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Evolutionary-base finite element model updating and damage detection using modal testing results

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Abstract. This research focuses on finite element model updating and damage assessment of structures at element level based on global nondestructive test results. For this purpose, an optimization system is generated to minimize the structural dynamic parameters discrepancies between numerical and experimental models. Objective functions are selected based on the square of Euclidean norm error of vibration frequencies and modal assurance criterion of mode shapes. In order to update the finite element model and detect local damages within the structural members, modern optimization techniques is implemented according to the evolutionary algorithms to meet the global optimized solution. Using a simulated numerical example, application of genetic algorithm (GA), particle swarm (PSO) and artificial bee colony (ABC) algorithms are investigated in FE model updating and damage detection problems to consider their accuracy and convergence characteristics. Then, a hybrid multi stage optimization method is presented merging advantages of PSO and ABC methods in finding damage location and extent. The efficiency of the methods have been examined using two simulated numerical examples, a laboratory dynamic test and a high-rise building field ambient vibration test results. The implemented evolutionary updating methods show successful results in accuracy and speed considering the incomplete and noisy experimental measured data.

Keywords: Finite Element Model updating; GA; PSO; ABC; hybrid optimization; damage detection, modal analysis

1. Introduction

In recent decades, many researchers have been eager to concentrate on structural health monitoring of important mechanical, industrial and civil engineering structures (Gentile 2006, Rahbari *et al.* 2015, Petrovic-Kotur and Pavic 2016). Potential risks of natural disasters and high investments in reconstructing the infrastructures are the main reasons for great tendency of employers and industrial managers in evaluating the performance of current structures. Destructive and nondestructive tests are two main approaches concern about identifying the structural behavior. Despite all their advantages, destructive approaches may cause damage in the tested structural members, besides, they are limited to local identification of structural behavior. Localized nondestructive experimental methods such as acoustic or ultrasonic methods, magnetic or thermal field methods, require initial information about the damage zones and accessibility of inspected portion of the structure (Teughels *et al.* 2002). Therefore, the testing techniques which lead to global understanding about the characteristics of structures can be considered as reasonable choices for structural evaluation.

Vibration-based damage identification methods are used as the global approaches and integrated health monitoring systems to detect structural changes based on dynamic properties (Titurus and Friswell 2014, Zhang *et al.* 2016). Based on whether using an analytical model or not, these methods can be classified into model based (parametric) and non-model based (non-parametric) techniques, respectively. The parametric approaches are based on adjusting some parameters within the model of structure, i.e., their performances depend on the model's quality. Non-parametric methods do not need a detailed model of structure, but most of them lack a solid theoretical and physical bases, excluding damage locating vector technique (Bernal 2002) and local flexibility method (Reynders and De Roeck 2010, Reynders *et al.* 2010). This research focuses on model based damage identification method. Doebling *et al.* (1998) gave an extensive overview of vibration-based damage detection methods.

In this arena, damage is represented as the decrease in structural stiffness, which causes the changing in the vibrational data of structure (Teughels and De Roeck 2004). Natural frequencies and mode shapes are the most common dynamic parameters used in damage recognition (Fox 1992, Bicanic and Chen 1997, Salawu 1997). Using the natural frequencies has the advantage of high accuracy in measurement, moreover, its usage is simple and low cost. Therefore, it can be practical in localizing the damaged zones. The location and severity of deficiencies can be better approximated applying mode shapes along with

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natural frequencies (Jaishi and Ren 2006). However, considering the measurement of mode shapes, many sensors are needed for large scale and complex structure. Moreover, it has less accuracy comparing to measurement of dynamic frequencies (Salawu and Williams 1995, Lam and Yin 2011). Other damage indicators presented in the literature are mode shape curvature (Wahab and De Roeck 1999), modal strain energy (Ren and De Roeck 2002, Shi *et al.* 2002, Wang *et al.* 2013), modal flexibility (Jaishi and Ren 2006, Du *et al.* 2017) and multiple modal residuals (Titurus and Friswell 2014).

Model based damage estimation techniques often use an inverse algorithm, generally called model updating, with the aim of adjusting analytical model to the experimental vibration results (Wei and Lv 2015, Lu *et al.* 2017). Two main approaches, direct and iterative methods, are applied for updating Finite Element (FE) models (Carvalho *et al.* 2007, Liu *et al.* 2013). In the former, a closed form direct solution is used to calculate global stiffness and/or mass matrices and offer the models capable of representing measured modal data exactly. No updated physical parameter is taken into account in the direct methods. This may cause deficiency in the connectivity of nodes, and generate asymmetric and populated (instead of banded) updated stiffness and/or mass matrices, all these are physically meaningless (Marwala 2010). Iterative methods update physical properties of FE model through optimization process to minimize the difference between numerical and experimental vibrational data. The success of these methods is related to the definition of objective function, capability of selected optimization method and quality of measured data. Mottershead and Frisswell (2013) have represented a complete overview of Finite Element Model Updating (FEMU) approaches.

Optimization techniques are used in the iterative finite element model updating methods to adjust uncertain parameters through matching updated numerical model response to the measured one. Classical optimization methods such as Quasi Newton, Gauss Newton, nonlinear conjugate gradient and Nelder Mead Simplex have disadvantages of converging the results to local minimums in multiple extremum optimization problems. Modern evolutionary based methods have random initial assumed solutions and try to converge the results to the best optimum solution within the whole search space. By the advent of computers with higher performance, the slow speed of these methods due to the high number of function evaluation and iterations is less problematic. Because of the good capability of finding the global minimum, they can yield to better results. Perera and Torres (2006) used Genetic Algorithm (GA) for structural damage detection considering eigenvalue equation along with modified total modal assurance criterion as the goal function. Saada *et al.* (2013) implemented Particle Swarm Optimization (PSO) method in damage identification in an experimental beam. In order to combine the efficiency of different methods, some hybrid methods have been designed and studied. Begambre and Laier (2009) integrated an iterative and a direct method in structural damage identification by using a hybrid PSO and Nelder-Mead algorithm. Artificial Bees Colony (ABC),

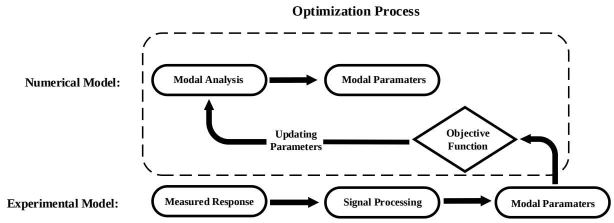


Fig. 1 Finite element model updating algorithm

proposed by Karaboga and Basturk (2007), was applied in different electrical and industrial engineering problems (Hemamalini and Simon 2011) and rarely used in structural and mechanical engineering field.

In this research, a finite element model updating method for damage detection is explored, using intelligent optimization techniques. The objective function has been selected as the square of Euclidean norm error between numerical and experimental eigenvalues as well as Modal Assurance Criteria (MAC) (Allemang 2003) value of mode shapes. Here, the main aim is to investigate application and characteristics of ABC, PSO and GA optimization methods in structural model updating problems. After exploring advantages and disadvantages of these methods, a hybrid optimization method is presented using positive points of ABC and PSO in optimization. As it is concentrated on beam elements, the updating parameters are selected as elemental bending stiffness. In ABC-PSO hybrid method, the speed of convergence is increased by reducing the number of updating parameters. Concerning the experimental conditions, only the information of first few modes can be measured. Furthermore, the presence of noise in the measured data can surely affect the accuracy of final results. This report is organized in four sections.

Section 1: introduction,

Section 2: finite element model updating procedure including objective function and optimization methods,

Section 3: Verifying the effectiveness of proposed method and its application in structural engineering using a numerical simulation and some experimental investigation data.

Section 4: brief summary of the obtained results.

2. Finite Element Model updating procedure

Finite element model updating method is an optimization process minimizing the differences between experimental and numerical model data. Many factors such as construction faults, aging and external unpredictable loads may cause discrepancy between the current measured structural characteristics and the initial FE model correspond to the as-built documents. As the experimental data represent the current realistic behavior of structure, the goal is to minimize an objective function by adjusting the uncertain properties of FE model (updating parameters). Fig. 1 presents the flowchart of FEMU algorithm.

Different FEMU methods within the literature have almost the same algorithms (Friswell and Mottershead 2013). During these algorithms, the goal function is considered based on updating uncertain parameters to match structural desired characteristics. This research focuses on the dynamic responses as the updating criteria. The experimental real behavior data may be achieved by performing field full scale or lab tests. After recording the responses using appropriate sensors, the signal processing methods will be implemented to reach the required dynamic properties. By constructing the objective function using numerical and experimental dynamic results, an appropriate optimization method should be selected to reach the fittest FE model for the studied structure. Two main parts of FE model updating algorithm are explained in the following.

2.1 Objective function

The finite element representation of an undamped multi degree of freedom structural system consisted of n second order coupled differential equations in matrix form, defined as follows

$$\mathrm{M}.\ddot{\mathrm{u}} + \mathrm{K}.\mathrm{u} = \mathrm{f}(t) \tag{1}$$

Where, M and K are global stiffness and mass matrices, respectively.

The modal characteristics of this system are described by eigenvalue equations as follows

$$(\mathrm{K} - \omega_i^{\,2}\mathrm{M})\,\Phi_i = 0, \quad i = 1, \ldots, \mathrm{n} \tag{2}$$

Here, ω_i is the natural frequencies corresponding to vibration mode shapes (Φ_i), and n is the total number of degrees of freedom.

The objective function is selected as square of Euclidean distance between numerical and measured square of natural frequencies. It can be written as follows

$$f = \sum_{j=1}^{m} \left[\frac{\omega_j^{\,2} - \widetilde{\omega_j}^{2}}{\widetilde{\omega_j}^{2}} \right]^2 \tag{3}$$

Where, $\widetilde{\omega_j}$ is the measured natural frequency of j-th mode, and m is the number of measured modes. The accuracy of measuring natural frequencies are higher than that of other dynamic parameters such as mode shapes and FRFs (Marwala 2010). Eq. (3) is selected as the nonlinear least square function of eigenvalues for the structure to meet more reliable results for the updated FE model.

In this research damage has been defined as the reduction in stiffness of each element of numerical model. Physical properties of each element have been corrected during the updating procedure using the damage index (α). For an Euler Bernoulli beam element, α can be written as

$$\alpha^i = 1 - \frac{{(EI)_d}^i}{{(EI)_0}^i} \tag{4}$$

Where, E is the Young modulus of elasticity, I is the second moment of area, and superscript d is the damaged status of the i^{th} element. The reduction factor (α), a dimensionless vector with a size between 0 and 1, is used as the updating parameter in the FEMU optimization process. The damage index (α) is linearly related to the stiffness matrix of the structure, presented as follows

$$\mathrm{K}_d^i = K_0^i\left(1 - \alpha^i\right) \tag{5}$$

Where, K_d^i and K_0^i are updated and initial element stiffness matrices respectively. The global stiffness matrix can be assembled using elemental stiffness matrices. The objective function mentioned in Eq. (3) can be rewritten as

$$f_1(\alpha) = \sum_{j=1}^{m} \left[\frac{{\omega(\alpha)_j}^2 - \widetilde{\omega_j}^{2}}{\widetilde{\omega_j}^{2}} \right]^2 \tag{6}$$

Where, $f_1(\alpha)$ is the amount of objective function 1 with respect to the parameter α. For the initial undamaged model, α is a vector of zeroes with a size equal to the number of elements.

As the structures get more complex, considering mode shapes beside the natural frequencies help the optimization problem to reach unique converged optimum solution. Therefore, the modal assurance criterion is applied in addition to the regular objective function mentioned in Eq. (6) to better converge damage index vectors to the final result (Friswell and Mottershead 2013). The MAC value for the i-th mode shape is as follows

$$MAC(i) = \frac{\left|\{\varphi_A\}_i^{\mathrm{T}}\{\varphi_E\}_i\right|^2}{(\{\varphi_A\}_i^{\mathrm{T}}\{\varphi_A\}_i)(\{\varphi_E\}_i^{\mathrm{T}}\{\varphi_E\}_i)} \tag{7}$$

Where $\{\varphi_A\}_i$ and $\{\varphi_E\}_i$ are the ith mode shapes of the analytical and experimental models, respectively.

The objective function $f_2(\alpha)$ can be completed as

$$f_2(\alpha) = \sum_{j=1}^{m} \left[\frac{{\omega(\alpha)_j}^2 - \widetilde{\omega_j}^{2}}{\widetilde{\omega_j}^{2}} \right]^2 + r\sum_{i=1}^{n}[1 - MAC(i)] \tag{8}$$

Where r is a scaling coefficient, helping optimization problem become more stable.

2.2 Optimization method

According to FEMU flow chart Fig. 1, after calculating the goal function from initial updating parameters, an appropriate optimization method should be selected to find the best-minimized solution for the problem. In this research, modern optimization methods are implemented based on the modern heuristic algorithms for finding global minimum solution in the search space by using n number of individuals (n also names as number of population size), each of having a dimension of D. The population evolves during a succession of iterations called generations until a termination criterion is satisfied. Such criteria take the quality of priori solutions into account. During each generation, a succession of operators is applied to the individuals of a population to generate the new population for the next generation. The fitness values of all members within the population are evaluated by a goal function after modal analysis of structure using modified stiffness. The limit number of iterations, the limit of the best fitness

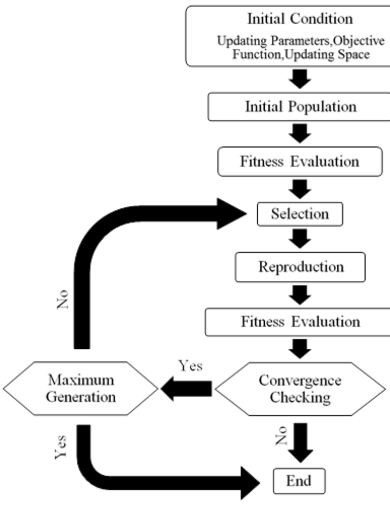


Fig. 2 Evolutionary base optimization general algorithm

function value and the limit of the average relative change in the best fitness function value, are the criteria evaluated at the end of each iteration.

In model updating problem, an optimization problem will be solved after constructing an objective function which is explained in section 2.1. The optimization parameters are selected as the damage index according to Eq. 4. The optimization methods applied in this section tries to find the best optimum solution for the problem.

The general procedure of finding global minimum solution is as below in Fig. 2.

2.2.1 Genetic algorithm (GA)

According to Darwin theory (Darwin 1859), competition to survive in each generation causes the populations, with better characteristics of fitting to the environment, overcome the weaker ones. Genetic algorithm is a model of machine learning derived from evolution mechanism in the nature (Holland 1992). The algorithm starts with creating an initial random population of solutions (chromosomes) with a uniform distribution considering the bound constraints. Each chromosome consists of a set of genes equal to D. In this research, a simple GA is applied which includes three main operators as selection, crossover and mutation.

Selection

The algorithm selects fitter chromosomes within the existing population to breed next generation of chromosomes. The selected solutions (names as parents) combine to create new group of solutions (names as children). Stochastic uniform selection method (the MathWorks 2014) is used in this research, which assures the diversity in the selected solutions. Each parent corresponds to a section of a line with its length proportional to its scaled value. The algorithm moves along the line in equal steps, in each of which allocating a parent from the section it lands on. Solutions with higher fitness have more probability to be selected.

Crossover

Single point crossover method (Dréo, Pétrowski *et al.* 2006) is used in this research. This operator mixes genetic information of a fraction of the population by cutting pairs of chromosomes at random points along their lengths and exchanging the cut sections. In this way, parents' characteristics will be combined to create the children with higher capabilities of fitness. Solutions are combined with the aim that new generation of solutions have higher fitness than their parents.

Mutation

To prevent converging the solution to the local optima, one or more chromosomes' genes will be changed with low probability to create new materials in the population. Here, adaptive feasible mutation method (the MathWorks 2014) is used that randomly generates directions that are adaptive with respect to the last successful or unsuccessful generation. The mutation chooses a direction and step length that satisfies bounds. Mutation operator tries to provide global searching within solution search space considering bounds. In every generation, a number of best individuals are transferred to the next generation in order to avoid being eliminated in the iteration.

2.2.2 Artificial bee colony (ABC)

Honey bee swarm have a unique foraging strategy. Artificial Bee Colony (Karaboga and Basturk 2007), an intelligent optimization algorithm, simulates this behavior considering three different groups of artificial bees: employed, onlooker and scout bees. For a problem with n variables, a population of preliminary bees that equals the half of the total population are generated randomly, each of them consisting of n variables. In an iterative procedure that is explained below, ABC finds the best solution for an optimization problem.

Employed bees

Each preliminary solution is given to an employed bee. It selects another solution (x_k) in the colony and uses it to make some changes to its own solution as

$$v_{ij} = x_{ij} + \varphi.(x_{ij} - x_{kj}) \qquad k \neq i \tag{9}$$

Where, φ is a random number between [-1 1]. If the modification has a better cost function, the bee saves the new solution, otherwise it keeps its own solution.

Onlooker bees

Each Onlooker bee inherits a solution from an employed bee. It modifies this solution to achieve better cost functions similar to the employed bees. However, they select the other solution wisely by using a selection method. (Here, Roulette Wheel selection methods is selected). In this method, solutions with better cost functions have higher probability to be selected. The probability of selecting the i-th bee can be shown as mentioned in Eqs. (10) and (11). The results of the modification by Onlooker bees will be reported to Scout bees.

$$fit(i) = \frac{1}{1+f(i)} \tag{10}$$

$$P(i)=\frac{fit(i)}{\sum_{j=1}^{n} fit(j)} \tag{11}$$

Scout bees

If a solution remains unchanged through a specific number of iterations called Limit, then the relative Scout bee selects a new random solution to keep the diversity of the solutions.

2.2.3 Particle swarm optimization (PSO)

PSO, first proposed by Kennedy and Eberhart in 1995 (Kennedy 2011), is a metaheuristic algorithm that is based on the special foraging approach of the swarms of birds. A swarm with a population equal to n, spreads in the domain of optimization problem. Position of each bird is stored in a vector with size equal to the number of variables of the problem (xi). In every iteration, the position of each bird is updated via the below procedure in order to converge to the best answer

$$v_i^{k+1}=w^k\times v_i^k+c_1\times r_1\times(p_i-x_i^k)+c_2\times r_2\times(g^k-x_i^k) \tag{12}$$

$$x_i^{k+1}=x_i^k+v_i^{k+1} \tag{13}$$

where v is the velocity of the bird, w is the learning factor that is in the Inertia range, c_1 and c_2 are positive constants as the self-adjustment and the social-adjustment factors respectively, r_1 and r_2 are uniformly distributed random vectors between [0, 1], p_i is the best position of birds till now and g^k is the best position attained among all of the birds so far.

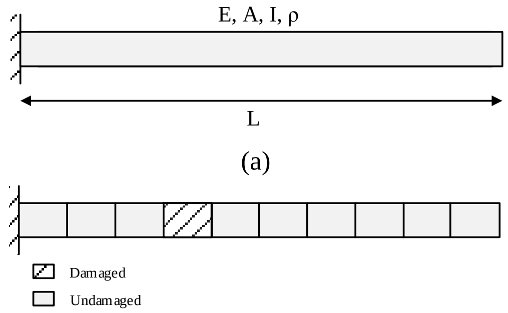
Some of the consumptions of the aforementioned algorithms used in the research are shown in the Table 1.

2.2.4 ABC-PSO hybrid optimization

As it is mentioned in literature, PSO is one of the best metaheuristic algorithms for optimization, however, it is reported that it has a disadvantage of low accuracy in objective functions that are too noisy which makes it susceptible to stick into a local minima (Aote, Raghuwanshi *et al.* 2013). On the other way, PSO is one of the best optimization methods in convergence speed when the solutions are near the global minimum zone. In order to investigate more accurate with less calculation cost updating method, after consideration of advantages and disadvantages of application of ABC and PSO optimization methods within several numerical and experimental examples, a new hybrid optimization method in updating procedure is proposed trying to merge the positive points of these optimization methods. ABC is powerful in dealing with noisy objective functions (Karaboga and Basturk 2008) and PSO is capable of digging deep into optimum solutions. In this way, the hybrid algorithm starts with ABC as the primary function, finding damage zones within the updating variables, continued by PSO as the function conducted in the last iterations to converge the solution to final results, by finding the amount of damage within candidates for damaged elements. After considering some termination criteria, like number of consecutive iterations' limit or cost function tolerance, it can be concluded that obtained solutions are near the global minimum damage zone. At this step, concerning engineering judgment, the algorithm can reduce the number of updating parameters by elimination of damage indices that are near zero. Therefore, ABC method finds the damaged elements candidates and helps the optimization algorithm to disregard undamaged elements in continue within optimization process. In the next step, PSO tries to converge the result to final solution with lower number of updating variables which represents finding severity of damage in candidate elements. The ABC- PSO optimization method can be so efficient in accuracy and speed in damage detection problems with a lot of updating variables.

Table 1 Parameter settings of the optimization algorithms used in the research

PSO		ABC		GA	
Population Size	n	Colony size	n	Population Size	n
Inertia Range	1.1→0.1	No. of Onlooker Bees	0.5×n	Crossover Fraction	0.8
Self-Adjustment Weight	1.49	No. of Employed Bees	0.5×n	Number of Elites	0.05×n
Social-Adjustment Weight	1.49	Limit	0.5×n×D		



(b)

Fig. 3 Simulated numerical example

3. Examples

3.1 Numerical Example 1 - Simulated Cantilever Beam

A numerical simulation has been performed to examine the proposed model updating procedure in damage detection problem using incomplete measured data. To this end, SAP2000 (CSI 2013) software verification example 1-014 is used as the basic model. A cantilever beam with the length of $\mathrm{L}=2.44\ \mathrm{m}$, and a rectangular cross section ($\mathrm{b}\times\mathrm{h}=0.30\times0.46\ \mathrm{m}$) has been modeled using ten 2D Euler Bernoulli beam elements in MATLAB. The beam is assumed to have young's modulus (E) of $24.71\ \mathrm{GPa}$ and the density (ρ) of $2400\ \mathrm{kg/m^3}$. Regarding the fourth element, 50% reduction in bending stiffness is assumed as

Table 2 Simulated measured, initial and updated frequencies for example 1, comparison between GA, ABC and PSO

Mode				1	2	3	4	5
Simulated measured Freq. [Hz]	damaged case			37.45	237.88	654.06	1346.07	2138.24
Numerical Freq. [Hz]	Initial FE model			39.48	247.40	692.89	1358.74	2249.62
			error %	5.42	4.01	5.94	0.94	5.21
			MAC	0.99722	0.98151	0.98476	0.95286	0.93421
	Updated FE model	GA	n=50	37.48	237.69	654.70	1343.63	2137.25
			error %	0.10	-0.08	0.10	-0.18	-0.05
			MAC	0.99996	0.99985	0.99984	0.99992	0.99953
			n=100	37.46	237.72	654.51	1338.76	2137.26
			error %	0.040	-0.065	0.068	-0.543	-0.046
			MAC	0.99994	0.99979	0.99977	0.99993	0.99957
		ABC	n=20	37.54	237.88	654.50	1343.97	2135.54
			error %	0.253	0.003	0.067	-0.156	-0.127
			MAC	0.99996	0.99979	0.99983	0.99993	0.99921
			n=50	37.46	237.97	654.31	1344.68	2138.18
			error %	0.039	0.041	0.038	-0.103	-0.003
			MAC	1.00000	0.99999	0.99999	1.00000	0.99999
			n=100	37.45	237.76	654.47	1345.33	2139.37
			error %	0.014	-0.048	0.062	-0.055	0.053
			MAC	1.00000	1.00000	0.99999	1.00000	0.99999
		PSO	n=20	37.45	237.88	654.06	1346.07	2138.24
			error %	0*	0	0	0	0
			MAC	1.00000	1.00000	1.00000	1.00000	1.00000
			n=50	37.45	237.87	654.06	1346.06	2138.22
			error %	0	0	0	0	0
			MAC	1.00000	1.00000	1.00000	1.00000	1.00000
			n=100	37.45	237.88	654.06	1346.07	2138.24
			error %	0	0	0	0	0
			MAC	1.00000	1.00000	1.00000	1.00000	1.00000

*Errors less than 10^{-4} % is considered as 0.

the damaged beam state. Simulated numerical example. Fig. 3 shows geometrical and material properties of damaged and undamaged beams. Numerical modal analysis of the damaged beam is considered as the measured modal response of the beam after damage. Only first five modal frequencies are selected as the measured response data in order to consider the limitation of incompleteness of experimental data. The effect of noise on measured information is ignored in this example. Here, the main goal is to verify the developed FE model updating codes and

Table 3 simulated damaged and updated frequencies for example 1 with objective function f1 and f2

Mode no.	Simulated damaged Frequencies [Hz]	Updated Model Frequencies [Hz]					
		$f1$	error %	MAC	$f2$	error %	MAC
1	37.45	37.46	0.040	0.999943	37.45	0.011	1.000000
2	237.88	237.72	-0.065	0.999791	237.78	-0.041	0.999999
3	654.06	654.51	0.068	0.999774	654.30	0.037	0.999999
4	1346.07	1338.76	-0.543	0.999926	1345.10	-0.072	0.999998
5	2138.24	2137.26	-0.046	0.999573	2138.27	0.001	0.999996

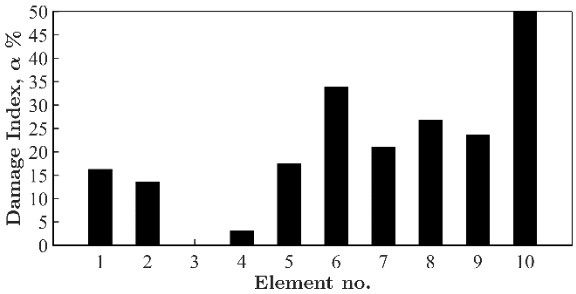
Table 4 simulated damaged and updated frequencies for three span continues beam example

		Initial FE Model			Updated FE Model		
Mode	fm [Hz]	fi [Hz]	error %	MAC	fu [Hz]	error %	MAC
1	24.15	24.54	1.650	0.9840	24.15	0*	1.0000
2	31.44	31.45	0.049	0.9805	31.44	0	1.0000
3	45.07	45.93	1.918	0.9786	45.07	0	1.0000
4	97.88	98.20	0.327	0.7931	97.88	0	1.0000
5	111.63	111.92	0.263	0.8447	111.63	0	1.0000

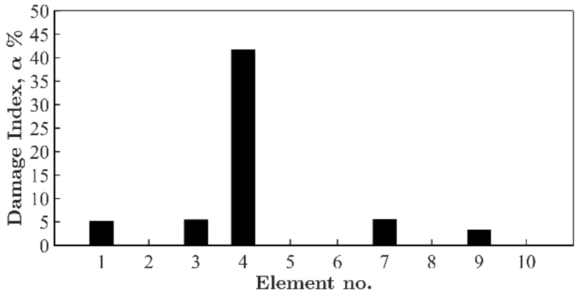
*Error less than 10^{-5} % is considered as 0.

investigate efficiencies of ABC, PSO and GA in these problems.

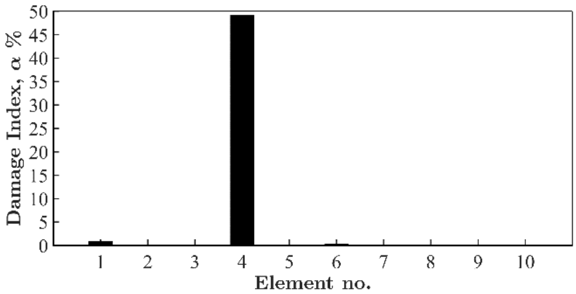
This problem has 20 degrees of freedom with ten updating parameters (damage index vector α) as the studied beam has been divided into ten beam elements. The goal is to update initial FE model to match the simulated numerical data of damaged beam. Considering five first modal information, ABC, PSO and GA optimization methods are applied to find the optimum solution of the example. The results of comparison between these methods considering different population sizes are presented in Table 2. This table shows the simulated measured modal information as well as the initial and updated one and compares the updated results using two accuracy criteria (relative error and MAC value). Considering maximum number of generations as 100, three different population sizes n=20, 50 and 100 are conducted for each optimization method. Only GA method with population size of n=20 was not successful to converge to acceptable results finding the damaged element, thus it is not included in the Table 2. It seems ABC and PSO has ability to reach the global minima with less population sizes and therefore less calculation costs than GA (Karaboga and Basturk 2008). Moreover, obtained results shows better accuracy of ABC and PSO in comparison with GA. Fig. 4(a) to (c) shows ABC method results at different iterations, trend of converging to final result. The final results of damage indices considering ABC and PSO methods are shown in Fig. 4(c) and (d). Concerning the influence of considering mode shapes plus modal frequencies within the objective function in comparison with only frequencies in accuracy of converged solutions, Table 3, Fig. 4(e) and (f) report the results of using objective function f1 and f2 using GA as optimization method. The population size is assumed to be n=100 within Fig. 4 and Table 3.



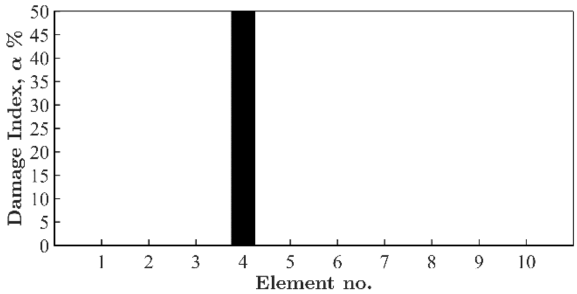
(a) ABC (10th iteration)



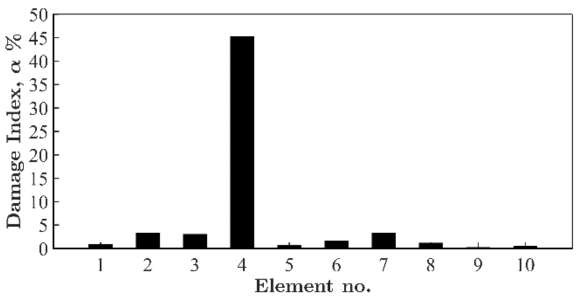
(b) ABC (25th iteration)



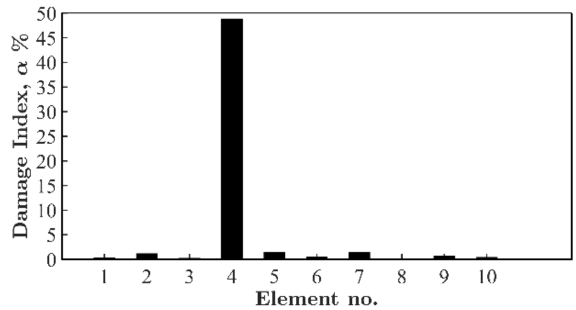
(c) ABC (100th iteration- converged result)



(d) PSO (100th iteration- converged result)



(e) GA (objective function f1)



(f) GA (with the objective function f2)

Fig. 4 Identified damaged indices for numerical example 1

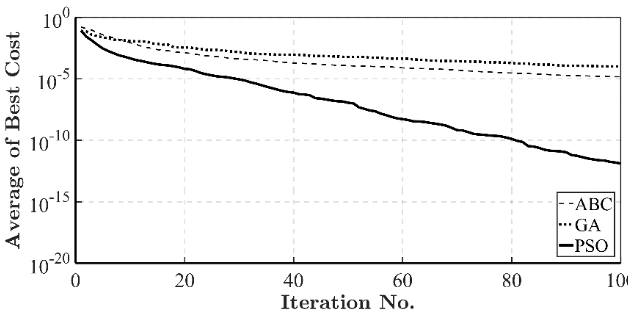


Fig. 5 Average of the results in every iteration for the algorithms with population=100

In order to investigate the trend of convergence of mentioned optimization methods to optimum solutions and minimizing the cost function, this problem is solved repeatedly for 30 times for each method considering population size of 100. According to random initial solution in each solution, average of cost function values in each iteration step is calculated. Fig. 5 shows average value of cost functions for best solution in each iteration. The results shows the ability of PSO to minimize the cost function fast.

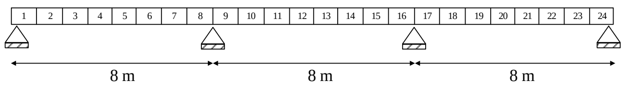
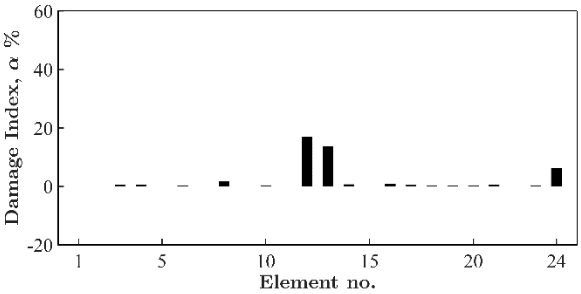


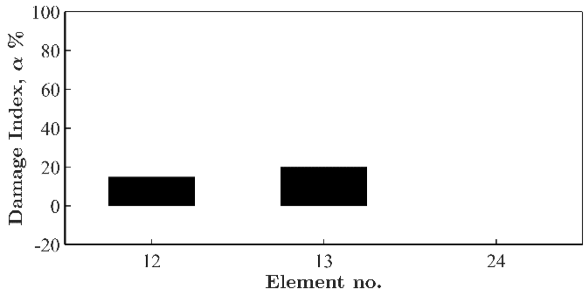
Fig. 6 Three span continues beam

3.2 Numerical example 2 - Three span continuous beam

This example is generated to explain damage detection procedure using presented ABC- PSO hybrid optimization method. A three span continues beam is selected in this section as a numerical simulation of damage detection problem with multi damaged elements. The dimensions and specification of the mentioned beam is considered the same as a similar numerical example previously conducted by Zhu *et al*. (2015). The beam has 24 meter length with 0.5×1.0 *m* cross section which is shown in Fig. 6. The physical parameters of the beam is considered as Young's modulus E = 30 GPa and mass density ρ = 2.8 ton/m^3. The beam is modeled using 24 Euler-Bernoulli beam elements (Fig. 6). Damage extent of 15% and 20% is assumed to be accrue in 12th and 13th elements respectively.

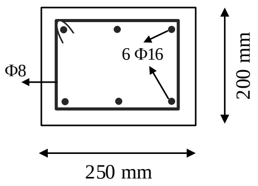


(a) ABC finds damage zones

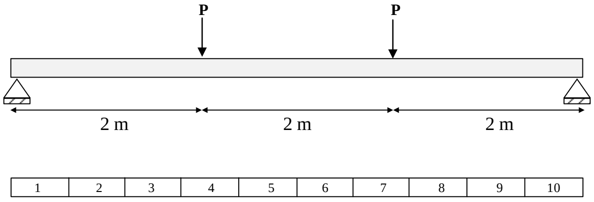


(b) PSO converges to final solution value

Fig. 7 Identified damage indices for numerical example 2



(a) Cross section



(b) Beam Elements and static loads arrangement

Fig. 8 Concrete beam specifications

Table 5 Measured, initial and updated Frequencies of concrete beam example- ABC method

	Reference State							Damaged State						
		Initial FE Model			Updated FE Model				Initial FE Model			Updated FE Model		
Mode	*f* m [Hz]	*f* i [Hz]	error %	MAC	*f u* [Hz]	error %	MAC	*f* m [Hz]	*f* i [Hz]	error %	MAC	*f u* [Hz]	error %	MAC
1	21.90	22.22	1.44	0.9989	21.90	0.00	0.9996	18.01	21.90	21.64	0.9976	17.98	-0.13	0.9991
2	60.33	61.26	1.55	0.9934	60.33	0.00	0.9959	50.20	60.33	20.16	0.9979	50.23	0.06	0.9984
3	117.02	120.18	2.70	0.9952	117.01	-0.01	0.9967	98.22	117.01	19.15	0.9862	98.10	-0.12	0.9990
4	192.03	198.96	3.61	0.9951	192.05	0.01	0.9982	161.88	192.05	18.64	0.8971	162.09	0.13	0.9979

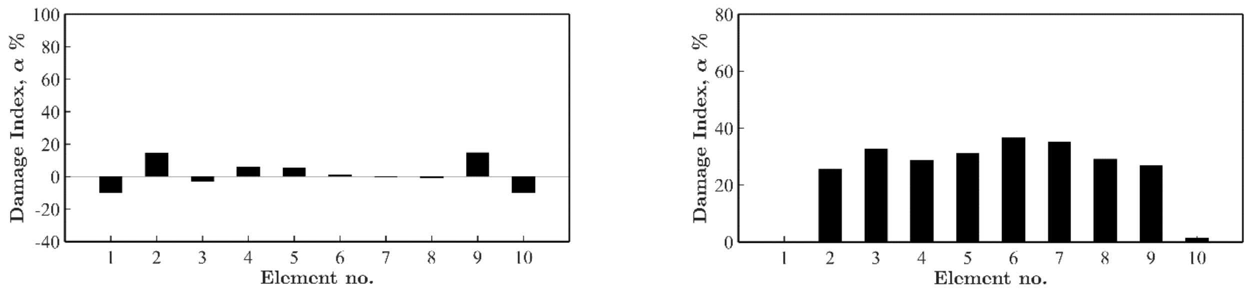
f m: Measured Frequency, *f* i: Initial Frequency, *f* u: Updated Frequency

To simulate the experimental results of the damaged beam, modal analysis is conducted on finite element model of the damaged beam and first five frequencies and mode shapes of this analysis are used as the algorithm input data. In this example, objective function and optimization method are considered as equation 8 and ABC- PSO hybrid method, respectively. The ABC- PSO algorithm consists of two main optimization stage. In the first stage, ABC algorithm starts with random initial solutions and tries to find the best solution by changing the damage indices of all 24 elements. Fig. 7(a) shows damage zones results from this optimization step. After 108 iterations and converging to the solution which satisfy termination criteria (the last 20 iterations had relative changes in their cost function values less than 0.0001), the elements that were found to have damage index value higher than 3% are selected for the next optimization stage (elements no. 12, 13 and 24). Then, PSO algorithm tries to find the final solution by modifying damage indices in estimated damage zones. In 60 iterations, the cost function value reduced from 6.80×10^{-5} to 1.67×10^{-17}. As it is shown in Fig. 7(b), in the second stage, PSO achieves to the exact amount of damage in the selected zones in a high-speed procedure. Severity of damage is found 15%, 20% and zero in elements no 12, 13 and 24, respectively.

The ABC-PSO method starts solving the optimization process with 24 numbers of updating parameters. As the algorithm estimates damage zones after the first step, number of updating parameters are reduced to 3 candidate damage elements for the second step. The reduction in the number of updating variables in ABC-PSO optimization method causes less computational cost for solving the problem comparing to application of only PSO or ABC method with total number of updating parameters. Moreover, number of population size in the second step of optimization process in ABC-PSO method can be reduced according to number of selected updating variables.

3.3 Experimental example - Concrete beam lab test

This example which is previously investigated in some relative researches like (Maeck *et al.* 1999, Ren and De Roeck 2002, Teughels *et al.* 2002, Jaishi and Ren 2006), is selected to verify the explained model updating method in structural damage detection problem using real structural behavior data. A reinforced concrete beam with 6 m length and 0.2×0.25 m cross section was tested in laboratory before and after damage. The beam was statically loaded in



(a) Reference State (b) Damaged State

Fig. 9 Identified damage Indices for concrete beam lab test

Table 6 Measured, initial and updated Frequencies of concrete beam example-PSO method

Reference State								Damaged State						
		Initial FE Model			Updated FE Model				Initial FE Model			Updated FE Model		
Mode	*f* m [Hz]	*f* i [Hz]	error %	MAC	*f* u [Hz]	error %	MAC	*f* m [Hz]	*f* i [Hz]	error %	MAC	*f* u [Hz]	error %	MAC
1	21.90	22.22	1.44	0.9989	21.90	-0.01	0.9996	18.01	21.90	21.64	0.9976	17.99	-0.11	0.9991
2	60.33	61.26	1.55	0.9934	60.33	0.00	0.9958	50.20	60.33	20.16	0.9979	50.20	-0.01	0.9985
3	117.02	120.18	2.70	0.9952	117.02	0.00	0.9967	98.22	117.02	19.15	0.9862	98.21	-0.01	0.9991
4	192.03	198.96	3.61	0.9951	192.05	0.01	0.9982	161.88	192.05	18.64	0.8971	162.07	0.12	0.9979

f m: Measured Frequency, *f* i: Initial Frequency, *f* u: Updated Frequency

two points with 2 m distance from beam edges. This static load was applied in six steps with final load of 24 kN causing cracks within the beam length. After this step, the damaged beam was unloaded and its modal parameters was extracted by hammer test on free- free boundary condition. Detailed information about the concrete beam, testing conditions and obtained dynamic results was reported by Jaishi and Ren (Jaishi and Ren 2006) which were concentrated on updating the model of this example using a sensitivity base model updating method with Trust Region Newton algorithm for optimization procedure. Here, the aim is to identify damage pattern of the beam using measured modal parameters before and after damage. The damage identification process is actually consisted of two model updating steps. First, the initial numerical finite element model modal properties are updated to match the experimental model modal data before damage as the reference model data. Then, the reference model data are updated to match the measured damaged modal information. The modification indices obtained in the second updating step demonstrate the damage pattern of the tested damaged beam. The geometrical information of the reinforced concrete beam is shown in Fig. 8. Only first four modal frequencies and mode shapes, reported by (Jaishi and Ren 2006), are used in the model updating processes.

The beam is modelled using ten euler-bernoulli finite element beam elements in Matlab. The elastic modulus and the moment of inertia for the initial model's elements are considered as 38GPa and 1.66×10^{-5} m4, respectively. Experimental first four frequencies of both the reference and damaged state of the tested beam are shown in Table 5 column no. 2 and 9.

In this example, during each model updating steps, Eq. 8 is considered as objective function and ABC and PSO methods with population size of 100 are used as optimization techniques.

Tables 5 and 6 show measured, initial and updated frequencies at reference and damaged state for ABC and PSO method respectively. Concerning these tables, the amount of relative errors for modal frequencies and MAC value for mode shapes are significantly improved which shows success of presented model updating algorithm in fitting the numerical model to experimental one. The updated results in the first updating step is the input (as initial model) for the second updating procedure verifying the numerical model in first step and finding damage severity and location within elements in the second one. Obtained modification indices for the reference and damaged state model updating are presented in Fig. 9.

In this research, in order to verify the presented updating algorithm and developed codes, obtained results for the concrete beam is compared to one reported by Jaishi and Ren (Jaishi and Ren 2006). As it is mentioned in this report, the errors in frequencies were between [-0.155%, 1.119%] for the reference state and [-1.144%, 6.854%] for the damaged state. However, the results of this research shows that the errors for the reference and damaged states are between [-0.011%, 0.013%] and [-0.134%, 0.129%],

Table 7 Measured frequencies for Nanjing TV tower [Hz]

Mode	Tang *et al.*'s results	Feng *et al.*'s results
1	0.234	0.237
2	0.73	0.727
3	1.266	1.271
4	1.596	1.588

Table 8 Initial and updated frequencies for Nanjing tower calculated by ABC, PSO and ABC-PSO

Initial FE Baseline Model					Updated FE Baseline Model									
					ABC			PSO			ABC-PSO			
Mode	f exp [Hz]	f i [Hz]	error %	MAC	f u [Hz]	error %	MAC	f u [Hz]	error %	MAC	f u [Hz]	error %	MAC
1	0.234	0.199	-14.99	0.999	0.233	-0.63	0.999	0.233	-0.59	0.999	0.232	-0.67	0.999
2	0.730	0.627	-14.06	0.952	0.727	-0.38	0.973	0.727	-0.37	0.973	0.727	-0.40	0.973
3	1.266	1.226	-3.13	-	1.266	0.02	-	1.266	0.01	-	1.267	0.11	-
4	1.596	1.623	1.70	-	1.620	1.49	-	1.620	1.52	-	1.622	1.61	-

respectively. Fig. 9 shows the damage index of each element in the reference and damaged state. The obtained damage distribution pattern is without any assumed damage pattern or damage function and the presented updating algorithm can be successful to find damaged elements.

3.4 Experimental example-Nanjing TV tower

As a benchmark problem, many researchers were previously concentrated on dynamic evaluation and structural health monitoring of Nanjing TV tower (Tang *et al.* 1995, Feng and Zhang 1997, Feng *et al.* 1998). Here, the 310m height tower's test results and base line finite element model is used to verify the efficiency of the presented model updating method. Tang *et al.* (1995) was reported ambient vibration test results of the structure in a technical report of southeast university of china. They extracted first four natural frequencies and first two mode shapes of the tower using measured acceleration data. Feng *et al.* also conducted vibration tests on the tower with less measurement points using the measured information to investigate dynamic characteristics of the structure (Feng *et al.* 1998). They reported first eight natural frequencies and four mode shapes of the structure and finally developed a base line model for the tower which consists of 17 beam elements with 18 nodes, 17 lumped mass and 34 degrees of freedom (two degrees of freedom for each node- lateral and rotational displacements). Wu and Li (Wu and Li 2004) concentrated on finite element model updating of this high rise structure using the mentioned base line model. They completely investigated the experimental measurements and modal parameters reported by two pervious works. As Wu and Li illustrated the most reliable modal parameters in their report, first four vibration frequencies obtained by Tang *et al*. (1995) with two mode shapes are considered as the experimental measured modal parameters in this example. Table 7 shows considered modal frequencies in this research. The frequencies from both researches are shown in this table. One can find detailed information about the tower's structural and geometrical specification, vibration test results, extracted mode shapes and finite element base-line model considering mentioned references.

ABC, PSO and hybrid ABC-PSO methods are conducted for this problem with population size equal to 170 for each method (population size for ABC-PSO method is 170 initially, reducing to 85 at the end). The lower and upper bounds of the damage indices are considered as mentioned in Wu and Lee (2004). Using Eq. (8) as objective function, obtained initial and updated frequencies are shown in Table 8. The r coefficient is considered equal to 0.1.

All three optimization algorithms reached to almost similar acceptable results fitting modal parameters of the base line model to the experimental data successfully. PSO converged to the results with the best objective function value out of the three algorithms. Concerning number of iterations to obtain the converged results, the hybrid ABC-PSO algorithm reached to the results after 210 iterations faster than the others, leaving the PSO as the slowest algorithm with 301 iterations.

Comparing obtained results with results reported by Wu & Li for the updated baseline model, two major criteria is considered as relative error of frequency results an MAC value of mode shapes. Relative error of frequencies can be described as percentage of relative difference between experimental and numerical data. According to Table 8, the range of frequency errors for the updated model are obtained within the bound of [-0.9%, 1.54%] which are significantly lower than the range reported by Wu & Li [-2.19%, 7.65%] for updated baseline model. Moreover, The MAC value for the second mode shape of the updated baseline model (0.973) in this research is more fitted to the measured data in comparison to MAC value 0.958 reported by Wu and Li (Wu and Li 2004).

4. Conclusions

In this paper, a finite element model updating method is described based on the evolutionary algorithm optimization techniques, including ABC, PSO, GA and ABC-PSO hybrid method. Two different objective functions are used based on the sum of squares of eigenvalues and MAC value of mode shapes and compared to each other. Numerical example 1 was generated to compare the efficiency and characteristics of the mentioned evolutionary optimization methods. With similar parameter settings, it can be concluded that ABC and PSO have a better functionality in model updating than GA concerning accuracy and reliability of converged results. Comparing the objective functions affirms the fact that MAC can positively improve the accuracy of the updated model and helps the optimization algorithm to find global optimum solution.

In real problems, the existence of uncertainty, noises and incompleteness of modal information are not negligible. Therefore, the model updating is conducted on an experimental beam and a real high rise building to examine the effects. The results show that the PSO has the best cost function values although ABC can achieve almost similar accurate results. According to the obtained results, the evolutionary algorithms are qualified in estimating the damage locations and offering acceptable results for amount of damage within elements. The updated models have higher accuracies in matching the experimental results with respect to improvement in frequency errors and MAC value for mode.

Regarding the stated disadvantage of PSO in dealing with the objective functions with high level of noises within optimization literature, a hybrid method is proposed merging the advantages of ABC and PSO in optimization. The method is initially conducted on a simple numerical simulation to explain the algorithm procedure details, and later applied for updating of baseline model of Nanjing TV tower to consider its robustness and speed. In ABC-PSO method, ABC attempts to find possible damage zones while PSO trying to achieve globally optimum value for severity of damage in predicted damaged elements candidates. Reducing the number of updating parameters in the second updating stage, the method is beneficial in dealing with the regular disadvantage of evolutionary algorithms encounter with problems with high number of updating variables. It can be concluded that the ABC- PSO method is successful to achieve accurate and reliable results with less computational effort with higher convergence speed.

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