

# Influence of shear preload on wave propagation in small-scale plates with nanofibers

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**Abstract.** In the present work, an attempt is made to explore the effects of shear in-plane preload on the wave propagation response of small-scale plates containing nanofibers. The small-scale system is assumed to be embedded in an elastic matrix. The nonlocal elasticity is utilized in order to develop a size-dependent model of plates. The proposed plate model is able to describe both nanofiber effects and the influences of being at small-scales on the wave propagation response. The size-dependent differential equations are derived for motions along all directions. The size-dependent coupled equations are solved analytically to obtain the phase and group velocities of the small-scale plate under a shear in-plane preload. The effects of this shear preload in conjunction with nanofiber and size effects as well as the influences of the elastic matrix on the wave propagation response are analyzed in detail.

**Keywords:** shear preload; nanofibers; small-scale plates; size effects

## 1. Introduction

Small-scale plates and beams such as microscale and nanoscale sheets and tubes operate as the fundamental parts of many ultrasmall devices since they display excellent electromechanical properties. In many applications, there are electromechanical loads exerted on small-scale plates and beams, which are originated from different sources such as electromagnetic fields and initial stresses. To better design the manufacturing process, it is important to understand the mechanical response of small-scale plates and beams under different loading conditions.

The mechanical response of nanostructures including nanoplates (Asemi *et al.* 2014, Bakhadda *et al.* 2018, Bouadi *et al.* 2018, Kadari *et al.* 2018, Mokhtar *et al.* 2018, Yazid *et al.* 2018), nanobeams (Chaht *et al.* 2015, Zemri *et al.* 2015, Ahouel *et al.* 2016, Bellifa *et al.* 2017, Nejad *et al.* 2017, Farajpour *et al.* 2018, Hamza-Cherif *et al.* 2018) and nanoshells (Farajpour and Rastgoo 2017, Karami *et al.* 2018) has been analyzed in the literature using a number of size-dependent continuum models such as nonlocal four variable model (Belkorissat *et al.* 2015), trigonometric theory of shear deformations (Besseghier *et al.* 2017, Khetir *et al.* 2017, Mouffoki *et al.* 2017), nonlocal zeroth-order theory of shear deformations (Bounouara *et al.* 2016), nonlocal quasi-3D theory (Bouafia *et al.* 2017), surface elasticity (Youcef, Kaci *et al.* 2018) and nonlocal strain gradient theory (Farajpour and Rastgoo 2017, Farajpour *et al.* 2018). On the other hand, in addition to the wave propagation analysis of macroscale structures (Yahia *et al.*

2015, Fourn *et al.* 2018), wave propagation characteristics of small-scale structures (Karami *et al.* 2018) using size-dependent theories (Karami *et al.* 2017, Karami *et al.* 2018) have been analyzed.

The influences of mechanical preload caused by initial stresses on the mechanical behaviors of small-scale beams were analyzed in the last decade using continuum-based models. Classical scale-free continuum mechanics cannot be utilized for nanobeams under mechanical preload since it does not capture size effects (Chakraverty and Behera 2015, Kiani *et al.* 2017, Ebrahimi and Barati 2018, Ebrahimi and Barati 2018, Ebrahimi and Barati 2018, Ebrahimi and Heidari 2018, Ma *et al.* 2018). A few size-dependent continuum-based models have been proposed for analyzing the stability, vibration and bending of small-scale structures (Malekzadeh and Shojaei 2013, Farajpour *et al.* 2014, Shen and Malekzadeh 2016, Farajpour *et al.* 2017, Ebrahimi and Barati 2018). Wang and Cai (Wang and Cai 2006) explored the effects of mechanical preload on the vibrations of a system of nanoscale tubes with the help of a continuum-based theory. In another study conducted by Song *et al.* (Song *et al.* 2010), the influences of axial preload on the wave propagations in nanoscale tubes were scrutinized employing the nonlocal theory. In addition, sound wave propagation characteristics of a single nanotube were extracted by Heireche *et al.* (Heireche *et al.* 2008) using a nonlocal model together with an analytical solution approach. The influences of axial preload as well as the effects of a magnetic field on the vibration of nanotubes were also studied in Ref. (Güven 2014). Moreover, the effects of compressive preload on the wave propagation in nanoscale tubes were investigated with the help of a continuum model (Selim *et al.* 2009). A nonlocal beam model was also proposed in the literature for analyzing the effects of axial preload on the frequency shifts of a

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nanomechanical sensor using two vibrating nanotubes (Shen *et al.* 2012).

In addition to the application of size-dependent models in analyzing small-scale tubes under axial preload, the mechanical behaviors of small-scale plates subjected to in-plane preload have been examined. A size-dependent model was introduced by Asemi *et al.* (Asemi *et al.* 2014) to investigate the effects of in-plane preload caused by initial stresses on the vibrations of nanoscale plates made of piezoelectric materials with the help of the nonlocal theory. Furthermore, the wave propagation features of small-scale plates subjected to in-plane preload were obtained in Ref. (Wang *et al.* 2010) employing the nonlocal theory. In another study reported in Ref. (Murmu and Pradhan 2009), the transverse vibration of nanoscale plates under uniaxial preload was investigated applying the nonlocal theory. The effects of an elastic substrate in conjunction with the influences of in-plane preload on the wave propagation in graphene sheets were scrutinized by Karami *et al.* (Karami *et al.* 2018). Mohammadi *et al.* (Mohammadi *et al.* 2014) examined the influences of an elastic medium and the effects of shear in-plane preload on the natural frequencies of small-scale plates. In another study, Ebrahimi and Shafiei (Ebrahimi and Shafiei 2017) used a combination of the nonlocal theory and Reddy's shear deformation model of plates to analyze the vibrations of small-scale plates subjected to in-plane preload.

In all of the above-stated valuable works, only the influence of initial load on the mechanics of a simple homogeneous small-scale structure without any kind of reinforcement is investigated. With the development of advanced manufacturing techniques at small-scales, more complex micro/nanoscale structures with superior electromechanical properties have successfully been synthesized. For instance, shape memory alloy (SMA) properties have lately been observed in a couple of small-scale fundamental structures such as nanofibers and nanofilms (Kahn *et al.* 1998). From the literature review, it can be seen that no size-dependent continuum model has been developed for investigating the influences of shear in-plane preload on the elastic wave dispersion in small-scale plates containing nanofibers. To ensure consistency between the continuum model and real operating conditions, it is assumed that the plate is embedded in an elastic matrix. The nonlocal elasticity, as a modified scale-dependent theory, is used to model the problem. The size-dependent motion equations are then derived along all directions. The phase and group velocities of the small-scale system subjected to shear in-plane preload are analytically obtained. The influences of shear preload and elastic matrix as well as size and nanofiber effects on the elastic wave dispersion in the nanosystem containing nanofibers are explored.

## 2. Small-scale plates containing nanofibers

In the following, the elastic wave dispersion in small-scale plates containing nanofibers is modeled based on a combination of Brinson's model and nonlocal theory. Figure 1 indicates a system of five small-scale plates and nanofibers. It is assumed that a uniform shear in-plane

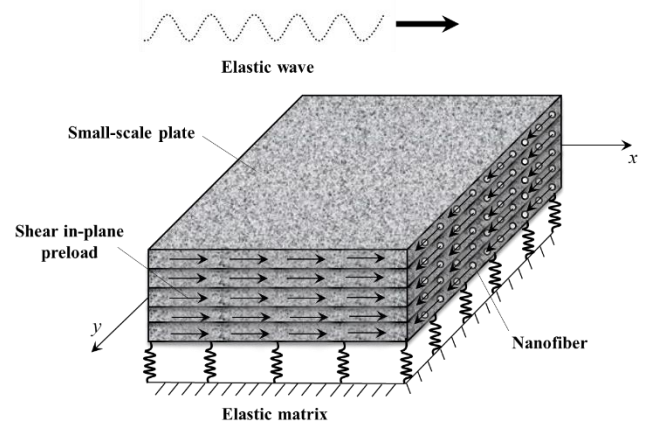


Fig. 1 Elastic waves in small-scale plates under shear preload containing nanofibers

preload is exerted on the system as shown in the figure. In addition, the system is embedded in an elastic matrix with shear stiffness coefficient  $k_s$  and normal stiffness coefficient  $k_n$  (Akgöz and Civalek 2013, Civalek 2013).

For the geometrical properties of each layer, we assume that (length, thickness, width) =  $(a, h, b)$ . Furthermore, for the elastic and physical properties, one can write (shear modulus, elasticity modulus, density, Poisson's ratio) =  $(G_{12}^{sys}, E_i^{sys}, \rho_{sys}, \nu_{12}^{sys})$  in which "sys" is an abbreviation for "system". These elastic and physical properties are written as (Park *et al.* 2004)

$$\begin{aligned} E_1^{sys}(\zeta) &= (V_F(\zeta))E_F + (1 - V_F(\zeta))E_L, \\ \rho_{sys}(\zeta) &= (V_F(\zeta))\rho_F + (1 - V_F(\zeta))\rho_L, \\ \nu_{12}^{sys}(\zeta) &= (V_F(\zeta))\nu_F + (1 - V_F(\zeta))\nu_L, \\ E_2^{sys}(\zeta) &= \frac{E_F E_L}{(V_F(\zeta))E_L + (1 - V_F(\zeta))E_F}, \\ G_{12}^{sys}(\zeta) &= \frac{G_F G_L}{(V_F(\zeta))G_L + (1 - V_F(\zeta))G_F}, \end{aligned} \quad (1)$$

in which "F" is employed to represent nanoscale fibers whereas "L" denotes each layer without nanofibers,  $V_F$  and  $\zeta$  represent the volume fraction of fibers and martensite fraction, respectively. In Appendix A, the basic equations for SMA nanofibers, as a particular type of nanofibers, are given. The in-plane strains of the small-scale system are as (Reddy 2010, Aksencer and Aydogdu 2011)

$$\begin{aligned} \epsilon_{xx} &= e_{xx}^0 - \zeta \kappa_{xx}, \\ \epsilon_{yy} &= e_{yy}^0 - \zeta \kappa_{yy}, \\ \gamma_{xy} &= \gamma_{xy}^0 - \zeta \kappa_{xy}, \end{aligned} \quad (2)$$

where

$$\begin{aligned} e_{xx}^0 &= \frac{\partial u}{\partial x}, \quad e_{yy}^0 = \frac{\partial v}{\partial y}, \quad \gamma_{xy}^0 = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}, \\ \kappa_{xx} &= \frac{\partial^2 w}{\partial x^2}, \quad \kappa_{yy} = \frac{\partial^2 w}{\partial y^2}, \quad \kappa_{xy} = 2 \frac{\partial^2 w}{\partial y \partial x}. \end{aligned} \quad (3)$$

In Eq. (3),  $v$ ,  $w$  and  $u$  are utilized to represent the mid-plane displacements in  $y$ ,  $z$  and  $x$  axes, respectively (Bouderba *et al.* 2013, Zenkour and Sobhy 2013). Using the nonlocal theory (Reddy 2007, Reddy and Pang 2008, Aydogdu 2009, Aydogdu and Arda 2016), the size-dependent basic equation of the  $k$ th layer can be written as

$$\{\boldsymbol{\sigma}^{(k)}\} - \eta_{nl} \nabla^2 \{\boldsymbol{\sigma}^{(k)}\} = [\tilde{\mathbf{C}}^{(k)}(\zeta, \phi_k)] \{\mathbf{e}^0\} - \zeta [\tilde{\mathbf{C}}^{(k)}(\phi_k, \zeta)] \{\boldsymbol{\kappa}\} + \sigma_{RS}^{(k)} V_{F,k}^{SMA} \{\boldsymbol{\mu}(\phi_k)\}, \quad (4)$$

where

$$\{\boldsymbol{\sigma}^{(k)}\} = \begin{Bmatrix} \sigma_{xx}^{(k)} \\ \sigma_{yy}^{(k)} \\ \sigma_{xy}^{(k)} \end{Bmatrix}, \quad \{\mathbf{e}^0\} = \begin{Bmatrix} e_{xx}^0 \\ e_{yy}^0 \\ \gamma_{xy}^0 \end{Bmatrix}, \quad \{\boldsymbol{\kappa}\} = \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix}, \quad (5)$$

$$\{\boldsymbol{\mu}(\phi_k)\} = \begin{Bmatrix} \mu_1(\phi_k) \\ \mu_2(\phi_k) \\ \mu_3(\phi_k) \end{Bmatrix} = \begin{Bmatrix} \cos^2(\phi_k) \\ \sin^2(\phi_k) \\ \cos(\phi_k) \sin(\phi_k) \end{Bmatrix},$$

and

$$[\tilde{\mathbf{C}}^{(k)}(\phi_k, \zeta)] = \begin{bmatrix} \tilde{C}_{11}^{(k)}(\phi_k, \zeta) & \tilde{C}_{12}^{(k)}(\phi_k, \zeta) & \tilde{C}_{16}^{(k)}(\phi_k, \zeta) \\ \tilde{C}_{12}^{(k)}(\phi_k, \zeta) & \tilde{C}_{22}^{(k)}(\phi_k, \zeta) & \tilde{C}_{26}^{(k)}(\phi_k, \zeta) \\ \tilde{C}_{16}^{(k)}(\phi_k, \zeta) & \tilde{C}_{26}^{(k)}(\phi_k, \zeta) & \tilde{C}_{66}^{(k)}(\phi_k, \zeta) \end{bmatrix}, \quad (6)$$

in which  $\eta_{nl} = (e_0 a_c)^2$  stands for the scale parameter (Benzair *et al.* 2008, Benguediab *et al.* 2014, Farajpour *et al.* 2018, Zenkour 2018),  $\sigma$  is the stress,  $a_c$  and  $e_0$ , respectively, represent an internal characteristic dimension and a calibration coefficient (Malekzadeh and Shojaei 2013, Farajpour *et al.* 2017). Also, the elastic constant of the  $k$ th plate, nanofiber angle and recovery stresses are indicated by  $\tilde{C}_{ij}^{(k)}$ ,  $\phi_k$  and  $\sigma_{RS}^{(k)}$ , respectively. Finally,  $\nabla^2$  is utilized to denote the Laplace operator (Asemi and Farajpour 2014, Zenkour and Sobhy 2015, Farajpour *et al.* 2016). Let us consider  $n$  small-scale plates containing nanofibers. The recovery stress resultants and nonlocal stress resultants are

$$\{\mathbf{N}_{RS}\} = \sum_{l=1}^n \int_{z_{l-1}}^{z_l} \sigma_{RS}^{(l)} V_{F,l}^{SMA} \{\boldsymbol{\mu}(\phi_l)\} dz, \quad (7)$$

$$\{\mathbf{M}_{RS}\} = \sum_{l=1}^n \int_{z_{l-1}}^{z_l} \sigma_{RS}^{(l)} V_{F,l}^{SMA} \{\boldsymbol{\mu}(\phi_l)\} z dz,$$

$$\{\mathbf{N}\} = \sum_{l=1}^n \int_{z_{l-1}}^{z_l} \{\boldsymbol{\sigma}^{(l)}\} dz, \quad \{\mathbf{M}\} = \sum_{l=1}^n \int_{z_{l-1}}^{z_l} \{\boldsymbol{\sigma}^{(l)}\} z dz, \quad (8)$$

Where

$$\{\mathbf{N}\} = \begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix}, \quad \{\mathbf{M}\} = \begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix}, \quad (9)$$

$$\{\mathbf{N}_{RS}\} = \begin{Bmatrix} N_{xx}^{RS} \\ N_{yy}^{RS} \\ N_{xy}^{RS} \end{Bmatrix}, \quad \{\mathbf{M}_{RS}\} = \begin{Bmatrix} M_{xx}^{RS} \\ M_{yy}^{RS} \\ M_{xy}^{RS} \end{Bmatrix},$$

In view of the above equations (i.e. Eqs. (4)-(9)), one obtains the recovery stress resultants and nonlocal stress resultants as follows

$$\{\mathbf{N}\} - \eta_{nl} \nabla^2 \{\mathbf{N}\} = [\tilde{\mathbf{K}}] \{\mathbf{e}^0\} - [\tilde{\mathbf{F}}] \{\boldsymbol{\kappa}\} + \{\mathbf{N}_{RS}\}, \quad (10)$$

$$\{\mathbf{M}\} - \eta_{nl} \nabla^2 \{\mathbf{M}\} = [\tilde{\mathbf{F}}] \{\mathbf{e}^0\} - [\tilde{\mathbf{S}}] \{\boldsymbol{\kappa}\} + \{\mathbf{M}_{RS}\}, \quad (11)$$

where

$$[\tilde{\mathbf{K}}] = \begin{bmatrix} \tilde{K}_{11} & \tilde{K}_{12} & \tilde{K}_{16} \\ \tilde{K}_{12} & \tilde{K}_{22} & \tilde{K}_{26} \\ \tilde{K}_{16} & \tilde{K}_{26} & \tilde{K}_{66} \end{bmatrix}, \quad (12)$$

$$[\tilde{\mathbf{S}}] = \begin{bmatrix} \tilde{S}_{11} & \tilde{S}_{12} & \tilde{S}_{16} \\ \tilde{S}_{12} & \tilde{S}_{22} & \tilde{S}_{26} \\ \tilde{S}_{16} & \tilde{S}_{26} & \tilde{S}_{66} \end{bmatrix},$$

$$[\tilde{\mathbf{F}}] = \begin{bmatrix} \tilde{F}_{11} & \tilde{F}_{12} & \tilde{F}_{16} \\ \tilde{F}_{12} & \tilde{F}_{22} & \tilde{F}_{26} \\ \tilde{F}_{16} & \tilde{F}_{26} & \tilde{F}_{66} \end{bmatrix},$$

and

$$\tilde{K}_{ij} = \sum_{l=1}^n \tilde{C}_{ij}^{(l)} (z_l - z_{l-1}),$$

$$\tilde{F}_{ij} = \frac{1}{2} \sum_{l=1}^n \tilde{C}_{ij}^{(l)} (z_l^2 - z_{l-1}^2), \quad (13)$$

$$\tilde{S}_{ij} = \frac{1}{3} \sum_{l=1}^n \tilde{C}_{ij}^{(l)} (z_l^3 - z_{l-1}^3).$$

The motion equations of the small-scale system in terms of nonlocal stress resultants are as

$$\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} - m_{sys} \frac{\partial^2 u}{\partial t^2} = 0, \quad (14)$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} - m_{sys} \frac{\partial^2 v}{\partial t^2} = 0, \quad (15)$$

$$\frac{\partial^2 M_{xx}}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_{yy}}{\partial y^2} + \frac{\partial}{\partial y} \left( N_{xy} \frac{\partial w}{\partial x} + N_{yy} \frac{\partial w}{\partial y} \right) + \frac{\partial}{\partial x} \left( N_{xx} \frac{\partial w}{\partial x} + N_{xy} \frac{\partial w}{\partial y} \right) + q_{em} - m_{sys} \frac{\partial^2 w}{\partial t^2} = 0. \quad (16)$$

Here  $q_{em}$  and  $m_{sys}$  are respectively the distributed load induced by the elastic matrix and the system mass per unit area. The distributed load can be expressed as (Akgöz and Civalek 2017, Akgöz and Civalek 2018)

$$q_{em} = -k_n w + k_s \frac{\partial^2 w}{\partial y^2} + k_s \frac{\partial^2 w}{\partial x^2}. \quad (17)$$

Equation (17) is in consistent with the equation of the elastic medium reported in the literature (Beldjelili *et al.* 2016, Bounouara *et al.* 2016, Attia *et al.* 2018, Kadari *et al.* 2018). Substituting Eqs. (10), (11) and (17) into Eqs. (14)-(16), the differential equations for the elastic wave dispersion in the system are derived as

$$\begin{aligned} & \tilde{K}_{11} \frac{\partial^2 u}{\partial x^2} + \tilde{K}_{66} \frac{\partial^2 u}{\partial y^2} + 2\tilde{K}_{16} \frac{\partial^2 u}{\partial y \partial x} + (\tilde{K}_{66} + \tilde{K}_{12}) \frac{\partial^2 v}{\partial y \partial x} \\ & + \tilde{K}_{16} \frac{\partial^2 v}{\partial x^2} + \tilde{K}_{26} \frac{\partial^2 v}{\partial y^2} - \left( \tilde{F}_{11} \frac{\partial^3 w}{\partial x^3} + (\tilde{F}_{12} + 2\tilde{F}_{66}) \frac{\partial^3 w}{\partial y^2 \partial x} \right. \\ & \left. + 3\tilde{F}_{16} \frac{\partial^3 w}{\partial y \partial x^2} + \tilde{F}_{26} \frac{\partial^3 w}{\partial y^3} \right) = m_{sys} \frac{\partial^2 u}{\partial t^2} - m_{sys} \eta_{nl} \nabla^2 \frac{\partial^2 u}{\partial t^2}, \end{aligned} \quad (18)$$

$$\begin{aligned} & \tilde{K}_{16} \frac{\partial^2 u}{\partial x^2} + \tilde{K}_{26} \frac{\partial^2 u}{\partial y^2} + (\tilde{K}_{66} + \tilde{K}_{12}) \frac{\partial^2 u}{\partial y \partial x} \\ & + \tilde{K}_{66} \frac{\partial^2 v}{\partial x^2} + \tilde{K}_{22} \frac{\partial^2 v}{\partial y^2} + 2\tilde{K}_{26} \frac{\partial^2 v}{\partial y \partial x} \\ & - \left( \tilde{F}_{16} \frac{\partial^3 w}{\partial x^3} + 3\tilde{F}_{26} \frac{\partial^3 w}{\partial y^2 \partial x} + (\tilde{F}_{12} + 2\tilde{F}_{66}) \frac{\partial^3 w}{\partial y \partial x^2} + \tilde{F}_{22} \frac{\partial^3 w}{\partial y^3} \right) \\ & = m_{sys} \frac{\partial^2 v}{\partial t^2} - m_{sys} \eta_{nl} \nabla^2 \frac{\partial^2 v}{\partial t^2}, \end{aligned} \quad (19)$$

$$\begin{aligned} & \tilde{F}_{11} \frac{\partial^3 u}{\partial x^3} + \tilde{F}_{26} \frac{\partial^3 u}{\partial y^3} + 3\tilde{F}_{16} \frac{\partial^3 u}{\partial y \partial x^2} \\ & + (2\tilde{F}_{66} + \tilde{F}_{12}) \frac{\partial^3 u}{\partial x \partial y^2} + \tilde{F}_{16} \frac{\partial^3 v}{\partial x^3} + \\ & \tilde{F}_{22} \frac{\partial^3 v}{\partial y^3} + (2\tilde{F}_{66} + \tilde{F}_{12}) \frac{\partial^3 v}{\partial y \partial x^2} + 3\tilde{F}_{26} \frac{\partial^3 v}{\partial y^2 \partial x} \\ & - \left( \tilde{S}_{11} \frac{\partial^4 w}{\partial x^4} + \tilde{S}_{22} \frac{\partial^4 w}{\partial y^4} + 2(2\tilde{S}_{66} + \tilde{S}_{12}) \frac{\partial^4 w}{\partial y^2 \partial x^2} + 4\tilde{S}_{16} \frac{\partial^4 w}{\partial y \partial x^3} \right. \\ & \left. + 4\tilde{S}_{26} \frac{\partial^4 w}{\partial x \partial y^3} \right) + N_{xx}^{RS} \frac{\partial^2 w}{\partial x^2} + N_{yy}^{RS} \frac{\partial^2 w}{\partial y^2} \\ & + 2(N_{xy}^{RS} + N_{xy}^{SP}) \frac{\partial^2 w}{\partial x \partial y} \\ & - \mu_{nl} \left( N_{xx}^{RS} \frac{\partial^4 w}{\partial x^4} + N_{yy}^{RS} \frac{\partial^4 w}{\partial y^4} + 2(N_{xy}^{RS} + N_{xy}^{SP}) \frac{\partial^4 w}{\partial x^3 \partial y} \right. \\ & \left. + (N_{xx}^{RS} + N_{yy}^{RS}) \frac{\partial^4 w}{\partial y^2 \partial x^2} + 2(N_{xy}^{RS} + N_{xy}^{SP}) \frac{\partial^4 w}{\partial x \partial y^3} \right) \\ & - k_n w + k_n \eta_{nl} \left( \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial x^2} \right) + k_s \left( \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial x^2} \right) \\ & - k_s \eta_{nl} \left( \frac{\partial^4 w}{\partial y^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial x^4} \right) = m_{sys} \frac{\partial^2 w}{\partial t^2} - m_{sys} \eta_{nl} \nabla^2 \frac{\partial^2 w}{\partial t^2}. \end{aligned} \quad (20)$$

More detail about the derivation of the above equations is given in Appendix B.  $N_{xy}^{SP}$  denotes the shear preload.

$N_{xy}^{SP}$  is related to the shear initial stress ( $\sigma_{xy(l)}^0$ ) by the following relation

$$N_{xy}^{SP} = \sum_{l=1}^n \int_{z_{l-1}}^{z_l} \sigma_{xy(l)}^0 dz. \quad (21)$$

The following expressions are assumed for the displacement components of the small-scale system containing nanofibers so as to extract the phase and group velocities (Ebrahimi *et al.* 2016)

$$\begin{aligned} u(x, y, t) &= \hat{A} \exp(-i\omega t + ik_y y + ik_x x), \\ v(x, y, t) &= \hat{B} \exp(-i\omega t + ik_y y + ik_x x), \\ w(x, y, t) &= \hat{C} \exp(-i\omega t + ik_y y + ik_x x), \end{aligned} \quad (22)$$

where  $k_x$  and  $k_y$  are, respectively, utilized to indicate the wave numbers in longitudinal and width directions,  $\hat{A}$ ,  $\hat{B}$  and  $\hat{C}$  represent wave amplitude constants, also,  $\omega$  denotes the system frequency. Substituting Eq. (22) into Eqs. (18)-(20) yields

$$[\tilde{\Sigma}]\{\Delta\} - \omega^2[\tilde{\Gamma}]\{\Delta\} = 0, \quad (23)$$

where

$$\begin{aligned} [\tilde{\Sigma}] &= \begin{bmatrix} \tilde{S}_{11} & \tilde{S}_{12} & \tilde{S}_{13} \\ \tilde{S}_{21} & \tilde{S}_{22} & \tilde{S}_{23} \\ \tilde{S}_{31} & \tilde{S}_{32} & \tilde{S}_{33} \end{bmatrix}, \\ [\tilde{\Gamma}] &= \begin{bmatrix} \tilde{F}_{11} & \tilde{F}_{12} & \tilde{F}_{13} \\ \tilde{F}_{21} & \tilde{F}_{22} & \tilde{F}_{23} \\ \tilde{F}_{31} & \tilde{F}_{32} & \tilde{F}_{33} \end{bmatrix}, \\ \{\hat{\Delta}\} &= \begin{Bmatrix} \hat{A} \\ \hat{B} \\ \hat{C} \end{Bmatrix}. \end{aligned} \quad (24)$$

The size-dependent dispersion relation for the small-scale system containing nanofibers is expressed as

$$[[\tilde{\Sigma}] - \omega^2[\tilde{\Gamma}]] = 0. \quad (25)$$

In Eq. (25), “[|]” is the determinant of a matrix. Assuming that the wave numbers in longitudinal and width directions are the same ( $k=k_x=k_y$ ), we have  $K = \sqrt{k_k^2 + k_y^2} = \sqrt{2}k$  in which  $K$  is the general wave number. The phase and group velocities are obtained as

$$c_p = \frac{\omega}{K}, \quad (26)$$

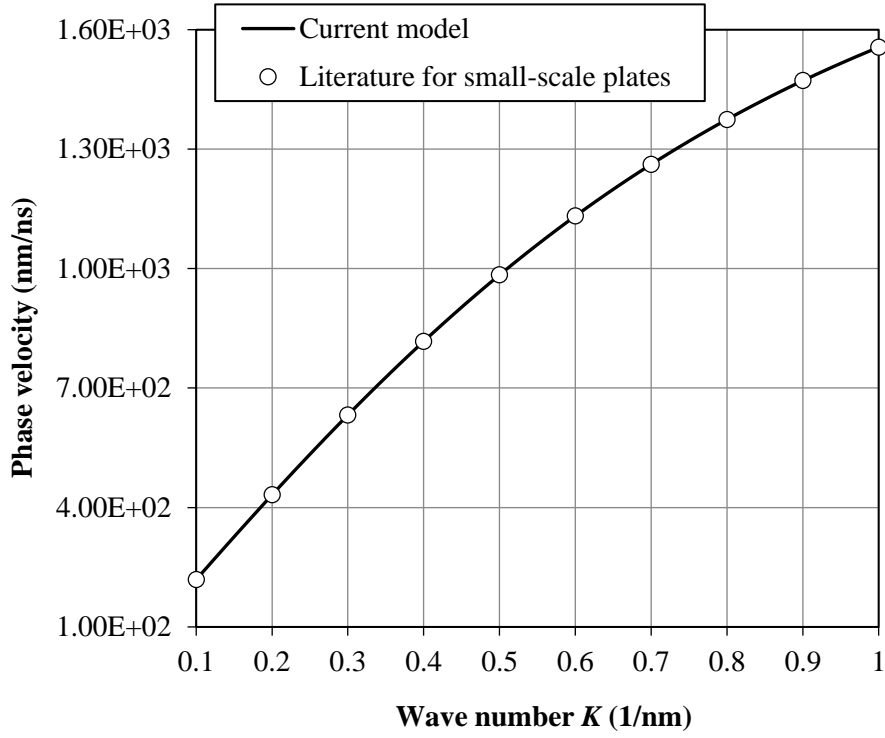


Fig. 2 Phase velocities of the current model in comparison with literature data (Wang *et al.* 2010)

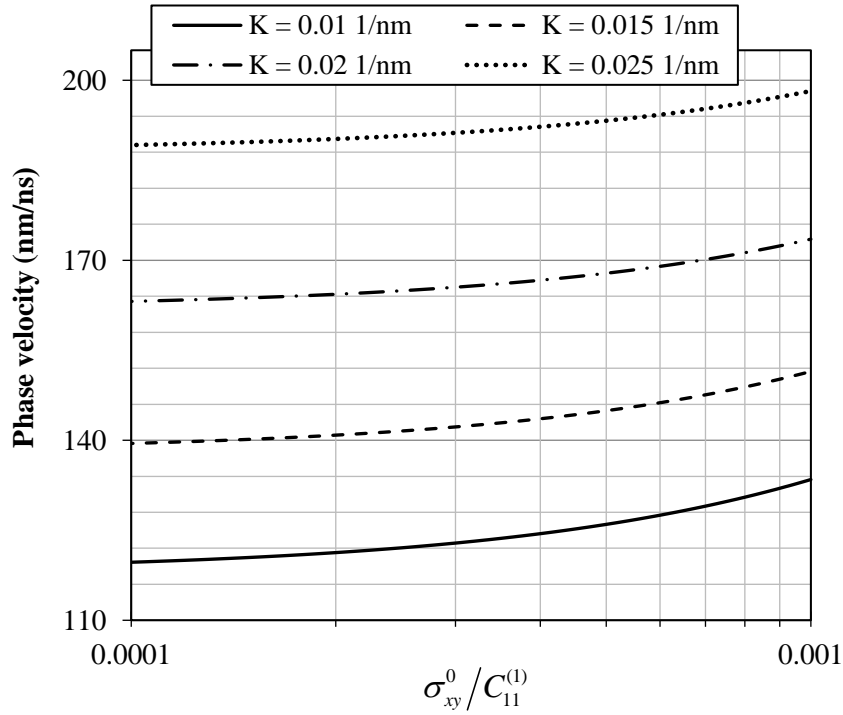


Fig. 3 Influences of shear in-plane preload on the phase velocity of the plate containing nanofibers

$$c_g = \frac{d\omega}{dK}. \quad (27)$$

Here the group velocity is indicated by  $c_g$  while  $c_p$  denotes the phase velocity of the small-scale system containing nanofibers

### 3. Results and discussion

For comparison purposes, the variation of phase velocities with the wave number is plotted in Fig. 2, the results are compared with those extracted by Wang *et al.* (Wang *et al.* 2010) for small-scale plates. To make a

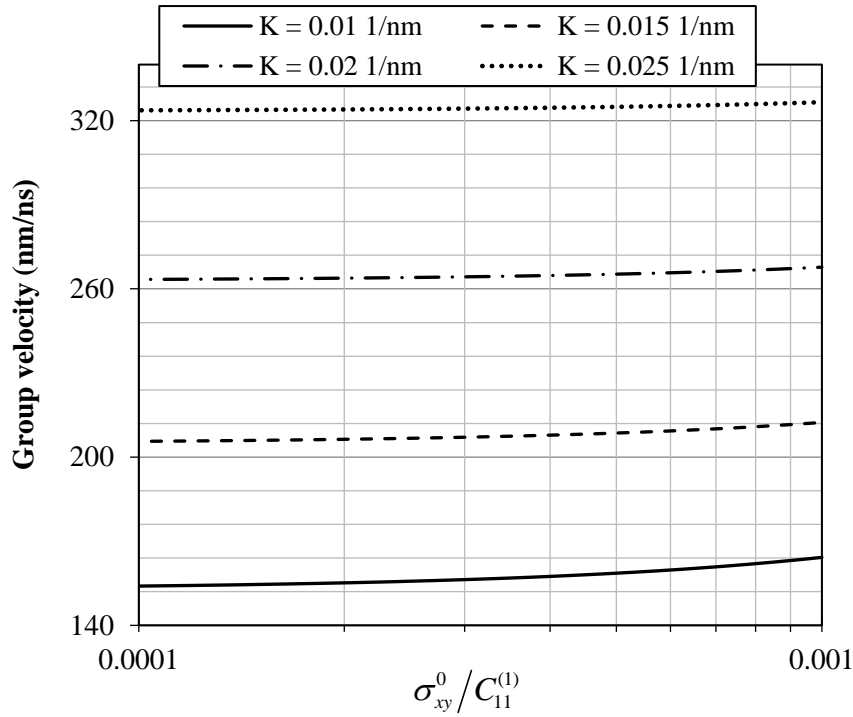


Fig. 4 Influences of shear in-plane preload on the group velocity of the plate containing nanofibers

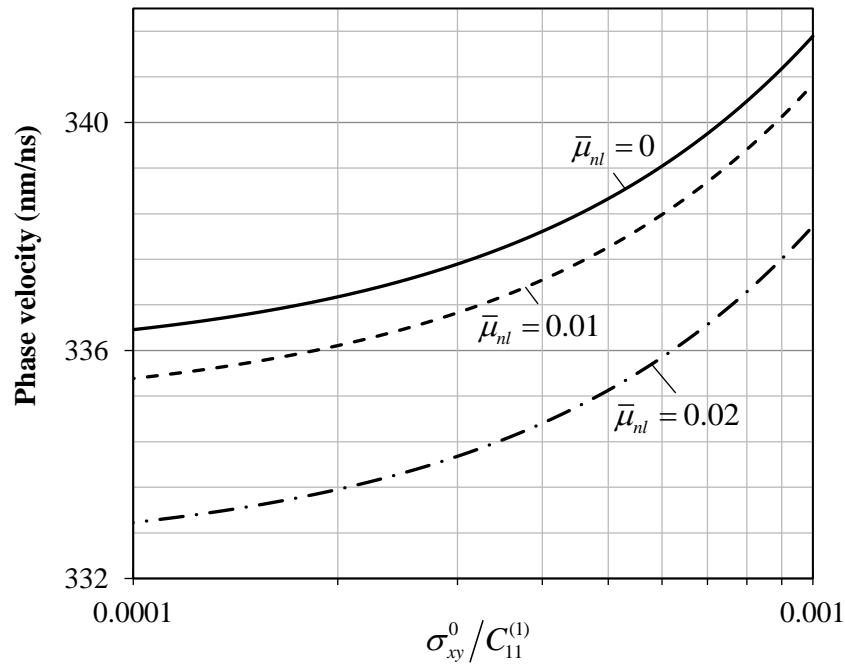


Fig. 5 Influences of scale parameter and shear preload on the phase velocity of the plate containing nanofibers

reasonable comparison, nanofiber effects are ignored in this figure. For the ultrasmall plate, we assume that the plate thickness and nonlocal parameter are 0.34 and 1 nm, respectively. Furthermore, the density, Poisson's ratio and elasticity modulus are chosen as 2250 kg/m<sup>3</sup>, 0.25, 1.06 TPa, respectively (Wang *et al.* 2010). An excellent

agreement is seen between the results and those available in the literature.

Figures 3 and 4 indicate the influences of shear prestress on the phase and group velocities of the small-scale plate containing nanofibers. The shear in-plane preload is assumed to be uniform. The nanoscale system consists of

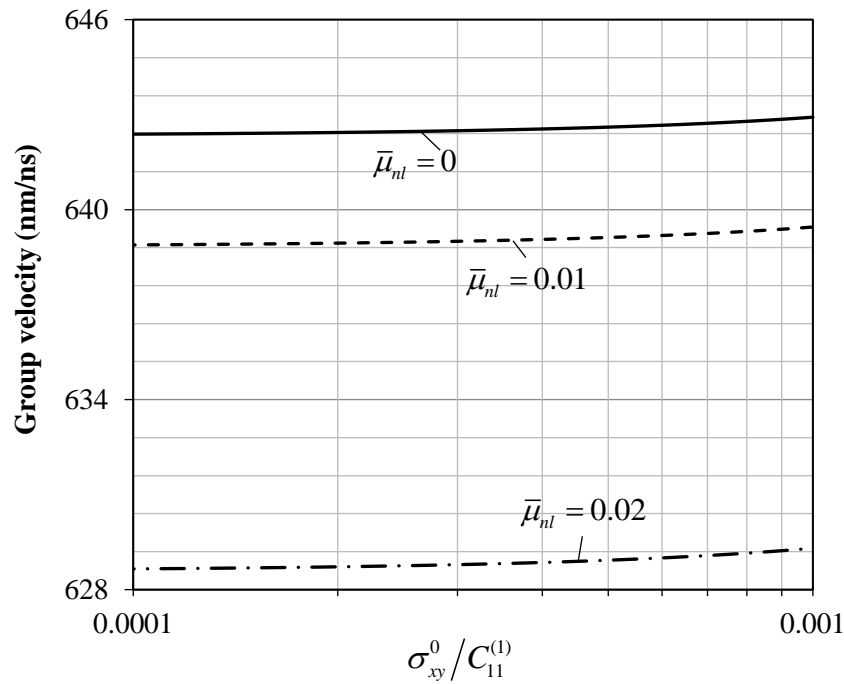


Fig. 6 Influences of scale parameter and shear preload on the group velocity of the plate containing nanofibers

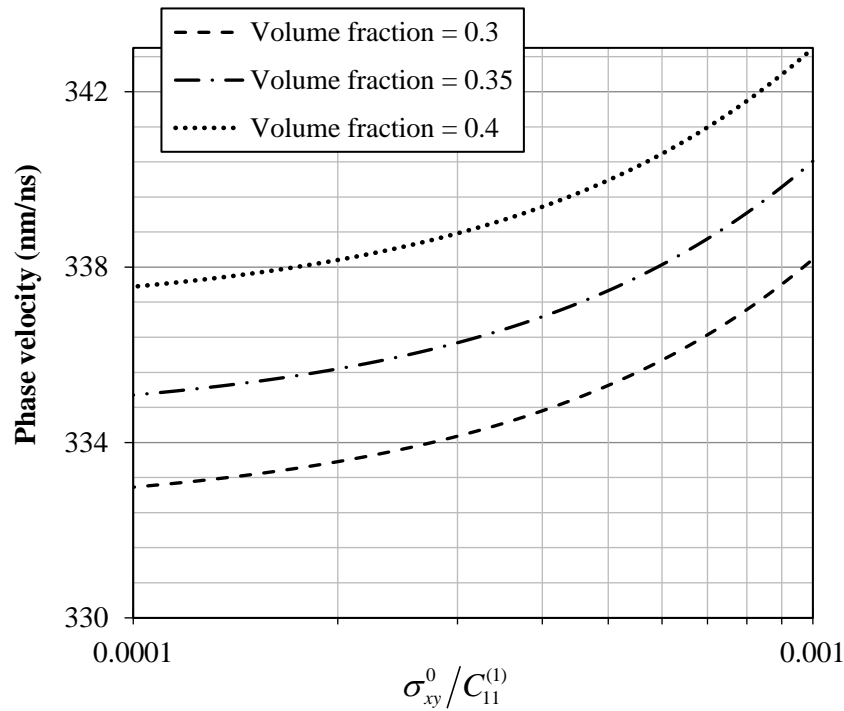


Fig. 7 Influences of volume fraction and shear preload on the phase velocity of the plate containing nanofibers

five plates containing nanofibers with the same material properties ( $v_F=0.3$ ,  $\rho_F=6450$  kg/m<sup>3</sup>,  $\sigma_{RS}=0.2$  GPa,  $E_F=30$  GPa,  $V_F=0.3$ , and  $\phi=0^\circ$  for nanofibers;  $v_L=0.3$ ,  $\rho_L=1600$  kg/m<sup>3</sup> and  $E_L=3.44$  GPa for each plate (Farajpour *et al.* 2018)). Each layer's aspect ratio, length and thickness are 1, 150 nm and 3 nm, respectively. The elastic medium is

not taken into consideration. The nonlocal parameter-to-length ratio is considered as 0.02. From the results given in Fig. 3, it is seen that the phase velocity of small-scale plates is greater for greater values of shear prestress. In addition, the shear in-plane preload affects the group velocity. Greater values of shear preload result in slightly greater

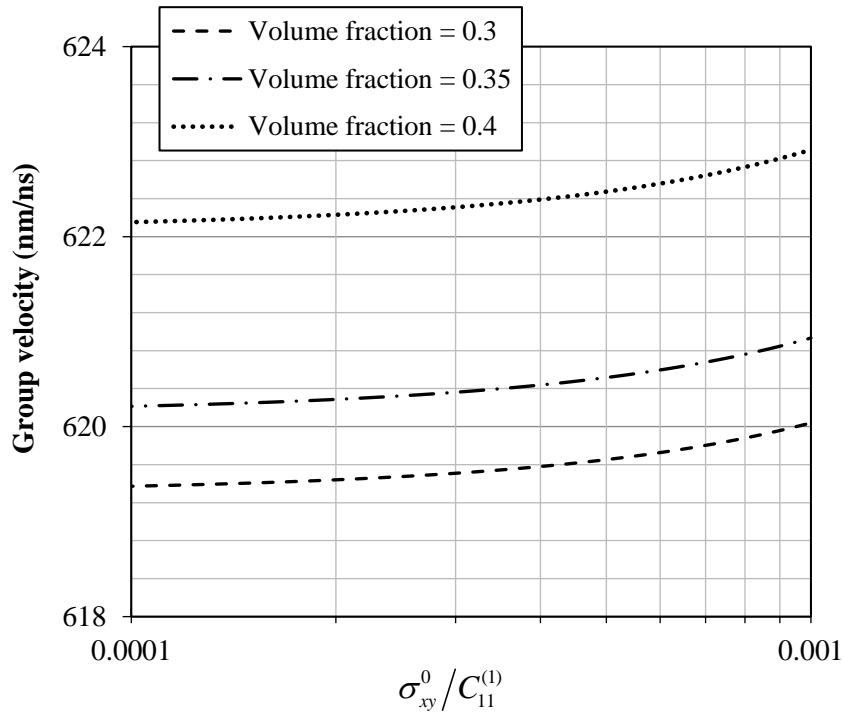


Fig. 8 Influences of volume fraction and shear preload on the group velocity of the plate containing nanofibers

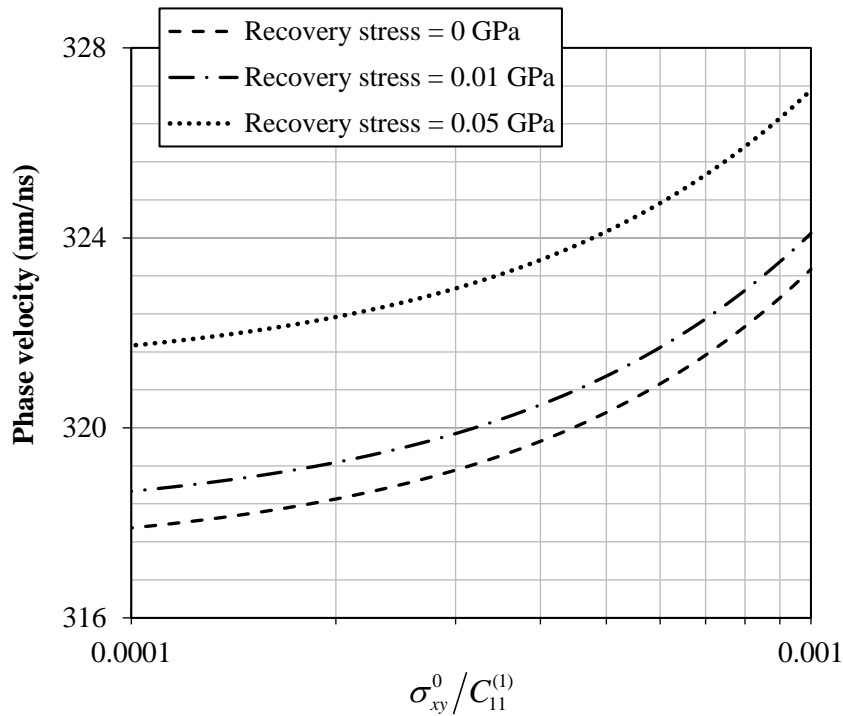


Fig. 9 Influences of recovery stress and shear preload on the phase velocity of the plate containing nanofibers

group velocities.

The influences of shear in-plane preload together with scale influences are indicated in Figs. 5 and 6. The total wave number is 0.05 1/nm. The elastic medium is not taken into consideration. Stronger scale effects yield lower group and phase velocities. It is rooted in the fact that stronger

scale effects correspond to a lower total stiffness for the small-scale system; consequently, lower values of structural stiffness lead to lower frequencies. From Eq. (26), it can be seen that lower frequencies result in lower phase velocities.

Figures 7 and 8, respectively, depict the effects of shear prestress and volume fraction on the phase and group



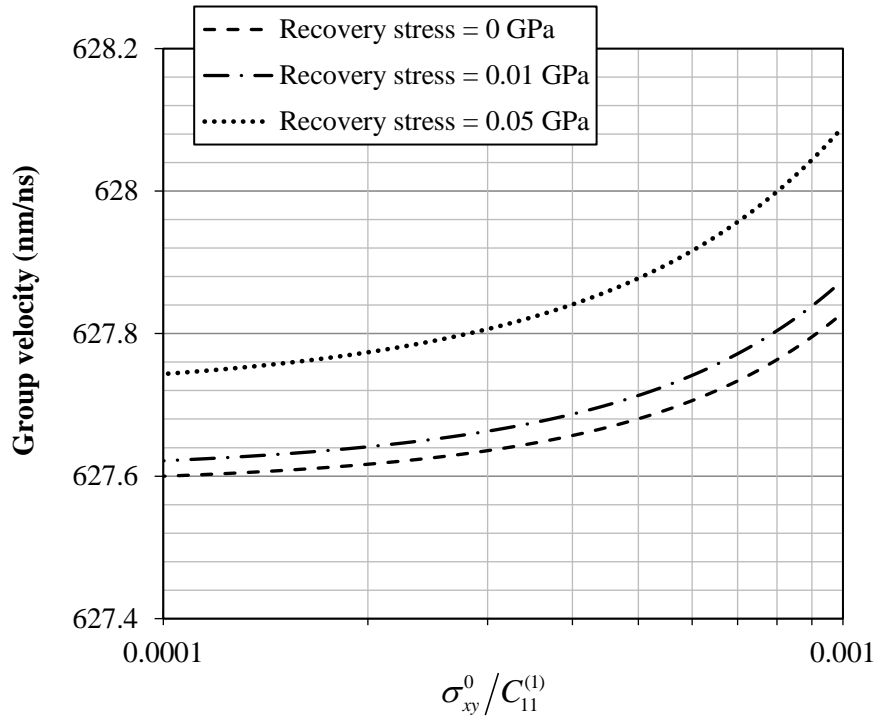


Fig. 10 Influences of recovery stress and shear preload on the group velocity of the plate containing nanofibers

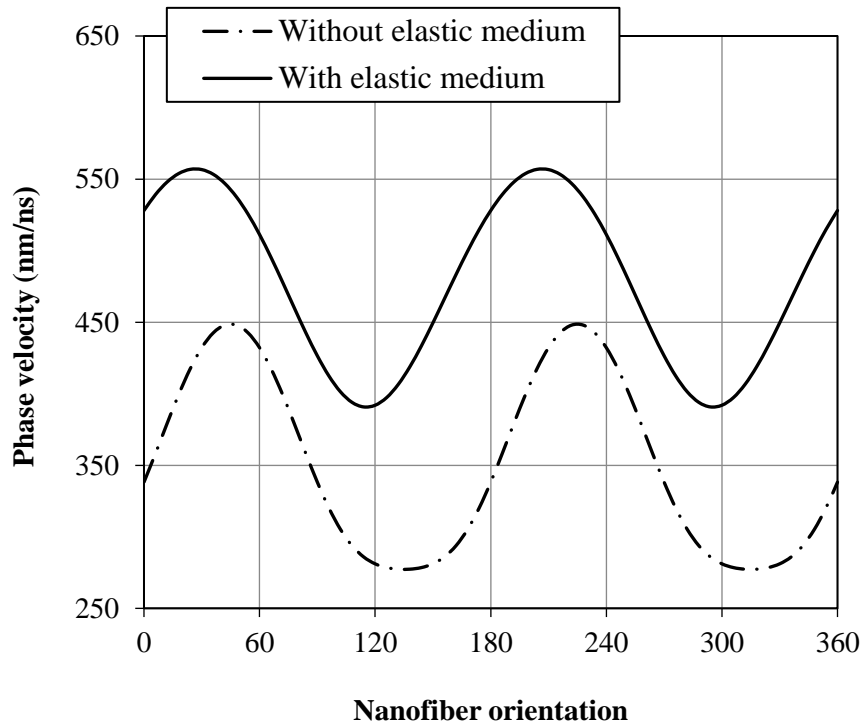


Fig. 11 Influences of elastic medium on the phase velocity of the plate containing nanofibers  $\sigma_{xy}^0 / C_{11}^{(1)} = 0.001$

velocities of the small-scale plate containing nanofibers. The dimensionless scale parameter and total wave number are, respectively, given by 0.02 and 0.05 1/nm. Generally, greater volume fractions yield greater phase and group velocities. This is because increasing the volume fraction of nanofibers increases the stiffness of the composite structure.

To understand the influence of recovery stress on the

phase and group velocities of small-scale plates containing nanofibers, Figs. 9 and 10 are plotted. The dimensionless scale parameter, total wave number and volume fraction are, respectively, given by 0.02, 0.05 1/nm and 0.3. Comparing Figs. 7 and 8 with Figs. 9 and 10, one can conclude that the wave propagation characteristics, especially the group velocity, are less sensitive to the

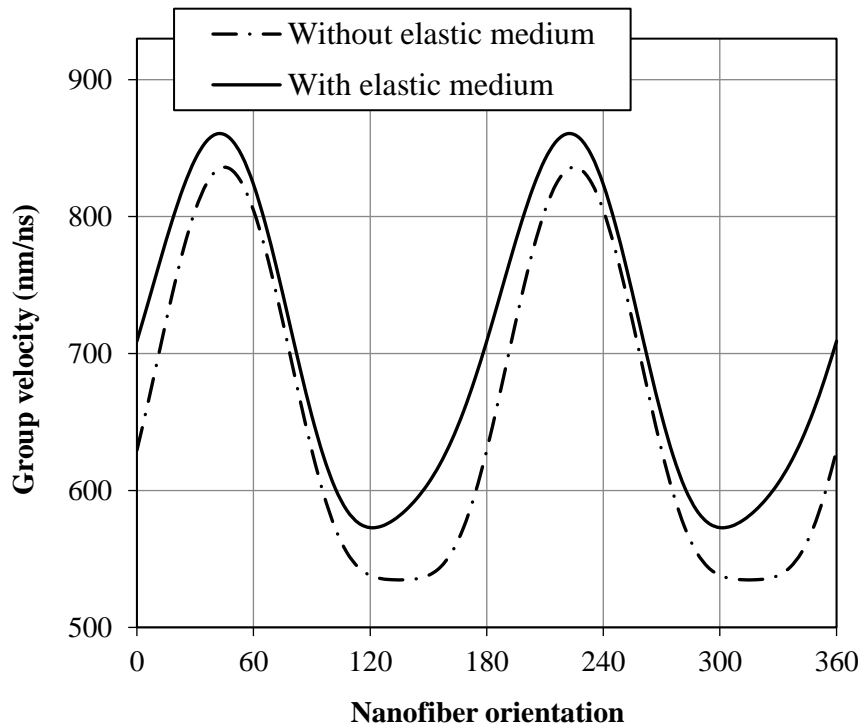


Fig. 12 Influences of elastic medium on the group velocity of the plate containing nanofibers  $\sigma_{xy}^0/C_{11}^{(1)} = 0.001$

recovery stress in comparison with the volume fraction. Nevertheless, the phase and group velocities for nanofibers with high recovery stresses are higher than those for small recovery stresses.

Figure 11 is plotted to illustrate the influence of elastic medium on the phase velocity while Fig. 12 is aimed at illustrating the influence of elastic medium on the group velocity. The dimensionless scale parameter, total wave number, volume fraction and recovery stress are, respectively, given by 0.02, 0.05 1/nm, 0.3 and 0.2 GPa. The normal and shear stiffness coefficients of the elastic medium are  $k_n a^4/\hat{S}_{II} = 50$  and  $k_s a^2/\hat{S}_{II} = 50$ , respectively. It can be concluded that the phase and group velocities are enhanced when the small-scale plate is embedded in an elastic medium. This is due to the fact that utilizing an elastic medium increases the total structural stiffness of the system, and this consequently increases the value of phase and group velocities.

#### 4. Conclusions

The effects of shear in-plane preload on the wave propagation response of small-scale plate containing nanofibers were explored. To develop a more realistic model, the small-scale system was assumed to be embedded in an elastic matrix. A size-dependent model of plates was proposed for the problem via nonlocal elasticity. The size-dependent differential equations were derived for motions along all directions. The differential equations were analytically solved so as to extract the phase and group velocities of the plate subjected to shear preload. The present results indicated that the phase velocity of small-

scale plates containing nanofibers is greater for greater values of shear preload. Furthermore, greater values of shear preload result in slightly greater group velocities. It was also indicated that stronger scale effects yield lower group and phase velocities. The wave propagation characteristics are more sensitive to the volume fraction in comparison with the recovery stress. Moreover, as the plate is embedded in an elastic medium, both phase and group velocities are enhanced.

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## Appendix A

For a particular type of nanofibers such as shape memory alloy nanofibers, Brinson's model can be used to obtain the elasticity modulus of nanofibers (Brinson 1993)

$$E_F(\zeta) = \frac{E_{aus} E_{mar}}{\zeta E_{aus} + (1 - \zeta) E_{mar}}, \quad (A1)$$

where "aus" and "mar" represent the nanofiber austenite and martensite phases, respectively.  $\zeta$  can be expressed as

$$\zeta = \zeta_{str} + \zeta_{tem}. \quad (A2)$$

where

$$\begin{aligned} \zeta_{str} &= \zeta_{str0} - \frac{(\zeta_0 - \zeta)}{\zeta_0} \zeta_{str0}, \\ \zeta_{tem} &= \zeta_{tem0} - \frac{(\zeta_0 - \zeta)}{\zeta_0} \zeta_{tem0}. \end{aligned} \quad (A3)$$

In the above equations, "0", "tem" and "str" stand for the initial condition, temperature and stress, respectively. Applying Brinson's model, the following equation is also introduced for  $\zeta$

$$\zeta = \frac{\zeta_0}{2} \left( 1 + \cos \left[ \gamma_A (C_A T - \sigma - C_A A_{sta}) \right] \right) \quad (A4)$$

for  $T > A_{sta}$  and  $C_A (T - A_{fin}) < \sigma < C_A (T - A_{sta})$ ,

where

$$\gamma_A = \pi \left[ C_A (A_{fin} - A_{sta}) \right]^{-1}. \quad (A5)$$

Here  $\sigma$  and  $T$  are respectively the stress and temperature;  $C_A$ ,  $A_{fin}$  and  $A_{sta}$ , respectively, indicate the critical stress slope, finish and start temperatures for the austenite phase.

## Appendix B

To give more detail about the derivation of the governing differential equations, the motion equation along the  $x$  direction, as an example, is derived in the following. Other motion equations are derived in a similar way. Using Eqs. (5), (9), (10) and (12), the force resultants are obtained as

$$\begin{aligned} N_{xx} &= \tilde{K}_{11} e_{xx}^0 + \tilde{K}_{12} e_{yy}^0 + \tilde{K}_{16} \gamma_{xy}^0 \\ &- \left( \tilde{F}_{11} \kappa_{xx} + \tilde{F}_{12} \kappa_{yy} + \tilde{F}_{16} \kappa_{xy} \right) + N_{xx}^{RS} + \eta_{nl} \nabla^2 N_{xx}, \\ N_{yy} &= \tilde{K}_{12} e_{xx}^0 + \tilde{K}_{22} e_{yy}^0 + \tilde{K}_{26} \gamma_{xy}^0 \\ &- \left( \tilde{F}_{12} \kappa_{xx} + \tilde{F}_{22} \kappa_{yy} + \tilde{F}_{26} \kappa_{xy} \right) + N_{yy}^{RS} + \eta_{nl} \nabla^2 N_{yy}, \\ N_{xy} &= \tilde{K}_{16} e_{xx}^0 + \tilde{K}_{26} e_{yy}^0 + \tilde{K}_{66} \gamma_{xy}^0 \\ &- \left( \tilde{F}_{16} \kappa_{xx} + \tilde{F}_{26} \kappa_{yy} + \tilde{F}_{66} \kappa_{xy} \right) + N_{xy}^{RS} + \eta_{nl} \nabla^2 N_{xy}. \end{aligned} \quad (B1)$$

Substituting the right-hand sides of Eq. (B1) into Eq. (14), one obtains

$$\begin{aligned} &\tilde{K}_{11} \frac{\partial^2 u}{\partial x^2} + \tilde{K}_{66} \frac{\partial^2 u}{\partial y^2} + 2\tilde{K}_{16} \frac{\partial^2 u}{\partial x \partial y} \\ &+ \tilde{K}_{16} \frac{\partial^2 v}{\partial x^2} + \tilde{K}_{26} \frac{\partial^2 v}{\partial y^2} + (\tilde{K}_{12} + \tilde{K}_{66}) \frac{\partial^2 v}{\partial x \partial y} \\ &- \left( \tilde{F}_{11} \frac{\partial^3 w}{\partial x^3} + (\tilde{F}_{12} + 2\tilde{F}_{66}) \frac{\partial^3 w}{\partial x \partial y^2} + 3\tilde{F}_{16} \frac{\partial^3 w}{\partial y \partial x^2} + \tilde{F}_{26} \frac{\partial^3 w}{\partial y^3} \right) \\ &+ \eta_{nl} \nabla^2 \left( \frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} \right) = m_{sys} \frac{\partial^2 u}{\partial t^2}. \end{aligned} \quad (B2)$$

Again using Eq. (14), the last term on the right-hand side of Eq. (B2) is obtained as

$$\eta_{nl} \nabla^2 \left( \frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} \right) = m_{sys} \eta_{nl} \nabla^2 \frac{\partial^2 u}{\partial t^2}. \quad (B3)$$

Now substituting Eq. (B3) into Eq. (B2) gives the motion equation along the  $x$  direction

$$\begin{aligned} &\tilde{K}_{11} \frac{\partial^2 u}{\partial x^2} + \tilde{K}_{66} \frac{\partial^2 u}{\partial y^2} + 2\tilde{K}_{16} \frac{\partial^2 u}{\partial x \partial y} + (\tilde{K}_{66} + \tilde{K}_{12}) \frac{\partial^2 v}{\partial x \partial y} \\ &+ \tilde{K}_{16} \frac{\partial^2 v}{\partial x^2} + \tilde{K}_{26} \frac{\partial^2 v}{\partial y^2} - \left( \tilde{F}_{11} \frac{\partial^3 w}{\partial x^3} + (\tilde{F}_{12} + 2\tilde{F}_{66}) \frac{\partial^3 w}{\partial x \partial y^2} \right. \\ &\left. + 3\tilde{F}_{16} \frac{\partial^3 w}{\partial y \partial x^2} + \tilde{F}_{26} \frac{\partial^3 w}{\partial y^3} \right) = m_{sys} \frac{\partial^2 u}{\partial t^2} - m_{sys} \eta_{nl} \nabla^2 \frac{\partial^2 u}{\partial t^2}. \end{aligned} \quad (B4)$$

It should be noticed that the differential equations of motions along the  $y$  and  $z$  directions are derived in a similar way.