**Structural Engineering and Mechanics**, *Vol. 70, No. 1 (2019) 81-96*
DOI: https://doi.org/10.12989/sem.2019.70.1.081 

# System RBDO of truss structures considering interval distribution parameters

Mohammad Zaeimi[a] and Ali Ghoddosian*

*Faculty of Mechanical Engineering, Semnan University, Semnan, Iran*

*(Received November 10, 2018, Revised January 24, 2019, Accepted February 8, 2019)*

**Abstract.** In this paper, a hybrid uncertain model is applied to system reliability based design optimization (RBDO) of trusses. All random variables are described by random distributions but some key distribution parameters of them which lack information are defined by variation intervals. For system RBDO of trusses, the first order reliability method, as well as monotonicity analysis and the branch and bound method, are utilized to determine the system failure probability; and Improved (μ + λ) constrained differential evolution (ICDE) is employed for the optimization process. System reliability assessment of several numerical examples and system RBDO of different truss structures are proposed to verify our results. Moreover, the effect of different classes of interval distribution parameters on the optimum weight of the structure and the reliability index are also investigated. The results indicate that the weight of the structure is increased by increasing the uncertainty level. Moreover, it is shown that for a certain random variable, the optimum weight is more increased by the translation interval parameters than the rotation ones.

**Keywords:** reliability based design optimization; interval distribution parameters; improved ($\mu$+$\lambda$) constrained differential evolution (ICDE); structural optimization

## 1. Introduction

In traditional optimization procedure, most engineers assume that the design variables in the problem are deterministic. However, different kinds of uncertainties are presented and needed to be accounted for the design optimization process. Reliability based design optimization (RBDO) is a method that takes into account uncertainty due to the presence of random variables during the design process. The aim is to obtain a trade-off between a higher safety and a lower cost which is normally satisfied by setting a maximum allowed probability of failure (Thoft-Christensen and Murotsu 1986).

Several attempts have been made in RBDO problems of truss structures by considering the displacement constraints and the stress limits of the components. Luo and Grandhi (1997) proposed a reliability based multidisciplinary structural analysis for optimizing truss structures. Mathakari and Gardoni (2007) developed a hybrid methodology by combining multi-objective genetic algorithms (MOGA) and finite element reliability analysis. The weight and reliability index of an electrical transmission tower are considered as the two objective functions for MOGA. The finite element reliability analysis performed by OpenSees software. Togan *et al.* (2011) used the harmony search optimization algorithm and double-loop strategy to perform RBDO based on the reliability index and performance measurement approaches. Yand and Hsieh (2011) solved the discrete and non-smooth RBDO problem by integrating subset simulation with a new particle swarm optimization algorithm (PSO). Shayanfar *et al.* (2014) developed a double-loop strategy for reliability-based optimization of structures by employing genetic algorithm (GA) as an optimization approach and OpenSees software for finite element reliability analysis. Safaeian Hamzehkolaei *et al.* (2016) proposed a decoupled RBDO method based on a safety factor concept, PSO and weighed simulation method. A new hybrid method, namely the SORA-ICDE, was developed by HoHuu *et al.* (2016) by integrating the sequential optimization and reliability assessment (SORA) with an improved constrained differential evolution algorithm (ICDE) for solving RBDO of engineering problems. Dizangian and Ghasemi (2016) proposed a decoupled strategy for the reliable optimum design of trusses based on a design amplification factor combined with response surface method. Ho-Huu *et al.* 2018) proposed a global single-loop deterministic approach by combining a single-loop deterministic method (SLDM) and an improved different evolution (IDE), showed its capability to solve RBDO problems with both the continuous and discrete design variables.

To solve RBDO of statically determinate truss structures, some studies can be found in the literature by modeling the structural system as a series system. Dimou and Koumousis (2003) proposed a new version of GA, namely competitive GA, and showed its application to the reliability based optimal design of trusses. In a similar study, they utilized PSO algorithm instead of GA (Dimou

*Corresponding author, Associate Professor
E-mail: aghoddosian@semnan.ac.ir
[a]Ph.D. Student


 ISSN: 1225-4568 (Print), 1598-6217 (Online)

and Koumousis 2009). Togan and Daloglu (2006) optimized a roof truss system by various optimization methods including sequential quadratic programming (SQP), evolution strategy (EVOL), and GA. Zhang *et al.* (2017) proposed a novel approach, namely time-dependent reliability analysis with response surface (TRARS), to estimate the time-dependent reliability for nondeterministic structures under stochastic loads. The basic idea is replacing the performance function with such as response surface. In similar studies, Meng *et al.* (2018) proposed an adaptive directional boundary sampling (ADBS) method (Meng *et al.* 2018) and a novel importance learning method (ILM) based on the active learning technique to obtain more accurate and efficient reliability analysis for some engineering problems (Meng *et al.* 2018).

It should be noted that for a statically indeterminate truss, the assumption of a series system is inapplicable since the structural failure may occur after more than one component fail. To overcome this problem, some researchers modeled them as series-parallel systems and determined the failure of the system based on the failure paths concept. Hendawi and Frangopol (1994) developed a probabilistic redundancy factor to estimate the first yielding of the structure. Natarajan and Santhakumar (1993) utilized the branch and bound method and developed a formulation for RBDO of a transmission line towers. Thampan and Krishnamoorthy (2001) proposed a modified branch-and-bound (MBB) method to perform the system reliability assessment of truss structures. They coupled MBB and GA to minimize the total expected cost of the structure. Based on some fundamental assumptions, Park *et al.* (2004) proposed an efficient technique to determine the system reliability of a complex structure directly from the reliabilities of its members. A single-loop method based on the matrix-based system reliability (MSR) technique was proposed by Nguyen *et al.* (2010) to solve RBDO of a 6-member indeterminate truss structure. The MSR determines the probabilities of the system events by the matrix formulation. Liu *et al.* (2015) proposed a system reliability based design optimization for truss structures by integrating GA and Monte Carlo simulation together with the trained radial basis function (RBF) neural networks. Okasha (2016) considered nonlinear material behavior in RBDO problems of indeterminate trusses. The system reliability determined by weighted average simulation method (WASM), and the optimization process performed by using the firefly algorithm.

As can be found from the previous studies, although they proposed some valuable and efficient methods in solving different kinds of RBDO problems, some key features in the design process of truss structures are ignored. On the one hand, most of the previous works are dealt with the statically determinate trusses, whereas statically indeterminate truss are more practical because the distribution of the applied between the structural members loads is done better and also some elements can be failed without compromising the viability of the structure (Macdonald 2001). On the other hand, research on solving the system RBDO problems of truss structures is focused on describing uncertainty by random distributions that have deterministic distribution parameters. However, finding precise random distributions require a large amount of information. Since in practical applications sufficient experimental samples are expensive and difficult to achieve, some assumptions should be made when using a probability model to perform the reliability analysis. However, several studies revealed that even a small deviation of the distribution parameters from the real values can result in very large errors in the reliability analysis (Ben-Haim and Elishakoff 1990).

In order to overcome this problem, two hybrid uncertain models have been proposed by integrating the traditional probability approach and the non-probability interval analysis. The concept of these models is based on the use of variation intervals in the face of insufficient uncertainty information. In this way, a more accurate reliability analysis can be achieved by eliminating errors from assumptions on the probability distributions (Jiang 2011).

In the first model, random variables with sufficient information and ones lacking enough uncertainty information are treated as random distributions and intervals, respectively. While in the second one, all random variables are described by random distributions but some key distribution parameters of them which lack information are defined by variation intervals. For a good overview about hybrid uncertain models, the interested reader is referred to the work by Jiang *et al.* (2018).

For the first hybrid uncertain model, a number of studies have been published. Du *et al.* (2005) proposed a single-loop RBDO to deal with the uncertain variables described by the mixture of probability distributions and intervals. Du (2007) formulated a reliability analysis framework with the aim of investigating computational tools to determine the effects of random and interval inputs on direct and inverse reliability analysis results. By considering both random variables and interval variables, a sensitivity analysis method was proposed by Guo and Du (2009). They introduced six sensitivity indices based on the first-order reliability method (FORM) to investigate the sensitivity of the average reliability and reliability bounds. Based on the probabilistic reliability model and interval arithmetic, Qiu and Wang (2010) developed a new model to improve interval estimation for reliability of the hybrid structural system. Jiang *et al.* (2012) proposed an equivalent model for reliability analysis with random and interval variables. By changing the interval variables to corresponding uniform distributions, the original problem was converted into a conventional reliability analysis problem with only random variables. Xie *et al.* (2015) proposed a new hybrid reliability analysis by decomposing the probability analysis loop and interval analysis loop into two separate loops. Furthermore, a new interval analysis method is formulated based on the monotonicity of limit-state function.

The second hybrid uncertain model was first proposed by Elishakoff and Colombi (1993) by formulating an anti-optimization problem. Non-linear buckling of a column with initial imperfection was investigated by Elishakoff *et al.* (1994) based on the probability and non-probability approaches. Moreover, they showed that the results from both of them were critically contrasted. Qiu *et al.* (2008)

proposed an approach to obtain the interval of the system failure probability from the statistical parameter intervals of the basic variables. Jiang *et al.* (2011) proposed a new structural reliability analysis by conducting a monotonicity analysis for the probability transformation process, and two efficient algorithms were formulated based on the reliability index and performance measurement approach. In another work, they presented a detailed description of the effects of interval parameters on the limit state function (Jiang *et al.* 2012). Huang *et al.* (2017) developed a decoupled RBDO algorithm by utilizing the reliability analysis presented in previous work by Jiang *et al.* (2012) and an incremental shifting vector (ISV) technique.

In this paper, for the first time, the second hybrid uncertain model is considered to system RBDO problems of truss structures; and the effect of different types of interval distribution parameter on the optimum weight of the structure is investigated which can be beneficial to select and define a proper variation intervals of the distribution parameters in the design process. The reliability analysis we use is based on the work by Jiang *et al.* (2012) and the optimization process is performed by using the ICDE algorithm. The rest of this paper is organized as follows. Section 2 presents the ICDE algorithm. In section 3 a brief description of the reliability assessment with and without interval distribution parameters, system reliability analysis for truss structures as well as reliability based design optimization are presented. Numerical and structural examples are proposed in section 4. Finally, the conclusion is presented in section 5.

## 2. The improved $(\mu+\lambda)$ constraint differential evolution (ICDE) algorithm

According to the relatively fast convergence rate, low standard deviation from the mean value in different runs of algorithm and ease of implementation (Zaeimi and Ghoddosian 2018), we used ICDE algorithm for optimization process in this study. ICDE is a robust version of differential evolution algorithm to solve the constrained optimization problem. Two main parts of ICDE are the improved $(\mu+\lambda)$- differential evolution (IDE) and the archiving-based adaptive tradeoff model (Jia *et al.* 2013). The next sections briefly introduce the main parts algorithm.

### *2.1 Differential evolution (DE) algorithm*

The differential evolution (DE) was first developed by Storn and Price (1997). It is one of the most successful and widely used metaheuristic algorithms to solve continuous optimization problem. DE is a population-based algorithm which uses three evolutionary operators, i.e., mutation, crossover, and selection operators. Note that the diversity of the population is guaranteed by using the mutation and crossover operators. The main steps of DE algorithm are described below.

***Step 1 - Generating initial population:*** In this step, an initial population containing $\mu$ parents are generated randomly in the search space as follows

$$\boldsymbol{X}_i^t = \boldsymbol{LB} + rand\,.(\boldsymbol{UB} - \boldsymbol{LB}),\;\; i = 1,2,\ldots,\mu \tag{1}$$

in which $\boldsymbol{X}_i^t$ is the current individual in the $t$-th generation (in the initial population t is equal to zero), *rand* is used to create a random number in [0,1] and $\boldsymbol{LB}$ and $\boldsymbol{UB}$ indicate the lower and upper bound of the design variables respectively.

***Step 2 - Generating the mutant vectors:*** In each generation, mutant vectors $\boldsymbol{V}_i$ are generated based on the following four strategies (Jia *et al.* 2013)

$$\textit{Rand/1:}\; \boldsymbol{V}_i^t = \boldsymbol{X}_{r_1}^t + F\cdot(\boldsymbol{X}_{r_2}^t - \boldsymbol{X}_{r_3}^t) \tag{2}$$

$$\textit{Rand/2:}\; \boldsymbol{V}_i^t = \boldsymbol{X}_{r_1}^t + F\cdot\left(\boldsymbol{X}_{r_2}^t - \boldsymbol{X}_{r_3}^t\right) + F\cdot(\boldsymbol{X}_{r_4}^t - \boldsymbol{X}_{r_5}^t) \tag{3}$$

$$\textit{Current-to-rand/1:}\; \boldsymbol{V}_i^t = \boldsymbol{X}_i^t + F\cdot\left(\boldsymbol{X}_{r_1}^t - \boldsymbol{X}_i^t\right) + F\cdot(\boldsymbol{X}_{r_2}^t - \boldsymbol{X}_{r_3}^t) \tag{4}$$

$$\textit{Current-to-best/1:}\; \boldsymbol{V}_i^t = \boldsymbol{X}_i^t + F\cdot(\boldsymbol{X}_{best}^t - \boldsymbol{X}_i^t) + F\cdot(\boldsymbol{X}_{r_1}^t - \boldsymbol{X}_{r_2}^t) \tag{5}$$

where $r_1, r_2, r_3, r_4$ and $r_5$ are different integers selected from the set $\{1,2,\ldots,\mu\}$ and satisfy$\{r_1 \neq r_2 \neq r_3 \neq r_4 \neq r_5\}$, $\boldsymbol{X}_{best}^t$ and $\boldsymbol{X}_i^t$ are respectively the best and the current individual in t-*th* generation and scale factor *F*, is selected randomly between 0 and 1. After generation of mutant vectors, they are checked against the boundary constraints and the following modification is performed

$$V_{i,j}^t = \begin{cases} 2X_j^l - V_{i,j}^t & if\; V_{i,j}^t < X_j^l \\ 2X_j^u - V_{i,j}^t & if\; V_{i,j}^t > X_j^u \\ V_{i,j}^t & otherwise \end{cases} \tag{6}$$

***Step 3 - generating trial vectors by crossover operator:*** In this step, by using the binomial crossover, some elements of the current vector is replaced by some elements of mutant vector to produce the trial vector $\boldsymbol{U}_i$

$$U_{i,j}^t = \begin{cases} V_{i,j}^t & if\;\; rand \le CR\;\; or\;\; j = j_{rand} \\ X_{i,j}^t & otherwise \end{cases} \tag{7}$$

***Step 4 - comparing the trial vector and current vector:*** Finally, the trial vector compares with the current vector according to their objective function values and the better one with better objective value will survive in the next generation

$$\boldsymbol{X}_i^{t+1} = \begin{cases} \boldsymbol{U}_i^t & if\;\; f(\boldsymbol{U}_i^t) \le f(\boldsymbol{X}_i^t) \\ \boldsymbol{X}_i^t & otherwise \end{cases} \tag{8}$$

### *2.2 Improved $(\mu+\lambda)$ -differential evolution (IDE) algorithm*

The ICDE utilizes an improved version of the DE, called IDE, which have better population diversity. In IDE, the offspring population $Q_t$is generated from the current population $P_t$ based on the following three steps. At the end of these steps, the offspring population will have $\lambda =$

$3\mu$ individuals.

***Step 1:*** set $Q_t = \emptyset$;

***Step 2:*** generate three offspring for each individual in $P_t$:

- for the first offspring $\boldsymbol{y}_1$, use the "rand/1" mutation strategy and the binomial crossover;
- for the second offspring $\boldsymbol{y}_2$, use the "rand/2" mutation strategy and the binomial crossover;
- for the third offspring $\boldsymbol{y}_3$, use the "current-to-best/1" strategy and improved breeder genetic algorithm (iBGA) (Jia *et al.* 2013).

***Step 3:*** update the offspring population, $Q_t = Q_t \cup \boldsymbol{y}_1 \cup \boldsymbol{y}_2 \cup \boldsymbol{y}_3$;

In order to obtain a good balance between the population diversity and convergence of the population, two different mutation strategies are performed in the "current-to-rand/best/1" in step 2. In the first one, the "current-to-rand/1" strategy is used to increase the global search of the algorithm, while the second one increases the convergence rate of the population toward the global optimum. When the generation number is more than a threshold value, the second phase begins.

### *2.3 Archiving-based adaptive tradeoff model (ArATM)*

In constraint optimization, three possible situations may exist in a combined population, $H_t$, resulting from the combination of the offspring population $Q_t$ and the parent population $P_t$. These situations are the infeasible, semi-feasible and feasible situations and each of them has different constraint-handling method in the ArATM. By performing the procedure described below, ArATM simultaneously satisfies the constraints and optimizes the objective function of the problem.

In the infeasible situation, since all individuals violate the constraints, the population should be guided toward the feasible region very quickly to maintain the population diversity. For this purpose, the original problem transformed to a bi-objective optimization problem and a good tradeoff between two objectives, the objective function and the degree of constraint violation is made. Moreover, the individuals that have no chance to survive into the next generation will be stored in an archive to compete with the individuals of the next combined population $H_{t+1}$. In this way, the population diversity can be increased during the optimization process.

In the semi-feasible situation, the combined population contains both feasible and infeasible individuals. In this situation, the algorithm benefits from not only the feasible individuals but also some infeasible ones since they may have important information to find the global optimum. To fulfill this aim, by using an adaptive fitness transformation scheme, some feasible individuals with small fitness values along with some infeasible individuals with both small degree of constraint violation and small fitness values are selected to survive into the next generation.

In the feasible situation, all individuals are feasible and the comparison between them is performed only based on their fitness values. Therefore, those with better fitness value constitute the next population.

## 3. Reliability and reliability based design optimization

### *3.1 Reliability analysis*

The Failure probability of a limit state function (or failure mode) can be calculated using a probabilistic reliability analysis

$$P_f = P(G(\boldsymbol{X}) \leq 0) = \int_{G(\boldsymbol{X})\leq 0} f_{\boldsymbol{X}}(\boldsymbol{X})\, d\boldsymbol{X} \tag{9}$$

where $P_f$ is the failure probability, $G(\boldsymbol{X})$ indicates the limit state function which is a function of random variables $\boldsymbol{X}$, and $f_{\boldsymbol{X}}(\boldsymbol{X})$ is the joint probability density function of $\boldsymbol{X}$. The reliability $R$ is defined as

$$R = 1 - P_f \tag{10}$$

Because of some difficulties in computing the above multi-dimensional integral (Elegbede 2005), one of the most commonly used approximation methods called first order reliability method (FORM) is used in this study.

The ease of the computational difficulties is provided through the simplifying the integrand $f_x(\boldsymbol{x})$ and approximating $G(\boldsymbol{X})$. First, the shape of the $f_x(\boldsymbol{x})$ is simplified by mapping $X$ into the independent standard normal space (i.e., U-space) (Rosenblatt 1952, Jiang *et al.* 2011)

$$\Phi(U_i) = F_{X_i}(X_i), \quad U_i = \Phi^{-1}\left[F_{X_i}(X_i)\right], \quad i = 1,2,\ldots,n \tag{11}$$

in which $F_{X_i}$ and $\Phi^{-1}$ are cumulative distribution function (CDF) and inverse standard normal CDF, respectively. The limit state function can be written in U-space as follows

$$G(\boldsymbol{X}) = G\big(T(\boldsymbol{U})\big) = G(\boldsymbol{U}) \tag{12}$$

where $T$ indicates a probability transformation and $G(\boldsymbol{U})$ is the transformed limit sate function in the U-space. Next, the limit state function is approximated by the first order Taylor expansion at a point with the highest probability density on the limit state function in U-space (Ang and Tang 2007). Geometrically, it is a point, namely the most probable point (MPP), with the shortest distance from the origin of U-space to $G(\boldsymbol{U}) = 0$. This minimum distance determines the reliability index (Hasofer and Lind 1974b)

$$\begin{cases} \beta = \min\limits_{U} \|\boldsymbol{U}\| \\ s.t. \quad G(\boldsymbol{U}) = 0 \end{cases} \tag{13}$$

where $\beta$ is the reliability index. Now, failure probability in Eq. (9) can be determined by the following equation

$$P_f = \Phi(-\beta) \tag{14}$$

where $\Phi$ is the standard normal CDF.

### *3.2 Reliability analysis for random variables with interval parameters*

When interval distribution parameters are involved in the distribution function of random variables X, Eq. (12) can be rewritten as follows (Jiang *et al.* 2011)

$$G(\boldsymbol{X}) = G\big(T(\boldsymbol{U},\boldsymbol{Y})\big) = G(\boldsymbol{U},\boldsymbol{Y}) \tag{15}$$

where $\boldsymbol{Y}$ is an $m$-dimensional vector containing all the interval distribution parameters

$$\boldsymbol{Y} \in [\boldsymbol{Y^L},\boldsymbol{Y^R}],\ \ Y_i \in [Y_i^L, Y_i^R] \quad i = 1,2,\dots,m \tag{16}$$

From Eq. (15), it can be found that the limit state function is related to not only the random variables $\boldsymbol{U}$ but also the interval variables $\boldsymbol{Y}$. Moreover, due to the interval variables, the transformed limit state described by $G(\boldsymbol{U},\boldsymbol{Y}) = 0$ will not be a single surface, but a strip enclosed by two bounding surfaces $S_L$ and $S_R$

$$S_L :\ \min_Y G(U,Y) = 0\ ,\quad S_R :\ \max_Y G(U,Y) = 0 \tag{17}$$

As shown in Fig. 1, $S_L$ and $S_R$ are respectively the lower and upper bounding surfaces of $G(\boldsymbol{U},\boldsymbol{Y})$ as $\boldsymbol{Y}$ changes. Jiang *et al.* (2011, 2012) proposed that for different classes of interval distribution parameters, there are two typical forms for the above bounding surfaces. For each bounding surface, we can obtain reliability indexes through FORM and define a hybrid reliability index $\beta^h$ (Jiang *et al.* 2011)

$$\boldsymbol{\beta^h} \in [\boldsymbol{\beta^L},\boldsymbol{\beta^R}] \tag{18}$$

where $\beta^L$ and $\beta^R$ indicate the reliability index of the lower and upper bounding surfaces, respectively. $\beta^h$ is not a deterministic value but a possible variation range of the reliability index formed by interval parameters $\boldsymbol{Y}$. Moreover, the failure probability of the structure will belong to an interval

$$\boldsymbol{P_f} \in \left[\boldsymbol{P_f^L},\boldsymbol{P_f^R}\right] = [\boldsymbol{\phi}(-\boldsymbol{\beta^L}),\boldsymbol{\phi}(-\boldsymbol{\beta^R})] \tag{19}$$

Since a strict reliability requirement can be satisfied only by focusing on the worst case, our most concern in this study is practically the upper bound of this interval. Therefore, according to Eqs. (18)-(19), only $\beta^L$ or $P_f^R$ should be determined to show the reliability degree of a structure. By using Eqs. (13), (15) and (17), the following optimization problem can be solved to compute $\beta^L$

$$\begin{cases}\beta^L = \min\limits_{\boldsymbol{U}}\|\boldsymbol{U}\| \\ subject\ to :\ S_L = \min\limits_{\boldsymbol{Y}} G(\boldsymbol{U},\boldsymbol{Y}) = 0\end{cases} \tag{20}$$

This problem can be decomposed into the following two-layer nesting optimization:

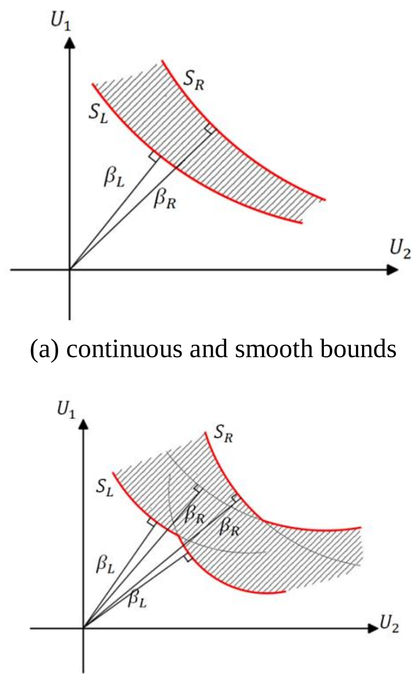
Outer layer

$$\begin{cases}\beta^L = \min\limits_{\boldsymbol{U}}\|\boldsymbol{U}\| \\ subject\ to\ \ G(\boldsymbol{U},\boldsymbol{Y}^*) = 0\end{cases} \tag{21}$$

Inner layer

$$\begin{cases}G(\boldsymbol{U},\boldsymbol{Y}^*) = \min\limits_{\boldsymbol{Y}} G(\boldsymbol{U},\boldsymbol{Y}) \\ subject\ to\ \ Y_i^L \le Y_i \le Y_i^R, i = 1,2,\dots,m\end{cases} \tag{22}$$

The inner-layer optimization is used to find the extreme values of the limit-state function in terms of $\boldsymbol{Y}$, and $\boldsymbol{Y}^*$ indicates the corresponding optimum values for interval




(b) non-continuous but smooth bounds

Fig. 1 limit-state strip by considering the interval parameters

variables. For practical engineering problems, solving the above nesting optimization leads to an extreme computational cost. Jiang *et al.* (2011) carried out a monotonicity analysis and reported that if CDF of a random variable $X$ is a monotonic function with respect to its own interval distribution parameter $Y$, the optimum values of the limit state function will be related to the bound combinations of all the interval parameters.

By investigating some widely used distribution functions, they classified all the distribution parameters into two main classes. For the first class, the monotonicity of the CDF with respect to this class of parameters remains unchanged; while for the second class, the monotonically increasing or decreasing of CDF is dependent on the value of the random variable $X$, and there is an inflection point of $X$. Note that, parameters in the first and second class are respectively called “translation distribution parameter” and “rotation distribution parameter” (Jiang *et al.* 2012).

For example, in normal distribution function, $\mu$ is a translation distribution parameter, while another parameter, $\sigma$, is a rotation one. Note that, since we assume that all random variables follow normal distributions in this work, only normal distribution parameters are mentioned. For a detailed classification of all the parameters in other distribution functions, the interested reader is referred to Jiang *et al.* (2011).

Considering a limit state function in standard normal space ($G(\boldsymbol{U},\boldsymbol{Y}) = 0$) with only a single interval variable $Y$ in CDF of $X_j$. As shown in Fig. 2, two situations can be considered as follows (Jiang *et al.* 2012):

(i) ***Y* is an interval translation parameter:** in this situation, the values of $Y$ on each bound surfaces will

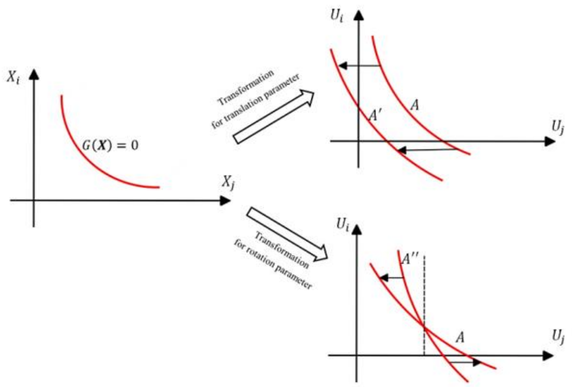



Fig. 2 transformation of the limit state function with interval distribution parameters

remain constant. For any fixed $U_i$ the values of $U_j$ will decrease with increasing of $Y$, if the CDF is a monotonically decreasing function with respect to $Y$. On the other hand, if the CDF is monotonically increasing, for any fixed $U_i$ the values of $U_j$ will decrease with increasing of $Y$. Thus, in this situation, the limit state strip has two continuous and smooth bounding surfaces ($A$ and $A'$ in Fig. 2).

(ii) ***Y is an interval rotation parameter:*** in contrast to the previous situation, the values of $Y$ will be changed on each bound. On one side of the inflection point where the CDF is monotonically decreasing with respect to $Y$, $U_j$ decreases as the interval parameter $Y$ increases for any fixed $U_i$, while on the opposite side $U_j$ becomes larger with increasing $Y$. With this behavior, $G(U,Y)=0$ will rotate around a hyper-plane (or an axis in a two-dimensional plane) as $Y$ changes ($A$ and $A''$ in Fig. 2, and thus the limit state strip will have continues but non-smooth bounding surfaces.

For problems where random variables have normal distributions with interval parameters, reliability analysis is performed in the following steps (Jiang *et al.* 2012):

***Step1:*** Divide all interval parameters into two classes, the interval translation parameters $Y_{ti}$, $i=1,2,\dots n_1$ and interval rotation parameters $Y_{rj}$, $j=1,2,\dots n_2$ where $n_1+n_2$ is equal to the total number of interval parameters.

***Step2:*** For interval translation parameters $Y_{ti}$, based on the gradient $\left.\frac{\partial G}{\partial U_j}\right|_{U=0}$, select appropriate values for $Y_{ti}$ (i.e., from lower or upper bound of $Y_{ti}$) which are on the lower bounding surface ($S_L$). For each $Y_{ti}$, if $\left.\frac{\partial G}{\partial U_k}\right|_{U=0}>0$, use lower bound of $Y_{ti}$, otherwise use upper bound of $Y_{ti}$. Then, collect all these selected value to form vector $\{Y_t\}$. Note that, $U_k$ is a standard normal variable which is defined by the interval translation parameter $Y_{ti}$.

***Step3:*** Generate all possible combinations of the bounds of the interval rotation parameters $Y_{rj}$ then collect them to form $\boldsymbol{Y}_r^i$, $i=1,2,\dots,2^{n_2}$, where $2^{n_2}$ is the total number of the combinatiton.

***Step4:*** Calculate the reliability index for all limit state functions in the following equation using FORM and determine $\beta_L$ (i.e., minimum value of calculated reliability indexes)

$$G(U,\{Y_t\},\boldsymbol{Y}_r^i)=0, i=1,2,\dots,2^{n_2} \tag{23}$$

***Step5:*** Compute the maximum probability of failure for $\beta_L$ through Eq. (14).

### *3.3 System reliability analysis for truss structures*

For a truss structure, a member fails when the internal force exceeds the strength of the member. It can be written as follows

$$G_i=R_iA_i-S_i \tag{24}$$

where $R_i$ is the allowable stress, $A_i$ and $S_i$ are respectively the cross sectional areas and the internal force of the i-th member. By using the FORM, failure probability of member $i$ can be evaluated as follows

$$P_i=P(G_i\le 0)=\Phi(-\beta_i) \tag{25}$$

where $\beta_i$ is the reliability index for $i$-th member. The internal force vector can be formulated as (Thoft-Christensen and Murotsu 1986)

$$S_i=\sum_{j=1}^{3l} b_{ij}L_j \tag{26}$$

where $L_j$ is the external load applied to the structure, $l$ is the number of nodes and $b_{ij}$ is the load coefficient of member $i$ with respect to $L_j$. For a statically determinate truss, $b_{ij}$ are constant while those of a statically indeterminate truss become functions of $A_i$. According to the nature of the structure, system failure occurs in parallel, series or a combination of both. For parallel failure, the system failure takes place when all failure modes in that system fail. While for series system, system failure results from the failure in any failure mode.

In case of a statically determinate truss, failure of the structure happens when any member fails. Therefore, the structural failure probability is estimated by modeling the structural system as a series system. However, in case of a statically indeterminate truss, estimating of the structural failure is very complex because failure of a member will not always result in failure of the whole system. In this case, we deal with a system composed of combinations of series and parallel subsystems. When failure of a member occurs, redistribution of loads takes place and thus the external loads are sustained by the members in survival. By repeating this process, system failure of the structure results when a specified number of members are failed and structure is turned into a mechanism. Failure of the structure is defined by investigating the singularity of the total structure stiffness matrix formed by the remaining members. After failure in $p$ members, by using the matrix method, stress analysis is carried out and the internal forces of the remaining members are determined as follows (Thoft-Christensen and Murotsu 1986)

$$S_{i\,(e_1,e_2,\dots,e_p)}=\sum_{j=1}^{3l} b_{ij}L_j-a_{ie_1}r_{e_1}-\dots-a_{ie_p}r_{e_p} \tag{27}$$

where suffix $(e_1,\ e_2, \ldots, e_p)$ shows a failure path including a set of failed members and their sequential order of failure, $a_{ij}$ are the coefficients of influence and $r_j$ indicated the residual strength of the $j$-th failed member. It should be noted that, the residual strength for a member of a brittle material is zero, while for a member of a ductile material is equal to yield strength (in tension) or buckling strength (in compression).

In this study, we utilize one of the most popular approaches called branch and bound method, to find system failure probability of statically indeterminate trusses. It is based on the failure paths which result in a failure mode. In this method, lower and upper bounds of the structural failure probability are determined by selecting dominant failure paths and discarding the failure paths that have negligible occurrence probability.

Note that the system failure probability is evaluated based on the Cornell's upper bound in this study. For statically determinate truss, it is equal to the sum of the failure probability of the members; while for statically indeterminate truss, it is equal to the sum of the upper bound of each failure path.

### *3.4 Reliability based design optimization*

RBDO formulations can be classified into component and system reliability. In the first one, only one single structural member with a single failure mode is investigated. However, in a real structure, more than one member can fail because of the existence of a large number of possible failure modes. In order to obtain the second one, it is required to know the component reliability and the relationship between the system and its components. Optimization problem under the component reliability constraints can be stated as (Tsompanakis *et al.* 2008, Kharmanda *et al.* 2013)

$$\begin{cases} min \quad W = \sum_{i=1}^{m} \rho L_i A_i \\ Subject\ to\!: \qquad P_i \leq P_i^t \end{cases} \tag{28}$$

where $W$ is the weight of the structure, $\rho$ is the density of the material, $L_i$, $A_i$ are the length and cross-section area of member $i$, respectively; $P_i$ and $P_i^t$ are respectively the failure probability and the target failure probability of member $i$. For system reliability based optimization, the above formulation can be expressed as follows

$$\begin{cases} min \quad W = \sum_{i=1}^{m} \rho L_i A_i \\ Subject\ to \qquad P_{sys} \leq P_{sys}^t \end{cases} \tag{29}$$

In which $P_{sys}$ and $P_{sys}^t$ are the failure probability and the target failure probability of the system. RBDO is based on three parts, including structural analysis, optimization procedure and reliability analysis. The first one is needed to obtain the response of the structure; the second one is used to find the design variables with minimum objective function; and the last one is performed to compute the reliability constraints of the RBDO. There are different strategies in the literature to link these parts together, e.g., single-loop, double-loop and decoupled, each having its own advantages and disadvantages.

Single-loop and decoupled strategies, despite the acceptable computational efficiency, have some disadvantages that limit their usage. Both of them avoid the reliability analysis by defining equivalent optimality conditions. In single-loop strategies, the most probable point is approximated based on approximation methods such as lower-order polynomial functions or derivatives of the performance functions. Since they are approximation methods, there is no guarantee to obtain accurate results (Yang and Hsieh 2011).

Decoupled strategies break the reliability analysis and the optimization procedure into sequential cycles. In these methods, the original problem is transformed to a deterministic optimization with constraints that are changed based on the reliability analysis cycle. They begin with a deterministic optimum and then try to find a close feasible solution satisfying the reliability constraint. When there exists multiple local optima and the reliable solution is not close to the deterministic optimum, they cannot provide satisfactory performance (Yang and Hsieh 2011).

Although it suffers from the computational effort compared to other strategies, we use the double-loop strategy based on the reliability index approach (RIA) because of its simplicity and accuracy (Kuschel and Rackwitz 1997, Cheng *et al.* 2006, Togan and Daloglu 2006).

As shown in Fig. 3, the double-loop strategy is a nested optimization problem where the inner loop deals with reliability assessment and structural analysis, while the outer loop deals with the minimization of the objective function. In this work, we utilize ICDE optimization algorithm in the outer loop of the RBDO. According to Fig. 3, the process of solving RBDO problems is briefly described with the following steps:

1. Generate an initial parent population ($P_t$) with $\mu$ individuals, which are selected uniformly and randomly from the search space.

2. Perform the system reliability analysis for each individual as follows:

(i) Generate the limit state function for each member of the truss structure using Eqs. (24) and (27).

(ii) Calculate the reliability indec (or probability of failure) for each limit state functions defined before. If there are no interval distribution parameters, use Eq. (13); otherwise, carry out the monotonicity analysis for translation rotation distribution parameters based on the five steps which are proposed in the last part of the section 3.2.

(iii) Calculate the failure probability of the system using Cornell's upper bound.

3. Compute the objective function (i.e., weight of each individual or the truss structure), constraint violation (i.e., the difference between the maximum allowable of structural system failure probability and the failure probability of the system) for each individual.

4. Generate the offspring population ( $Q_t$ ) with $\lambda$ offspring by performing IDE on all the individuals in $P_t$ (see section 2.2).

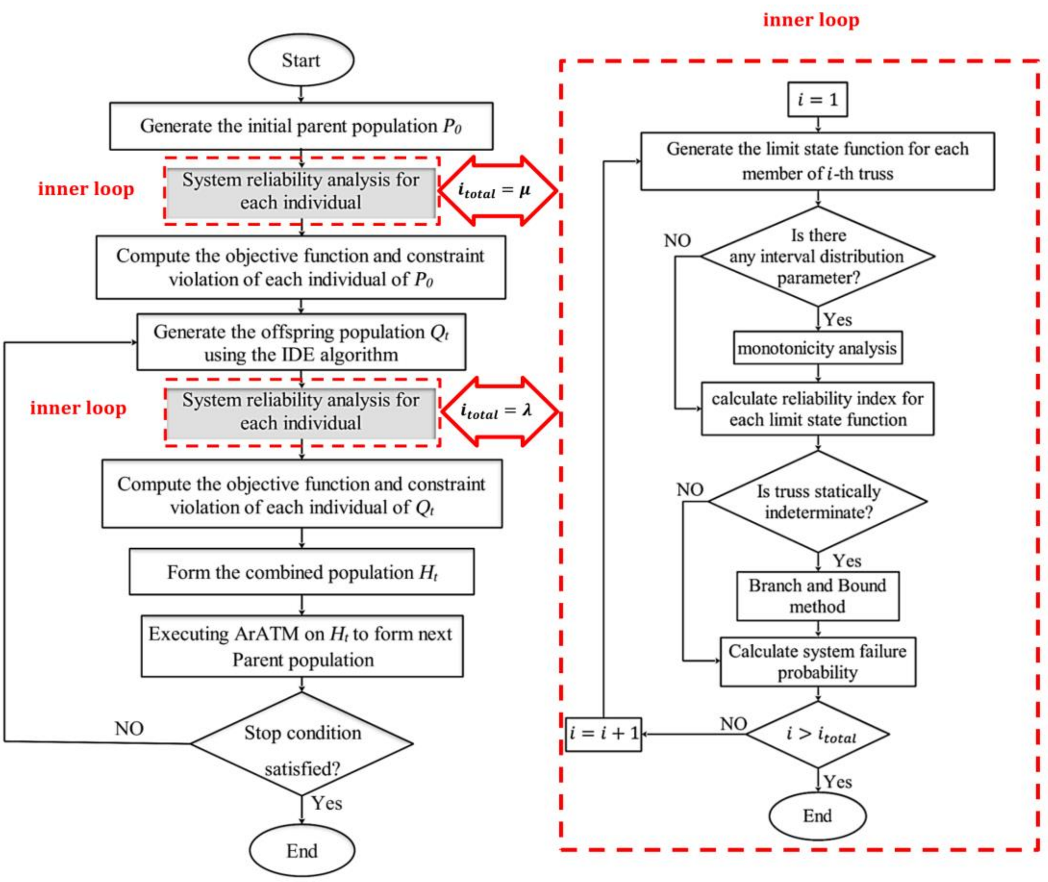



Fig. 3 Flowchart of RBDO using ICDE algorithm

5. Execute step 2 for each individual in $Q_t$.

6. Combine $P_t$ with $Q_t$ to obtain a combined population ( $H_t$).

7. Select $\mu$ potential individuals from $H_t$ to form the next population by ArATM strategy (see Section 2.3).

8. Check the termination criterion. If it is not satisfied go to Step 4; otherwise, stop and output the best individual in $P_t$.

## 4. Numerical results

In this section, several mathematical and engineering design benchmarks taken from the literature are investigated. The computing machine used for this work consists of an Intel Xeon at 2.9 GHz with 40 cores and 40 GB RAM. The ICDE algorithm is coded in MATLAB programming software and the parameters are set as follows: $\mu = 20, CR = 0.8, F = 0.9$ and $\delta = 0.0001$. The optimization process will be terminated when there is no improvement of the solutions after 150 iterations. Note that, for all considered truss structure examples, failure of the members is assumed to occur under tension or compression. However, the allowable tension and compression stresses are taken to be the same.

### *4.1 System reliability analysis of mathematical examples without interval distribution parameters*

Table 1 Limit state functions with independent standard normal distribution

| Limit state function description | |
|---|---|
| Parallel system | Series system |
| $G_1(x) = \max(g_1, g_2, g_3, g_4)$ $\begin{cases} g_1(x) = 2.677 - x_1 - x_2 \\ g_2(x) = 2.500 - x_2 - x_3 \\ g_3(x) = 2.323 - x_3 - x_4 \\ g_4(x) = 2.250 - x_4 - x_5 \end{cases}$ | $G_2(x) = \min(g_1, g_2)$ $\begin{cases} g_1(x) = -x_1 - x_2 - x_3 + 3\sqrt{3} \\ g_2(x) = -x_3 + 3 \end{cases}$ |
| $G_3(x) = \max(g_1, g_2)$ $\begin{cases} g_1(x) = -x_1 - x_2 - x_3 + 3\sqrt{3} \\ g_2(x) = -x_3 + 3 \end{cases}$ | $G_4(x) = \min(g_1, g_2)$ $\begin{cases} g_1(x) = 2 - x_2 + \exp(-0.1x_1^2) + (0.2\,x_1)^4 \\ g_2(x) = 4.5 - x_1 x_2 \end{cases}$ |
| $G_5(x) = \max(g_1, g_2)$ $\begin{cases} g_1(x) = 2 - x_2 + \exp(-0.1x_1^2) + (0.2\,x_1)^4 \\ g_2(x) = 4.5 - x_1 x_2 \end{cases}$ | $G_6(x) = \min(g_1, g_2, g_3, g_4)$ $\begin{cases} g_1(x) = 0.1(x_1 - x_2)^2 - \frac{(x_1 + x_2)}{\sqrt{2}} + 3 \\ g_2(x) = 0.1(x_1 - x_2)^2 + \frac{(x_1 + x_2)}{\sqrt{2}} + 3 \\ g_3(x) = x_1 - x_2 + 3.5\sqrt{2} \\ g_4(x) = -x_1 + x_2 + 3.5\sqrt{2} \end{cases}$ |

In this example, six limit state functions with independent standard normal random variables are considered. As shown in Table 1, they contain a set of equations that describe a parallel or a series system. The optimum reliability index and the corresponding probability of failure obtained by different algorithms are proposed in Table 2.

It can be seen that all metaheuristic methods converge to the almost same results with those obtained by ARBIS (adaptive radial-based importance sampling), which is more efficient than the Monet-Carlo simulation (MSC)

Table 2 Comparison of reliability index and failure probability obtained by different methods

| | | ICDE | DPSO (Kaveh and Ilchi Ghazaan 2015) | DPSO (Kaveh and Ilchi Ghazaan 2015) | CBO (Kaveh and Ilchi Ghazaan 2015) | ECBO (Kaveh and Ilchi Ghazaan 2015) | iBA (Chakri, Khelif *et al.* 2016) | ARBIS (Grooteman 2008) | HL-RF (Chakri, Khelif *et al.* 2016) |
|---|---|---|---|---|---|---|---|---|---|
| $G_1$ | $P_f$ | 0.003587 | 0.003529 | 0.003442 | 0.003439 | 0.003363 | NA | 0.000211 | NC |
| | β | 2.68862 | 2.69400 | 2.70230 | 2.70260 | 2.71000 | 2.73224 | 2.73800 | NC |
| $G_2$ | $P_f$ | 0.001350 | 0.001350 | 0.001350 | 0.001350 | 0.001350 | NA | 0.002570 | NA |
| | β | 2.99990 | 2.99990 | 2.99990 | 2.99990 | 2.99990 | NA | 2.95300 | NA |
| $G_3$ | $P_f$ | 0.000365 | 0.000365 | 0.000364 | 0.000365 | 0.000364 | NA | 0.000123 | NA |
| | β | 3.37801 | 3.37800 | 3.37810 | 3.37800 | 3.37820 | NA | 3.43400 | NA |
| $G_4$ | $P_f$ | 0.001350 | 0.001350 | 0.001350 | 0.001350 | 0.001350 | NA | 0.003540 | NA |
| | β | 2.99990 | 2.99990 | 2.99990 | 2.99990 | 2.99990 | 3.00355 | 2.92500 | 3.00000 |
| $G_5$ | $P_f$ | 0.000647 | 0.000647 | 0.000647 | 0.000647 | 0.000647 | NA | 0.000250 | NC* |
| | β | 3.21716 | 3.21710 | 3.21720 | 3.21710 | 3.21720 | 3.21749 | 3.21900 | NC |
| $G_6$ | $P_f$ | 0.001350 | 0.001350 | 0.001350 | 0.001350 | 0.001350 | NA | 0.002180 | NC |
| | β | 2.99990 | 2.99990 | 2.99990 | 2.99990 | 2.99990 | 3.00150 | 2.92500 | NC |
| $G_7$ | $P_f$ | 0.012624 | NA | NA | NA | NA | NA | NA | NA |
| | β | 2.23758 | NA | NA | NA | NA | NA | NA | NA |

*NC: not converged, NA: not available

Table 3 Random variables for statically determinate 13-member truss structure

| Variable | Description | Distribution | $\mu$ | Coefficient of variation |
|---|---|---|---|---|
| $L_1$ | Load (KN) | Normal | 66.72 | 0.16 |
| $L_2$ | Load (KN) | Normal | 66.72 | 0.16 |
| $L_3$ | Load (KN) | Normal | 66.72 | 0.16 |
| $\sigma_{yield}$ | Yield stress (KN/cm$^2$) | Normal | 25.31 | 0.12 |

Table 4 Results for statically determinate 13-member truss structure

| Cross section (cm$^2$) | ICDE | Interior penalty function (Nakib and Frangopol 1990) | Feasible direction (Nakib and Frangopol 1990) |
|---|---|---|---|
| $A_1 = A_{12}$ | 7.521 | 7.594 | 7.458 |
| $A_2 = A_{13}$ | 11.703 | 11.903 | 12.187 |
| $A_3 = A_{11}$ | 7.41 | 7.348 | 7.529 |
| $A_4 = A_8$ | 7.594 | 7.594 | 7.452 |
| $A_5 = A_9$ | 2.356 | 2.316 | 2.271 |
| $A_6 = A_{10}$ | 8.403 | 8.432 | 8.4 |
| $A_7$ | 5 | 5.142 | 5.219 |
| Weight (kg) | 362.3048 | 364.79 | 367.05 |

(Grooteman 2008). From Table 2, HL-RF is not converged for $G_5$ and $G_6$, and thus showing the weakest performance among the considered methods.

### *4.2 System reliability based design optimization without interval distribution parameters*

#### *4.2.1 Statically determinate 13-member truss structure*

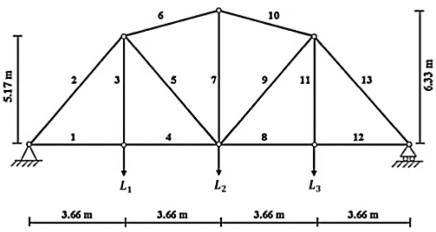



Fig. 4 statically determinate 13-member truss structure (Nakib and Frangopol 1990)

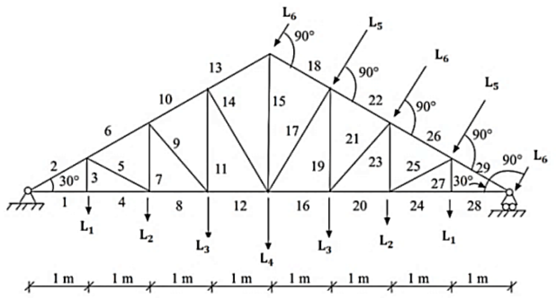



Fig. 5 statically determinate 29-member truss structure (Togan and Daloglu 2006)

A well-known 13-bar truss (Nakib and Frangopol 1990) is considered, as shown in Fig. 4. It is a statically determinate structure which shows a series system; hence the failure of any element leads to the failure of the whole system. The Young's Modulus E of the truss is $206.84$ GPa and the density $\rho$ is $7850\ \mathrm{kg/m^3}$. It is optimized for weight and cross-sectional areas are continuous design variables in which $A_i \in [0, 20]$, $i = 1, 2, \ldots, 13$.

The maximum allowable of structural system failure probability ($P_{sys}^t$) is equal to $1 \times 10^{-5}$. As shown in Table 3, the yield stress and the applied loads are considered as statically independent norm l random variables, and other variables are deterministic quantities. The structure includes 13 members, which are divided into seven groups as follows: (1) $A_1 = A_{12}$, (2) $A_2 = A_{13}$, (3) $A_3 = A_{11}$, (4) $A_4 = A_8$, (5) $A_5 = A_9$, (6) $A_6 = A_{10}$, (7) $A_7$.

The optimum results obtained by different methods are listed in Table 4; it can be observed that the ICDE converges to better optimum weight than the other methods.

#### *4.2.2 Statically determinate 29-member truss structure*

In this example, the size optimization of the roof truss, shown in Fig. 5, is performed. The collapse of the structure is considered based on a series system model. It is assumed that the yield stress of the members and the applied loads are statically independent normal random variables, while the cross-sectional areas of the members and the geometry of the truss are deterministic. The cross-sectional areas are continuous variables with values in the interval $[0, 15]$ and the distribution parameters of random variables are given in Table 5. The material density is $7850\ \mathrm{kg/m^3}$ and the modulus of elasticity is $206.84$ GPa.

Table 5 Random variables for statically determinate 29-member truss structure

| Variable | Description | Distribution | $\mu$ | Coefficient of variation |
|---|---|---|---|---|
| $L_1$ | Load (KN) | Normal | 0.29 | 0.2 |
| $L_2$ | Load (KN) | Normal | 0.69 | 0.2 |
| $L_3$ | Load (KN) | Normal | 1.08 | 0.2 |
| $L_4$ | Load (KN) | Normal | 1.37 | 0.2 |
| $L_5$ | Load (KN) | Normal | 19.6 | 0.2 |
| $L_6$ | Load (KN) | Normal | 9.8 | 0.2 |
| $\sigma_{yield}$ | Yield stress ($KN/cm^2$) | Normal | 27.60 | 0.05 |

Table 6 Results for statically determinate 29-member truss structure

| | Method | Cross section ($cm^2$) | Weight (Kg) |
|---|---|---|---|
| Optimally allocated design | ICDE | (0.92, 2.74, 0.04, 0.94, 0.04, 2.72, 0.06, 0.92, 0.06, 2.7, 0.12, 0.9, 2.66, 0.1, 2.1, 1.86, 2.28, 2.42, 1.44, 2.86, 1.84, 3.2, 0.8, 3.88, 1.52, 3.94, 0.04, 3.88, 4.66) | 17.044642 |
| | Indirect method (Thoft-Christensen and Murotsu 1986) | (0.57, 2.96, 0.02, 0.57, 0.02, 2.95, 0.07, 0.56, 0.05, 2.92, 0.12, 0.53, 2.88, 0.1, 2.3, 1.6, 2.53, 2.58, 1.59, 2.65, 2.07, 3.28, 0.91, 3.88, 1.74, 3.98, 0.02, 3.88, 4.7) | 17.4176 |
| Equally allocated design | ICDE | (0.98, 2.72, 0.04, 0.91, 0.04, 2.8, 0.08, 0.98, 0.06, 2.72, 0.12, 0.96, 2.74, 0.1, 2.16, 1.81, 2.36, 2.38, 1.46, 2.96, 1.882, 3.3, 0.84, 3.79, 1.58, 4.08, 0.04, 4.02, 4.8) | 17.4127 |
| | SQP (Togan and Daloglu 2006) | (0.85, 2.82, 1.049, 0.876, 0.1068, 2.807, 0.065, 0.863, 0.0562, 2.785, 0.1169, 0.88, 2.745, 0.095, 2.21, 1.799, 2.418, 2.5, 1.499, 2.82, 1.9127, 3.34, 0.846, 3.895, 1.6067, 4.13, 0.0838, 3.92, 4.91) | 17.7089 |
| | EVOL (Togan and Daloglu 2006) | (0.848, 2.823, 0.022, 0.848, 0.0236, 2.808, 0.0597, 0.858, 0.053, 2.783, 0.11, 0.876, 2.745, 0.095, 2.19, 1.78, 2.385, 2.49, 1.486, 2.818, 1.903, 3.34, 0.82, 3.88, 1.585, 4.13, 0.022, 3.88, 4.91) | 17.4183 |
| | GA (Togan and Daloglu 2006) | (1.61, 3.18, 1.61, 1.61, 1.61, 3.18, 1.61, 1.61, 1.61, 2.79, 1.61, 1.61, 2.79, 1.61, 2.79, 2.06, 2.79, 2.79, 1.61, 3.18, 2.06, 4.12, 1.61, 4.12, 1.61, 4.31, 1.61, 4.12, 5.15) | 23.8495 |

The system is optimized for $P_{sys}^t = 1 \times 10^{-5}$ and the results are compared by those obtained from different algorithms in the literature, including sequential quadratic programming (SQP), evolution strategy (EVOL), and GA. The optimization procedure is performed for two cases: equally allocated and optimum allocated design. In contrast to the last one, the probabilities of failure are equally allocated to the members of truss. As shown in Table 6, ICDE converges to the better results in both cases.

### *4.2.3 Statically indeterminate 6-member truss structure*

In this example, we consider a statically indeterminate 6-member truss with one degree of redundancy as shown in Fig. 6 (Thoft-Christensen and Murotsu 1986). The material density and the modulus of elasticity are respectively $2700\ kg/m^3$ and $70.6$ GPa. The cross-sectional areas are continuous variables which are taken from the

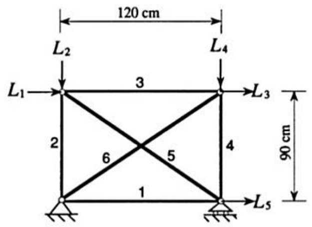



Fig. 6 statically indeterminate 6-member truss structure (Thoft-Christensen and Murotsu 1986)

Table 7 Random variables for statically indeterminate 6-member truss structure

| Variable | Description | Distribution | $\mu$ | Coefficient of variation |
|---|---|---|---|---|
| $L_1$ | Load (KN) | Normal | 50 | 0.2 |
| $L_2, L_4$ | Load (KN) | Normal | 50 | 0.2 |
| $L_3, L_5$ | Load (KN) | Normal | 50 | 0.2 |
| $\sigma_{yield}$ | Yield stress ($KN/cm^2$) | Normal | 27.60 | 0.05 |

Table 8 Results for statically indeterminate 6-member truss structure obtained by ICDE and reference (Thoft-Christensen and Murotsu 1986)

| Cross section ($cm^2$) | ICDE | reference | ICDE | reference | ICDE | reference | ICDE | reference |
|---|---|---|---|---|---|---|---|---|
| $A_1$ | 2.7500 | 2.91 | 2.9332 | 3.14 | 3.01683 | 3.33 | 3.1432 | 3.66 |
| $A_2$ | 0.4566 | 0.61 | 0.6011 | 0.8 | 0.68705 | 0.96 | 0.8141 | 1.23 |
| $A_3$ | 0.8235 | 0.6 | 0.9541 | 0.76 | 1.11722 | 0.89 | 1.4302 | 1.12 |
| $A_4$ | 2.5236 | 2.42 | 2.7030 | 2.68 | 2.91235 | 2.9 | 3.2749 | 3.27 |
| $A_5$ | 2.3620 | 2.6 | 2.5452 | 2.83 | 2.64864 | 3.02 | 2.7737 | 3.35 |
| $A_6$ | 1.9070 | 1.76 | 2.1361 | 2.0 | 2.38329 | 2.2 | 2.8435 | 2.55 |
| Weight (kg) | 3.61 | 3.64 | 3.958 | 4.07 | 4.252 | 4.42 | 4.750 | 5.03 |
| $P_{sys}^t$ | $10^{-1}$ | | $10^{-2}$ | | $10^{-3}$ | | $10^{-5}$ | |

interval $[0, 10]$.

By assuming that the yield stress and applied loads are statically independent normal random variables, for different values of $P_{sys}^t$, weight optimization of the structure is performed. Table 7 shows the distribution parameters of random variables.

Since it is a statically indeterminate structure, we use branch and bound method to determine failure paths and their corresponding failure probabilities. As can be observed I Table 8, for all values of $P_{sys}^t$, our results converge to better optimum value for weight of the structure.

### *4.3 System reliability analysis of mathematical examples with interval distribution parameters*

#### *4.3.1 4-member truss structure*

Consider a 4-bar truss shown in Fig. 7. The members 1-3 have a same cross-sectional area $A_1$ and the member 4 has an area $A_2$. The vertical displacement at the tip joint caused by two vertical forces should be less than an allowable value $\delta_a = 1.7\ mm$. The limit state function is defined as follow (Jiang *et al.* 2011)

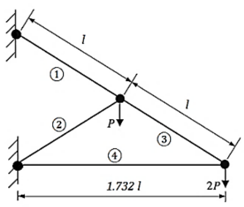



Fig. 7 4-bar truss

Table 9 Random variables for 4-member truss

| Distribution parameters | | Random variables |
|---|---|---|
| $\sigma_{A_1} = 70.65$ | $\mu_{A_1} \in [635.85, 777.15]$ | $A_1\ (mm^2)$ |
| $\sigma_{A_2} \in [169.56, 207.24]$ | $\mu_{A_2} = 1256$ | $A_2\ (mm^2)$ |

Table 10 Reliability analysis for 4-member truss

| Distribution parameters | | | | Bounds of hybrid reliability index | |
|---|---|---|---|---|---|
| $\mu_{A_1}$ | $\mu_{A_2}$ | $\sigma_{A_1}$ | $\sigma_{A_2}$ | This work | Ref (Jiang *et al.* 2011) |
| 635.85 | 1256 | 70.65 | 207.24 | $\beta^L = 1.477$ | $\beta^L = 1.62$ |
| 777.15 | 1256 | 70.65 | 169.56 | $\beta^R = 3.408$ | $\beta^R = 3.13$ |

$$G(X) = \delta_a - \delta = \delta_a - \frac{6Pl}{E}\left(\frac{3}{A_1} + \frac{\sqrt{3}}{A_2}\right) \tag{30}$$

where $A_1$ and $A_2$ are independent random variables following normal distribution. As shown in Table 9, the mean $\mu_{A_1}$ and standard deviation $\sigma_{A_2}$ are interval parameters, and their uncertainty levels are 10% off from their midpoints.

From the reliability analysis results proposed in Table 10, our results are almost the same as those reported by Jiang *et al.* (2011). It can be seen that due to the interval parameters, the reliability index of the structure will belong to an interval.

In order to propose the effect of the uncertainty levels of the interval parameters on the hybrid reliability index, the reliability analysis is performed under different uncertainty levels. As shown in Table 11, the width of the reliability index increase is increased by increasing the uncertainty level. It can be seen that the worst case reliability (i.e., lower bound of $\beta^h$) will decrease with increasing the uncertainty level, and thus the maximum failure probability will increase.

### *4.4 System reliability based design optimization with interval distribution parameters*

In order to performing system RBDO, the branch and bound method is integrated with monotonicity analysis of interval distribution parameters, to determine the system failure probability of each individual in the ICDE algorithm (i.e., each solution candidate). As mentioned before, the failure probability of the structural system $P_{sys}$ should be less than or equal to the target system failure probability $P_{sys}^t$.

Table 11 Effect of the uncertainty levels of the interval parameters on the hybrid reliability index for 4-member truss

| | Distribution parameters | | This work | | Jiang *et al.* (2011) | |
|---|---|---|---|---|---|---|
| Uncertainty levels (%) | $\mu_{A_1}$ | $\sigma_{A_2}$ | $\beta^h$ | Width of $\beta^h$ | $\beta^h$ | Width of $\beta^h$ |
| 3 | [685.31,727.70] | [182.75,194.05] | [2.147, 2.725] | 0.578 | [2.21,2.66] | 0.45 |
| 5 | [671.18,741.83] | [178.98,197.82] | [1.95, 2.92] | 0.965 | [2.04,2.80] | 0.76 |
| 10 | [635.85, 777.15] | [169.56, 207.24] | [1.477, 3.408] | 1.931 | [1.62,3.13] | 1.51 |
| 15 | [600.53,812.48] | [160.14,216.66] | [1.001, 3.902] | 2.901 | [1.43,3.44] | 2.01 |
| 20 | [565.20,847.80] | [150.72,226.08] | [0.5264, 4.4016] | 3.877 | [0.81,3.72] | 2.91 |

Table 12 System reliability based design optimization of the 6-bar truss for case 1

| Uncertainty level (%) | 0 | 5 | 10 | 15 | 25 | 25 | $P_{sys}^t$ |
|---|---|---|---|---|---|---|---|
| Weight (kg) | 3.61 | 3.800319 (5.3%) | 4.010717 (11.1%) | 4.24577 (17.6%) | 4.51063 (24.9%) | 4.811059 (33.3%) | $10^{-1}$ |
| | 3.958 | 4.165168 (5.2%) | 4.395546 (11.1%) | 4.653049 (17.6%) | 4.943365 (24.9%) | 5.273079 (33.2%) | $10^{-2}$ |
| | 4.252 | 4.474441 (5.2%) | 4.721356 (11.0%) | 4.997874 (17.5%) | 5.310326 (24.9%) | 5.664872 (33.2%) | $10^{-3}$ |

Table 13 System reliability based design optimization of the 6-bar truss for case 2

| Uncertainty level (%) | 0 | 5 | 10 | 15 | 25 | 25 | $P_{sys}^t$ |
|---|---|---|---|---|---|---|---|
| Weight (kg) | 3.61 | 3.780693 (4.7%) | 3.950865 (9.4%) | 4.11953 (14.1%) | 4.2907405 (18.9%) | 4.462212 (23.6%) | $10^{-1}$ |
| | 3.958 | 4.129032 (4.3%) | 4.298432 (8.6%) | 4.4686122 (12.9%) | 4.640522 (17.24%) | 4.814539 (21.64%) | $10^{-2}$ |
| | 4.252 | 4.423126 (4.0%) | 4.593848 (8.0%) | 4.765213 (12.1%) | 4.9377768 (16.1%) | 5.112963 (20.2%) | $10^{-3}$ |

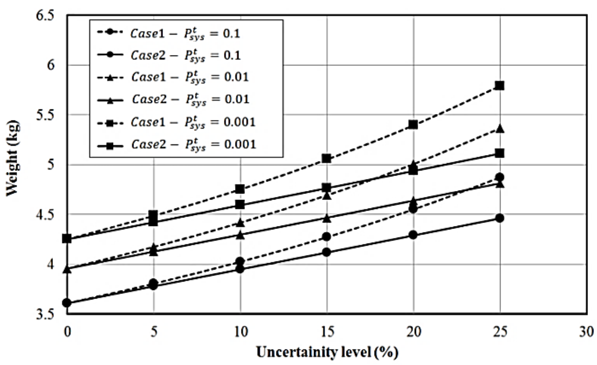



Fig. 8 effect of the uncertainty levels on the optimum weight of the 6-bar truss for cases 1 and 2

#### *4.4.1 Statically indeterminate 6-member truss structure*

In this example, the size optimization of the 6-bar truss structure in Fig. 6 is performed under different cases of interval distribution parameters. Each case shows which distribution parameter of a random variable is considered as an interval distribution parameter.

They are defined as follows: (1) the mean of the yield stress (2) the mean of the loads (3) the standard deviation of the yield stress (4) the standard deviation of the loads (5) the standard deviation of both the yield stress and loads (6) the mean of both the yield stress and loads (7) the mean and standard deviation of all random variables. According to the mean and standard deviation values presented in Table 7,

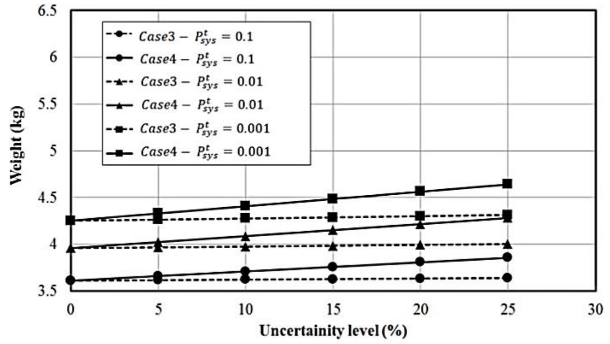



Fig. 9 effect of the uncertainty levels on the optimum weight of the 6-bar truss for cases 3 and 4

for different uncertainty levels, the intervals are defined as follows

$$\begin{aligned} \mu_X &\in \left[\mu_{X_0}(1-\alpha), \mu_{X_0}(1+\alpha)\right] \\ \sigma_X &\in \left[\sigma_{X_0}(1-\alpha), \sigma_{X_0}(1+\alpha)\right] \end{aligned} \tag{31}$$

where $X$ is a random variable with interval distribution parameter(s) and $X_0$ is a random variable without interval distribution parameter. In this example, $\mu_{X_0}$ and $\sigma_{X_0}$ are equal to the mean and standard deviation described in Table 7. The results are presented in Tables 12-16 and Figs. 8-10. Note that the percentage values in Tables 12-16 indicate the percentage increases in optimum weight for different levels of uncertainty.

For cases 1 and 2, the RBDO results are shown in Tables 12-13 and Fig. 8. It can be found that for all considered values of $P_{sys}^{t}$, the optimum weight increases with increasing the uncertainty level value. This behavior can be explained as an effect of the reliability index of the structural members. As mentioned in the previous section, the lower bound of the hybrid reliability index will decrease with increasing the uncertainty level. As a consequence of decreasing the reliability of the members, the reliability of the structural system can be decreased. Therefore, to have a reliable design, the cross-sectional area of the members and thus the weight of the structure will increase.

From Fig. 8, we can see that the interval distribution parameters defined in case 1 (mean of the yield stress) lead to greater increase in structural weight than those in case 2 (mean of the loads).

The optimization results for cases 3 and 4 are proposed in Tables 14-15 and Fig. 9. It can be observed that the effect of uncertainty levels on the optimum weight of the truss is the same as in cases 1 and 2. By comparing Figs. 8-9 for a certain class of interval parameters (i.e., $\mu$ or $\sigma$), it is clear that the slope of the curves in Fig. 9 is less than their corresponding ones in Fig. 8. Therefore, in this example, the sensitivity of the optimum weight (or system failure probability) to interval rotation parameters is less than interval translation ones. As noted previously, in normal distribution function, $\sigma$ is a rotation parameter and $\mu$ is a translation one.

From the percentage values in Tables 12-15, for a certain target system failure probability, the mean and standard deviation of the yield stresses have respectively the most and least impact on the optimum weight when described by interval parameters.

Table 14 System reliability based design optimization of the 6-bar truss for case 3

| Uncertainty level (%) | 0 | 5 | 10 | 15 | 25 | 25 | $P_{sys}^{t}$ |
|---|---|---|---|---|---|---|---|
| Weight (kg) | 3.61 | 3.616058 (0.2%) | 3.621117 (0.3%) | 3.626629 (0.5%) | 3.6326219 (0.6%) | 3.638508 (0.8%) | $10^{-1}$ |
| | 3.958 | 3.966111 (0.2%) | 3.974245 (0.4%) | 3.983311 (0.6%) | 3.99238 (0.9%) | 4.0017993 (1.1%) | $10^{-2}$ |
| | 4.252 | 4.262807 (0.3%) | 4.274679 (0.5%) | 4.286634 (0.8%) | 4.299827 (1.1%) | 4.313586 (1.5%) | $10^{-3}$ |

Table 15 System reliability based design optimization of the 6-bar truss for case 4

| Uncertainty level (%) | 0 | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | $P_{sys}^{t}$ |
|---|---|---|---|---|---|---|---|
| Weight (kg) | 3.61 | 3.659517 (1.4%) | 3.708476 (2.7%) | 3.756793 (4.1%) | 3.8057736 (5.4%) | 3.8548031 (6.8%) | $10^{-1}$ |
| | 3.958 | 4.022237 (1.6%) | 4.086791 (3.3%) | 4.15093 (4.9%) | 4.215584 (6.5%) | 4.28048 (8.2%) | $10^{-2}$ |
| | 4.252 | 4.329052 (1.8%) | 4.406535 (3.6%) | 4.484301 (5.5%) | 4.56195 (7.3%) | 4.6395682 (9.1%) | $10^{-3}$ |

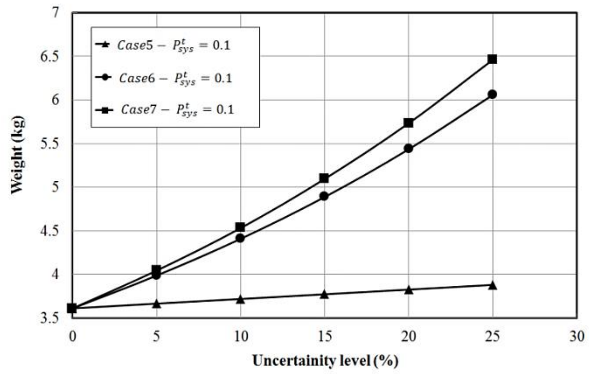



Fig. 10 effect of the uncertainty levels on the optimum weight of the 6-bar truss for cases 5-7 for $P_{sys}^{t} = 0.1$

Table 16 and Fig. 10 indicate the RBDO results for cases 5-7. In order to investigate the effect of the different class of interval parameters on run time of the optimization process, for each case, average run time for each generation is also listed in Table16. It can be seen that RBDO under the interval translation parameters (case 6) has the least processing time because of the benefit of using simple monotonicity analysis related to these parameters. In contrast to the interval rotation parameters, we only perform one reliability analysis for all the interval translation parameters. While for each combination of the bounds of the interval rotation parameters, a reliability analysis should be performed.

By comparing the first and second rows of Table 16, it can be observed that the interval rotation parameters need more computational time. Also, from the first and third rows, there is no change on run time process when interval translation parameters are considered in addition to the interval rotation ones. For $P_{sys}^{t} = 0.1$, effect of the different uncertainty levels on the optimum weight is shown in Fig. 10, which is similar to those of cases 1-4.

#### *4.4.2 Statically determinate 13-member truss structure*

The weight optimization of statically determinate 13-

Table 16 System reliability based design optimization of the 6-bar truss for cases 5-7 with $P_{sys}^t = 0.1$

| Uncertainty level (%) | 0 | 5 | 10 | 15 | 25 | 25 | Average run time for each generation (sec) | Case # |
|---|---|---|---|---|---|---|---|---|
| Weight (kg) | 3.61 | 3.6647159 (1.5%) | 3.718006 (3.0%) | 3.771944 (4.5%) | 3.8256165 (6.0%) | 3.8795249 (7.5%) | 25.52 | 5 |
| | 3.61 | 3.9866509 (10.4%) | 4.407838 (22.1%) | 4.8876867 (35.4%) | 5.4351523 (50.6%) | 6.0582078 (67.8%) | 8.721 | 6 |
| | 3.61 | 4.0439586 (12.0%) | 4.5308193 (25.5%) | 5.0953232 (41.1%) | 5.73375 (58.8%) | 6.461247 (79.0%) | 25.501 | 7 |

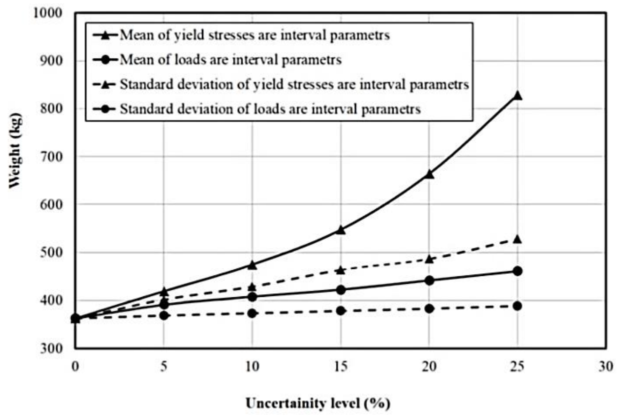



Fig. 11 effect of the uncertainty levels of mean values on the optimum weight of the 13-bar truss considering mean values as interval parameters

member truss, shown in Fig. 4, is investigated for $P_{sys}^t = 1 \times 10^{-5}$ under different uncertainty levels. Mean and standard deviation of the external loads and the yield stress of the members are considered separately as interval distribution parameters. The interval parameters are defined using Eq. (31) and the distribution parameters listed in Table 3.

The results for different uncertainty levels are presented in Tables 17-18 and Fig. 11. It can be found that for all considered interval parameters the optimum weight of the structure behaves an increasing trend as the uncertainty level increases.

Similar to the previous example, for a certain random variable (i.e., yield stress of the members or applied loads), the optimum weight is more influenced by the translation interval parameters than the rotation ones. But in this example, as can be observed from Fig. 11 and the percentage values in Tables 17-19, the impact of the standard deviation of the yield stress on the optimum weight is more than the mean of the loads. This can be due to the difference between the number of interval random variables related to the yield stress (i.e., number of the members) and the number of those related to the loads, which is much higher in this example.

### *4.4.3 Statically indeterminate 16-member truss structure*

Consider a statically indeterminate 16-member truss with three degree of redundancy shown in Fig. 12. The Young's Modulus E, the density $\rho$, and the lengths $L_1$ and $L_2$ are respectively 206 GPa, $2700\ \mathrm{kg/m^3}$, $121.9\ cm$

Table 17 system reliability based design optimization for the 13-bar truss with interval translation parameters for $\mathrm{P_{sys}^t} = 1 \times 10^{-5}$

| Interval parameter | Mean of yield stress | | | | | | Mean of Loads | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Uncertainty Level (%) | 0 | 5 | 10 | 15 | 20 | 25 | 5 | 10 | 15 | 20 | 25 |
| $\mathbf{A_1 = A_{12}}$ | 7.521 | 8.244 | 8.702 | 10.109 | 13.071 | 17.162 | 8.384 | 8.327 | 8.368 | 10.086 | 9.962 |
| $\mathbf{A_2 = A_{13}}$ | 11.703 | 12.560 | 15.055 | 17.245 | 21.949 | 25.762 | 12.095 | 12.968 | 12.975 | 13.943 | 16.155 |
| $\mathbf{A_3 = A_{11}}$ | 7.410 | 9.415 | 10.610 | 11.529 | 12.166 | 16.577 | 7.331 | 8.231 | 9.053 | 8.380 | 8.687 |
| $\mathbf{A_4 = A_8}$ | 7.594 | 9.354 | 9.812 | 11.718 | 15.809 | 18.516 | 8.866 | 8.626 | 8.439 | 8.983 | 8.708 |
| $\mathbf{A_5 = A_9}$ | 2.356 | 2.659 | 3.147 | 3.221 | 3.500 | 4.768 | 2.568 | 3.635 | 3.008 | 3.441 | 3.690 |
| $\mathbf{A_6 = A_{10}}$ | 8.403 | 9.672 | 10.336 | 14.534 | 15.892 | 19.966 | 9.556 | 8.713 | 10.973 | 9.707 | 9.429 |
| $\mathbf{A_7}$ | 5.000 | 6.419 | 7.809 | 7.501 | 10.150 | 13.085 | 5.803 | 5.234 | 5.474 | 6.600 | 5.827 |
| Weight (kg) | 362.305 | 419.080 (15.7%) | 474.644 (31.0%) | 547.398 (51.1%) | 663.999 (83.27%) | 828.545 (128.7%) | 390.862 (7.9%) | 407.858 (12.6%) | 422.335 (16.6%) | 441.773 (21.9%) | 460.925 (27.2%) |

Table 18 System reliability based design optimization for the 13-bar truss with interval rotation parameters for $\mathrm{P_{sys}^t} = 1 \times 10^{-5}$

| Interval parameter | Standard deviation of yield stress | | | | | | Standard deviation of Loads | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Uncertainty Level (%) | 0 | 5 | 10 | 15 | 20 | 25 | 5 | 10 | 15 | 20 | 25 |
| $\mathbf{A_1 = A_{12}}$ | 7.521 | 7.682 | 8.152 | 10.615 | 10.158 | 11.268 | 7.974 | 8.200 | 7.682 | 7.276 | 8.945 |
| $\mathbf{A_2 = A_{13}}$ | 11.703 | 13.000 | 13.207 | 14.159 | 14.562 | 16.528 | 12.254 | 11.799 | 12.047 | 12.928 | 12.217 |
| $\mathbf{A_3 = A_{11}}$ | 7.410 | 7.814 | 8.820 | 9.666 | 10.540 | 10.868 | 7.433 | 7.666 | 8.000 | 8.017 | 7.619 |
| $\mathbf{A_4 = A_8}$ | 7.594 | 8.815 | 9.709 | 10.423 | 10.695 | 10.680 | 7.103 | 7.407 | 7.114 | 7.532 | 8.749 |
| $\mathbf{A_5 = A_9}$ | 2.356 | 2.274 | 2.488 | 2.633 | 3.379 | 2.921 | 2.739 | 2.491 | 2.690 | 3.185 | 2.601 |
| $\mathbf{A_6 = A_{10}}$ | 8.403 | 10.441 | 9.189 | 9.922 | 11.058 | 11.802 | 8.131 | 8.195 | 8.590 | 7.947 | 8.770 |
| $\mathbf{A_7}$ | 5.000 | 5.724 | 8.655 | 7.525 | 7.212 | 9.896 | 4.690 | 5.977 | 5.963 | 4.924 | 4.956 |
| Weight (kg) | 362.305 | 401.485 (10.8%) | 428.700 (18.3%) | 463.537 (27.9%) | 486.281 (34.2%) | 528.060 (45.7%) | 368.381 (1.7%) | 373.107 (3.0%) | 377.923 (4.3%) | 382.767 (5.6%) | 388.346 (7.1%) |

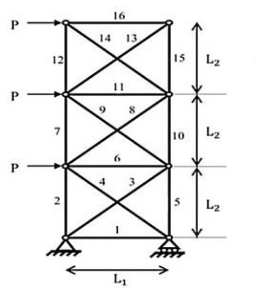



Fig. 12 statically indeterminate 16-member truss structure

and $91.44\ cm$. According to the distribution parameters of random variables presented in Table 19, the size optimization of the truss structure is performed for $P_{sys}^t = 1 \times 10^{-5}$. The cross-sectional areas are continues design variables taken from [0, 15].

The system RBDO is performed with, and without, considering interval distribution parameters for different uncertainty levels. The interval parameters are described using Eq. (31) and the distribution parameters presented in Table 19. Note that for $\alpha = 0$ in Eq. (31), the distribution parameters of the random variables have deterministic

Table 19 random variables for statically indeterminate 16-member truss structure

| Variable | Description | Distribution | $\mu$ | Coefficient of variation |
|---|---|---|---|---|
| **P** | Load (KN) | Normal | 44.45 | 0.1 |
| $\boldsymbol{\sigma_{yield}}$ | Yield stress ($\mathrm{KN/cm^2}$) | Normal | 27.60 | 0.1 |

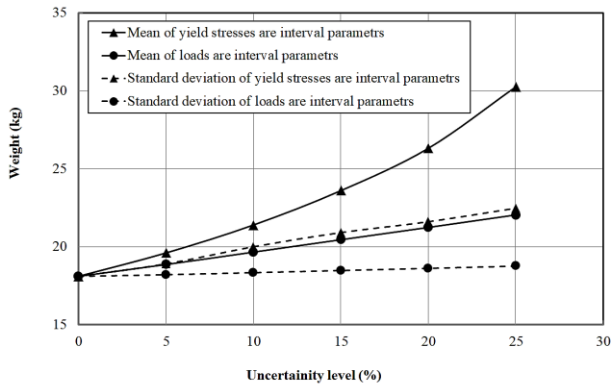



Fig. 13 effect of the uncertainty levels of mean values on the optimum weight of the 16-bar truss considering mean values as interval parameters

Table 20 System reliability based design optimization for the 16-bar truss with interval translation parameters for $\mathrm{P_{sys}^{t}} = 1 \times 10^{-5}$

| Interval parameter | Mean of yield stress | | | | | | Mean of Loads | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Uncertainty Level (%) | 0 | 5 | 10 | 15 | 20 | 25 | 5 | 10 | 15 | 20 | 25 |
| $A_1$ | 3.525 | 3.218 | 4.003 | 4.252 | 4.933 | 6.775 | 3.273 | 3.378 | 3.627 | 4.087 | 4.261 |
| $A_2$ | 9.461 | 9.523 | 10.994 | 11.992 | 13.535 | 14.999 | 9.366 | 9.698 | 10.201 | 10.897 | 11.355 |
| $A_3$ | 5.606 | 6.880 | 6.742 | 7.604 | 8.099 | 8.424 | 6.361 | 6.708 | 6.813 | 6.652 | 6.856 |
| $A_4$ | 4.305 | 3.899 | 4.851 | 5.168 | 6.044 | 8.037 | 3.996 | 4.107 | 4.452 | 4.974 | 5.221 |
| $A_5$ | 9.983 | 11.587 | 12.185 | 13.621 | 14.974 | 14.999 | 10.911 | 11.452 | 11.754 | 11.937 | 12.356 |
| $A_6$ | 0.979 | 1.593 | 1.239 | 1.526 | 1.866 | 2.638 | 1.424 | 1.510 | 1.590 | 1.252 | 1.345 |
| $A_7$ | 5.273 | 5.715 | 6.313 | 6.996 | 7.643 | 8.282 | 5.460 | 5.702 | 5.810 | 6.102 | 6.300 |
| $A_8$ | 2.280 | 2.381 | 2.645 | 2.895 | 3.441 | 4.571 | 2.335 | 2.403 | 2.629 | 2.725 | 2.832 |
| $A_9$ | 4.700 | 5.186 | 5.511 | 6.098 | 6.355 | 6.682 | 4.920 | 5.124 | 5.213 | 5.426 | 5.571 |
| $A_{10}$ | 3.668 | 3.880 | 4.225 | 4.666 | 5.431 | 6.503 | 3.769 | 3.882 | 4.124 | 4.211 | 4.366 |
| $A_{11}$ | 0.657 | 0.757 | 0.951 | 1.043 | 1.116 | 1.658 | 0.684 | 0.796 | 0.761 | 0.980 | 1.047 |
| $A_{12}$ | 1.797 | 1.918 | 2.140 | 2.316 | 2.703 | 3.448 | 1.842 | 1.952 | 2.029 | 2.200 | 2.295 |
| $A_{13}$ | 0.993 | 1.105 | 1.159 | 1.322 | 1.382 | 1.271 | 1.074 | 1.080 | 1.094 | 1.042 | 1.043 |
| $A_{14}$ | 2.833 | 3.014 | 3.404 | 3.676 | 4.251 | 5.372 | 2.920 | 3.082 | 3.240 | 3.456 | 3.637 |
| $A_{15}$ | 0.653 | 0.711 | 0.772 | 0.866 | 0.923 | 0.861 | 0.697 | 0.699 | 0.694 | 0.673 | 0.674 |
| $A_{16}$ | 0.827 | 0.925 | 0.961 | 1.091 | 1.170 | 1.092 | 0.892 | 0.880 | 0.907 | 0.867 | 0.862 |
| Weight (kg) | 18.108 | 19.610 (8.3%) | 21.400 (18.2%) | 23.604 (30.3%) | 26.318 (45.3%) | 30.259 (67.2%) | 18.866 (4.2%) | 19.662 (8.6%) | 20.457 (13.0%) | 21.246 (17.3%) | 22.045 (21.7%) |

values. For different uncertainty levels, the optimum weight and cross sectional areas of the structure are presented in Tables 20-21. Also, the change of the optimum weight with respect to changes in uncertainty level is shown in Fig. 13.

It can be observed that the optimum weight of the truss is increased by increasing the uncertainty level for each interval parameter. Furthermore, like the previous example, the optimum weight is affected most by interval distribution parameters of the yield stresses. As can be seen in Tables 20-22, when the uncertainty level increase to a large value 25%, the optimum weight is increased 67.2% and 24.1% for mean and standard deviation of the yield stresses, respectively. Whereas, for interval distribution parameters of the loads, the maximum increase in optimum weight is equal to 3.6% and 21.7% for standard deviation and mean of the loads, respectively.

Table 21 System reliability based design optimization for the 16-bar truss with interval rotation parameters for $\mathrm{P_{sys}^{t}} = 1 \times 10^{-5}$

| Interval parameter | Standard deviation of yield stress | | | | | | Standard deviation of Loads | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Uncertainty Level (%) | 0 | 5 | 10 | 15 | 20 | 25 | 5 | 10 | 15 | 20 | 25 |
| $A_1$ | 3.525 | 3.460 | 4.081 | 5.060 | 5.210 | 4.597 | 3.070 | 3.189 | 3.152 | 3.214 | 3.226 |
| $A_2$ | 9.461 | 9.550 | 9.771 | 10.013 | 11.613 | 12.224 | 8.926 | 9.111 | 9.095 | 9.182 | 9.234 |
| $A_3$ | 5.606 | 6.182 | 6.481 | 6.652 | 5.655 | 5.578 | 6.254 | 6.141 | 6.269 | 6.277 | 6.276 |
| $A_4$ | 4.305 | 4.048 | 4.269 | 4.170 | 5.652 | 6.455 | 3.703 | 3.898 | 3.812 | 3.889 | 3.940 |
| $A_5$ | 9.983 | 10.708 | 11.006 | 11.272 | 11.031 | 11.940 | 10.625 | 10.564 | 10.726 | 10.732 | 10.788 |
| $A_6$ | 0.979 | 1.188 | 1.814 | 1.761 | 0.862 | 0.408 | 1.429 | 1.387 | 1.412 | 1.385 | 1.382 |
| $A_7$ | 5.273 | 5.712 | 5.113 | 6.102 | 5.490 | 6.331 | 5.272 | 5.235 | 5.341 | 5.383 | 5.423 |
| $A_8$ | 2.280 | 2.556 | 3.109 | 3.422 | 2.580 | 2.954 | 2.263 | 2.332 | 2.292 | 2.318 | 2.325 |
| $A_9$ | 4.700 | 4.876 | 4.161 | 4.441 | 5.283 | 4.867 | 4.778 | 4.700 | 4.831 | 4.835 | 4.903 |
| $A_{10}$ | 3.668 | 3.556 | 4.627 | 4.587 | 4.605 | 4.762 | 3.661 | 3.740 | 3.747 | 3.792 | 3.813 |
| $A_{11}$ | 0.657 | 0.527 | 0.810 | 0.769 | 0.608 | 0.658 | 0.683 | 0.678 | 0.690 | 0.705 | 0.739 |
| $A_{12}$ | 1.797 | 1.952 | 1.837 | 2.628 | 2.118 | 2.732 | 1.786 | 1.868 | 1.814 | 1.864 | 1.895 |
| $A_{13}$ | 0.993 | 1.281 | 1.665 | 1.179 | 1.294 | 1.973 | 1.033 | 0.992 | 1.073 | 1.054 | 1.065 |
| $A_{14}$ | 2.833 | 2.634 | 2.600 | 3.057 | 3.139 | 2.984 | 2.837 | 2.961 | 2.873 | 2.957 | 3.001 |
| $A_{15}$ | 0.653 | 0.886 | 0.765 | 0.811 | 2.299 | 1.699 | 0.652 | 0.650 | 0.690 | 0.682 | 0.695 |
| $A_{16}$ | 0.827 | 1.012 | 1.372 | 0.738 | 1.597 | 1.848 | 0.867 | 0.818 | 0.891 | 0.870 | 0.880 |
| Weight (kg) | 18.108 | 18.904 (4.4%) | 20.005 (10.5%) | 20.915 (15.5%) | 21.609 (19.3%) | 22.480 (24.1%) | 18.211 (0.5%) | 18.343 (1.3%) | 18.480 (2.1%) | 18.618 (2.8%) | 18.762 (3.6%) |

## 5. Conclusions

Traditional system reliability based design optimization (RBDO) of truss structures is generally focused on describing uncertainty by probability approach that requires a large amount of information to determine precise distributions of random variables. In this study, the second hybrid uncertain model is applied to the system RBDO of trusses. All random variables are described by random distributions but some key distribution parameters of them which lack information are defined by variation intervals. For all the considered test problems, the effect of the uncertainty level and different classes of the interval distribution parameter on the optimum weight of the structure is investigated. The results show that by increasing the uncertainty level, the lower bound of the hybrid reliability index for each member of the structure is decreased, which causes an increase in the system failure probability. As a consequence, to have a reliable design, the cross-sectional area of the members and thus the weight of the structure will increase. Moreover, it has shown that for a certain random variable, the optimum weight is more increased by the translation interval parameters than the rotation ones. In our future work, we intend to consider the first hybrid uncertain model to system RBDO problems of truss structures.

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