)	Horizontal distance between points	Symbol	Value (m)	Unstrained cable length	Symbol	Value (m)
C_1 and C_2	Δh_m	0.120	D_1 and D_2	L_m	730.000	Main span	S_m	746.779
I_1 and C_1	Δh_{s1}	68.517	I_1 and D_1	L_{s1}	238.711	Left side span	$S_{s,L}$	248.965
C_2 and I_2	Δh_{s2}	48.105	D_2 and I_2	L_{s2}	118.436	Right side span	$S_{s,R}$	129.104
A_1 and I_1	Δh_{a1}	6.164	A_1 and I_1	L_{a1}	17.673	Left anchor span	$S_{a,L}$	19.931
I_2 and A_2	Δh_{a2}	7.841	I_2 and A_2	L_{a2}	16.886	Right anchor span	$S_{a,R}$	19.929

Table 9 Calculated parameters of the cable configuration in the unloaded state

Parameter			Left anchor span	Left side span	Main span	Right side span	Right anchor span
Horizontal component of the cable tension		Symbol	$H'_{a,L}$	$H'_{s,L}$	H'_m	$H'_{s,R}$	$H'_{a,R}$
		Value (kN)	12345.43	14177.03	14177.03	14177.03	11550.51
		Eq.			(80)		
Parameters of the catenary equation		Symbol	$a'_{a,L}$	$a'_{s,L}$	a'_m	$a'_{s,R}$	$a'_{a,R}$
		Value	0.65978	0.39931	0.35510	0.45613	0.76952
		Eq.			(80)		
		Symbol	$b'_{a,L}$	$b'_{s,L}$	b'_m	$b'_{s,R}$	$b'_{a,R}$
		Value (m)	1096.047	1109.810	1092.217	1135.586	1096.874
		Eq.			$b'=-c'\cosh a'$		
		Symbol	$c'_{a,L}$	$c'_{s,L}$	c'_m	$c'_{s,R}$	$c'_{a,R}$
		Value (m)	-894.272	-1026.949	-1026.949	-1026.949	-836.690
		Eq.			$c'=-H'/q'$		
Length of the orthogonal projection of the catenary segment on the horizontal plane		Symbol	$l'_{a,L}$	$l'_{s,L}$	l'_m	$l'_{s,R}$	$l'_{a,R}$
		Value (m)	15.743	234.958	728.094	116.089	14.748
		Eq.			(80)		
Elevation difference between the catenary segment's two endpoints		Symbol	$\Delta h'_{c,a,L}$	$\Delta h'_{c,s,L}$	$\Delta h'_{c,m}$	$\Delta h'_{c,s,R}$	$\Delta h'_{c,a,R}$
		Value (m)	10.987	67.939	0.127	47.645	12.332
		Eq.			$\Delta h'=c'[\cosh(l'/c'+a')-\cosh a']$		
Unstrained cable length	Left arc segment	Symbol	\	$S'_{D'_3Q_1}$	$S'_{D'_1F'_1}$	$S'_{D'_2E'_2}$	$S'_{D'_4J_2}$
		Value (m)	\	1.629	1.683	2.363	0.717
		Eq.	\	(61-2)	(57-2)	(65-2)	(78-2)
	Catenary segment	Symbol	$S'_{c,a,L}$	$S'_{c,s,L}$	$S'_{c,m}$	$S'_{c,s,R}$	$S'_{c,a,R}$
		Value (m)	19.189	244.972	743.129	125.486	19.216
		Eq.	(71-1)	(61-1)	(57-1)	(65-1)	(78-1)
	Right arc segment	Symbol	$S'_{J_1D'_3}$	$S'_{E'_1D'_1}$	$S'_{F'_2D'_2}$	$S'_{Q_2D'_4}$	\
		Value (m)	0.744	2.365	1.969	1.254	\
		Eq.	(71-2)	(61-3)	(57-3)	(65-3)	\

Table 10 Calculated angles of contingence, β'

Tangent point	Left splay saddle		Left tower saddle		Right tower saddle		Right splay saddle	
	J_1	Q_1	E'_1	F'_1	F'_2	E'_2	Q_2	J_2
Symbol	β'_1	β'_2	β'_3	β'_4	β'_5	β'_6	β'_7	β'_8
Value (°)	35.323	9.709	22.281	19.908	19.889	25.266	19.276	40.289

4.3 Cable configuration in the unloaded state

Several parameters from the calculated cable configuration under final dead load are needed to determinethe cable configuration in the unloaded state, as listed in Table 8. The weight per unit length of the free cable is q'=13.805 kN/m.

Based on Eq. (80), the 17 unknown quantities for determining the cable configuration in the unloaded state can be calculated. Then other parameters can be derived. The calculated parameters of the cable configuration in the unloaded state are listed in Table 9. The Calculated angles of contingence are list in Table 10. An angle of contingence is the angle between the vertical line and the segment connecting the tangent point and the center of circular arc. The calculated pre-offsets of saddles are listed in Table 11.

The proposed method and above-mentioned results have been fully and successfully employed in the design and construction control of the Jindong bridge, which has passed the authorities' tests and is ready to be opened for traffic.

Table 11 Calculated pre-offsets of saddles

Pre-offset		Symbol	Unit	Value	Eq.
Angle	Left splay saddle	$\alpha_{s,L}$	°	0.851	(80)
	Right splay saddle	$\alpha_{s,R}$	°	0.850	
Distance	Left tower saddle	$\Delta_{m,L}$	m	1.183	
	Right tower saddle	$\Delta_{m,R}$	m	0.489	

5. Conclusions

Analytical methods for calculating a suspension bridge's main cable configuration under final dead load and in the unloaded state and relevant construction parameters are proposed based on the segmental catenary theory. The analytical results can be used as a reference for bridge design and construction control. This method has a number of strengths:

(1) It is capable of determining the unstrained cable length, unstrained hanger lengths, pre-offsets for tower and splay saddles, and other critical design and construction parameters.

(2) It takes into account the effects of cable strands over the anchor spans, arc-shaped tops of tower/splay saddles, and tower top pre-uplifts.

(3) In terms of cable configuration under final dead load, mechanical equilibrium conditions and geometric relationships are utilized to calculate the parameters of catenary equations for the cable segments over each span, coordinates of tangent points, and angles of contingence. The calculations are first performed for the main span, followed by the side spans and then anchor spans. Hanger tensile forces and unstrained hanger lengths are calculated by iteratively solving the equations governing hanger tensile forces and the cable configuration, which gives careful consideration to the effect of hanger weight.

(4) Equations for calculating cable configuration in the unloaded state are derived from the cable configuration under final dead load and the conditions for unstrained cable length to be conserved. By simultaneously solving the equations for the main span, two side spans and two anchor spans, we can obtain the pre-offsets for saddles, parameters of the catenary equation for the cable segment over each span, coordinates of tangent points, and angles of contingence.

(5) The coupled nonlinear equations are converted to objective functions in an unconstrained optimization problem and then solved by GRG for nonlinear programming. This avoids complicated iterative process for solving coupled equations. For example, the number of coupled equations involved in calculating cable configuration in the unloaded state reaches up to 17.

(6) Compared to finite element methods, the proposed analytical method does not require complex simulations, and features explicit physical concepts and easily adjustable parameters. With general applicability, it is expected to be popularized.

The feasibility and validity of the proposed method have been demonstrated through a numerical example of a suspension bridge spanned as 240m+730m+120m.

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