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Probabilistic analysis of RC beams according to IS456:2000 in limit state of collapse

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Abstract. This paper investigates the probability of failure of reinforced concrete beams for limit state of collapse for flexure and shear. The influence of randomness of the variables on the failure probability is also examined. The Indian standard code for plain and reinforced concrete IS456:2000 is used for the design of beams. Probabilistic models are developed for flexure and shear according to IS456:2000. The loads considered acting on the beam are live load and dead load only. Random variables associated with the limit state equation such as grade of concrete, grade of steel, live load and dead load are identified. Probability of failure is evaluated based on the limit state equation using First Order Reliability Method (FORM). Importance of the random variables on the limit state equations are observed and the variables are accordingly reduced. The effect of the reduced parameters is checked on the probability of failure. The results show the role of each parameter on the design of beam. Thus, the Indian standard guidelines for plain and reinforced concrete IS456:2000 is investigated with the probabilistic and risk-based analysis and design for a simple beam. The results obtained are also compared with the literature and accordingly some suggestions are made.

Keywords: flexure; FORM; IS456:2000; probabilistic; reinforced concrete; shear

1. Introduction

In recent times the probabilistic analysis or reliability analysis is increasingly being applied in the field of structural engineering. The focus of the analysis in the structural engineering is mainly on the study of source of randomness in structures which includes the variability of resistance and loads. As a result of this many design guidelines and codes are being modified to incorporate the reliability-based analysis and design. The Indian Standard code for plain and reinforced concrete (IS456:2000) needs to be updated to reliability-based design approach. Studies have investigated the reliability of beams exposed to fire (Eamon and Jensen 2013, Balaji *et al*. 2016, Kmet *et al*. 2016) since reinforced concrete structures are vulnerable to high temperature conditions such as those during fire (Kmet *et al*. 2016) and for limit state of serviceability (Galambos and Ellingwood 1986, Hossain and Stewart 2001, Honfi *et al*. 2012, Stewart 1996b, Khor *et al*. 2001, Torii and Machado 2010, Lu *et al*. 1994). Number of serviceability issues related mainly to excessive deflection of structural floor elements such as beams and columns and the field data for serviceability damage have been collected and studied (Hossain and Stewart 2001). The probabilistic concept was applied for Indian code specifications on RC beams for exposure to fire (Balaji *et al*. 2016), after designing it as per limit state design method. For Australian and US codes, target reliability index is suggested for serviceability reliabilities for structural steel beams in flexure (Stewart 1996a). Reliability analysis of reinforced concrete beams with respect of limit state of crack width under different loading condition was carried out using Monte Carlo technique (Desayi and Rao 1989). Reliability of corroded reinforced beams and columns was reviewed and parametric study on the serviceability and collapse limit state has been done and its effect on various parameters was observed (El-Reedy 2012). Reliability based methodology is also being used in accessing the damages in the reinforced concrete structures using statistics of random variables in the limit state functions (Sakka *et al*. 2018).

The present paper aims to determine the reliability index and probability of failure of IS456:2000 specifications for limit state of collapse for flexure and shear considering reinforced concrete beam subjected to normal loadings. The random variables associated with the limit state functions are identified based on the literatures. The statistical data for the parameters is collected for the Indian conditions and were used for the probabilistic analysis. The loading is restricted to live load and dead load. The beams are designed as per the specifications of IS code. First Order Reliability Method (FORM) is used for the analysis. From the analysis, reliability index and probability of failure are calculated. The sensitivity of the random variables associated with the limit state functions is shown. The intention here is to show the difference in the probability of failure considered for different standards and Indian standard and the importance of the random variables in the design method.

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2. Design according to IS456:2000

The Indian Standard code for plain and reinforced concrete (IS456:2000) is based on the design methodology of limit state method and working stress method, the choice of either is being left to the designer. However, majority follow the limit state method of design. In case of American, European and other international codes, as a result of extensive efforts by different engineering disciplines during the last three decades, design guidelines and codes are being modified or already have been modified to incorporate the concept of risk-based analysis and design (Haldar and Mahadevan 2000). Therefore, Indian standard is lacking in this effort of application of risk and reliability.

2.1 Limit state of flexure

The limit state of flexure is considered in terms of the moment of resistance and bending moment (Ranganathan 1990, Balaji *et al*. 2016) of singly reinforced RC beam. The moment of resistance of the simply supported beam without considering the factor of safety and load factor is given by Eq. (1)

$$M_r = f_y A_{st} d \left[1 - \frac{0.42\, f_y A_{st}}{0.54\, f_c bd}\right] \tag{1}$$

The bending moment due to external loading on the simply supported beam is given by Eq. (2).

$$M_s = \frac{(w_l + w_d)}{8} l^2 \tag{2}$$

where f_y is the yield strength of steel, f_c is the mean strength of concrete, b is the breadth of beam, d is the effective depth of the beam, w_l is the live load, w_d is the dead load, A_{st} is the area of tension steel. Random variables identified are f_y, f_c, w_l and w_d.

2.2 Limit state of shear

Resistance against shear for beam (Ranganathan 1990, Balaji *et al*. 2016) is given by Eq. (3).

$$V_R = \tau_c bd + V_{us} \tag{3}$$

where τ_c is shear strength of concrete and is calculated as per IS456:2000. V_{us} is the shear resistance offered by shear reinforcement, calculated as per IS456:2000 shown by Eq. (4).

$$V_{us} = \frac{f_y A_{sv} d}{s_v} \tag{4}$$

s_v is the spacing of stirrups, A_{sv} is the area of shear reinforcement. Shear force due to external load on the simply supported beam is calculated and is given by Eq. (5).

$$V_s = \frac{(w_l + w_d)}{2} l \tag{5}$$

3. Reliability analysis

Reliability analysis is defined as a probabilistic approach to determine safety level of the system or a structure to perform its functions under given conditions (Ghasemi and Nowak 2017). The basic step for the reliability analysis using First Order Reliability Method (FORM) is the formulation of failure functions based on various failure criteria using the relevant load and resistance parameters, called as variables X_i, (Haldar and Mahadevan 2000, Ceribasi 2017).

$$Z = g\,(X_1, X_2, \ldots\ldots, X_n) \tag{6}$$

The failure function or limit state of interest can then be defined as Z = 0. This is the boundary between the safe and unsafe regions in the design parameter space, and it also represents a state beyond which a structure or component can no longer fulfill the function for which it was designed (Haldar and Mahadevan 2000).

$$\beta = -\frac{\sum_{i=1}^{n} x_i^* \left(\frac{\partial g}{\partial X_i}\right)^*}{\sqrt{\sum_{i=1}^{n} \left(\frac{\partial g}{\partial X_i}\right)^{2*}}} \tag{7}$$

Where $(\partial g/\partial X_i)^*$ is the i[th] partial derivative evaluated at design point with coordinates (x_1^*, x_2^*, .., x_n^*). The design point is given by Eq. (8).

$$x_i^* = -\alpha_i \beta \tag{8}$$

Where,

$$\alpha_i = \frac{\left(\frac{\partial g}{\partial X_i}\right)^*}{\sqrt{\sum_{i=1}^{n} \left(\frac{\partial g}{\partial X_i}\right)^{2*}}} \tag{9}$$

Where α_i are the direction cosines. Direction cosines give the sensitivity of the random variables in the limit state equation. By knowing the sensitivity, the number of random variables can be reduced. Using Eq. (8), design point can be found out as given by Eq. (10).

$$x_i^* = \mu_{x_i} - \alpha_i \sigma_{x_i} \beta \tag{10}$$

Where μ_{x_i} is the mean values of the variable and σ_{x_i} is standard deviation value of the variable. Following are the steps to evaluate β for a limit state equation:

Step 1. Define appropriate limit state function.

Step 2. Assume initial values of design point x_i^*.

Step 3. Evaluate $(\partial g/\partial X_i)^*$ and α_i at x_i^*.

Step 4. Obtain new design point x_i^* in terms of β as in Eq. (8).

Step 5. Substitute the new x_i^* in limit state equation g(x_i^*)=0 and solve for β.

Step 6. Using β value obtained in Step 5, reevaluate $x_i^* = -\alpha_i \beta$.

Step 7. Repeat Steps 3 through 6 until β converges.

The probability of failure in terms of reliability index, β is given by Eq. (11), (Haldar and Mahadevan 2000, Zhao *et al*. 2016, Ceribasi 2017, Ghasemi and Nowak 2017).

$$p_f = \Phi(-\beta) = 1 - \Phi(\beta) \tag{11}$$

Where Φ is the cumulative distribution function (CDF) of

Table 1 Statistical distribution of random variables of beam (Ranganathan 1990 and Balaji *et al*. 2016)

0	Distribution Type	Parameters		Units
		Mean	COV	
f_y	Normal	468.9	0.05	MPa
f_c	Normal	30.28	0.145	MPa
w_l	Extreme Largest, Type-1 (Gumble Max)	10	0.3	kN/m
w_d	Normal	5+Self weight	0.05	kN/m

where f_y is the grade of steel, f_c is the grade of concrete, w_l is live load and w_d is dead load. Where COV is the Coefficient of Variation which is the ratio of standard deviation to the mean.

Table 2 Statistical distribution of grade of concrete (IS456:2000)

Grade of Concrete	Standard Deviation (MPa)	Mean (MPa)
M20	4	26.6
M25	4	31.6
M30	5	38.25
M35	5	43.25
M40	5	48.25
M45	5	53.25
M50	5	58.25

For designing the beam section, grade of steel used is Fe415.

the standard normal variate. Alternatively, p_f is same as given Eq. (12) when failure occurs (Z<0), (Haldar and Mahadevan 2000, Sakka *et al*. 2018)

$$p_f = \int \dots \int_{g(\ )<0} f_X(x_1, x_2, \dots, x_n) dx_1 dx_2 \dots dx_n \quad (12)$$

In which $f_X(x_1, x_2, \dots, x_n)$ is joint probability density function (PDF) for the basic random variables X_1, X_2, …, X_n and integration is performed over the failure region g()<0.

The work carried is divided in two parts as i) Reliability analysis for limit state of flexure and ii) Reliability analysis for limit state of shear. The failure functions are based on Indian codes and the literature available (IS456:2000, Ranganathan 1990, Balaji *et al.* 2016). The beam is assumed as simply supported beam. The dimensions of the beam are taken according to the specifications of IS 456:2000, Jain and Jain (2002) and Varghese (2002) which are dependent on the span of the beam. Similarly, as per the beam dimensions obtained minimum tension steel required is provided.

3.1 Statistical Data

For Indian conditions, the statistical data for material properties and loadings is specified and used in Ranganathan (1990) and Balaji *et al*. (2016). The statistical data for material properties and loading properties of the beam under consideration are given in Table 1.

Along with the values mentioned in Table 1, IS456:2000 has also suggested the statistical data for grade of concrete (f_c) and are given in Table 2. As per IS code, the mean strength of concrete mix should be equal to the characteristic strength plus 1.65 times the standard deviation.

3.2 Reliability analysis for limit state of flexure

Failure function for limit state of collapse with respect to flexure using Eqs. (1), (2) and (6) is given by Eq. (13),

$$g(x) = f_y A_{st} d \left[1 - \frac{0.42\, f_y A_{st}}{0.54\, f_c bd}\right] - \frac{(w_l + w_d)}{8} l^2 \quad (13)$$

First Order Reliability Method (FORM) is used for analysis and is carried out using software package COMREL (Version 9) (Honfi *et al*. 2012). In the reliability analysis the length of the beam is varied and for a particular length, the dimensions of the beam are obtained as per the provisions of IS 456:2000. Similarly, the area of tension steel is initially kept at minimum as per the provisions of IS456:2000 and is given by Eq. (14) and hanger bars are provided in compression zone, and reliability index is calculated. Minimum percentage of tension steel or tension reinforcement is calculated using Eq. (15). If the index is negative, it indicates that the beam has failed. Then the area of tension steel is increased and is shown in terms of percentage of tension steel (p_t), until the positive reliability index is obtained indicating that the beam is safe. The results are obtained for statistical data as per Balaji *et al.* (2016) and IS456:2000. The work is divided as per the statistical data. For data as per Balaji *et al.* (2016) all the parameters given are used. Whereas for the study as per IS456:2000 the statistical data for grade of concrete is available in the code and the remaining data is taken as per Balaji *et al.* (2016) that is grade of steel, dead load and live load.

$$A_{st\,min} = \frac{0.85bd}{f_y} \quad (14)$$

$$p_t = \frac{100A_{st}}{bD} \quad (15)$$

Where $A_{st\,min}$ is the minimum area of tension reinforcement, D is the overall depth of beam and p_t is percentage of tension reinforcement. As per NPTEL (2009), width to overall depth ratio is maintained between 0.5 and 0.67. The width or breadth of beam is kept as 150, 200, 230, 250, 300 mm which satisfies most of the practical aspects. As per IS456:2000, the exposure condition for the beams is assumed to be mild and accordingly the nominal cover for beams is taken as 20 mm.

Fig. 1 (a)–(h) are showing the reliability index values for varying percentage of steel. If the reliability index value is negative, then it indicates that in the limit state equation, the action on the members is greater than the resistance offered by the member. As reliability index value changes to

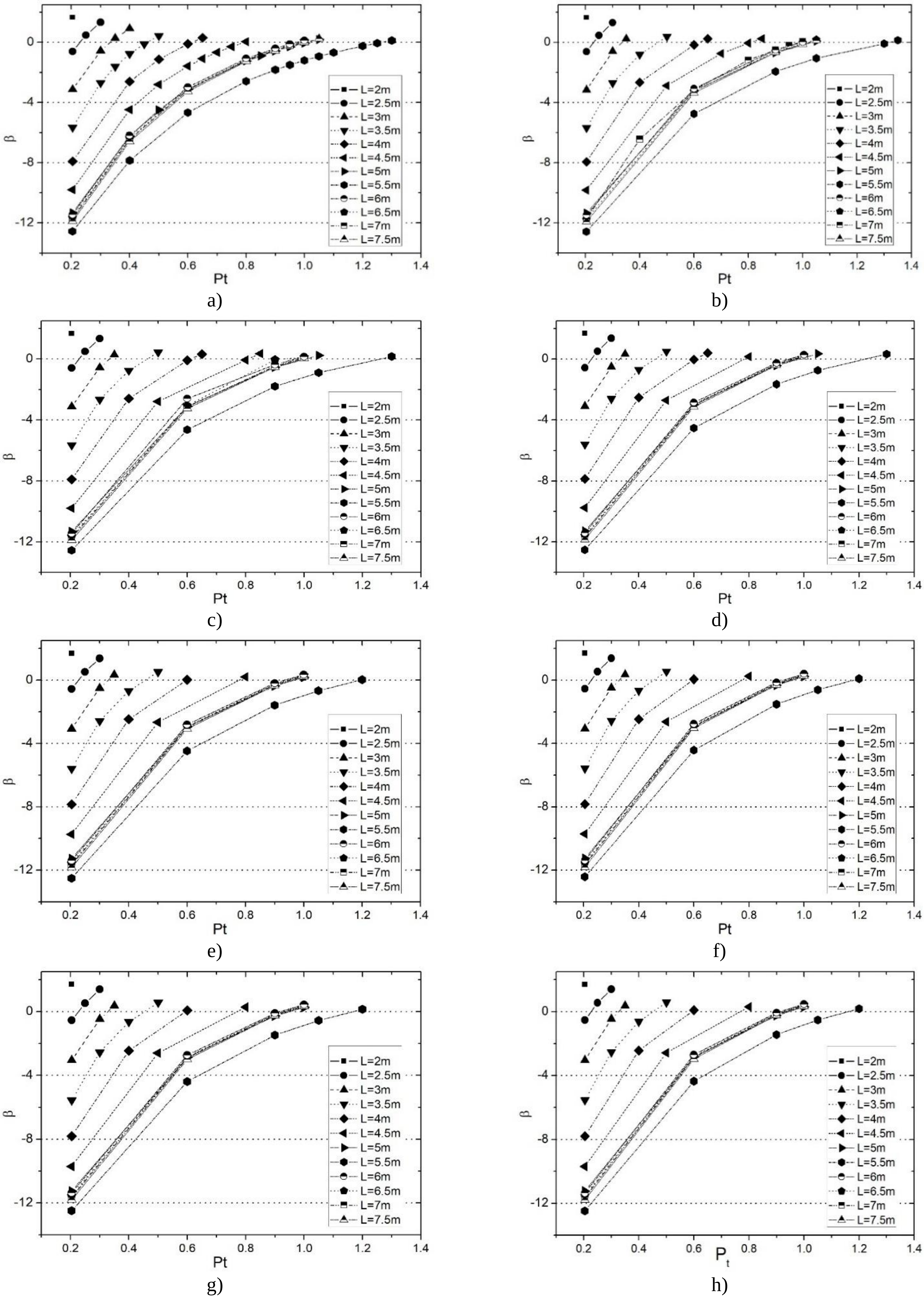

Fig. 1 Percentage of Tension Steel (P_t) vs Reliability Index (β) – a) Balaji *et al*. (2016), b) M20-IS456:2000, c) M25-IS456:2000, d) M30-IS456:2000, e) M35-IS456:2000, f) M40-IS456:2000, g) M45-IS456:2000, h) M50-IS456:2000

Table 3 Probability of failure (p_f) for limit state of flexure (As per Balaji *et al.* 2016)

Length of Beam (m)	Percentage of Tension Steel (p_t)	β	p_f
2	0.204	1.665	4.8460 x 10^{-2}
2.5	0.25	0.484	3.1561 x 10^{-1}
3	0.35	0.272	3.9358 x 10^{-1}
3.5	0.5	0.426	3.3724 x 10^{-1}
4	0.65	0.300	3.8209 x 10^{-1}
4.5	0.8	0.041	4.8405 x 10^{-1}
5	1.05	0.190	4.2465 x 10^{-1}
5.5	1.3	0.124	4.5224 x 10^{-1}
6	1	0.135	4.4828 x 10^{-1}
6.5	1	0.085	4.6812 x 10^{-1}
7	1	0.049	4.8405 x 10^{-1}
7.5	1.05	0.237	4.0905 x 10^{-1}

positive, indicating that the member will be able to offer some resistance to the action on it. If β is negative, indicating failure, if β is equal to zero, it means action is equal to reaction (capacity is equal to demand) and if β is positive, indicating safety. Therefore, reliability index values are indicating that the beams are failing or on the verge of being safe. Table 3 gives an idea about the probability of failure (p_f) corresponding to the reliability index values obtained when the beams are safe along with percentage of steel required as per the statistical data of Balaji *et al.*, 2016.

From Table 3 it is observed that the probability of failure is varying from 10^{-1} to 10^{-2}. Generally, the range of probability of failure as per many literatures (Delgado *et al.* 2000 and Haldar and Mahadevan 2000) is 10^{-3} to 10^{-5}, same is considered for Eurocode beam design specifications (Delgado *et al.* 2000). Thus, even though the beams are safe, but they are verge of failure. It can also be observed that the minimum percentage of steel reinforcement criteria as per IS456:2000 is not satisfied for the average loading criteria.

3.2.1 Important random variables for limit state of flexure

After obtaining the reliability index for the limit state of flexure for different lengths and percentage of tension steel in the safe region, it is to be seen which random variable is the most significant. Direction cosines obtained in the FORM indicate the importance of the random variables used in the limit state equation. The direction cosines given here are for statistical data as per Balaji *et al.* (2016) and IS456:2000. The direction cosines obtained are positive and negative values. Positive value indicates that the parameter is resistance variable and negative value indicates that it is a load variable. Therefore, the direction cosine values are squared and represented in the form of bar chart to know the influence of each parameter in the limit state equation.

From Fig. 2 (a)-(h) it can be seen that live load is the most significant random variable and is followed by grade of steel. Grade of concrete and dead load are also important parameters but from the study it is observed that they are not as significant in case of limit state of flexure of beam. Thus, grade of concrete and dead load can be taken as deterministic values. For the safe beams only, the grade of concrete and dead load are taken as their mean values and reliability analysis is carried out and the results obtained are shown in the graph (Fig. 3).

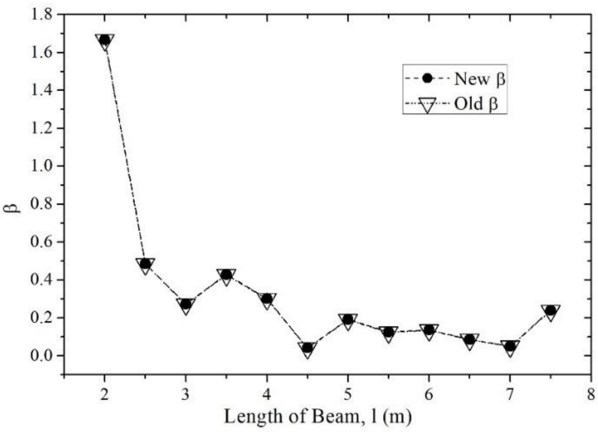

Fig. 3 Comparison of reliability indices for flexure

In Fig. 3, a comparison is shown between the reliability indices obtained when the variables were four (f_y, f_c, w_l, and w_d) and are represented by old β, and when the variables were reduced to two (f_y and w_l) keeping f_c and w_d as deterministic and represented by new β. It can be seen that there is no change in reliability indices even if the variables are reduced. From the Fig. 3, it can be concluded that the grade of concrete and dead load has least effect on the reliability index for limit state of flexure of beam.

3.3 Reliability Analysis for Limit State of Shear

Failure function for limit state of collapse with respect to flexure using Eqs. (3), (4), (5) and (6) is given by Eq. (16),

$$g(x) = \tau_c bd + \frac{f_y A_{sv} d}{s_v} - \frac{(w_l + w_d)}{2} l \qquad (16)$$

For the limit state of shear, it is assumed that the stirrups

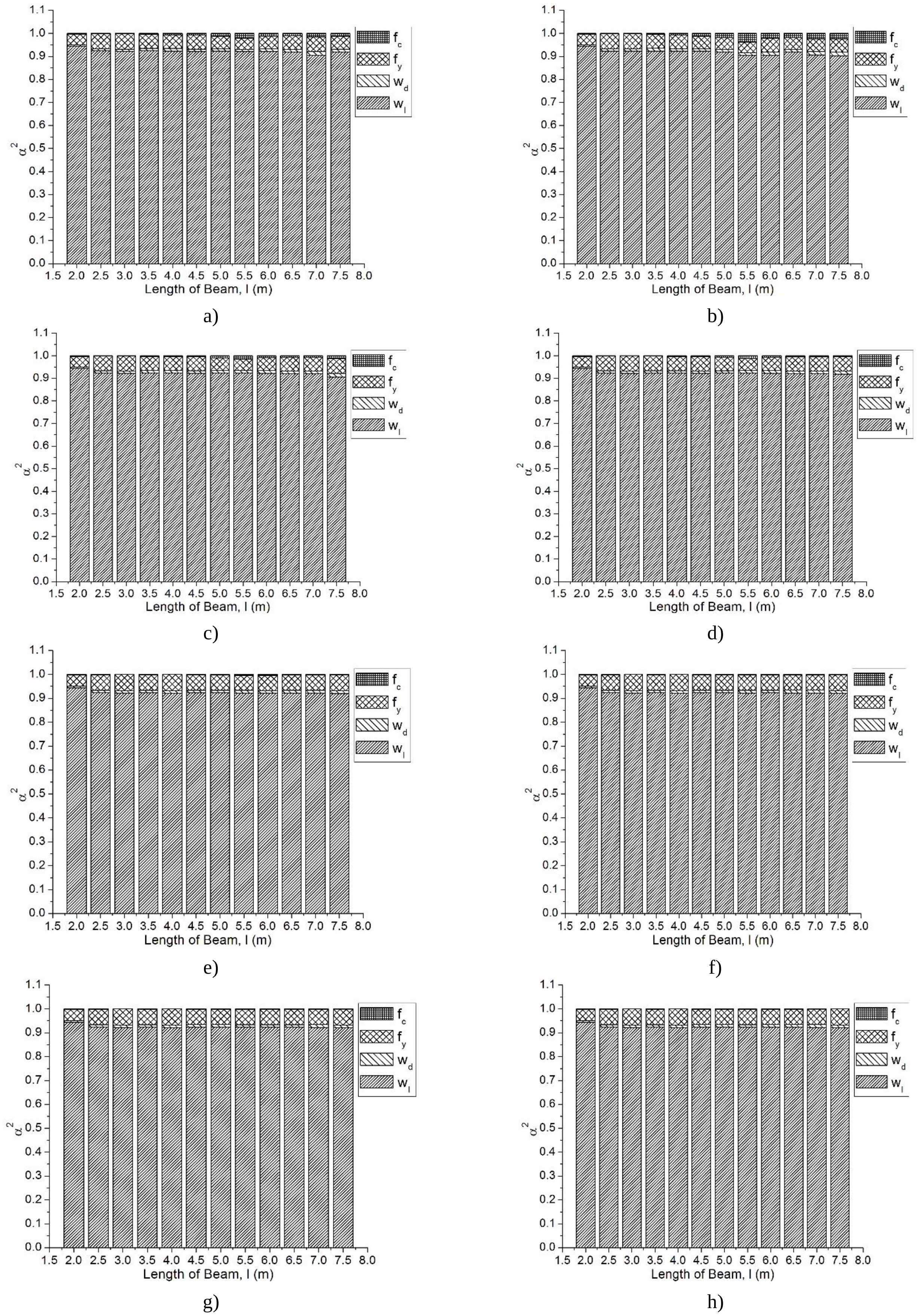

Fig. 2 Plot for Squared Direction Cosine values vs. Length of Beam for limit state of flexure - a) Balaji *et al*., 2016, b) M20-IS456:2000, c) M25-IS456:2000, d) M30-IS456:2000, e) M35-IS456:2000, f) M40-IS456:2000, g) M45-IS456:2000, h) M50-IS456:2000

Table 4 Reliability index for limit state of shear

Length of Beam (m)	Percentage of Tension Steel (p_t)	β	p_f
2	0.204	4.584	2.3249×10^{-6}
2.5	0.204	3.708	1.0780×10^{-4}
3	0.204	3.000	1.3500×10^{-3}
3.5	0.204	2.391	8.4200×10^{-3}
4	0.204	1.840	3.2880×10^{-2}
4.5	0.204	1.325	9.3420×10^{-2}
5	0.204	0.827	2.0611×10^{-1}
5.5	0.204	0.338	3.1193×10^{-1}
6	0.204	0.593	2.7760×10^{-1}
6.5	0.3	0.093	4.6414×10^{-1}
7	0.3	0.058	4.8006×10^{-1}
7.5	0.3	0.004	4.9800×10^{-1}

are 2 legged and 6mm diameter. The spacing of stirrups is as per the provisions of IS 456:2000. The maximum spacing of stirrups is 0.75d and in no case it is exceeding 300 mm. The statistical data is used as per Balaji *et al*. (2016) only, since the grade of concrete is not present in the limit state equation.

From Table 4 it can be clearly seen that for limit state of shear the beam as per the provisions of IS456:2000 is safe for length upto 6m, but as the length is increased the minimum percentage of tension steel is not sufficient. Slight increase in the percentage of tension steel, is bringing the limit state of shear in the safe region. Percentage of tension steel is given in Table 4. If that percentage of tension steel is increased then the β value will also be increasing for that particular length of beam. For length upto 3.5m the probability of failure is quite acceptable but as length is increased the beams are on the verge of failure.

3.3.1 Important random variables for limit state of shear

After obtaining the minimum reliability index for the limit state of shear for different lengths and percentage of tension steel in the safe region, it is to be seen which random variable is the most significant. Direction cosines obtained in the FORM indicate the importance of the random variables used in the limit state equation.

From Fig. 4, in case of limit state of shear, live load is the most significant random variable and is followed by grade of steel. Dead load is also an important parameter but from the study it is observed that it is not as significant in case of limit state of shear of beam. Thus, dead load can be taken as deterministic value. For the safe beams only, dead load is taken as the mean values and reliability analysis is carried out and the results obtained are shown in the graph below.

In Fig. 5, a comparison is shown between the reliability indices obtained when the variables were three (f_y, w_l, and w_d) and are represented by old β, and when the variables were reduced to two (f_y and w_l) keeping w_d as deterministic and represented by new β. It can be seen that there is no change in reliability indices even if the variables are reduced. From the Fig. 5, it can be concluded that dead load has the least effect on the reliability index for limit state of shear of beam.

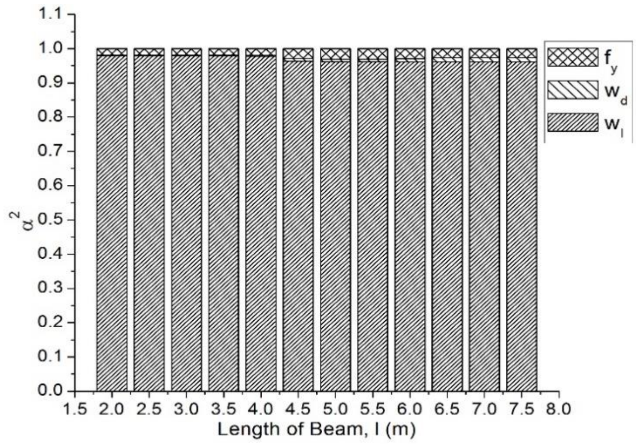

Fig. 4 Plot for Squared Direction Cosine values vs. Length of Beam for limit state of Shear (Balaji *et al*. 2016)

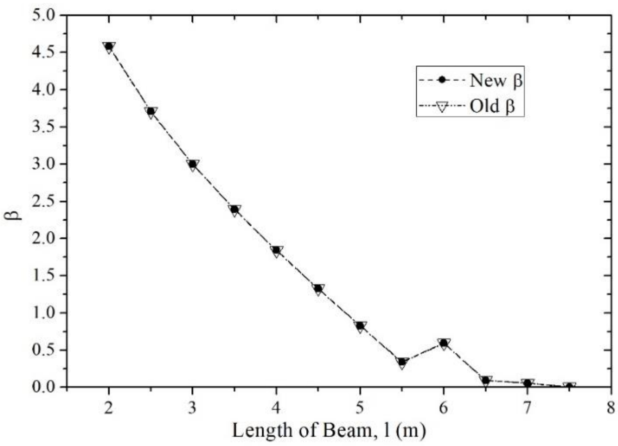

Fig. 5 Comparison of reliability indices for shear

4. Results

For limit state of flexure following results are summarized,

- For L = 2m, the beam is safe for given minimum area of tension steel in limit state of flexure. But for lengths above 2m the beam is failing for minimum percentage of steel.
- The minimum area of tension steel as per IS456:2000 is not sufficient for the limit state of flexure for average loading (Dead load and Live Load).
- It is also observed that change in dimensions (minimum criteria as per IS456:2000 and Jain and Jain 2002, maintaining b/D = 0.5 to 0.67, cover = 20mm and minimum width criteria) of the beam affects on the value of reliability index. For example, L = 5.5m (b = 150mm and d = 280mm) the percentage of tension steel at which the beam is safe is 1.3 percent, whereas for L = 6m (b = 200mm and d = 300mm) the beam is safe at 1 percent.
- The above observations made are almost similar for statistical data of Balaji *et al*. (2016) and IS456:2000. For higher grade of concrete (M50) the reliability index values are higher when compared with lower grade (M20) for same length of beam and percentage of tension steel.
- The probability of failure calculated is high (10^{-1} to 10^{-2}) when compared with Eurocode (10^{-5}) (Delgado *et al*. 2000)
- It is observed that grade of concrete and dead load are not very significant parameters as compared to grade of steel and live load, even if the deterministic values of grade of concrete and dead load are used, there is no change in the values of reliability indices.

For limit state of shear following results are summarized,

- For beams upto 3.5m, the minimum tension steel provided is sufficient and probability of failure is also less and within the range of probability of failures in comparison with Delgado *et al*. (2000).
- For beams having length more than 3.5m, the probability of failure is ranging from 10^{-1} to 10^{-2}, which is indicating that failure risk is high when compared with Eurocode (Delgado *et al*. 2000) having probability of failure ranging from 10^{-4} to 10^{-5}.
- It is observed that, dead load acting on the beam is not very significant for limit state of shear, even if the deterministic value is used, there is no change in the reliability indices.

5. Conclusions

Probabilistic analysis method was applied to Indian standard code for plain and reinforced concrete IS456:2000 for checking the probability of failure of singly reinforced simply supported beam. It can be concluded that the minimum tension steel for beam design as per the provisions of IS 456:2000, is not sufficient when limit state of flexure and shear are taken into account for designing. Whereas dead load and grade of concrete are not very significant factors while designing as compared to grade of steel and live load. The probability of failure calculated for the beams in limit state of collapse is high as compared with the Eurocode standards. The reliability index values obtained for beams are highly fluctuating. Thus, it is to be noted that the Indian standard IS456:2000 is lacking in the application of probabilistic concept. The target reliability levels should be set for the design of beams in accordance with different standards and guidelines. This will lead to a more safe as well as bring uniformity in the design specifications.

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