Structural Engineering and Mechanics, *Vol. 70, No. 1 (2019) 55-66*
DOI: https://doi.org/10.12989/sem.2019.70.1.055

Wave propagation of functionally graded anisotropic nanoplates resting on Winkler-Pasternak foundation

Behrouz Karami*[1], Maziar Janghorban[1a] and Abdelouahed Tounsi[2b]

[1]*Department of Mechanical Engineering, Marvdasht Branch, Islamic Azad University, Marvdasht, Iran*
[2]*Material and Hydrology Laboratory, University of Sidi Bel Abbes, Faculty of Technology, Civil Engineering Department, Algeria*

(Received June 15, 2018, Revised February 11, 2018, Accepted February 12, 2019)

Abstract. This work deals with the size-dependent wave propagation analysis of functionally graded (FG) anisotropic nanoplates based on a nonlocal strain gradient refined plate model. The present model incorporates two scale coefficients to examine wave dispersion relations more accurately. Material properties of FG anisotropic nanoplates are exponentially varying in the z-direction. In order to solve the governing equations for bulk waves, an analytical method is performed and wave frequencies and phase velocities are obtained as a function of wave number. The influences of several important parameters such as material graduation exponent, geometry, Winkler-Pasternak foundation parameters and wave number on the wave propagation of FG anisotropic nanoplates resting on the elastic foundation are investigated and discussed in detail. It is concluded that these parameters play significant roles on the wave propagation behavior of the nanoplates. From the best knowledge of authors, it is the first time that FG nanoplate made of anisotropic materials is investigated, so, presented numerical results can serve as benchmarks for future analysis of such structures.

Keywords: wave propagation; functionally graded anisotropic materials; nonlocal strain gradient theory; four variable shear deformation refined plate theory; elastic foundation

1. Introduction

Classical plate theory CPT as the simplest plate theory, cannot model thick/moderately thin plates accurately because of several assumptions for simplifications used in this theory (Lagnese *et al.* 2015, Farokhi and Ghayesh 2015). On the other hand, first order plate theory (FSDT) with less simplifications with two main unknowns give us better results for thick/moderately thin plates although the results still have some inaccuracy. Besides, this theory also needs shear correction factor. For solving these two problems, several higher order shear deformation theories (HSDTs) have been developed. The problem with these HSDTs is the complexity of them. Some of them have more than six main unknowns which cause several long equations. In this research, we choose one of the theories which has some of the advantages and disadvantages mentioned above. This theory, known as refined plate theory (RPT), does not need any shear correction factor and it is simple (i.e. with only two unknowns) with results more accurate than CPT which can be a candidate for engineering applications, (see some applications of HSDTs and RPTs in Refs. (Shimpi and Patel 2006, Mehar *et al.* 2018b, Katariya *et al.* 2017a, Dutta *et al.* 2017, Sahoo *et al.* 2017, Singh and Panda 2017, Mehar *et al.* 2017, Mahapatra *et al.* 2017, Mehar and Panda 2017a, Mehar *et al.* 2018c, Hirwani and Panda 2019, Hirwani and Panda 2018, Dash *et al.* 2018b, Mehar *et al.* 2018a, Hirwani *et al.* 2018, Mehar and Panda 2018, Katariya *et al.* 2018, Katariya *et al.* 2017b, Dash *et al.* 2018a, Mehar and Panda 2017b, Sayyad and Ghugal 2018).

In the past decades, with the development of nanotechnology, we need to investigate the nanostructures that old models such as CPT, HSDTs and RPTs, etc. cannot analyzed the mechanical behavior of such structures accurately. Hence, to consider the size-dependent effect, nanostructures usually modelled based on the molecular dynamics (MD) simulation and non-classical theories. Eringen nonlocal model (Eringen 1983) is one of the non-classical theories in which stress at a reference point are assumed to depend not only on the strains at the reference point but also on the strains at all other points in the body. As an example of the merits of this theory, one can mention the non-monotonicity of dispersion between phase velocity and wave number predicted by MD simulation. Several studies have been done based on this size-dependent model (Karami *et al.* 2018f, Karami *et al.* 2019b, Shahsavari *et al.* 2018a, Shahsavari *et al.* 2017, Shahsavari and Janghorban 2017, Apuzzo *et al.* 2017, Barretta *et al.* 2018, Romano and Barretta 2017, Romano *et al.* 2017, Shahsavari *et al.* 2018e, Karami *et al.* 2018l). Free vibration analysis of FG nanoplates resting on elastic foundation using a nonlocal zeroth-order shear deformation theory were studied by

*Corresponding author, Ph.D. Student
E-mail: behrouz.karami@miau.ac.ir
[a]Ph.D.
E-mail: maziar.janghorban@miau.ac.ir
[b]Professor
E-mail: tou_abdel@yahoo.com

http://www.techno-press.com/journals/sem&subpage=7 ISSN: 1225-4568 (Print), 1598-6217 (Online)

(Bounouara *et al.* 2016). (Ebrahimi and Barati 2016) investigated the free vibration analysis of FG nanobeams third-order shear deformation plate theory and nonlocal elasticity theory. Bending analysis of bi-directional FG Euler-Bernoulli nano-beams using Eringen's non-local elasticity theory were presented by (Nejad and Hadi 2016). Based on the first-order plate theory in conjunction with nonlocal elasticity theory, guided wave propagation analysis of fully-clamped porous FG nanoplates were studied by (Karami *et al.* 2018a) for the first time. As can be seen, there are numerous applications on the mechanical analysis of nanostructures based on Eringen nonlocal model (Moradweysi *et al.* 2018, Ehyaei *et al.* 2017, Khetir *et al.* 2017, Chaht *et al.* 2015, Simsek 2011, Rahmani *et al.* 2017, Mirjavadi *et al.* 2017, Bellifa *et al.* 2017, Sobhy 2017). Nevertheless, nonlocal elasticity theory cannot always predict the manner of materials at nanoscale well, and therefore the sufficiency of nonlocal elasticity theory for predicting physical behavior is questionable for some special cases. Among the examples that can be mentioned there are may be some limitations to its capability of identifying size-dependent stiffness (Ma *et al.* 2008, Lim *et al.* 2015). Moreover, still, the problem remains why, in some cases, the results of bending of nonlocal nanobeams are ineffective at all (Challamel and Wang 2008). Besides, it can be seen that the stiffness enhancement effects seen from experimental data and the strain gradient model (Lam *et al.* 2003) cannot be included well by adopting the Eringen's nonlocal elasticity theory. It is worth noting that in other size-dependent studies, we observe the importance of non-classical boundary conditions, but in this theory we see that in some cases the same classical boundary conditions suffice, which is a question for some researchers (Volokh and Hutchinson 2002, Polizzotto 2003). Another one is the gradient model of elasticity (Mindlin 1964, Aifantis 1992) which considers additional higher-order strain gradient terms by adopting the fact that the materials should be modelled as atoms with higher-order deformation mechanism at nanoscale rather than set of points. A modified gradient elasticity theory, which states the strain energy density should be considered as a function of both the strain tensor conjugated with stress tensor and the curvature tensor conjugated with couple stress tensor were presented by (Yang *et al.* 2002). Initial stress affected wave propagation behavior of nanoplates were investigated by (Nami and Janghorban 2014) using gradient elasticity theory. (Karami and Janghorban 2016) investigated the wave propagation in rectangular nanoplates via gradient elasticity theory and two-variable RPT.

With regard to the above literature, it is easy to conclude that the nonlocal and gradient models of elasticity explain two entirely different physical characteristics of materials at small scale. More recently, to study the influence of the two small scale parameters on the structural responses, (Lim *et al.* 2015) presented the nonlocal strain gradient theory with the approach of the thermodynamics framework to consider both of the nonlocal and gradient parameters into a single theory. Many researches have been carried out on the mentioned theory (She *et al.* 2018c, She *et al.* 2018a, Karami *et al.* 2018d, Barati 2017, Zhu and Li 2017, Xiao *et al.* 2017, She *et al.* 2019, She *et al.* 2017, Shafiei and She 2018, She *et al.* 2018b, Karami *et al.* 2018e, Shahsavari *et al.* 2018d, Karami *et al.* 2018j, Karami *et al.* 2018g, Karami *et al.* 2018k, Karami *et al.* 2018b, Karami *et al.* 2018i, Shahsavari *et al.* 2018b, Shahsavari *et al.* 2018c, Karami *et al.* 2019a, Karami *et al.* 2018c, Ansari *et al.* 2011, Sahmani *et al.* 2018b, Sahmani *et al.* 2018a, Karami and Karami 2019, Nami and Janghorban 2015, Karami and Janghorban 2019). Li *et al*. (2016), reported that a good matching of the dispersive curve of molecular dynamic simulations can be obtained with nonlocal beams and illustrated that the stiffness softening effects or the stiffness enhancement effects can be obtained depend on the values of the nonlocal parameter and the strain gradient length scale parameter. Also, in another study a good agreement is achieved for wave dispersion curves of graphene and experimental data using nonlocal strain gradient second-order plate theory by (Karami *et al.* 2018h). Based on the nonlocal strain gradient theory, (Li and Hu 2015) investigated the stability analysis of nonlinear nanobeams and showed that the stiffness softening effects or the stiffness enhancement effects can be obtained depend on the values of the nonlocal parameter and the strain gradient length scale parameter. The nonlocal strain gradient theory can be considered among other theories as one of the most accurate modeling for the study of nanostructures with considering both nonlocal and gradient effects.

FGMs have received a considerable attention in several engineering applications due to its wonderful properties in different environments. This type of materials is characterized by its properties that vary continuously or exponentially through a direction like thickness. Generally, FG structures are made of isotropic or anisotropic materials in which to manufacturing FG anisotropic materials, transport phenomena are used to create compositional and microstructural gradients during the production of a component.

A few number of studies have been conducted on the static and dynamic behavior of shell and plates in micro/nano-scale made of different anisotropic materials (i.e., anisotropic and FG anisotropic). Vibrational behavior of three-dimensional anisotropic layered composite nanoplates using modified couple-stress theory were studied by (Guo *et al.* 2017). (Sahmani and Fattahi 2017) investigated the nonlocal anisotropic shear deformable plate model for uniaxial instability of 3D metallic carbon nanosheets. In their study a calibration is presented for nonlocal anisotropic shear deformable plate using molecular dynamic simulation. The wave propagation analysis of anisotropic plate using trigonometric shear deformation theory were studied by (Aminipour and Janghorban 2017). (Karami *et al.* 2017) presented three dimensional nonlocal strain gradient plate model for wave propagation behavior of anisotropic nanoplates under the influences of triaxial magnetic field. (Aminipour *et al.* 2018) investigated the wave propagation in FG anisotropic shells based on a new model for wave analysis.

In the current study, due to the lack of any study on the mechanics of graded nanoplates made of anisotropic materials, we prepared an article to investigate the wave

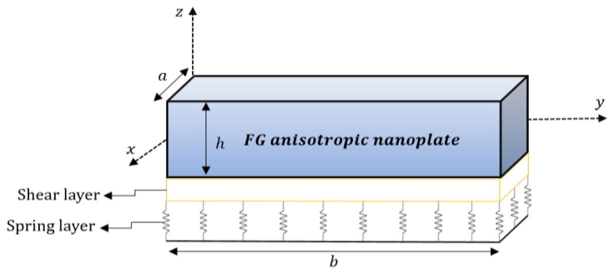


Fig. 1 Geometry of FG anisotropic nanoplate rested on an elastic foundation

propagation analysis of FG anisotropic nanoplate. Four-variable RPT without any shear correction factor is used to model the plate as a continuum model. For more accurate design, the proposed modeling of nanoplates incorporates a nonlocal stress field parameter as well as a length scale parameter related to strain gradient. Thus, stiffness enhancement or reduction observed in nanostructures are considered. Governing equations are obtained using Hamilton's principle. By solving the governing equations using an analytical method, wave frequencies as well as phase velocities of the FG anisotropic nanoplate are obtained as a function of wave number. The results show that wave characteristics of FG anisotropic nanoplate is significantly influenced by the nonlocality, strain gradient parameter, material composition, Winkler-Pasternak foundation and geometrical parameters. Obtained results can be used as benchmark results in the analysis of FG anisotropic nanoplates modeled by nonlocal and strain-gradient theories.

2. Theoretical formulation

2.1 The material properties of FG anisotropic elastic nanoplates

Rectangular nanosize plate made of FG anisotropic materials is considered here (Fig. 1). A Cartesian coordinate system (x, y, z) is assumed; a and b are length and width of plate and h is the plate thickness.

The constitutive relation for each material can be written as

$$\sigma = C\varepsilon \tag{1}$$

where σ and ε respectively, denote stress and strain terms and C denotes the elastic constant that is different in various structures. For FGMs with exponentially varying material properties in the z-direction, the elastic stiffness matrix in Eq. (1) can be expressed as (Pan 2003, Karami *et al.* 2018c)

$$C(z) = C^0 \exp(\lambda z) \tag{2}$$

where λ is the exponential factor defined the material gradient's degree in the thickness direction. It is also mentioned that the exponential factor λ has the dimension $1/L$, in which L defined as the characteristic length of the problem. In the current study, to investigate the wave propagation of graded nano-size plates, a hexagonal system (beryllium crystal) is used. It has an axis of symmetry such that a rotation of the crystal through 60° about that axis brings the space lattice into coincidence with its original configuration (Batra *et al.* 2004). The elastic constants are,

$$C^0 = \begin{bmatrix} 298.2 & 27.7 & 0 & 0 & 0 \\ & 298.2 & 0 & 0 & 0 \\ & & 165.5 & 0 & 0 \\ & \text{sym.} & & 165.5 & 0 \\ & & & & 135.3 \end{bmatrix} \times 10^9\,\text{Pa} \tag{3}$$

and the density of beryllium equals to $\rho = 1850\dfrac{\text{kg}}{\text{m}^3}$.

2.2 Nonlocal strain gradient elasticity

The total nonlocal strain gradient stress tensor can be expressed as below (Lim *et al.* 2015),

$$\sigma_{ij} = \sigma_{ij}^{(0)} - \nabla\partial\sigma_{ij}^{(1)} \tag{4}$$

in which $\sigma_{ij}^{(0)}, \sigma_{ij}^{(1)}$ in order are related to strain ε_{ij} and strain gradient $\nabla\varepsilon_{ij}$ and are defined as

$$\sigma_{ij}^{(0)} = \int_0^L C_{ijkl}\alpha_0(x,x',e_0a)\varepsilon'_{kl}(x')dx' \tag{5}$$

$$\sigma_{ij}^{(1)} = l^2\int_0^L C_{ijkl}\alpha_1(x,x',e_1a)\nabla\varepsilon'_{kl}(x')dx' \tag{6}$$

where C_{ijkl} are the elastic constants and e_0a and e_1a consider the influence of nonlocal stress field and l is the strain gradient small scale parameter which defines the effects of higher order strain gradient stress field. As the conditions by Eringen are satisfied by the nonlocal functions $\alpha_0(x,x',e_0a)$ and $\alpha_0(x,x',e_1a)$, the constitutive equation of nonlocal strain gradient theory can be written in following format

$$\begin{aligned} &\left[1-(e_1a)^2\nabla^2\right]\left[1-(e_0a)^2\nabla^2\right]\sigma_{ij} \\ &= C_{ijkl}\left[1-(e_1a)^2\nabla^2\right]\varepsilon_{kl} - C_{ijkl}l^2\left[1-(e_0a)^2\nabla^2\right]\nabla^2\varepsilon_{kl} \end{aligned} \tag{7}$$

in which ∇^2 is represents Laplacian operator in Cartesian coordinates. Supposing $e_1 = e_0 = e$ and discarding terms of order $O(\nabla^2)$, the general constitutive relation in Eq. (7) can be rewritten as (Lim *et al.* 2015)

$$\left[1-(ea)^2\nabla^2\right]\sigma_{ij} = C_{ijkl}\left[1-l^2\nabla^2\right]\varepsilon_{kl} \tag{8}$$

The equivalent form of Eq. (8) is presented as

$$\mathrm{L}_\mu\sigma_{ij} = C_{ijkl}\mathrm{L}_l\varepsilon_{kl} \tag{9}$$

in which the linear operators are defined as

$$\mathrm{L}_\mu = (1-\mu^2\nabla^2), \mathrm{L}_l = (1-l^2\nabla^2) \tag{10}$$

where $\mu = ea$.

2.3 Refined plate theory (RPT)

According to the plate theories, the refined model initially proposed by (Shimpi 2002) is widely used as a reliable theory in which no shear correction factor is required. İn Shimpi's theory, the displacement field, for $z \in [-h/2; h/2]$, is defined as

$$u(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \tag{11}$$

$$v(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w_b}{\partial y} - f(z)\frac{\partial w_s}{\partial y} \tag{12}$$

$$w(x,y,z,t) = w_b(x,y,t) + w_s(x,y,t) \tag{13}$$

where u_0, v_0, w_b and w_s reperesent, respectivly, in-plane displacments andthe bending and shear transverse displacements. The shape function of transverse shear deformation is considered as

$$f(z) = -\frac{z}{4} + \frac{5z^3}{3h^2} \tag{14}$$

Nonzero strains according to the four-variable RPT can be written as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} K_x^b \\ K_y^b \\ K_{xy}^b \end{Bmatrix} + f\begin{Bmatrix} K_x^s \\ K_y^s \\ K_{xy}^s \end{Bmatrix}$$
$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g\begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix},\; g = 1 - \frac{\partial f}{\partial z} \tag{15}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix},\; \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\frac{\partial^2 w_b}{\partial x^2} \\ -\frac{\partial^2 w_b}{\partial y^2} \\ -2\frac{\partial^2 w_b}{\partial x \partial y} \end{Bmatrix}$$
$$\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} -\frac{\partial^2 w_s}{\partial x^2} \\ -\frac{\partial^2 w_s}{\partial y^2} \\ -2\frac{\partial^2 w_s}{\partial x \partial y} \end{Bmatrix},\; \begin{Bmatrix} \gamma_{xz}^s \\ \gamma_{yz}^s \end{Bmatrix} = \begin{Bmatrix} \frac{\partial w_s}{\partial x} \\ \frac{\partial w_s}{\partial y} \end{Bmatrix} \tag{16}$$

Also, the extended Hamilton principle introduces as

$$\int_0^t \delta(U - T + V)dt = 0 \tag{17}$$

where U is strain energy, T is kinetic energy and V is work of external (applied) forces. With substituting the corresponding terms, the first variation of strain energy can be written as

$$\begin{aligned} \delta U &= \int_V \left[\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \tau_{yz}\varepsilon_{yz} + \tau_{xz}\varepsilon_{xz} + \tau_{xy}\varepsilon_{xy}\right]dV \\ &= \int_0^L \left[N_x \delta\varepsilon_x^0 + N_y \delta\varepsilon_y^0 + N_{xy}\delta\gamma_{xy}^0 + M_x^b \delta k_x^b + M_y^b \delta k_y^b\right. \\ &+ M_{xy}^b \delta k_{xy}^b + M_x^s \delta k_x^s + M_y^s \delta k_y^s + M_{xy}^s \delta k_{xy}^s + Q_{yz}^s \gamma_{yz}^0 \\ &\left.+ Q_{xz}^s \gamma_{xz}^0\right]dx = 0 \end{aligned} \tag{18}$$

where N, M, and Q are the stress resultants which are defined as

$$\begin{Bmatrix} \{N\} \\ \{M^b\} \\ \{M^s\} \end{Bmatrix} = \begin{bmatrix} [A] & [B] & [B^s] \\ [B] & [D] & [D^s] \\ [B^s] & [D^s] & [H^s] \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ k^b \\ k^s \end{Bmatrix}$$
$$\begin{Bmatrix} \{Q_{xz}\} \\ \{Q_{yz}\} \end{Bmatrix} = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix} \begin{Bmatrix} \gamma_{xz}^s \\ \gamma_{Yz}^s \end{Bmatrix} \tag{19}$$

in which

$$(A, B, B^s, D, D^s, H^s) = \int_{-\frac{h}{2}}^{\frac{h}{2}} C_{ij}(1, z, f, z^2, zf, f^2)dz \tag{20}$$

and

$$(A_{44}^s, A_{55}^s) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (C_{44}, C_{55})g^2 dz \tag{21}$$

The first variation of work done by applied forces can be stated as

$$\begin{aligned} \delta V &= \int_{-h/2}^{h/2}\int_A \left[-k_W \delta(w_b + w_s) + k_P\left(\frac{\partial(w_b + w_s)}{\partial x}\frac{\partial\delta(w_b + w_s)}{\partial x}\right.\right. \\ &\left.\left. + \frac{\partial(w_b + w_s)}{\partial y}\frac{\partial\delta(w_b + w_s)}{\partial y}\right)\right]dA\,dz \end{aligned} \tag{22}$$

where k_W and k_P represent, respectively, the linear and shear coefficient of elastic foundation. The variation of kinetic energy according to the present theory can be achieved as

$$\begin{aligned} \delta K &= \int_{-h/2}^{h/2}\int_A \left[\dot{u}_0 \delta\dot{u}_0 + \dot{v}_0 \delta\dot{v}_0 + \dot{w}\,\delta\dot{w}\right]dA\,dz \\ &= \int_A \Big\{I_0\left(\dot{u}_0\delta\dot{u}_0 + \dot{v}_0\delta\dot{v}_0 + (\dot{w}_b + \dot{w}_s)(\delta\dot{w}_b + \delta\dot{w}_s)\right) \\ &- I_1\left(\dot{u}_0\frac{\partial\delta\dot{w}_b}{\partial x} + \frac{\partial\dot{w}_b}{\partial x}\delta\dot{u}_0 + \dot{v}_0\frac{\partial\delta\dot{w}_s}{\partial y} + \frac{\partial\dot{w}_s}{\partial y}\delta\dot{v}_0\right) \\ &- J_1\left(\dot{u}_0\frac{\partial\delta\dot{w}_s}{\partial x} + \frac{\partial\dot{w}_s}{\partial x}\delta\dot{u}_0 + \dot{v}_0\frac{\partial\delta\dot{w}_s}{\partial y} + \frac{\partial\dot{w}_s}{\partial y}\delta\dot{v}_0\right) \\ &+ I_2\left(\frac{\partial\delta\dot{w}_b}{\partial x}\frac{\partial\delta\dot{w}_b}{\partial x} + \frac{\partial\delta\dot{w}_b}{\partial y}\frac{\partial\delta\dot{w}_b}{\partial y}\right) \\ &+ K_2\left(\frac{\partial\delta\dot{w}_s}{\partial x}\frac{\partial\delta\dot{w}_s}{\partial x} + \frac{\partial\delta\dot{w}_s}{\partial y}\frac{\partial\delta\dot{w}_s}{\partial y}\right) \\ &+ J_2\left(\frac{\partial\dot{w}_b}{\partial x}\frac{\partial\delta\dot{w}_s}{\partial x} + \frac{\partial\dot{w}_s}{\partial x}\frac{\partial\delta\dot{w}_b}{\partial x} + \frac{\partial\dot{w}_b}{\partial y}\frac{\partial\delta\dot{w}_s}{\partial y} + \frac{\partial\dot{w}_s}{\partial y}\frac{\partial\delta\dot{w}_b}{\partial y}\right)\Big\}dA \end{aligned} \tag{23}$$

In above relation, the differentiation with respect to time is introduced with the dot-superscript; and $(I_0, I_1, J_1, I_2, J_2, K_2)$ are mass inertias as below

$$\{I_0, I_1, J_1, I_2, J_2, K_2\} = \int_{-h/2}^{h/2} \{1, z, f, z^2, zf, f^2\} \rho(z) dz \tag{24}$$

Inserting the expressions for δU, δV, and δK from Eqs. (17), (20), and (23) into Eq. (16) and integrating by parts from them, with collecting the coefficients of δu_0, δv_0, δw_b, and δw_s equilibrium equations are derived as

$$\delta u_0 : \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \ddot{u}_0 - I_1 \frac{\partial \ddot{w}_b}{\partial x} - J_1 \frac{\partial \ddot{w}_s}{\partial x} \tag{25}$$

$$\delta v_0 : \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0 \ddot{v}_0 - I_1 \frac{\partial \ddot{w}_b}{\partial y} - J_1 \frac{\partial \ddot{w}_s}{\partial y} \tag{26}$$

$$\begin{aligned} &\delta w_b : \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} - k_W w + k_P \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \\ &= I_0 (\ddot{w}_b + \ddot{w}_s) + I_1 \left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y}\right) - I_2 \nabla^2 \ddot{w}_b - J_2 \nabla^2 \ddot{w}_s \end{aligned} \tag{27}$$

$$\begin{aligned} &\delta w_s : \frac{\partial^2 M_x^s}{\partial x^2} + 2\frac{\partial^2 M_{xy}^s}{\partial x \partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial Q_{xz}^s}{\partial x} + \frac{\partial Q_{yz}^s}{\partial y} - k_W w \\ &+ k_P \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = I_0 (\ddot{w}_b + \ddot{w}_s) + J_1 \left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y}\right) \\ &- J_2 \nabla^2 \ddot{w}_b - K_2 \nabla^2 \ddot{w}_s \end{aligned} \tag{28}$$

2.4 Equations of motion

Based on the nonlocal strain gradient theory, the constitutive relations of presented higher order nanoplate can be stated as

$$L_\mu \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xz} \\ \tau_{yz} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \times L_l \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xz} \\ \gamma_{yz} \\ \gamma_{xy} \end{bmatrix} \tag{29}$$

in which $(\sigma_x, \sigma_y, \tau_{yz}, \tau_{xz}, \tau_{xy})$ and $(\varepsilon_x, \varepsilon_y, \gamma_{yz}, \gamma_{xz}, \gamma_{xy})$ denote the stress and strain components, respectively. Integrating Eq. (32), the force strain and the moment-strain of gradient refined FG plate model can be presented as

$$\begin{aligned} &L_l \left\{ A_{11} \frac{\partial^2 u_0}{\partial x^2} + A_{66} \frac{\partial^2 u_0}{\partial y^2} + (A_{12} + A_{66}) \frac{\partial^2 v_0}{\partial x \partial y} - B_{11} \frac{\partial^3 w_b}{\partial x^3} \right\} \\ &+ L_l \left\{ -(B_{12} + 2B_{66}) \frac{\partial^3 w_b}{\partial x \partial y^2} - B_{11}^s \frac{\partial^3 w_s}{\partial x^3} - (B_{12}^s + 2B_{66}^s) \frac{\partial^3 w_s}{\partial x \partial y^2} \right\} \\ &= L_\mu \left\{ I_0 \ddot{u}_0 - I_1 \frac{\partial \ddot{w}_b}{\partial x} - J_1 \frac{\partial \ddot{w}_s}{\partial x} \right\} \end{aligned} \tag{30}$$

$$\begin{aligned} &L_l \left\{ A_{66} \frac{\partial^2 v_0}{\partial x^2} + A_{22} \frac{\partial^2 v_0}{\partial y^2} + (A_{12} + A_{66}) \frac{\partial^2 u_0}{\partial x \partial y} - B_{22} \frac{\partial^3 w_b}{\partial y^3} \right\} \\ &+ L_l \left\{ -(B_{12} + 2B_{66}) \frac{\partial^3 w_b}{\partial x^2 \partial y} - B_{22}^s \frac{\partial^3 w_s}{\partial x^3} - (B_{12}^s + 2B_{66}^s) \frac{\partial^3 w_s}{\partial x^2 \partial y} \right\} \\ &= L_\mu \left\{ I_0 \ddot{v}_0 - I_1 \frac{\partial \ddot{w}_b}{\partial y} - J_1 \frac{\partial \ddot{w}_s}{\partial y} \right\} \end{aligned} \tag{31}$$

$$\begin{aligned} &L_l \left\{ B_{11} \frac{\partial^3 u_0}{\partial x^3} + (B_{12} + 2B_{66}) \frac{\partial^3 u_0}{\partial x \partial y^2} + (B_{12} + 2B_{66}) \frac{\partial^3 v_0}{\partial x^2 \partial y} \right\} \\ &+ L_l \left\{ + B_{22} \frac{\partial^3 v_0}{\partial y^3} - D_{11} \frac{\partial^4 w_b}{\partial x^4} - 2(D_{12} + 2D_{66}) \frac{\partial^4 w_b}{\partial x^2 \partial y^2} - D_{22} \frac{\partial^4 w_b}{\partial x^4} \right\} \\ &+ L_l \left\{ -D_{11}^s \frac{\partial^4 w_s}{\partial x^4} - 2(D_{12}^s + 2D_{66}^s) \frac{\partial^4 w_s}{\partial x^2 \partial y^2} - D_{22}^s \frac{\partial^4 w_s}{\partial y^4} \right\} \\ &= L_\mu \left\{ \begin{aligned} &k_W w - k_P \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) + I_0 (\ddot{w}_b + \ddot{w}_s) + I_1 \left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y}\right) \\ &- I_2 \nabla^2 \ddot{w}_b - J_2 \nabla^2 \ddot{w}_s \end{aligned} \right\} \end{aligned} \tag{32}$$

$$\begin{aligned} &L_l \left\{ B_{11}^s \frac{\partial^3 u_0}{\partial x^3} + (B_{12}^s + 2B_{66}^s) \frac{\partial^3 u_0}{\partial x \partial y^2} + (B_{12}^s + 2B_{66}^s) \frac{\partial^3 v_0}{\partial x^2 \partial y} \right\} \\ &+ L_l \left\{ + B_{22} \frac{\partial^3 v_0}{\partial y^3} - D_{11}^s \frac{\partial^4 w_b}{\partial x^4} A_{55}^s \frac{\partial^2 w_s}{\partial x^2} + A_{44}^s \frac{\partial^2 w_s}{\partial y^2} - \right\} \\ &+ L_l \left\{ 2(D_{12}^s + 2D_{66}^s) \frac{\partial^4 w_b}{\partial x^2 \partial y^2} - D_{22}^s \frac{\partial^4 w_b}{\partial x^4} - H_{11}^s \frac{\partial^4 w_s}{\partial x^4} \right\} \\ &+ L_l \left\{ -2(H_{12}^s + 2H_{66}^s) \frac{\partial^4 w_s}{\partial x^2 \partial y^2} - H_{22}^s \frac{\partial^4 w_s}{\partial y^4} \right\} \\ &= L_\mu \left\{ \begin{aligned} &k_W w - k_P \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) + I_0 (\ddot{w}_b + \ddot{w}_s) J_1 \left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y}\right) \\ &- J_2 \nabla^2 \ddot{w}_b - K_2 \nabla^2 \ddot{w}_s \end{aligned} \right\} \end{aligned} \tag{33}$$

3. Solution procedure

In order to study wave propagation in FG anisotropic nanoplate, displacement along u_0, v_0, w_b and w_s is expressed as

$$\begin{aligned} u_0 &= A_1 e^{i(xk_1 + yk_2 - \omega t)} \\ v_0 &= A_2 e^{i(xk_1 + yk_2 - \omega t)} \\ w_b &= A_3 e^{i(xk_1 + yk_2 - \omega t)} \\ w_s &= A_4 e^{i(xk_1 + yk_2 - \omega t)} \end{aligned} \tag{34}$$

where A_1, A_2, A_3 and A_4 are wave amplitudes; k_1 and k_2 are wave numbers along *x*, *y* directions.

$$\left([K] - \omega^2 [M]\right)\{\Delta\} = 0 \tag{35}$$

where [*K*] and [*M*], respectively, are the stiffness matrix and mass matrix; ω is circular frequency. Substituting Eq. (34) into Eqs. (30)-(33), the equations of motion are written in matrix form as follows

$$\left|[K] - \omega^2 [M]\right| = 0 \tag{36}$$

To find the wave eigenfrequencies ω, the determinant of above matrix is set to zero. However, phase velocity of waves propagating in a structure is an important parameter in analysis of wave propagation. Phase velocity depends on the obtained wave frequency as well as wave number according to the following relation

$$c = \omega / k \tag{37}$$

in which $k_1 = k_2 = k$.

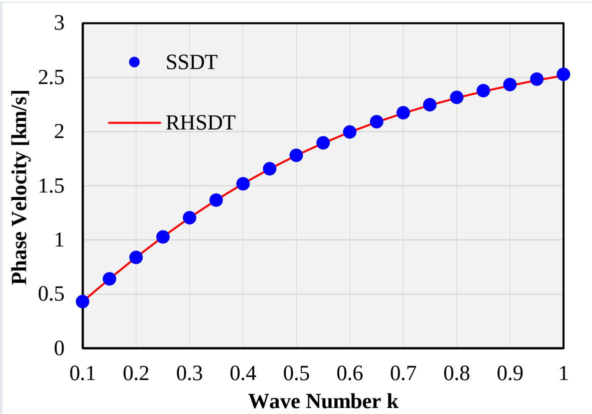
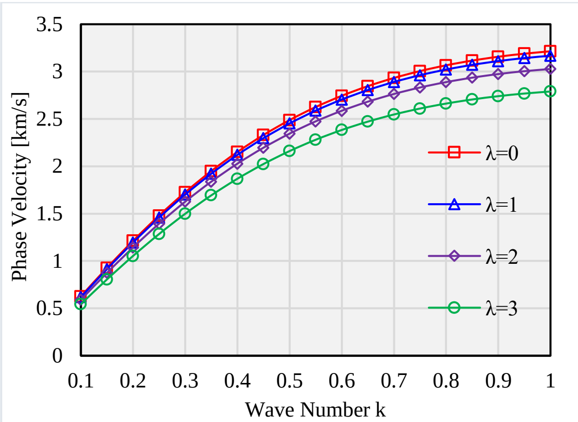
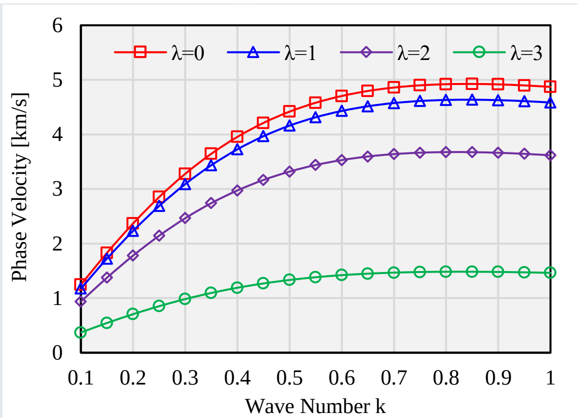


Fig. 2 Comparison of nonlocal strain gradient wave dispersion curves for rectangular nanoplate. ($l=0.2$nm, $\mu=1$nm)



(a) h=1 nm



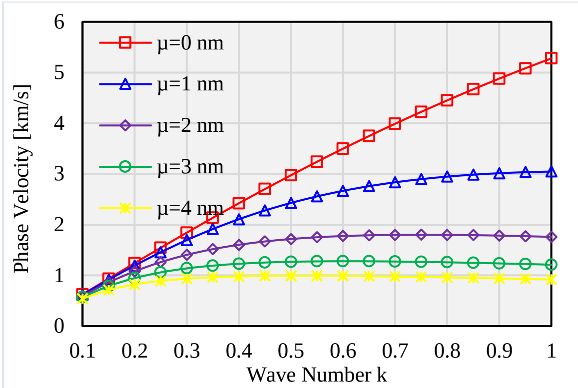
(b) h=2 nm

Fig. 3 Influence of exponential factor on the wave dispersion curves of FG anisotropic nanoplate. ($l=0.2$nm, $\mu=1$nm)

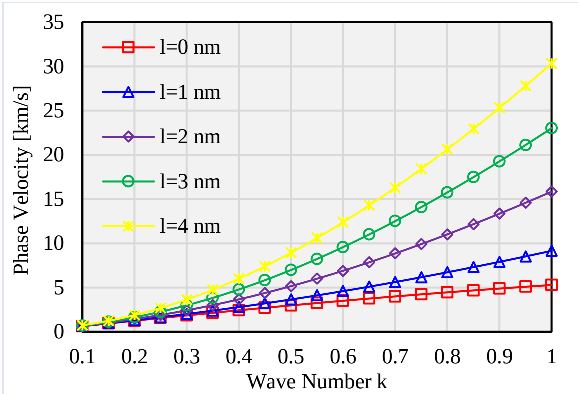
4. Numerical results and discussions

After proposing analytical states for nonlocal strain gradient refined model of FG anisotropic nanoplates, selected numerical results are presented in this section.

Firstly, the efficiency of present methodology is verified. Because in accordance with the best authors knowledge there is no study in the open literature on the size-dependent wave propagation of FG anisotropic nanoplates. Considering the nonlocal strain gradient effects the wave propagation of nanoplates made of an isotropic material with no associated to boundary conditions (bulk waves) are illustrated in Fig. 2 and compared with those obtained by Karami *et al.* (2018h) using second order shear deformation theory (SSDT). An excellent agreement is found between two types of modeling process which confirms the validity of the given preceding numerical results.



(a)



(b)

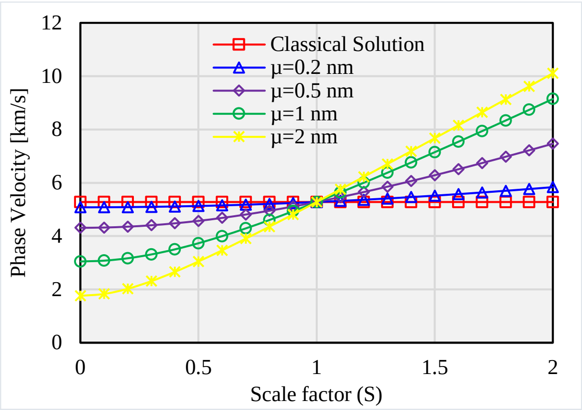
Fig. 4 Nonlocal strain gradient wave dispersion curves of FG anisotropic nanoplate. (a) l=0 nm, (b) $\mu=0$nm

4.1 Thickness of nanoplates and exponential factor

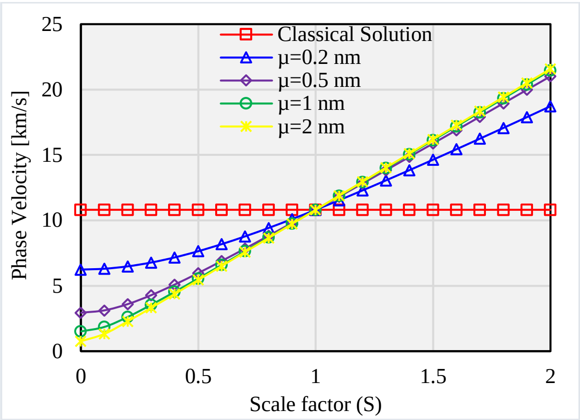
Fig. 3 displays the nonlocal strain gradient phase velocity curves of FG anisotropic nanoplates corresponding to various values of wave numbers. Also, varying of thicknesses in nanoplates and exponential factor effects are shown. It is found that the with increasing exponential factor and thickness of nanoplate, the phase velocity of FG anisotropic nanoplate will decrease and increase respectively. As a consequence, the exponential factor effect on the phase velocity of FG anisotropic nanoplate is more in the high value of the thickness of nanoplate.

4.2 Nonlocal constant and strain gradient length scale parameter

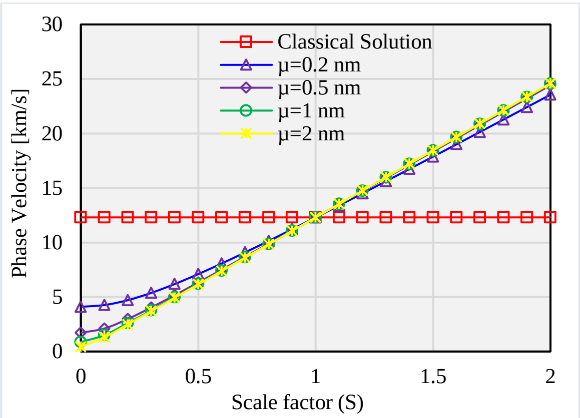
Fig. 4 depicts the nonlocal strain gradient phase velocity



(a) k=1 1/nm



(b) k=5 1/nm



(c) k=10 1/nm

Fig. 5 Variation of phase velocities of FG anisotropic nanoplate with respect to scale factor and nonlocal parameter

curves of FG anisotropic nanoplates with different small scale parameters. It is seen that the softening-stiffness effect of nonlocality causes to reduce the phase velocity, while the hardening-stiffness of strain gradient size dependency leads to enhance them. In addition, it can be observed that both types of size effect have more influence on the phase velocity in higher values of wave number.

Nonlocality effects are shown in Fig. 5 for variations of phase velocities in FG anisotropic nanoplate as a function of scale factor (S), where $S = \dfrac{l}{\mu}$.

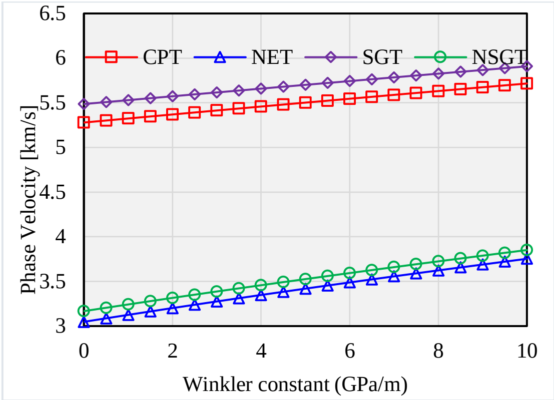


Fig. 6 Phase velocity based on Winkler constant and in different theory

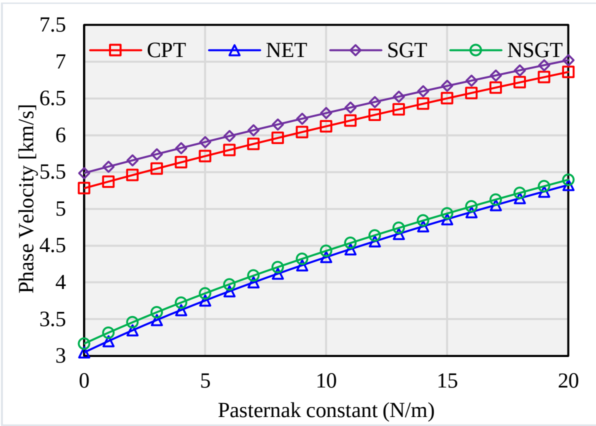


Fig. 7 Phase velocity based on Pasternak constant and in different theory

As we see in figure, when the scale factor (S) is less than unity, the nanoplate provides softer response and the size-dependent phase velocities are smaller than those from classical model. Also, it can be concluded that for values of nonlocal parameter (μ) smaller than gradient parameter (l), the results achieved from present theory are larger than those from the classical model. Besides, more changes can be seen in lower value of wave number with the variation of the scale factor (S). It is mentioned that by neglecting the scale factor (S), the present results are the same as the results from Eringen's nonlocal theory.

4.3 Winkler constant

Fig. 6 demonstrates the effect of Winkler constant of FG anisotropic nanoplate on the phase velocity in different theories (CPT; Nonlocal Elasticity Theory (NET); Strain Gradient Theory (SGT); Nonlocal Strain Gradient Theory (NSGT)). Increased Winkler constants leads to increased nanoplate rigidity, which in turn leads to increase the phase velocity. It can be seen that the phase velocity has the highest and lowest values for the strain gradient theory and the nonlocal elasticity theory, respectively. It means that with increasing the gradient parameter, the rigidity of FG anisotropic nanoplate will increase.

Table 1 Frequency of rectangular FG anisotropic nanoplate

k (1/nm)	λ	μ (nm)	l (nm)	h= 1 nm				h= 2 nm			
				T_1	T_2	T_3	T_4	T_1	T_2	T_3	T_4
1	0	0	0	5.6111	2.3478	2.3478	0.8526	3.7096	2.3478	2.3478	1.2920
			1.5	13.1592	5.5060	5.5060	1.9995	8.6997	5.5060	5.5060	3.0301
			3	24.4582	10.2337	10.2337	3.7164	16.1696	10.2337	10.2337	5.6319
		1	0	3.2396	1.3555	1.3555	0.4922	2.1417	1.3555	1.3555	0.7460
			1.5	7.5975	3.1789	3.1789	1.1544	5.0228	3.1789	3.1789	1.7494
			3	14.1210	5.9084	5.9084	2.1457	9.3355	5.9084	5.9084	3.2516
		2	0	1.8704	0.7826	0.7826	0.2842	1.2365	0.7826	0.7826	0.4307
			1.5	4.3864	1.8353	1.8353	0.6665	2.8999	1.8353	1.8353	1.0100
			3	8.1527	3.4112	3.4112	1.2388	5.3899	3.4112	3.4112	1.8773
	1	0	0	5.6989	2.3893	2.3893	0.8402	4.1868	2.4418	2.4418	1.2161
			1.5	13.3651	5.6034	5.6034	1.9703	9.8188	5.7264	5.7264	2.8520
			3	24.8409	10.4147	10.4147	3.6622	18.2496	10.6434	10.6434	5.3008
		1	0	3.2903	1.37947	1.37947	0.4851	2.4172	1.4097	1.4097	0.7021
			1.5	7.7163	3.23514	3.23514	1.1376	5.6689	3.3062	3.3062	1.6466
			3	14.3419	6.01295	6.01295	2.1143	10.5364	6.1449	6.1449	3.0604
		2	0	1.8996	0.7964	0.7964	0.2801	1.3956	0.8139	0.8139	0.4054
			1.5	4.4550	1.8678	1.8678	0.6568	3.2729	1.9088	1.9088	0.9507
			3	8.2803	3.4716	3.4716	1.2207	6.0832	3.5478	3.5478	1.7669
	2	0	0	5.9668	2.5135	2.5135	0.8031	5.4980	2.8023	2.8023	0.9597
			1.5	13.9933	5.8948	5.8948	1.8835	12.8939	6.5720	6.5720	2.2506
			3	26.0085	10.9562	10.9562	3.5007	23.9651	12.2151	12.2151	4.1831
		1	0	3.4449	1.4512	1.4512	0.4637	3.1743	1.6179	1.6179	0.5541
			1.5	8.0790	3.4033	3.4033	1.0874	7.4443	3.7944	3.7944	1.2994
			3	15.0160	6.3256	6.3256	2.0211	13.8363	7.0524	7.0524	2.4151
		2	0	1.9889	0.8378	0.8378	0.2677	1.8327	0.9341	0.9341	0.3199
			1.5	4.6644	1.9649	1.9649	0.6278	4.2980	2.1907	2.1907	0.7502
			3	8.6695	3.6521	3.6521	1.1669	7.9884	4.0717	4.0717	1.3944
2	0	0	0	7.4191	4.6955	4.6955	2.5841	5.8536	4.6955	4.6955	3.2988
			1.5	32.3392	20.4674	20.4674	11.2638	25.5151	20.4674	20.4674	14.3791
			3	63.3890	40.1187	40.1187	22.0786	50.0129	40.1187	40.1187	28.1848
		1	0	2.4730	1.5652	1.5652	0.8614	1.9512	1.5652	1.5652	1.0996
			1.5	10.7797	6.8225	6.8225	3.7546	8.5050	6.8225	6.8225	4.7930
			3	21.1297	13.3729	13.3729	7.3595	16.6710	13.3729	13.3729	9.3949
		2	0	1.2915	0.8174	0.8174	0.4498	1.0190	0.8174	0.8174	0.5742
			1.5	5.6295	3.5630	3.5630	1.9608	4.4416	3.5629	3.5629	2.5031
			3	11.0346	6.9838	6.9838	3.8434	8.7061	6.9838	6.9838	4.9063
	1	0	0	7.6682	4.7373	4.7373	2.5466	7.2849	4.6515	4.6515	3.1131
			1.5	33.4247	20.6493	20.6493	11.1005	31.7541	20.2752	20.2752	13.5697
			3	65.5168	40.4754	40.4754	21.7584	62.2421	39.7420	39.7420	26.5983
		1	0	2.5561	1.5791	1.5791	0.8489	2.4283	1.5505	1.5505	1.0377
			1.5	11.1416	6.8831	6.8831	3.7002	10.5847	6.7584	6.7584	4.5232
			3	21.8389	13.4918	13.4918	7.2528	20.7474	13.2473	13.2473	8.8661
		2	0	1.3349	0.8247	0.8247	0.4433	1.2681	0.8097	0.8097	0.5419
			1.5	5.8185	3.5946	3.5946	1.9324	5.5277	3.5295	3.5295	2.3622
			3	11.4050	7.0459	7.0459	3.7877	10.8350	6.9182	6.9182	4.6302
	2	0	0	8.3735	4.8835	4.8835	2.4322	10.1712	5.6995	4.9249	2.3575

Table 1 Continued

	1.5	36.4993	21.2867	21.2867	10.6015	44.3353	24.8435	21.4673	10.2762
	3	71.5433	41.7247	41.7247	20.7803	86.9030	48.6964	42.0786	20.1427
1	0	2.7912	1.6278	1.6278	0.8107	3.3904	1.8998	1.6416	0.7858
	1.5	12.1664	7.0956	7.0956	3.5338	14.7784	8.2812	7.1558	3.4254
	3	23.8478	13.9082	13.9082	6.9268	28.9677	16.2321	14.0262	6.7142
2	0	1.4576	0.8501	0.8501	0.4234	1.7706	0.9922	0.8573	0.4104
	1.5	6.3537	3.7055	3.7055	1.8455	7.7178	4.3247	3.7370	1.7889
	3	12.4541	7.2633	7.2633	3.6174	15.1279	8.4770	7.3249	3.5064

4.4 Pasternak constant

Fig. 7 demonstrates the effects of Pasternak constant of FG anisotropic nanoplate on the phase velocity in different theories. An increase in Pasternak constant leads to an increase in the phase velocity. In this case, increasing in Pasternak constant and gradient parameter is accompanied by an increased in the rigidity of the FG anisotropic nanoplate while the nonlocal theory has the behavior contrary to them.

4.5 Four different modes for wave analysis

One of the aims of this study is to provide the wave behavior of rectangular FG anisotropic nanoplates with respect to nonlocal parameter (μ) and strain gradient length scale parameter (l). So, as a benchmark table, wave characteristics of FG anisotropic nanoplate for the wave frequency for different nonlocal parameter (μ), strain gradient length scale (l), wave number, exponential factor (λ) and thickness of the plate (h) has been tabulated in Table 1. To provide results, results are explored for four pairs of distinct dispersion modes, (T_1, T_2, T_3, T_4) . As can be seen, the wave frequency of FG anisotropic nanoplate rises with the increment of the strain gradient length scale (l) and reduces with the increment of the nonlocal parameter (μ).

5. Conclusions

The objective of present work was to analyze the size-dependent wave propagation behavior of rectangular FG anisotropic nanoplates rested on elastic foundation including both of hardening-stiffness and softening-stiffness size effects. To this end, the nonlocal strain gradient theory of elasticity was incorporated to the RPT the governing equations of wave motion was obtained employing the Hamiltonian principle. After that, an analytical technique was used to solve the governing equations for wave propagation of FG anisotropic nanoplates as a function of wave number.

It was revealed that the softening-stiffness influence of nonlocality causes to reduce the phase velocity, while the hardening-stiffness of strain gradient size dependency leads to enhance them. Moreover, it was observed that the wave number cause enhances the phase velocity of FG anisotropic nanoplate. It was revealed that the softening-stiffness effect of nonlocality causes to reduce the phase velocity, while the hardening-stiffness of strain gradient size dependency leads to enhance them. Moreover, it was observed that the wave number cause enhances the phase velocity of FG anisotropic nanoplate. Additionally, it was seen that both of the electric foundation parameters cause to increase of the phase velocity of FG anisotropic nanoplates. Furthermore, increasing the exponential factor induces a reduction effect on the obtained results for phase velocity. In addition, it was demonstrated that the thickness of plate had the more important role on the wave behavior of FG anisotropic nanoplate.

References

Aifantis, E.C. (1992), "On the role of gradients in the localization of deformation and fracture", *Int. J. Eng. Sci.*, **30**(10), 1279-1299.

Aminipour, H. and Janghorban, M. (2017), "Wave propagation in anisotropic plates using trigonometric shear deformation theory", *Mech. Adv. Mater. Struct.*, **24**(13), 1135-1144.

Aminipour, H., Janghorban, M. and Li, L. (2018), "A new model for wave propagation in functionally graded anisotropic doubly-curved shells", *Compos. Struct.*, **190**, 91-111.

Ansari, R., Rouhi, H. and Sahmani, S. (2011), "Calibration of the analytical nonlocal shell model for vibrations of double-walled carbon nanotubes with arbitrary boundary conditions using molecular dynamics", *Int. J. Mech. Sci.*, **53**(9), 786-792.

Apuzzo, A., Barretta, R., Luciano, R., De Sciarra, F.M. and Penna, R. (2017), "Free vibrations of Bernoulli-Euler nano-beams by the stress-driven nonlocal integral model", *Compos. Part B: Eng.*, **123**, 105-111.

Barati, M.R. (2017), "Vibration analysis of FG nanoplates with nanovoids on viscoelastic substrate under hygro-thermo-mechanical loading using nonlocal strain gradient theory", *Struct. Eng. Mech.*, **64**(6), 683-693.

Barretta, R., Luciano, R., De Sciarra, F.M. and Ruta, G. (2018), "Stress-driven nonlocal integral model for Timoshenko elastic nano-beams", *Eur. J. Mech.-A/Sol.*, **72**, 275-286.

Batra, R., Qian, L. and Chen, L. (2004), "Natural frequencies of thick square plates made of orthotropic, trigonal, monoclinic, hexagonal and triclinic materials", *J. Sound Vibr.*, **270**(4-5), 1074-1086.

Bellifa, H., Benrahou, K.H., Bousahla, A.A., Tounsi, A. and Mahmoud, S. (2017), "A nonlocal zeroth-order shear deformation theory for nonlinear postbuckling of nanobeams", *Struct. Eng. Mech.*, **62**(6), 695-702.

Bounouara, F., Benrahou, K.H., Belkorissat, I. and Tounsi, A. (2016), "A nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates resting on elastic foundation", *Steel Compos. Struct.*, **20**(2), 227-249.

Chaht, F.L., Kaci, A., Houari, M.S.A., Tounsi, A., Bég, O.A. and Mahmoud, S. (2015), "Bending and buckling analyses of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect", *Steel Compos. Struct.*, **18**(2), 425-442.

Challamel, N. and Wang, C. (2008), "The small length scale effect for a non-local cantilever beam: A paradox solved", *Nanotechnol.*, **19**(34), 345703.

Dash, S., Mehar, K., Sharma, N., Mahapatra, T.R. and Panda, S.K. (2018a), "Modal analysis of FG sandwich doubly curved shell structure", *Struct. Eng. Mech.*, **68**(6), 721-733.

Dash, S., Sharma, N., Mahapatra, T., Panda, S. and Sahu, P. (2018b), "Free vibration analysis of functionally graded sandwich flat panel", *Mater. Sci. Eng.*, **377**(1).

Dutta, G., Panda, S.K., Mahapatra, T.R. and Singh, V.K. (2017), "Electro-magneto-elastic response of laminated composite plate: A finite element approach", *Int. J. Appl. Comput. Math.*, **3**(3), 2573-2592.

Ebrahimi, F. and Barati, M.R. (2016), "A nonlocal higher-order shear deformation beam theory for vibration analysis of size-dependent functionally graded nanobeams", *Arab. J. Sci. Eng.*, **41**(5), 1679-1690.

Ehyaei, J., Farazmandnia, N. and Jafari, A. (2017), "Rotating effects on hygro-mechanical vibration analysis of FG beams based on Euler-Bernoulli beam theory", *Struct. Eng. Mech.*, **63**(4), 471-480.

Eringen, A.C. (1983), "On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves", *J. Appl. Phys.*, **54**(9), 4703-4710.

Farokhi, H. and Ghayesh, M.H. (2015), "Nonlinear dynamical behaviour of geometrically imperfect microplates based on modified couple stress theory", *Int. J. Mech. Sci.*, **90**, 133-144.

Gholipour, A., Farokhi, H. and Ghayesh, M.H. (2015), "In-plane and out-of-plane nonlinear size-dependent dynamics of microplates", *Nonlin. Dyn.*, **79**(3), 1771-1785.

Guo, J., Chen, J. and Pan, E. (2017), "Free vibration of three-dimensional anisotropic layered composite nanoplates based on modified couple-stress theory", *Phys. E: Low-Dimens. Syst. Nanostruct.*, **87**, 98-106.

Hirwani, C., Biswash, S., Mehar, K. and Panda, S.K. (2018), "Numerical flexural strength analysis of thermally stressed delaminated composite structure under sinusoidal loading", *Mater. Sci. Eng.*, 012019.

Hirwani, C.K. and Panda, S.K. (2018), "Numerical and experimental validation of nonlinear deflection and stress responses of pre-damaged glass-fibre reinforced composite structure", *Ocean Eng.*, **159**, 237-252.

Hirwani, C.K. and Panda, S.K. (2019), "Nonlinear finite element solutions of thermoelastic deflection and stress responses of internally damaged curved panel structure", *Appl. Math. Modell.*, **65**, 303-317.

Karami, B. and Janghorban, M. (2016), "Effect of magnetic field on the wave propagation in nanoplates based on strain gradient theory with one parameter and two-variable refined plate theory", *Mod. Phys. Lett. B*, **30**(36), 1650421.

Karami, B. and Janghorban, M. (2019), "On the dynamics of porous nanotubes with variable material properties and variable thickness", *Int. J. Eng. Sci.*, **136**, 53-66.

Karami, B., Janghorban, M. and Li, L. (2018a), "On guided wave propagation in fully clamped porous functionally graded nanoplates", *Acta Astronaut.*, **143**, 380-390.

Karami, B., Janghorban, M., Shahsavari, D. and Tounsi, A. (2018b), "A size-dependent quasi-3D model for wave dispersion analysis of FG nanoplates", *Steel Compos. Struct.*, **28**(1), 99-110.

Karami, B., Janghorban, M. and Tounsi, A. (2017), "Effects of triaxial magnetic field on the anisotropic nanoplates", *Steel Compos. Struct.*, **25**(3), 361-374.

Karami, B., Janghorban, M. and Tounsi, A. (2018c), "Galerkin's approach for buckling analysis of functionally graded anisotropic nanoplates/different boundary conditions", *Eng. Comput.*, 1-20.

Karami, B., Janghorban, M. and Tounsi, A. (2018d), "Nonlocal strain gradient 3D elasticity theory for anisotropic spherical nanoparticles", *Steel Compos. Struct.*, **27**(2), 201-216.

Karami, B., Janghorban, M. and Tounsi, A. (2018e), "Variational approach for wave dispersion in anisotropic doubly-curved nanoshells based on a new nonlocal strain gradient higher order shell theory", *Thin-Wall. Struct.*, **129**, 251-264.

Karami, B. and Karami, S. (2019), "Buckling analysis of nanoplate-type temperature-dependent heterogeneous materials", *Adv. Nano Res.*, **7**(1), 51-61.

Karami, B., Shahsavari, D. and Janghorban, M. (2018f), "A comprehensive analytical study on functionally graded carbon nanotube-reinforced composite plates", *Aerosp. Sci. Technol.*, **82**, 499-512.

Karami, B., Shahsavari, D. and Janghorban, M. (2018g), "Wave propagation analysis in functionally graded (FG) nanoplates under in-plane magnetic field based on nonlocal strain gradient theory and four variable refined plate theory", *Mech. Adv. Mater. Struct.*, **25**(12), 1047-1057.

Karami, B., Shahsavari, D., Janghorban, M., Dimitri, R. and Tornabene, F. (2019a), "Wave propagation of porous nanoshells", *Nanomater.*, **9**(1), 22.

Karami, B., Shahsavari, D., Janghorban, M. and Li, L. (2018h), "Wave dispersion of mounted graphene with initial stress", *Thin-Wall. Struct.*, **122**, 102-111.

Karami, B., Shahsavari, D., Karami, M. and Li, L. (2019), "Hygrothermal wave characteristic of nanobeam-type inhomogeneous materials with porosity under magnetic field", *J. Mech. Eng. Sci.*, **233**(6), 2149-2169.

Karami, B., Shahsavari, D. and Li, L. (2018j), "Hygrothermal wave propagation in viscoelastic graphene under in-plane magnetic field based on nonlocal strain gradient theory", *Phys. E: Low-Dimens. Syst. Nanostruct.*, **97**, 317-327.

Karami, B., Shahsavari, D. and Li, L. (2018k), "Temperature-dependent flexural wave propagation in nanoplate-type porous heterogenous material subjected to in-plane magnetic field", *J. Therm. Stress.*, **41**(4), 483-499.

Karami, B., Shahsavari, D., Li, L., Karami, M. and Janghorban, M. (2019b), "Thermal buckling of embedded sandwich piezoelectric nanoplates with functionally graded core by a nonlocal second-order shear deformation theory", *J. Mech. Eng. Sci.*, **233**(1), 287-301.

Karami, B., Shahsavari, D., Nazemosadat, S.M.R., Li, L. and Ebrahimi, A. (2018l), "Thermal buckling of smart porous functionally graded nanobeam rested on Kerr foundation", *Steel Compos. Struct.*, **29**(3), 349-362.

Katariya, P.V., Hirwani, C.K. and Panda, S.K. (2018), "Geometrically nonlinear deflection and stress analysis of skew sandwich shell panel using higher-order theory", *Eng. Comput.*, 1-19.

Katariya, P.V., Panda, S.K., Hirwani, C.K., Mehar, K. and Thakare, O. (2017a), "Enhancement of thermal buckling strength of laminated sandwich composite panel structure embedded with shape memory alloy fibre", *Smart Struct. Syst.*, **20**(5), 595-605.

Katariya, P.V., Panda, S.K. and Mahapatra, T.R. (2017b), "Nonlinear thermal buckling behaviour of laminated composite panel structure including the stretching effect and higher-order finite element", *Adv. Mater. Res.*, **6**(4), 349-361.

Khetir, H., Bouiadjra, M.B., Houari, M.S.A., Tounsi, A. and

Mahmoud, S. (2017), "A new nonlocal trigonometric shear deformation theory for thermal buckling analysis of embedded nanosize FG plates", *Struct. Eng. Mech.*, **64**(4), 391-402.

Lagnese, J. (1989), *Boundary Stabilization of Thin Plates, Philadelphia*, SIAM Studies in Applied Mathematics, **10**.

Lam, D.C., Yang, F., Chong, A., Wang, J. and Tong, P. (2003), "Experiments and theory in strain gradient elasticity", *J. Mech. Phys. Sol.*, **51**(8), 1477-1508.

Li, L. and Hu, Y. (2015), "Buckling analysis of size-dependent nonlinear beams based on a nonlocal strain gradient theory", *Int. J. Eng. Sci.*, **97**, 84-94.

Li, L., Hu, Y. and Ling, L. (2016), "Wave propagation in viscoelastic single-walled carbon nanotubes with surface effect under magnetic field based on nonlocal strain gradient theory", *Phys. E: Low-Dimes. Syst. Nanostruct.*, **75**, 118-124.

Lim, C., Zhang, G. and Reddy, J. (2015), "A higher-order nonlocal elasticity and strain gradient theory and its applications in wave propagation", *J. Mech. Phys. Sol.*, **78**, 298-313.

Ma, H., Gao, X.L. and Reddy, J. (2008), "A microstructure-dependent Timoshenko beam model based on a modified couple stress theory", *J. Mech. Phys. Sol.*, **56**(12), 3379-3391.

Mahapatra, T.R., Mehar, K., Panda, S.K., Dewangan, S. and Dash, S. (2017), "Flexural strength of functionally graded nanotube reinforced sandwich spherical panel", *Mater. Sci. Eng.*, **178**(1), 012031.

Mehar, K. and Panda, S.K. (2017a), "Numerical investigation of nonlinear thermomechanical deflection of functionally graded CNT reinforced doubly curved composite shell panel under different mechanical loads", *Compos. Struct.*, **161**, 287-298.

Mehar, K. and Panda, S.K. (2017b), "Thermoelastic analysis of FG-CNT reinforced shear deformable composite plate under various loadings", *Int. J. Comput. Meth.*, **14**(2), 1750019.

Mehar, K. and Panda, S.K. (2018), "Thermoelastic flexural analysis of FG-CNT doubly curved shell panel", *Aircraft Eng. Aerosp. Technol.*, **90**(1), 11-23.

Mehar, K., Panda, S.K. and Mahapatra, T.R. (2018a), "Large deformation bending responses of nanotube-reinforced polymer composite panel structure: Numerical and experimental analyses", *J. Aerosp. Eng.*, 0954410018761.

Mehar, K., Panda, S.K. and Mahapatra, T.R. (2018b), "Thermoelastic deflection responses of CNT reinforced sandwich shell structure using finite element method", *Sci. Iranic.*, **25**(5), 2722-2737.

Mehar, K., Panda, S.K. and Patle, B.K. (2017), "Thermoelastic vibration and flexural behavior of FG-CNT reinforced composite curved panel", *Int. J. Appl. Mech.*, **9**(4), 1750046.

Mehar, K., Panda, S.K. and Patle, B.K. (2018c), "Stress, deflection, and frequency analysis of CNT reinforced graded sandwich plate under uniform and linear thermal environment: A finite element approach", *Polym. Compos.*, **39**(10), 3792-3809.

Mindlin, R.D. (1964), "Micro-structure in linear elasticity", *Arch. Rat. Mech. Analy.*, **16**(1), 51-78.

Mirjavadi, S.S., Afshari, B.M., Shafiei, N., Hamouda, A. and Kazemi, M. (2017), "Thermal vibration of two-dimensional functionally graded (2D-FG) porous Timoshenko nanobeams", *Steel Compos. Struct.*, **25**(4), 415-426.

Moradweysi, P., Ansari, R., Hosseini, K. and Sadeghi, F. (2018), "Application of modified Adomian decomposition method to pull-in instability of nano-switches using nonlocal Timoshenko beam theory", *Appl. Math. Modell.*, **54**, 594-604.

Nami, M.R. and Janghorban, M. (2014), "Wave propagation in rectangular nanoplates based on strain gradient theory with one gradient parameter with considering initial stress", *Mod. Phys. Lett. B*, **28**(3), 1450021.

Nami, M.R. and Janghorban, M. (2015), "Free vibration analysis of rectangular nanoplates based on two-variable refined plate theory using a new strain gradient elasticity theory", *J. Brazil. Soc. Mech. Sci. Eng.*, **37**(1), 313-324.

Nejad, M.Z. and Hadi, A. (2016), "Eringen's non-local elasticity theory for bending analysis of bi-directional functionally graded Euler-Bernoulli nano-beams", *Int. J. Eng. Sci.*, **106**, 1-9.

Pan, E. (2003), "Exact solution for functionally graded anisotropic elastic composite laminates", *J. Compos. Mater.*, **37**(21), 1903-1920.

Polizzotto, C. (2003), "Gradient elasticity and nonstandard boundary conditions", *Int. J. Sol. Struct.*, **40**(26), 7399-7423.

Rahmani, O., Refaeinejad, V. and Hosseini, S. (2017), "Assessment of various nonlocal higher order theories for the bending and buckling behavior of functionally graded nanobeams", *Steel Compos. Struct.*, **23**(3), 339-350.

Romano, G. and Barretta, R. (2017), "Nonlocal elasticity in nanobeams: The stress-driven integral model", *Int. J. Eng. Sci.*, **115**, 14-27.

Romano, G., Barretta, R. and Diaco, M. (2017), "On nonlocal integral models for elastic nano-beams", *Int. J. Mech. Sci.*, **131**, 490-499.

Sahmani, S., Aghdam, M.M. and Rabczuk, T. (2018a), "Nonlinear bending of functionally graded porous micro/nano-beams reinforced with graphene platelets based upon nonlocal strain gradient theory", *Compos. Struct.*, **186**, 68-78.

Sahmani, S., Aghdam, M.M. and Rabczuk, T. (2018b), "Nonlocal strain gradient plate model for nonlinear large-amplitude vibrations of functionally graded porous micro/nano-plates reinforced with GPLs", *Compos. Struct.*, **198**, 51-62.

Sahmani, S. and Fattahi, A. (2017), "Calibration of developed nonlocal anisotropic shear deformable plate model for uniaxial instability of 3D metallic carbon nanosheets using MD simulations", *Comput. Meth. Appl. Mech. Eng.*, **322**, 187-207.

Sahoo, S.S., Panda, S.K. and Singh, V.K. (2017), "Experimental and numerical investigation of static and free vibration responses of woven glass/epoxy laminated composite plate", *J. Mater.: Des. Appl.*, **231**(5), 463-478.

Sayyad, A.S. and Ghugal, Y.M. (2018), "An inverse hyperbolic theory for FG beams resting on Winkler-Pasternak elastic foundation", *Adv. Aircr. Spacecr. Sci.*, **5**(6), 671-689.

Shafiei, N. and She, G.L. (2018), "On vibration of functionally graded nano-tubes in the thermal environment", *Int. J. Eng. Sci.*, **133**, 84-98.

Shahsavari, D. and Janghorban, M. (2017), "Bending and shearing responses for dynamic analysis of single-layer graphene sheets under moving load", *J. Brazil. Soc. Mech. Sci. Eng.*, **39**(10), 3849-3861.

Shahsavari, D., Karami, B., Fahham, H.R. and Li, L. (2018a), "On the shear buckling of porous nanoplates using a new size-dependent quasi-3D shear deformation theory", *Acta Mech.*, **229**(11), 4549-4573.

Shahsavari, D., Karami, B., Janghorban, M. and Li, L. (2017), "Dynamic characteristics of viscoelastic nanoplates under moving load embedded within visco-Pasternak substrate and hygrothermal environment", *Mater. Res. Expr.*, **4**(8), 085013.

Shahsavari, D., Karami, B. and Li, L. (2018b), "Damped vibration of a graphene sheet using a higher-order nonlocal strain-gradient Kirchhoff plate model", *Compt. Rend. Mécan.*, **346**(12), 1216-1232.

Shahsavari, D., Karami, B. and Li, L. (2018c), "A high-order gradient model for wave propagation analysis of porous FG nanoplates", *Steel Compos. Struct.*, **29**(1), 53-66.

Shahsavari, D., Karami, B. and Mansouri, S. (2018d), "Shear buckling of single layer graphene sheets in hygrothermal environment resting on elastic foundation based on different nonlocal strain gradient theories", *Eur. J. Mech.-A/Sol.*, **67**, 200-214.

Shahsavari, D., Shahsavari, M., Li, L. and Karami, B. (2018e), "A

novel quasi-3D hyperbolic theory for free vibration of FG plates with porosities resting on Winkler/Pasternak/Kerr foundation", *Aerosp. Sci. Technol.*, **72**, 134-149.
She, G.L., Ren, Y.R., Yuan, F.G. and Xiao, W.S. (2018a), "On vibrations of porous nanotubes", *Int. J. Eng. Sci.*, **125**, 23-35.
She, G.L., Yan, K.M., Zhang, Y.L., Liu, H.B. and Ren, Y.R. (2018b), "Wave propagation of functionally graded porous nanobeams based on non-local strain gradient theory", *Eur. Phys. J. Plus*, **133**(9), 368.
She, G.L., Yuan, F.G., Karami, B., Ren, Y.R. and Xiao, W.S. (2019), "On nonlinear bending behavior of FG porous curved nanotubes", *Int. J. Eng. Sci.*, **135**, 58-74.
She, G.L., Yuan, F.G. and Ren, Y.R. (2018c), "On wave propagation of porous nanotubes", *Int. J. Eng. Sci.*, **130**, 62-74.
She, G.L., Yuan, F.G., Ren, Y.R. and Xiao, W.S. (2017), "On buckling and postbuckling behavior of nanotubes", *Int. J. Eng. Sci.*, **121**, 130-142.
Shimpi, R. and Patel, H. (2006), "Free vibrations of plate using two variable refined plate theory", *J. Sound Vibr.*, **296**(4), 979-999.
Shimpi, R.P. (2002), "Refined plate theory and its variants", *AIAA J.*, **40**(1), 137-146.
Simsek, M. (2011), "Forced vibration of an embedded single-walled carbon nanotube traversed by a moving load using nonlocal Timoshenko beam theory", *Steel Compos. Struct.*, **11**(1), 59-76.
Singh, V.K. and Panda, S.K. (2017), "Geometrical nonlinear free vibration analysis of laminated composite doubly curved shell panels embedded with piezoelectric layers", *J. Vibr. Contr.*, **23**(13), 2078-2093.
Sobhy, M. (2017), "Hygro-thermo-mechanical vibration and buckling of exponentially graded nanoplates resting on elastic foundations via nonlocal elasticity theory", *Struct. Eng. Mech.*, **63**(3), 401-415.
Volokh, K.Y. and Hutchinson, J. (2002), "Are lower-order gradient theories of plasticity really lower order?", *J. Appl. Mech.*, **69**(6), 862-864.
Xiao, W., Li, L. and Wang, M. (2017), "Propagation of in-plane wave in viscoelastic monolayer graphene via nonlocal strain gradient theory", *Appl. Phys. A*, **123**(6), 388.
Yang, F., Chong, A., Lam, D.C.C. and Tong, P. (2002), "Couple stress based strain gradient theory for elasticity", *International J. Sol. Struct.*, **39**(10), 2731-2743.
Zhu, X. and Li, L. (2017), "Closed form solution for a nonlocal strain gradient rod in tension", *Int. J. Eng. Sci.*, **119**, 16-28.

CC