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A multi-objective decision making model based on TLBO for the time – cost trade-off problems

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Abstract. In a project schedule, it is possible to reduce the time required to complete a project by allocating extra resources for critical activities. However, accelerating a project causes additional expense. This issue is addressed by finding optimal set of time-cost alternatives and is known as the time-cost trade-off problem in the literature. The aim of this study is to identify the optimal set of time-cost alternatives using a multiobjective teaching-learning-based optimization (TLBO) algorithm integrated with the non-dominated sorting concept and is applied to successfully optimize the projects ranging from a small to medium large projects. Numerical simulations indicate that the utilized model searches and identifies optimal / near optimal trade-offs between project time and cost in construction engineering and management. Therefore, it is concluded that the developed TLBO-based multiobjective approach offers satisfactorily solutions for time–cost trade-off optimization problems.

Keywords: critical path method (CPM); multi objective optimization; meta-heuristic algorithm; time cost trade-off problem (TCTP); construction management

1. Introduction

Being in a highly competitive sector, construction project professionals are always kept on their toes to minimize the project time, cost and other resources, which affects their profitability and margins. Therefore, they try to identify the best balance between the potentially conflicting objectives. In the field of construction management, optimization is a very useful tool to meet the desired objectives under the given constraints. Through optimization, it is possible to increase the productivity of different components of project. Importance of the optimization in construction project was noticed several decades and was used for finding the ideal plan and schedule to complete a project. Since then, solution for time-cost trade-off problem (TCTP) has been found using various optimization processes proposed. These processes involved-exact, heuristic and metaheuristic algorithms. However, recently, usage of the optimization algorithm inspired from the natural phenomena had become popular among researchers. Meyer and Shaffer (1963) has unraveled time-cost trade-off problem (TCTP) considering both linear and discrete relationship between time and cost by utilizing mixed integer programming. However, integer programming requires more computational effort when the numbers of options for activity increases. Trade-off during the project planning is not restricted to time and cost. So, variants of time-cost trade-off problem are also analyzed. Babu and Suresh (1996), Khang and Myint (1999), Tareghian and Taheri (2006), Zhang and Xing (2010), Kimet *et al*. (2012), Mungle *et al.* (2013) and Monghasemi *et al*.(2015) added quality to TCT problem and solved the time cost-quality trade-off problems. TCT problem is solved by assuming availability of infinite resource. If available resources are limited, then the problem becomes a multi-mode resource constrained project scheduling. Keeping the availability of resources in mind, Hegazy (1999), Liu and Wang (2008), Ghoddousi *et al*. (2013), Afruzi *et al.* (2014) and Rostami *et al*. (2014) solved TCTP with limited resource.

Sönmez and Bettemir (2012) developed a hybrid strategy based on genetic algorithm (GA), simulated annealing and quantum simulated annealing techniques for TCT optimization problem. Being first introduced by Colorni *et al.* (1994), Ng and Zhang (2008) used an evolutionary-based optimization algorithm known as the ant colony (ACO) to analyze the multi-objective TCT problem. They concluded that ACO can solve TCT problem with less computational effort. Aminbakhsh and Sönmez (2016) introduced a discrete particle swarm optimization (DPSO) to solve the large-scale discrete TCT problem (DTCTP). Their computational experiment results indicate that the introduced new method outperforms the methods previously proposed, both in terms of the quality of the solution and time required for completion, particularly for medium and large-scale problems.

Toğan and Eirgash (2019) applied MAWA-TLBO algorithm to obtain the optimum solutions for the small 7 and 18 activities with three and five modes problems. For the small 7 and 18 activities with three modes, the MAWA-TLBO was able to achieve Pareto front solutions with an

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average deviation of %0 from the optimal costs. Similarly, 18 activity problem with five modes could provide Pareto front solutions with an average deviation of %0.4 from the optimal costs. On the other hand, it is observed that the quality of the obtained solutions for 18 activities with five modes and large example problem of 63 activities slightly deteriorate as they are exposed to larger daily indirect costs as well as with mode increments. More specifically, the diversity in population can't be preserved and staging to local optima because of the MAWA's drawback.

Bettemir and Birgönül (2016) proposed fast, simple and optimum converging network analysis algorithm inspired by minimum cost-slope method, for the solution of discrete TCT problem. Their study has shown that the minimum cost slope method provides optimum solution for continuous TCT problems. However, the algorithm is not suitable for discrete TCT problems because- discrete crashing options prevent the formation of linear cost functions.

Kotb *et al.* (2016) have described that a project is a series of complex and connected activities with a singular purpose of being completed within a specific time frame, budget, quality and specification. That a project must be completed within schedule, budget and according to specification denotes that the project is constrained by all of these parameters hence an effective control is needed in place to meet proposed planned time and budget for purposes of accomplishing the stated objectives.

Eirgash, M.A (2018) has observed that the proposed sole MAWA-TLBO algorithm is not able to find out the optimum solutions for the 18-activity and a more complex 63-activity problems. Thereby, in the present study, a more promising non-dominating sorting approach is applied to further investigate the exploration capacity of the proposed algorithm.

Maksym *et al.* (2019) has presented modern and an efficient optimization algorithm called Jaya for the optimum mass of braced dome structures with natural frequency constraints. The Jaya algorithm has been programmed in MATLAB to optimize braced dome. The finding result shows that the utilized algorithm is an effective tool for detecting the optimum design of structure with frequency constraints.

Crashed time and cost according to Biswas, *et al*. (2016) are conflicting factors, in that the reduction of one increases the other.

Hasan, *et al.* (2017) have utilized time-cost and quality of project network activities determined by some experts through fuzzy theory and linguistic variables and a novel Genetic Algorithm; Super Genetic Algorithm (SGA) is introduced to solve the problem. A new algorithm is proposed to calculating the project network paths which is very useful for complex project networks and a new method is applied to ranking fuzzy numbers.

By reviewing the technical literature, it can be recognized that most preferred metaheuristic algorithms used to solve DTCTP problems are GA, ACO, PSO and improved or hybridized version of them. However, many other new optimization methods have been developed and new methods continue to be invented. They were applied to the problems encountered in many engineering fields in order to validate the developed optimization algorithms. One of the recently developed optimization algorithm is Teaching-Learning Based Optimization (TLBO) built up by Rao *et al*. (2011). TLBO stimulate the inherent fact, which mirrors teaching learning process in a class between the educator and the students (learners). Rao *et al.* (2011) have demonstrated that the TLBO algorithm is more successful and effective than many other optimization methods. TLBO algorithm has been effectively exerted to numerous engineering optimization problems. Among them, TLBO algorithm has been effectively utilized for electric power generators under multiple constraints, for example, energy cost, emission, electrical energy misfortunes, voltage deviations and so forth (Azizipanah-Abarghooee *et al.* 2012, Niknam *et al.* 2012a, 2012b). It has also been used for thermoelectric cooler by Rao and Patel (2013). It had been applied to some structural engineering problems, i.e., truss frameworks, I-beams, grillage structures are done underweight obeying stress, deflection and frequency constraints by (Toğan 2012, Toğan 2013, Dede 2013, Dede and Toğan 2015). In addition to these applications, Teaching-learning-based optimization (TLBO) was applied to solve multi-objective optimization problems related to robotic arms, mechanical system, structural system, etc. (Rao and Patel 2012a, 2012b). However, based on our knowledge, it might be stated that the performance of the TLBO has not been tested Time cost trade-off problems (TCTP) problems since its inception.

In the literature, small TCT problems ranging from 7 to 18 activities (Afshar *et al.* 2009, Elbeltagi *et al*. 2007, Eshtehardian *et al*. 2008, Feng *et al*. 1997, Hegazy 1999, Ng and Zhang 2008, Xiong and Kuang 2008, Zhang and Xing 2010) and medium-larger TCT problem consisting 63 activities (Sönmez and Bettemir 2012, Aminbakhsh and Sönmez 2016) were investigated to assess the efficiency of the suggested metaheuristic optimization algorithms.

Due to the some drawbacks of MAWA, today instead of MAWA approach an effective approach known as non-dominating sorting (NDS) approach (Deb 2001) is broadly being preferred for solving the mentioned TCTP problems. This approach seek a satisfactory solution from the non-dominated solutions depending on the experience and knowledge of decision-makers.

The main objective of this paper is to fill the gap in the multiobjective DTCTP literature by developing Teaching-Learning Based Optimization (TLBO) algorithm combining with the non-dominating sorting approach that can achieve successful Pareto front solutions. The paper also aims to contribute to the efficiency and performance of the proposed TLBO-based multiobjective optimization model which is being adopted for the first time in construction management filed. Also, the applied algorithm could provide a high number of Pareto front solutions with minimal deviations from the optimal solutions. Moreover, NDS-TLBO is utilized for unraveling the more complex medium scale project of 63 as well as 630 activities for the first time in the literature which was practiced by few of the non-dominating sorting methods previously. As it is obvious that, non-dominating sorting approach is superior to the modified adaptive weight approach traditional approach. Findings indicate that the developed model provides an attractive alternative to solve construction management time–cost trade-off optimization problem.

The presentation of this study is as follows: basic formulations for the time–cost optimization are shown first. Subsequently, the non-dominating sorting approach along with characteristics of the utilized teaching learning based optimizers to solve the time-cost trade-off problems (TCTP) for construction projects is going to be presented. To demonstrate the NDS-TLBO efficiency in practical projects, benchmark TCT optimization problems are then examined and finally, numerical results.

2. Time–Cost optimization

The main goal of a discrete TCT optimization problem is to determine a set of time-cost alternatives which provide an optimal balance between the time and cost for project scheduling under the specific conditions. Since both of direct and indirect costs are tried to optimize simultaneously in this optimization problem, it can be expressed mathematically as follows

$$\text{minimize } y \equiv (D, TC) \tag{1}$$

where y is the bi-objective function, D and TC, respectively, are the total duration and the total cost of the project. The project duration D is calculated by using critical path method depending on the defined activity relationships for that project. The total cost of a project consists of two parts: direct cost and indirect cost. Direct cost is determined by the sum of direct cost of all activities within a project network. On the other hand, indirect cost depends heavily upon the project duration, i.e., the longer the duration, the higher the indirect cost. Subsequently, Eqns. (2)–(4) are put forward to compute the total cost of a project.

$$DC = \sum_{i=0}^{n+1} dc_i^{(k)}\ x_i^{(k)} \tag{2}$$

$$IC = D + ICR \tag{3}$$

$$TC = DC + IC \tag{4}$$

where DC = total direct cost of a project; IC = total indirect cost of a project; TC = total cost of a project; $dc_i^{(k)}\ x_i^{(k)}$ = direct cost of activity i under the kth option; and ICR = indirect cost rate of a project. If $x_i^{(k)}=1$, then activity i performs the kth option, while $x_i^{(k)} = 0$ means not.

3. Non-dominated sorting TLBO algorithm for multiobjective optimization

The domination concept defined as: design A dominates design B if it is better in at least one criteria and not worse in all other objectives Deb (2001). The process of sorting designs based on dominance concept is called non-dominated sorting (NDS). At any phase in an optimization run, a population or repository of "current" designs is kept up. At each progression, every feasible design that is not dominated by some other designs in the population (or archive) is given the rank of 1. These are the just non-dominated designs in the population. At that point, these designs are adroitly expelled from the repository and the rest of the designs are judged for domination. Those that are not dominated by any of the rest of the designs are given the rank of 2. The method is repeated, re-positioning the rest of the designs after eliminating non-dominated designs, to build up ranks 3, 4 and so on. As the run progresses, new designs will dominate and replace other designs on a series of local Pareto fronts. The final result will regularly be a combination of variables that are not overwhelmed by any other designs and converge towards the Pareto front. From this bunch of designs, one can pick up the design that best suits the present requirements or those that move towards hunting. To maintain and preserve a diverse population in order to prevent premature convergence, crowded comparison operator is used. If the algorithm is already able to locate diverse solutions along the front, so no need to use a diversifier. Crowding distance metric is used to determine the superiority of individuals if the individuals have same rank. Therefore, calculated crowding distances are not valid for the next or previous front.

NDS-TLBO algorithm is suitable for unraveling multi-objective optimization problems and to keep up a set of diverse solutions. The NDS-TLBO algorithm comprises of teacher and learner stages like the TLBO algorithm. In any case, in order to handle objectives effective and efficiently, TLBO algorithm is associated with non-dominated sorting approach and crowding distance computation mechanism proposed by Deb (2001). The teacher and learner stages guarantee the great investigation and hunting of the sparse region while non-dominated sorting approach verifies that the determination procedure is dependably towards the better solutions and in each iteration, the population is pushed towards the Pareto front. The crowding distance assignment mechanism ensures the selection of teacher from a sparse region of the exploration space in this manner turning away any shot of untimely convergence of the algorithm at local optima.

In this approach, the learners are updated based on the teacher as well as learner phases of the TLBO algorithm. However, in the optimization of single objective it is easy to make up the mind which solution is better than the other depending upon the objective function value. But in the existence of various objectives determining the best solution from a set of solutions is not a straightforward job.

Toward the starting, an initial population is arbitrarily produced with P number of solutions (learners). This initial population is then sorted and ranked depending on the non-dominance concept. The learner with the highest rank (rank = 1) is picked up as the teacher of the class. In case, there exists more than one learner in rank=1 then the learner with the highest value of crowding distance is selected as the teacher of the class. However, in the case of the learners have the same crowding distance metric, any of learners becomes teacher. Once the teacher is chosen the mean of the learners is computed and the learners are updated according to the teacher phase of the TLBO algorithm. The recombination of the updated learners with the initial population to acquire a set of *2P* solutions happen after the teacher phase. According to the non-dominating sorting concept, the learners are re-sorted and re-ranked and the crowding distance value is calculated for each learner. Considering the new ranking and crowding distance value, *P* number of best learners are chosen. Depending upon the learner phase of the TLBO algorithm these learners are further updated.

4. Teaching-Learning Based Optimization (TLBO)

Teaching-learning-based optimization (TLBO) is a population-based algorithm which simulates the teaching-learning process of the classroom. This algorithm requires only the common control parameters such as the population size and the number of generations and does not require any algorithm-specific control parameters. All evolutionary and swarm intelligence based optimization algorithms require common control parameters like population size, number of generations, elite size, etc. Besides the common control parameters, different algorithms require their own algorithm-specific parameters. For example, GA uses mutation probability and crossover probability and selection operator; PSO uses inertia weight and social and cognitive parameters; ABC algorithm uses number of bees (scout, onlooker and employed) and limit; and NSGA-II requires crossover probability, mutation probability and distribution index. Proper tuning of these algorithm-specific parameters is a very crucial factor which affects the performance of the algorithms. The improper tuning of algorithm-specific parameters either increases the computational effort or yields a local optimal solution. In addition to the tuning of algorithm-specific parameters, the common control parameters also need to be tuned which further enhances the effort. Thus, there is a need to develop an algorithm which does not require any algorithm-specific parameters and teaching-learning-based optimization (TLBO) is such an algorithm. The TLBO algorithm is a teaching-learning process inspired algorithm proposed by Rao *et al*. (2011, 2012a) and Rao and Savsani (2012) based on the effect of influence of a teacher on the output of learners in a class. TLBO has emerged as one of the simple and efficient techniques for solving single-objective benchmark problems and real life application problems in which it has been empirically shown to perform well on many optimization problems (Rao *et al*. 2012a, b, Rao and Patel 2011, Rao and Kalyankar 2013, Toğan 2012). These are precisely the characteristics of TLBO that make it attractive to extend it to solve MOPs (Rao and Patel 2011, 2013, Niknam *et al*. 2012a, b, Satapathy *et al*. 2012).

The algorithm describes two basic modes of the learning: (i) through teacher (known as teacher phase) and (ii) through interaction with the other learners (known as learner phase). In this optimization algorithm, a group of learners is considered as population and different subjects offered to the learners are considered as different design variables of the optimization problem and a learner's result is analogous to the 'fitness' value of the optimization problem. The best solution in the entire population is considered as the teacher. The design variables are actually the parameters involved in the objective function of the given optimization problem and the best solution is the best value of the objective function.

4.1 Optimum solution of TCTP via NDS-TLBO algorithm

The solution of TCTP employing NDS-TLBO process is summarized in five steps as follows:

Step I: Define the number of learners (population size) in the class and the maximum number of iterations (stopping criteria) to initialize the TLBO algorithm.

Step II: Fill the initial matrix (class; *CL*) with *pn* (student or population size) number of solution vectors that contains *dn* number of randomly generated design variables (X_i) between the upper (X_i^{max}) and lower (X_i^{min}) limit of the solution range (Eq. (5)).

$$X_i^{\min} \le X_i \le X_i^{\max} \qquad i = 1, \cdots, dn \tag{5}$$

Thus, initial matrix (*CL*) can be written as:

$$CL = \begin{bmatrix} X_{1,1} & X_{1,2} & \cdots & X_{1,dn} \\ X_{2,1} & X_{2,2} & \cdots & X_{2,dn} \\ \vdots & \vdots & \vdots & \vdots \\ X_{pn-1,1} & X_{pn-1,2} & \cdots & X_{pn-1,dn} \\ X_{pn,1} & X_{pn,2} & \cdots & X_{pn,dn} \end{bmatrix} \tag{6}$$

In which each row of the matrix is a candidate solution of TCTP problem that is corresponded two objective function values associated with time (f_t ($\mathbf{X}$)) and cost (($f_c(\mathbf{X})$)).

$$f(\mathbf{X}) = \begin{bmatrix} f_t(\mathbf{X}_1),\ f_c(\mathbf{X}_1) \\ f_t(\mathbf{X}_2),\ f_c(\mathbf{X}_2) \\ \vdots \\ f_t(\mathbf{X}_{pn-1}),\ f_c(\mathbf{X}_{pn-1}) \\ f_t(\mathbf{X}_{pn}),\ f_c(\mathbf{X}_{pn}) \end{bmatrix} \tag{7}$$

Perform a non-dominated sorting on *CL*. Then calculate the crowded distance values of solutions in the front(s) and sort them. Keep the sorted solution in an external achieve.

Step III: Apply "teaching phase (t_p)" of the TLBO algorithm. Due to the fact that teacher has the best knowledge, the best learner in the class is assigned as a teacher ($\mathbf{X}_{teacher}$) of the class based on non-dominated sorting and crowding distance metric.

$$\mathbf{X}_{teacher} = \mathbf{X}_i \mid \text{in front 1 and max. crowded distance} \tag{8}$$

Then, knowledge of the teacher is used to increase the capacity of the whole class. The main aim is to increase of the mean ($\mathbf{X}_{mean}$) of the class. For that reason the equation of new students is found, according to teacher and mean of the class as seen in Eq. (9).

$$\mathbf{X}^{tp}_{new,\,i} = \mathbf{X}_{old,\,i} + \text{rand}\ (0,\,1).\ (\mathbf{X}_{teacher} - T_F.\ \mathbf{X}_{mean}) \tag{9}$$

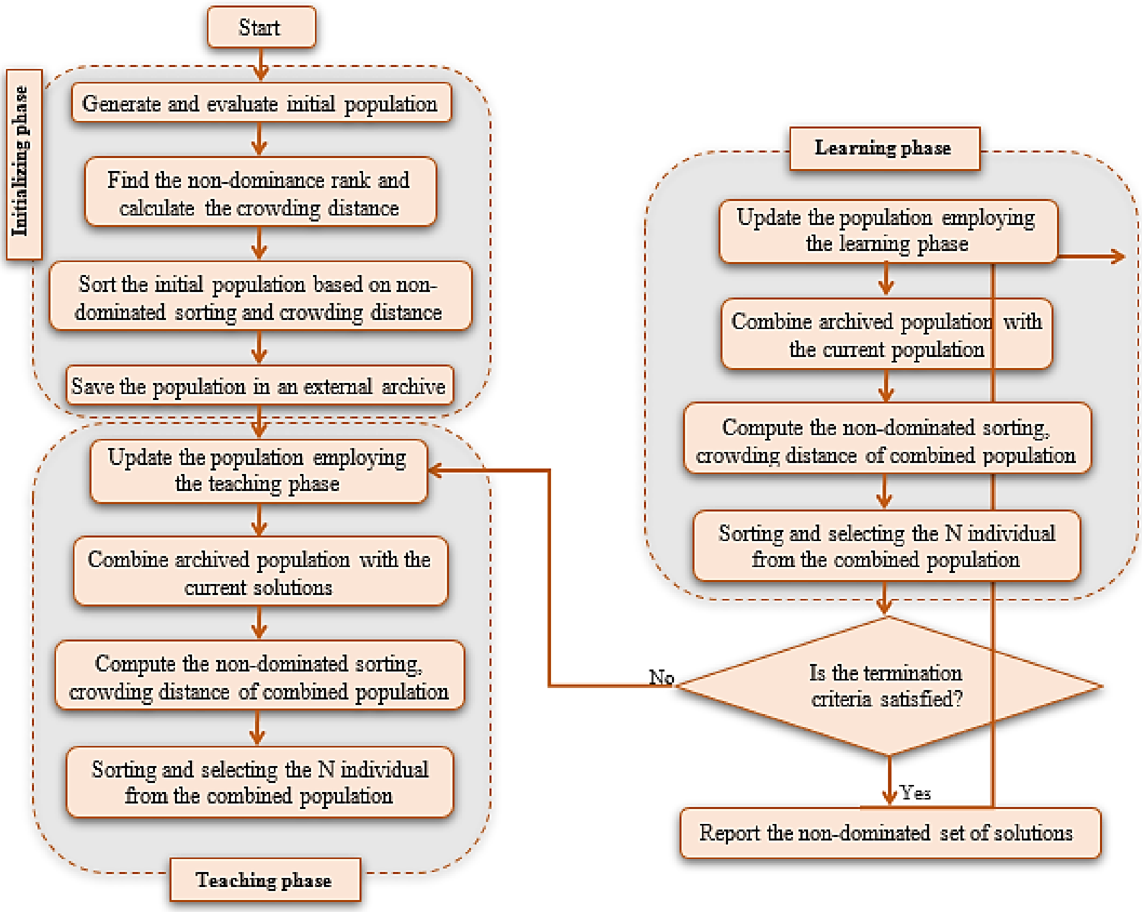

Fig. 1 Flowchart of the NDS-TLBO algorithm for TCTP

where T_F represents teaching factor defined as

$$T_F = \text{round}\,[1 + \text{rand}\,(0.1)] \quad \rightarrow \{1\text{-}2\} \tag{10}$$

and it takes a value 1 or 2 based on the uniformly distributed random numbers that are within the range [0, 1]. If the new solution ($\mathbf{X}^{tp}_{new,\,i}$) is better than the old one, the new solution is accepted.

After employing the teaching phase, combine the current population with the archived one. Perform a non-dominated sorting on the combined population. Then calculate the crowded distance values of solutions in the front(s) and sort them. Select N individual from it.

Step IV: Proceed with the "learning phase (l_p)" of the TLBO algorithm. As it is stated above, students also have an important role in the learning process by communication, interaction, investigation, etc. This interaction can be expressed as follows:

$$\mathbf{X}^{l_p}_{new,i} = \begin{cases} \mathbf{X}_{old,i} + \text{rand}(0,1)\left(\mathbf{X}_i - \mathbf{X}_j\right) \\ \text{if} \quad \mathbf{X}_i \text{ lies on a better non-dominated front than } \mathbf{X}_j \\ \mathbf{X}_{old,i} + \text{rand}(0,1)\left(\mathbf{X}_j - \mathbf{X}_i\right) \\ \text{if} \quad \mathbf{X}_j \text{ lies on a better non-dominated front than } \mathbf{X}_i \end{cases} \tag{11}$$

where $\mathbf{X}_i$ and $\boldsymbol{X}_j$ are randomly selected learners that are different from each other. If the new solution ($\mathbf{X}^{lp}_{new,\,i}$) is better, it is replaced with old one.

Combine the current population with the one that is used at the starting of the phase. Perform a non-dominated sorting on the combined population. Then calculate the crowded distance values of solutions in the front(s) and sort them. Select N individual from it.

Step V: Check the stopping criterion. This criterion usually is defined as the maximum iteration number. If the stopping is satisfied, the optimization process is terminated, otherwise the iteration process continues from the step III. The flowchart of the process can be seen in Fig. 1.

5. Numerical Examples

For performance evaluation of the NDS-TLBO method, a small-scale problem, as well as a more complex medium scale problems are evaluated. The algorithm was implemented in MATLAB (R2015a) and was carried out on a personal computer having Intel (R) Core (TM) i3 CPU 2.40 GHz and 3GB RAM. Total number of iterations was used as stopping criteria. Consecutive experimental run number is adopted as 10 for the entire instances.

5.1 Empirical example of 18-activities project

The first test problem involves an 18-activity network, details of which can be derived from Feng *et al.* (1997) incorporating the time-cost alternatives defined in Hegazy (1999). Majority of the previous research (Ng and Zhang 2008, Afshar *et al.* 2009, Zhang and Ng 2012) used this problem to evaluate the performances of their multi-

Table 1 Case study (adapted from Feng *et al*. 2000)

Activities		5 Methods of Construction - Normal to Crash									
Activity Number	Precedent Activity	Option / Mode1		Option / Mode 2		Option / Mode 3		Option/Mode4		Option/Mode5	
		Dur (day)	Direct Cost	Dur (day)	Direct Cost	Dur (day)	Direct Cost	Dur (day)	Direct Cost	Dur (day)	Direct Cost
1	-	14	2400	15	2150	16	2400	21	1500	24	1200
2	-	15	300	18	2400	20	1900	23	1500	25	1000
3	-	15	4500	22	4000	33	1800				
4	-	12	45000	16	35000	20	3200				
5	1	22	20000	24	17500	28	30000	30	10000		
6	1	14	40000	18	32000	24	15000				
7	5	9	30000	15	24000	18	18000				
8	6	14	220	15	21	16	22000	21		24	
9	6	15	300	18	240	20	200	23	208	25	120
10	2 , 6	15	450	22	400	33	180		150		100
11	7 , 8	12	450	16	350	20	320				
12	5 , 9 , 10	22	2000	24	1750	28	1500	30			
13	3	14	4000	18	3200	24	1800				
14	4 , 10	9	3000	15	2400	18	2200				
15	12	12	4500	16	3500						
16	13 , 14	20	3000	22	2000	24	1750	28	1500	30	1000
17	11,14,15	14	4000	18	3200	24	1800				1200
18	16,17	9	3000	15	2400	18	2200				1000

Table 2 Comparison of Pareto Fronts located for 18 activities problem

Duration (days)	Ng and Zhang (2008)	Afshar *et al*. (2009)	Zhang and Ng (2012)	Aminbakhsh and Sönmez (2016)	NDS-TLBO (This paper)
100	283320	283320	285400	283320	283320
101	279820	279820	282508	279820	279820
104	276320	276320	277200	276320	276320
110	271320	271270	273165	271270	271270
Pop. size	10	50	10	80	40
Num. of iterations	200	300	200	100	100
Num. of function evaluation	2000	15000	2000	8000	8040

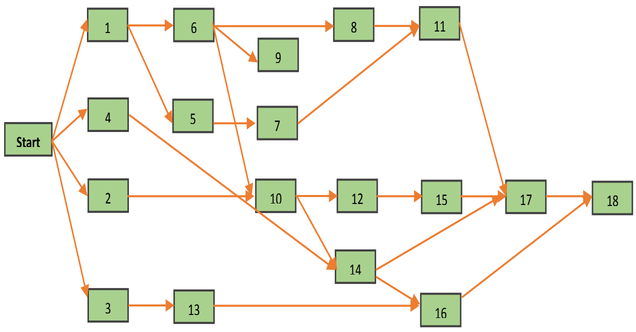

Fig. 2 Activity relationships for the model project of 18 activities

objective metaheuristic optimization models. The activity-on-node diagram for the project is presented in Fig. 2. Their associated time and cost are presented in Table 1. This problem with a total of $4.72\text{x}10^9$ possible schedules is examined with a daily indirect cost of $1,500.

Table 2 summarizes the results of the TLBO along with the performance of five previous meta-heuristic algorithms for the 18-activity problem. For 110 days, ACS-TCO of Ng and Zhang (2008) and ACS of Zhang and Ng (2012) provide a solution which costs more than the proposed TLBO's result. The Pareto front solutions reported for NA-ACO by Afshar *et al.* (2009) and for NDS- PSO by Aminbakhsh and Sönmez (2016) are identical to the results acquired by the TLBO method. However, the utilized algorithm exhibit its competency and accuracy by exploring a tiny portion [$5640/4.72\text{x}10^9$ =0.00012%] of the solution space. This reveals a remarkable reduction in number of function evaluations of the proposed algorithm compared to NA-ACO of Afshar *et al.* (2009) and NDS- PSO of Aminbakhsh and Sönmez (2016). Graphical representations for Pareto front for the current problems are given in Figs. 3 and 4. From Fig. 4 it is clear that the global optimum solutions are achieved in the 1^{st} run analysis and could explore 100 days, $283320 six times, 101days, $279820 five times, 104 day, $276320 four times and 110 days, $271270 three times. This indicates the success of the proposed algorithm. The comparison of TLBO with the contemporary methods discloses that proposed NDS-TLBO

Table 3 Options selected and solution generated for 18 activity TCTP problem with five modes

PF Sol	Proj. Time (day)	Project Cost ($)	Selected duration of the corresponding activity (days) 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	100	283320	14	25	33	20	28	14	18	24	15	15	16	22	24	18	12	30	14	9
2	101	279820	14	25	33	20	30	14	18	24	15	15	16	22	24	18	12	30	14	9
3	104	276320	14	25	33	20	30	18	18	24	15	15	16	22	24	18	12	30	14	9
4	110	271270	14	25	33	20	30	24	18	24	15	15	20	22	24	18	12	30	14	9

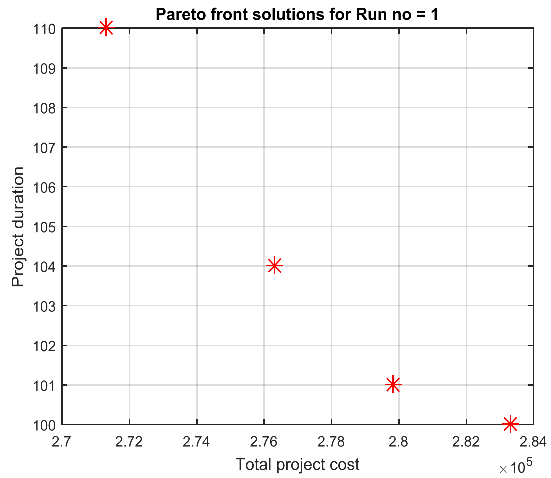

Fig. 3 Pareto optimal solutions of 18 activity problem obtained by NDS-TLBO algorithm

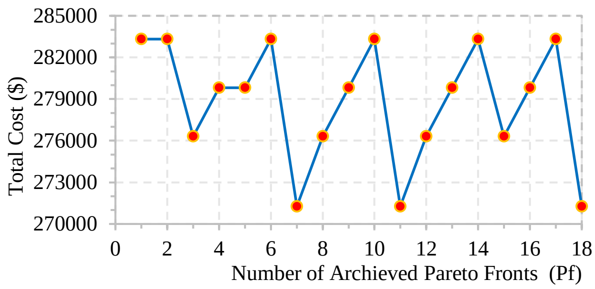

Fig. 4 Graphical representation of first run analysis of 18-activity TCTP problem with 0.3 Pareto fraction

is among the most effective algorithms for Pareto front optimization of the more complex small-scale TCTPs. The Pareto front along with selected duration of corresponding 18 activities is illustrated in Table 3.

5.2 Medium-scale test problem

A medium scale project with 63 activities taken from Bettemir (2009) is examined as a second test project to exhibit the performance of proposed NDS-TLBO. The activity-on-node diagram for the project is presented in Fig. 5 and time–cost optional modes are given in Table 4. The costs in Table 4 are given in US Dollars and the durations are given in days.

The project involves two activities with three modes, 15 activities with four modes and 46 activities with five modes. The number of total possible time–cost alternatives for the project is 1.4E+42. The project was tested under two cases: in the first case (63a), the indirect cost is taken as $2300/day, whereas it is adopted as $3500/day in the second case (63b). The optimal solutions obtained using integer programming for both of the cases were reported in Bettemir (2009) as 630 days with $5421120 as cost for 63a and 621 days with $6176170 as cost for 63b. Bettemir (2009) utilized eight metaheuristic algorithms out of which three core algorithms and five hybrid algorithms which are incorporated with the non-dominating sorting approach to solve the TCTP problem. Aminbakhsh and Sönmez (2016) have also reported the best Pareto front solutions obtained by applying the modified particle swarm optimization method for the same problem.

As previously mentioned, since 63-activity problem has not been practiced more by the researchers, the results obtained in this study by using NDS-TLBO are compared with the solutions acquired through core NDS-GA, NDS-ACO and NDS-PSO models of Bettemir's (2009) only. The results are not compared with Aminbakhsh and Sönmez's (2016) model although Aminbakhsh and Sönmez (2016) have also reported the best Pareto front solutions of the same 63-activity problem, because of fact that a cluster of predefined initial population is fed into models to accelerate the search process.

The compared results of 63a and 63b activity problems are tabulated in Table 5 and 7, respectively. In addition, Table 8 illustrates Pareto front results of ten consecutive experimental runs with corresponding average percent deviations (%APD) from the optima. Graphical representations of the Pareto front solution of the problem is given in Fig. 6 and 7. The selected duration of corresponding activities is given in Table 6.

The NDS-TLBO searched 162180 (= 180 x 450 x 2 + 180) possible different schedules, only searching a negligible portion of the solution space [162180/1.4E+42]. Population and number of iterations are adopted as 180 and 450, respectively.

5.3 Large-scale test problems 630a and 630b

To investigate the efficiency of core TLBO integrated with non-dominating sorting approach on a project with 630 activities taken from Bettemir (2009) is resolved by the proposed algorithm. In the literature, the largest project whose global optimum is obtained by metaheuristic

Table 4 Data for the 63-activity TCT problem

Activity Number	Precedent Activity	Option / Mode 1		Option / Mode 2		Option / Mode 3		Option / Mode 4		Option / Mode 5	
		Dur (days)	Cost ($)	Dur (days)	Cost ($)	Dur (days)	Cost ($)	Dur (days)	Cost ($)	Dur (days)	Cost ($)
1	-	14	3700	12	4250	10	5400	9	6250		
2	-	21	11250	18	14800	17	16200	15	19650		
3	-	24	22450	22	24900	19	27950	17	31650		
4	-	19	17800	17	19400	15	21600	-			
5	-	28	31180	26	34200	23	38250	21	41400		
6	1	44	54260	42	58450	38	63225	35	68150		
7	1	39	47600	36	50750	33	54800	30	59750		
8	2	52	62140	47	69700	44	72600	39	81750		
9	3	63	72750	59	79450	55	86250	51	91500	49	99500
10	4	57	66500	53	70250	50	75800	46	80750	41	86450
11	5	63	83100	59	89450	55	97800	50	104250	45	112400
12	6	68	75500	62	82000	58	87500	53	91800	49	96550
13	7	40	34250	37	38500	33	43950	31	48750		
14	8	33	52750	30	58450	27	63400	25	66250		
15	9	47	38140	40	41500	35	47650	32	54100		
16	9 , 10	75	94600	70	101250	66	112750	61	124500	57	132850
17	10	60	78450	55	84500	49	91250	47	94640		
18	10, 11	81	127150	73	143250	66	154600	47	161900		
19	11	36	82500	34	94800	30	101700	-			
20	12	41	48350	37	53250	34	59450	32	66800		
21	13	64	85250	60	92600	57	99800	53	107500	49	113750
22	14	58	74250	53	79100	50	86700	47	91500	42	97400
23	15	43	66450	41	69800	37	75800	33	81400	30	88450
24	16	66	72500	62	78500	58	83700	53	89350	49	96400
25	17	54	66650	50	70100	47	74800	43	79500	40	86800
26	18	84	93500	79	102500	73	111250	68	119750	62	128500
27	20	67	78500	60	86450	57	89100	56	91500	53	94750
28	21	66	85000	63	89750	60	92500	58	96800	54	100500
29	22	76	92700	71	98500	67	104600	64	109900	60	115600
30	23	34	27500	32	29800	29	31750	27	33800	26	36200
31	19, 25	96	145000	89	154800	83	168650	77	179500	72	189100
32	26	43	43150	40	48300	37	51450	35	54600	33	61450
33	26	52	61250	49	64350	44	68750	41	74500	38	79500
34	28, 30	74	89250	71	93800	66	99750	62	105100	57	114250
35	24, 27, 29	138	183000	126	201500	115	238000	103	283750	98	297500
36	24	54	47500	49	50750	42	56800	38	62750	33	68250
37	31	34	22500	32	24100	29	26750	27	29800	24	31600
38	32	51	61250	47	65800	44	71250	41	76500	38	80400
39	33	67	81150	61	87600	57	92100	52	97450	49	102800
40	34	41	45250	39	48400	36	51200	33	54700	31	58200
41	35	37	17500	31	21200	27	26850	23	32300		
42	36	44	36400	41	39750	38	42800	32	48300	30	50250
43	36	75	66800	69	71200	63	76400	59	81300	54	86200
44	37	82	102750	76	109500	70	127000	66	136800	63	146000
45	39	59	847500	55	91400	51	101300	47	126500	43	142750
46	39	66	94250	63	99500	59	108250	55	118500	50	136000
47	40	54	73500	51	78500	47	83600	44	88700	41	93400
48	42	41	36750	39	39800	37	43800	34	48500	31	53950
49	38, 41, 44	173	267500	159	289700	147	312000	138	352500	121	397750
50	45	101	47800	74	61300	63	76800	49	91500		
51	46	83	84600	77	93650	72	98500	65	104600	61	113200
52	47	31	23150	28	27600	26	29800	24	32750	21	35200
53	43, 48	39	31500	36	34250	33	37800	29	41250	26	44600
54	49	23	16500	22	17800	21	19750	20	21200	18	24300
55	52, 53	29	23400	27	25250	26	26900	24	29400	22	32500
56	50, 53	38	41250	35	44650	33	47800	31	51400	29	55450
57	51, 54	41	37800	38	41250	35	45600	32	49750	30	53400
58	52	24	12500	22	13600	20	15250	18	16800	16	19450
59	55	27	34600	24	37500	22	41250	19	46750	17	50750
60	56	31	28500	29	30500	27	33250	25	38000	21	43800
61	56, 57	29	22500	27	24750	25	27250	22	29800	20	33500
62	60	25	38750	23	41200	21	44750	19	49800	17	51100
63	61	27	9500	26	9700	25	10100	24	10800	22	12700

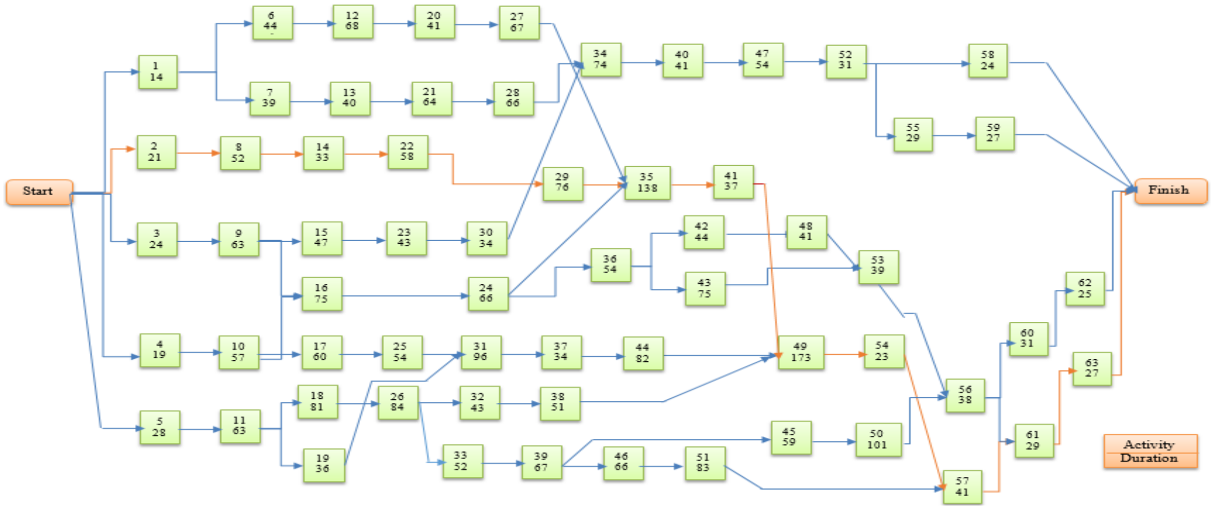

Fig. 5 Network representation of the 63 activity network

Table 6 Option selected and solution generated for 63a activity TCTP problem of NDS approach (Indirect cost=$2300)

P-F Sol.	Project time (days)	Project total cost ($)	Selected duration of the corresponding activity (days)															
			1	2	3	4	5	6	------------------				57	58	59	60	61	63
1	630	6427770	12	18	24	19	28	44	39	52	63	57	63	68	40	33	47	75
			60	81	36	41	64	53	43	66	50	84	67	66	76	34	96	43
			52	74	138	54	29	51	67	41	23	44	75	82	55	66	54	41
			147	101	83	31	39	18	29	38	30	24	27	31	20	25	22	

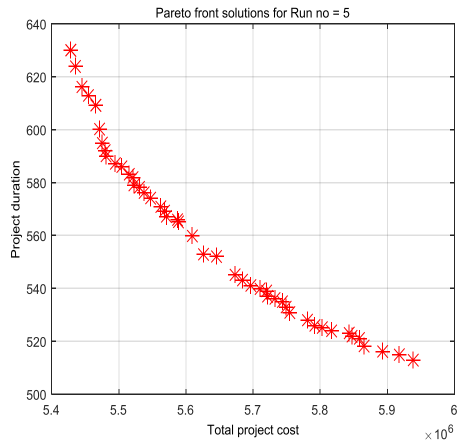

Fig. 6 Pareto front solutions of the solved problem using NDS-TLBO algorithm

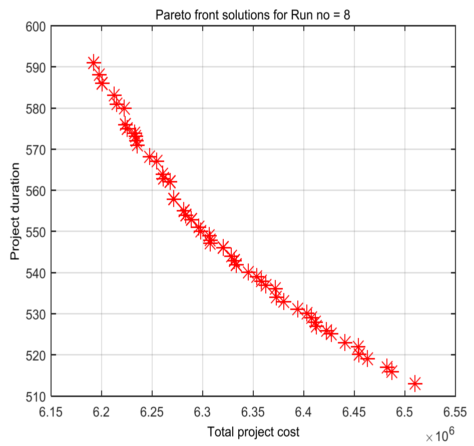

Fig. 7 Pareto front solutions of the solved problem using NDS-TLBO algorithm

Table 5 Analysis results of 63a-Activity project for the Case 1 (daily indirect cost of $2300)

Sr. No	Bettemir (2009)						(This paper)	
	NDS-GA		NDS-ACO		NDS-PSO		NDS-TLBO	
	Dur	Cost	Dur	Cost	Dur	Cost	Dur	Cost
1	641	5704200	635	5490120	637	5421620	630	5428870
2	661	5712485	653	5494410	644	5428920	630	5428120
3	650	5722260	638	5491180	651	5439620	630	5427770
4	653	5713450	657	5491620	634	5422920	630	5428120
5	645	5699650	644	5494920	651	5440570	630	5428920
6	639	5684295	626	5486630	633	5421320	637	5428220
7	640	5695655	664	5495080	633	5421320	633	5428870
8	621	5707600	661	5490350	633	5421620	628	5428170
9	641	5693015	643	5490680	633	5421320	633	5428470
10	623	5690790	635	5492210	633	5421320	633	5428720
Pop. size	500		500		500		180	
Num. of iterations	500		500		500		450	
NFE	250000		250000		250000		162180	

algorithm was only 63-activity project. In the current study, by duplicating the 63-Activity project 9 times, 630-Activity project is formed and analyzed by the proposed algorithm. Global optimum of the case 1 is $54211200 and case 2 is $61761700.

The compared mean values of 10 run of 630a and 630b problems are presented in Tables 10 and 12, respectively. Also %APD of the case 1 and case 2 with the previous models are tabulated in table 13. In addition, Table 9 and 11 illustrate Pareto front results of ten consecutive

Table 7 Analysis results of 63b-Activity project for the Case 2 (daily indirect cost of $3500)

Sr. No	Bettemir (2009)						(This paper)	
	NDS-GA		NDS-ACO		NDS-PSO		NDS-TLBO	
	Dur	Cost	Dur	Cost	Dur	Cost	Dur	Cost
1	617	6462580	631	6219220	644	6201720	612	6192140
2	651	6411540	632	6205850	629	6217470	617	6184820
3	647	6442440	626	6234520	644	6210170	590	6188690
4	639	6420500	640	6223830	648	6218170	588	6195910
5	648	6447900	617	6231440	649	6216020	591	6191490
6	627	6433810	627	6197070	647	6207870	586	6196840
7	618	6439240	604	6247850	651	6216220	592	6189140
8	623	6449790	635	6231860	649	6215420	589	6199870
9	630	6443805	623	6198650	645	6208920	617	6187390
10	629	6450065	651	6262830	642	6198520	616	6190570
Pop. size	500		500		500		180	
Num. of iterations	500		500		500		450	
Num. of function evaluation	250000		250000		250000		162180	

Table 8. Average deviations from the optimal for problems 63a and 63b

Algorithms	63a		63b	
	No of Runs	APD (%)	No of Runs	APD (%)
GA (Bettemir, 2009)	10	5.86	10	5.16
ACO (Bettemir,2009)	10	1.2	10	0.7
PSO (Bettemir, 2009)	10	0.152	10	0.2
NDS-TLBO (This paper)	10	0.128	10	0.14

Table 9 Analysis results of 630-Activity project for the Case 1 (Indirect Cost = $2300)

(This paper) NDS-TLBO		%PD	Rank	Crowding Distance
Duration	Cost ($)			
6373	54611340	0.74	1	0.0423
6387	54775880	1.04	1	0.0397
6383	54805960	1.09	1	0.0154
6364	54829460	1.14	1	0.0250
6360	54856620	1.19	1	0.0126
6302	54943070	1.35	1	0.0119
6377	54692200	0.88	1	0.0451
6388	54705310	0.91	1	0.0416
6346	54849940	1.17	1	0.0119
6300	54992260	1.44	3	0.0137
Pop. size	250			
Num. of iterations	500	%APD =1.1		
NFE	250000			

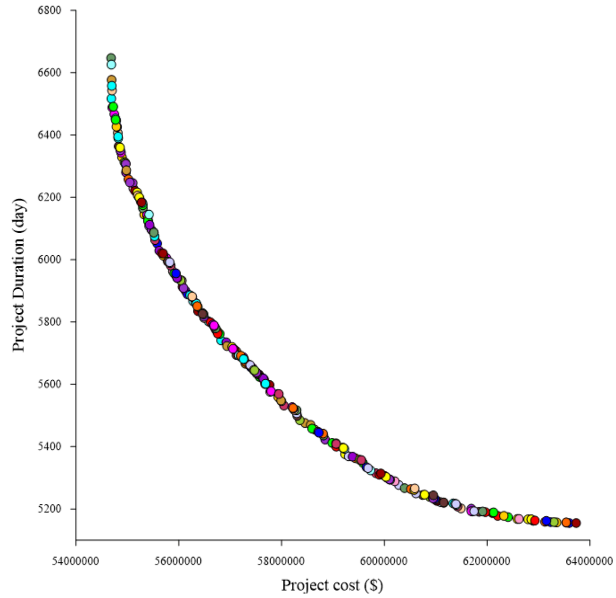

Fig. 8. Pareto front solutions of the solved problem using NDS approach

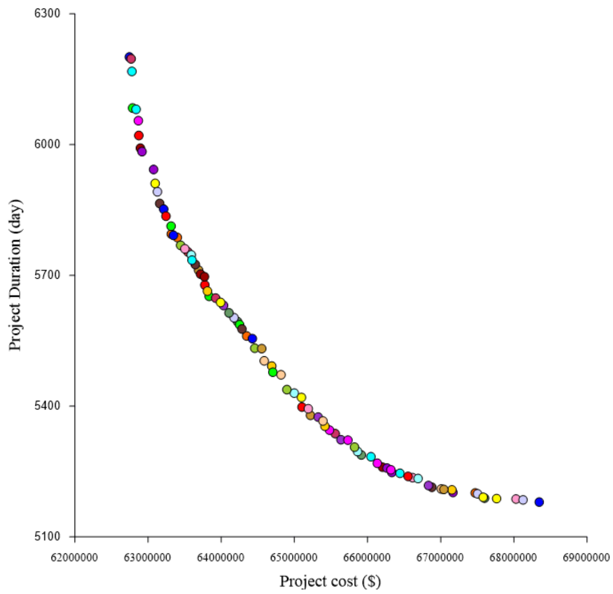

Fig. 9 Pareto front solutions of the solved problem using NDS approach

experimental runs with corresponding average percent deviations (%APD) from the optima. Graphical representations of the Pareto front solution of the solved problems are demonstrated in Figs. 8 and 9.

The APD values are %0.128 and %0.14 for case 63a and 63b. This implies that both the number of function evaluation as well as average percent deviation of the NDS-TLBO model are less than those of the Bettemir's (2009) models. Thereby, this demonstrates that the proposed algorithm has more exploration capability.

The APD values of NDS-TLBO for problems 630a and 630b were 0.74 and 1.51 and were smaller than the APD values of NDS-GA, NDS-ACO and NDS-PSO. In both of the test problems NDS-TLBO performed better than the NDS-GA, NDS-ACO and NDS-PSO.

Table 10 Comparison of Mean values of 10 run of 630-Activity problem

Sr. No	Bettemir (2009)			(This paper)
	Mean values of 10 run of 630-Activity			
	NDS-GA	NDS-ACO	NDS-PSO	NDS-TLBO
10	58983147	56703583	54815790	54806204
Pop. size	250	250	250	700
Num. of iterations	1000	1000	1000	350
NFE	250000	250000	250000	490000

Table 11 Analysis results of 630-Activity project for the Case 2 (daily indirect cost of $3500)

(This paper) NDS-TLBO Duration	Cost ($)	%PD	Rank	Crowding Distance
6212	62793865	1.67	1	0.0649
6220	62750580	1.60	1	0.0621
6204	62591490	1.34	1	0.1022
6232	62692340	1.50	1	Inf (∞)
6236	62741130	1.58	1	Inf (∞)
6225	62586260	1.33	1	Inf (∞)
6201	62744310	1.59	1	0.0418
6127	62650570	1.43	1	0.0876
6190	62699400	1.51	1	Inf (∞)
6279	62734550	1.57	1	Inf (∞)
Pop. size	250			
Num. of iterations	450	%APD =1.51		
NFE	225000			

Table 12 Comparison of Mean values of 10 run of 630-Activity problem

Sr. No	Bettemir (2009)			(This paper)
	Mean values of 10 run of 630-Activity			
	NDS-GA	NDS-ACO	NDS-PSO	NDS-TLBO
10	66395840	64574989	63121500	62698449
Pop. Size	250	250	250	250
Num. of iterations	1000	1000	1000	450
NFE	250000	250000	250000	225000

The performances of previous meta-heuristics for larger networks are not available. However, in the current study, for a more complex numerical simulation of 630-activity, NDS-TLBO was able to obtain non-dominated solutions, for the first time. The results of NDS-TLBO for large networks indicate that NDS-TLBO as a rule provides adequate optimal and near-optimal solutions for the TCTP problems. Hence, NDS-TLBO is among the top performing algorithms, providing a powerful alternative for the time cost trade-off problems.

6. Conclusions

Time-cost trade-off optimization problems encountered in the construction management field cannot be solved by linear programming or other analytical methods. Therefore, different metaheuristic optimization algorithms have been applied to optimize those problems. This study describes a newly developed Pareto-based NDS-TLBO algorithm to confirm the suitability of the proposed model for solving multi-objective optimization problems. Validation of NDS-TLBO algorithm is tested on a small test project consisting of 18-activity, medium-scale project with 63- activity and more complex large-scale problem with 630-activity. Based on the numerical results, it can be indicated that NDS-TLBO based model produces alternative Pareto front solution with less both the total number of function evaluations and average percent deviation than those of the previously proposed models. Consequently, optimization results clearly demonstrate the applicability and efficiency of the TLBO application for the first time on solving TCTP Problems in construction management field. The results also indicate that the TLBO has a great potential for solving simultaneous optimization of large TCTP problems e.g. 630-activity project. Moreover, the simplicity can be taken into account as strength point of existing method.

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