Structural Engineering and Mechanics, *Vol. 70, No. 3 (2019) 269-277*
DOI: https://doi.org/10.12989/sem.2019.70.3.269

Theoretical analysis of chirality and scale effects on critical buckling load of zigzag triple walled carbon nanotubes under axial compression embedded in polymeric matrix

Tayeb Bensattalah*[1,2], Mohamed Zidour[1,2], Tahar Hassaine Daouadji[1,2] and Khaled Bouakaz[1,2]

[1]*Département de Génie Civil, Université Ibn Khaldoun Tiaret, BP 78 Zaaroura, Tiaret, Algérie*
[2]*Laboratoire de Géomatique et Développement Durable, Université de Tiaret, Algérie*

(Received August 4, 2018, Revised February 20, 2019, Accepted February 21, 2019)

Abstract. Using the non-local elasticity theory, Timoshenko beam model is developed to study the non- local buckling of Triple-walled carbon nanotubes (TWCNTs) embedded in an elastic medium under axial compression. The chirality and small scale effects are considered. The effects of the surrounding elastic medium based on a Winkler model and van der Waals' (vdW) forces between the inner and middle, also between the middle and outer nanotubes are taken into account. Considering the small-scale effects, the governing equilibrium equations are derived and the critical buckling loads under axial compression are obtained. The results show that the critical buckling load can be overestimated by the local beam model if the small-scale effect is overlooked for long nanotubes. In addition, significant dependence of the critical buckling loads on the chirality of zigzag carbon nanotube is confirmed. Furthermore, in order to estimate the impact of elastic medium on the non-local critical buckling load of TWCNTs under axial compression, the use of these findings are important in mechanical design considerations, improve and reinforcement of devices that use carbon nanotubes.

Keywords: triple-walled carbon nanotubes; elastic medium; nonlocal elasticity; chirality; buckling; Timoshenko beam theory

1. Introduction

In recent years, much research has been carried out on carbon nanotubes since their discovery in 1991, carbon nanotubes are the probable and hopeful source of one of the greatest technological revolution, and CNTs are tubular macromolecules. They exhibit superior mechanical, electronic, and thermal properties such as high elastic modulus and high strength, which make them attractive for a wide range of applications, for example, energy storage media, drug delivery, novel probes and sensors, ultrafine nanocomponents, etc. (Ajayan 2001, Zhou 2008, Adim 2016, Bouakaz 2014, Benferhat 2016). The recent progress and developing of double-walled carbon nanotubes (DWNTs), triple-walled carbon nanotubes (TWNTs) and multi-walled CNTs (MWCNTs), and may be understood as two, three and multiple concentric SWNTs, respectively. Its which interact with each other through interlayer interactions by weak van der Waals forces. The van der Waals interactions have a fundamental role in biology, physics and chemistry, in particular in the self-assembly and the ensuing function of nanostructured materials and these interlayer interactions will change according to the chirality combinations between the pairs (n, m)@(n1, m1) in a DWNT or among the set (n, m)@(n1, m1)@(n2, m2) in a TWNT (Charlier 2007, Ho 2004, Wei 2003, Tison 2008, Soto 2015, Saito 2001).

To the present, several experimental and theoretical study of the buckling behavior of a single walled carbon nanotube (SWCNT), multi-walled carbon nanotube (MWCNT) and nanocomposite (Abdelhak 2016, Demir 2016, Shahba 2011, Civalek 2011, Mercan 2017, Akgoz 2011, Tsiatas 2014, Adim 2018, Hassaine Daouadji 2016, Rabahi 2018, Tahar 2017, Benhenni 2018, Chaded 2017, Hassaine Daouadji 2017, Tayeb 2018, Benferhat 2018, Khelifa 2018, Bensattalah 2018) or bending are especially prone to buckling because of their high aspect ratios Lijima (1996). Can be the buckling of CNTs lead to potential applications as nanometer sized tunnel barriers for electron transport. Postma (2001), fluid-flow control nanovalves (Solares 2004, Tahar 2016, Zidour 2014, Rabahi 2017, Grujieric 2005) and instability of triple-walled carbon nanotubes (TWCNTs) conveying fluid (Yan 2009).

However, the technical difficulties involved in the manipulation of these nano-scale structures make the direct determination of their buckling properties a rather challenging task. Therefore, the theoretical methods, including atomistic simulations and continuum mechanics, are often applied for studying the buckling behavior of CNTs. For example, XinHao *et al*. (2008) employed the molecular dynamics (MD) simulations to simulate theaxially compressed buckling behaviors of SWCNTs and double-walled carbon nanotubes (DWCNTs). And Ranjbartoreh *et al.* (2007) used the classical theory of shells for analysed the buckling behavior and critical axial

*Corresponding author, Professor
E-mail: t_satal@yahoo.fr

http://www.techno-press.com/journals/sem&subpage=7
ISSN: 1225-4568 (Print), 1598-6217 (Online)

pressure of double-walled carbon nanotubes (a double-shells) with surrounding elastic medium. Using a systematic molecular mechanics (MM) by Cao and Chen (2006) for analysis the effect of the displacement of single-walled carbon nanotubes (SWCNTs) under the axial compressive loads.

However, the nonlocal continuum theory initiated by Eringen (1972, 1983), in this nonlocal continuum mechanics differs from classical continuum mechanics in two basic aspects: The proposed global form of the energy law and the stress at a point is influenced by the strain at all other points in the body (the small scale effects are taken into account). There are several studies on the axial and torsional buckling analyses behaviors of carbon nanotube with the nonlocal continuum theory in the literature. Sudak firstly studied the buckling of multiwalled carbon nanotubes using nonlocal multi-beam model (Sudak 2003). Zhang *et al.* (2004) firstly presented the nonlocal multi-shell model (Zhang 2004) and estimated a value of the scale effect parameter for nanotubes (Zhang 2005). As a result, the nonlocal continuum theory can present the more reliable analysis and show accurate results (Heriche 2008, Rabahi 2016, Simsek 2011, Lim 2010, Murmu 2010). He *et al.* (2005) used a cylindrical shell continuum model of modeling the van der Waals force for infinitesimal deformation of multi-walled carbon nanotubes to study the influence of the effect of vdW interaction between different layers of a CNT, and Ya (2010) is analysed the buckling of a simply supported nonlocal TWCNT of initial axial stress are analysed by Nonlocal shell model.

As yet, to the best of authors' knowledge, has not been studied the non-local buckling of Triple-walled carbon nanotubes (TWCNTs) embedded in an elastic medium under axial compression by nonlocal Timoshenko beam theory.

In this paper, the buckling behavior of zigzag TWCNT embedded in an elastic medium under axial compression is investigated based on nonlocal Timoshenko beam model. The chirality and small scale effects are considered. The effects of the surrounding elastic medium based on a Winkler model and van der Waals' (vdW) forces between the inner and middle, also between the middle and outer nanotubes are taken into account. The equivalent Young's modulus and shear modulus for zigzag TWCNTs used in this study are calculated by Bao *et al.* (2004), Civalek (2017), Mercan (2016) and by Tu and Ou-Yang (2002). The obtained results in this paper can provide useful guidance for the study and design of the next generation of nanodevices that make use of the mechanical buckling properties of zigzag Triple-walled carbon nanotubes.

2. Atomic structure of carbon nanotube

Carbon nanotubes are considered to be tubes formed by rolling a graphene sheet about the $\vec{T}$ vector. A vector perpendicular to the $\vec{T}$ is the chiral vector denoted by $\vec{C}_h$ .Using $\vec{T}\cdot\vec{C}_h = 0$

Translational vector $\vec{T}$, the chiral vector $\vec{C}_h$ and the corresponding chiral angle define the type of CNT, i.e., zigzag, armchair, chiral can be expressed with respect to two base vectors $\vec{a}_1$ and $\vec{a}_2$ as under (Charlier 2007, Civalek 2010)

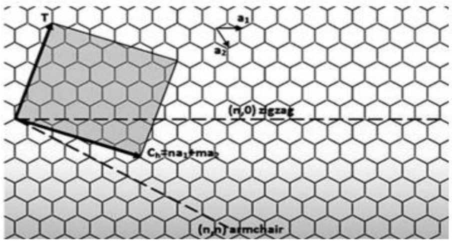

Fig. 1 Hexagonal lattice of graphene sheet including base vectors

$$\vec{C}_h = n\vec{a}_1 + m\vec{a}_2 \tag{1a}$$

$$\vec{T} = t_1\,\vec{a}_1 + t_2\,\vec{a}_2 \tag{1b}$$

$$t_1 = \frac{2m+n}{N_R} \ and \ t_2 = \frac{2n+m}{N_R} \tag{1c}$$

Where N_R is the greatest common divisor of ($2m+n$) and ($2n+m$), but n and m are the indices of translation, which decide the structure around the circumference. Fig. 1 depicts the lattice indices of translation (n, m) along with the base vectors, $\vec{a}_1$ and $\vec{a}_2$. If the indices of translation are such that $m=0$ and $n=m$ then the corresponding CNTs are categorized as zigzag and armchair, respectively. Considering the chirality the diameter and the chiral angle of the CNT can be calculated by the chiral vector for each nanostructure.

The radius of the zigzag nanotube in terms of the chiral vector components can be obtained from the relation (Tokio 1995)

$$R = \frac{na}{2\pi}\sqrt{3} \tag{2}$$

where "a" is the length of the carbon–carbon bond which is 1,42A^o.

3. The nonlocal Timoshenko beam model

The nonlocal continuum elasticity theory assumed that the stress at a reference point is considered to be a functional of the strain field at every point in the body proposed by Eringen (1972, 1983). The nonlocal elasticity theory is applied in various types of nanostructures (nano FGM structures, nanotube..) such as the static, free vibration, the buckling, wave propagation and thermo-mechanical analysis of (CNTs) (Zemri 2015, Civalek 2013, Aisssani 2015, Larbi 2015, Tounsi 2013, Bensattalah 2016, Wang 2003). The local or classical theory of elasticity is obtained when the effects of strains at points other than x are neglected. For homogeneous and isotropic elastic solids, the constitutive equation of non-local elasticity can be given by Eringen. Non-local stress tensor (t) at point (x′) is defined by

$$\sigma = \int_V K\left(|x-x'|,\ \tau\right) S(x')dx, \tag{3}$$

where $S(x')$ is the classical, macroscopic stress tensor at point x', $K(x\text{-}x'),\ \tau)$ is the kernel function and τ is a material constant that depends on internal and external characteristic length (such as the lattice spacing and wavelength).

Nonlocal constitutive relations for present nanobeams can be written as

$$\sigma_x - \mu \frac{\partial^2 \sigma_x}{\partial x^2} = E\varepsilon_x \tag{4a}$$

$$\tau_{xz} - \mu \frac{\partial^2 \tau_{xz}}{\partial x^2} = G\gamma_{xz} \tag{4b}$$

Where E and G are the Young's and shear moduli, respectively, and γ_{xz}, ε_x, are the shear strain in plane xz and the normal strain in the x direction, respectively.

where $\mu=(e_0a)^2$ is nonlocal parameter, a an internal characteristic length (e.g., length of C–C bond, lattice spacing, granular distance) and e0 a constant. Choice of e0a (in dimension of length) is crucial to ensure the validity of nonlocal models. This parameter was determined by matching the dispersion curves based on the atomic models (Eringen 1972, 1983). For a specific material, the corresponding nonlocal parameter can be estimated by fitting the results of atomic lattice dynamic and experiment.

Using the free body diagram of an infinitesimal element of the beam structure subjected to an axial loading P, the force equilibrium equations in vertical direction and the moment on the one-dimensional structure can be derived as follows

$$\frac{dV}{dx} = q(x)$$
$$V = \frac{dM}{dx} + P\frac{dw}{dx} \tag{5}$$

where P is the axial compression and q(x) is the distributed transverse force along axis x. w is the transverse displacement, M and V are the resultant bending moment and the resultant shear force, respectively.

Substituting the kinematics relationships, bending moment, and shear force into Eqs. (4), the bending moment M and the shear force V for the non-local model can be expressed as

$$M - \mu \frac{\partial^2 M}{\partial x^2} = EI\frac{d\psi}{dx} \tag{6a}$$

$$V - \mu \frac{\partial^2 V}{\partial x^2} = \beta\ GA\left(\psi - \frac{dw}{dx}\right) \tag{6b}$$

Where ψ is the rotation angle of cross-section of the beam. Also, A is the cross-section area of the beam, $I$the moment of inertia and β a correction factor depending on the shape of the cross-section of the considered beam.

Substituting Eqs. (6) into Eqs. (5) and eliminating ψ yield the following differential equation

$$EI\frac{d^4w}{dx^4} + \left(1-\mu\frac{d^2}{dx^2}\right)\left(1-\frac{EI}{\beta\ AG}\frac{d^2}{dx^2}\right)q(x) + \left(1-\mu\frac{d^2}{dx^2}\right)P\frac{d^2w}{dx^2} = 0 \tag{7}$$

The above equation is the equilibrium equation of a Timoshenko beam considering the non-local effects.

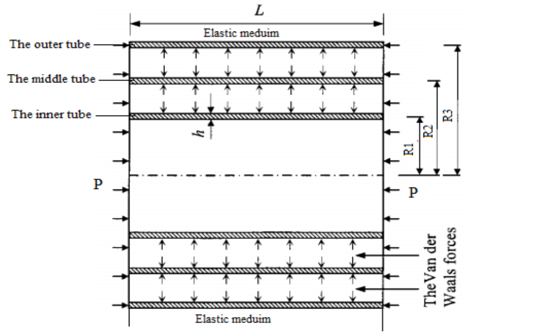

Fig. 2 A TWCNT under axial compression embedded in an elastic medium

4. Triple-walled carbon nanotubes

Fig. 2 shows a TWCNT under axial compression embedded in an elastic medium. In this figure R_1, R_2 and R_3 are: the radius of the inner, the middle and the outer tubes, respectively. Also, p is the buckling pressure of the TWCNT, Young's modulus E and h the thickness of the inner, the middle and outer nanotubes. We assume in throughout this section that the ends of all nanotubes are simply supported.

In this study, the buckling analysis of TWCNTs embedded in an elastic medium under axial compression has been investigated using the Timoshenko beam theory (TBT) based on the non-local continuum model. In this model, the effects of the surrounding elastic medium based on the Winkler model and the vdW force between the inner, middle and outer nanotubes are considered. The small-scale effect is clearly considered in the formulation.

For the outer tube which specified with subscript 3, the normal pressure $q_3(x)$ can be defined as

$$q_3(x) = q_3^W(x) + q_{32}^{vdW}(x) \tag{8}$$

Where $q_{32}^{vdW}(x)$ denotes the vdW force between the outer and middle tubes and $q_3^W(x)$ the interaction pressure due to the elastic medium. Based on the Winkler model, the elastic medium force can be written as

$$q_3^W(x) = -K_w \cdot w_3(x) \tag{9}$$

where K_w is the spring constant of the Winkler-type foundation.

For the middle tube which specified with subscript 2, the normal pressure $q_2(x)$ is due to the vdW force, then

$$q_2(x) = q_{23}^{vdW}(x) + q_{21}^{vdW}(x) \tag{10}$$

Where $q_{23}^{vdW}(x)$ denotes the vdW force between the middle and outer tubes and $q_{21}^{vdW}(x)$ the vdW force between the middle and inner tubes.

For the inner tube which specified with subscript 1, the normal pressure $q_1(x)$ is only due to the vdW force, then

$$q_1(x) = q_{12}^{vdW}(x) \tag{11}$$

The interaction forces (pressure) $q_{i\,(i+1)}^{vdW}(x)$, exerted on the i^{th} tube due to the $(i+1)^{th}$ tube, and the interaction forces (pressure) $q^{vdW}_{(i+1)\,i}(x)$, exerted on the $(i+1)^{th}$ tube due to the i^{th} tube, are related by

$$q^{vdW}_{i\,(i+1)}(x)\cdot R_i = -q^{vdW}_{(i+1)\,i}(x)\cdot R_{(i+1)} \quad i=1,2....n-1 \tag{12}$$

Using Eq. (12) we have

$$q^{vdW}_{12}(x)\cdot R_1 = -q^{vdW}_{21}(x)\cdot R_2 \;\; and \;\; q^{vdW}_{23}(x)\cdot R_2 = -q^{vdW}_{32}(x)\cdot R_3 \tag{13}$$

The van der Waals interaction potential, as a function of the interlayer spacing between two adjacent tubes, can be estimated by the Lennard-Jones model. The interlayer interaction potential between two adjacent tubes can be simply approximated by the potential obtained for two flat graphite monolayers, denoted by $g(\Delta)$, where Δ is the interlayer spacing (Girifalco 1956, 1991, Sears 2006, Heireche 2009, Batra 2007, Wang 2007). Since the interlayer spacing is equal or very close to initial equilibrium spacing, the initial van der Waals force is zero for each of the tubes provided they deform coaxially. Thus, for small-amplitude sound waves, the van der Waals pressure should be a linear function of the difference of the deflections of the two adjacent layers at the point as follows

$$q_{12} = tc_{12}(w_2 - w_1) \tag{14a}$$

$$q_{21} = -\frac{R_1}{R_2} tc_{12}(w_2 - w_1) \tag{14b}$$

$$q_{23} = tc_{23}(w_3 - w_2) \tag{14c}$$

$$q_{32} = -\frac{R_2}{R_3} tc_{23}(w_3 - w_2) \tag{14d}$$

Where w_i(i=1,2 and 3) shows the deflection of the i^{th} tube; and c_i(i+1) is the intertube interaction coefficient per unit length between two tubes, which can be estimated by Sudak (2003)

$$c_{i\,(i+1)} = \frac{320\,(2R_i)\,erg\,/\,cm^2}{0.16\,d^2}, (d = 0.142\ nm) \;\; and \;\; i=1,2...n-1 \tag{15}$$

where R_i the radius of i^{th}. The buckling pressure is the same for third tubes that is (Wang and Mioduchowski 2003)

$$P_1 = P_2 = P_3 = P \tag{16}$$

Using Eq. (7) and applying Eqs. (8), (10), (11) and (13), the governing equilibrium equations for the inner, middle and the outer tubes can be written as

$$EI_1\frac{d^4w_1}{dx^4} + \left(1-\mu\frac{d^2}{dx^2}\right)\left(1-\frac{EI_1}{\beta A_1 G}\frac{d^2}{dx^2}\right)\cdot q^{vdW}_{12}(x) + \left(1-\mu\frac{d^2}{dx^2}\right)P\frac{d^2w_1}{dx^2} = 0 \tag{17a}$$

$$EI_2\frac{d^4w_2}{dx^4} + \left(1-\mu\frac{d^2}{dx^2}\right)\left(1-\frac{EI_2}{\beta A_2 G}\frac{d^2}{dx^2}\right)\cdot\left(q^{vdW}_{23}(x) - q^{vdW}_{21}(x)\right) + \left(1-\mu\frac{d^2}{dx^2}\right)P\frac{d^2w_2}{dx^2} = 0 \tag{17b}$$

$$EI_3\frac{d^4w_3}{dx^4} + \left(1-\mu\frac{d^2}{dx^2}\right)\left(1-\frac{EI_3}{\beta A_3 G}\frac{d^2}{dx^2}\right)\cdot\left(q^{w}_{3}(x) - q^{vdW}_{32}(x)\right) + \left(1-\mu\frac{d^2}{dx^2}\right)P\frac{d^2w_3}{dx^2} = 0 \tag{17c}$$

Let us assume the buckling modes as (Heireche 2008).

$$w_1 = A\sin\left(\frac{m\pi}{L}x\right),\quad w_2 = B\sin\left(\frac{m\pi}{L}x\right),\quad w_3 = C\sin\left(\frac{m\pi}{L}x\right) \tag{18}$$

The above equations satisfy the simply supported boundary conditions which are

$$w_i = \frac{d^2w_i}{dx^2} = 0 \;\; \text{at} \;\; x = 0,\ L \quad (i = 1, 2, 3) \tag{19}$$

Replacing Eq. (17) into Eqs. (16), one can easily obtain

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix}\begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = 0 \tag{20}$$

where K_{11}, K_{12}, K_{13}, K_{21}, K_{22}, K_{23}, K_{31}, K_{32} and K_{33} in Eqs. (20) are defined as

$$K_{11} = EI_1\left(\frac{m\pi}{L}\right)^4 - \left\{tc_{12}\left[1+\frac{EI_1}{\beta A_1 G}\left(\frac{m\pi}{L}\right)^2\right] - P\left(\frac{m\pi}{L}\right)^2\right\}\left[1+\mu\cdot\left(\frac{m\pi}{L}\right)^2\right] \tag{21a}$$

$$K_{12} = tc_{12}\left[1+\frac{EI_1}{\beta A_1 G}\left(\frac{m\pi}{L}\right)^2\right]\left[1+\mu\cdot\left(\frac{m\pi}{L}\right)^2\right] \tag{21b}$$

$$K_{13} = 0 \tag{21c}$$

$$K_{21} = tc_{12}\frac{R_1}{R_2}\left[1+\frac{EI_2}{\beta A_2 G}\left(\frac{m\pi}{L}\right)^2\right]\left[1+\mu\cdot\left(\frac{m\pi}{L}\right)^2\right] \tag{21d}$$

$$K_{22} = EI_2\left(\frac{m\pi}{L}\right)^4 - \left\{\left(tc_{12}\frac{R_1}{R_2} + tc_{23}\right)\left[1+\frac{EI_2}{\beta A_2 G}\left(\frac{m\pi}{L}\right)^2\right] - P\left(\frac{m\pi}{L}\right)^2\right\}\left[1+\mu\cdot\left(\frac{m\pi}{L}\right)^2\right] \tag{21e}$$

$$K_{23} = tc_{23}\left[1+\frac{EI_2}{\beta A_2 G}\left(\frac{m\pi}{L}\right)^2\right]\left[1+\mu\cdot\left(\frac{m\pi}{L}\right)^2\right] \tag{21f}$$

$$K_{31} = 0 \tag{21g}$$

$$K_{32} = tc_{23}\frac{R_2}{R_3}\left[1+\frac{EI_3}{\beta A_3 G}\left(\frac{m\pi}{L}\right)^2\right]\left[1+\mu\cdot\left(\frac{m\pi}{L}\right)^2\right] \tag{21h}$$

$$K_{33} = EI_3\left(\frac{m\pi}{L}\right)^4 + \left\{\left(-tc_{23}\frac{R_2}{R_3} + K_w\right)\left[1+\frac{EI_3}{\beta A_3 G}\left(\frac{m\pi}{L}\right)^2\right] + P\left(\frac{m\pi}{L}\right)^2\right\}\left[1+\mu\cdot\left(\frac{m\pi}{L}\right)^2\right] \tag{21i}$$

For nontrivial solution, the determinant of the coefficient matrix in Eq. (20) must be zero. This gives the buckling pressure of the zigzag TWCNT in which the effects of the small scale and the van der Waals force between the inner and the outer tubes are shown.

5. Results and discussions

The Young's moduli used in this study of three types of single-walled carbon nanotubes(SWCNTs), armchair, zigzag and chiral tubules, are calculated by Bao *et al.* (2004) and Tu and Ou-Yang (2002) indicated that the relation between Young's modulus of multi-walled carbon nanotubes (MWCNTs) and the layer number N' can be expressed as

$$E_{MWNT} = \frac{N'}{N' - 1 + t/h}\frac{t}{h}E_{SWNT} \tag{22}$$

where E_{MWNT}, E_{SWNT}, t, N' and h are Young's modulus of multi-walled nanotubes, Young's modulus of single-walled nanotubes, effective wall thickness of single-walled nanotubes, number of layers and layer distance. E_{MWNT}=E_{SWNT} if N'=1, which corresponds to the case of single-walled carbon nanotubes. Based on the formulations obtained above with the nonlocal Timoshenko beam

Table 1 Lists the values of Young's modulus of single and double carbon nanotube for different chirality's

(n,m)	Young's modulus		
	SWNT-GPa (Bao2004)	DWNT-GPa (Tu2002)	TWNTGPa (Tu2002)
Zigzag			
(14,0) (23,0) (32,0)	939.032	856.397	831.992
(17,0) (26,0) (35,0)	938.553	855.960	831.568
(21,0) (30,0) (39,0)	936.936	854.486	830.135
(24,0) (33,0) (42,0)	934.201	851.991	827.712
(28,0) (37,0) (46,0)	932.626	850.555	826.316
(31,0) (40,0) (49,0)	932.598	850.529	826.291
(35,0) (44,0) (53,0)	933.061	850.952	826.702

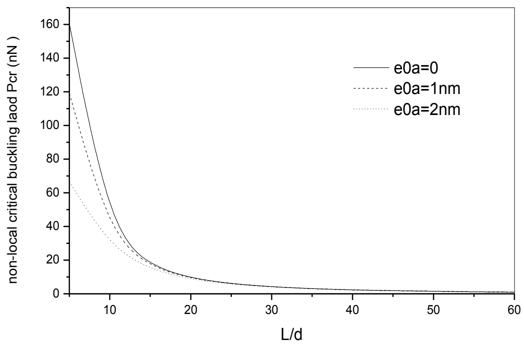

Fig. 3 Non-local critical buckling load of (14,0) (23,0) (31,0) TWCNT with different nonlocal values and first mode N=1 for different ration L/d based on Timoshenko beam model

models, the critical buckling loads of Triple-walled carbon nanotubes (TWCNT's) are discussed here. To investigate the critical buckling loads of (TWCNTs), the results including the aspect ratio of the (TWCNTs), the vibration al mode number and effect of nonlocal small-scale coefficient. In addition, to explore the effect of chirality, the critical buckling loads of different chiral of (TWCNTs) are compared. The parameters used in calculations for the zigzag SWCNTs, DWCNTs and TWCNTs are given as follows: the effective thickness of CNTs taken to be 0.258 nm (Wu *et al.* 2006), the mass density ρ=2,3g/cm^3 (2008). The shear coefficient β=9/10. It should be noted that according to the previous discussions about the values of e_0 and a in detail, e_0a is usually considered as the single scale coefficient which is smaller than 2.0 nm for nanostructures (Heireche 2008, Wang 2007).

The parameters such as the small scale parameter, the spring constant of elastic medium (i.., similar values of modulus parameters were taken by Ansari *et al.* (2015, 2012)) and the carbon nanotube aspect ratio affect the instability region of TWCNT. The effect of each parameter on the instability of TWCNT based on the nonlocal Timoshenko beam model is discussed here.

In Table 1, the Young's modulus of SWCNTs, DWCNTs and TWCNTs employed in this study, are

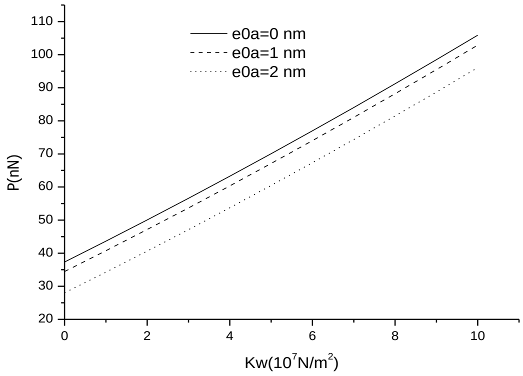

Fig. 4 Effect of the Winkler modulus parameter on critical buckling load for various small-scale coefficients (L/d=10, N=1)

calculated by Bai *et al.* (2004), Tu and Ou-Yang (2002) respectively. The results show the decreasing of Young's modulus (TWCNTs) for some chirality nanotube. The reason for this phenomenon is attributed to the weak van der Waals forces between the inner and outer tube.

Fig. 3 depicts the nonlocal critical buckling load of zigzag TWCNT with different nonlocal values and first mode N=1 for different respect to length-to-diameter ratio based on nonlocal Timoshenko beam model. The parameter value of e_0a=0 nm implies that the nonlocal effect is neglected. It can be seen that the effect of nonlocal parameter e_0a on the nonlocal critical buckling load is significant, especially at $L/d\leq20$. Increasing the nonlocal effect decreases the nonlocal critical buckling load.

The influences of the Winkler modulus parameter on critical buckling load of zigzag TWCNT on the non-local critical buckling load are shown in Figure 4. It is noticed that the critical buckling loads are sensitive to stiffness of the surrounding polymer elastic medium. As the Winkler modulus parameter increases (soft elastic medium to hard medium), the critical buckling loads also increase. This is because increasing the elastic medium constant makes the Triple-walled carbon nanotubes become stiffer.

The variation of non-local critical buckling loads of triple-walled carbon nanotubes (TWCNTs) zigzag chirality in the absence and presence of an elastic medium for the first and the sixth modes with different length-to-diameter ratios based on the non-local Timoshenko beam model are listed in Table 2. In this table, it is observed that as the mode number increases, the critical buckling load increases. In addition, it is observed, that the non-local critical buckling loads is more affected by elastic medium and the long of nanotube. If length-to-diameter ratios increase the non-local buckling load gets reduced and vice versa in case without an elastic medium and inversely in case with elastic medium. Finally, the results show the dependence of the different chirality's of carbon nanotube, Aspect Ratio and, vibrational mode number on the non-local critical buckling loads.

The values of the non-local critical buckling load (P_{cr}) for (TWNTC) type zigzag with different chiralities, in the absence and presence of an elastic medium, calculated by the nonlocal Timoshenko beam model, are listed in Table.

Table 2 Comparison between the values of non-local critical buckling load for different zigzag chirality's and Winkler modulus parameter of carbon nanotube, when the value of scale coefficients (e0a) is 2 nm and first mode N=1 and in sixth mode N=6

		N=1,	$P_{cr}(nN)$		
(TWCNTs) Zigzag	L/d	(Kw=0 N/m^2)	(Kw=10^{7}N/m^2)	(Kw=5.10^7 N/m^2)	(Kw=15.10^{7}N/m^2)
(14,0) (23,0) (32,0)	10	28.000	28.056	28.087	28.118
	100	0.37	3.177	5.985	8.793
(17,0) (26,0) (35,0)	10	29.729	29.773	29.816	29.859
	100	0.363	4.368	8.373	12.379
(21,0) (30,0) (39,0)	10	31.988	32.051	32.114	32.176
	100	0.366	6.27	12.173	18.078
(24,0) (33,0) (42,0)	10	33.600	33.751	33.831	33.911
	100	0.374	7.924	15.475	23.026
(28,0) (37,0) (46,0)	10	36.000	36.155	36.26	36.366
	100	0.39	10.431	20.472	30.515
(31,0) (40,0) (49,0)	10	37.921	38.048	38.175	38.302
	100	0.405	12.534	24.664	36.795
(35,0) (44,0) (53,0)	10	40.400	40.649	40.808	40.967
	100	0.427	15.633	30.841	46.049
		N=6,	Pcr (nN)		
(14,0) (23,0) (32,0)	10	21.593	21.594	21.594	21.595
	100	11.945	12.025	12.105	12.185
(17,0) (26,0) (35,0)	10	29.146	29.147	29.148	29.149
	100	12.102	12.216	12.330	12.444
(21,0) (30,0) (39,0)	10	39.665	39.667	39.668	39.670
	100	12.540	12.707	12.874	13.041
(24,0) (33,0) (42,0)	10	45.604	45.606	45.609	45.611
	100	12.961	13.175	13.388	13.602
(28,0) (37,0) (46,0)	10	48.008	48.011	48.015	48.019
	100	13.649	13.932	14.216	14.5
(31,0) (40,0) (49,0)	10	43.126	43.131	43.136	43.141
	100	14.232	14.574	14.917	16.353
(35,0) (44,0) (53,0)	10	23.503	23.51	23.517	23.523
	100	15.067	15.496	15.924	16.353

2. As is seen, the non-local critical buckling load (P_{cr}) increase as the diameter of carbon nanotubes increases at different chiralities.

Fig. 5 illustrates the non-local critical buckling load using nonlocal Timoshenko Beam Theory (TBT) under axial compression versus the respect to length-to-diameter ratio (*L/d*) for different Winkler elastic medium. It can be seen that the non-local critical buckling load under axial compression in the presence of the surrounding elastic medium $K_w \neq 0$ is higher than that in the absence of the surrounding elastic medium $K_w=0$ for long nanotubes. Since considering the Winkler elastic medium causes to stiff the outer tube. Also, the difference between the two cases increases with increasing *L/d*; whereas, for short nanotubes,

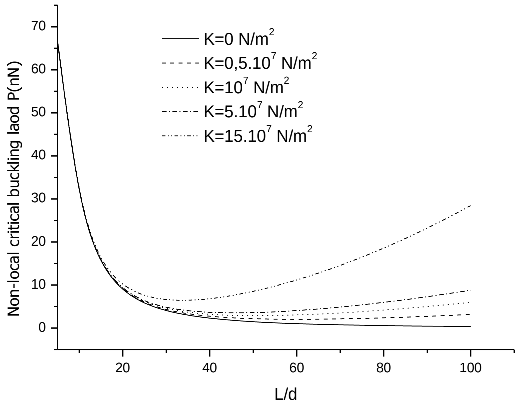

Fig. 5 Non-local critical buckling load of (14,0) (23,0) (31,0) TWCNT with different Winkler modulus of elastic medium and first mode N=1 for $e_0a=2nm$ based on Timoshenko beam model

the difference between the two cases is negligible. This means that for short nanotubes, the spring constant of the Winkler type can be ignored.

6. Conclusions

This paper studies the Influence of Winkler modulus parameter, non-local small-scale coefficient, the aspect ratio and the chirality of a carbon nanotube on the nonlocal critical buckling loads using non-local Timoshenko beam theory. The obtained numerical results show that:

- If Winkler modulus parameter increases, the critical buckling loads also increase. This is because increasing the elastic medium constant makes the Triple-walled carbon nanotubes become stiffer, mostly for long nanotubes.
- Increasing the value of scale coefficient decreased the critical buckling load, especially at higher L/d ratio.
- The effect of chirality on the non-local critical buckling load will diminish with the scale coefficients increasing.
- Increasing the value of mode the critical buckling load, also increase.

References

Abdelhak, Z., Hadji, L., Tahar, H.D. and Adda Bedia, E.A. (2016), "Thermal buckling response of functionally graded sandwich plates with clamped boundary conditions", *Smart Struct. Syst.*, **18**(2), 267-291. https://doi.org/10.12989/sss.2016.18.2.267.

Adim, B., Tahar, H.D., Abbas, B. and Rabahi, A. (2016), "Buckling and free vibration analysis of laminated composite plates using an efficient and simple higher order shear deformation theory", *J. Mech. Industr.*, **17**(5), 512. https://doi.org/10.1051/meca/2015112.

Adim, B., Tahar, H.D., Abderrezak, R., Amine, B.M., Mohamed, Z. and Boussad, A. (2018), "Mechanical buckling analysis of hybrid laminated composite plates under different boundary conditions", *Struct. Eng. Mech.*, **66**(6), 761-769. https://doi.org/10.12989/sem.2018.66.6.761.

Aissani, K., Bachir Bouiadjra, M., Ahouel, M. and Tounsi, A. (2015), "A new nonlocal hyperbolic shear deformation theory for nanobeams embedded in an elastic medium", *Struct*. *Eng*. *Mech*., **55**(4), 743-762. https://doi.org/10.12989/sem.2015.55.4.743.

Ajayan P.M. and Zhou, O.Z. (2001), "Applications of carbon nanotubes", *Appl*. *Phys*., **80**, 391-425.

Akgöz, B. and Civalek, O. (2011), "Buckling analysis of cantilever carbon nanotubes using the strain gradient elasticity and modified couple stress theories", *J*. *Comput*. *Theoret*. *Nanosci*., **8**(9), 1821-1827. https://doi.org/10.1166/jctn.2011.1888.

Ansari, R., Gholami, R. and Sahmani, S. (2012), "On the dynamic stability of embedded single-walled carbon nanotubes including thermal environment effects", *Sci*. *Iran*., **19**(3), 919-925. https://doi.org/10.1016/j.scient.2012.02.013.

Ansari, R., Gholami, R., Sahmani, S., Norouzzadeh, A. and Bazdid-Vahdati, M. (2015), "dynamic stability analysis of embedded multi-walled carbon nanotubes in thermal environment", *Acta Mech*. *Sol*. *Sinic*., **28**(6), 659-667. https://doi.org/10.1016/S0894-9166(16)30007-6.

Bao, W.X., Zhu, C.C. and Cui, W.Z. (2004), "Simulation of Young's modulus of single-walled carbon nanotubes by molecular dynamics", *Phys*. *B*, **352**(1-4), 156-163. https://doi.org/10.1016/j.physb.2004.07.005.

Batra, R.C. and Sears, A. (2007), "Continuum models of multi-walled carbon nanotubes", *Int*. *J*. *Sol*. *Stuct*., **44**(22-23), 7577-7596. https://doi.org/10.1016/j.ijsolstr.2007.04.029.

Benferhat, R., Rabahi, A., Tahar, H.D. Abbes, B., Adim, B. and Abbes, F. (2018), "Analytical analysis of the interfacial shear stress in RC beams strengthened with prestressed exponentially-varying properties plate", *Adv*. *Mater*. *Res*., **7**(1), 29-44. https://doi.org/10.12989/amr.2018.7.1.029.

Benferhat, R., Tahar, H.D. and Mansour, M.S. (2016), "Free vibration analysis of FG plates resting on the elastic foundation and based on the neutral surface concept using higher order shear deformation theory", *Compt*. *Rend*. *Mecaniq*., **344**(9), 631-641. https://doi.org/10.1016/j.crme.2016.03.002.

Benhenni, M., Tahar, H.D., Abbes, B., Li, Y.M. and Abbes, F. (2018), "Analytical and numerical results for free vibration of laminated composites plates", *Int*. *J*. *Chem*. *Molecul*. *Eng*., **12**(6), 300-304.

Bensattalah, T., Bouakkaz, K., Zidour, M. and Daouadji, T.H. (2018), "Critical buckling loads of carbon nanotube embedded in Kerr's medium", *Adv*. *Nano Res*., **6**(4), 339-356. https://10.12989/anr.2018.6.4.339.

Bensattalah, T., Mohamed, Z., Amar Meziane, M.A., Tahar, H.D. and Tounsi, A. (2018) "Critical buckling load of carbon nanotube with non-local Timoshenko beam using the differential transform method", *Int*. *J*. *Civil Environ*. *Eng*., **12**(6).

Bensattalah, T., Tahar, H.D., Zidour, M., Tounsi, A. and Adda Bedia, E.A. (2016), "Investigation of thermal and chirality effects on vibration of single-walled carbon nanotubes embedded in a polymeric matrix using nonlocal elasticity theories", *Mech*. *Compos*. *Mater*., **52**(4), 555-568. https://doi.org/10.1007/s11029-016-9606-z.

Bouakaz K., Tahar, H.D., Meftah, S.A., Ameur, M. and Adda Bedia, E.A. (2014), "A numerical analysis of steel beams strengthened with composite materials", *Mech*. *Compos*. *Mater*., **50**(4), 685-696. https://doi.org/10.1007/s11029-014-9435-x.

Cao, G. and Chen, X. (2006), "The effect of the displacement increment on the axial compressive buckling behaviours of single-walled carbon nanotubes", *Nanotechnol*., **17**(15), 3844.

Chaded, A., Tahar, H.D., Abderezak, R., Belkacem, A., Abbes, B., Rabia, B. and Abbes, F. (2017), "A high-order closed-form solution for interfacial stresses in externally sandwich FGM plated RC beams", *Adv*. *Mater*. *Res*., **6**(4), 317-328. https://doi.org/10.12989/amr.2017.6.4.317.

Charlier, J.C. Blase, X. and Roche, S. (2007), "Electronic and transport properties of nanotubes", *Rev*. *Mod*. *Phys*., **79**(2), 677-732. https://doi.org/10.1103/RevModPhys.79.677.

Chen, W.J. and Li, X.P. (2015), "Size-dependent free vibration analysis of composite laminated Timoshenko beam based on new modified couple stress theory", *Arch*. *Appl*. *Mech*., **83**(3), 431-444. https://doi.org/10.1007/s00419-012-0689-2.

Civalek, O. (2013), "Nonlinear dynamic response of laminated plates resting on nonlinear elastic foundations by the discrete singular convolution-differential quadrature coupled approaches", *Compos*. *Part B*, **50**, 171-179. https://doi.org/10.1016/j.compositesb.2013.01.027.

Civalek, O. (2017), "Free vibration of carbon nanotubes reinforced (CNTR) and functionally graded shells and plates based on FSDT via discrete singular convolution method", *Compos*. *Part B*, **111**, 45-59. https://doi.org/10.1016/j.compositesb.2016.11.030.

Civalek, O. and Demir, C. (2011), "Buckling and bending analyses of cantilever carbon nanotubes using the Euler-Bernoulli beam theory based on non-local continuum model", *Asian J*. *Civil Eng*., **12**(5), 651-661.

Civalek, O., Korkmaz, A. and Demir, C. (2010), "Discrete singular convolution approach for buckling analysis of rectangular Kirchhoff plates subjected to compressive loads on two-opposite edges", *Adv*. *Eng*. *Softw*., **41**(4), 557-560. https://doi.org/10.1016/j.advengsoft.2009.11.002.

Demir, C., Mercan, K. and Civalek, O. (2016), "Determination of critical buckling loads of isotropic, FGM and laminated truncated conical panel", *Compos*. *Part B*, **94**, 1-10. https://doi.org/10.1016/j.compositesb.2016.03.031.

Eringen, A.C. (1972), "Nonlocal polar elastic continua", *Int*. *J*. *Eng*. *Sci*., **10**(1), 1-16. https://doi.org/10.1016/0020-7225(72)90070-5.

Eringen, A.C. (1983), "On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves", *J*. *Appl*. *Phys*., **54**(9), 4703-4710. https://doi.org/10.1063/1.332803.

Girifalco L.A. and Lad, R.A. (1956), "Energy of cohesion, compressibility, and the potential energy functions of the graphite system", *J*. *Chem*. *Phys*., **25(4)**, 693-697. https://doi.org/10.1063/1.1743030.

Girifalco, L.A. (1991), "Interaction potential for carbon (C60) molecules", *J*. *Chem*. *Phys*., **95**(14), 5370-5371. https:// 10.1021/j100167a002.

Grujicic, M., Cao, G., Pandurangana, B. and Royb, W.N. (2005), "Finite element analysis-based design of a fluid-flow control nano-valve", *Mater*. *Sci*. *Eng*. *B*, **117**(1), 53-61. https://doi.org/10.1016/j.mseb.2004.10.020.

Hao, X., Qiang, H. and Xiaohu, Y. (2008), "Buckling of defective single-walled and double-walled carbon nanotubes under axial compression by molecular dynamics simulation", Compos. Sci. Technol., **68**(7-8), 1809-1814. https://doi.org/10.1016/j.compscitech.2008.01.013.

He, X.Q., Kitipornchai, S., Wang, C.M. and Liew, K.M. (2005), "Modeling of van der Waals force for infinitesimal deformation of multi-walled carbon nanotubes treated as cylindrical shells", *Int*. *J*. *Sol*. *Struct*., **42**(23), 6032-6047. https://doi.org/10.1016/j.ijsolstr.2005.03.045.

Heireche, H., Tounsi, A., Benzair, A. and Mechab, I. (2008), "Sound wave propagation in single-walled carbon nanotubes with initial axial stress", *J*. *Appl*. *Phys*., **104**(1), 014301. https://doi.org/10.1063/1.2949274.

Heireche, H., Tounsi, A., Benzair, A., Maachou, M., Adda Bedia, E.A. (2009), "Sound wave propagation in single-walled carbon nanotubes using nonlocal elasticity", *Phys*. E, **40**(8), 2791-2799. https://doi.org/10.1016/j.physe.2007.12.021.

Ho, Y.H., Chang, C.P., Shyu, F.L., Chen, R.B., Chen, S.C. and

Lin, M.F. (2004), “Electronic and optical properties of double-walled armchair carbon nanotubes”, *Carb*., **42**(15), 3159-3167. https://doi.org/10.1016/j.carbon.2004.07.027.

Khalifa, Z., Hadji, L., Tahar, H.D. and Bourada, M. (2018), “Buckling response with stretching effect of carbon nanotube-reinforced composite beams resting on elastic foundation”, *Struct. Eng. Mech.*, **67**(2), 125-130. https:// 10.12989/sem.2018.67.2.125.

Larbi Chaht, F., Kaci, A., Houari, M.S.A., Tounsi, A., Anwar Bég, O. and Mahmoud, S.R. (2015), “Bending and buckling analyses of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect”, *Steel Compos. Struct*., **18**(2), 425-442. https://doi.org/10.12989/scs.2015.18.2.425.

Lijima S., Brabec, C., Maiti, A. and Bernholc, J. (1996), “Structural flexibility of carbon nanotubes”, *J. Chem. Phys*., **104**(5), 2089-2092. https://doi.org/10.1063/1.470966.

Lim, C.W. (2010), “Is a nanorod (or nanotube) with a lower Young’s modulus stiffer? Is not Young’s modulus a stiffness indicator”, *Sci. Chin. Phys. Mech. Astronom*., **53**(4), 712-724. https://doi.org/10.1007/s11433-010-0170-6.

Mercan, K. and Civalek, O. (2016), “DSC method for buckling analysis of boron nitride nanotube (BNNT) surrounded by an elastic matrix”, *Compos. Struct*., **143**, 300-309. https://doi.org/10.1016/j.compstruct.2016.02.040.

Mercan, K. and Civalek, O. (2017), “Buckling analysis of Silicon carbide nanotubes (SiCNTs) with surface effect and nonlocal elasticity using the method of HDQ”, *Compos. Part B*, **114**, 34-45. https://doi.org/10.1016/j.compositesb.2017.01.067.

Murmu, T. and Adhikari, S. (2010), “Nonlocal transverse vibration of double-nanobeam-systems”, *J. Appl. Phys*., **108**(8), 083514. https://doi.org/10.1063/1.3496627.

Postma H.W.C., Teepen, T., Yao, Z., Grigoni, M. and Dekker, C. (2001), “Carbon nanotube single-electron transistors at room temperature”, *Sci*., **293**(5527), 76-79. https:// 10.1126/science.1061797.

Rabahi, A., Tahar, H.D., Abbes, B. and Adim, B. (2016), “Analytical and numerical solution of the interfacial stress in reinforced-concrete beams reinforced with bonded prestressed composite plate”, *J. Reinf. Plast. Compos*., **35**(3), 258-272. https://doi.org/10.1177/0731684415613633.

Rabahi, A., Tahar, H.D., Benferhat, R. and Adim, B. (2017), “Elastic analysis of interfacial stress concentrations in CFRP-RC hybrid beams: Effect of creep and shrinkage”, *Adv. Mater. Res*., **6**(3), 257-278. https://doi.org/10.12989/amr.2017.6.3.257.

Rabahi, A., Tahar, H.D., Benferhat, R. and Adim, B. (2018), “Nonlinear analysis of damaged RC beams strengthened with glass fiber reinforced polymer plate under symmetric loads”, *Earthq. Struct*., **15**(2), 113-122. https://doi.org/10.12989/eas.2018.15.2.113.

Ranjbartoreh, A.R., Ghorbanpour, A. and Soltani, B. (2007), “Double-walled carbon nanotube with surrounding elastic medium under axial pressure”, *Phys*. E, **39**(2), 230-239. https://doi.org/10.1016/j.physe.2007.04.010.

Saito, R., Matsuo, R., Kimura, T., Dresselhaus, G. and Dresselhaus, M.S. (2001), “Anomalous potential barrier of double-wall carbon nanotubes”, *Chem. Phys. Lett*., **348**(3-4), 187-193. https://doi.org/10.1016/S0009-2614(01)01127-7.

Sears, A. and Batra, R.C. (2006), “Buckling of multiwalled carbon nanotubes under axial compression”, *Phys. Rev. B*, **73**(8), 085410. https://doi.org/10.1103/PhysRevB.73.085410.

Shahba, A., Attarnejada, R. and Hajilar, S. (2011), “Free vibration and stability of axially functionally graded tapered Euler-Bernoulli beams”, *Shock Vibr*., **18**(5), 683-696. http://dx.doi.org/10.3233/SAV-2010-0589

Şimşek, M. (2011), “Forced vibration of an embedded single-walled carbon nanotube traversed by a moving load using nonlocal Timoshenko beam theory”, *Steel Compos. Struct*., **11**(1), 59-76. https://doi.org/10.12989/scs.2011.11.1.059.

Solares S.D., Blanco, M. and Goddard III, W.A. (2004), “Design of a nanomechanical fluid control valve based on functionalized silicon cantilevers: Coupling molecular mechanics with classical engineering design”, *Nanotechnol*., **15**(11), 1405.

Soto, M., Boyer, T.A., Biradar, S., Ge, L., Vajtai, R., Elías-Zúñiga, A., Ajayan, P.M. and Barrera, E.V. (2015), “Effect of interwall interaction on the electronic structure of double-walled carbon nanotubes”, *Nanotechnol*., **26**(16), 165201. https//:10.1088/0957-4484/26/16/165201.

Sudak, L.J. (2003), “Column buckling of multiwalled carbon nanotubes using nonlocal continuum mechanics”, *J. Appl. Phys*., **94**(11), 7281-7287. https://doi.org/10.1063/1.1625437.

Tahar, H.D. (2017), “Analytical and numerical modeling of interfacial stresses in beams bonded with a thin plate”, *Adv. Comput. Des*., **2**(1), 57-69.

Tahar, H.D. and Adim, B. (2016), “Theoretical analysis of composite beams under uniformly distributed load”, *Adv. Mater. Res*., **5**(1), 1-9. https://doi.org/10.12989/amr.2016.5.1.001.

Tahar, H.D. and Belkacem, A. (2017), “Mechanical behaviour of FGM sandwich plates using a quasi-3Dhigher order shear and normal deformation theory”, *Struct. Eng. Mech*., **61**(1), 49-63. https:// 10.12989/sem.2017.61.1.049.

Tahar, H.D., Rabahi, A., Abbes, B. and Adim, B. (2016), “Theoretical and finite element studies of interfacial stresses in reinforced concrete beams strengthened by externally FRP laminates plate”, *J. Adhes. Sci. Technol*., **30**(12), 1253-1280. https://doi.org/10.1080/01694243.2016.1140703.

Tison, Y., Giusca, C.E., Stolojan, V., Hayashi, Y. and Silva, R.P. (2008), “The inner shell influence on the electronic structure of double-walled carbon nanotubes”, *Adv. Mater*., **20**(1), 189-194. https://doi.org/10.1002/adma.200700399.

Tokio, Y. (1995), “Recent development of carbon nanotube”, *Synth Met*., **70**(1-3), 1511-1518. https://doi.org/10.1016/0379-6779(94)02939-V.

Tounsi, A., Houari, M.S.A., Benyoucef, S. and Adda Bedia, E.A. (2013), “A refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates”, *Aerosp. Sci. Technol*., **24**(1), 209-220. https://doi.org/10.1016/j.ast.2011.11.009.

Tsiatas, G.C. (2014), “A new efficient method to evaluate exact stiffness and mass matrices of non-uniform beams resting on an elastic foundation”, *Arch. Appl. Mech*., **84**(5), 615-623. https://doi.org/10.1007/s00419-014-0820-7.

Tu, Z.C. and Ou-Yang, Z.C. (2002), “Single-walled and multiwalled carbon nanotubes viewed as elastic tubes with the effective Young’s modulus dependent on layer number”, *Phys. Rev. B*, **65**(23), 233407. https://doi.org/10.1103/PhysRevB.65.233407.

Wang, C.Y., Ru, C.Q. and Mioduchowski, A. (2003), “Axially compressed buckling of pressured multiwall carbon nanotubes”, *Int. J. Sol. Struct*., **40**(15), 3893-3911. https://doi.org/10.1016/S0020-7683(03)00213-0.

Wang, Q. and Wang, C.M. (2007), “The constitutive relation and small scale parameter of nonlocal continuum mechanics for modelling carbon nanotubes”, *Nanotechnol*., **18**(7), 075702.

Wei, J., Zhu, H., Jiang, B., Ci, L. and Dehai, W.E. (2003), “Electronic properties of double-walled carbon nanotube films”, *Carb*., **41**(13), 2495-2500. https://doi.org/10.1016/S0008-6223(03)00295-1.

Wu, Y., Zhang, X., Leung, A.Y.T. and Zhong, W. (2006), “An energy-equivalent model on studying the mechanical properties of single-walled carbon nanotubes”, *Thin-Wall. Struct*., **44**(6), 667-676. https://doi.org/10.1016/j.tws.2006.05.003.

Yan, Y., He, X.Q., Zhang, L.X. and Wang, C.M, (2010), "Nonlocal effect on axially compressed buckling of triple-walled carbon nanotubes under temperature field", *Appl*. *Math*. *Model*., **34**(11), 3422-3429. https://doi.org/10.1016/j.apm.2010.02.031.

Yan, Y., He, X.Q., Zhang, L.X. and Wang, C.M. (2009), "Dynamic behavior of triple-walled carbon nanotubes conveying fluid", *J*. *Sound Vibr*., **319**(3-5), 1003-1018. https://doi.org/10.1016/j.jsv.2008.07.001.

Zemri, A., Houari, M.S.A., Bousahla, A.A. and Tounsi, A. (2015), "A mechanical response of functionally graded nanoscale beam: An assessment of a refined nonlocal shear deformation theory beam theory", *Struct*. *Eng*. *Mech*., **54**(4), 693-710. https://doi.org/10.12989/sem.2015.54.4.693.

Zhang, Y.Q., Liu, G.R. and Wang, J.S. (2004), "Small-scale effects on buckling of multiwalled carbon nanotubes under axial compression", *Phys*. *Rev*. *B*, **70**(20), 205430. https://doi.org/10.1103/PhysRevB.70.205430.

Zhang, Y.Q., Liu, G.R. and Xie, X.Y. (2005), "Free transverse vibrations of double-walled carbon nanotubes using a theory of nonlocal elasticity", *Phys*. *Rev*. *B*, **71**(19), 195404. https://doi.org/10.1103/PhysRevB.71.195404.

Zhou, Y. (2008), "Nanotubes: A new carrier for drug delivery systems", *Open Nanosci*. J 2, 1-5. DOI: 10.2174/1874140100802010001.

Zidour, M., Daouadji, T.H., Benrahou, K.H., Tounsi, A., Adda Bedia, E.A. and Hadji, L. (2014), "Buckling analysis of chiral single walled carbon nanotubes by the nonlocal Timoshenko beam theory", *Mech*. *Compos*. *Mater*., **50**(1), 95-104.

PL