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Effects of porosity models on static behavior of size dependent functionally graded beam

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Abstract. In this study, the mechanical bending behaviors of functionally graded porous nanobeams are investigated. Four types of porosity which are, the classical power porosity function, the symmetric with mid-plane cosine function, bottom surface distribution and top surface distribution are proposed in analysis of nanobeam for the first time. A comparison between four types of porosity are illustrated. The effect of nano-scale is described by the differential nonlocal continuum theory of Eringen by adding the length scale into the constitutive equations as a material parameter comprising information about nanoscopic forces and its interactions. The graded material is designated by a power function through the thickness of nanobeam. The beam is simply-supported and is assumed to be thin, and hence, the kinematic assumptions of Euler-Bernoulli beam theory are held. The mathematical model is solved numerically using the finite element method. Numerical results show effects of porosity type, material graduation, and nanoscale parameters on the static deflection of nanobeam.

Keywords: porosity models; static bending; functionally graded beam; nonlocal elasticity; finite element method

1. Introduction

Recently, functionally graded materials (FGMs), which are novel composite materials, have received more investigations from researchers because of their unique advantages offered by smoothing and continuously distribution of material along one or more directions. FGMs have potential applications in various fields such as aircraft, space vehicles, rocket engine, automotive industries, optics, barrier coating, nuclear reactors, propulsion systems and nanostructures (i.e., NEMS, thin films, shape memory alloys and atomic force microscopes), Hamed *et al*. (2016). During fabrication processing of FGM, micro-voids and porosities in the material can occur due to technical problems and poor quality productions. In sometimes, material porosity in the micro/nano-scales has been widely managed in different applications, such as lightweight structures, biomedical systems, catalysts in electrochemical actuators and fuel cells (Detsi *et al*. (2013)), a piezoelectric ceramic graded actuator (Li *et al*. (2003)), porous titanium dioxide nano-layers to improve material hydrophilicity, (Kim *et al*. (2009)).

The porosity is most significantly in the mechanical behaviors of structure since material loses its strength by increasing the porosity ratio. Therefore, understanding the mechanical behavior of structural elements with porosity is importance in designs.

There are many different physical models proposed to include the porosity in the formulation of equivalent elasticity of structures. In 1964 Biot studied buckling of a porous slab and its thermoelastic analogy. Detournay and Cheng (1995) presented fundamentals of poroelasticity and presented constitutive equation of poroelastic materials. Magnucki and Stasiewicz (2004) studied analytically elastic buckling of a porous isotropic beam by assuming, that the modulus of elasticity is minimal on the beam axis and assumes maximum values at its top and bottom surfaces. Magnucka-Blandzi (2008, 2009, and 2010) studied static deflection, buckling, linear and nonlinear dynamic stability of circular porous-cellular isotropic plate. Eltaher *et al*. (2014a&b) investigated static bending, buckling and vibration behaviors of nonlocal FG nanobeam without porosity. Eltaher *et al*. (2018b&c) studied analytically mechanical behaviors of nonlocal perforated isotropic nanobeams by using nonlocal elasticity.

The general model of a porosity assumed that, the porosity is a constant parameter multiply by average value of ceramics and metal properties. Yahia *et al*. (2015) studied wave propagation in FG plates with porosities using various higher-order shear deformation plate theories. Ebrahimi and Zia (2015) illustrated large amplitude of nonlinear vibration of FG Timoshenko beams with porosities by utilizing both Galerkin's method and the method of multiple scales. Zemri *et al*. (2015) and Bouafia *et al*. (2017) presented a nonlocal shear deformation beam theory for bending, buckling, and vibration of functionally graded (FG) nanobeams using the nonlocal differential constitutive relations of Eringen. Chaht *et al*. (2015) presented a nonlocal shear deformation beam theory for bending, buckling, and vibration of FG nanobeams. Belkorissat *et al*. (2015) and Belabed *et al*.

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(2018) proposed a new nonlocal hyperbolic refined plate model for free vibration properties of FG plates incorporated the length scale parameter. Al-Basyouni *et al*. (2015) and Ahouel *et al*. (2016) presented model capable of capturing both small scale effect and transverse shear deformation effects of FG nanobeams and included the neutral axis effect. Ebrahimi and Jafari (2016) studied thermomechanical vibration characteristics of FG Reddy beams made of porous material subjected to various thermal loadings by using Navier solution method. Mechab *et al*. (2016), Bounouara *et al*. (2016) and Besseghier *et al*. (2017) presented a free vibration analysis of FGM nano-plate resting on elastic foundations based on two-variable refned plate theories including the porosities effect. Ebrahimi and Daman (2017) analytical solution for free vibration of curved FG nonlocal beam supposed to thermal loading by considering porosity distribution via nonlocal elasticity theory. Al Rjoub and Hamad (2017) developed an analytical method to study the dynamic behavior of functionally graded porous beams by transfer matrix method. Zidi *et al*. (2017) proposed a novel simple hyperbolic shear deformation theory for bending and free vibration analysis of FG beams. Shafiei and Kazemi (2017a&b) illustrated an exhaustive analysis on the buckling and vibration behaviors of two-dimensional FG tapered Euler–Bernoulli beams made of porous materials in nano- and micro-scales. Hachemi *et al*. (2017) presented a new simple three-unknown shear deformation theory for bending analysis of FG plates resting on elastic foundations. Bellifa *et al*. (2017) presented a nonlocal zeroth-order shear deformation theory for the nonlinear postbuckling behavior of nanoscale beams. Shojaeefard *et al*. (2017) studied free vibration and thermal buckling of micro temperature dependent FG porous circular plate subjected to a nonlinear thermal load. Atmane *et al*. (2017) illustrated the effect of thickness stretching and porosity on mechanical response of a functionally graded beams resting on elastic foundations. Mouffoki *et al*. (2017) presented the effects of moisture and temperature on free vibration characteristics of FG nanobeams resting on elastic foundation by proposing a novel simple trigonometric shear deformation theory. Karami *et al*. (2017) implemented influences of triaxial magnetic field on the wave propagation behavior of anisotropic nanoplates by including small scale effects of nonlocal strain gradient theory.

Eltaher *et al*. (2018a) studied the mechanical bending and vibration of FG nonlocal porous nanobeams using finite elements method. Modified porosity model described the equivalent Young's modulus as a function of ratio of the mass density with porosity to that without porosity. Yousfi *et al*. (2018) studied a free vibration of FGM plates with porosity by a shear deformation theory with four variables. Hamza-Cherif *et al*. (2018) investigated the vibration behaviors of nanobeam using differential transform method including thermal effect. Kadari *et al*. (2018) presented the buckling investigation of embedded orthotropic nanoplates by using a new hyperbolic plate theory and nonlocal small-scale effects. Bouadi *et al*. (2018) presented a new nonlocal higher order shear deformation for stability analysis of single layer graphene sheet. Mokhtar *et al*. (2018) and Yazid *et al*. (2018) presented the buckling of embedded orthotropic nanoplates graphene by employing a new refined plate theory and nonlocal differential constitutive relations of Eringen. Bakhadda *et al*. (2018) examined vibration and bending response of carbon nanotube-reinforced composite plates resting on the Pasternak elastic foundation. Four types of distributions of uni-axially aligned SWCNT are considered to reinforce the plates. Karami *et al*. (2018a) presented a variational approach for the wave dispersion in anisotropic doubly-curved nanoshells. Karami *et al*. (2018b) developed a 3D elasticity theory in conjunction with nonlocal strain gradient theory (NSGT) for mechanical analysis of anisotropic nanoparticles. Bourada *et al*. (2019) investigated dynamic of porous FG beam using a sinusoidal shear deformation theory.

The porosity effect is included in the equivalent modulus along the thickness by a constant parameter multiplies by cosine function by many authors. Chen *et al*. (2015 & 2016) presented elastic buckling, static bending, free and forced vibrations of shear deformable FG porous Timoshenko beams by using Ritz trial functions and Newmark-β method in the time domain. Galeban *et al*. (2016) investigated free vibration of beam made of porous beam in three situations: poro/nonlinear nonsymmetric distribution, poro/nonlinear symmetric distribution, and poro/monotonous distribution. Houari *et al*. (2016) developed a new simple higher-order shear deformation theory for bending and free vibration analysis of functionally graded (FG) plates. Akbas (2017) analyzed post-buckling of FG beams with porosity effect under compression load. Material properties of the beam change in the thickness direction according to power-law distributions with different porosity models. Khetir *et al*. (2017) proposed a new nonlocal trigonometric shear deformation theory for thermal buckling response of nanosize FG nano-plates resting on two-parameter elastic foundation under various types of thermal environments. Chen (2017) investigated the nonlinear free vibration and postbuckling behaviors of multilayer FG porous nanocomposite beams made of metal foams reinforced by graphene platelets. Kitipornchai *et al*. (2017) investigated buckling and vibration of FG porous beam reinforced by graphene platelets by using Halpin-Tsai micromechanics model. Mirjavadi *et al*. (2017) studied thermo-mechanical vibration behavior of two dimensional functionally graded (2D-FG) porous nanobeam. Kaci *et al*. (2018) studied post-buckling analysis of shear-deformable composite beams using a novel simple two-unknown beam theory. Heshmati and Daneshmand (2018) presented a semi-analytical approach to study vibrational properties of weight-efficient plates made of FG porous materials. She *et al*. (2018a) predicted wave propagation behaviors of functionally graded materials (FG) porous nanobeams based on Reddy's higher-order shear deformation beam theory in conjunction with the non-local strain gradient theory. Thang *et al*. (2018) investigated elastic buckling and free vibration of porous-cellular first-order shear deformation plates with uniform and non-uniform porosity distributions. For the first time, She *et al*. (2018b) analyzed the nonlinear bending

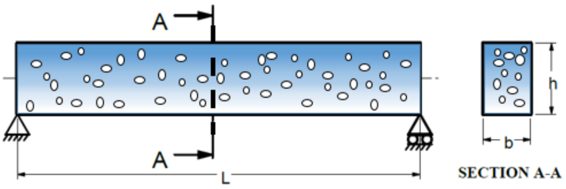


Fig 1 A porous FG simply-supported nanobeam

and vibrational characteristics of porous tubes within the framework of the nonlocal strain gradient theory. Kim *et al*. (2019) studied analytically static bending and buckling and free vibration of functionally graded porous micro-plates using first-order shear deformation plate theories by Navier solution technique. She *et al*. (2019) illustrated nonlinear bending behavior of porous functionally graded (FG) curved nanotubes by using the nonlocal strain gradient theory. Eltaher *et al*. (2019) characterized Mechanical behaviors of SWCNTs by equivalent-continuum mechanics approach. Eltaher *et al*. (accepted) presented a novel numerical procedure to predict nonlinear buckling and postbuckling stability of imperfect clamped–clamped SWCNT surrounded by nonlinear elastic foundation.

According to the author's knowledge, the static bending of nonlocal elastic FG nanobeam with different porosity models not included elsewhere. So, this paper tends to fill this gap and illustrates effects of power porosity, symmetric porosity, bottom surface distribution and top surface distribution on the bending behaviors of porous FGM nanobeams. The rest of paper is organized as follows: Section 2 describes mathematical modelling of FG porous nanobeam. Porosity models, geometrical fit conditions, nonlocal constitutive equation, and equation of motions are presented in details through the modeling section. A numerical finite element model developed to solve the governing equations is shown in Section 3. Numerical results and parametric studies are illustrated and discussed in Section 4. Section 5 summarizes the concluding remarks.

2. Mathematical formulation

A geometrical description of a FG simply supported nanobeam of length (L), width (b), and thickness (h), and having a random porosity distribution is illustrated in Fig. 1.

2.1 Material Graduation Functions

To describe the graduation of material constituents accurately, a simple homogenization Voigt rule is used, Hamed *et al*. (2016). The volume fraction of materials are graded across the beam thickness (z) by the following functions: (Komijani *et al*. (2014))

$$V_c = \left(\frac{1}{2} + \frac{z}{h}\right)^n \qquad (0 \le n < \infty)$$
$$V_c + V_m = 1 \tag{1}$$

in which V, n, h are, respectively, the volume fraction, positive power exponent, and beam thickness. Subscript c represents the ceramic while m represents the metal. Hence, the equivalent Young's modulus (E) and can be represented by a power function as

$$E(z) = [E_c - E_m]\left(\frac{1}{2} + \frac{z}{h}\right)^n + E_m \tag{2}$$

2.2 Porosity model

It is noted from experimental investigation that linear variation of porosity is insufficient to consider a reduction in the rigidity of a structure. In this work, four types of porosity's distributions are proposed. The first model proposed by Wattanasakulpong and Ungbhakorn (2014), assumed that the porosity is uniform distributed through the beam thickness and its effect can be described by

$$E(z) = [E_c - E_m]\left(\frac{1}{2} + \frac{z}{h}\right)^n + E_m - \frac{\alpha}{2}\,[E_c + E_m] \tag{3}$$

Type (1)

where α is the volume fraction of porosity in the material. The last term of the equation represents the porosity content in both metal and ceramic constituents. It is assumed that the porosity is distributed evenly in the material, i.e., half in the ceramic and another half in the metal.

The second model of porosity assumed that the porosity is distributed symmetric around mid-axis and its distribution is concentrated near to mid-axis and decreased as approach to top or bottom surface. In this case, the elasticity distribution included the symmetric effect of porosity can be represented by

$$E(z) = \left[[E_c - E_m]\left(\frac{1}{2} + \frac{z}{h}\right)^n + E_m\right]\left[1 - \alpha\cos\left[\pi\left(\frac{z}{h}\right)\right]\right] \tag{4}$$

Type (2)

The third and fourth models assumed that the porosity are concentrated at the bottom and top surfaces, respectively, according to the following functions:

$$E(z) = \left[[E_c - E_m]\left(\frac{1}{2} + \frac{z}{h}\right)^n + E_m\right]\left[1 - \alpha\cos\left[\frac{\pi}{2}\left(\frac{z}{h} + \frac{1}{2}\right)\right]\right] \tag{5}$$

Type (3)

$$E(z) = \left[[E_c - E_m]\left(\frac{1}{2} + \frac{z}{h}\right)^n + E_m\right]\left[1 - \alpha\cos\left[\frac{\pi}{2}\left(\frac{z}{h} - \frac{1}{2}\right)\right]\right] \tag{6}$$

Type (4)

The normalized distribution of the porosity for these different models through the beam thickness are presented in Fig. 2. As shown, Type 1 indicates that the distribution of porosity is constant through thickness. However, porosity Type 2 has a peak value at mid-plane and zero porosity at the top and bottom surface. In case of Type 3, the porosity is maximum at the bottom surface and decreased gradually as moving to top surface, that has a zero porosity. Porosity of Type 4 describes the decreasing of porosity distribution as moving from top surface to the bottom.

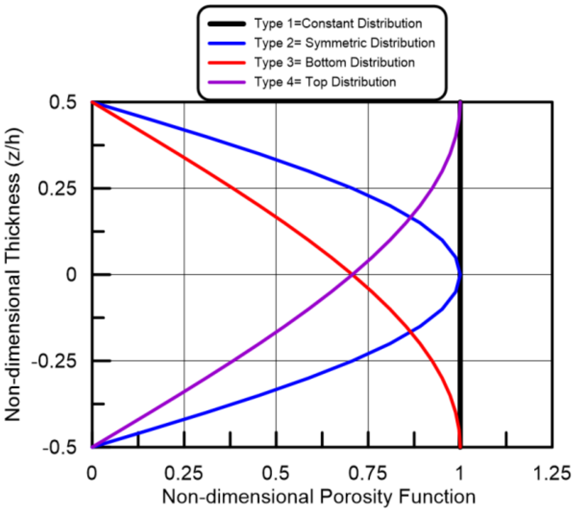


Fig. 2 Profile distribution of normalized porosity through the beam thickness for different models

Through this analysis, FG constituent of nanobeam are assumed to be metal steel with following Young modulus E_m = 210 GPa and alumina ceramics with E_c = 390 GPa. According to these values, the normalized distribution of elasticity through beam thickness at a specific porosity (α) and distribution (n) values for four types of porosity functions are presented in Fig 3. As shown, the only linear distribution of elasticity occurs for Type 1 of porosity at $n = 1.0$, otherwise, all distributions are nonlinear variations. It is noticed that, the symmetric and bottom distributions have the same elasticity at the top surface. However, the symmetric and top distributions have the same elasticity at the bottom surface as presented in Fig. 3.

2.3 Kinematics and Governing Equations

Based on Euler–Bernoulli assumptions, geometrical fit conditions can be described as for by

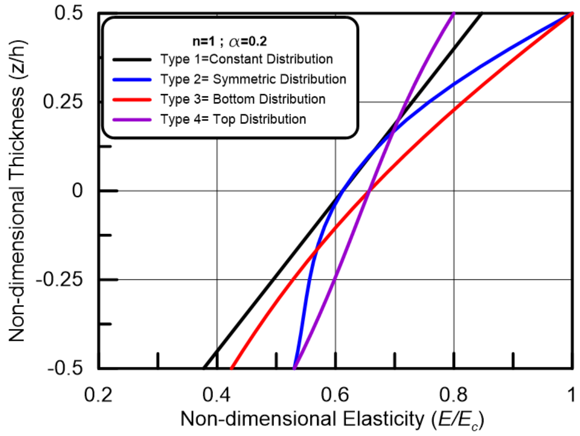
$$u(x,z) = u_0(x) - z\frac{dw_0}{dx} \quad \& \quad w(x,z,t) = w_0(x) \tag{7}$$

in which u and w are in-plane and transverse displacements at any generic point on the beam. u_0 is mid-plane axial displacement and w_0 is the and transverse displacements at the neutral axis. The only nonzero axial strains and related normal classical stress are described by the following:

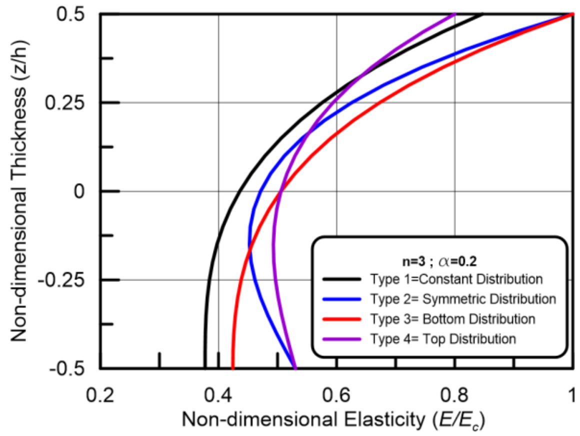
$$\varepsilon_{xx}(x,z,t) = \frac{du_0(x,t)}{dx} - z\frac{d^2w_0}{dx^2} \tag{8}$$

$$\sigma_{xx}(x,z,t) = E(z)\,\varepsilon_{xx}(x,z,t) \tag{9}$$

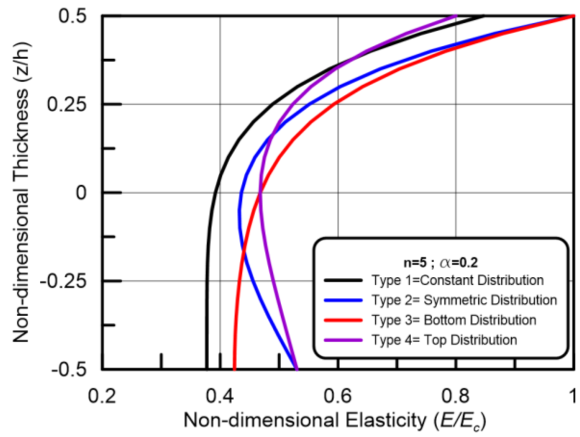
Since, the classical mechanics theory can't capture the effect of nanosscale and size dependency of nanostructure, a modified nonlocal continuum mechanics of Eringen is proposed to overcome this deficiency. The nonlocal electricity theory states that, the stress tensor at a reference point depends not only on the strain components at same position but also on all other points of the body (Eltaher *et al*. (2013, 2016a,b)). The nonlocal constitutive behavior for



a) Material distribution parameter n=1.0



b) Material distribution parameter n=3.0



c) Material distribution parameter n=5.0

Fig. 3 Distribution of Young modulus along the beam thickness at porosity constant α=0.2 for different porosity models

the nonlocal Eringen model is given by

$$\sigma_{ij} = \int_V \alpha(|X' - X|, \tau)\, C_{ijkl}\varepsilon_{kl}(X') \tag{10}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{11}$$

where σ_{ij}, ε_{ij}, and u_i are the stress, strain, and displacement components, respectively; C_{ijkl}, is the fourth-order elasticity tensor. The function $\alpha(|x' - x|, \tau)$ represents the nonlocal modulus, $|x_0 - x|$ is the Euclidean distance. The nonlocal integral form of Eringen can be simplified to the following differential constitutive relations

$$\sigma_{ij} - (e_0 a)^2 \nabla^2 \sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{12}$$

in which ∇^2 is the Laplace operator. $\tau = e_0 a/l$ is defined as the scale coefficient that incorporates the small scale factor, where e_0 is a material constant. Internal characteristic lengths (e.g. lattice parameter, granular size) are represented by a, while l represents external characteristic lengths (e.g. crack length, wavelength) of the nanostructures. In case of Euler-Bernoulli nonlocal FG beam, Eq. (12) can be written as

$$\sigma_{xx} - \mu \frac{d^2\sigma_{xx}}{dx^2} = E(z)\varepsilon_{xx}, \quad (\mu = e_0^2 a^2) \tag{13}$$

Therefore, the equations of motion of a nonlocal FG porous Euler-Bernoulli nanobeam can be described as, Eltaher *et al.* (2013)

$$A\frac{d^2u_0}{dx^2} - B\frac{d^3w_o}{dx^3} + \left(1 - \mu\frac{d^2}{dx^2}\right)f = 0 \tag{14}$$

$$B\frac{d^3u_0}{dx^3} - D\frac{d^4w_0}{dx^4} + \left(1 - \mu\frac{d^2}{dx^2}\right)q = 0 \tag{15}$$

where f and q are distributed axial and transverse applied loads, respectively, whereas stiffness parameters $A, B,$ and D can be determined by

$$A = \int_{-\frac{h}{2}}^{\frac{h}{2}} E(z)dz,$$

$$B = \int_{-\frac{h}{2}}^{\frac{h}{2}} z\,E(z)dz \tag{16}$$

$$D = \int_{-\frac{h}{2}}^{\frac{h}{2}} z^2 E(z)dz$$

3. Numerical formulation

The variational form of the equilibrium equations in terms of the displacements for the porous FG beam can be represented by, (Alshorbagy (2011) and Eltaher *et al.* (2013))

$$\begin{aligned}\int_0^L \Bigg\{ & \left(A\frac{du_0}{dx}\frac{d\delta u_0}{dx} + B\frac{d^2w_o}{dx^2}\frac{d\delta u_0}{dx} + B\frac{d^2\delta w_0}{dx^2} + D\frac{d^2w_0}{dx^2}\frac{d^2\delta w_0}{dx^2}\right) \\ & + \left(f\delta u_0 + \mu\frac{df}{dx}\frac{d\delta u_0}{dx}\right) \\ & + \left(q\delta w_0 - \mu q\frac{d^2\delta w_0}{dx^2}\right)\Bigg\} dxdt \\ & + \left[\bar{N}_B\,\delta u_0 + \bar{V}_B\,\delta w_0 + \bar{M}_B\,\frac{\partial\delta w_0}{\partial x}\right]_0^L = 0\end{aligned} \tag{17}$$

where $\bar{N}_B$, $\bar{V}_B$, and $\bar{M}_B$ detonate specified stress resultants at the boundary points $x = 0, L$. To discretize the nonlocal porous FG nanobeam to elements, the in-plane and transverse displacement components at the neutral axis of the beam are depicted by, (Hamed *et al.* (2016) and Eltaher *et al.* (2018),)

$$u_0^{(e)}(x,t) = \sum_{i=1}^{2} N_i U_i(t) = N_1U_1(t) + N_2U_2(t) \tag{18}$$

$$\begin{aligned} w_0^{(e)}(x,t) &= \sum_{k=1}^{4} \tilde{N}_k \tilde{W}_k \\ &= \tilde{N}_1 W_1 + \tilde{N}_2\theta_1 + \tilde{N}_3 W_2 + \tilde{N}_4\theta_2 \end{aligned} \tag{19}$$

where U, W and θ are in plane displacement, transverse displacement and slope at the nodal points,respectively; and N_i and $\tilde{N}_k$ are the Lagrangian and Hermetian interpolation shape functions for in-plane and transverse displacements, respectively. Substituting Eqs.(18&19) into Eq. (17) and integrating over the domain, the following equilibrium equation can be represented in matrix form as

$$\begin{bmatrix} K_{uu} & K_{uw} \\ K_{uw} & K_{ww} \end{bmatrix}\begin{Bmatrix} U \\ W \end{Bmatrix} = \begin{Bmatrix} F \\ Q \end{Bmatrix}_L + \begin{Bmatrix} Fn \\ Qn \end{Bmatrix}_{NL} \tag{20}$$

in which K_{uu} and K_{ww} are the axial stiffness and bending stiffness, respectively. K_{uw} is the axial-bending stifffness. F and Q are the axial and transverse load. Subscripts *L*, and *NL* indicate the effect of load due to local and nonlocal.

4. Numerical results

Through this section, effects of porosity model, porosity content, size effect, material graduation on the static deflection of simply-supported nonlocal nanobeam are investigated and discussed in details. The material constituents and their distributions are given in section 2. The geometrical dimensions of the beam are b×h×L= 1×0.1×10 nm. The coupled equations (Eq. 20) are solved and the non-dimensional center deflection for the simply-supported beam is determined by the following term $W_{max} = 100\frac{E_c I}{qL^4}(\max(w_o))$.

4.1 Effect of porosity volume fraction

On the day of the beam test, the respective control the effect of porosity volume fraction on the static deflection of local beam for various porosity models are illustrated in Fig. 4. As shown for all types of porosity, as the volume fraction of porosity increased the static deflection is increased, due to decreasing of the beam stiffness. It is also noted that, as the material graduation increased, the static deflection is increased. So that, the static deflection is proportional with both porosity parameter and material graduation. As show, the porosity parameter is most significant rather than material graduation in case of power porosity distribution (Type 1), however, the material

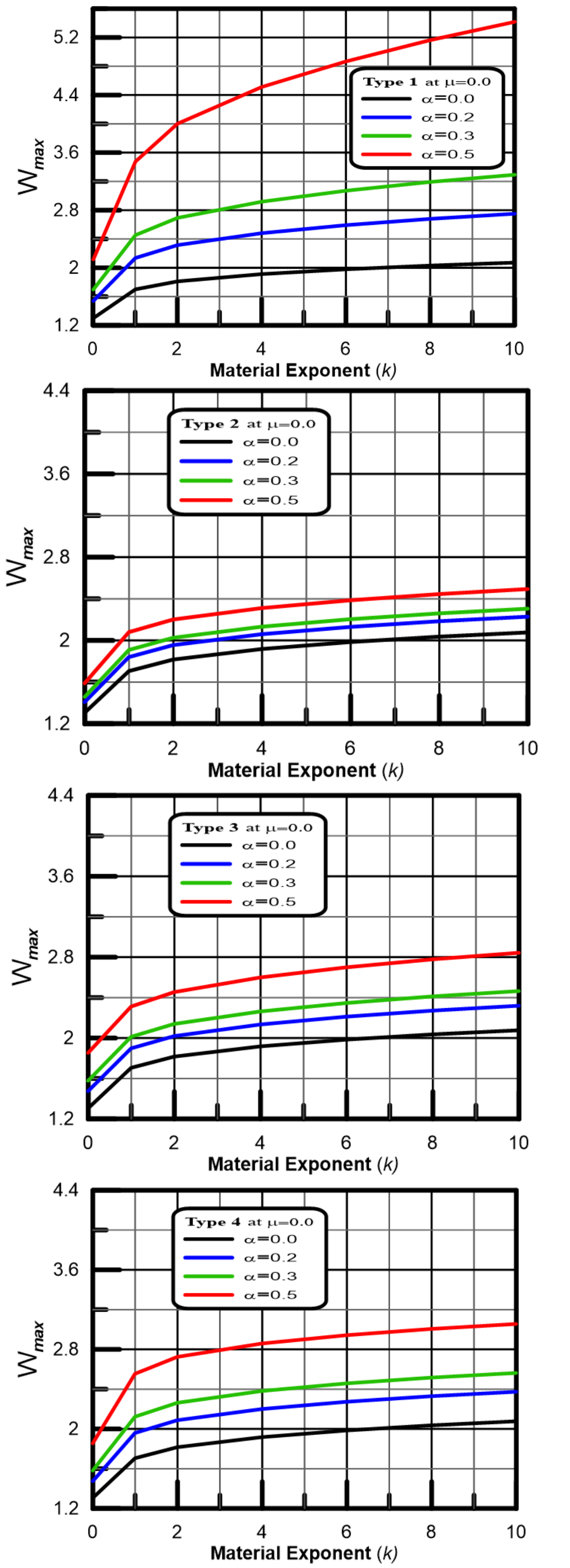


Fig. 4 Effect of porosity on the static deflection of local beam or different porosity models

graduation is most significant in case of symmetric distribution (Type 2) rather than the porosity parameter. The porosity parameter and material graduations are equal on their effect on the static deflection in case of porosity models Type 3 and Type 4.

4.2 Effect nonlocal size-dependent parameter

The effect of nonlocal size-dependent parameter on bending of simply supported nonlocal FG nanobeam for different porosity models are presented in Fig. 5. It is noted that, as the nonlocal parameter increased, the stiffness of nanobeam is decreased and hence the static deflection increased for all porosity models as shown in Fig 5. As the nonlocal parameter increase to 1, 2, 3 and 4, the static deflection increased by 4.8%, 9.6%, 14.3% and 19.1%, respectively for all models. However, as the material graduation parameter (n) increased from 0 to 1, 2, 4, 6, 8, and 10 the increasing of deflection is observed by 38.6%, 50.2%, 61%, 68.3%, 74.1% and 78.7% , respectively, for porosity Type 1 and , 30.7%, 38.9%, 46.4%, 51.4% 55.3% , and 58.45% , respectively for porosity Type 2. So that, the graduation effect is more pronounced than the nonlocal parameter for all types. The graduation parameter is more significant for a power distribution Type 1, than symmetric distribution (Type 2).

4.3 Effect porosity type

The influence of porosity distribution models on the static bending deflections are presented in Fig. 6. As depicted in Fig 6, at porosity parameter $\alpha = 0.2$ and material graduation n=10, the static deflection increased by 4.2% in case of Type 3, 6.8% in case of Type 4, and 23.6% in case of Type 1, relative to the porosity Type 2. So that, the most significant model to the porosity is the power porosity model (Type 1), after that Top surface Type 4, then bottom surface Type 3. The symmetric distribution model (Type 2) is the lowest model affected by the porosity volume fraction coefficient. As depicted in the figure at= 0.5 , the non-dimensional static deflection may reach to 5.4, 3.05, 2.84, 2.5, in case of porosity Type 1, Type3, Type 4, and Type 2, respectively. For a porosity Type 1, increasing porosity contents to 0.2, 0.3 and 0.5 tend to increase the deflection by 33%, 59% and 161% respectively. However, for porosity type 2, the increasing of porosity contents tend to increasing a deflection a little bit by 7%, 11% and 20%, respectively.

5. Conclusions

The static bending of nonlocal elastic FG nanobeam with different porosity models including power, symmetric power, bottom surface, and top surface porosity distributions are studied. The thin nanobeam is governed by Euler-Bernoulli theory for kinematic relations, and differential nonlocal Eringen model for constitutive equation that includes a size effect. A rule of mixture is proposed to describe the relation between metal and ceramics constituents. An equilibrium equations for a functional graded nonlocal porous nanobeam is derived and

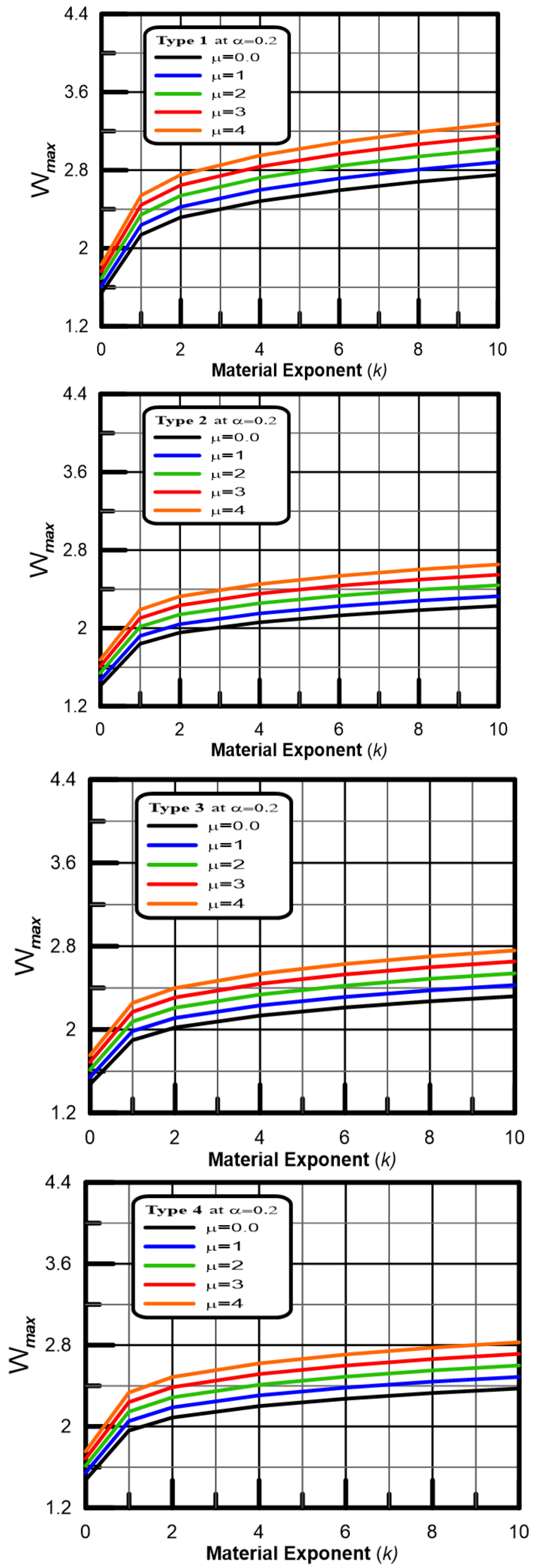
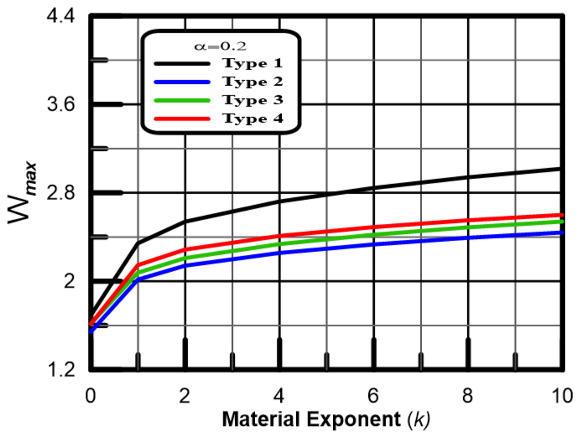
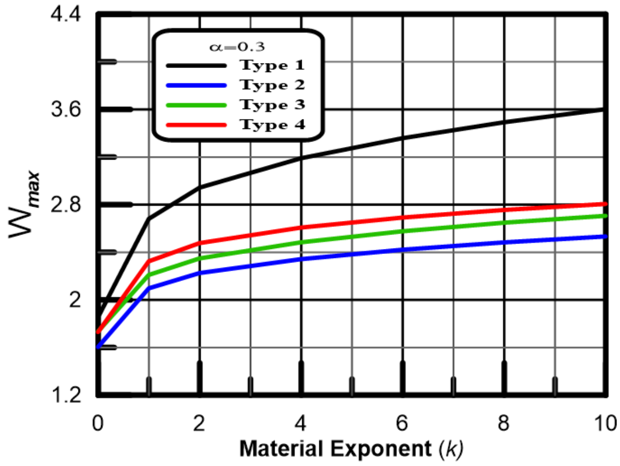


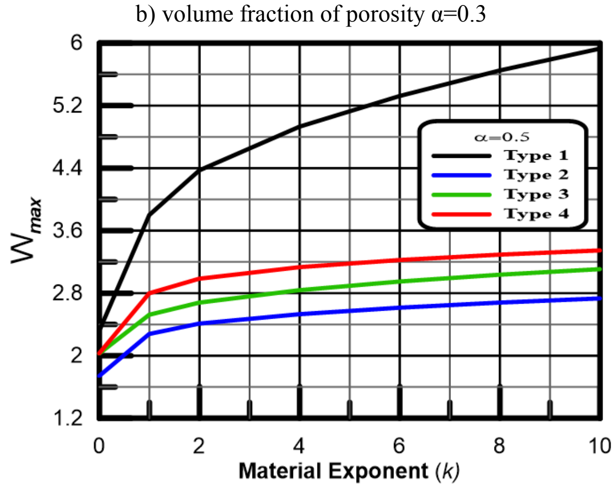
Fig. 5 Effect of nonlocal parameter on the static deflection of nanobeam for different porosity models



a) volume fraction of porosity α=0.2



b) volume fraction of porosity α=0.3



c) volume fraction of porosity α=0.5

Fig. 6 Effect of porosity type on the static deflection of nanobeam for different porosity parameters and constant nonlocal parameter =4

numerical finite element solution is proposed to extract a numerical solution of the problem. The results show that the porosity models, material graduation, porosity parameter, and nonlocal size dependent parameter have great influences on the static deflection of FG porous nanobeams. The main conclusions can be summarized as follow:

- Material distribution parameter, nonlocal size parameter, and porosity parameter; all of them tend to reduce the stiffness and hence increase the static deflection.
- The most significant model to the porosity parameter is the power porosity model (Type 1), after that Top surface Type 4, then bottom surface Type 3, and the last is symmetric porosity distribution (Type 2).
- The porosity parameter is most significant rather than material graduation in case of power porosity distribution (Type 1), however, the material graduation is most significant in case of symmetric distribution (Type 2) rather than the porosity parameter.
- The porosity parameter and material graduations have the same effects on the static deflection in case of porosity models Type 3 and Type 4.

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