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Bending analysis of composite skew cylindrical shell panel

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Abstract. A nine node isoparametric plate bending element is used for bending analysis of laminated composite skew cylindrical shell panels. Both thick and thin shell panels are solved. Rotary inertia and shear deformation are incorporated by considering first order shear deformation theory. The analysis is performed considering shallow shell theory. Both shallow and moderately deep skew cylindrical shells are investigated. Skew cylindrical shell panels having different thickness ratios (h/a), radius to length ratios (R/a), ply angle orientations, number of layers, aspect ratio (b/a), boundary conditions and various loading (concentrated, uniformly distributed, linear varying and doubly sinusoidal varying) conditions are analysed. Various new results are presented.

Keywords: bending; FSDT; linear varying load; shallow shell theory; sinusoidal load; skew shell panel

1. Introduction

Plates and shells are of significant interest as structural members in civil and aerospace industries. They are used in applications ranging from bridge decks to solar panels of satellites. Due to their potentially endless applications, it is imperative for any structural design engineer to understand their static and dynamic behaviours. In this regard, the finite element method (FEM) is regarded as one of the most versatile analysis tools specifically in structural mechanics problems (Zienkiewicz *et al.* 1977). In fact, analysis of plates and shells is one of the first problems where FEM was applied. The initial attempts were made with Kirchoff's hypothesis where a number of problems were faced. The major problem concerned the satisfaction of normal slope continuity at the element edges which could not be solved satisfactorily. In the current study, the above problem is avoided by adopting Mindlin's hypothesis where the effect of shear deformation is considered.

A considerable amount of literature is available on free vibration of skew plates and shells. But works on bending behaviour of skew cylindrical shell panels are rare. Mizusawa (1994) presented spline element method to analyse the bending of skew plates subjected to transverse uniform load and concentrated load with arbitrary boundary conditions. Sheikh *et al.* (2002) presented a high precision shear deformable element for bending analysis of plates of different shapes. They analysed plates (rectangular, skew, triangular, trapezoidal) having different boundary conditions, thickness ratios, number of layers and fiber orientations. Muhammad and Singh (2004) presented an energy method for the linear static analysis of first-order shear deformable plates of various shapes. Sahoo and Chakravorty (2004) presented a finite element procedure with an eight-node isoparametric curved quadratic element for investigating the static behaviour of laminated composite hyperbolic paraboloid shell. They analysed the shell considering differing boundary conditions subjected to a uniform distributed load. An improved finite element model for the bending and free vibration analysis of doubly curved laminated composite shells was presented by Latifa and Sinha (2005). Achryya *et al*. (2009) developed a formulation using an eight-node isoparametric shell element to analyse bending behaviour of delaminated composite shallow cylindrical shells. Effect of various boundary conditions, lamination, curvature and extent of delamination area was studied by them. A finite element analysis of composite conoidal shells with delamination damage was carried out by Kumari and Chakravorty (2010) to examine the bending behaviour of such shell configurations. They used isoparametric shell bending element for analysis of shell and validated the formulation through a solution of benchmark problems. Shariyat (2011) proposed an accurate high-order global-local theory for bending and vibration of cylindrical shells subjected to thermo-mechanical loads. Bending response of functionally graded skew sandwich plates was analysed by Taj *et al*. (2014) using a C^0 finite element formulation with higher order shear deformation theory. They studied the plate considering different skew angle, aspect ratio, thickness ratio and boundary conditions. Maleki and Tahani (2014) investigated the bendingbehaviour of composite conical shell panels subjected to different distributed mechanical loads with various types of orthotropy. A C^0 finite element formulation based on higher order shear deformation theory was developed by Kumar *et al.* (2015) for free vibration analysis

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of composite skew cylindrical shells. They used a nine-node curved isoparametric element for analysis of composite skew cylindrical shells having different geometry, boundary conditions, ply orientation and skew angles. Biswal *et al*. (2016) used a first-order shear deformation theory (FSDT) based FEA analysis to study effects of moisture and temperature on buckling of laminated composite cylindrical shell panels. A similar approach was adopted by Sofiyev et al. to study dynamic instability of shells (Sofiyev *et al.* 2017, Sofiyev and Kuruoglu 2016, Sofiyev and Kuruoglu 2015). Najafov *et al.* (2014) studied the effect of a Pasternak elastic foundation on the stability of exponentially graded cylindrical shells under hydrostatic pressure. Recently, an energy-oriented modified Fourier method has been used to perform dynamics analysis of isotropic and composite plates and shells with general boundary conditions. Dynamic analysis of laminated cylindrical shells (Jin *et al*. 2013), laminated plates (Ye *et al*. 2014), composite cylindrical shells with general elastic boundary conditions (Jin *et al*. 2013), functionally graded cylindrical shells (Jin *et al*. 2014), composite laminated structure elements of revolution (Jin *et al*. 2014) has been carried out by this method.

As seen from the literature review, there is a very limited number of works available on bending behaviour of isotropic skew shells. Literature on bending behaviour of laminated composite skew cylindrical shell panels subjected to different types of mechanical loading is even rarer. The main objective of the present study is to investigate the effect of different types of mechanical loadings (concentrated, uniform distributed, linear varying distribute and sinusoidal) on bending behaviour of laminated composite skew cylindrical shell panels having various skew angles, fibre orientations, number of layers, radius to length ratios, thickness ratios and boundary conditions. In the present work, a finite element code is written using nine-node isoparametric element with the concept of shallow shell theory. The finite element code is validated against benchmark solutions. For future research, various new results are presented.

2. Finite element formulation

The formulation is based on shallow shell theory. The effect of shear deformation is taken into account following the Mindlin's hypothesis where it is assumed that the normal to the middle plane of the shell before bending remains straight but not necessarily normal to the middle plane of the shell after bending. Taking the middle surface of the shell as the reference surface, the formulation is carried out following the usual assumptions of linear elastic analysis.

Element used in the present work is the nine-node isoparametric element. In the recent past, the authors have used this element to conduct free vibration studies on plates (Kalita and Haldar 2017, Kalita *et al.* 2018). One of the major advantages of the element is that any plate shape can be nicely handled through a simple mapping technique which may be defined as

$$x = \sum_{r=1}^{9} N_r x_r \; and \; y = \sum_{r=1}^{9} N_r y_r \tag{1}$$

Where (x, y) are the coordinates of any point within the element, (x_r, y_r) are the coordinates of the r^{th} nodal point and N_r is the corresponding interpolation function of the element. In this element, Lagrangian interpolation function is used for N_r by Zienkiewicz (1977). Taking the bending rotations as independent field variables, the effect of shear deformation may be expressed as

$$\begin{Bmatrix} \phi_x \\ \phi_y \end{Bmatrix} = \begin{Bmatrix} \theta_x - \dfrac{\partial w}{\partial x} \\ \theta_y - \dfrac{\partial w}{\partial y} \end{Bmatrix} \tag{2}$$

Where, ϕ_x and ϕ_y are the average shear rotation over the entire shell thickness and θ_x and θ_y are the total rotations in bending. The other independent field variables are u, v and w, where w is the transverse displacement while u and v are the corresponding in-plane displacements.

The interpolation functions used for the representation of element geometry, Eq. (1) are used to express the displacement field at a point within the element in terms of nodal variables as

$$u = \sum_{r=1}^{9} N_r u_r \;, v = \sum_{r=1}^{9} N_r v_r \;, w = \sum_{r=1}^{9} N_r w_r \;, \theta_x = \sum_{r=1}^{9} N_r \theta_{xr} \text{ and } \theta_y = \sum_{r=1}^{9} N_r \theta_{yr} \tag{3}$$

The generalized stress-strain relationship with respect to its reference plane may be expressed as

$$\{\sigma\} = [D]\{\varepsilon\} \tag{4}$$

Where

$$\{\sigma\}^T = [N_x N_y \; N_{xy} \; M_x \; M_y \; M_{xy} \; Q_x \; Q_y] \tag{5}$$

$$\{\varepsilon\} = \begin{Bmatrix} \partial u/\partial x + w/R_x \\ \partial v/\partial y + w/R_y \\ \partial u/\partial y + \partial v/\partial x \\ -\partial \theta_x/\partial x \\ -\partial \theta_y/\partial y \\ -\partial \theta_x/\partial y - \partial \theta_y/\partial x \\ \partial w/\partial x - \theta_x \\ \partial w/\partial y - \theta_y \end{Bmatrix} \tag{6}$$

and

$$[D] = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} & 0 & 0 \\ A_{61} & A_{62} & A_{66} & B_{61} & B_{62} & B_{66} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} & 0 & 0 \\ B_{61} & B_{62} & B_{66} & D_{61} & D_{62} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{55} & A_{54} \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{45} & A_{44} \end{bmatrix} \tag{7}$$

Table 1 Central deflection (w^*=100wD/qa^4) and bending moments (M^*_{max}=10M_{max}/qa^2 and M^*_{min}=10M_{min}/qa^2) of a simply supported isotropic skew plate under uniform distributed and concentrated load. (b/a=1, $h/2a$=0.01, ν=0.3, α=45°)

Load	Source	w^*	M^*_{max}	M^*_{min}
UDL	Present (12x12)#	2.093	1.520	0.993
	Present (16x16)	2.101	1.468	0.982
	Present (20x20)	2.103	1.452	0.974
	Butalia *et al*. (1990)	1.912	1.227	0.780
	Sengupta (1995)	2.203	1.326	0.901
Concentrated load	Present (12x12)	2.532	-	
	Present (14x14)	2.537		
	Present (16x16)	2.540		
	Butalia *et al.* (1990)	2.309		
	Aggarwal (1966)	2.509		

#Present result with different mesh divisions

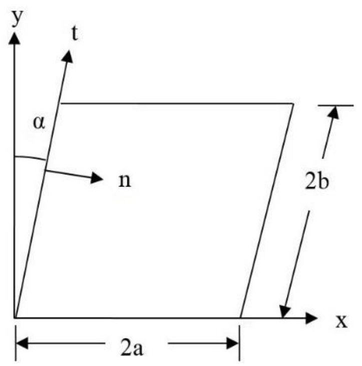

Fig. 1 Skew plate

The rigidity matrix $[D]$ constitutes the contributions of individual orthotropic layers oriented in different directions. Using the material properties and fiber orientations of these layers, it can be easily obtained following the steps available in any standard text on mechanics of fiber reinforced laminated composites.

With the help of Eqs. (3) and (6) the strain vector may be expressed in terms of the nodal displacement vector {δ} as

$$\{\varepsilon\} = \sum_{r=1}^{9} [B]_r \{\delta_r\}_e \tag{8}$$

Where $[B]$ is the strain displacement matrix containing interpolation functions and their derivatives and $\{\delta\}$ is the nodal displacement vector.

Once the matrices $[B]$ and $[D]$ are obtained, the stiffness matrix of an element $[K]_e$ can be easily derived with the help of virtual work method which may be expressed as

$$[K] = t \int_{-1}^{+1} \int_{-1}^{+1} [B]^T [D][B]|J| d\xi d\eta \tag{9}$$

Table 2 Deflections (w^*) and bending moments (M^*_x and M^*_y) for an isotropic plate

Source	b/a	w^*=10$^4 wqa^4/D$	M^*_x = 10$^3 M_x/qa^2$	
			M^*_x	M^*_y
Present	1	36.85	32.56	0
Thin plate solution (Timoshenko and Woinowsky-Krieger 1959)		36.8	32.5	0
Present	2	29.09	25.9	0
Thin plate solution (Timoshenko and Woinowsky-Krieger 1959)		29.1	25.8	0

In Eq. (9) the Jacobian matrix $|J|$ is derived from Eq. (1). The degrees of freedom of the inclined edges are transformed from global to local axes. This transformation is done in element level. In a similarmanner, the load vector $\{P_e\}$ may be expressed as

$$\{P_e\} = \iint q\,[N]^T |J| d\xi d\eta \tag{10}$$

The integration of the above Eqs. (9) and (10) is carried out numerically following the Gauss-quadrature rule.

The stiffness matrix and load vector having an order of forty-five are evaluated for all the elements and they are assembled together to form the overall stiffness matrix $[K]$ and load vector $\{P\}$ respectively.

Different boundary conditions used are simply-supported (S), clamped or fixed (C) and free (F).

Simply-supported (S) boundary condition is realized by using,

$$\begin{aligned} w = \theta_x = 0, &\text{At boundary line parallel to x} - \text{axis} \\ w = \theta_y = 0, &\text{At boundary line parallel to y} - \text{axi} \end{aligned} \tag{11}$$

Clamped or fixed (C) boundary condition is realized using,

$$w = \theta_x = \theta_y = 0 \tag{12}$$

Free (F) boundary condition is as follows,

$$w \neq 0, \theta_x \neq 0, \qquad \theta_y \neq 0 \tag{13}$$

After incorporating the boundary conditions in the overall system of equations it is solved to get the nodal displacements of the structure. Once nodal displacements are obtained, the stresses at any point within an element can be evaluated with the above equations.

3. Results and discussions

3.1 Isotropic skew plate

A simply supported isotropic skew plate as shown in Fig. 1 under uniform transverse distributed and centrally applied concentrated load with skew angle α=45°, aspect ratio b/a=1 and thickness ratio $h/2a$=0.01 is analysed. As the two sides are inclined to the global axis system (x-y), the necessary transformation is made to express the degree of freedom of the nodes on these two sides along t-x axis system (Fig. 1). The non-dimensional deflection and principal bending moments are given in Table 1 with those

Table 3 Deflection ($w^*=1000wE_2h^3/qa^4$), bending moments ($M^*=100M/qa^2$) and in-plane stresses ($\sigma^*=10h^2\sigma/qa^2$) at the centre of a simply supported cross ply (0/90/0) cylindrical shell panel under uniform distributed load. (b/a=1, R/a=3, h/a=0.1)

Source	w^*	M^*_x	M^*_y	σ^*_x	σ^*_y	σ^*_x	σ^*_y
				Top surface		Bottom surface	
Present (12x12)	9.09	11.85	1.13	7.57	0.41	-7.57	-0.41
Present (16x16)	9.09	11.97	1.15	7.65	0.42	-7.65	-0.42
Present (20x20)	9.09	12.15	1.17	7.76	0.42	-7.76	-0.42
Analytical (Seide and Chaudhuri, 1987)	8.69	12.06	1.089	7.32	0.40	-7.33	-0.37
FEM (Seide and Chaudhuri 1987)	10.84	11.83	1.281	8.40	0.46	-8.03	-0.43

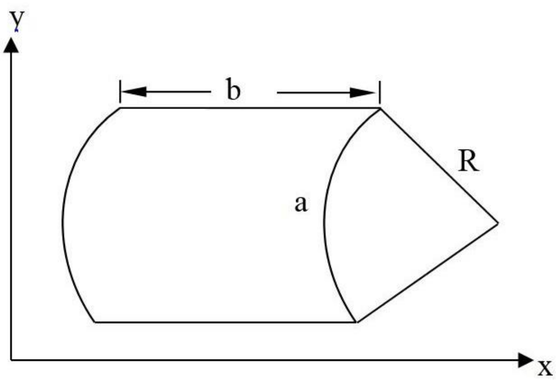

Fig. 2 Cylindrical shell panel

of Butalia *et al*. (1990), Sengupta (1995) and Aggarwal (1966). Butalia *et al.* (1990) used an eight-node isoparametric element whereas Sengupta (1995) used a 16-node triangular element and the fourthorder polynomial for transverse displacement. For the convergence test, present results are given with different mesh divisions. The present results are very close to the published results.

Next, an isotropic plate under transverse hydrostatic load is investigated. The left side of the plate is free and other three edges are simply supported. The deflections, bending moments and in-plane stresses at x=0 and y=a/2 are presented in Table 2 with the thin plate solutions of (Timoshenko and Woinowsky-Krieger 1959). There is an excellent agreement between the present results and that of Timoshenko and Woinowsky-Krieger (1959) for isotropic plate.

3.2 Laminated composite cylindrical shell panel

A three-layer symmetric cross-ply (0/90/0) cylindrical shell panel (Fig. 2) with aspect ratio b/a=1, radius to side ratio R/a=3 and thickness ratio h/a=0.1 is analysed. The shell is simply supported along all the four edges and subjected to a uniform distributed transverse load. The non-dimensional values of deflection, bending moments and in-

Table 4 Deflection ($w^*=1000wE_2h^3/qa^4$), bending moments ($M^*=100M/qa^2$) and in-plane stresses ($\sigma^*=10h^2\sigma/qa^2$) at the centre of a simply supported cross ply (0/90) skew cylindrical shell panel under doubly sinusoidal load. (b/a=1, R/a=3, h/a=0.1)

Skew angle	R/a	Source	w^*	M^*_x	M^*_y	σ^*_x	σ^*_y
						Bottom surface	
0	Infinity	Present	12.17			-7.20	
		FSDT (Reddy 1989)	12.37	4.21	4.21	-7.16	-0.848
		HSDT (Reddy 1989)	12.14			-7.47	
15	3	Present	10.29	4.07	3.59	-6.55	-0.782
30			7.78	3.33	4.59	-5.26	-0.726
45			5.06	2.28	7.33	-3.58	-0.705
60			2.64	1.30	8.11	-1.99	-0.662

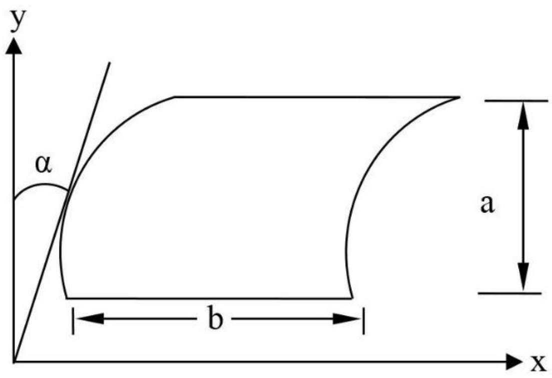

Fig. 3 Skew cylindrical shell panel

plane normal stresses obtained at the center of the shell panel are presented with those of Seide and Chaudhury (1987) in Table 3. Seide and Chaudhury (1987) analysed the shell with a quadratic C^o triangular curved element as well as analytically. The Table 3 shows that the present results are in good agreement with those of Seide and Chaudhury. The material property used in the example are $E_1=25E_2$, $G_{12}=G_{13}=G_{23}=0.5E_2$ and $\upsilon_{12}=0.25$. Since the present finite element formulation gives good results for the isotropic skew plate and composite shell panels, it can be concluded that present formulation can be applied for composite skew shell panels also.

3.3 Laminated composite skew cylindrical shell panel

A two-layer anti-symmetric cross-ply (0/90) composite skew cylindrical shell panel (Fig. 3) having aspect ratio b/a=1 and h/a=0.1 is studied. The material property used in the example are $E_1=25\ E_2$, $G_{12}=G_{13}=0.5\ E_2$, $G_{23}=0.2\ E_2$ and $\upsilon_{12}=0.25$. Same material properties are used for rest of the examples as well. The shell is simply supported along all the four edges and subjected to a transversedouble sinusoidal varying load. The analysis is performed considering R/a=3. The present results are given in Table 4 with the finite element solution of Reddy (1989). The present results are very close to the solution of Reddy

Table 5 Deflection (w^*=1000wE_2h^3/qa^4), bending moments (M^*=100M/qa^2) and in-plane stresses (σ^*=10$h^2\sigma/qa^2$) at the centre of a skew cylindrical shell panel fixed along all the edges under doubly sinusoidal load. (b/a=1, R/a=3, h/a=0.1, α=45°)

Ply orientation	w^*	M^*_x	M^*_y	σ^*_x	σ^*_y	σ^*_x	σ^*_y
				Bottom surface		Middle surface	
Anti-symmetric							
0/90/0/90	1.76	1.16	0.95	-1.22	-0.15	0.335	-0.050
75/-75/75/-75	1.46	0.28	1.91	-0.21	-1.59	-0.037	-0.477
60/-60/60/-60	1.59	0.66	1.74	-0.48	-1.34	-0.074	-0.257
45/-45/45/-45	1.87	1.35	1.43	-0.90	-1.00	-0.039	-0.087
30/-30/30/-30	2.30	2.21	1.02	-1.26	-0.65	0.066	-0.034
Symmetric							
0/90/90/0	2.00	1.68	1.17	-1.15	-0.22	-0.01	-0.97
75/-75/-75/75	1.53	0.31	1.86	-0.22	-1.68	-0.05	-0.66
60/-60/-60/60	1.76	0.70	1.77	-0.51	-1.37	-0.15	-0.49
45/-45/-45/45	2.13	1.47	1.56	-0.93	-1.05	-0.25	-0.30
30/-30/-30/30	2.30	2.12	1.02	-1.26	-0.65	0.07	-0.03

Table 6 Deflection (w^*=1000wE_2h^3/qa^4), bending moments (M^*=100M/qa^2) and in-plane stresses (σ^*=10$h^2\sigma/qa^2$) at the centre of a skew cylindrical shell panel fixed along two straight edges and two curved edges are free under uniform distributed load. (b/a=1, R/a=3, h/a=0.2, α=45°)

No. of Plies	w^*	M^*_x	M^*_y	σ^*_x	σ^*_y
				Bottom surface	
$(0/90)_2$	11.64	-0.471	2.075	0.680	-0.588
$(0/90)_4$	10.54	-0.578	2.545	0.684	-0.281
$(0/90)_6$	10.46	-0.580	2.705	0.712	-0.247
$(0/90)_8$	10.43	-0.578	2.785	0.730	-0.233
$(0/90)_{10}$	10.42	-0.576	2.832	0.742	-0.226

(1989). The deflections, bending moments and in-plane stresses for different skew angles are presented as new results. It is seen that as skew angle increases, the shell becomes stiffer and subsequently deflection reduces.

A four-layer symmetric and anti-symmetric cross and angle ply skew composite cylindrical shell panel fixed along all the four edges subjected to a sinusoidal load having b/a=1, h/a=0.1, R/a=3 and α=45° is analysed. The analysis is performed considering different ply orientations of the shell panel. The non-dimensional deflection, bending moments and in-plane stresses at the center are presented in Table 5. From the results, it is seen that for the same thickness and number of plies, the symmetric laminate is stiffer compared to anti-symmetric lamination.

In the next example, anti-symmetric cross-ply skew cylindrical shell panel subjected to the uniform distributed load having a different number of layers is analysed taking thickness ratios h/a=0.2. The panel is fixed supported along the two straight edges and other two inclined edges are free. The non-dimensional deflection, bending moments and in-plane stresses at the center are presented in Table 6. It is seen that the shell becomes stiffer as the number of layers increases.

Table 7 Deflections (w^*=1000wE_2h^3/qa^4), bending moments (M^*_x and M^*_y) and in-plane stresses (σ^*=10$h^2\sigma/qa^2$) of composite (0/90/0) skew cylindrical shell panel (having left curved edge free and other edges are simply supported) subjected to transverse hydrostatic load. α=30°, h/a=0.1

R/a	b/a	w^*	M^*=100M/qa^2		σ^* Bottom Surface	
			M^*_x	M^*_y	σ^*_x	σ^*_y
10		33.55	-188.56	8.71	-0.517	-0.367
5	1	33.07	-185.62	8.58	-1.05	-0.712
4		32.73	-183.22	8.05	-1.25	-0.881
10		19.85	-71.60	0.75	-0.571	-0.048
5	2	19.75	-71.15	0.75	-1.13	-0.095
4		19.68	-70.81	0.75	-1.19	-1.20

Table 8 Deflection (w^*=1000wE_2h^3/qa^4), bending moments (M^*=100M/qa^2) and in-plane stresses (σ^*=10$h^2\sigma/qa^2$) at centre of a simply supported skewed cross ply (0/90/0) cylindrical shell panel under uniformly distributed load. (b/a=1, α=45° and R/a=5)

h/a	w^*	M^*		σ^*_x	σ^*_y
		M^*_x	M^*_y	Bottom surface	
0.01	2. 298	4.408	0.830	-3.220	-0.252
0.03	3.555	6.751	1.288	-4.441	-0.416
0.06	4.243	6.990	1.428	-4.492	-0.466
0.09	5.118	6.844	1.572	-4.372	-0.516
0.12	6.259	6.590	1.737	-4.204	-0.565
0.15	7.659	6.308	1.903	-4.026	-0.614
0.18	9.311	6.032	2.056	-3.856	-0.664
0.21	11.21	5.779	2.191	-3.702	-0.706

In the next example, a three-layer symmetric (0/90/0) cross-ply laminated skew cylindrical shell panel (Fig. 3) subjected to transverse hydrostatic load is analysed. The load varies linearly with the intensity of the load at the left side is zero and maximum at the right side (q). The non-dimensional deflection, bending moment and in-plane stresses at the center are presented in Table 7.

Next, a simply supported three-layer symmetric angle-ply (0/90/0) skew cylindrical shell panel (Fig. 3) having various thickness ratios is analysed. The panel is subjected to a transverse uniformly distributed load. The aspect ratio, skew angle and radius to side ratio of the panel are b/a=1, α=45° and R/a=5 respectively. The non-dimensional deflection, bending moment and in-plane stresses at the center are presented in Table 8.

Next, a simply supported skew unsymmetrical angle ply cylindrical shell panel (Fig. 3) subjected to doubly sinusoidal distributed load having aspect ratio a/b=1.0, thickness ratio h/a=1.0, skew angle α=45° and R/a=3 is analysed. Here number of plies varies from two to ten with various fiber angle orientations (from 15° to 75°). The results obtained in terms of non-dimensional deflection (w^*)

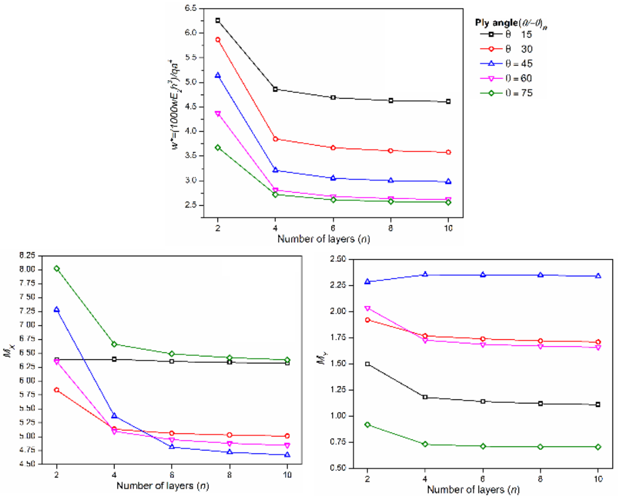

Fig. 4 Variation of non-dimensional deflection (w^*) and bending moments (M^*_x and M^*_y) at different number of layers

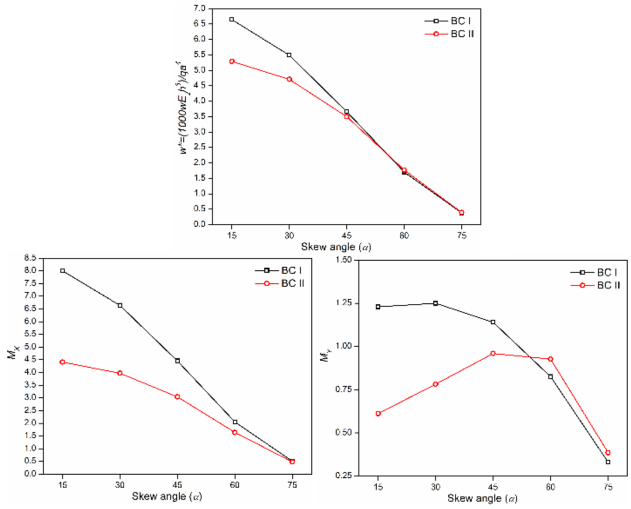

Fig. 5 Variation of non-dimensional deflection (w^*) and bending moments (M^*_x and M^*_y) at different skew angles

and bending moments (M^*_x and M^*_y) is presented in graphical form as shown in Fig. 4. From Fig. 4, it is seen that as number of layers increases deflection and bending moments decrease for all fiber angle orientations. As number of layers increases stiffness of the shell increases and consequently deflection and moments decrease. It is also seen that when fiber angle orientations changes from 15° to 75° deflection decreases. This is because as fiber angle orientation increases, stiffness of the shell also decreases.

In the last example, a three-layer symmetric cross ply (0/90/0) cylindrical shell panel subjected to uniformly distributed load having aspect ratio a/b=1.0, thickness ratio h/a=1.0 and R/a=5 with varying skew angle (α=15° to 75°) is analysed. Here two types of boundary conditions are used. In one case the two straight edges are fixed and other two curved edges are free (represented as BC I) and in the second case curves edges are fixed and straight edges are free (represented as BC II). From Fig. 5, it is seen that as skew angle increases the shell becomes more and more stiff and subsequently deflection decreases. From Fig. 5 it is also clear that shell having two curved edges fixed and straight edges free is stiffer compared to another boundary condition.

4. Conclusions

The present analysis is performed considering the first order shear deformation theory. The entire analysis is carried out by using a nine node isoparametric element. Skew cylindrical shell panels with different fiber orientation angles, number of layers, radius to side ratio, thickness ratio, boundary condition and loading condition are analysed. The present formulation is validated with number of published results related to isotropic skewed plate and composite cylindrical shell panels. In all the cases the results obtained by the present formulations are veryclose to the published solutions. Few new examples of laminated composite skew cylindrical shell panels are presented which will be useful for future research in this field. It is seen that in general as number of layers is increased, the deflection and moments decrease. Also, as the skewness of the shell increases, the deflection decreases.

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