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Analytical study of bending and free vibration responses of functionally graded beams resting on elastic foundation

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Abstract. In this investigation, study of the static and dynamic behaviors of functionally graded beams (FGB) is presented using a hyperbolic shear deformation theory (HySDT). The simply supported FG-beam is resting on the elastic foundation (Winkler-Pasternak types). The properties of the FG-beam vary according to exponential (E-FGB) and power-law (P-FGB) distributions. The governing equations are determined via Hamilton's principle and solved by using Navier's method. To show the accuracy of this model (HySDT), the current results are compared with those available in the literature. Also, various numerical results are discussed to show the influence of the variation of the volume fraction of the materials, the power index, the slenderness ratio and the effect of Winkler spring constant on the fundamental frequency, center deflection, normal and shear stress of FG-beam.

Keywords: functionally graded beam; bending; stress; natural frequency; shear deformation theory; Winkler-Pasternak foundation parameters

1. Introduction

Novel class of advanced composites materials (FGMs) has been of great importance to several researchers worldwide these last years because of their large range of applications in structural mechanics (Chen, 2005, Woo *et al.* 2006, Efraim and Eisenberger 2007, Zhao *et al.* 2009, Mohammadi and Saidi 2010, Kiani *et al.* 2011, Ghannadpour *et al.* 2012, Bouderba *et al.* 2013, Jha *et al.* 2013, Bousahla *et al.* 2014, Mahi *et al.* 2015, Bourada *et al.* 2015, Kar and Panda, 2015, Kolahchi *et al.* 2016a, Arani and Kolahchi, 2016, Madani *et al.* 2016, Meftah *et al.* 2017, Aldousari, 2017, Avcar and Mohammed 2018, Bouhadra *et al.* 2018, Zine *et al.* 2018, Karami *et al.* 2019ab). This advanced FG material has gradual variation of the volume fraction which gives a non-uniform microstructure with continuously macro properties such as density, conductivity and elasticity modulus. Three types of volume fraction were found in the literature such as exponential model "E-FGM" (Ravichandran 1995, Ait Atmane *et al.* 2010 and Chakraborty *et al.* 2003) power-law model "P-FGM" (Zidi *et al.* 2014, Zemri *et al.* 2015, Yahia *et al.* 2015, Ahouel *et al.* 2016, Abdelaziz *et al.* 2017, Abualnour *et al.* 2018) and Sigmoid model "S-FGM" (Chung and Chi 2001, Hamed *et al.* 2016, Jung *et al.* 2016, Duc and Cong 2015, Bourada *et al.* 2018). Several research works have been published to study the static and dynamic behaviors of functionally graded beams and plates. Sankar (2001) developed an elasticity solution for flexural FG-beams based on Euler–Bernoulli beam theory (EBT). The free and forced vibration analysis of FG-Beams under moving load has been investigated by Şimşek and Kocaturk (2009) using Euler Bernoulli theory. The exact solutions for static and dynamic characteristics of FG beams resting elastic foundation was presented by Ying *et al*. (2008). Buckling analysis of FG clamped plate under thermal load has been published by Kiani *et al*. (2011). The stability of FG-plates under non-uniform compression was investigated by Mahdavian (2009) using the classical plate theory (CPT) and Fourier solutions. Civalek and Öztürk (2010) examined the free vibration response of tapered beam-column with pinned ends embedded in Winkler-Pasternak elastic foundation by using EBT. Ghomshei and Abbasi (2013) examined the thermal buckling of functionally graded annular plates with variable thickness using the CPT and the FE method. Bilouei *et al*. (2016) used EBT and Differential quadrature method (DQM) is used in order to obtain the buckling load of concrete columns retrofitted with Nano-Fiber Reinforced Polymer. Avcar (2016a)

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studied the free vibration of non-homogeneous beam subjected to axial force resting on Pasternak foundation using EBT. Since the EBT and CPT over predicts the fundamental frequencies as well as buckling loads deflection of short beams and moderately thick plate. Reissner (1945) and Mindlin (1951) have extended the classical plate theory to the first shear deformation theory by introducing the transverse shear effect. Several works have been presented to study the bending and the free vibration of functionally graded structures with taking into account the transverse shear deformation. The static bending and transverse vibration of FG-Timoshenko beams have been studied by Li (2008). Koochaki (2011) investigated on the dynamic response of FG Timoshenko beam. A new four variables first shear deformation theory for static and vibration analysis of FG- plates has been proposed (Xiang and Shi 2011, Meksi *et al.* 2015, Mantari and Granados 2015, Bellifa *et al.* 2016, Bennoun *et al.* 2016, Hadji *et al*. 2016, Avcar 2016b, Draiche *et al.* 2016, Boukhari *et al.* 2016, Bousahla *et al.* 2016, Beldjelili *et al.* 2016, Bouderba *et al.* 2016, El-Haina *et al.* 2017, Fahsi *et al.* 2017, Chikh *et al.* 2017, Zamanian *et al.* 2017, Hajmohammad *et al.* 2018a, Luo *et al.* 2018, Bouadi *et al.* 2018, Fourn *et al.* 2018, Yazid *et al.* 2018, Avcar 2019). To avoid the limitations of CPT and FSDT, the high order shear deformation theory take into account the transverse shear effect without using the shear correction factors. This theory (HSDTs) is widely used in many research investigations. Şimşek (2010) has studied the dynamic analysis of an FG beam using different higher order beam theories. The wave propagation of FG porous plate has been examined by Yahia *et al*. (2015) using a high order shear deformation theory. Recently, a novel high shear deformation theories (HSDT) for vibrational behavior of FG-plates has been proposed by Younsi *et al.* (2018) and Zaoui *et al*. (2019). Also, Bourada *et al.* (2019a) have investigated on the effect of the porosity on fundamental frequencies of power law FG-beams using a sinusoidal shear deformation theory. Other HSDTs or FSDT can be documented in literature (Avcar 2015, Hamidi *et al.* 2015, Kolahchi and MoniriBidgoli 2016, Abdelbari *et al.* 2016, Kolahchi *et al.* 2016b, Kolahchi *et al.* 2017abc, Kolahchi and Cheraghbak 2017, Bellifa *et al.* 2017a, Hajmohammad *et al.* 2017, Kolahchi 2017, Hajmohammad *et al.* 2018bc, Fakhar and Kolahchi 2018, Amnieh *et al.* 2018, Kadari *et al.* 2018, Attia *et al.* 2018, Bakhadda *et al.* 2018, Golabchi *et al.* 2018, Karami *et al.* 2018abcd, Hosseini and Kolahchi 2018, Abazid *et al.* 2018, Hadji *et al.* 2018, Boukhlif *et al.* 2019, Khiloun *et al.* 2019).

The aim of this paper is to study the bending and free vibration responses of functionally graded beams using a hyperbolic shear deformation theory (HySDT). The present theory involves just three unknowns in which the transverse displacement is divided into both bending and shears components. It also accounts for a hyperbolic distribution of the transverse shear stresses through the thickness that satisfies the zero traction boundary conditions on the lower and upper beam surfaces without including a shear correction factor. The equations of motion of simply supported beam subjected to uniform loads are obtained using Hamilton's principle and solved via Navier's method. A parametric study was made to investigate the effect of the power-law volume fraction, the material length scale parameter and the Winkler parameter on the deflection, stresses and natural frequencies. It is concluded that the present theory is not only efficient but can achieve the same accuracy of the other higher order shear deformation theories that contain more number of unknowns.

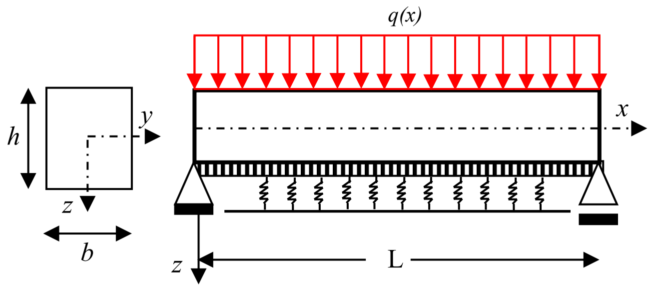

Fig. 1 Geometry and coordinates of a functionally graded beam resting on the elastic foundation

2. Theoretical formulation

In this study, we consider a simply supported functionally graded beam of a length L, width b and the thickness h subjected to a uniform distributed loads and resting on Winkler-Pasternak type elastic foundation as shown in Fig.1.

2.1 Effective material properties

The material characteristics of the FGM are assumed to vary continuously through the beam thickness based on the power-law and exponential forms. For the power law form, the volume fraction of the P-FGM beam change smoothly through the thickness of the beam with a simple power law form, (Bao and Wang 1995, Tounsi *et al.* 2013, Hebali *et al.* 2014, Belkorissat *et al.* 2015, Al-Basyouni *et al.* 2015, Attia *et al.* 2015, Larbi Chaht *et al.* 2015, Houari *et al.* 2016, Menasria *et al.* 2017, Benahmed *et al.* 2017, Meksi *et al.* 2019) which is stated as follow:

$$P(z) = P_m + (P_c - P_m)\left(\frac{1}{2} + \frac{z}{h}\right)^k \tag{1}$$

Where P represents the effective material property, P_m and P_c denote the Young's modulus and mass density metal and ceramic on the top and bottom surfaces of the beam. k is the power-law exponent that defines the material variation profile within the beam thickness. In the case of the exponential variation (Delale and Erdogan 1983), the effective Young's modulus of FG beam can be calculated using the exponential form as presented below:

$$E(z) = E_0 e^{k(z+h/2)} \tag{2}$$

Where E_0 is the Young's modulus of homogeneous material. The Poisson's ratio v is considered to be constant.

2.2 Kinematic and equations of motion

Based on the same higher order shear deformation theory proposed by Ould Larbi *et al*. (2013) and Meziane *et al*. (2014), the displacement field of the present model can be written as

$$u(x,z,t) = u_0(x,t) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \tag{3a}$$

$$w(x,z,t) = w_b(x,t) + w_s(x,t) \tag{3b}$$

Where u_0 denotes in-plane displacement and the transverse displacement w include two components: the bending component w_b, and shear component w_s. In this study, a hyperbolic shape function $f(z)$ is used:

$$f(z) = z\left(1+\frac{3\pi}{2}\sec h^2\left(\frac{1}{2}\right)\right) - \frac{3\pi}{2}h\tanh\left(\frac{z}{h}\right) \tag{4}$$

The corresponding strains-displacement relations are given by the following expressions

$$\varepsilon_x = \varepsilon_x^0 + z\,k_x^b + f(z)k_x^s \tag{5a}$$

$$\gamma_{xz} = g(z)\gamma_{xz}^s \tag{5b}$$

where

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial x},\ k_x^b = -\frac{\partial^2 w_b}{\partial x^2},\ k_x^s = -\frac{\partial^2 w_s}{\partial x^2},\ \gamma_{xz}^0 = \frac{\partial w_s}{\partial x} \tag{6a}$$

$$g(z) = 1 - f'(z) \quad and \quad f'(z) = \frac{\partial f(z)}{\partial z} \tag{6b}$$

The materials of FG beams are supposed to obey Hooke's Law. So, the beam's stresses can be expressed as

$$\sigma_x = Q_{11}(z)\varepsilon_x \tag{7a}$$

$$\tau_{xz} = Q_{55}(z)\gamma_{xz} \tag{7b}$$

In which, Q_{ij} are the elastic coefficients as below

$$Q_{11}(z) = E(z) \quad \text{and} \quad Q_{55}(z) = \frac{E(z)}{2(1+\nu)} \tag{8}$$

In order to obtain the equations of motion, Hamilton's principle is used herein as stated in the following analytical form (Belabed *et al.* 2014, Larbi Chaht *et al.* 2015, Zemri *et al.* 2015, Mahi *et al.* 2015, Bounouara *et al.* 2016, Bellifa *et al.* 2017b, Khetir *et al.* 2017, Klouche *et al.* 2017, Zidi *et al.* 2017, Hachemi *et al.* 2017, Belabed *et al.* 2018, Mokhtar *et al.* 2018, Cherif *et al.* 2018, Kaci *et al.* 2018, Semmah *et al.* 2019, Bourada *et al.* 2019b, Tlidji *et al.* 2019)

$$\int_0^T (\delta U + \delta V - \delta K)dt = 0 \tag{9}$$

where δU is the virtual variation of the strain energy, δV is the virtual variation of the potential energy and δK is the virtual variation of the kinetic energy.

The variation of strain energy of the FG beam is given by

$$\begin{aligned}\delta U &= \int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(\sigma_x\,\delta\varepsilon_x + \tau_{xz}\,\delta\gamma_{xz}\right)dz\,dx \\ &= \int_0^L \left(N_x\,\delta\varepsilon_x^0 + M_x^b\delta k_x^b + M_x^s\delta k_x^s + Q_{xz}^s\,\delta\gamma_{xz}^0\right)dx\end{aligned} \tag{10}$$

Where the stress resultants N_x, M_x^b, M_x^s and Q_{xz}^s are determined by

$$\left(N_x, M_x^b, M_x^s\right) = \int_{-h/2}^{h/2}(1, z, f)\sigma_x dz \quad \text{and} \quad Q_{xz}^s = \int_{-h/2}^{h/2} g(z)\,\tau_{xz}\,dz \tag{11}$$

The variation of the potential energy of external load can be stated by

$$\delta V = -\int_0^L q\,\delta w_0\,dx \tag{12}$$

The variation of kinetic energy of the beam can be written as (Mouffoki *et al.* 2017, Sekkal *et al.* 2017ab, Benadouda *et al.* 2017, Besseghier *et al.* 2017, Bouafia *et al.* 2017, Youcef *et al.* 2018)

$$\begin{aligned}\delta K &= \int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[\dot{u}\delta\dot{u} + \dot{w}\delta\dot{w}\right]\rho(z)\,dz\,dx \\ &= \int_0^L \Big\{ I_0\left(\dot{u}_0\delta\dot{u}_0 + (\dot{w}_b + \dot{w}_s)(\delta\dot{w}_b + \delta\dot{w}_s)\right) \\ &\quad - I_1\left(\dot{u}_0\frac{\partial\delta\dot{w}_b}{\partial x} + \frac{\partial\dot{w}_b}{\partial x}\delta\dot{u}_0\right) + I_2\left(\frac{\partial\dot{w}_b}{\partial x}\frac{\partial\delta\dot{w}_b}{\partial x}\right) \\ &\quad - J_1\left(\dot{u}_0\frac{\partial\delta\dot{w}_s}{\partial x} + \frac{\partial\dot{w}_s}{\partial x}\delta\dot{u}_0\right) + K_2\left(\frac{\partial\dot{w}_s}{\partial x}\frac{\partial\delta\dot{w}_s}{\partial x}\right) \\ &\quad + J_2\left(\frac{\partial\dot{w}_b}{\partial x}\frac{\partial\delta\dot{w}_s}{\partial x} + \frac{\partial\dot{w}_s}{\partial x}\frac{\partial\delta\dot{w}_b}{\partial x}\right)\Big\}dx\end{aligned} \tag{13}$$

Where dot-superscript convention indicates the differentiation with respect to the time variable t; $\rho(z)$ is the mass density; and (I_i, J_i, K_i) are mass inertias defined as

$$(I_0, I_1, J_1, I_2, J_2, K_2) = \int_{-h/2}^{h/2}\left(1, z, f, z^2, zf, f^2\right)\rho(z)\,dz \tag{14}$$

Substituting Eqs. (10), (12), and (13) into Eq. (9), integrating by parts, and collecting the coefficients of δu_0, δw_b and δw_s, the equations of motion are obtained in terms of efforts as given below

$$\delta u_0 : \frac{\partial N_x}{\partial x} = I_0\ddot{u}_0 - I_1\frac{\partial\ddot{w}_b}{\partial x} - J_1\frac{\partial\ddot{w}_s}{\partial x} \tag{15a}$$

$$\delta w_0 : \frac{\partial^2 M_x^b}{\partial x^2} + q = I_0\left(\ddot{w}_b + \ddot{w}_s\right) + I_1\frac{\partial\ddot{u}_0}{\partial x} - I_2\frac{\partial^2\ddot{w}_b}{\partial x^2} - J_2\frac{\partial^2\ddot{w}_s}{\partial x^2} \tag{15b}$$

$$\delta\phi\ :\ -\frac{\partial^2 M_x^s}{\partial x^2}+\frac{\partial Q_{xz}^s}{\partial x}+q_z=I_0\left(\ddot{w}_b+\ddot{w}_s\right)+J_1\frac{\partial \ddot{u}_0}{\partial x}-J_2\frac{\partial^2 \ddot{w}_b}{\partial x^2} -K_2\frac{\partial^2 \ddot{w}_s}{\partial x^2} \tag{15c}$$

Substituting Eq. (5) into Eq. (7) and the subsequent results into Eq. (11), the stress resultants can be expressed in terms of generalized displacements (u_0 , w_0 , ϕ) as

$$\begin{Bmatrix} N_x \\ M_x^b \\ M_x^s \end{Bmatrix}=\begin{bmatrix} A_{11} & B_{11} & B_{11}^s \\ B_{11} & D_{11} & D_{11}^s \\ B_{11}^s & D_{11}^s & H_{11}^s \end{bmatrix}\begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ -\dfrac{\partial^2 w_b}{\partial x^2} \\ -\dfrac{\partial^2 w_s}{\partial x^2} \end{Bmatrix},\ Q_{xz}^s=A_{55}^s\frac{\partial w_s}{\partial x} \tag{16}$$

Where A, D, B^s, D^s, etc… are the stiffnesses of the FG beam given by

$$\left(A_{11},\quad D_{11},\quad B_{11}^s,\quad D_{11}^s,\quad H_{11}^s\right)= \int_{-h/2}^{h/2} Q_{11}(1,z^2,f,zf,f^2)dz \tag{17a}$$

$$A_{55}^s=\int_{-h/2}^{h/2} Q_{55}(g(z))^2dz \tag{17b}$$

Substituting Eqs. (16) into Eqs. (15), the equations of motion of the can be expressed in terms of displacements (u_0 , w_b , w_s) as

$$\delta u_0\ :\ A_{11}\frac{\partial^2 u_0}{\partial x^2}-B_{11}^s\frac{\partial^3 w_s}{\partial x^3}=I_0\ddot{u}_0-I_1\frac{\partial \ddot{w}_b}{\partial x}+J_1\frac{\partial \ddot{w}_s}{\partial x} \tag{18a}$$

$$\delta w_b:\ -D_{11}\frac{\partial^4 w_b}{\partial x^4}-D_{11}^s\frac{\partial^4 w_s}{\partial x^4}+q=I_0\left(\ddot{w}_b+\ddot{w}_s\right)+I_1\frac{\partial \ddot{u}_0}{\partial x} -I_2\frac{\partial^2 \ddot{w}_b}{\partial x^2}-J_2\frac{\partial^2 \ddot{w}_s}{\partial x^2} \tag{18b}$$

$$\delta w_s\ :\ B_{11}^s\frac{\partial^3 u_0}{\partial x^3}-D_{11}^s\frac{\partial^4 w_b}{\partial x^4}-H_{11}^s\frac{\partial^4 w_s}{\partial x^4}+A_{55}^s\frac{\partial^2 w_s}{\partial x^2}+q= I_0\left(\ddot{w}_b+\ddot{w}_s\right)+J_1\frac{\partial \ddot{u}_0}{\partial x}-J_2\frac{\partial^2 \ddot{w}_b}{\partial x^2}-K_2\frac{\partial^2 \ddot{w}_s}{\partial x^2} \tag{18c}$$

3. Closed-form solutions

The equations of motion have been solved by using Navier's procedure to satisfy the boundary conditions. The solution of the displacement variables are expanded in the double-Fourier sine series as (Benchohra *et al.* 2018):

$$\begin{Bmatrix} u_0 \\ w_b \\ w_s \end{Bmatrix}=\sum_{m=1}^{\infty}\begin{Bmatrix} U_m\cos(\lambda\,x)e^{i\omega t} \\ W_{bm}\sin(\lambda\,x)e^{i\omega t} \\ W_{sm}\sin(\lambda\,x)e^{i\omega t} \end{Bmatrix} \tag{19}$$

Where (U_m , W_{bm} , W_{sm}) are unknown functions to be determined, ω is the natural frequency and λ is expressed as

$$\lambda=m\pi/L \tag{20}$$

The transverse load q is also expanded in the double-Fourier sine series as

$$q(x)=\sum_{m=1}^{\infty}q_m\sin(\lambda\,x) \tag{21}$$

Where q_m is the intensity of the load calculated from the Eq. (22)

$$q_m=\frac{2}{L}\int_0^L q(x)\sin(\lambda x)dx \tag{22}$$

For sinusoidal distributed load

$$q_m=q_0 \tag{23}$$

For uniform distributed load

$$q_m=\frac{4q_0}{m\pi},\qquad (m=1,3,5,.....) \tag{24}$$

Substituting Eqs. (19) and (21) into equations of motion (18), the closed-form solutions can be obtained from the following equations:

$$\left(\begin{bmatrix} s_{11} & 0 & s_{13} \\ 0 & s_{22} & s_{23} \\ s_{13} & s_{23} & s_{33} \end{bmatrix}-\omega^2\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{bmatrix}\right)\begin{Bmatrix} U_m \\ W_{bm} \\ W_{sm} \end{Bmatrix}=\begin{Bmatrix} 0 \\ q_m \\ q_m \end{Bmatrix} \tag{25}$$

Where

$$s_{11}=\lambda^2 A_{11},\qquad s_{13}=-\lambda^3 B_{11}^s,\qquad s_{22}=\lambda^4 D_{11},\qquad s_{23}=\lambda^4 D_{11}^s,\qquad s_{33}=\lambda^4 H_{11}^s+\lambda^2 A_{55}^s \tag{26a}$$

$$m_{11}=I_0,\qquad m_{12}=-\lambda\,I_1,\qquad m_{13}=-\lambda\,J_1,\qquad m_{22}=I_0+\lambda^2 I_2,\qquad m_{23}=I_0+\lambda^2 J_2,\qquad m_{33}=I_0+\lambda^2 K_2 \tag{26b}$$

4. Numerical results and discussions

In this section, a number of numerical examples are presented to check the accuracy of the present theory and to examine the influences of material index, side-to-thickness ratio and the elastic foundation parameter on the axial displacement, deflection, stresses and the natural frequencies of the FG beams. The FG beams studied in this paper are made of metal (Aluminum) and ceramics (Alumina) which their mechanical properties are listed in Table 1. For convenience, the following non-dimensional parameters are used: For the interpretation of the results, the following non-dimensional parameters were used:

Table 1Material properties of metal and ceramic

Material	Properties		
	Young's modulus (GPa)	Poisson's ratio	Mass density (kg/m³)
Aluminium (Al)	70	0.3	2702
Alumina (Al_2O_3)	380	0.3	3800

$$\bar{w} = 100\frac{h^3 E_m}{L^4 q_0} w\left(\frac{L}{2}\right),\ \bar{u} = 100\frac{h^3 E_m}{L^4 q_0} u\left(0, -\frac{h}{2}\right),$$

$$\bar{\sigma}_x = \frac{h}{Lq_0}\sigma_x\left(\frac{L}{2},\frac{h}{2}\right),\quad \bar{\tau}_{xz} = \frac{h}{Lq_0}\tau_{xz}(0,0),$$

$$\bar{K}_W = \frac{k_w a^4}{D_m},\ D_m = \frac{E_m h^3}{12(1-\upsilon^2)},\ \bar{K}_s = \frac{k_s a^2}{D_m}, \tag{27}$$

$$\bar{\omega} = \frac{\omega L^2}{h}\sqrt{\frac{\rho_m}{E_m}}$$

4.1 Bending analysis

Example 1. The objective of this example is to investigate the accuracy of the proposed theory in predicting the bending response of simply supported FG beams using a power law variation of Young's modulus. The dimensionless center deflection, axial displacements and stresses of thick and thin FG beam under a uniform distributed load for different power-law index "*k*" and side-to-thickness ratio "*L*/*h*" are carried out in Table 2. The calculated values are compared with those of Li *et al*. (2010) and Ould Larbi *et al*. (2013) based on 2D shear deformation theories. Form this table, it can be seen that results are close to each other which means that the proposed hyperbolic higher order shear theory can predict the static response of simply supported FG beams. To check the effect of power law index on the bending response, Fig. 2 presents the axial displacement and stresses distributions through the thickness of simply supported Al/Al_2O_3 FG beam. From these results, it can be noticed that the axial displacement $\bar{u}$, in-plane stress $\bar{\sigma}_x$ and shear stress τ_{xz} increase with the increasing value of power law index "*k*".

Example 2. This example is performed to analyze a simply supported beam using an exponential form (see Eq. 2) to define the material properties of the beam. Table 3 shows the non-dimensional deflection of FG beam with different values of span-to-depth ratio "*L*/*h*" and power law index "*k*". It can be seen that, the increasing values of power law index and side-to-thickness ratio lead to a decreasing in deflection. It is means that the elastic modulus increases with the increase of the value of exponent index. And consequently, the FG beam becomes stiffer.

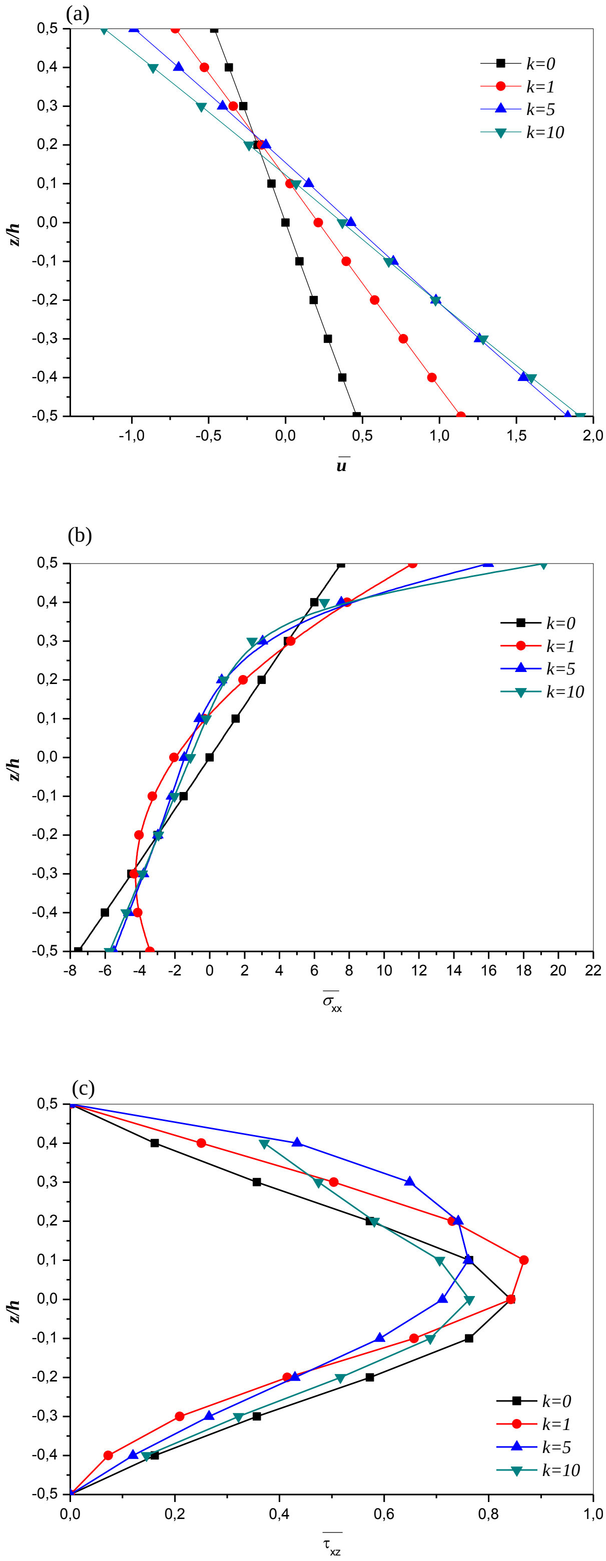

Fig. 2 Variation of displacements and stresses through the thickness of P-FGM beam (L/h = 10)

Example 3. To illustrate the effect of the Winkler and Pasternak foundation parameters on the beam deflection, Fig. 3 depicts the variation of non-dimensional deflection within foundation parameters and power law index. From

Table 2 Non-dimensional deflections and stresses of P-FGM beams under uniform load

k	Method	$L/h=5$				$L/h=20$			
		$\bar{w}$	$\bar{u}$	$\bar{\sigma}_x$	$\bar{\tau}_{xz}$	$\bar{w}$	$\bar{u}$	$\bar{\sigma}_x$	$\bar{\tau}_{xz}$
0	Li *et al*. (2010)	3.1657	0.9402	3.8020	0.7500	2.8962	0.2306	15.0130	0.7500
	Ould Larbi *et al*. (2013)	3.1651	0.9406	3.8043	0.7489	2.8962	0.2305	15.0136	0.7625
	Present	3.1581	0.9434	3.8128	0.8285	2.8962	0.2305	15.0138	0.7692
0.5	Li *et al*. (2010)	4.8292	1.6603	4.9925	0.7676	4.4645	0.4087	19.7005	0.7676
	Ould Larbi *et al*. (2013)	4.8282	1.6608	4.9956	0.7660	4.4644	0.4087	19.7013	0.7795
	Present	4.8188	1.6649	5.0074	0.8459	4.4638	0.4087	19.7042	0.8662
1	Li *et al*. (2010)	6.2599	2.3045	5.8837	0.7500	5.8049	0.5686	23.2054	0.7500
	Ould Larbi *et al*. (2013)	6.2590	2.3052	5.8875	0.7489	5.8049	0.5685	23.2063	0.7625
	Present	6.2472	2.3100	5.9019	0.8285	5.8041	0.5686	23.2099	0.8491
2	Li *et al*. (2010)	8.0602	3.1134	6.8812	0.6787	7.4415	0.7691	27.0989	0.6787
	Ould Larbi *et al*. (2013)	8.0683	3.1146	6.8878	0.6870	7.4421	0.7691	27.1005	0.7005
	Present	8.0566	3.1202	6.9071	0.7692	7.4414	0.7692	27.1053	0.7897
5	Li *et al*. (2010)	9.7802	3.7089	8.1030	0.5790	8.8151	0.9133	31.8112	0.5790
	Ould Larbi *et al*. (2013)	9.8345	3.7128	8.1187	0.6084	8.8186	0.9134	31.8151	0.6218
	Present	9.8378	3.7230	8.1488	0.6979	8.8189	0.9135	31.8226	0.7186
10	Li *et al*. (2010)	10.8979	3.8860	9.7063	0.6436	9.6879	0.9536	38.1372	0.6436
	Ould Larbi *et al*. (2013)	10.9413	3.8898	9.7203	0.6640	9.6907	0.9537	38.1408	0.6788
	Present	10.9205	3.9019	9.7484	0.7482	9.6895	0.9539	38.1477	0.7703

Table 3 Non-dimensional deflections of an E-FGM beams under sinusoidal load

L/h	Exponent index k					
	0.1	0.3	0.5	0.7	1	1.5
2	4.3839	3.9656	3.5862	3.2422	2.7853	2.1587
4	3.1536	2.8544	2.5845	2.3408	2.0187	1.5788
5	3.0043	2.7197	2.4630	2.2315	1.9257	1.5084
10	2.8048	2.5394	2.3005	2.0853	1.8013	1.4142

these figures, it can be observed that the dimensionless deflection diminishes with the increasing of the foundation parameters. This indicates that the inclusion of foundation parameters will increase the rigidity of the beam, and thus, lead to a decrease of deflection. Also, it can be noticed that the effect of the shear foundation parameter is more significant than the Winkler foundation parameter.

4.2 free vibration analysis

Example 4. This example is conserved to illustrate the exactitude of the present hyperbolic shear deformation theory for dynamic analysis of simply supported FG-beams. From Table 4, the actual results are compared with those obtained by Şimşek (2010) based on Timoshenko beam theory and given by Ould Larbi *et al*. (2013) based on high order shear deformation theory. It can be seen that the results obtained by the proposed model are in good agreement with those found in the literature for slender and short FG-beams.

Example 5. Fig.4 Show the variation of fundamental natural frequency ϖ versus power-law index "k" for different side-to-thickness ratio "L/h" of simply supported Power law FG-beam.it can be noted from the fig.4 that the fundamental natural frequency ϖ decreases with increasing of material index "k". It can be seen also that the natural frequency ϖ is in direct correlation relation with slenderness ratio "L/h".

5. Conclusions

In this paper, a static and free vibration behavior of functionally graded beams has been analyzed by using a two dimensional higher order shear deformation theory. The equations of motion of the present model are obtained through the Hamilton's principle. These equations are analytically solved by utilizing Navier's procedure. The obtained results were compared with the solutions of several theories such as Timoshenko beams theory (Li *et al.* 2010), higher order shear deformation theory (Şimşek 2010 and Ould Larbi *et al*. 2013), its remarkable that the different results of the proposed model has an excellent agreement with the other theories existing in the literature. In conclusion, it can be said that the present theory (HySDT) is not only accurate but also efficient in predicting the deflection, axial displacement, in-plane stress, shear stress and fundamental frequency of functionally graded beams.

Table 4 non-dimensional frequencies ϖ of P-FGM beams

L/h	Method	Power-law index k					
		0	0.5	1	2	5	10
5	Present	5.1633	4.4180	3.9963	3.6303	3.4004	3.2846
	Ould Larbi *et al*. (2013)	5.1529	4.4108	3.9905	3.6263	3.4001	3.2812
	TBT (Şimşek 2010)	5.1527	4.4111	3.9904	3.6264	3.4012	3.2816
20	Present	5.4657	4.6546	4.2076	3.8378	3.6491	3.5395
	Ould Larbi *et al*. (2013)	5.4603	4.6511	4.2051	3.8361	3.6484	3.5389
	TBT (Şimşek 2010)	5.4603	4.6511	4.2051	3.8361	3.6485	3.5390

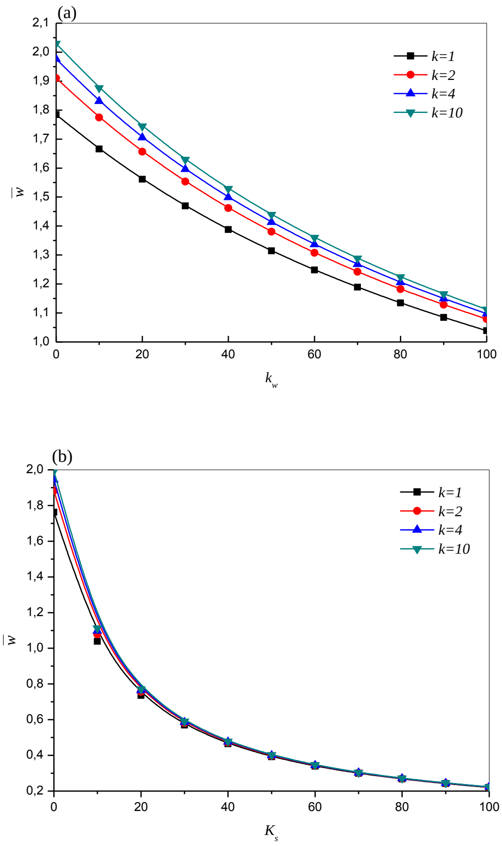

Fig. 3 Effect of the elastic foundation parameters ($\bar{K}_w$, $\bar{K}_s$) on the non-dimensional deflection of FG beam

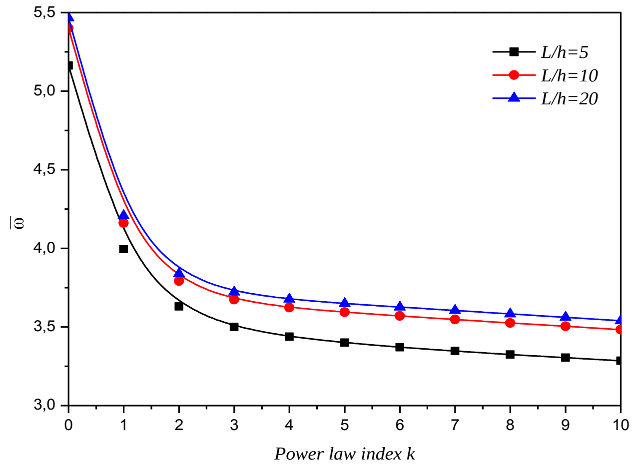

Fig. 4 Variation of fundamental natural frequency ϖ versus power-law index (k) for differentside-to-thickness ratio L/h of P-FGM beam

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