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Modelling of shear deformation and bond slip in reinforced concrete joints

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Abstract. A macro-element model is developed to account for shear deformation and bond slip of reinforcement bars in the beam-column joint region of reinforced concrete structures. The joint region is idealized by two springs in series, one representing shear deformation and the other representing bond slip. The softened truss model theory is adopted to establish the shear force-shear deformation relationship and to determine the shear capacity of the joint. A detailed model for the bond slip of the reinforcing bars at the beam-column interface is presented. The proposed macro-element model of the joint is validated using available experimental data on beam-column connections representing exterior joints in ductile and nonductile frames.

Key words: reinforced concrete; connections; shear strength; bond slip; modelling; shear deformation; joints.

1. Introduction

The philosophy for a desirable reinforced concrete beam-column joint design is that joint deformation should not significantly increase the story drift. Properly designed and detailed joints may be considered to behave elastically. Existing reinforced concrete frames that were built before current seismic codes, may have inadequate joint reinforcement and detailing. This deficiency may result in local joint shear failure before other flexural elements reach their capacity. When reinforced concrete frames are subjected to earthquake motion, joint zone distortions can have significant impact on the story drift and the overall deflection of the structure. In the seismic analysis of moment resisting frames, joint deformations are routinely ignored and the joints are normally assumed rigid. This assumption implies that the members remain at right angles even after the joint has undergone severe shear deformation. It is, therefore, important to develop models capable of simulating the hysteretic behaviour of the shear deformation in joints and bar bond slip at beam-column interfaces.

Joint deformation is due to two different effects: a) shear deformation of the joint core due to applied shear forces; and b) slippage of the longitudinal reinforcing bars anchored in or passing through the joint core due to bond degradation. Joint shear deformation results in rotations of the side faces of the joint. The shear deformation depends on the distortion angle of the joint core and on the joint dimensions. Bond slip causes pullout of the bars with cracks opening at the beam-

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joint interface, which induces fixed-end rotations of the beams framing into the joint (Monti 1995).

Numerous experimental studies on the hysteretic behaviour of joints have been conducted, however, analytical studies are limited. Atrach (1992) presented a finite element model for incorporating inelastic shear distortions in the joint region. This approach is useful in explaining the joint behaviour but it is cumbersome for practical analysis of structures with a large number of joints.

The objective of this study is to develop a simple yet sufficiently accurate macro-model for the joint whose shear-distortion properties under reversed cyclic loading and bar bond slip are specified explicitly. The proposed joint element is implemented in the nonlinear dynamic analysis program SARCF (Chung *et al.* 1988) developed for the seismic analysis of reinforced concrete frames.

2. Beam-column joint behaviour

Joints in reinforced concrete frames may be subjected to severe cyclic shear under seismic excitations. Experiments on beam-column joints designed to earlier codes (Ghobarah *et al.* 1996) have shown that joint deformation may contribute more than 50% of the story drift. Satisfactory joint behaviour is associated with minimal contribution of joint distortion to the overall lateral drift of the structure. Ideally, joint contribution to drift should decrease with increasing total lateral drift as the adjacent beams develop plastic hinges and undergo the bulk of the inelastic action. The majority of existing structures that were designed before seismic codes were available have inadequate joint design and detailing. In these cases, the contribution of joint distortion to the total drift increases with increasing total lateral drift until joint failure occurs (Ghobarah *et al.* 1996). In addition to the code requirement for a strong column-weak beam design, special measures are needed to ensure that the joint responds elastically during earthquakes.

Adequately detailed joint mobilizes two self equilibrating mechanisms of shear transfer. The first mechanism is the truss mechanism, in which the core concrete supplies the necessary diagonal compression field, balanced by the boundary forces and horizontal and vertical tension in the reinforcement passing through the joint core. Once the bond deteriorates, the principal component of the second mechanism which is the diagonal compression strut forming across the compression zones of the framing members and directly transmitting shear in the form of inclined compression. Since joint shear can cause dilation of the joint core, the steel ties in the joint region provide confinement to the core as well as participate in the joint shear resisting mechanism (Pantazopoulou and Bonacci 1992).

When beam reinforcement steel has favourable bond conditions, high shear forces are transferred to the joint which require large amount of joint hoop reinforcement. However, in the case of poor bond conditions along the positive moment reinforcement bars in the beam, the joint rotates as a rigid body to accommodate the excessive slippage of the bars which could not be counteracted by increased transverse reinforcement (Pantazopoulou and Bonacci 1994). In this case, the shear in the joint is reduced and the contribution of bond-slip to story drift is accompanied by reduction in the contribution of joint distortion to drift.

Several experimental studies have been conducted on reinforced concrete beam-column joint subassemblages (Durrani and Wight 1985, Leon 1990, Ghobarah *et al.* 1996). The measured response of the specimens is affected by the wide range of influencing parameters such as transverse beams, slabs and percentage of beam reinforcement. Several conceptual models have been developed based on equilibrium requirements (Paulay 1989). However, these models can only predict the shear capacity of the joint. More recently, mathematical formulations for interior joints (Pantazopoulou and Bonacci 1992) were conducted considering joint kinematics and material response. However, they are limited to interior joints that are well designed and detailed to satisfy code reinforcement requirements.

3. Modelling of shear deformation

Experimental studies showed that joints can be modelled as a two dimensional panel in the direction of loading, reinforced with column longitudinal reinforcement in the vertical direction and hoop reinforcement in the transverse direction. The element is acted upon by in-plane shear stresses and normal stresses as shown in Figs. 1a and 1b. A mechanical model using the softened truss model theory (Hsu 1988) is used to satisfy both equilibrium of stress resultants and compatibility of deformations within the joint taking into account the constitutive laws of concrete and reinforcement. The actual shape of the shear stress-shear strain relationship is obtained without the need for experimental calibration. An accurate prediction by the softened truss model strongly depends on the constitutive material laws for concrete and steel in the element.

The equations used in the softened truss model theory are: a) equilibrium equations assuming steel bars to resist only axial stresses as shown in Figs. 1c, 1d and 1e; b) compatibility equations of Collins (1978) to determine the angle of inclination of the concrete struts, as shown in Figs. 1f and 1g; and c) constitutive laws of materials. It is important in the model that the nonlinear stress-strain relationship for concrete represents the softening of concrete in compression caused by cracking due to tension in the perpendicular direction (Vecchio and Collins 1986).

The average principal tensile stress in concrete, f_{c1}, is related to the principal tensile strain, ε_1. Prior to cracking, f_{c1} is given by the following expression:

$$f_{c1} = E_c\ \varepsilon_1 \tag{1}$$

After cracking, f_{c1} is given by:

$$f_{c1} = \frac{\alpha_1 \alpha_2 f_{cr}}{1+\sqrt{500\varepsilon_1}}, \quad \text{where } f_{cr} = 0.33\sqrt{f_c'}\ \text{MPa} \tag{2}$$

where α_1 and α_2 are factors accounting for the bond characteristics of reinforcement and the type of loading. The value of α_1 is 1.0 for deformed reinforcement bars, 0.7 for plain bars and 0.0 for unbonded reinforcement. The value of α_2 is taken 1.0 for monotonic loading and 0.7 for cyclic loading. The concrete compressive strength in MPa is denoted f_c'.

The described analysis considers average stresses and average strains while local variations are ignored. At cracks, there is no tensile stress in the concrete while the tensile stresses in the reinforcement are greater than their average values. The shear capacity of the joint may be limited by its ability to transmit forces across the crack. To maintain static equilibrium between the average stresses and the localized stresses it is required to limit the value of f_{c1} as follows:

$$f_{c1} = \frac{0.18\sqrt{f_c'}]\tan\theta}{0.3+\dfrac{24w}{a+16}}\ \text{MPa} \tag{3}$$

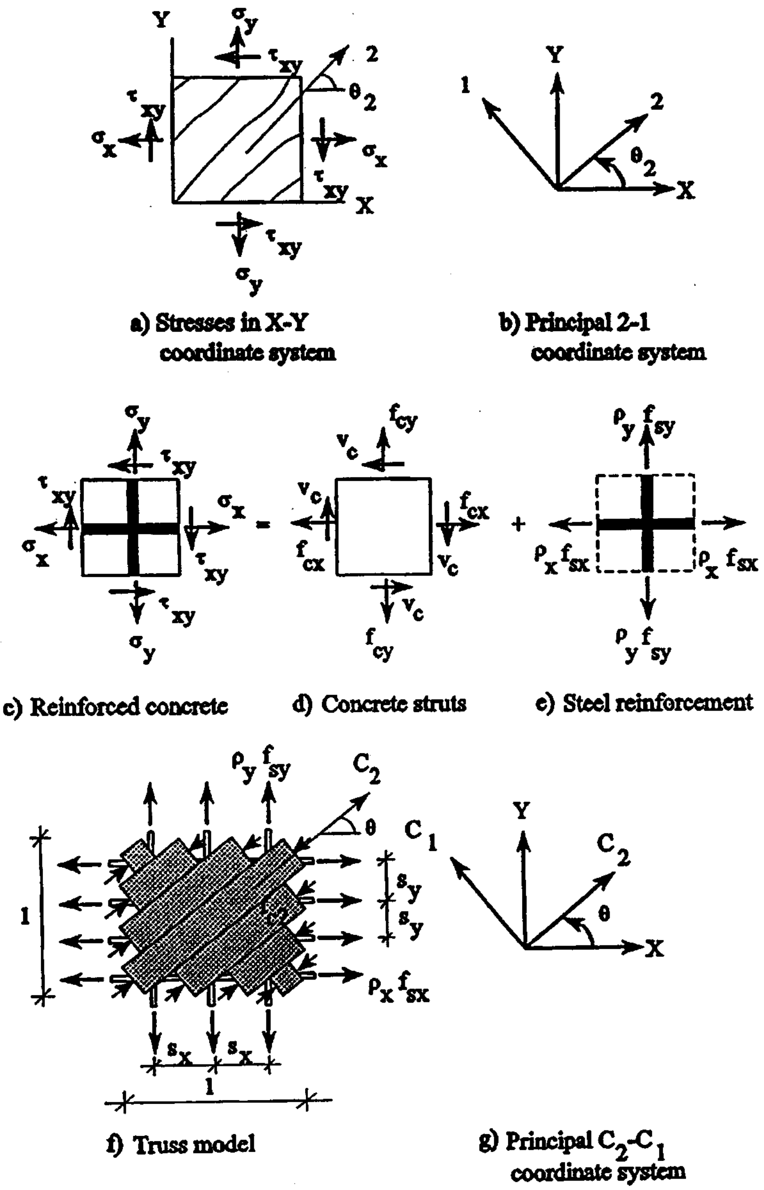


Fig. 1 Definitions of stresses and coordinate systems

where a is the maximum aggregate size (in mm), the crack width, $w=\varepsilon_1\, S_{m\theta}$ (in mm) and $S_{m\theta}$ is the spacing of the inclined cracks, which depends on the crack control characteristics of the reinforcement in both directions and the inclination of the cracks.

The following procedure is proposed for the calculation of $S_{m\theta}$ based on the element tests by Vecchio and Collins (1986) and the current concrete design code (CSA A23.3-94, 1994) approach.

1) In a code designed joint, the required cross sectional area of the tie bars, A_{tie}, in the joint within a vertical distance S is calculated as follows:

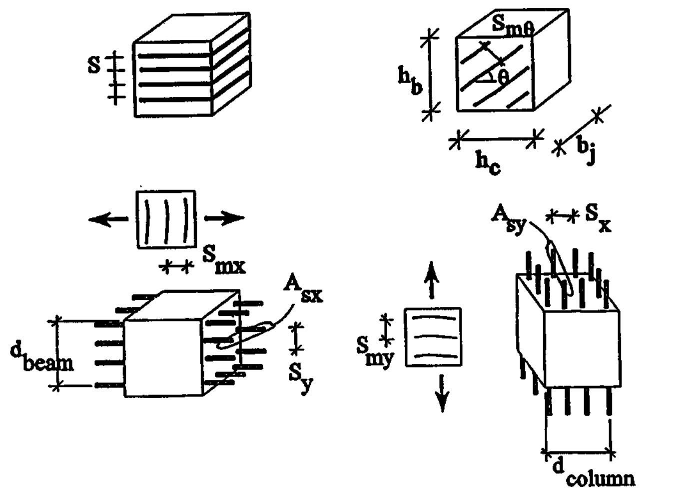
$$A_{tie} = 0.06 \frac{\sqrt{f_c{}'}}{f_y} b_j \, S \tag{4}$$

where b_j is the joint width and S is the vertical spacing between the ties (in mm) and f_y is the yield stress of the steel reinforcement in MPa.

2) The cross sectional area of the ties in an existing joint is compared to the required cross sectional area of the ties A_{tie} given by Eq. (4). If the cross sectional area of the ties in the joint is less than the required area A_{tie}, then:

$$S_{m\theta} = \frac{1}{\dfrac{\sin\theta}{S_{mx}} + \dfrac{\cos\theta}{S_{my}}} \tag{5}$$

where S_{mx} and S_{my} are the average crack spacing that would result if the element was subjected to a horizontal and vertical tension, respectively, as shown in Fig. 2. The crack spacing S_{my} depends on the distribution and cross sectional area of the horizontal reinforcement. If the cross sectional area of the horizontal reinforcement is greater than or equal to 0.004 b_j S_y then S_{my}=S_y; otherwise S_{my}=0.9 the depth of the beam d_{beam}. The spacing between the longitudinal reinforcement is S_y. A similar approach is adopted in the case of the vertical reinforcement. If the cross sectional area of the ties in the joint is greater than or equal to the required area of ties A_{tie}, then $S_{m\theta}$=300 mm.



If $A_{sx} < 0.004\ b_j\ S_y$ then $S_{mx} = 0.9\ d_{beam}$

If $A_{sx} \geq 0.004\ b_j\ S_y$ then $S_{mx} = S_y$

If $A_{sy} < 0.004\ b_j\ S_x$ then $S_{my} = 0.9\ d_{column}$

If $A_{sy} \geq 0.004\ b_j\ S_x$ then $S_{my} = S_x$

where $S_{m\theta}$=the average crack spacing
S_{mx}=the average crack spacing that would result if the element was subjected to horizontal tension
S_{my}=the average crack spacing that would result if the element was subjected to vertical tension

Fig. 2 Joint geometry

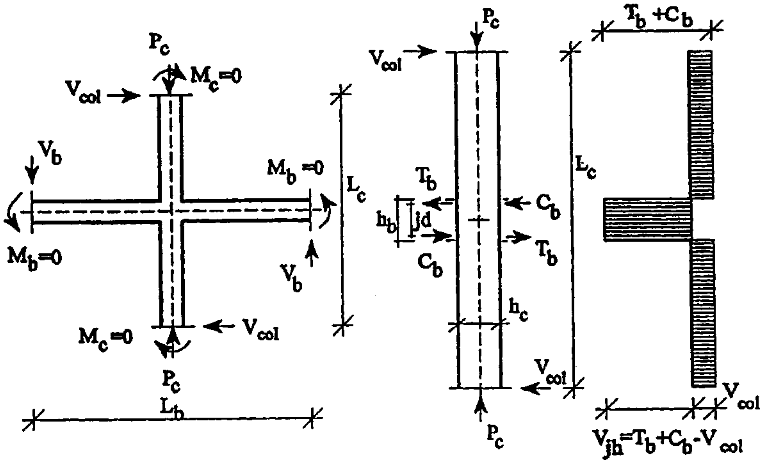
3.1. Criterion for shear failure

The shear failure of the joint element can be categorized as tensile failure or compressive failure. As the compressive strength of concrete deteriorates due to the existence of cracks, it is more appropriate to define the compressive failure by the maximum strain in the concrete. The failure criterion in compression is expressed by, $\varepsilon_c \geq \varepsilon_{cc}'$, where ε_c is the compressive strain of concrete and ε_{cc}' is the compressive strain corresponding to the maximum strength. Tensile failure will occur when the reinforcing bar reaches the limit state. It is observed experimentally (Izumo 1992) that the strain affects the maximum strength of the reinforced concrete panel less than the stress. Thus, the maximum tensile strain of the reinforcing bar of 0.03 is selected to correspond to the tensile failure of the reinforced concrete joints.

3.2. Equilibrium of joint forces

In the current study, the joint shear deformation is represented by a rotational spring. First, the softening truss model is used to determine the shear stress-shear deformation relationship of the joint. This relationship has a corresponding moment-rotation relationship that defines the characteristics for the rotational spring to represent the shear deformation in the joint. The moment transmitted by the element is the moment transferred from beams to the column. The rotation of the element represents the shear deformation of the joint which is the change in the angle between the connected beams and columns. Fig. 3a shows a free body diagram of an interior beam-column assemblage between its points of contra flexure while Fig. 3b shows the equilibrium of an interior column with equal and opposite axial forces substituted for the beam moment. The length along the beam between the contra flexure points is L_b and that along the column is L_c. The shear diagram is plotted in Fig. 3c. Equilibrium of the forces in Fig. 3b in the horizontal direction gives:

$$V_{jh} = C_b + T_b - V_{col}, \quad C_b = T_b = \frac{M_b}{jd} \tag{6}$$



a) Beam-column connection **b) Column equilibrium** **c) Shear diagram**

Fig. 3 Equilibrium of interior beam-column joint where jd, is the moment arm of the beam. Summing the moments in Fig. 3a and expressing the shear in terms of an equivalent beam moment results in:

$$V_{jh} = 2M_b \left[\frac{1}{jd} - \frac{1}{L_c(1-\frac{h_c}{L_b})} \right] \tag{7}$$

Defining ΣM_b as the total beam moment to be transferred to the column by joint shear, then:

$$\sum M_b = 2M_b = \frac{V_{jh}}{\frac{1}{jd} - \frac{1}{L_c(1-\frac{h_c}{L_b})}} \tag{8}$$

Using the same procedure of calculating the total beam moment ΣM_b, as a function of the joint shear force, formulations for the ΣM_b can be made for interior and exterior top floor beam-column connections including corner joints.

3.3. Model verification

The calculated shear-deformation envelopes of the reinforced concrete joints are compared to the test results of six beam-column joints by Ghobarah *et al.* (1996), eight beam-column joints by Fujii and Morita (1991) and eighteen beam-column joints by Kaku and Asakusa (1991). Some of the comparisons are plotted in Figs. 4, 5 and 6. The analytical results compare satisfactorily with experiments if the connection failed by joint shear. For a well designed joint as in the case of specimen J2 in Fig. 4 and specimen #1 in Fig. 6, no more joint deformation occurs in the specimens after the beam hinging. Most of the damage and eventual failure occurred in the beam hinge zone. The joint contribution to drift in case of specimen J1 was more than 50% compared with less than 20% joint contribution to drift in case of J2 (Ghobarah *et al.* 1996). The proposed model is shown to be effective for the analysis of reinforced concrete joints under monotonic loading. A tri-linear idealization is assumed. The hysteretic model for the shear-distortion relationship proposed by Sakata and Wada (1991) (Fig. 7) is used in this analysis.

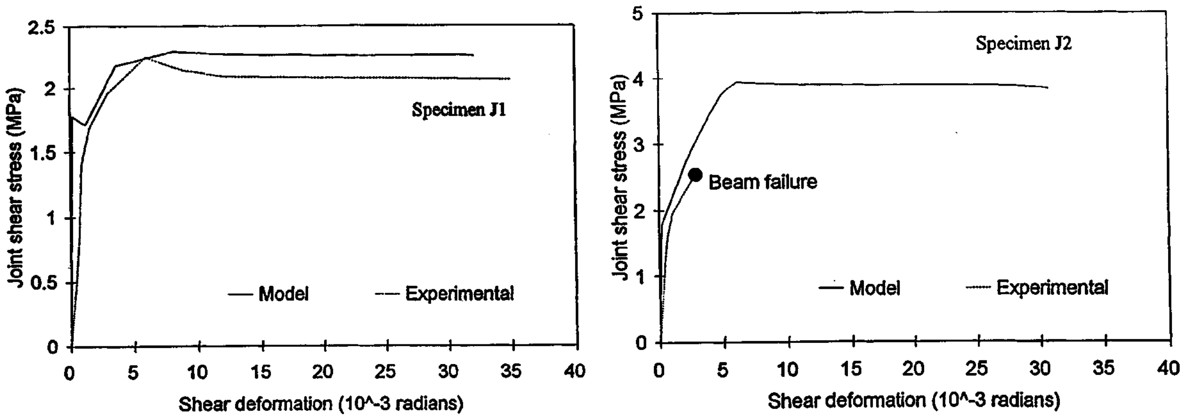


Fig. 4 Analytical results compared to experimental results (Ghobarah *et al.* 1966)

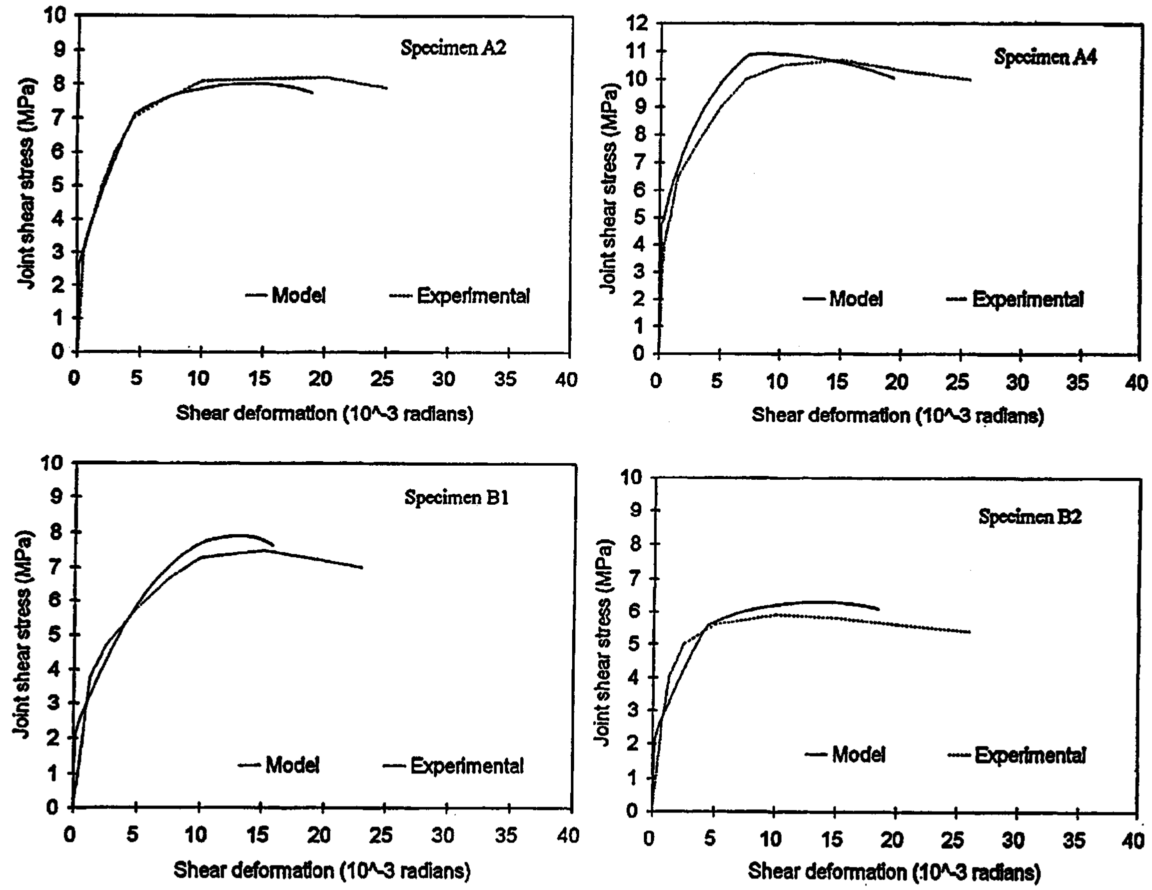


Fig. 5 Analytical results compared to experimental results of Fujii and Morita (1991)

4. Modelling of bond slip

Based on experimental results, several investigators (Kaku and Morita 1978) indicated that additional rotations caused by the anchorage slip of the reinforcing bars from the beam-column joints contribute significantly to the total inelastic deformations of reinforced concrete multistorey frames subjected to strong earthquake motions. Analytical approaches to determine the load-displacement relationship of reinforced concrete beam-column subassemblages and frames are developed considering these additional rotations (Ichinose 1983). More complex bond-slip models have also been proposed (Hawkins and Lin 1979, Filippou *et al.* 1986, Filippou 1986 and Alsiwat and Saatcioglu 1992).

4.1. Moment-rotation relationship under monotonic loading

In the current study, a simple moment-additional rotation relationship is proposed. The moment-rotation relationship due to bond-slip is defined by the critical points of cracking (M_{cr}), yielding

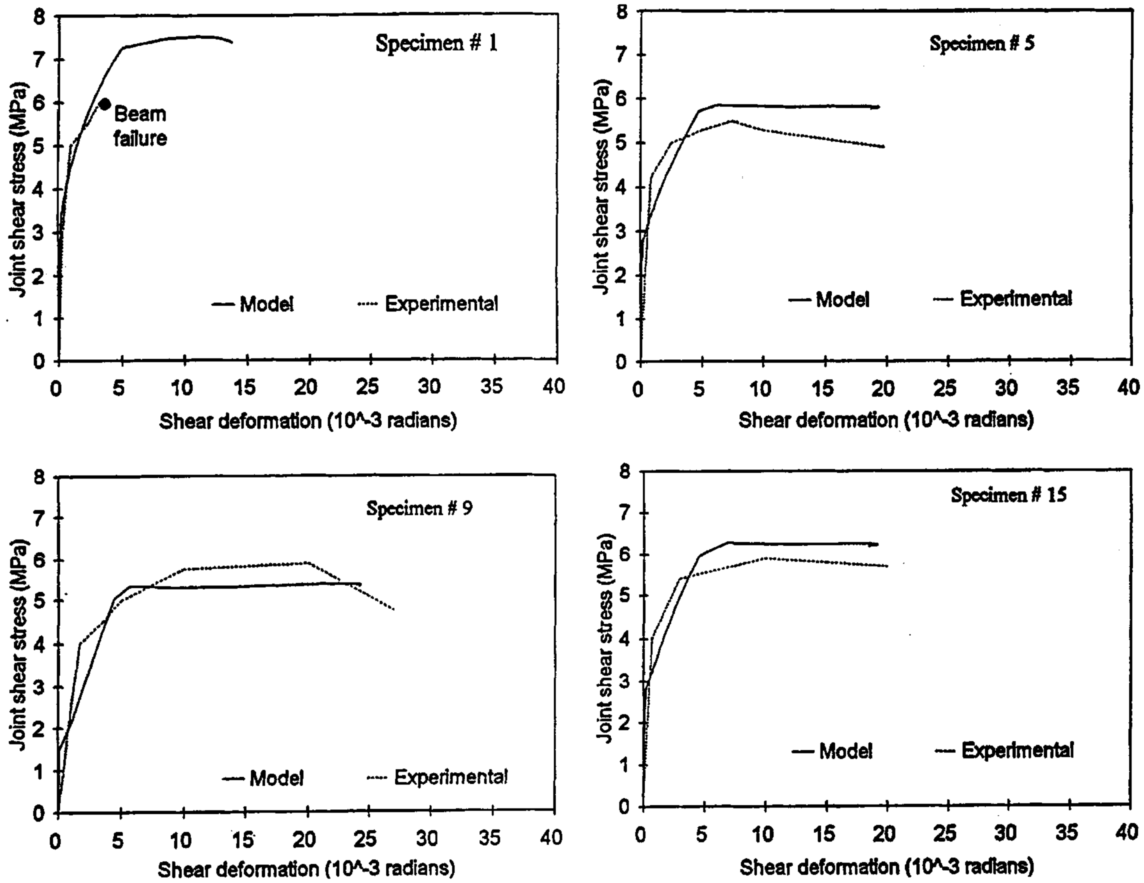
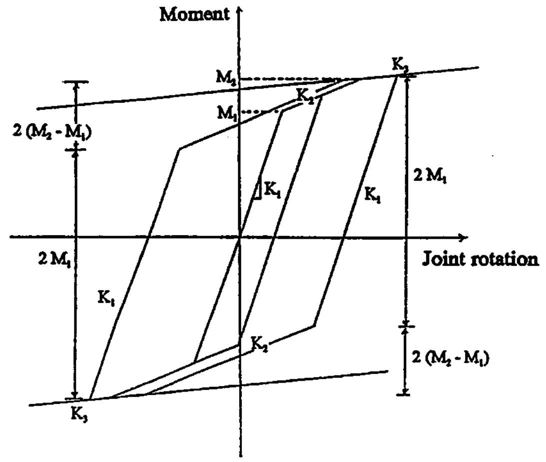


Fig. 6 Analytical results compared to experimental results of Kaku and Asakusa (1991)



Hysteretic model

Fig. 7 Typical moment-rotation relationship for the shear spring

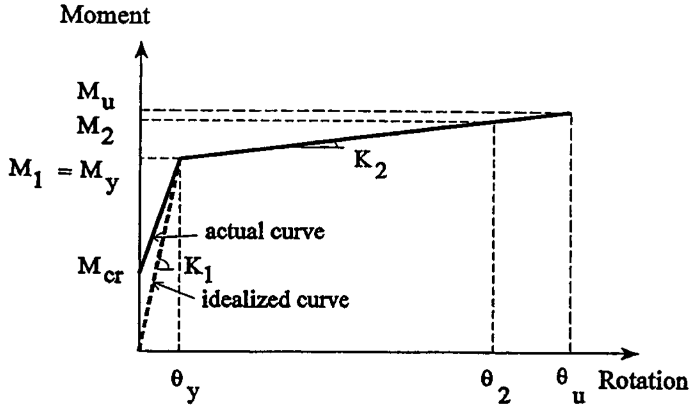


Fig. 8 Proposed moment-rotation relationship due to bond slip

(M_y, θ_y) and ultimate conditions (M_u, θ_u). A bi-linear idealization is assumed as shown in Fig. 8. The advantage of this simple relationship is its easy application and introduction into a nonlinear frame analysis. The various constants used in this formulation are determined based on analysis and experimental data.

Neglecting the concrete strain along the bar in the joint, the slippage Δ_s at the beam-column interface is given by the integration of the steel strain distribution over a length L_s where L_s represents the distance from the beam-column interface to the point at which the bar begins to slip (Fig. 9a). In the figure, the strain distribution along the slipping length of the bar is plotted. The maximum strain at the beam-column interface is denoted ε_s. From the stress-strain relationship for steel, the corresponding distribution of stress in the bars along the slipping length L_s, is also shown.

Before bar yielding ($\varepsilon_s \leq \varepsilon_y$), it is assumed that the steel stress and strain distributions are linear and that the concrete bond stress is uniform as given by the expression:

$$f_b = 600\, \varepsilon_s \sqrt{f_c'} \ \text{MPa} \tag{9}$$

The constant value of 600 was determined from experimental measurements of Morita and Kaku

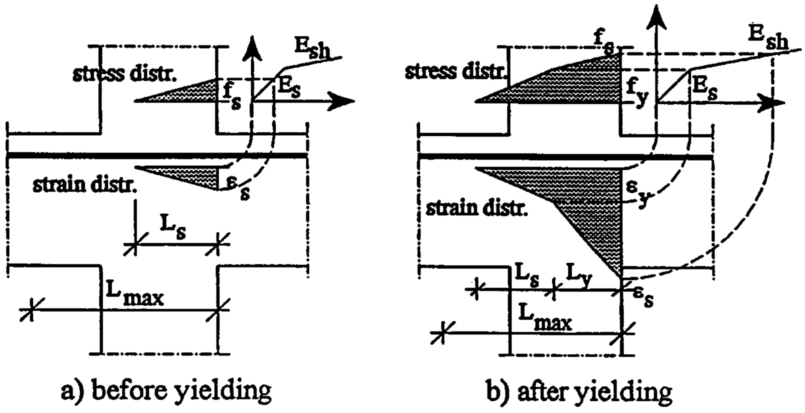


Fig. 9 Stress and strain distribution assumption in the joint

(1972). This value also gives good agreement between the bond stress and that calculated by the ACI Committee 408 (1979). From the equilibrium of the bar axial force, L_s can be calculated as:

$$L_s = \frac{f_s A_s}{\sum \text{perimeter. } f_b} = \frac{E_s\ d_b}{2400\sqrt{f_c'}} \tag{10}$$

The bar slippage from the joint in mm is given by:

$$\Delta_s = \frac{L_s\ \varepsilon_s}{2} = \frac{f_s\ d_b}{4800\sqrt{f_c'}} \tag{11}$$

where d_b is the diameter of the bar in mm, E_s is the Young's modulus of steel in MPa and f_s is the beam reinforcing bar stress in MPa.

After the yielding of the steel bar, ($\varepsilon_s > \varepsilon_y$), it is assumed that the bar stress-strain relationship includes no plastic flow and that strain hardening immediately follows. The stress and strain distributions in the joint may be assumed bilinear as shown in Fig. 9b. After yielding, the slope of the bilinear M-θ relationship depends on the strain-hardening characteristics of the reinforcing bars, the thickness of the concrete cover relative to the dimensions of the cross section confined by the transverse steel, the amount of transverse steel, the size of the yielding region, the penetration of yielding into the beam-column connection as well as other factors. The upper limit length of the slip region is denoted L_{max} and is defined in Fig. 10 for interior and exterior joints. The yield development length is denoted L_y in Fig. 9b and was determined from the strain measurements of bars in the joints of the beam-column subassemblages tested by Morita and Kaku (1984) as follows:

$$\chi = \frac{f_s}{f_y} \leq \frac{f_{ult}}{f_y} \tag{12}$$

for $1 < \chi \leq 1.05$ $\quad L_y = 2000\,\chi - 2000$ mm
$\chi > 1.05$ $\quad L_y = 500\,\chi - 425$ mm

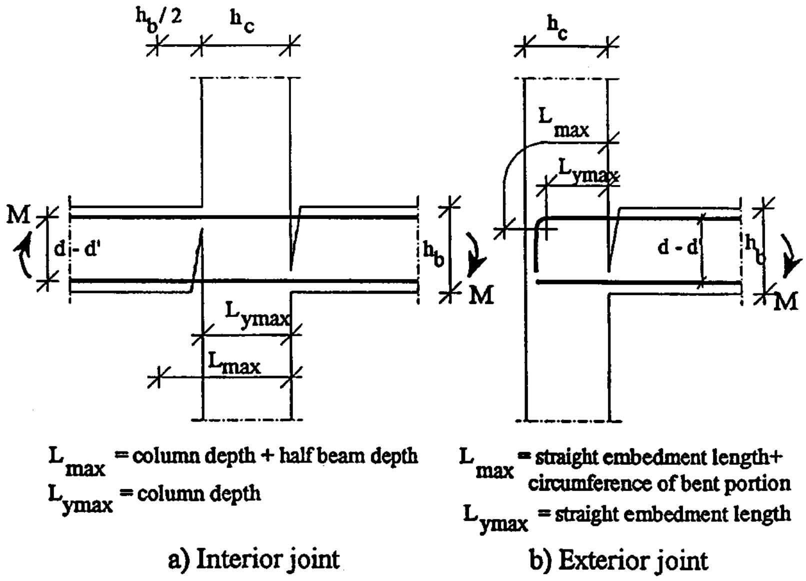


Fig. 10 Definition of L_{max} and L_{ymax}

Therefore, the value of bar slippage from the joint can be calculated as:

$$\Delta_s = \left[\frac{(L_s + 2L_y)}{E_s} + (\chi - 1)\frac{L_y}{E_{sh}} \right] \frac{f_y}{2} \tag{13}$$

where E_{sh} is the slope of the strain hardening branch of the stress-strain diagram of steel and f_{ult} is the ultimate strength of the reinforcing bars. The yield development length L_y, should not exceed the maximum limit (L_{ymax}) which is defined in Fig. 10 for interior and exterior joints (Morita and Kaku 1984). If $L_s+L_y > L_{max}$ or $L_y > L_{ymax}$, then L_{max} or L_{ymax} are substituted for (L_s+L_y) or L_y in the above equations, respectively.

The situation is either yielding of the reinforcement before slippage ($L_s \leq L_{max}$) or pullout before yielding ($L_s > L_{max}$). In the first case of yielding then slippage, the stiffness before yielding K_1 as shown in Fig. 8, is calculated as:

$$K_1 = \frac{M_y}{\theta_y} = \frac{n\, A_s\, f_y}{\Delta_y}(d-d')^2$$
$$= 1200\,\pi\, n\, d_b\,(d-d')^2\sqrt{f_c'} \quad \text{MPa} \tag{14}$$

where θ_y and Δ_y are the rotation and bar slippage at yielding, (d-d') is the effective depth of the beam, and d_b and n are the bar diameter and number of tensile reinforcing bars in the beam, respectively.

After yielding, the stiffness K_2 is assumed to be a function of χ. Consider a point on the moment-rotation relationship (Fig. 8) with coordinates (θ_2, M_2) identified by L_y=L_{ymax}. The factor at this point (denoted χ_2) and the stiffness K_2 can be calculated as follows:

$$\chi_2 = \frac{f_{s2}}{f_y} = \frac{M_2}{M_y} \leq \frac{f_{ult}}{f_y} \tag{15}$$

$$K_2 = \frac{M_s - M_y}{\theta_2 - \theta_y} = \frac{(\chi_2 - 1)\, n\, A_s\, f_y\,(d-d')^2}{\Delta_2 - \Delta_y} \tag{16}$$

$$K_2 = \frac{2(\chi_2 - 1)\, n\, A_s\,(d-d')^2 E_s}{2L_{ymax} + (\frac{\alpha\chi_2 - 1}{\zeta})\, L_{ymax}} \tag{17}$$

where χ=E_{sh}/E_s. If L_s+$L_{ymax} > L_{max}$ then L_y=L_{max}-L_s

$$K_2 = \frac{2(\chi_2 - 1)\, n\, A_s\,(d-d')^2 E_s}{(L_{\max} - L_s)\,[2 + (\frac{\chi_2 - 1}{\zeta})]} \tag{18}$$

In the second case of bar pullout before yielding in which $L_s > L_{max}$ and K_2=0, the moment M_1 and the stiffness K_1 can be calculated as follows:

$$M_1 = n\, A_s\, f_y\,(d - d')\frac{L_{\max}}{L_s} \tag{19}$$

$$K_1 = \frac{2\, n\, A_s\, E_s\,(d-d')^2}{L_{\max}} \tag{20}$$

4.2. Moment-rotation relationship under cyclic loading

The model developed by Chung *et al.* (1987) is adopted to represent the hysteretic behaviour of the rotation spring representing the bond slip. The model is capable of including the effects of stiffness degradation, strength deterioration and pinching. An input parameter which ranges between 0.0 and 0.1 governs the strength deterioration, where a value of 0.0 signifies no strength deterioration. In the present study, it is selected to be 0.1.

The pinching effect is introduced in the loops by an input parameter α^* as shown in Fig. 11. The parameter α^* ranges between 0.0 and 1.0. A value of 1.0 signifies no pinching. The value of α^* depends on several parameters such as the development length of the beam bars in the joint, the amount of shear reinforcement in the joint, the existence of transverse beams and number of loading cycles. Based on Morita and Kaku's (1984) work, the proposed values for the parameter α^* are:

for interior columns

$$\begin{aligned} &L_s \leq h_c \alpha^* = 1.0 \text{ (no pinching)} \\ &h_c < L_s\ \alpha^* L_{max} = 1.0 - 0.5\ [(L_s - h_c)/(L_{max} - h_c)] \\ &L_s > L_{max}\ \alpha^* = 0.5 \end{aligned} \tag{21}$$

for exterior column

$$\begin{aligned} &L_s \leq h_c \alpha^* = 1.0 \text{ (no pinching)} \\ &h_c < L_s\ \alpha^* L_{max} = 1.0 - 0.25\ [(L_s - h_c)/(L_{max} - h_c)] \\ &L_s > L_{max}\ \alpha^* = 0.75 \end{aligned} \tag{22}$$

where h_c represents the column depth.

5. Joint macro-element

The joint is represented by two rotational springs in series, one representing the joint shear deformation and the second represents the reinforcing bars bond-slip. The two springs are influenced by the relative rotational displacement between the nodes only as shown in Figs. 12a and 12b. The moment transmitted by the element is the moment transferred from the beams to the column. The deformation of the element represents either the shear deformation of the joint (the angle between the connected beams and columns) or the additional joint rotation due to bond-slip of the longitudinal bars of the beam. In this element, translational displacements of the nodes at both ends of the element are constrained to be identical. To satisfy equilibrium, the nodal coordinates are taken to be identical.

6. Computational tool: SARCF

A modified version of the computer program Seismic Analysis of Reinforced Concrete Frames, SARCF (Chung *et al.* 1988), was used in the dynamic analysis. The program is based on DRAIN-2D (Kanaan and Powell 1973) with various enhancements. SARCF has an improved model for reinforced concrete elements which simulate the strength and stiffness degradation, possibility to interrupt the analysis to compute mode shapes and natural frequencies using the tangent stiffness method, as well as displacement control analysis. The original version of SARCF includes beam and column elements only. The program was modified to include the developed joint element simulating the shear deformations in joints and the reinforcing bars bond slip at the beam-column interface. The modified program is suitable for calculating the nonlinear static and dynamic response of reinforced concrete frames.

7. Correlation with test results

To establish the validity and accuracy of the proposed models, the program is used in the simulation of the hysteretic behaviour of beam-column subassemblages (Biddah *et al.* 1997). The analytical model used to simulate the behaviour of the specimens consists of a girder element, two column elements, joint shear element and bond slip element. The properties of the constituent elements are derived from the material and geometric properties of the specimens. Using the measured stress-strain relationships of concrete and reinforcing steel, the section geometry and the reinforcement layout of the girder, the monotonic moment-curvature relationship (envelope) of a typical girder section can be established. The analytical and experimental load-displacement relationships of the specimens are shown in Fig. 13. From these results, the following observations are made:

a) In spite of the simplicity of the hysteretic model used in the different elements, good agreement between the analytical local behaviour of beam-column subassemblages and the test results is found.

b) The analytical model is capable of correctly representing the strength and stiffness of the subassemblage. This is important in the case of the post-yield stiffness where slight discrepancies

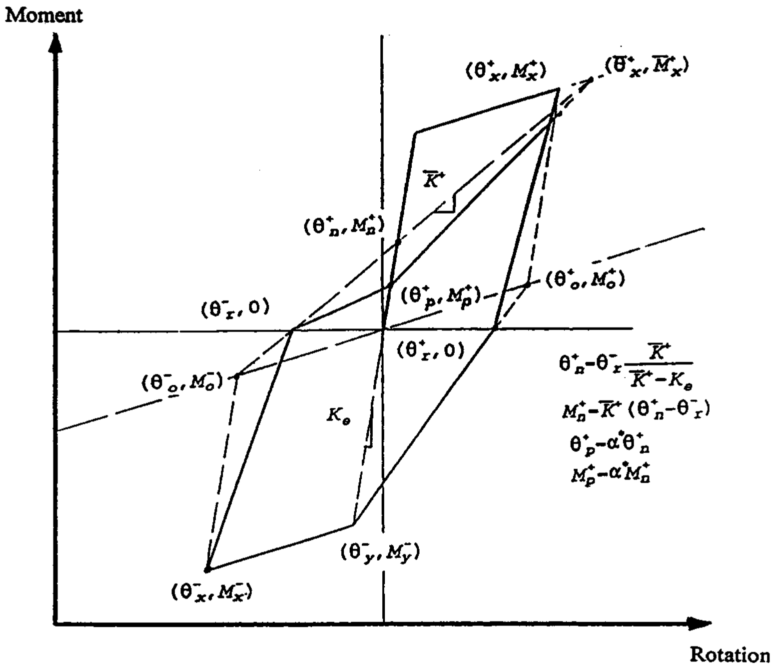


Fig. 11 Computational procedure for pinching effect

can lead to substantial deviations between observed and predicted forces. The strength degradation in the model agrees with the experimental measurements.

c) The pinching of the hysteretic loops of the girder caused by the interaction of shear forces with the opening and closing of the cracks, is simulated by the analytical model. This effect is particularly important in short span members.

d) A limitation of the hysteretic models appears to be the value of the unloading stiffness which is consistently lower than that observed in experiments. There is, however, as yet no rational

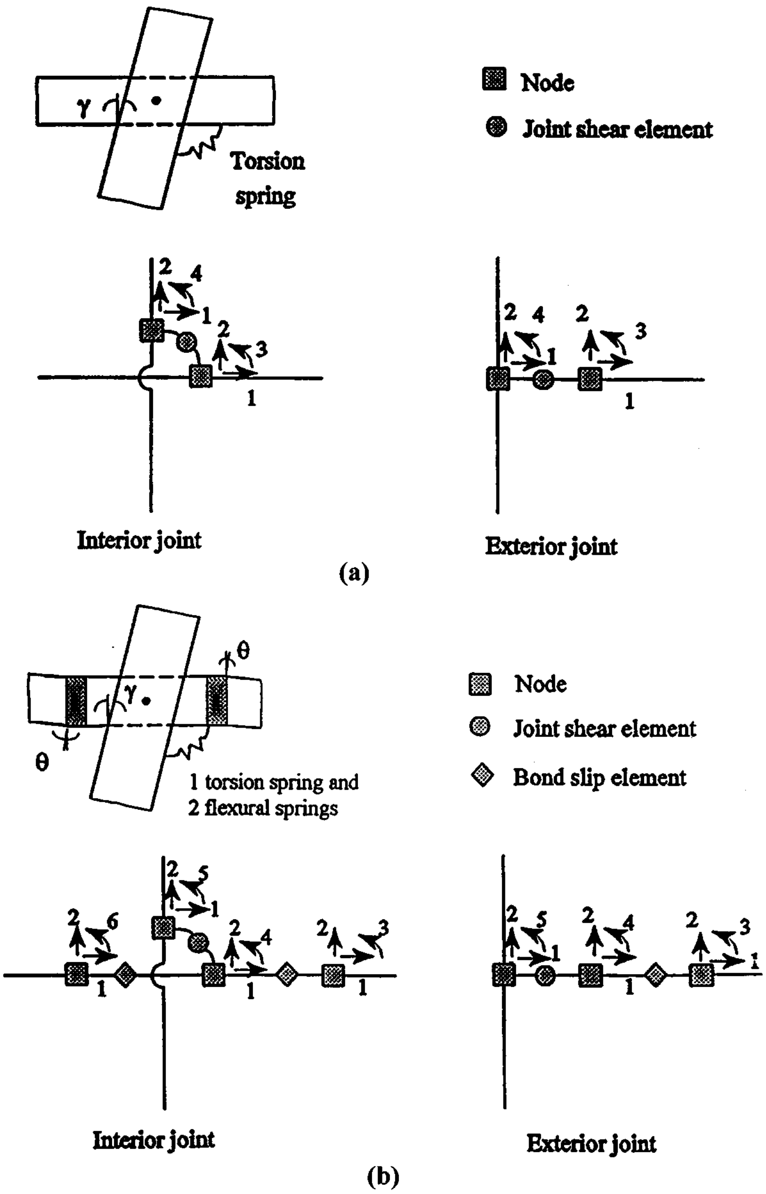


Fig. 12 a) Idealization of the joint shear element in frame analysis, b) Idealization of the joint shear and bond slip elements in frame analysis model for predicting the unloading stiffness in the inelastic regions of the girder.

8. Discussion

So far, the proposed joint model did not address two important issues. These are the confining effect of transverse beams and the contribution of the slab to joint shear resistance. The contribution of the transverse beams and the floor slab can be taken into account on the basis of

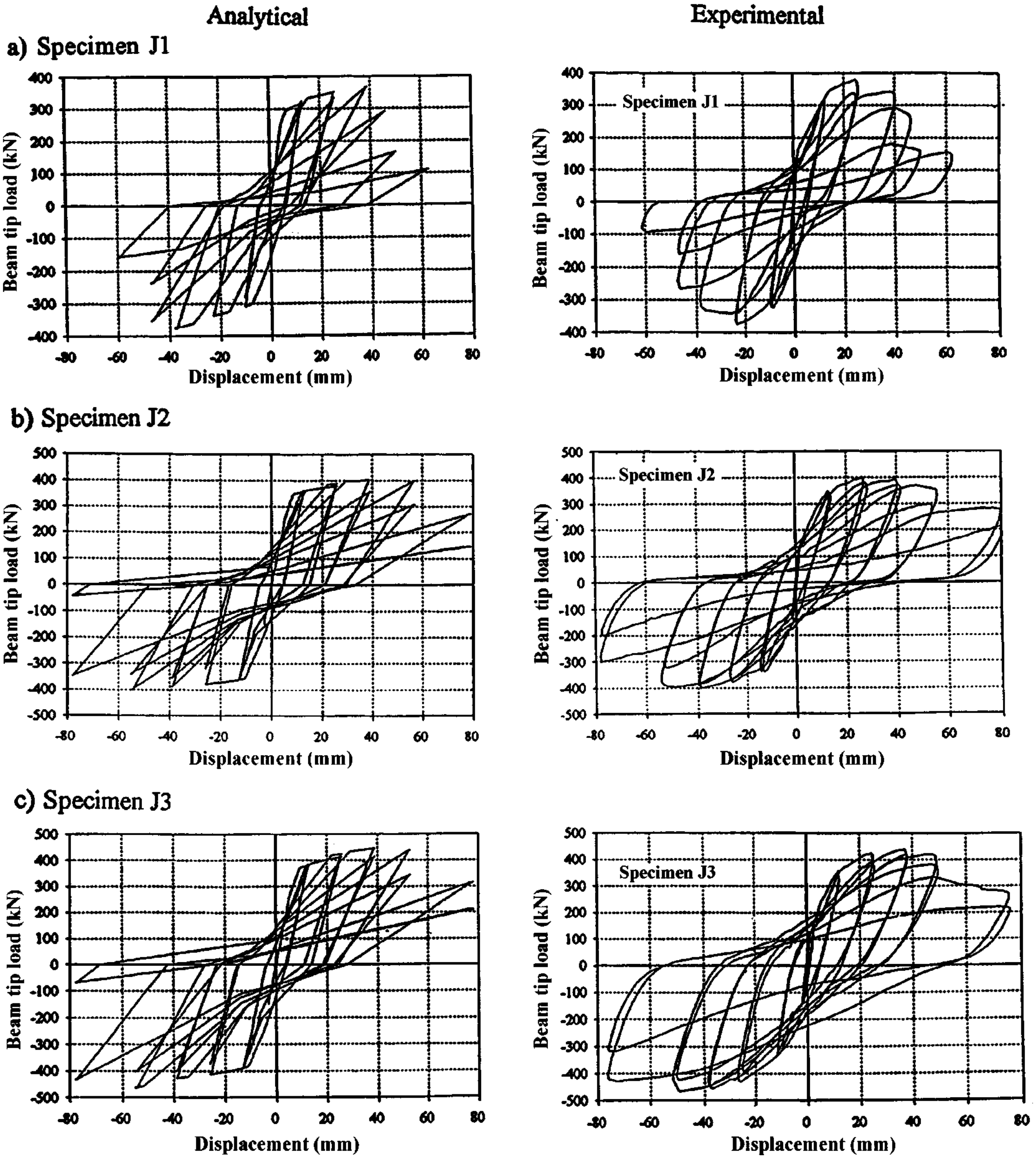


Fig. 13 Comparison between experimental and analytical load-deflection curves of the tested specimens of Biddah *et al.* (1997)

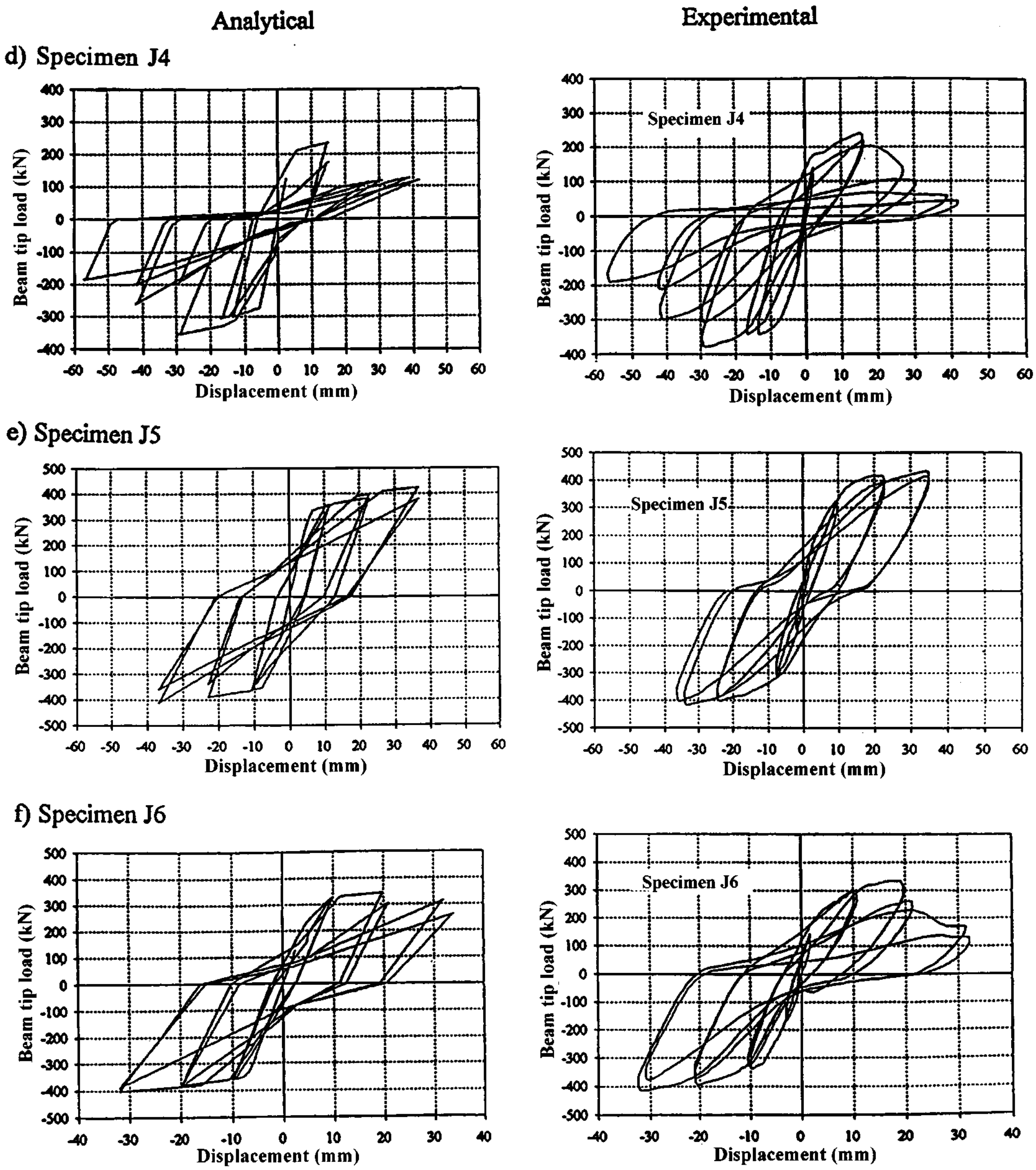


Fig. 13 Continued

available test results.

Kurose (1987) and Kitayama *et al.* (1991) tested interior beam-column joints with and without transverse beams and with and without ties. It was observed that transverse beams, even when loaded cyclically to flexural yielding, could enhance the joint shear strength by at least 1.2 times the shear strength in the case of no transverse beams. The increase in joint confinement is attributed to the longitudinal reinforcement in the transverse beams.

The Architectural Institute of Japan AIJ, Standards (1988) increases the nominal shear strength of interior beam-column joints by 1.3 times that in the case without transverse beams or slab (i.e. from 0.

25 f_c' to 0.33 f_c'). This recommendation is based on the assumption that transverse beams and slab are cracked and that beams frame into four vertical faces of the joint and that at least two-thirds of each joint face is covered by framing beams. The AIJ guidelines for earthquake resistant reinforced concrete buildings (AIJ 1988) proposes that for transverse beam effects, the shear resistance may be increased by a factor β such that

$$\beta = 1 + 0.3\,\lambda, \quad \lambda = \frac{\sum b_T D_T}{2(D_L\ D_c)} \quad \text{and } D_T \le D_L \tag{23}$$

where D_L, D_c, D_T and b_T are the total depth of the longitudinal beam, the total depth of the column, the total depth of the transverse beam and the width of the transverse beam, respectively. The effect of the transverse beam shall not be considered if the beam exists only on one side of the joint.

The shear resistance in this study is proposed to be increased by a factor β^* such that:

$\beta^* = 1.4$	for the cracking shear stress
$\beta^* = 1.0 + 0.35\,\lambda^* < 1.3$	for the ultimate shear strength in interior joints
$\beta^* < 1.4$	for the ultimate shear strength in exterior joints

where

$$\rho = \frac{\sum \text{Longitudinal reinforcement in transverse beams}}{\text{average area of transverse beams}}$$

$$\lambda^* = \frac{\sum b_T D_T}{D_c\ (D_{L[left]} + D_{L[right]})} \quad D_T \le D_L \tag{24}$$

The 1.4 value for the increase in cracking shear stress is chosen according to the ACI Committee 352 (1976) where the concrete strength consists of factor of 1.0 for no transverse beams and 1.4 for transverse beams. This value agrees well with the test results of Kitayama *et al.* (1991). On the basis of the test results, the β parameter proposed by the AIJ guidelines (1988) is replaced by β^* to obtain more accurate results. This is because the joint shear strength was found to be affected by the dimensions of beams and the percentage of reinforcement in the transverse beams.

The influence of the floor slabs on the shear strength of interior joints is studied by Kitayama *et al.* (1991). The average joint shear stress is compared for specimens with and without floor slabs. Joint shear strength was increased by 10% due to the floor slabs for interior connections (5% in case of exterior connection). Some of the reasons that contribute to the shear strength enhancement are:

a) the stress concentration from the joint diagonal compression strut is relieved; b) the slab concrete adjacent to the upper part of a joint without transverse beams provides added shear resistance; and c) the top part of the joint is confined by the slab.

It is also concluded that the shear strength of the joint with both transverse beams and floor slabs was more than 30% higher than that of the specimen without transverse beams or floor slab.

10. Conclusions

A bi-linear moment-rotation analytical model representing beam reinforcing bar bond-slip was selected for the case of monotonic loading and to establish the envelope of reversed cyclic loading. A tri-linear moment-rotation analytical model representing joint shear was developed for the case of monotonic loading and to establish the envelope of reversed cyclic loading. The model sensitivity was assessed by comparing its simulation to experimental measurements of reinforced concrete joints. The developed model was found to be capable of accurately simulating the behaviour of reinforced concrete joints subjected to in-plane stresses. A reasonable analytical representation of the hysteretic behaviour of the tested specimens was achieved. The proposed joint element representing joint shear deformation and bars bond slip is simple and easy to incorporate into currently available software.

The proposed model can be used to predict the contribution of well designed joints to the overall deformation of frames as well as to represent the distortion and failure modes of nonductile joints in existing structures.

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