

Effect of rotation and inclined load on transversely isotropic magneto thermoelastic solid

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Abstract. In present research, we have considered transversely isotropic magneto thermoelastic solid with two temperature and without energy dissipation due to inclined load. The mathematical model has been formulated using Lord-Shulman theory. The Laplace and Fourier transform techniques have been used to find the solution to the problem. The displacement components, stress components and conductive temperature distribution with the horizontal distance are computed in the transformed domain and further calculated in the physical domain using numerical inversion techniques. The effect of rotation and angle of inclination of inclined load is depicted graphically on the resulting quantities.

Keywords: transversely isotropic thermoelastic; Laplace and Fourier transform; concentrated and distributed sources; inclined load

1. Introduction

During the past few years, wide spread attention has been given to thermoelasticity theories that defines the deformation and heat flow in a continuum. When a material body is subjected to an external force or loads, it transmits mechanical waves. For example, if a sudden heat is applied in a solid body, it will create a mechanical wave through thermal expansion. The study of interaction between mechanical and thermal fields is one of the most extensive and productive areas of continuum dynamics. The proposed model is helpful for finding the type of interaction between mechanical and thermal fields as most of the structural elements of heavy industries are frequently related to mechanical and thermal stresses at a higher temperature.

Ailawalia and Narah (2009) had studied the deformation of a rotating generalized thermoelastic solid beneath the impact of gravity with a superimposing infinite thermoelastic fluid due to different forces acting along the interface. Ailawalia *et al.* (2010) had studied a rotating generalized thermoelastic medium with two temperatures beneath hydrostatic stress and gravity with different types of sources using integral transforms. Marin (1997) had proved the Cesaro means of the kinetic and strain energies of dipolar bodies with finite energy. Sharma and Kaur (2010) presented the propagation of Rayleigh waves in a generalized thermoelastic half-space with voids. The surface chosen is stress-free and thermally insulated. They detected the elliptical paths during the Rayleigh wave

motion without rotation. Kumar and Devi (2011) considered the thermoelastic material in Lord and Shulman theory (L-S) and coupled theory (CT) of thermoelasticity with one relaxation time having voids depending upon modulus of elasticity and thermal conductivity. Abd-Alla *et al.* (2012) investigated the Rayleigh waves propagation in a homogeneous orthotropic elastic medium with impact of rotation, initial stress and gravity field by Lamé's potentials and governing equations.

Singh and Yadav (2012) solved the transversely isotropic rotating magnetothermoelastic medium equations by cubic velocity equation of three plane waves without anisotropy, rotation, and thermal and magnetic effects. Banik and Kanoria (2012) studied the thermoelastic interaction in an isotropic infinite elastic body with a spherical cavity for the three-phase-lag heat equation with two-temperature generalized thermoelasticity theory and has shown dissimilarities between two models: the two-temperature Green-Naghdi theory with energy dissipation and two-temperature three-phase-lag model and has shown the effects of ramping parameters and two-temperature. Mahmoud (2012) had considered the influence of rotation, magnetic field, relaxation times, initial stress and gravity field on attenuation coefficient and Rayleigh waves in an elastic half-space of granular medium and obtained the analytical solution of Rayleigh waves velocity by using Lamé's potential techniques. The reflection of plane periodic wave's occurrence on the surface of generalized thermoelastic micropolar transversely isotropic medium had been studied by Kumar and Gupta (2012) to calculate complex velocities of the four waves i.e., quasi-longitudinal displacement (qLD) wave, quasi-transverse displacement (qTD) wave, quasi-transverse microrotational (qTM) wave and quasi thermal (qT) waves from the complex roots of a quartic equation.

Abouelregal (2013) had investigated the induced

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displacement, temperature, and stress fields in an infinite transversely isotropic boundless medium with cylindrical cavity due to a moving heat source and harmonically varying heat in reference to the linear theory of generalized thermoelasticity with dual phase lag model. Abd-Alla and Alshaikh (2015) had discussed the effect of rotation and magnetic field on plane waves in transversely isotropic thermoelastic medium under the Green-Lindsay theory with two relaxation times of generalized thermoelasticity to show the presence of three quasi plane waves in the medium. Marin *et al.* (2013) has modelled a micro stretch thermoelastic body with two temperatures and eliminated divergences among the classical elasticity and research. Gupta and Gupta (2014) studied the effect of rotation on the propagation of plane waves in a transversely isotropic medium in the context of thermoelasticity theory of GN theory of types II and III. Gupta and Gupta (2014) studied the effect of initial stress in a rotating transversely isotropic medium for GN theory of type-II and III on the propagation of plane waves and, obtained three waves a quasi-longitudinal wave, a thermal wave and a quasi-transverse and obtained the amplitudes of their reflection coefficients. Mahmoud *et al.* (2015) studied the impact of the initial stress and rotation on harmonic waves propagation in a human long dry bone (transversely isotropic material). They solved the equations of elastodynamic in terms of displacements. Sharma *et al.* (2015) investigated the two dimensional deformation in a homogeneous, transversely isotropic thermoelastic solids with two temperatures in Green-Naghdi-II theory with an inclined load (linear combination of normal load and tangential load). Shaw and Mukhopadhyay (2015) exemplified the generalized theory of thermoelasticity including the thermal relaxation time, electric displacement current, and the coupling of heat transfer and microrotation of the material to study the propagation of plane harmonic waves in an infinitely long, isotropic, micropolar plate a uniform magnetic field. Two potential functions were used to determine the effect of the presence of thermal and magnetic fields on the phase velocity.

Kumar *et al.* (2016) investigated the effects of Hall current in a transversely isotropic magneto-thermoelastic with and without energy dissipation due to normal force. Kumar *et al.* (2016) studied the conflicts caused by thermomechanical sources in a homogeneous transversely isotropic thermoelastic rotating medium with magnetic effect and two temperature and applied this to the thermoelasticity Green-Naghdi theories with and without energy dissipation using thermomechanical sources. Bijarnia and Singh (2016) studied the propagation of plane waves using Lord and Shulman theory of generalized thermoelasticity in a transversely isotropic thermoelastic solid half-space with voids and rotation and solved to illustrate the existence of four plane waves and its reflection from thermally insulated stress free surface. Kumar *et al.* (2016) illustrated the effect of Hall current and magnetic field due to thermomechanical sources on GN-II and GN-III theories in a rotating transversely isotropic homogeneous thermoelastic medium with two temperatures. Lata *et al.* (2016) studied two temperature and rotation aspect for GN-

II and GN-III theory of thermoelasticity in a homogeneous transversely isotropic magneto-thermoelastic medium for the case of the plane wave propagation and reflection. Mona and SE (2017) compared the theory of thermoelasticity with two relaxation times and without energy dissipation. Kumar *et al.* (2017) considered a thick circular plate with axisymmetric heat supply with traction free lower and upper surfaces of the plate. Ezzat *et al.* (2017) proposed a mathematical model of electro-thermoelasticity for heat conduction with memory-dependent derivative. Kumar *et al.* (2017) analyzed the Rayleigh waves in a homogeneous transversely isotropic magneto-thermoelastic medium with two temperature, with Hall current and rotation. Marin *et al.* (2017) studied the GN-thermoelastic theory for a dipolar body using mixed initial BVP and proved a result of Hölder's-type stability. Parveen Lata (2018) studied the effect of energy dissipation on plane waves in sandwiched layered thermoelastic medium of uniform thickness, with combined effects of two temperature, rotation and Hall current in the context of GN Type-II and Type-III theory of thermoelasticity. Ezzat and El-Bary (2017) gave mathematical model of phase-lag G-N magneto-thermoelasticity theories for perfectly conducting media based on fractional derivative heat transfer in the presence of a constant magnetic field. Ezzat and El-Bary (2017) had applied the magneto-thermoelasticity model to a one-dimensional thermal shock problem of functionally graded half-space of based on memory-dependent derivative.

In spite of these, not much work has been carried out in magneto-thermoelastic transversely isotropic solid with the combined effects of rotation and two temperatures in generalized thermoelasticity without energy dissipation. Keeping these considerations in mind, analytic expressions for the displacements, stresses and temperature distribution in two-dimensional homogeneous, transversely isotropic magneto-thermoelastic solids with two temperatures and rotation because inclined load have been obtained.

2. Basic equations

The constitutive relations for a transversely isotropic thermoelastic medium are given by

$$t_{ij} = C_{ijkl}e_{kl} - \beta_{ij}T. \quad (1)$$

Equation of motion for a transversely isotropic thermoelastic medium rotating uniformly with an angular velocity $\Omega = \Omega n$, where n is a unit vector representing the direction of axis of rotation and taking into account Lorentz force

$$t_{ij,j} + F_i = \rho\{\ddot{u}_i + (\Omega \times (\Omega \times u))_i + (2\Omega \times \dot{u})_i\}, \quad (2)$$

where $F_i = \mu_0(j \times \vec{H}_0)_i$ are the components of Lorentz force, $\vec{H}_0$ is the external applied magnetic field intensity vector, $\vec{j}$ is the current density vector, $\vec{u}$ is the displacement vector, μ_0 and ε_0 are the magnetic and electric permeabilities respectively.

The heat conduction equation without energy dissipation using Lord-Shulman model is

$$K_{ij}T_{,ij} + \rho(Q + \tau_0\dot{Q}) = \beta_{ij}T_0(\dot{e}_{ij} + \tau_0\ddot{e}_{ij}) + \rho C_E(\dot{T} + \tau_0\ddot{T}), \quad (3)$$

where

$$\beta_{ij} = C_{ijkl} \alpha_{ij}, \quad (4)$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), i, j = 1, 2, 3. \quad (5)$$

$\beta_{ij} = \beta_i \delta_{ij}$, $K_{ij} = K_i \delta_{ij}$, i is not summed.

Here C_{ijkl} ($C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk}$) are elastic parameters, β_{ij} is the thermal elastic coupling tensor, T is the absolute temperature, T_0 is the reference temperature, φ is the conductive temperature, t_{ij} are the components of stress tensor, e_{ij} are the components of strain tensor, u_i are the displacement components, ρ is the density, C_E is the specific heat, K_{ij} is the materialistic constant, α_{ij} are the two temperature parameters, α_{ij} is the coefficient of linear thermal expansion, τ_0 is the relaxation time, which is the time required to maintain steady state heat conduction in an element of volume of an elastic body when sudden temperature gradient is imposed on that volume element, δ_{ij} is the Kronecker delta and Ω is the angular velocity of the solid.

3. Formulation and solution of the problem

We consider a homogeneous transversely isotropic magnetothermoelastic medium, permeated by an initial magnetic field $\vec{H}_0 = (0, H_0, 0)$ acting along y -axis. The rectangular Cartesian co-ordinate system (x, y, z) having origin on the surface ($z = 0$) with z -axis pointing vertically into the medium is introduced. The surface of the half-space is subjected to an inclined load acting at $z = 0$.

We also assume that

$$\Omega = (0, \Omega, 0).$$

In From the generalized Ohm's law

$$J_2 = 0.$$

The current density components J_1 and J_3 are given as

$$J_1 = -\varepsilon_0 \mu_0 H_0 \frac{\partial^2 w}{\partial t^2}, \quad (6)$$

$$J_3 = \varepsilon_0 \mu_0 H_0 \frac{\partial^2 u}{\partial t^2}. \quad (7)$$

Following Slaughter (2002), using appropriate transformations, on the set of Eqs. (1) and (2), we derive the basic equations for transversely isotropic thermoelastic solid. The components of displacement vector $(\vec{u}, \vec{v}, \vec{w})$ and conductive temperature φ for the two dimensional problem have the form

$$\vec{u} = u(x, z, t), \vec{v} = 0, \vec{w} = w(x, z, t) \text{ and } \varphi = \varphi(x, z, t). \quad (8)$$

Eqs. (1)-(3) with the aid of (8), yield

$$C_{11} \frac{\partial^2 u}{\partial x^2} + C_{13} \frac{\partial^2 w}{\partial x \partial z} + C_{44} \left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x \partial z} \right) - \beta_1 \frac{\partial}{\partial x} \left\{ \varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x^2} + a_3 \frac{\partial^2 \varphi}{\partial z^2} \right) \right\} - \mu_0 J_3 H_0 = \rho \left(\frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega \frac{\partial w}{\partial t} \right), \quad (9)$$

$$(C_{13} + C_{44}) \frac{\partial^2 u}{\partial x \partial z} + C_{44} \frac{\partial^2 w}{\partial x^2} + C_{33} \frac{\partial^2 w}{\partial z^2} - \beta_3 \frac{\partial}{\partial z} \left\{ \varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x^2} + a_3 \frac{\partial^2 \varphi}{\partial z^2} \right) \right\} - \mu_0 J_1 H_0 = \rho \left(\frac{\partial^2 w}{\partial t^2} - \Omega^2 w - 2\Omega \frac{\partial u}{\partial t} \right), \quad (10)$$

$$K_1 \frac{\partial^2 \varphi}{\partial x^2} + K_3 \frac{\partial^2 \varphi}{\partial z^2} + \rho(Q + \tau_0 \dot{Q}) = \rho C_E (\dot{T} + \tau_0 \ddot{T}) + T_0 \frac{\partial}{\partial t} \left\{ \beta_1 \left(1 + \tau_0 \frac{\partial}{\partial t} \right) \frac{\partial u}{\partial x} + \beta_3 \left(1 + \tau_0 \frac{\partial}{\partial t} \right) \frac{\partial w}{\partial z} \right\}, \quad (11)$$

and

$$t_{11} = C_{11} e_{11} + C_{13} e_{13} - \beta_1 T, \quad (12)$$

$$t_{33} = C_{13} e_{11} + C_{33} e_{33} - \beta_3 T, \quad (13)$$

$$t_{13} = 2C_{44} e_{13}, \quad (14)$$

where

$$T = \varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x^2} + a_3 \frac{\partial^2 \varphi}{\partial z^2} \right),$$

$$\beta_1 = (C_{11} + C_{12})\alpha_1 + C_{13}\alpha_3,$$

$$\beta_3 = 2C_{13}\alpha_1 + C_{33}.$$

We assume that medium is initially at rest. The undisturbed state is maintained at reference temperature. Then we have the initial and regularity conditions as given by

$$u(x, z, 0) = 0 = \dot{u}(x, z, 0), \\ w(x, z, 0) = 0 = \dot{w}(x, z, 0), \varphi(x, z, 0) = 0 \\ = \dot{\varphi}(x, z, 0) \text{ for } z \geq 0, -\infty < x < \infty, \\ u(x, z, t) = w(x, z, t) = \varphi(x, z, t) = 0 \text{ for } t > 0 \text{ when } z \rightarrow \infty.$$

To facilitate the solution, following dimensionless quantities are introduced

$$x' = \frac{x}{L}, \quad z' = \frac{z}{L}, \quad t' = \frac{c_1}{L} t, \quad u' = \frac{\rho c_1^2}{L \beta_1 T_0} u, \quad w' = \frac{\rho c_1^2}{L \beta_1 T_0} w, \quad T' = \frac{T}{T_0}, \quad t'_{11} = \frac{t_{11}}{\beta_1 T_0}, \quad t'_{33} = \frac{t_{33}}{\beta_1 T_0}, \quad t'_{31} = \frac{t_{31}}{\beta_1 T_0}, \quad \varphi' = \frac{\varphi}{T_0}, \quad a'_1 = \frac{a_1}{L^2}, \quad a'_3 = \frac{a_3}{L^2}, \quad h' = \frac{h}{H_0}, \quad \Omega' = \frac{\Omega}{c_1}. \quad (15)$$

Making use of (15) in Eqs. (9)-(11), after suppressing the primes, yield

$$\frac{\partial^2 u}{\partial x^2} + \delta_4 \frac{\partial^2 w}{\partial x \partial z} + \delta_2 \left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x \partial z} \right) - \frac{\partial}{\partial x} \left\{ \varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x^2} + a_3 \frac{\partial^2 \varphi}{\partial z^2} \right) \right\} = \left(\frac{\varepsilon_0 \mu_0^2 H_0^2}{\rho} + 1 \right) \frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega \frac{\partial w}{\partial t}, \quad (16)$$

$$\delta_1 \frac{\partial^2 u}{\partial x \partial z} + \delta_2 \frac{\partial^2 w}{\partial x^2} + \delta_3 \frac{\partial^2 w}{\partial z^2} - \frac{\beta_3}{\beta_1} \frac{\partial}{\partial z} \left\{ \varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x^2} + a_3 \frac{\partial^2 \varphi}{\partial z^2} \right) \right\} = \left(\frac{\varepsilon_0 \mu_0^2 H_0^2}{\rho} + 1 \right) \frac{\partial^2 w}{\partial t^2} - \Omega^2 w + 2\Omega \frac{\partial u}{\partial t}, \quad (17)$$

$$\begin{aligned} \frac{\partial^2 \varphi}{\partial x^2} + \frac{K_3}{K_1} \frac{\partial^2 \varphi}{\partial z^2} + \rho \left(1 + \tau_0 \frac{c_1}{L} \frac{\partial}{\partial t} \right) Q \\ = \delta_5 \frac{\partial}{\partial t} \left(1 + \tau_0 \frac{c_1}{L} \frac{\partial}{\partial t} \right) \left[\varphi - a_1 \frac{\partial^2 \varphi}{\partial x^2} - a_3 \frac{\partial^2 \varphi}{\partial z^2} \right] \\ + \delta_6 \frac{\partial}{\partial t} \left(1 + \tau_0 \frac{c_1}{L} \frac{\partial}{\partial t} \right) \left[\beta_1 \frac{\partial u}{\partial x} + \beta_3 \frac{\partial w}{\partial z} \right], \end{aligned} \quad (18)$$

where

$$\delta_1 = \frac{c_{13} + c_{44}}{c_{11}}, \delta_2 = \frac{c_{44}}{c_{11}}, \delta_3 = \frac{c_{33}}{c_{11}}, \delta_4 = \frac{c_{13}}{c_{11}}, \\ \delta_5 = \frac{\rho C_E C_1 L}{K_1}, \delta_6 = \frac{T_0 \beta_1 L}{\rho C_1 K_1}.$$

Apply Laplace and Fourier transforms defined by

$$\tilde{f}(x, z, s) = \int_0^\infty f(x, z, t) e^{-st} dt, \quad (19)$$

$$\hat{f}(\xi, z, s) = \int_{-\infty}^\infty \tilde{f}(x, z, s) e^{i\xi x} dx. \quad (20)$$

On Eqs. (16)-(18), we obtain a system of equations

$$\begin{aligned} [-\xi^2 + \delta_2 D^2 - \delta_7 s^2 + \Omega^2] \hat{u}(\xi, z, s) \\ + [\delta_4 D i \xi + \delta_2 D i \xi - 2\Omega s] \hat{w}(\xi, z, s) \\ + (-i\xi) [1 + a_1 \xi^2 - a_3 D^2] \hat{\varphi}(\xi, z, s) \\ = 0, \end{aligned} \quad (21)$$

$$\begin{aligned} [\delta_1 D i \xi + 2\Omega s] \hat{u}(\xi, z, s) \\ + [-\delta_2 \xi^2 + \delta_3 D^2 - \delta_7 s^2 \\ + \Omega^2] \hat{w}(\xi, z, s) \\ - \frac{\beta_3}{\beta_1} D [1 + a_1 \xi^2 - a_3 D^2] \hat{\varphi}(\xi, z, s) \\ = 0, \end{aligned} \quad (22)$$

$$\begin{aligned} [\delta_6 s \delta_8 \beta_1 i \xi] \hat{u}(\xi, z, s) + [\delta_6 s \delta_8 \beta_3 D] \hat{w}(\xi, z, s) \\ + \left[\xi^2 - \frac{K_3}{K_1} D^2 \right. \\ \left. + \delta_5 \delta_8 s (1 + a_1 \xi^2 \right. \\ \left. - a_3 D^2) \right] \hat{\varphi}(\xi, z, s) = \rho \delta_8 \hat{Q}(\xi, z, s), \end{aligned} \quad (23)$$

$$\text{where } \delta_7 = \frac{\varepsilon_0 \mu_0^2 H_0^2}{\rho} + 1, \quad \delta_8 = 1 + \tau_0 \frac{c_1}{L} s.$$

Without considering internal heat source and setting $\hat{Q}(\xi, z, s) = 0$ we yield a set of homogeneous equations which will have a non trivial solution if determinant of coefficient $(\hat{u}, \hat{w}, \hat{\varphi})$ vanishes and we obtain the following characteristic equation

$$AD^6 + BD^4 + CD^2 + E = 0 \quad (24)$$

where

$$A = \delta_2 \delta_3 \zeta_7 - \zeta_5 \delta_2 \frac{\beta_3}{\beta_1} a_3,$$

$$B = \delta_3 \zeta_1 \zeta_7 - a_3 \zeta_1 \zeta_5 \frac{\beta_3}{\beta_1} + \delta_2 \delta_3 \zeta_6 + \delta_2 \zeta_7 \zeta_3 - \zeta_5 \zeta_9 \delta_2 - \zeta_8 \delta_1 i \xi \zeta_7 + \zeta_8 \zeta_4 \frac{\beta_3}{\beta_1} a_3 - a_3 \xi^2 \zeta_5 \delta_1 - a_3 \delta_3 \zeta_4 i \xi,$$

$$C = \delta_3 \zeta_1 \zeta_6 + \zeta_1 \zeta_3 \zeta_7 - \zeta_1 \zeta_5 \zeta_9 + \delta_2 \zeta_6 \zeta_3 + \zeta_4 \zeta_8 \zeta_9 - \zeta_8 \delta_1 i \xi \zeta_6 + 4\Omega^2 s^2 \zeta_7 + \zeta_2 \delta_1 i \xi \zeta_5 - \zeta_2 \zeta_4 \delta_3 - a_3 \zeta_4 i \xi \zeta_3,$$

$$E = \zeta_3 \zeta_1 \zeta_6 + 4\Omega^2 s^2 \zeta_6 - \zeta_2 \zeta_4 \zeta_3,$$

$$\zeta_1 = \xi^2 - \delta_7 s^2 + \Omega^2, \zeta_2 = -i\xi(1 + a_1 \xi^2), \zeta_3 = -\delta_2 \xi^2 - \delta_7 s^2 + \Omega^2, \zeta_4 = \delta_6 \delta_8 s \beta_1 i \xi,$$

$$\zeta_5 = \delta_6 \delta_8 s \beta_3, \zeta_6 = \xi^2 + \delta_5 \delta_8 s (1 + a_1 \xi^2), \zeta_7 = -\frac{K_3}{K_1} -$$

$$a_3 \delta_5 \delta_8 s, \zeta_8 = \delta_1 i \xi, \zeta_9 = -(1 + a_1 \xi^2) \frac{\beta_3}{\beta_1}.$$

The roots of the Eq. (24) are $\pm \lambda_i$, ($i = 1, 2, 3$), the solution of the Eq. (24) satisfying the radiation condition that $\tilde{u}, \tilde{v}, \tilde{w}$ can be written as

$$\bar{u}(\xi, z, s) = \sum_{i=1}^3 A_i e^{-\lambda_i z}, \quad (25)$$

$$\bar{w}(\xi, z, s) = \sum_{i=1}^3 d_i A_i e^{-\lambda_i z}, \quad (26)$$

$$\bar{\varphi}(\xi, z, s) = \sum_{i=1}^3 l_i A_i e^{-\lambda_i z}, \quad (27)$$

where A_i , $i = 1, 2, 3$ being undetermined constants and d_i and l_i are given by

$$d_i = \frac{\delta_2 \zeta_7 \lambda_i^4 + (\zeta_7 \zeta_1 - a_3 \zeta_4 i \xi + \delta_2 \zeta_6) \lambda_i^2 + \zeta_1 \zeta_6 - \zeta_4 \zeta_2}{\left(\delta_3 \zeta_7 - \frac{\beta_3}{\beta_1} a_3 \zeta_5 \right) \lambda_i^4 + (\delta_3 \zeta_6 + \zeta_3 \zeta_7 - \zeta_5 \zeta_9) \lambda_i^2 + \zeta_3 \zeta_6}$$

$$l_i = \frac{\delta_2 \delta_3 \lambda_i^4 + (\delta_2 \zeta_3 + \zeta_1 \delta_3 - \delta_1 \zeta_8 i \xi) \lambda_i^2 + 4\Omega^2 s^2 + \zeta_3 \zeta_1}{\left(\delta_3 \zeta_7 - \frac{\beta_3}{\beta_1} a_3 \zeta_5 \right) \lambda_i^4 + (\delta_3 \zeta_6 + \zeta_3 \zeta_7 - \zeta_5 \zeta_9) \lambda_i^2 + \zeta_3 \zeta_6}$$

4. Boundary conditions

We consider a normal line load F_1 per unit length acting in the positive z -axis on the plane boundary $z=0$ along the y -axis and a tangential load F_2 per unit length, acting at the origin in the positive x axis. The appropriate boundary conditions are

$$i \quad t_{33}(x, z, t) = -F_1 \psi_1(x) H(t), \quad (28)$$

$$\text{ii} \quad t_{31}(x, z, t) = -F_2 \psi_2(x) H(t), \quad (29)$$

$$\text{iii} \quad \frac{\partial \varphi}{\partial z}(x, z, t) = 0. \quad (30)$$

where F_1 and F_2 are the magnitude of the forces applied, $\psi_1(x)$ and $\psi_2(x)$ specify the vertical and horizontal load distribution function along x -axis, and $H(t)$ is the Heaviside unit step function and is given by

$$H(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}.$$

Applying the Laplace and Fourier transform defined by (19) and (20) on the boundary conditions (28)-(30), (13)-(14) and with the help of Eqs. (25)-(27), we obtain the components of displacement, normal stress, tangential stress, and conductive temperature as

$$\hat{u} = \frac{F_1 \hat{\psi}_1(\xi)}{s\Lambda} \left[\sum_{i=1}^3 \Lambda_{1i} e^{-\lambda_i z} \right] + \frac{F_2 \hat{\psi}_2(\xi)}{s\Lambda} \left[\sum_{i=1}^3 \Lambda_{2i} e^{-\lambda_i z} \right] \quad (31)$$

$$\hat{w} = \frac{F_1 \hat{\psi}_1(\xi)}{s\Lambda} \left[\sum_{i=1}^3 d_i \Lambda_{1i} e^{-\lambda_i z} \right] + \frac{F_2 \hat{\psi}_2(\xi)}{s\Lambda} \left[\sum_{i=1}^3 d_i \Lambda_{2i} e^{-\lambda_i z} \right], \quad (32)$$

$$\hat{\phi} = \frac{F_1 \hat{\psi}_1(\xi)}{s\Lambda} \left[\sum_{i=1}^3 l_i \Lambda_{1i} e^{-\lambda_i z} \right] + \frac{F_2 \hat{\psi}_2(\xi)}{s\Lambda} \left[\sum_{i=1}^3 l_i \Lambda_{2i} e^{-\lambda_i z} \right], \quad (33)$$

$$\hat{t}_{11} = \frac{F_1 \hat{\psi}_1(\xi)}{s\Lambda} \left[\sum_{i=1}^3 S_i \Lambda_{1i} e^{-\lambda_i z} \right] + \frac{F_2 \hat{\psi}_2(\xi)}{s\Lambda} \left[\sum_{i=1}^3 S_i \Lambda_{2i} e^{-\lambda_i z} \right], \quad (34)$$

$$\hat{t}_{13} = \frac{F_1 \hat{\psi}_1(\xi)}{s\Lambda} \left[\sum_{i=1}^3 N_i \Lambda_{1i} e^{-\lambda_i z} \right] + \frac{F_2 \hat{\psi}_2(\xi)}{s\Lambda} \left[\sum_{i=1}^3 N_i \Lambda_{2i} e^{-\lambda_i z} \right], \quad (35)$$

$$\hat{t}_{33} = \frac{F_1 \hat{\psi}_1(\xi)}{s\Lambda} \left[\sum_{i=1}^3 M_i \Lambda_{1i} e^{-\lambda_i z} \right] + \frac{F_2 \hat{\psi}_2(\xi)}{s\Lambda} \left[\sum_{i=1}^3 M_i \Lambda_{2i} e^{-\lambda_i z} \right], \quad (36)$$

where

$$\begin{aligned} \Lambda_{11} &= -N_2 R_3 + R_2 N_3, \\ \Lambda_{12} &= N_1 R_3 - R_1 N_3, \\ \Lambda_{13} &= -N_1 R_2 + R_1 N_2, \\ \Lambda_{21} &= M_2 R_3 - R_2 M_3, \\ \Lambda_{22} &= -M_1 R_3 + R_1 M_3, \\ \Lambda_{23} &= M_1 R_2 - R_1 M_2, \\ \Lambda &= -M_1 \Lambda_{11} - M_2 \Lambda_{12} - M_3 \Lambda_{13}, \\ N_j &= -\delta_2 \lambda_j + i \xi d_j, \\ M_j &= i \xi - \delta_3 d_j \lambda_j - \frac{\beta_3}{\beta_1} l_j [(1 + a_1 \xi^2) - a_3 \lambda_j^2], \\ R_j &= -\lambda_j l_j [(1 + a_1 \xi^2) - a_3 \lambda_j^2], \\ S_j &= -i \xi - \delta_4 d_j \lambda_j - l_j [(1 + a_1 \xi^2) - a_3 \lambda_j^2]. \end{aligned}$$

5. Special cases

5.1 Concentrated force

The solution due to concentrated normal force on the half space is obtained by setting

$$\psi_1(x) = \delta(x), \psi_2(x) = \delta(x), \quad (37)$$

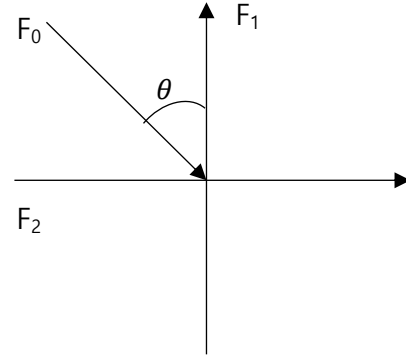


Fig. 1 Inclined load over a transversely isotropic magneto-thermoelastic solid

where $\delta(x)$ is dirac delta function.

Applying Fourier transform defined by (19)-(20) and (37), we obtain

$$\hat{\psi}_1(\xi) = 1, \hat{\psi}_2(\xi) = 1. \quad (38)$$

Using (38) in (31)-(36), the components of displacement, stress and conductive temperature are obtained.

5.2 Uniformly distributed force

The solution due to uniformly distributed force applied on the half space is obtained by setting

$$\psi_1(x), \psi_2(x) = \begin{cases} 1 & \text{if } |x| \leq m \\ 0 & \text{if } |x| > m \end{cases}. \quad (39)$$

The Fourier transforms of $\psi_1(x)$ and $\psi_2(x)$ with respect to the pair (x, ξ) for the case of a uniform strip load of non-dimensional width $2m$ applied at origin of co-ordinate system $x = z = 0$ in the dimensionless form after suppressing the primes becomes

$$\hat{\psi}_1(\xi) = \hat{\psi}_2(\xi) = \left\{ \frac{2 \sin(\xi m)}{\xi} \right\}, \xi \neq 0. \quad (40)$$

Using (40) in (31)-(36), the components of displacement, stress and conductive temperature are obtained.

5.3 Linearly distributed force

The solution due to linearly distributed force applied on the half space is obtained by setting

$$\{\psi_1(x), \psi_2(x)\} = \begin{cases} 1 - \frac{|x|}{m} & \text{if } |x| \leq m \\ 0 & \text{if } |x| > m \end{cases} \quad (41)$$

Here $2m$ is the width of the strip load, using (15) and applying the transform defined by (20) on (41), we get

$$\hat{\psi}_1(\xi) = \hat{\psi}_2(\xi) = \left\{ \frac{2 \{1 - \cos(\xi m)\}}{\xi^2 m} \right\}, \xi \neq 0. \quad (42)$$

Using (42) in (31)-(36), the components of displacement, stress and conductive temperature are obtained.

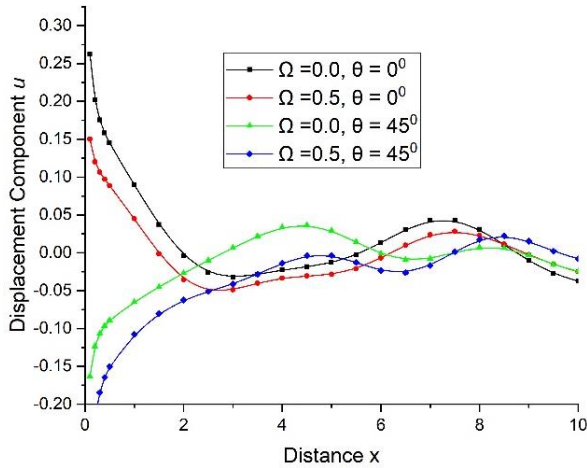


Fig. 2 variations of displacement component u with distance x

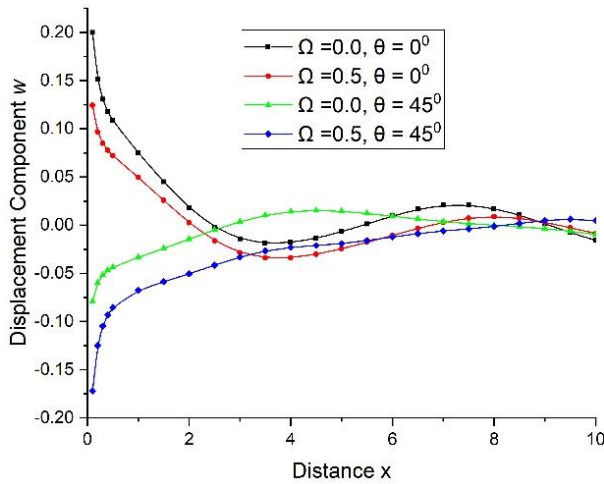


Fig. 3 variations of displacement component w with distance x

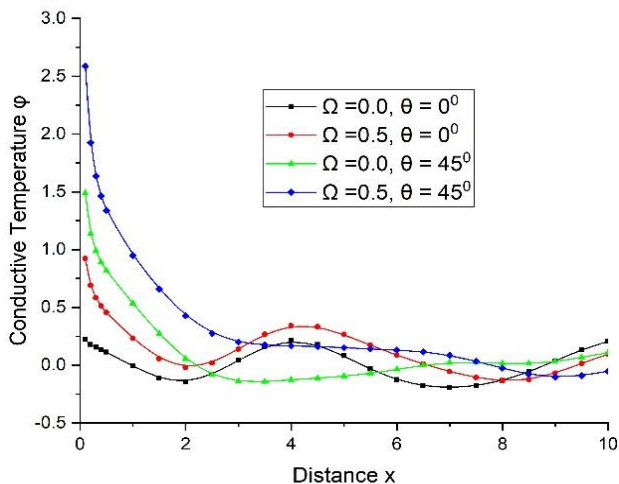


Fig. 4 variations of conductive temperature ϕ with distance x

6. Special cases

Suppose an inclined load, F_0 per unit length is acting on

the y -axis and its inclination with z -axis is θ (see Fig. 1), have

$$F_1 = F_0 \cos \theta \text{ and } F_2 = F_0 \sin \theta. \quad (43)$$

Using Eq. (43) in Eqs. (31)-(36) and with aid of Eqs. (37)-(42) we obtain the expressions for displacements, and stresses and conductive temperature for concentrated force, uniformly distributed force and linearly distributed force on the surface of transversely isotropic magneto-thermoelastic body without energy dissipation.

7. Inversion of the transformation

Using To find the solution of the problem in physical domain, we must invert the transforms in Eqs. (31)-(36). Here the displacement components, normal and tangential stresses and conductive temperature are functions of z , the parameters of Laplace and Fourier transforms s and ξ respectively and hence are of the form $\hat{f}(\xi, z, s)$. To find the function $\tilde{f}(x, z, t)$ in the physical domain, we first invert the Fourier transform using

$$\tilde{f}(x, z, s) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\xi x} \hat{f}(\xi, z, s) d\xi = \frac{1}{2\pi} \int_{-\infty}^{\infty} [\cos(\xi x) f_e - i \sin(\xi x) f_o] d\xi, \quad (43)$$

where f_e and f_o are respectively the odd and even parts of $\hat{f}(\xi, z, s)$. Following Honig and Hirdes (1984), the Laplace transform function $\tilde{f}(x, z, s)$ can be inverted to $f(x, z, t)$ by

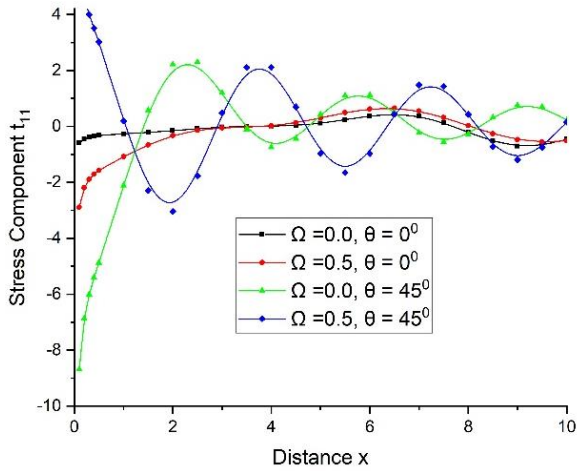
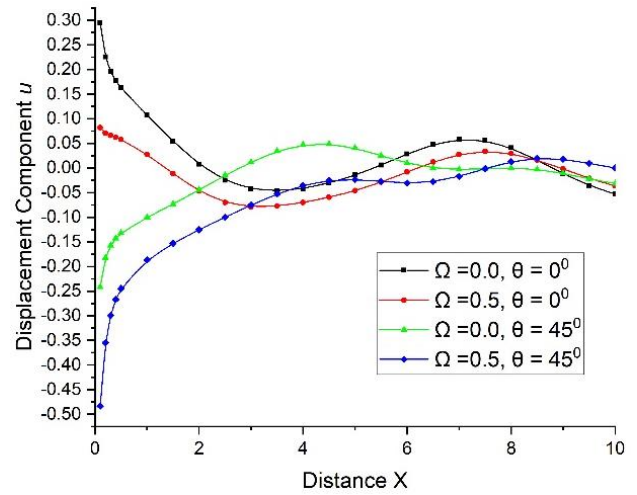
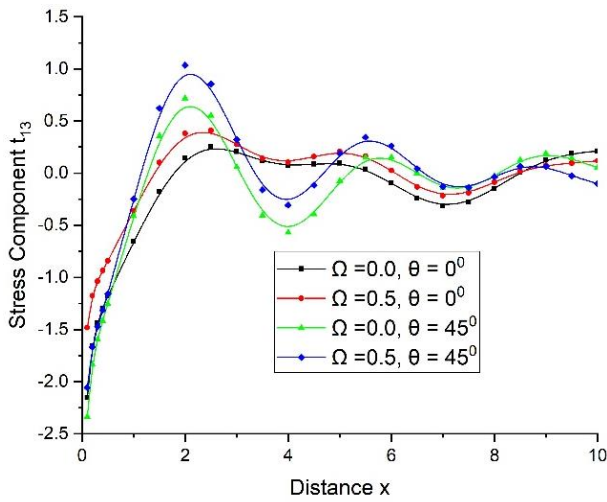
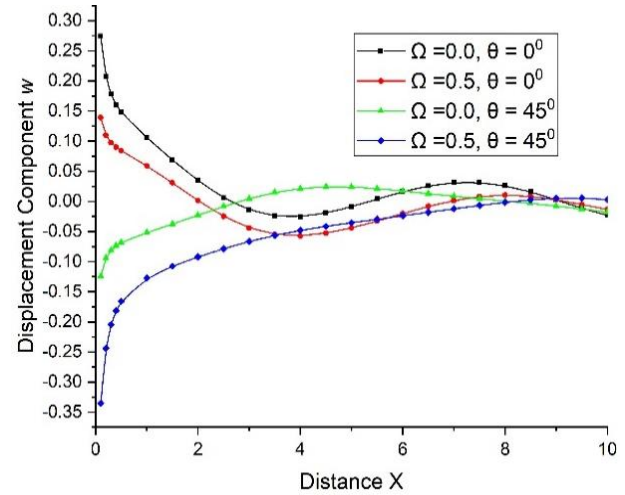
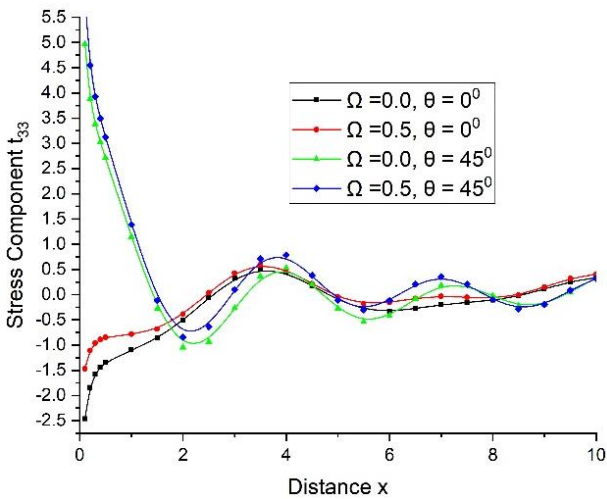
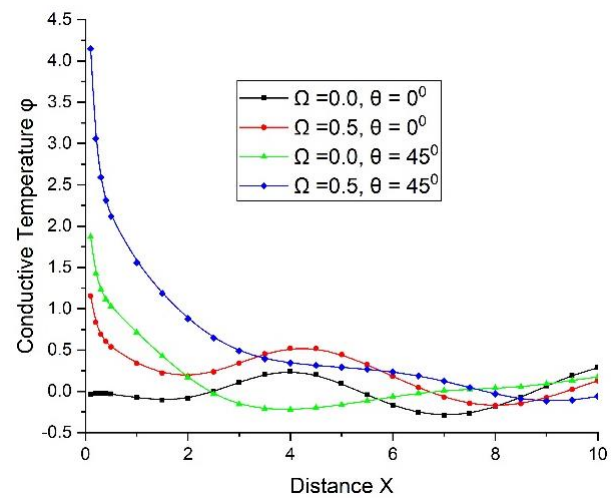
$$f(x, z, t) = \frac{1}{2\pi i} \int_{e^{-i\infty}}^{e^{+i\infty}} \tilde{f}(x, z, s) e^{-st} ds. \quad (44)$$

The last step is to calculate the integral in Eq. (43). The method for evaluating this integral is described in Press *et al.* (1986). It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

8. Numerical results and discussion

In order to illustrate our theoretical results in the proceeding section and to show the effect of two temperature and rotation, we now present some numerical results. Following Dhaliwal and Singh (1980), cobalt material has been taken for thermoelastic material as $c_{11} = 3.07 \times 10^{11} \text{Nm}^{-2}$, $c_{33} = 3.581 \times 10^{11} \text{Nm}^{-2}$, $c_{13} = 1.027 \times 10^{10} \text{Nm}^{-2}$, $c_{44} = 1.510 \times 10^{11} \text{Nm}^{-2}$, $\beta_1 = 7.04 \times 10^6 \text{Nm}^{-2} \text{deg}^{-1}$, $\beta_3 = 6.90 \times 10^6 \text{Nm}^{-2} \text{deg}^{-1}$, $\rho = 8.836 \times 10^3 \text{Kgm}^{-3}$, $C_E = 4.27 \times 10^2 \text{Jkg}^{-1} \text{deg}^{-1}$, $K_1 = 0.690 \times 10^2 \text{Wm}^{-1} \text{Kdeg}^{-1}$, $K_3 = 0.690 \times 10^2 \text{Wm}^{-1} \text{K}^{-1}$, $T_0 = 298 \text{K}$, $H_0 = 1 \text{Jm}^{-1} \text{nb}^{-1}$, $\epsilon_0 = 8.838 \times 10^{-12} \text{Fm}^{-1}$, $L = 1$.

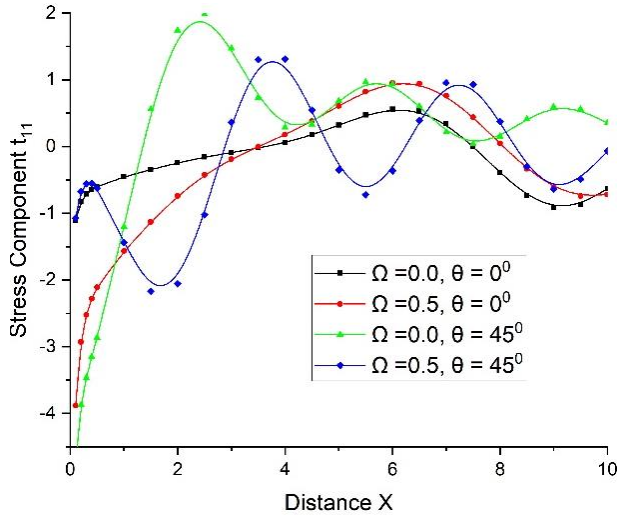
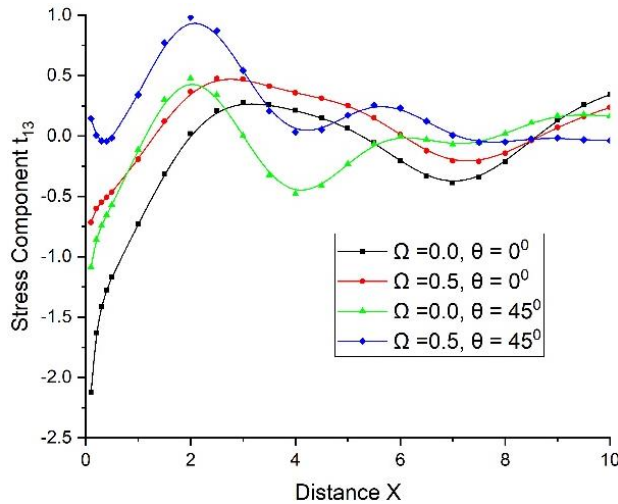
Using the above values, the graphical representations of displacement component u , normal displacement w ,

Fig. 5 variations of stress component t_{11} with distance x Fig. 8 variations of displacement component u with distance x Fig. 6 variations of stress component t_{13} with distance x Fig. 9 variations of displacement component w with distance x Fig. 7 variations of stress component t_{33} with distance x Fig. 10 variations of conductive temperature ϕ with distance x

conductive temperature ϕ , stress components t_{11} , t_{13} and t_{33} for transversely isotropic thermoelastic medium have been investigated and the effect of inclination and rotation has been depicted.

i. The black solid line with square symbols corresponds

to transversely isotropic magneto-thermoelastic medium with $\Omega = 0$ and $\theta = 0^\circ$ and $a_1 = 0.02$, $a_3 = 0.04$.

Fig. 11 variations of stress component t_{11} with distance x Fig. 12 variations of stress component t_{13} with distance x

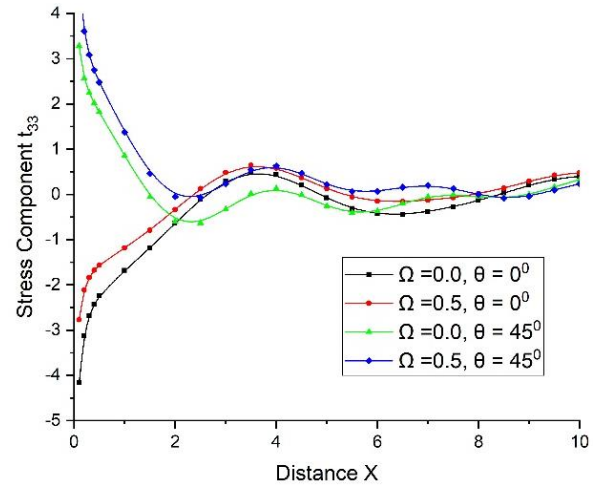
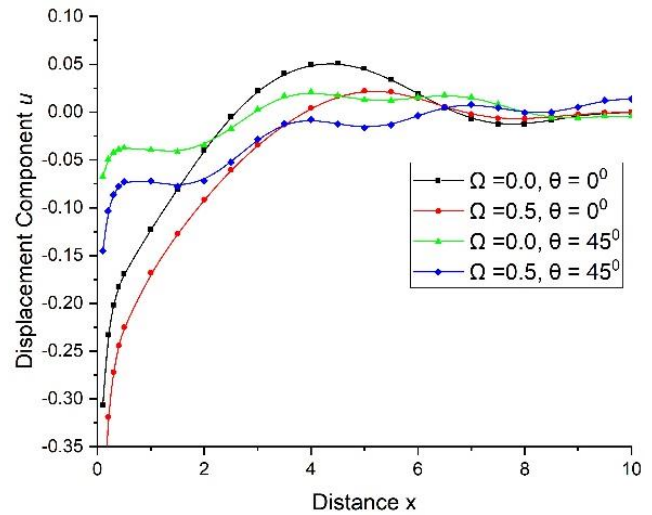
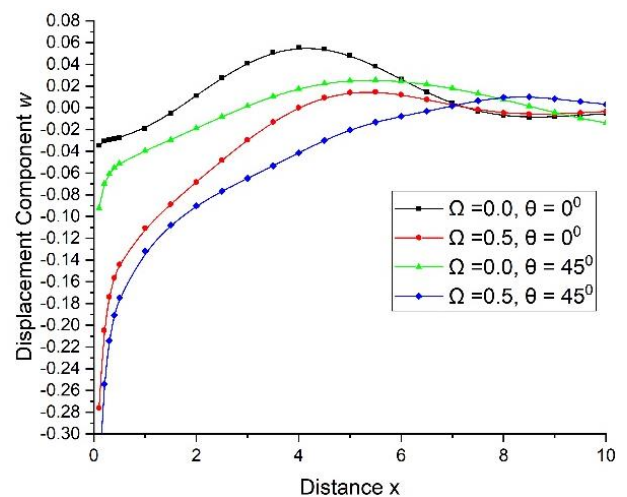
ii. The red solid line with circle symbols corresponds to transversely isotropic magneto-thermoelastic medium with $\Omega = 0.5$ and $\theta = 0^\circ$ and $a_1 = 0.02$, $a_3 = 0.04$.

iii. The green solid line with triangle symbols corresponds to transversely isotropic magneto-thermoelastic medium with $\Omega = 0.0$ and $\theta = 45^\circ$ and $a_1 = 0.02$, $a_3 = 0.04$.

iv. The blue solid line with diamond symbols corresponds to transversely isotropic magneto-thermoelastic medium with $\Omega = 0.5$ and $\theta = 45^\circ$ and $a_1 = 0.02$, $a_3 = 0.04$.

Case 1: Concentrated force due to inclined load with rotation

Fig. 2 shows the variations of the displacement component u for transversely isotropic magneto-thermoelastic medium with rotation. The values of displacement component u , decreases for $\theta = 0^\circ$ and $\Omega = 0, 0.5$ and increases for $\theta = 45^\circ$ and $\Omega = 0, 0.5$ for the initial values of distance and follow oscillatory pattern for rest of the range of distance. Fig.3 depicts variations of the displacement component w for transversely isotropic thermoelastic medium with rotation. The values of displacement component w , decreases for $\theta = 0^\circ$ and $\Omega = 0, 0.5$ and increases for $\theta = 45^\circ$ and $\Omega = 0, 0.5$ for the initial values of distance and follow oscillatory pattern for

Fig. 13 variations of stress component t_{33} with distance x Fig. 14 variations of displacement component u with distance x Fig. 15 variations of displacement component w with distance x

rest of the range of distance. Fig. 4 represents the variations of the conductive temperature φ for transversely isotropic

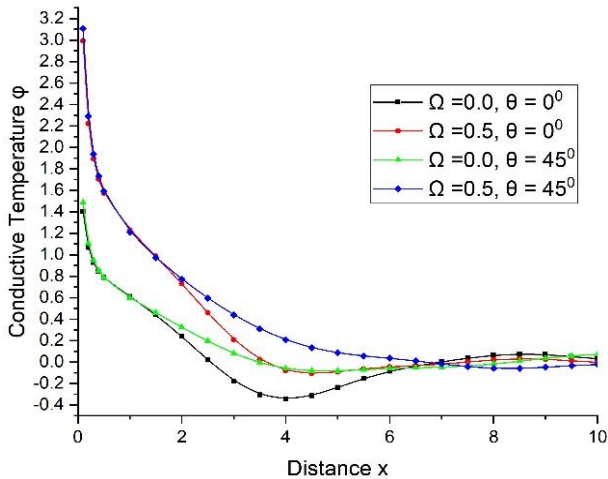


Fig. 16 variations of conductive temperature ϕ with distance x

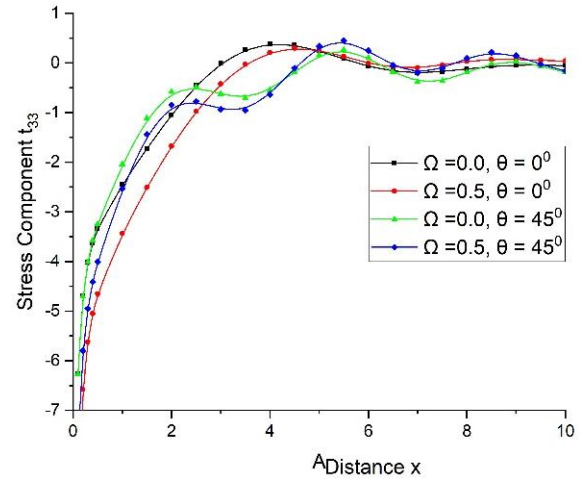


Fig. 19 variations of stress component t_{33} with distance x

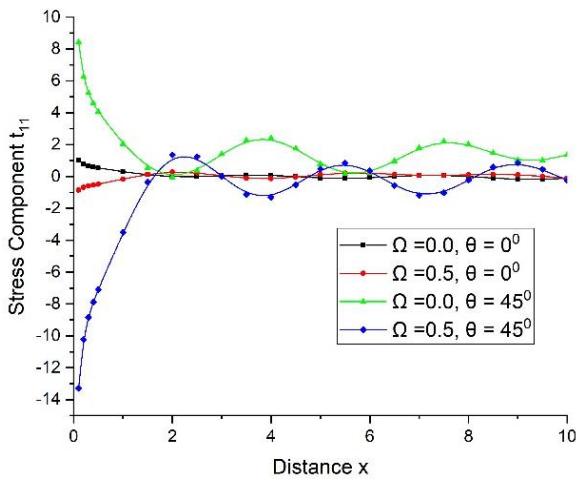


Fig. 17 variations of stress component t_{11} with distance x

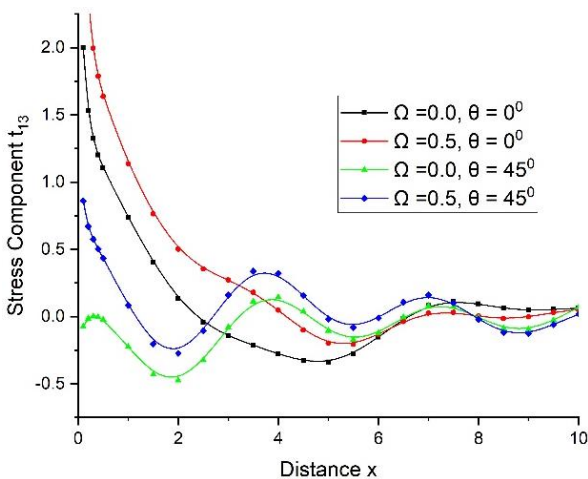


Fig. 18 variations of stress component t_{13} with distance x

thermoelastic medium with rotation. The values of conductive temperature ϕ , decreases and follow small oscillatory pattern for rest of the range of distance. Fig. 5 represents the values of stress component t_{11} . Near the loading surface, the values of t_{11} for $\theta = 0^\circ$ and $\Omega =$

0,0.5 remains same and have minor change but for $\theta = 45^\circ$, t_{11} decreases sharply corresponding to the rotation $\Omega = 0$ and increase sharply corresponding to the rotation $\Omega = 0.5$, and after that oscillate for $\Omega = 0.0, 0.5$. Fig. 6 describes the variations of stress component t_{13} . Near the loading surface, the values of t_{13} increase sharply and then somehow oscillates. Fig. 7 interprets the variations of stress component t_{33} . The values increase sharply near the loading surface with $\theta = 0^\circ$ and $\Omega = 0, 0.5$ and decrease sharply near the loading surface with $\theta = 45^\circ$ and $\Omega = 0, 0.5$ follow small oscillatory pattern for rest of the range.

Case ii. Uniformly distributed force due to inclined load with rotation

Fig. 8 shows the variations of the displacement component u for uniformly distributed force for transversely isotropic magneto-thermoelastic medium with rotation. The values of displacement component u , decreases for $\theta = 0^\circ$ and $\Omega = 0, 0.5$ and increases for $\theta = 45^\circ$ and $\Omega = 0, 0.5$ for the initial values of distance and follow oscillatory pattern for rest of the range of distance. Fig. 9 depicts variations of the displacement component w for transversely isotropic thermoelastic medium with rotation. The values of displacement component w , decreases for $\theta = 0^\circ$ and $\Omega = 0, 0.5$ and increases for $\theta = 45^\circ$ and $\Omega = 0, 0.5$ for the initial values of distance and follow small oscillatory pattern for rest of the range of distance. Fig. 10 represents the variations of the conductive temperature ϕ for transversely isotropic thermoelastic medium with rotation. The values of conductive temperature ϕ , sharply decreases for $\theta = 45^\circ$ and $\Omega = 0, 0.5$ and becomes stable for rest of the range of distance and for $\theta = 0^\circ$ and $\Omega = 0, 0.5$ decrease very small and then follows oscillatory pattern. Fig. 11 represents the values of stress component t_{11} . Near the loading surface, the values of t_{11} increase sharply for $\theta = 45^\circ$ and $\Omega = 0, 0.5$ and then oscillates but for $\theta = 0^\circ$ and $\Omega = 0, 0.5$ increase very little and then follows oscillatory pattern. Fig. 12 describes the variations of stress component t_{13} . Near the loading surface, the values of t_{13} sharply increases for all the cases and then somehow oscillates. Fig. 13 interprets the variations of stress

component t_{33} . It sharply decreases sharply decreases for $\theta = 45^\circ$ and $\Omega = 0, 0.5$ and for $\theta = 0^\circ$ and $\Omega = 0, 0.5$ increases and then follows oscillatory pattern in all cases of the range.

Case iii. Linearly distributed force due to inclined load with rotation

Fig. 14 shows the variations of the displacement component u for transversely isotropic magneto-thermoelastic medium with linearly distributed force with rotation. The values of displacement component u , sharply increases for all the cases in the initial values of distance and follow oscillatory pattern for isotropic magneto-thermoelastic medium with rotation. The values of displacement component w , sharply increases for all the cases in the initial values of distance and follow oscillatory pattern for rest of the range of distance. Fig. 16 represents the variations of the conductive temperature ϕ for transversely isotropic magneto-thermoelastic medium with rotation. The values of conductive temperature ϕ , sharply decreases for all the cases in the initial values of distance and follow small oscillatory pattern for rest of the range of distance. Fig. 17 represents the values of stress component t_{11} . Near the loading surface, the values of t_{11} decrease sharply corresponding to $\theta = 45^\circ$ and $\Omega = 0$ and increases for $\theta = 45^\circ$ and $\Omega = 0.5$ and after that these oscillate for rest of the range and for $\theta = 0^\circ$ and $\Omega = 0, 0.5$ remains stable for all range. Fig. 18 describes the variations of stress component t_{13} . Near the loading surface, the values of t_{13} decrease sharply and then somehow oscillates. Fig. 19 interprets the variations of stress component t_{33} . The values increase sharply near the loading surface with all the values of rotation then small oscillatory pattern for rest of the range.

9. Conclusions

From above investigation, it is observed that the magnetic effect of rotation as well as the angle of inclination of the applied load plays a major role in the distribution of all the physical quantities. The amplitude of all the physical quantities differ (either increase or decrease) with increase in rotation as well as the angle of inclined load. Presence of rotation confines the quantities to oscillate near the point of application of source as well as away from the source. In presence of rotation and inclined load, the displacement components and stress components show an oscillatory nature with increasing amplitude with respect to x . The inclined load plays a significant role in the distribution of all the physical quantities. The result gives an inspiration to study magneto-thermoelastic materials as an innovative domain of applicable thermoelastic solids. The results of this paper become useful for those researchers who works in material science, inventors of new materials, in addition to those working on the magneto-thermoelasticity and in real life as in geophysics, acoustics, geomagnetic etc. The proposed methods in this research is relevant to a wide range of problems in thermodynamics and thermoelasticity.

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