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A simple HSDT for bending, buckling and dynamic behavior of laminated composite plates

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Abstract. In the present article, cross ply laminated composite plates are considered and a simple sinusoidal shear deformation model is tested for analyzing their flexural, stability and dynamic behaviors. The model contains only four unknown variables that are five in the first order shear deformation theory (FSDT) or other higher order models. The in-plane kinematic utilizes undetermined integral terms to quantitatively express the shear deformation influence. In the proposed theory, the conditions of zero shear stress are respected at bottom and top faces of plates without considering the shear correction coefficient. Equations of motion according to the proposed formulation are deduced by employing the virtual work principle in its dynamic version. The analytical solution is determined via double trigonometric series proposed by Navier. The stresses, displacements, natural frequencies and critical buckling forces computed using present method are compared with other published data where a good agreement between results is demonstrated.

Keywords: shear deformation; flexural, buckling; dynamic; cross-ply laminates; laminated plates

1. Introduction

Since the structures made of composite materials are used in a growing way in many engineering industries because of their attractive characteristics, such as strength, stiffness, reduced weight, thermal characteristics, resistance to corrosion, fatigue life, and resistance to wear. The plates fabricated with these materials require accurate structural investigation to predict the correct flexural behavior.

The influence of transverse shear strain is more important in thick structures than in thin ones. Thus, various plate models have been proposed by scientists to predict correct flexural behavior of thick structures. The conventional plate theory (CPT) of Kirchhoff (1850) is not applicable to thick structures because of the neglect of transverse shear deformation (Sofiyev and Avcar 2010, Sofiyev *et al.* 2008, 2012, Bilouei *et al.* 2016, Avcar and Mohammed 2018, Cherif *et al.* 2018). The first order shear deformation theory (FSDT) proposed by Mindlin (1951) is also not interesting for the investigation since it does not verify the conditions of zero stress at upper and bottom faces of the plate and requires factors of shear correction (Avcar 2015, Al-Basyouni *et al.* 2015, Arani and Kolahchi 2016, Bellifa *et al.* 2016, Madani *et al.* 2016, Kolahchi *et al.* 2016a, b, Bouderba *et al.* 2016, Zamanian *et al.* 2017, Kolahchi 2017, Amnieh *et al.* 2018, Youcef *et al.* 2018). Thus, many high order shear deformation theories have been proposed for the investigation of plates (Reddy 1984, Reddy and Khdeir 1989, Touratier 1991, Soldatos 1992, Metin 2009, Karama *et al.* 2009, Sayyad and Ghugal 2012, Rezaiee-Pajand *et al.* 2012, Sayyad 2013, Belabed *et al.* 2014, Sayyad and Ghugal 2014, Bousahla *et al.* 2014, Ghugal and Sayyad 2013, Ahmed 2014, Hebali *et al.* 2014, Zemri *et al.* 2015, Mahi *et al.* 2015, Panda and Katariya 2015, Kolahchi and Moniri Bidgoli 2016, Bennoun *et al.* 2016, Beldjelili *et al.* 2016, Bousahla *et al.* 2016, Akavci 2016, Baseri *et al.* 2016, Kolahchi *et al.* 2017a, b, c, Abdelaziz *et al.* 2017, Hajmohammad *et al.* 2017, Aldousari 2017, Kolahchi and Cheraghbak 2017, Mouffoki *et al.* 2017, Bouafia *et al.* 2017, Hajmohammad *et al.* 2018a, b, c, Golabchi *et al.* 2018, Salami and Dariushi, 2018, Kadari *et al.* 2018, Karami *et al.* 2018a, b, Bouadi *et al.* 2018, Mokhtar *et al.* 2018, Fakhar and Kolahchi 2018, Hosseini and Kolahchi 2018, Taleb *et al.* 2018).

In the recent decade, a novel class of plate models has been proposed by scientists in which kinematic involves only four variables. Shimpi and Patel (2006) are the first to use a plate model involving two variables for bending and dynamic study of orthotropic plates. This model is further extended by Bouderba *et al.* (2013), Tounsi *et al.* (2013), Zidi *et al.* (2014) and Hamidi *et al.* (2015) for the thermo-

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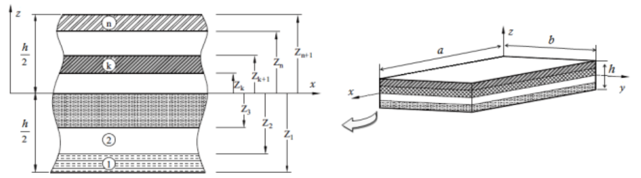

Fig. 1 Coordinate system and layer numbering used for a typical laminated plate

mechanical effect on bending response of functionally graded plates considering four and five variables. Meziane *et al.* (2014) also employed this theory for the stability and dynamic of exponentially graded sandwich structures under various boundary conditions. Yahia *et al.* (2015) used this theory for analyzing wave propagation in functionally graded plates with porosities. Draiche *et al.* (2016) presented a refined theory with stretching effect for the flexure analysis of laminated composite plates. Karami *et al.* (2017) used a nonlocal strain gradient theory and four variable refined plate theory to present the wave propagation analysis in functionally graded (FG) nanoplates under in-plane magnetic field.

Recently, another new class of plate models has been presented in literature in which in the kinematic is introduced undetermined integral terms to reduce the number of governing equations (El-Haina *et al.* 2017, Menasria *et al.* 2017, Chikh *et al.* 2017, Benahmed *et al.* 2017, Bellifa *et al.* 2017a, Khetir *et al.* 2017, Besseghier *et al.* 2017, Fahsi *et al.* 2017, Abualnour *et al.* 2018, Benchohra *et al.* 2018, Bouhadra *et al.* 2018, Attia *et al.* 2018).

In the present work, an attempt is carried out to verify the efficiency of four variable refined shear deformation theory for the flexural, stability and dynamic investigation of composite plates. Undetermined integral terms in in-plane displacements are employed in the kinematics of the model to consider the shear deformation influences. The model respects conditions of zero shear stress at bottom and top surfaces of the plates. The model does not require problem dependent shear correction coefficient. Equations of motion are determined by considering the virtual work principle. Analytical solution is found by using a double trigonometric series method proposed by Navier. Finally, the results computed by utilizing present model are compared with exact elasticity solutions reported by Pagano (1970) for bending, Noor (1973) for dynamic and Noor (1975) for stability analysis of laminated composite plates.

2. Theoretical formulation

Consider a rectangular plate of the sides a and b, a constant thickness h and origin o as shown in Fig. 1. The plate consists of n number of homogenous layers which are perfectly bounded and made up of linearly elastic and orthotropic material. The plate occupies the region $0 \le x \le a$, $0 \le y \le b$, $-h/2 \le z \le h/2$ in Cartesian coordinate system. A transverse load q(x, y) is applied on the upper surface of the plate.

2.1 Kinematics and strains

In the unified shear-deformable plate theory, the displacement field at a point in the laminated plate is expressed as (Bakhadda *et al.* 2018)

$$
\begin{aligned}
u(x,y,z) &= u_0(x,y) - z\frac{\partial w_0}{\partial x} + f(z)\varphi_x(x,y) \\
v(x,y,z) &= v_0(x,y) - z\frac{\partial w_0}{\partial y} + f(z)\varphi_y(x,y) \\
w(x,y,z) &= w_0(x,y)
\end{aligned}
\tag{1}
$$

where u, v, and w are the displacements along x, y, and z directions, respectively, u_0, v_0, w_0, φ_x, and φ_y are five unknown displacement functions of the mid-plane of the plate. By assuming $\varphi_x(x,y) = \int \theta(x,y)dx$ and $\varphi_y(x,y) = \int \theta(x,y)dy$, the displacement field of the present theory can be rewritten with four unknowns is expressed as (Sekkal *et al.* 2017a, b, Bourada *et al.* 2018, Fourn *et al.* 2018, Yazid *et al.* 2018, Younsi *et al.* 2018, Zine *et al.* 2018, Meksi *et al.* 2019, Zaoui *et al.* 2019)

$$
\begin{aligned}
u(x,y,z) &= u_0(x,y) - z\frac{\partial w_0}{\partial x} + k_1 f(z)\int \theta(x,y)dx \\
v(x,y,z) &= v_0(x,y) - z\frac{\partial w_0}{\partial y} + k_2 f(z)\int \theta(x,y)dy \\
w(x,y,z) &= w_0(x,y)
\end{aligned}
\tag{2}
$$

It is clear that $\theta(x,y)$ is a mathematical term that allows obtaining the rotations of the normal to the midplate about the x and y axes. The constants k_1 and k_2 depends on the geometry and $f(z)$ denote a shape function determining the changes in the transverse shear strain and the stress distribution along the thickness of the plate and is defined as (Touratier 1991).

$$
f(z) = \left(\frac{h}{\pi}\right)\sin\left(\frac{\pi z}{h}\right) \tag{3}
$$

The strains associated with the present theory are obtained using strain-displacement relationship from theory of elasticity

$$
\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} \varepsilon_x^1 \\ \varepsilon_y^1 \\ \gamma_{xy}^1 \end{Bmatrix} + f(z)\begin{Bmatrix} \varepsilon_x^2 \\ \varepsilon_y^2 \\ \gamma_{xy}^2 \end{Bmatrix} \tag{4a}
$$

$$
\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial v}{\partial z} + \dfrac{\partial w}{\partial y} \\ \dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial x} \end{Bmatrix} = g(z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \tag{4b}
$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} \varepsilon_x^1 \\ \varepsilon_y^1 \\ \gamma_{xy}^1 \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \tag{5a}$$

$$\begin{Bmatrix} \varepsilon_x^2 \\ \varepsilon_y^2 \\ \gamma_{xy}^2 \end{Bmatrix} = \begin{Bmatrix} k_1 \theta \\ k_2 \theta \\ k_1 \dfrac{\partial}{\partial y} \int \theta dx + k_2 \dfrac{\partial}{\partial x} \int \theta dy \end{Bmatrix},$$

$$\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_2 \int \theta dy \\ k_1 \int \theta dx \end{Bmatrix} \tag{5b}$$

And

$$g(z) = \frac{df(z)}{dz} \tag{6}$$

The integrals used in the above relations shall be resolved by a Navier solution and can be expressed by

$$\frac{\partial}{\partial y}\int \theta dx = A' \frac{\partial^2 \theta}{\partial x \partial y}, \quad \frac{\partial}{\partial x}\int \theta dy = B' \frac{\partial^2 \theta}{\partial x \partial y}, \\ \int \theta dx = A' \frac{\partial \theta}{\partial x}, \quad \int \theta dy = B' \frac{\partial \theta}{\partial y} \tag{7}$$

where the parameters A' and B' are defined according to the type of solution employed, in this case via Navier. Hence, A' and B' are expressed by

$$A' = -\frac{1}{\alpha^2}, \qquad B' = -\frac{1}{\beta^2}, \qquad k_1 = \alpha^2, \\ k_2 = \beta^2 \tag{8}$$

where α and β are defined in expression (21).

2.2 Constitutive relations

Since the laminate is made of several orthotropic layers, the stress-strain relations in the kth layer of the laminated plate in the material coordinate axes are given by

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}^{(k)} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} & 0 & 0 \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} & 0 & 0 \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \overline{Q}_{44} & 0 \\ 0 & 0 & 0 & 0 & \overline{Q}_{55} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^{(k)} \tag{9}$$

where $\overline{Q}_{ij}$ are the transformed material constants of the kth orthotropic layer expressed as

$$\begin{aligned}
\overline{Q}_{11}^{k} &= Q_{11}\cos^4\theta_k + 2(Q_{12}+2Q_{66})\sin^2\theta_k\cos^2\theta_k + Q_{22}\sin^4\theta_k \\
\overline{Q}_{12}^{k} &= (Q_{11}+Q_{22}-4Q_{66})\sin^2\theta_k\cos^2\theta_k + Q_{12}(\sin^4\theta_k+\cos^4\theta_k) \\
\overline{Q}_{16}^{k} &= (Q_{11}-Q_{12}-2Q_{66})\sin\theta_k\cos^3\theta_k + (Q_{12}-Q_{22}+2Q_{66})\sin^3\theta_k\cos\theta_k \\
\overline{Q}_{22}^{k} &= Q_{11}\sin^4\theta_k + 2(Q_{12}+2Q_{66})\sin^2\theta_k\cos^2\theta_k + Q_{22}\cos^4\theta_k \\
\overline{Q}_{26}^{k} &= (Q_{11}-Q_{12}-2Q_{66})\sin\theta_k\cos^3\theta_k + (Q_{12}-Q_{22}+2Q_{66})\cos\theta_k + \sin^3\theta_k \\
\overline{Q}_{66}^{k} &= (Q_{11}+Q_{22}-2Q_{12}-2Q_{66})\sin^2\theta_k\cos^2\theta_k + Q_{66}(\sin^4\theta_k+\cos^4\theta_k) \\
\overline{Q}_{44}^{k} &= Q_{44}\cos^2\theta_k + Q_{55}\sin^2\theta_k \\
\overline{Q}_{45}^{k} &= (Q_{55}-Q_{44})\cos\theta_k\sin\theta_k \\
\overline{Q}_{55}^{k} &= Q_{55}\cos^2\theta_k + Q_{44}\sin^2\theta_k
\end{aligned} \tag{10}$$

where θ_k is the angle of material axes with the reference coordinate axes of each lamina and Q_{ij} are the plane stress-reduced stiffness coefficients defined in terms of the engineering constants in the material axes of the layer

$$Q_{11} = \frac{E_{11}}{1-\nu_{12}\nu_{21}}, Q_{22} = \frac{E_{22}}{1-\nu_{12}\nu_{21}}, Q_{12} = \frac{\nu_{12}E_{11}}{1-\nu_{12}\nu_{21}}, \tag{11a}$$

$$Q_{66} = G_{12}, Q_{44} = G_{23}, \quad Q_{44} = G_{23}, Q_{55} = G_{13} \tag{11b}$$

2.3 Equations of motion

The governing equations and boundary conditions of the present higher order shear deformation theory are derived using principle of virtual work. The principle of virtual work is applied in the following analytical form (Attia *et al.* 2015, Larbi Chaht *et al.* 2015, Bourada *et al.* 2015, Belkorissat *et al.* 2015, Ahouel *et al.* 2016, Boukhari *et al.* 2016, Bounouara *et al.* 2016, Houari *et al.* 2016, Hachemi *et al.* 2017, Bellifa *et al.* 2017b, Klouche *et al.* 2017, Zidi *et al.* 2017, Benadouda *et al.* 2017, Karami *et al.* 2018c, Kaci *et al.* 2018, Belabed *et al.* 2018, Bourada *et al.* 2019)

$$\begin{aligned}
&\int_{-h/2}^{h/2}\int_A \left(\sigma_x\delta\varepsilon_x + \sigma_y\delta\varepsilon_y + \tau_{xy}\delta\gamma_{xy} + \tau_{yz}\delta\gamma_{yz} + \tau_{xz}\delta\gamma_{xz}\right) dA dz + \\
&\int_{-h/2}^{h/2}\int_A \rho\left(\frac{\partial^2 u}{\partial t^2}\delta u + \frac{\partial^2 v}{\partial t^2}\delta v + \frac{\partial^2 w}{\partial t^2}\delta w\right) dA dz \\
&- \int_A q(x,y)\delta w dA - \int_A \left(N_x^0\frac{\partial^2 w}{\partial x^2} + N_y^0\frac{\partial^2 w}{\partial y^2} + 2N_{xy}^0\frac{\partial^2 w}{\partial x \partial y}\right)\delta w dA = 0
\end{aligned} \tag{12}$$

where A is the area of the top surface of the plate, ρ is the density of material, $q(x,y)$ and N_x^0, N_y^0, N_{xy}^0 are transverse and in-plane applied loads, respectively. The symbol δ denotes the variational operator. Substituting expressions for stresses and virtual strains into the principle of virtual work and integrating Eq. (12) by parts and collecting the coefficients of δu_0, δv_0, δw_0 and $\delta\theta$, the following equations of motion of the plate are obtained in terms of stress resultants

$$\delta u_0:\ \frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=I_0\frac{\partial^2 u_0}{\partial^2 t}-I_1\frac{\partial^3 w_0}{\partial x\partial t^2}+k_1A'I_3\frac{\partial^3\theta}{\partial x\partial t^2} \tag{13a}$$

$$\delta v_0:\ \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=I_0\frac{\partial^2 v_0}{\partial^2 t}-I_1\frac{\partial^3 w_0}{\partial y\partial t^2}+k_2B'I_3\frac{\partial^3\theta}{\partial y\partial t^2} \tag{13b}$$

$$\delta w_0:\ \frac{\partial^2 M_x^b}{\partial x^2}+2\frac{\partial^2 M_{xy}^b}{\partial x\partial y}+\frac{\partial^2 M_y^b}{\partial y^2}+N_x^0\frac{\partial^2 w_0}{\partial x^2}+N_y^0\frac{\partial^2 w_0}{\partial y^2}+2N_{xy}^0\frac{\partial^2 w_0}{\partial x\partial y}+q=I_0\frac{\partial^2 w_0}{\partial t^2}+I_1\frac{\partial^2}{\partial t^2}\left(\frac{\partial u_0}{\partial x}+\frac{\partial v_0}{\partial y}\right)$$
$$-I_2\nabla^2\frac{\partial^2 w_0}{\partial t^2}+k_1A'I_4\frac{\partial^4\theta}{\partial x^2\partial t^2}+k_2B'I_4\frac{\partial^4\theta}{\partial y^2\partial t^2} \tag{13c}$$

$$\delta\theta:\ k_1A'\frac{\partial^2 M_x^s}{\partial x^2}+\left(k_1A'+k_2B'\right)\frac{\partial^2 M_{xy}^s}{\partial x\partial y}+k_2B'\frac{\partial^2 M_y^s}{\partial y^2}-k_1A'\frac{\partial Q_{xz}^s}{\partial x}-k_2B'\frac{\partial Q_{yz}^s}{\partial y}=-I_3\left(k_1A'\frac{\partial^3 u_0}{\partial x\partial t^2}+k_2B'\frac{\partial^3 v_0}{\partial y\partial t^2}\right)$$
$$+I_4\left(k_1A'\frac{\partial^4 w_0}{\partial x^2\partial t^2}+k_2B'\frac{\partial^4 w_0}{\partial y^2\partial t^2}\right)-I_5\left(\left(k_1A'\right)^2\frac{\partial^4\theta}{\partial x^2\partial t^2}+\left(k_2B'\right)^2\frac{\partial^4\theta}{\partial y^2\partial t^2}\right) \tag{13d}$$

where ∇^2 is the Laplacian operator in two-dimensional Cartesian coordinate system and the stress resultants $(N_x,\ N_y,\ N_{xy})$, $(M_x^b,\ M_y^b,\ M_{xy}^b)$, $(M_x^s,\ M_y^s,\ M_{xy}^s)$ and $(S_{xz}^s,\ S_{yz}^s)$ are defined as

$$\begin{aligned}
\left(N_x,N_y,N_{xy}\right)&=\sum_{k=1}^{n}\int_{z_k}^{z_{k+1}}\left(\sigma_x,\sigma_y,\tau_{xy}\right)dz,\\
\left(M_x^b,M_y^b,M_{xy}^b\right)&=\sum_{k=1}^{n}\int_{z_k}^{z_{k+1}}\left(\sigma_x,\sigma_y,\tau_{xy}\right)z\,dz,\\
\left(M_x^s,M_y^s,M_{xy}^s\right)&=\sum_{k=1}^{n}\int_{z_k}^{z_{k+1}}\left(\sigma_x,\sigma_y,\tau_{xy}\right)f(z)\,dz,\\
\left(Q_{xz}^s,Q_{yz}^s\right)&=\sum_{k=1}^{n}\int_{z_k}^{z_{k+1}}\left(\tau_{xz},\tau_{yz}\right)g(z)\,dz
\end{aligned} \tag{14}$$

and the inertia constants (i=0, 2, 3, 4, 5) are defined by the following equations

$$\left(I_0,I_1,I_2,I_3,I_4,I_5\right)=\sum_{k=1}^{n}\int_{z_k}^{z_{k+1}}\rho^{(k)}\left(1,z,z^2,f(z),z\ f(z),\left[f(z)\right]^2\right)dz \tag{15}$$

by substituting the stress–strain relations expressed by Eq. (9) into the definitions of force and moment resultants of the present theory given in Eq. (14), the following equations are obtained

$$\begin{Bmatrix}\begin{Bmatrix}N_x\\N_y\\N_{xy}\end{Bmatrix}\\\begin{Bmatrix}M_x^b\\M_y^b\\M_{xy}^b\end{Bmatrix}\\\begin{Bmatrix}M_x^s\\M_y^s\\M_{xy}^s\end{Bmatrix}\end{Bmatrix}=\begin{bmatrix}
\begin{bmatrix}A_{11}&A_{12}&A_{16}\\A_{21}&A_{22}&A_{26}\\A_{61}&A_{62}&A_{66}\end{bmatrix}&\begin{bmatrix}B_{11}&B_{12}&B_{16}\\B_{21}&B_{22}&B_{26}\\B_{61}&B_{62}&B_{66}\end{bmatrix}&\begin{bmatrix}E_{11}&E_{12}&E_{16}\\E_{21}&E_{22}&E_{26}\\E_{61}&E_{62}&E_{66}\end{bmatrix}\\
\begin{bmatrix}B_{11}&B_{12}&B_{16}\\B_{21}&B_{22}&B_{26}\\B_{61}&A_{62}&B_{66}\end{bmatrix}&\begin{bmatrix}D_{11}&D_{12}&D_{16}\\D_{21}&D_{22}&D_{26}\\D_{61}&D_{62}&D_{66}\end{bmatrix}&\begin{bmatrix}F_{11}&F_{12}&F_{16}\\F_{21}&F_{22}&F_{26}\\F_{61}&F_{62}&F_{66}\end{bmatrix}\\
\begin{bmatrix}E_{11}&E_{12}&E_{16}\\E_{21}&E_{22}&E_{26}\\E_{61}&E_{62}&E_{66}\end{bmatrix}&\begin{bmatrix}F_{11}&F_{12}&F_{16}\\F_{21}&F_{22}&F_{26}\\F_{61}&F_{62}&F_{66}\end{bmatrix}&\begin{bmatrix}H_{11}&H_{12}&H_{16}\\H_{21}&H_{22}&H_{26}\\H_{61}&H_{62}&H_{66}\end{bmatrix}
\end{bmatrix}\begin{Bmatrix}\begin{Bmatrix}\varepsilon_x^0\\\varepsilon_y^0\\\gamma_{xy}^0\end{Bmatrix}\\\begin{Bmatrix}\varepsilon_x^1\\\varepsilon_y^1\\\gamma_{xy}^1\end{Bmatrix}\\\begin{Bmatrix}\varepsilon_x^2\\\varepsilon_y^2\\\gamma_{xy}^2\end{Bmatrix}\end{Bmatrix} \tag{16a}$$

and

$$\begin{Bmatrix}Q_{yz}^s\\Q_{xz}^s\end{Bmatrix}=\begin{bmatrix}A_{44}^s&A_{45}^s\\A_{45}^s&A_{55}^s\end{bmatrix}\begin{Bmatrix}\gamma_{yz}^0\\\gamma_{xz}^0\end{Bmatrix} \tag{16b}$$

where the plate stiffnesses A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij} and A_{ij}^s are defined as follows

$$\left(A_{ij},B_{ij},D_{ij},E_{ij},F_{ij},H_{ij}\right)=\sum_{k=1}^{n}\int_{z_k}^{z_{k+1}}\overline{Q}_{ij}^{(k)}\begin{pmatrix}1,z,z^2,f(z),z\ f(z),\\ \left[f(z)\right]^2\end{pmatrix}dz,\ i,j=1,2,6 \tag{17a}$$

$$A_{ij}^s=\sum_{k=1}^{n}\int_{z_k}^{z_{k+1}}\overline{Q}_{ij}^{(k)}\left[g(z)\right]^2dz,\quad i,j=4,5 \tag{17b}$$

Substituting the stress resultants in terms of unknown displacement variables from Eq. (16) into the Eq. (13), the equations of motion of the present theory can be rewritten as

$$\begin{aligned}
\delta u_0:\ &A_{11}\frac{\partial^2 u_0}{\partial x^2}+\left(A_{12}+A_{66}\right)\frac{\partial^2 v_0}{\partial x\partial y}+2A_{16}\frac{\partial^2 u_0}{\partial x\partial y}+A_{16}\frac{\partial^2 v_0}{\partial x^2}+A_{26}\frac{\partial^2 v_0}{\partial y^2}+A_{66}\frac{\partial^2 u_0}{\partial y^2}-B_{11}\frac{\partial^3 w_0}{\partial x^3}\\
&-\left(B_{12}+2B_{66}\right)\frac{\partial^2 w_0}{\partial x\partial y^2}-3B_{16}\frac{\partial^3 w_0}{\partial x^2\partial y}-B_{26}\frac{\partial^3 w_0}{\partial y^3}+k_1A'E_{11}\frac{\partial^3\theta}{\partial x^3}+k_2B'\left(E_{12}+E_{66}\right)\frac{\partial^3\theta}{\partial x\partial y^2}\\
&+\left(2k_1A'+k_2B'\right)E_{16}\frac{\partial^3\theta}{\partial x^2\partial y}+k_2B'E_{26}\frac{\partial^3\theta}{\partial y^3}+k_1A'E_{66}\frac{\partial^3\theta}{\partial x\partial y^2}=I_0\frac{\partial^2 u_0}{\partial t^2}-I_1\frac{\partial^3 w_0}{\partial x\partial t^2}+k_1A'I_3\frac{\partial^3\theta}{\partial x\partial t^2}
\end{aligned} \tag{18a}$$

$$\begin{aligned}
\delta v_0:\ &\left(A_{12}+A_{66}\right)\frac{\partial^2 u_0}{\partial x\partial y}+A_{16}\frac{\partial^2 u_0}{\partial x^2}+A_{22}\frac{\partial^2 v_0}{\partial y^2}+A_{26}\frac{\partial^2 u_0}{\partial y^2}+2A_{26}\frac{\partial^2 v_0}{\partial x\partial y}+A_{66}\frac{\partial^2 v_0}{\partial x^2}\\
&-\left(B_{12}+2B_{66}\right)\frac{\partial^2 w_0}{\partial x^2\partial y}-B_{16}\frac{\partial^3 w_0}{\partial x^3}-B_{22}\frac{\partial^3 w_0}{\partial y^3}-3B_{26}\frac{\partial^3 w_0}{\partial x\partial y^2}+k_1A'\left(E_{12}+E_{66}\right)\frac{\partial^3\theta}{\partial x^2\partial y}+k_1A'E_{16}\frac{\partial^3\theta}{\partial x^3}\\
&+k_2B'E_{22}\frac{\partial^3\theta}{\partial y^3}+\left(k_1A'+2k_2B'\right)E_{26}\frac{\partial^3\theta}{\partial x\partial y^2}+k_2B'E_{66}\frac{\partial^3\theta}{\partial x^2\partial y}=I_0\frac{\partial^2 v_0}{\partial t^2}-I_1\frac{\partial^3 w_0}{\partial y\partial t^2}+k_2B'I_3\frac{\partial^3\theta}{\partial y\partial t^2}
\end{aligned} \tag{18b}$$

$$\begin{aligned}
\delta w_0:\ &B_{11}\frac{\partial^3 u_0}{\partial x^3}+\left(B_{12}+2B_{66}\right)\frac{\partial^2 u_0}{\partial x\partial y^2}+\left(B_{12}+2B_{66}\right)\frac{\partial^2 v_0}{\partial x^2\partial y}+3B_{16}\frac{\partial^2 u_0}{\partial x^2\partial y}+B_{16}\frac{\partial^3 v_0}{\partial x^3}+B_{22}\frac{\partial^3 v_0}{\partial y^3}+B_{26}\frac{\partial^3 u_0}{\partial y^3}\\
&+B_{26}\frac{\partial^2 v_0}{\partial x\partial y^2}-D_{11}\frac{\partial^4 w_0}{\partial x^4}-2\left(D_{12}+2D_{66}\right)\frac{\partial^4 w_0}{\partial x^2\partial y^2}-4D_{16}\frac{\partial^4 w_0}{\partial x^3\partial y}-D_{22}\frac{\partial^4 w_0}{\partial y^4}-4D_{26}\frac{\partial^4 w_0}{\partial x\partial y^3}+k_1A'F_{11}\frac{\partial^4\theta}{\partial x^4}\\
&+k_1A'\left(F_{12}+2F_{66}\right)\frac{\partial^4\theta}{\partial x^2\partial y^2}+k_2B'\left(F_{12}+2F_{66}\right)\frac{\partial^4\theta}{\partial x^2\partial y^2}+\left(3k_1A'+k_2B'\right)F_{16}\frac{\partial^4\theta}{\partial x^3\partial y}+k_2B'F_{22}\frac{\partial^4\theta}{\partial y^4}\\
&+\left(k_1A'+3k_2B'\right)F_{26}\frac{\partial^4\theta}{\partial x\partial y^3}+N_x^0\frac{\partial^2 w_0}{\partial x^2}+N_y^0\frac{\partial^2 w_0}{\partial y^2}+2N_{xy}^0\frac{\partial^2 w_0}{\partial x\partial y}+q=I_0\frac{\partial^2 w_0}{\partial t^2}+I_1\left(\frac{\partial^3 u_0}{\partial x\partial t^2}+\frac{\partial^3 v_0}{\partial y\partial t^2}\right)\\
&-I_2\left(\frac{\partial^4 w_0}{\partial x^2\partial t^2}+\frac{\partial^4 w_0}{\partial y^2\partial t^2}\right)+I_4\left(k_1A'\frac{\partial^4\theta}{\partial x^2\partial t^2}+k_2B'\frac{\partial^4\theta}{\partial y^2\partial t^2}\right)
\end{aligned} \tag{18c}$$

$$\begin{aligned}
\delta\theta:\ &-k_1A'E_{11}\frac{\partial^3 u_0}{\partial x^3}-\left(k_2B'\left(E_{12}+E_{66}\right)+k_1A'E_{66}\right)\frac{\partial^3 u_0}{\partial x\partial y^2}-\left(2k_1A'+k_2B'\right)E_{16}\frac{\partial^3 u_0}{\partial x^2\partial y}-k_1A'E_{16}\frac{\partial^3 v_0}{\partial x^3}\\
&-k_2B'E_{22}\frac{\partial^3 v_0}{\partial y^3}-k_2B'E_{26}\frac{\partial^3 u_0}{\partial y^3}-\left(k_1A'+2k_2B'\right)E_{26}\frac{\partial^3 v_0}{\partial x\partial y^2}-k_1A'E_{66}\frac{\partial^3 v_0}{\partial x^2\partial y}-k_1A'F_{11}\frac{\partial^4 w_0}{\partial x^4}\\
&+k_1A'\left(F_{12}+2F_{66}\right)\frac{\partial^4 w_0}{\partial x^2\partial y^2}+\left(3k_1A'+k_2B'\right)F_{16}\frac{\partial^4 w_0}{\partial x^3\partial y}+k_2B'\left(F_{12}+2F_{66}\right)\frac{\partial^4 w_0}{\partial x^2\partial y^2}+k_2B'F_{22}\frac{\partial^4 w_0}{\partial y^4}\\
&+\left(k_1A'+3k_2B'\right)F_{26}\frac{\partial^4 w_0}{\partial x\partial y^3}-\left(k_1A'\right)^2H_{11}\frac{\partial^4\theta}{\partial x^4}-2k_1A'k_2B'\left(H_{12}+H_{66}\right)\frac{\partial^4\theta}{\partial x^2\partial y^2}-2\left(\left(k_1A'\right)^2+k_1A'k_2B'\right)H_{16}\frac{\partial^4\theta}{\partial x^3\partial y}\\
&-\left(k_2B'\right)^2H_{22}\frac{\partial^4\theta}{\partial y^4}-2\left(\left(k_2B'\right)^2+k_1A'k_2B'\right)H_{26}\frac{\partial^4\theta}{\partial x\partial y^3}-2\left(\left(k_1A'\right)^2+\left(k_2B'\right)^2\right)H_{66}\frac{\partial^4\theta}{\partial x^2\partial y^2}+\left(k_2B'\right)^2As_{44}\frac{\partial^2\theta}{\partial y^2}\\
&+2k_1A'k_2B'As_{45}\frac{\partial^2\theta}{\partial x\partial y}+\left(k_1A'\right)^2As_{55}\frac{\partial^2\theta}{\partial x^2}=-I_3\left(k_1A'\frac{\partial^3 u_0}{\partial x\partial t^2}+k_2B'\frac{\partial^3 v_0}{\partial y\partial t^2}\right)+I_4\left(k_1A'\frac{\partial^4 w_0}{\partial x^2\partial t^2}+k_2B'\frac{\partial^4 w_0}{\partial y^2\partial t^2}\right)\\
&-I_5\left(\left(k_1A'\right)^2\frac{\partial^4\theta}{\partial x^2\partial t^2}+\left(k_2B'\right)^2\frac{\partial^4\theta}{\partial y^2\partial t^2}\right)
\end{aligned} \tag{18d}$$

2.4 Analytical solution for laminated composite plates

The Navier approach is employed to determine the analytical solutions for the bending, buckling and free vibration analysis of laminated rectangular plates simply supported. The following simply supported boundary conditions at all four edges are given by

$$u_0 = w_0 = N_y = M_y^b = M_y^s = \theta = 0 \quad \text{on edges } (x{=}0,\ a) \tag{19a}$$

$$v_0 = w_0 = N_x = M_x^b = M_x^s = \theta = 0 \quad \text{on edges } (y{=}0,\ b) \tag{19b}$$

The displacement variables which automatically satisfy the above boundary conditions can be expressed in the following Fourier series

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn}\cos(\alpha\, x)\sin(\beta\, y)\, e^{i\omega t} \\ V_{mn}\sin(\alpha\, x)\cos(\beta\, y)\, e^{i\omega t} \\ W_{mn}\sin(\alpha\, x)\sin(\beta\, y)\, e^{i\omega t} \\ \Phi_{mn}\sin(\alpha\, x)\sin(\beta\, y)\, e^{i\omega t} \end{Bmatrix} \tag{20}$$

where U_{mn}, V_{mn}, W_{mn} and Φ_{mn} are the unknown Fourier coefficients to be determined for each (m, n) value, as well as the parameters α and β are defined as

$$\alpha = m\pi / a,\ \beta = n\pi / b \tag{21}$$

The transverse load $q(x, y)$ is expanded in the double-Fourier sine series as

$$q(x, y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} q_{mn}\sin(\alpha\, x)\cos(\beta\, y) \tag{22}$$

where $q_{mn}=q_0$ for sinusoidally distributed load m=1, n=1 and q_0 is the maximum intensity of distributed load at the centre of plate.

Substitution this form of solution Eq. (20) and transverse load Eq. (22) into the equations of motion (18) leads to the following matrix form

$$\left(\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{12} & K_{22} & K_{23} & K_{24} \\ K_{13} & K_{23} & K_{33}+N_{33} & K_{34} \\ K_{14} & K_{24} & K_{34} & K_{44} \end{bmatrix} - \omega^2 \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{12} & M_{22} & M_{23} & M_{24} \\ M_{13} & M_{23} & M_{33} & M_{34} \\ M_{14} & M_{24} & M_{34} & M_{44} \end{bmatrix} \right) \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ \Phi_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ q_{mn} \\ 0 \end{Bmatrix} \tag{23}$$

And the values of in-plane compressive forces are assumed as follows

$$N_{33} = -\left(N_x^0 \alpha^2 + N_y^0 \beta^2\right) \tag{24}$$

where the parameter N_{33} refers to the corresponding buckling load and can be expressed as

$$N_x^0 = k_1 N_0,\quad N_y^0 = k_2 N_0 \quad \text{and} \quad N_{xy}^0 = 0 \tag{25}$$

Eq. (23) is a general form for bending, buckling and free vibration analysis of simply supported laminated composite square and rectangular plates subjected to in-plane compressive and transverse loads. In order to solve bending problem, the in-plane compressive load N_0 and mass matrix $|M_{ij}|$ are set to zeros. However, the critical buckling loads N_{cr} can be obtained from the stability problem $|K_{ij}|$=0, while the free vibration problem is achieved by omitting both in-plane and transverse loads.

Table 1

a/h	Theory	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{\tau}_{xz}$	$\bar{\tau}_{yz}$
4	Present	0.0114	1.9766	0.9143	0.0889	0.0577	0.1274	0.1274
	Sayyad *et al.* (2016)	0.0114	1.9793	0.9154	0.0890	0.0578	0.0660	0.1276
	Sayyad and Ghugal (2014a)	0.0111	1.9424	0.9062	0.0964	0.0562	0.1270	0.1270
	Reddy (1984)	0.0114	2.0256	0.9172	0.0932	0.0713	0.1270	0.1270
	Mindlin (1951)	0.0088	1.9682	0.7157	0.0843	0.0525	0.0910	0.0910
	Kirchhoff (1850)	0.0088	1.0636	0.7157	0.0843	0.0525	—	—
	Pagano (1970)	—	2.0670	0.8410	0.1090	0.0591	0.1200	0.1350
10	Present	0.0093	1.2132	0.7483	0.0851	0.0533	0.1304	0.1304
	Sayyad *et al.* (2016)	0.0093	1.2135	0.7484	0.0851	0.0534	0.1270	0.1306
	Sayyad and Ghugal (2014a)	0.0092	1.2089	0.7471	0.0876	0.0530	0.1300	0.1300
	Reddy (1984)	0.0095	1.2479	0.7652	0.0889	0.0680	0.1310	0.1310
	Mindlin (1951)	0.0088	1.2083	0.7157	0.0843	0.0525	0.0910	0.0910
	Kirchhoff (1850)	0.0088	1.0636	0.7157	0.0843	0.0525	—	—
	Pagano (1970)	—	1.2250	0.7302	0.0886	0.0535	0.1210	0.1250

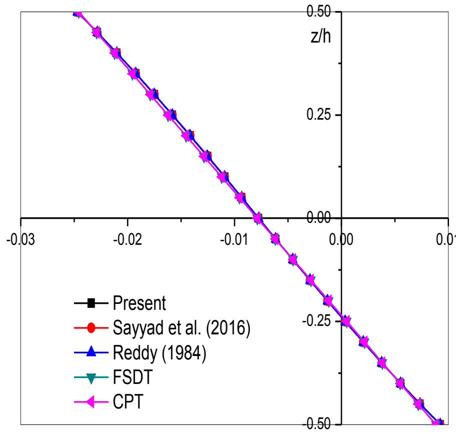

Fig. 2 Through thickness distribution of in-plane displacement ($\bar{u}$) for two layered (0°/90°) laminated composite plate subjected to sinusoidally distributed load (b/a=1, a/h= 10)

3. Discussion of numerical results

3.1 Bending analysis of laminated composite plates

The following material properties are employed for the bending investigation of simply supported anti-symmetric laminated composite square plates under sinusoidally distributed load.

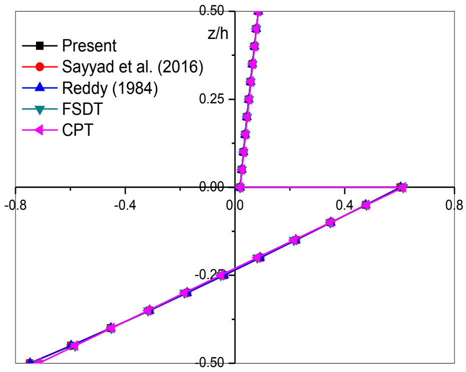

Fig. 3 Through thickness distribution of in-plane normal stress ($\bar{\sigma}_x$) for two layered (0°/90°) laminated composite plate subjected to sinusoidally distributed load (*b*/*a*=1, *a*/*h*= 10)

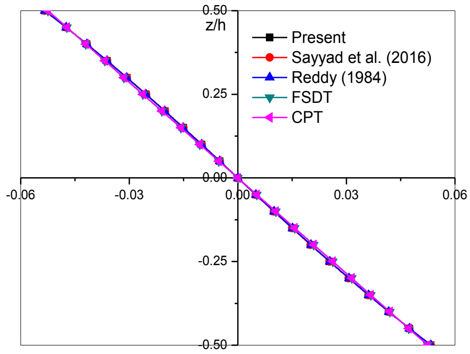

Fig. 4 Through thickness distribution of in-plane shear stress ($\bar{\tau}_{xy}$) for two layered (0°/90°) laminated composite plate subjected to sinusoidally distributed load (*b*/*a*=1, *a*/*h*= 10)

$$E_{11}=25E_{22},\quad G_{12}=G_{13}=0.5E_{22},$$
$$G_{23}=0.2E_{22},\quad \nu_{12}=0.25,\quad \nu_{21}=\frac{E_{22}}{E_{11}}\nu_{12} \tag{26}$$

The displacements and stresses are presented in the following non-dimensional form.

$$\bar{u}\left(0,\frac{b}{2},-\frac{h}{2}\right)=\frac{uE_2h^2}{q_0a^3},$$
$$\bar{w}\left(\frac{a}{2},\frac{b}{2},0\right)=\frac{100wh^3E_2}{q_0a^4},$$
$$\bar{\sigma}_x=\frac{h^2}{q_0a^2}\sigma_x\left(\frac{a}{2},\frac{b}{2},-\frac{h}{2}\right),$$
$$\bar{\sigma}_y=\frac{h^2}{q_0a^2}\sigma_y\left(\frac{a}{2},\frac{b}{2},-\frac{h}{2}\right), \tag{27}$$
$$\bar{\tau}_{xy}=\frac{h^2}{q_0a^2}\tau_{xy}\left(0,0,-\frac{h}{2}\right),$$
$$\bar{\tau}_{xz}=\frac{h}{q_0a}\tau_{xz}\left(0,\frac{b}{2},0\right),\quad \bar{\tau}_{yz}=\frac{h}{q_0a}\tau_{yz}\left(\frac{a}{2},0,0\right)$$

In this numerical example, the effectiveness of the current theory is demonstrated for the bending analysis of simply supported two layered (0°/90°) anti-symmetric laminated composite square plates subjected to sinusoidally distributed load. The non-dimensional displacement and stresses computed employing the present model are compared and discussed with those reported by (CPT) of Kirchhoff (1850), FSDT of Mindlin (1951), higher order shear deformation theory (HSDT) of Reddy (1984), sinusoidal shear and normal deformation theory (SSNDT) of Sayyad and Ghugal (2014a) and exact elasticity solution provided by Pagano (1970). The non-dimensional results are presented in Table 1.

It is seen that the in-plane displacement computed by present theory is in good agreement with other models. In-plane displacement is maximum in 90° layer whereas minimum in 0° layer (Fig. 2).

The proposed model underestimates the value of transverse displacement for aspect ratio 4 but it is in good

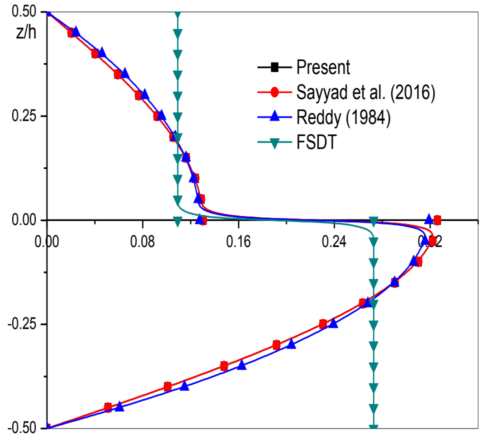

Fig. 5 Through thickness distribution of transverse shear stress ($\bar{\tau}_{xz}$) for two layered (0°/90°) laminated composite plate subjected to sinusoidally distributed load (*b*/*a*=1, *a*/*h*= 10)

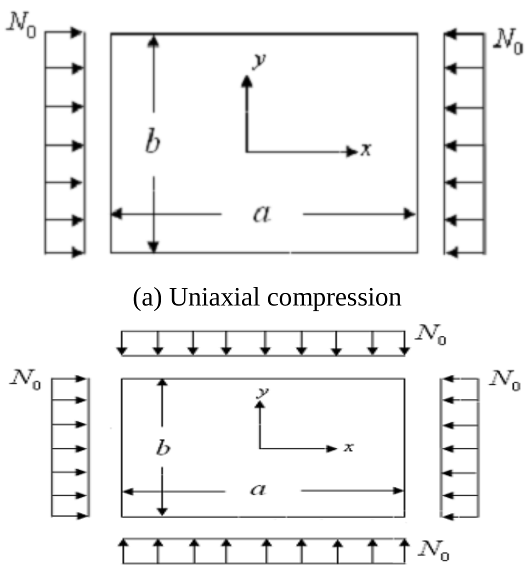

Fig. 6 A simply supported plate subjected to in-plane compressive forces

Table 2 Comparison of critical buckling load (N_{cr}) for simply supported laminated composite square plates under uniaxial and biaxial compression (a/h= 10)

Lay-up	Compression	(k_1,k_2)	Theory	E_{11}/E_{22}		
				20	30	40
0/90/0	Uniaxial	(1, 0)	Present	16.236	21.440	25.983
			Sayyad *et al.* (2016)	15.215	20.428	24.977
			Sayyad and Ghugal (2014b)	15.003	19.002	22.330
			Turan *et al.* (2017)	16.223	21.435	25.982
			Reddy (1984)	15.300	19.675	23.339
			FSDT	14.985	19.027	22.315
			CPT	19.712	27.936	36.160
			Noor (1975)	15.019	19.304	22.880
	Biaxial	(1, 1)	Present	8.1184	10.720	12.992
			Sayyad *et al.* (2016)	7.6075	10.214	12.488
			Sayyad and Ghugal (2014b)	7.5014	9.5009	11.165
			Turan *et al.* (2017)	7.6500	9.8376	11.669
			Reddy (1984)	7.4925	9.5135	11.157
			FSDT	9.8560	13.968	18.080
			CPT	7.5095	9.6520	11.440
0/90/0/90/0	Uniaxial	(1, 0)	Present	16.236	21.440	25.983
			Sayyad *et al.* (2016)	16.234	21.435	25.976
			Sayyad and Ghugal (2014b)	15.828	20.643	24.756
			Turan *et al.* (2017)	15.783	20.578	24.676
			Reddy (1984)	15.736	20.485	24.547
			FSDT	19.712	27.936	36.160
			CPT	15.653	20.466	24.593
	Biaxial	(1, 1)	Present	8.118	10.720	12.992
			Sayyad *et al.* (2016)	8.117	10.717	12.988
			Sayyad and Ghugal (2014b)	7.9140	10.321	12.378
			Turan *et al.* (2017)	7.8915	10.289	12.338
			Reddy (1984)	7.8680	10.240	12.273
			FSDT	9.8560	13.968	18.080
			CPT	7.8265	10.466	12.296

agreement with exact solution and other higher order models for aspect ratio 10.

Fig. 3 demonstrates that, in-plane normal stress ($\bar{\sigma}_x$) computed by the proposed theory is in close agreement with that of other models.

The in-plane shear stress ($\bar{\tau}_{xy}$) is presented in Fig. 4 where a good agreement is demonstrated between other theories. The proposed model predicts good values of transverse shear stress ($\bar{\tau}_{xz}$) as is shown in Fig. 5.

Table 3 Comparison of critical buckling load N_{cr}) for simply supported four layered (0°/90°/90°/0°) laminated composite rectangular plates under uniaxial and biaxial compression

Compression	(k_1,k_2)	a/h	Theory	b/a			
				1	2	3	4
Uniaxial	(1, 0)	5	Present	14.236	9.953	9.092	8.779
			Sayyad *et al.* (2016)	14.181	9.950	9.091	8.778
			Sayyad and Ghugal (2014b)	11.986	8.780	8.463	8.382
			FSDT	12.146	8.673	8.357	8.279
			CPT	36.160	29.833	29.259	29.102
		10	Present	25.983	19.792	18.708	18.315
			Sayyad *et al.* (2016)	25.908	19.785	18.705	18.313
			Sayyad and Ghugal (2014b)	23.387	18.500	18.057	17.941
			FSDT	23.453	18.398	17.962	17.849
			CPT	36.160	29.833	29.259	29.102
		100	Present	36.017	29.682	29.094	28.931
			Sayyad *et al.* (2016)	36.016	29.682	29.094	28.931
			Sayyad and Ghugal (2014b)	35.961	29.652	29.080	28.924
			FSDT	35.956	29.648	29.077	28.921
			CPT	36.160	29.833	29.259	29.102
Biaxial	(1, 1)	5	Present	7.1181	7.9624	8.1826	8.2625
			Sayyad *et al.* (2016)	7.0900	7.9600	8.1820	8.2620
			Sayyad and Ghugal (2014b)	5.9934	7.0244	7.6171	7.8896
			FSDT	6.0730	6.9387	7.5216	7.7928
			CPT	18.080	23.866	26.333	27.390
		10	Present	12.992	15.834	16.837	17.238
			Sayyad *et al.* (2016)	12.954	15.828	16.834	17.236
			Sayyad and Ghugal (2014b)	11.694	14.800	16.251	16.886
			FSDT	11.726	14.719	16.166	16.799
			CPT	18.080	23.866	26.333	27.390
		100	Present	18.009	23.746	26.185	27.229
			Sayyad *et al.* (2016)	18.008	23.746	26.185	27.229
			Sayyad and Ghugal (2014b)	17.980	23.722	26.172	27.223
			FSDT	17.978	23.718	26.169	27.219
			CPT	18.080	23.866	26.333	27.390

3.2 Buckling analysis of laminated composite plates

A simply supported laminated composite square and rectangular plates under the uniaxial and biaxial loading

Table 4 Comparison of non-dimensional natural frequencies of simply supported square laminated composite plates (a/h=10)

Lay-up	Theory	E_{11}/E_{22}			
		10	20	30	40
0/90	Present	0.27986	0.31352	0.34126	0.36494
	Sayyad *et al.* (2016)	0.27987	0.31354	0.34128	0.36498
	Sayyad and Ghugal (2015)	0.28060	0.31415	0.34181	0.36543
	Reddy (1984)	0.27955	0.31284	0.34020	0.36348
	FSDT	0.27757	0.30824	0.33284	0.35353
	CPT	0.30968	0.35422	0.39335	0.42884
	Noor (1975)	0.27938	0.30698	0.32705	0.34250
0/90/0	Present	0.34307	0.40639	0.44507	0.47162
	Sayyad *et al.* (2016)	0.34261	0.40623	0.44502	0.47162
	Sayyad and Ghugal (2015)	0.32696	0.37037	0.39498	0.41176
	Reddy (1984)	0.33095	0.38112	0.41094	0.43155
	FSDT	0.32739	0.37110	0.39540	0.41158
	CPT	0.42599	0.55793	0.66419	0.75565
	Noor (1975)	0.32841	0.38241	0.41089	0.43006
0/90/0/90	Present	0.3277	0.3848	0.4210	0.4465
	Sayyad *et al.* (2016)	0.3422	0.4055	0.4441	0.4706
	Sayyad and Ghugal (2015)	0.3319	0.3821	0.4119	0.4324
	Reddy (1984)	0.3308	0.3810	0.4108	0.4314
	FSDT	0.3319	0.3826	0.4130	0.4341
	CPT	0.4260	0.5579	0.6642	0.7556
	Noor (1975)	0.3257	0.3762	0.4066	0.4272
0/90/0/90/0	Present	0.3431	0.4064	0.4450	0.4716
	Sayyad *et al.* (2016)	0.3430	0.4063	0.4449	0.4715
	Sayyad and Ghugal (2015)	0.3384	0.3950	0.4287	0.4518
	Reddy (1984)	0.3399	0.3994	0.4350	0.4592
	FSDT	0.3368	0.3930	0.4271	0.4506
	CPT	0.4259	0.5579	0.6641	0.7556
	Noor (1975)	0.3408	0.3979	0.4314	0.4537

conditions, as presented in Fig. 6, is considered to prove the accuracy of the proposed model in predicting the buckling behavior. The following material properties are employed in the numerical study

$$N_{cr} = \left(N_0 a^2\right) / \left(E_{22} h^3\right) \tag{28}$$

A comparison of the critical buckling load parameters determined by the proposed model for a three layered (0°/90°/0°) and five layered (0°/90°/0°/90°/0°) symmetric cross-ply laminated composite square plates subjected to uniaxial and biaxial compressions for various modular ratios (E_{11}/E_{22}) is illustrated in Table 2.

The results of proposed model are compared with HSDT of Reddy (1984), SSNDT of Sayyad and Ghugal (2014b), Sayyad *et al.* (2016), Turan *et al.* (2017) FSDT of Mindlin (1951) and CPT of Kirchhoff (1850) and exact elasticity solution given by Noor (1975). It can be observed from Table 2 that the present results are in good agreement with other ones. It is also shown that the buckling loads given by CPT are significantly higher than those obtained by the proposed model. This is the consequence of omitting the transverse shear deformation influence in the CPT. It can be seen that the critical buckling loads in case of biaxial compression are exactly half of those of uniaxial compression for square plates.

Table 3 presents the critical buckling force parameter for four layered (0°/90°/90°/0°) symmetric laminated composite rectangular plate. The numerical results are determined for different values of b/a ratios and a/h ratios. From Table 3 it is seen that the critical buckling load increases with respect to increase in b/a and a/h ratios. It is also pointed out that the proposed model is in good agreement while predicting the buckling response of rectangular laminated composite plates.

3.3 Free vibration analysis of laminated composite plates

For this study, the material properties given by Eq. (28) are employed. Natural frequencies are presented in the following non-dimensional form

$$\bar{\omega} = \omega\sqrt{\rho h^2 / E_{22}} \tag{30}$$

In Table 4, non-dimensional natural frequencies of simply supported square laminated composite plates for different modular ratios (E_{11}/E_{22}) are given and compared with those predicted SSNDT of Sayyad and Ghugal (2015), Sayyad *et al.* (2016), HSDT of Reddy (1984), FSDT of Mindlin (1951), CPT of Kirchhoff (1850) and exact elasticity solution provided by Noor (1975). From the Table 4 it is seen that the proposed model is in good agreement while predicting the natural frequencies of laminated composite plates. The CPT overestimates the natural frequencies due to neglecting the transverse shear deformation influence. It is also noticed that the natural frequencies of laminated composite plates increase with respect to increase in modular ratios (E_{11}/E_{22}).

4. Conclusions

In the present investigation, a refined sinusoidal shear deformation theory is utilized for the bending, buckling and dynamic analysis of laminated composite plates. The most important feature of the present theory is that it has only four unknowns, contrary to the case of first order shear deformation theory and other higher order theories where five unknowns are required. The present theory satisfies the tensile conditions of the upper and lower surfaces of the plates without the use of a shear correction factor. From the mathematical formulation of the current theory, it has been observed that, due to four unknown variables, the current theory requires less computational effort compared to five and six variable shear deformation theories. The numerical results and discussion suggest that the current theory is in good agreement, while predicting the flexural, buckling and free vibration behavior of laminated composite plates.

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