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Optimum design of cable-stayed bridges

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Abstract. This paper presents a procedure to minimize the cost of materials of cable-stayed bridges with composite box girder and concrete tower. Two sets of iterations are included in the proposed procedure. The first set of iteration performs the structural analysis for a cable-stayed bridge. The second set of iteration performs the optimization process. The design is formulated as a general mathematical problem with the cost of the bridge as the objective function and bending forces, shear forces, fatigue stresses, buckling and deflection as constraints. The constraints are developed based on the Canadian National Standard CAN/CSA-S6-88. The finite element method is employed to perform the complicated nonlinear structural analysis of the cable-stayed bridges. The internal penalty function method is used in the optimization process. The limit states design method is used to determine the load capacity of the bridge. A computer program written in FORTRAN 77 is developed and its validity is verified by several practical-sized designs.

Key words: bridges; cables; structures; structural analysis; nonlinearity; optimization; optimum design; box girder; composite structures.

1. Introduction

Traditional methods for the design of cable-stayed bridges involve a trial-and-error procedure. A set of cross-sectional properties is first assumed with an accepted geometric layout for the given loads. Structural analysis is then carried out to obtain stresses and displacements. These stresses and displacements are then compared with the allowable values given by the chosen specifications. If the stresses and displacements satisfy the requirements of the specifications, the assumed members may be adopted. Otherwise, the cross-sectional dimensions are modified and the structural analysis is repeated until satisfactory results are reached. The above procedure is tedious and uncertainties exist as to whether or not the final design is either optimal or economic.

Since the introduction of the optimization technique, a lot of research has been done on the

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application of optimization in engineering. Application of optimization techniques in engineering makes it possible to design bridge structures, even complicated bridge structures such as cable-stayed bridges, automatically by computer. Some papers have been published on the optimal design of bridge structures. The dynamic programming method was applied to the optimization of continuous bridges (Busek 1971). The general geometric programming method has been applied to get the minimum weight for multispan plate girder bridges (Adeli and Chompooping 1988) and to the optimal design of simply supported I-beams (Burns and Ramamurthy 1988, Azad 1981). The feasible direction method was applied to the design of continuous highway bridges (Memari *et al.* 1991). A multi-level algorithm was applied to optimal design of structural concrete bridge systems (Cohn and Lounis 1994).

Research in the optimum design of cable-stayed bridges dates back two decades. Bhatti *et al.* (1985) presented a preliminary optimum design method of cable-stayed bridges based on the linearization algorithm which solves a quadratic programming problem to arrive at the optimum solution. Gimsing (1983) investigated the rational cable arrangement of cable-stayed bridges from the point of structural analysis. Nakamura and Wyatt (1988) studied the method to determine the cable's prestresses based on linear programing. Ohkubo *et al.* (1992) proposed a two-stage optimum design method for steel cable-stayed bridges to determine the optimum values of design variables based on the convex and linear approximations concept. Cable-stayed bridges are treated as elastic linear structures in all these studies.

This paper presents a method dealing with the cost-optimal design of cable-stayed bridges with composite superstructures. The design procedure involves two main tasks: structural analysis and optimization. The finite element method is used to perform the structural analysis and the nonlinearity of a cable-stayed bridge is taken into consideration. The internal penalty function algorithm for nonlinear programming is used in the optimization procedure. The load capacity of a cable-stayed bridge is determined based on the limit states design method.

2. General consideration of model

A cable-stayed bridge is modeled as a two-dimensional structure (Fig. 1). The finite element method is employed to perform the structural analysis. Cable stays are treated as truss members having no resistance to compression. The deck girder and tower are treated as beam elements.

One distinct characteristic of a cable-stayed bridge is its nonlinearity. The nonlinear behavior of a cable-stayed bridge can be observed in the following three aspects: the nonlinear axial force-deformation relationship for the inclined cable stays due to sag caused by their selfweight; the nonlinear axial and bending force-deformation due to interaction of large axial force and bending moment in the girder and tower; and the nonlinear behavior due to the change in geometry caused

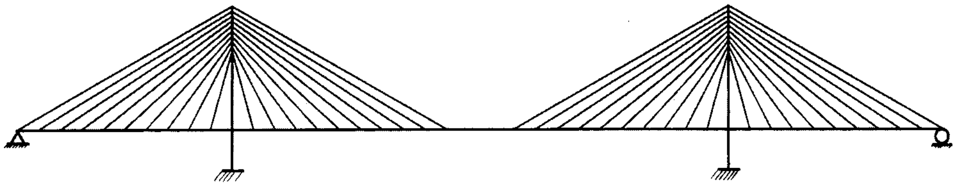


Fig. 1 Bridge model

by large displacements in the structure. Previous studies have shown that the stresses at control sections of a cable-stayed bridge analyzed by nonlinear theory are about 8-15 percent larger than those analyzed by linear theory (Lazar 1972, Rajaraman *et al.* 1980). Thus it becomes necessary to take into consideration the effect of nonlinearities when performing the analysis of a cable-stayed bridge.

It is convenient and accurate to account for the nonlinearity of a cable by modelling it as a straight chord element with an equivalent modulus of elasticity that effectively reflects the behavior of a sagged cable. The concept of the equivalent modulus of elasticity was first introduced by Ernst (1965) and has been universally adopted. The equivalent modulus for a cable can be expressed as:

$$E_{eq}=\frac{E}{1+\dfrac{(wl)^2}{12T^3}EA} \tag{1}$$

where E_{eq}=equivalent modulus of cable stay; w=uniformly distributed cable weight; E=modulus of elasticity of cable stay; A=cross-sectional area of cable stay, l=horizontal projected length of cable; T=cable tensile force due to load. The stiffness matrix for cable stays can therefore be written as:

$$k_m=\frac{E_{eq}A}{L}\begin{bmatrix} 1 & & & & & \\ 0 & 0 & & Sym & & \\ 0 & 0 & 0 & & & \\ -1 & 0 & 0 & 1 & & \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{2}$$

where L=length of cable stay

The interaction of axial and flexural effects yields nonlinear load-deformation relationships. Instead of using standard linear structural analysis methods, the nonlinearity can be taken into consideration by introducing the concept of stability functions (Harrison 1973, Weaver and Gere 1990). The stiffness matrix of the element shown in Fig. 2 can then be expressed as:

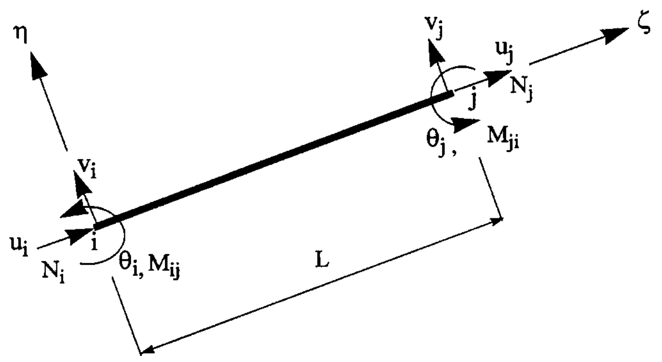


Fig. 2 Beam element

$$k_m = \frac{EI}{L}\begin{bmatrix} \frac{A}{I}S_0 & & & & & \\ 0 & \frac{12}{L^2}S_1 & & & Sym & \\ 0 & \frac{6}{L}S_2 & 4S_3 & & & \\ -\frac{A}{I}S_0 & 0 & 0 & \frac{A}{I}S_0 & & \\ 0 & -\frac{12}{L^2}S_1 & -\frac{6}{L}S_2 & 0 & \frac{12}{L^2}S_1 & \\ 0 & \frac{6}{L}S_2 & 2S_4 & 0 & -\frac{6}{L}S_2 & 4S_3 \end{bmatrix} \tag{3}$$

where A=cross-sectional area of deck girder or tower, I=moment of inertia of element, L=length of element. E=modulus of elasticity. The stability functions can be expressed in terms of the member axial force N (N is the average of N_i and N_j) and end moments M_{ij} and M_{ji}, as shown in Fig. 2, as follows:

For a compressive axial force

$$S_0=\frac{1}{1+\frac{EAR_{cm}}{4N^3L^2}}, \qquad S_1=\frac{(\mu L)^3\sin(\mu L)}{12R_c}, \qquad S_2=\frac{(\mu L)^2[1-\cos(\mu L)]}{6R_c} \tag{4abc}$$

$$S_3=\frac{(\mu L)[\sin(\mu L)-(\mu L)\cos(\mu L)]}{4R_c}, \qquad S_4=\frac{(\mu L)[(\mu L)-\sin(\mu L)]}{2R_c} \tag{4de}$$

where

$$\mu=\sqrt{\frac{N}{EI}} \tag{5}$$

$$R_c=2-2\cos(\mu L)-(\mu L)\sin(\mu L) \tag{6}$$

$$R_{cm}=(\mu L)(M_{ij}^2+M_{ji}^2)[\cot(\mu L)+(\mu L)\operatorname{cosec}^2(\mu L)]-2(M_{ij}+M_{ji})^2 \\ +M_{ij}M_{ji}[1+(\mu L)\cot(\mu L)][2(\mu L)\operatorname{cosec}(\mu L)] \tag{7}$$

For a tensile axial force

$$S_0=\frac{1}{1-\frac{EAR_{tm}}{4N^3L^2}} \tag{8a}$$

$$S_1=\frac{(\mu L)^3\sinh(\mu L)}{12R_t} \tag{8b}$$

$$S_2=\frac{(\mu L)^2[\cosh(\mu L)-1]}{6R_t} \tag{8c}$$

$$S_3=\frac{(\mu L)[(\mu L)\cosh(\mu L)-\sinh(\mu L)]}{4R_t} \tag{8d}$$

$$S_4=\frac{(\mu L)[\sinh(\mu L)-(\mu L)]}{2R_t} \tag{8e}$$

where μ has the same expression as in Eq. (5) and

$$R_t = 2 - 2\cosh(\mu L) + (\mu L)\sinh(\mu L) \tag{9}$$

$$R_{tm} = (\mu L)(M_{ij}^2 + M_{ji}^2)[\coth(\mu L) + (\mu L)\operatorname{cosech}^2(\mu L)] - 2(M_{ij} + M_{ji})^2 + M_{ij}M_{ji}[1 + (\mu L)\coth(\mu L)][2(\mu L)\operatorname{cosech}(\mu L)] \tag{10}$$

In linear structural analysis, it is assumed that the joint displacements of the structure under the design loads are small with respect to the original joint coordinates. Therefore, the geometric changes in the structure can be ignored and the stiffness of the structure in the deformed shape can be assumed to equal the stiffness of the undeformed structure. However, in cable-stayed bridges, very large displacements can occur under normal design loads and the effect of geometry changes in the structure could be significant. Therefore, the stiffness of the bridge in the deformed shape should be computed from the new geometry of the structure.

There are several techniques available for solving the nonlinear structures. These methods can be classified into:

(1) incremental or stepwise procedure,
(2) iteration of the Newton-Raphson procedure and
(3) mixed procedures.

The Newton-Raphson iteration procedure is used in this study since it gives accurate results and converges fast. In the first iteration, the problem is solved for the initial geometry and loadings. The deformations obtained are then used to update the stiffness matrix of the structure. The unbalance between the external loads and the internal forces is applied to the structure and solved for the revised stiffness of the structure. The procedure is repeated until the unbalanced loads reach a desired tolerance. The problem usually converges in 3 or 4 iterations.

Design loading, their combinations and applications, should be consistent with the appropriate specifications such as AASHTO, AREA or CAN/CSA Design of Highway Bridges. AASHTO specifications are applicable only to spans up to 500 feet (150 m), AREA specifications are applicable to spans of 400 feet and shorter. The Canadian Code, Design of Highway Bridges, is applicable only to spans up to 100m. For spans in excess of 500 feet, reductions may be used as listed in Table 1 (Ivy *et al.* 1954).

It is assumed that the steel box girder is first built segment after segment and suspended to the successive cables; then the concrete slab (precast or in-situ concrete) be put on segment after segment. Therefore the composite superstructure resists both the dead load and superimposed dead load together.

Table 1 Recommended live load for long bridges

Loaded length ft (m)		Uniform lane live load k/ft (kN/m) (3)	Concentrated live load kip (kN)	
Minimum (1)	Maximum (2)		For moment (4)	For shear (5)
0	600 (182.88)	640 (9.4)	18 (80.06)	26 (115.65)
600 (182.88)	800 (243.84)	640 (9.4)	9 (40.03)	13 (57.82)
800 (243.84)	1,000 (304.80)	640 (9.4)	0	0
1,000 (304.80)	1,200 (365.76)	600 (8.8)	0	0
1,200 (365.76)	∞	560 (8.2)	0	0

3. Optimization formulation

The goal of optimization is to select a set of design variables in such a way that the final design will produce the minimum cost or minimum weight. Here we consider a problem in which the geometry of the bridge structure is first determined according to natural conditions such as geology, clearance and traffic flow. We only deal with the selection of the cross-sectional dimensions of the members. Mathematically, optimization of the structure can be stated as:

$$\text{Minimize} \qquad F(\boldsymbol{x}) \tag{11}$$

$$\text{Subjected to} \qquad g_j(\boldsymbol{x}) \le 0.0 \qquad j=1,\cdots,m \tag{12}$$

$$\text{and} \qquad x_i \ge 0 \qquad i=1,\cdots,n \tag{13}$$

where $\boldsymbol{x}=\{x_i\}$ is the vector of design variables. $F(\boldsymbol{x})$ is the objective function which describes the cost, weight or volume of the system, $g_j(\boldsymbol{x})$ is the constraints imposed on the structure by the chosen specifications. Usually $g_j(\boldsymbol{x})$ is an implicit nonlinear function of design variables, therefore, Eqs. (11)-(13) represent a nonlinear optimization problem.

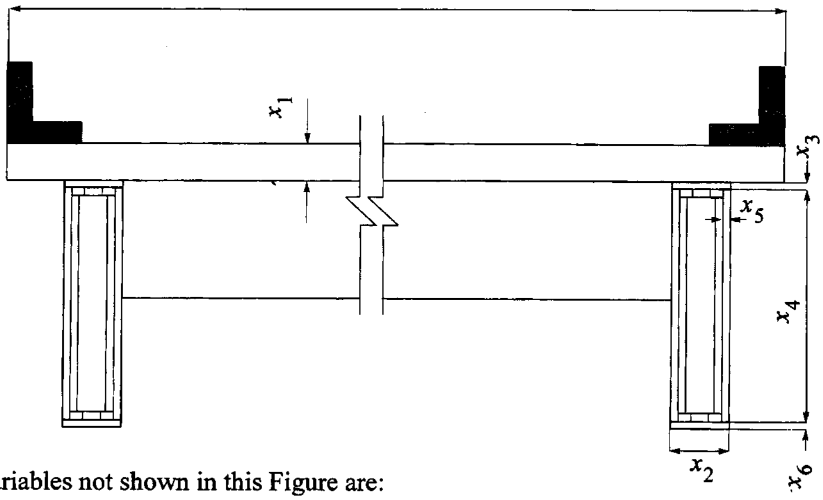
In the design of a cable-stayed bridge with the composite box girder, the width of the bridge roadway can be pre-decided according to the traffic flow, therefore, can be assumed as constant. The possible design variables which can be chosen are the thickness of the concrete deck; the width and thickness of the flanges and webs of the steel box girder; the width, thickness and spacing of longitudinal stiffeners and transverse stiffeners; the area of each cable stay and the dimensions of the pylons. Since in practice the girder cross-section of a cable-stayed bridge is usually prismatic, a prismatic section is assumed in this study. Figs. 3-4 show the chosen variables.

The objective of this study is to find the minimum cost of a bridge. The cost function should only reflect the amount and cost of the superstructure. The study includes the cost of the concrete, the steel box girder, the longitudinal and transverse stiffeners (ribs) for flanges and webs, the cable stays and the formworks. It should be noted that the floor system and shear studs can be dealt with separately in the design. For simplification, they are not involved in the cost function. The objective function may be expressed as:

$$\begin{aligned} F =\ & A_s L \rho_s P_s + V_c P_c + \sum A_{cl} L_{cl} \rho_s P_{cl} + A_{ls} L_{ls} N_{ls} \rho_s P_{ls} \\ & + A_{ts1} L_{ts1} N_{ts1} \rho_s P_{ts1} + A_{ts2} L_{ts2} N_{ts2} \rho_s P_{ts2} + A_{re} H \rho_s P_{re} + A_f P_f \end{aligned} \tag{14}$$

where A_s is the area of steel. L is the length of the bridge. P_s is the unit price of steel. V_c is the volume of concrete including the concrete slab and the concrete tower and P_c is the unit price of concrete. A_{cl}, L_{cl} and P_{cl} are, respectively, the area, length and the unit price of the cable stay. A_{ls}, L_{ls}, N_{ls} and P_{ls} are the area, length, number and the unit price of the longitudinal stiffeners for the flanges. A_{ts1}, L_{ts1}, N_{ts1} and P_{ts1} are the area, length, number and the unit price of transverse stiffeners for flanges. A_{ts2}, L_{ts2}, N_{ts2} and P_{ts2} are the area, length, number and the unit price of the transverse stiffeners for the web. A_{re} and H are the reinforcement area and the height of the tower. P_{re} is the unit price of reinforcement. A_f and P_f are the concrete surface area and the average unit price of formwork. ρ_s and ρ_c are, respectively, the density of steel and concrete.

As mentioned before, there are various minimization techniques which can be used to solve the stated standard optimization problem. A simple, yet powerful, method for structural optimization is the internal penalty function algorithm which is used here to solve the above optimization problem. The internal penalty function method has been successfully applied to the optimum design of engineering structures by researchers (Kavlie and Moe 1971, Metwally Abo-Hamd 1984,



Variables not shown in this Figure are:

x_7, x_8 and x_9 = width, thickness and spacing of longitudinal stiffeners

x_{10}, x_{11} and x_{12} = width, thickness and spacing of transverse stiffeners for flanges

x_{13}, x_{14} and x_{15} = width, thickness and spacing of transverse stiffeners for webs

Fig. 3 Design variables of composite girder

Abendroth and Salmon 1987). Fiacco and McCormic (1968) have made a great contribution to the development of the penalty function technique. The internal penalty function method is considered to be robust and well suited for the optimization of statically indeterminate structures (Gallagher and Zienkiewicz 1973, Kavlie and Moe 1971). One favorite fact is that although one feasible point is needed in order to start the internal penalty method, by means of the extended penalty-function technique or the internal penalty function method itself, it is possible to start the internal function algorithm from an infeasible point (Kowalik and Osborne 1968). This may be quite an important feature since for some complicated structures a feasible point is difficult to get at the beginning of the design.

The basic idea of the penalty function technique is that the constrained problem is transferred into an unconstrained one by adding a penalty term which takes care of the effect of the constraints to the objective function. There are several different forms of internal penalty functions and one of them used in this study has the following form (Carroll 1961):

$$P(x,\ r_k)=F(x)-r_k\sum_{j=1}^{m}\frac{1}{g_j(x)} \tag{15}$$

where r_k is the response factor which should approach zero as the penalty function converges to the minimum value. It is of great importance to properly chose the initial response factor r_1 for the efficiency of the internal penalty function algorithm. The strategy is to chose the initial response factor r_1 in such a way that the penalty term adds a certain percentage to the objective function at the starting point, that is:

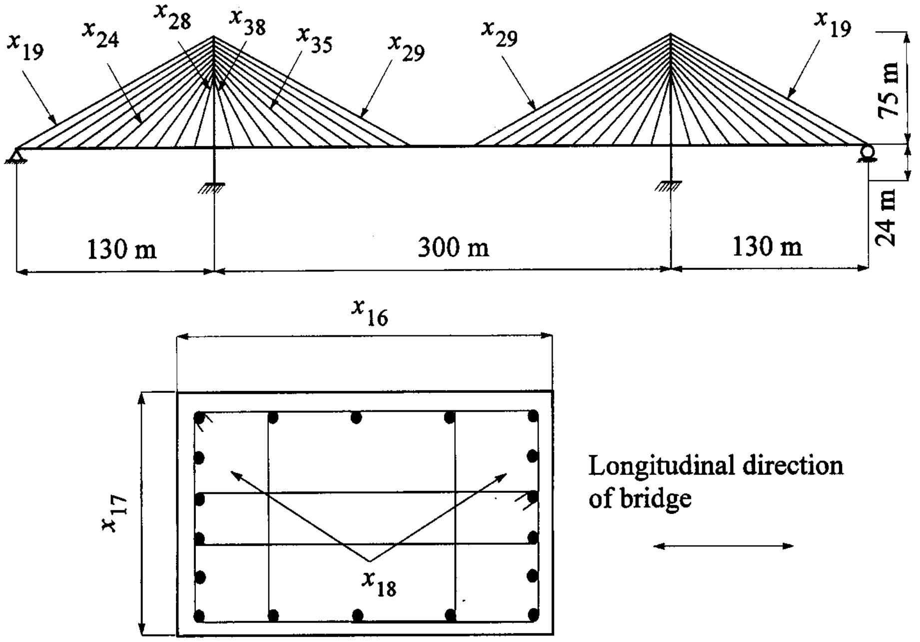


Fig. 4 Design variables of cables stays and tower

$$r_1 = \sum_{j=1}^{m} \frac{1}{g_j(x^0)} = \frac{p}{100} F(x^0) \tag{16}$$

where p is the selected percentage and p=50 is used in this study. The response factor r_k is reduced in such a way that the current response factor is 1/20 of the previous one.

If the initial design is not feasible, the method proposed by Fox (1971) is used to find a feasible point. That is, suppose the k^{th} constraint is violated under the initial design x^0, then a temporary objective function is defined and minimized as follows:

$$\text{minimize} \quad g_k(x) \tag{17}$$

$$\text{subjected to} \quad g_j(x) \leq 0 \quad j = 1, \cdots, m \tag{18}$$

$$\text{and} \quad g_k(x) - g_j(x^0) \leq 0 \tag{19}$$

The new point x obtained in this way will obviously satisfy the constraints. If more than one constraint is violated, the process is repeated until a feasible design point is obtained.

Once the constrained optimization problem is transferred into an unconstrained one, the direct search method (Powell 1964) or the Davidon-Fletcher-Powell (Fletcher and Powell 1963) method can be employed to solve it. The Powell method is called a direct method because it does not calculate the derivatives of the functions. The Davidon-Fletcher-Powell method, which is sometimes referred to as the variable metric method or quasi-Newton-Raphson method, is a modification of the steepest descent method and involves obtaining a matrix that approximates the inverse of the matrix of partial second derivatives of the function being minimized. Powell's method is used here to take advantage of the fact that the derivatives of the functions do not need to be calculated.

4. Design constraints

All design criteria have to be converted into constraints in order to perform optimum design. Except for those mentioned, all constraints are derived based on the design criteria of the Canadian standard: Design of Highway Bridges (Design 1988).

Cables are treated as truss elements that can resist only tension forces. Only axial tensile constraints are imposed. The allowable design load should be one-third of the ultimate breaking strength of the strand. Where fatigue effects may occur, the allowable load of the cable may be reduced to one-fifth of its ultimate strength (Task Committee 1977, Troitsky 1988). The material and cross-sectional dimensions of the cables should satisfy the equation:

$$T-\frac{1}{5}T_r \leq 0.0 \tag{20}$$

where T=factored tensile force in the cable. T_r=allowable design load.

The cable stay's ultimate strength depends on its diameter. Strand and rope are two types of stays used in practice. The ultimate strengths of different types of cable stays can be obtained by interpolating. For different diameters, the following formulas can be derived:

For strand:

$$T_r = 73.56d - 826.88 \tag{21}$$

For rope:

$$T_r = 56.44d - 491.20 \tag{22}$$

where d=diameter of the cable stay.

There exists a big positive moment at the center of the bridge and big negative moments at the sections over the intermediate supports and cable anchor places. During the optimization process, the locations of the biggest positive moment and the biggest negative moment may be changed as the design variables are changed. Therefore, the biggest positive and negative moment should be determined in each iteration. These moments must not exceed their corresponding limits. The moment resistance constraint can take the form:

$$M - M_r \leq 0.0 \tag{23}$$

where M is the factored moment due to loads. M_r is the factored moment resistance of the composite section.

For non-compact sections, in the positive moment region, where the depth of the compression portion of the web of the steel in the composite section does not exceed $685x_5/\sqrt{F_y}$, the factored moment resistance of the composite section can be calculated using a full plastic stress distribution. The position of the neutral axis of the section, measured from the top of the section, can be calculated as:

$$a_x = \frac{\phi A_s F - \phi_r A_s' f_y'}{0.85\phi_c f_c' b_e} \tag{24}$$

if $a_x \leq x_1$, The moment resistance can be calculated as:

$$M_r = C_c e_c + C_s e_s \tag{25}$$

where ϕ=0.95, ϕ_c=0.70, and ϕ_r=0.85 are capacity reduction factors for structural steel, concrete and reinforcement, respectively. F_y, f_c', and f_y' are specified yield strengths for steel, concrete, and reinforcement, respectively. A_s is the area of steel, A_s' is the area of reinforcement steel within the effective width of the concrete slab, and b_e is the effective width of the concrete slab. C_c and e_c are the factored compressive resistance and the lever arm of concrete, C_s and e_s are the factored compressive resistance and the lever arm of reinforcing steel.

If $a_x > x_1$, we have

$$C' = 0.5(\phi A_s F_y - C_s - C_c) \tag{26}$$

If $C' < \phi A_{tf} F_y = \phi x_2 x_3 F_y$, then the neutral axis lies in the flange of the steel section. Therefore,

$$a_x = \frac{C}{\phi x_2 F_y} + x_1 \tag{27}$$

If $C' \geq \phi A_{tf} F_y = \phi x_2 x_3 F_y$, then the neutral axis stays in the web of the steel section. Therefore, the location of the neutral axis can be shown as:

$$a_x = \frac{C' - \phi x_2 x_3 F_y}{4\phi x_5 F_y} + x_1 + x_3 \tag{28}$$

The moment resistance can be obtained as:

$$M_r = C_c e_c + C_s e_s + C' e' \tag{29}$$

where C' and e' are the factored compressive resistance and the lever arm of the compression portion of steel.

When the compressive portion of the steel web exceeds $685x_5/\sqrt{F_y}$ but the steel section satisfies the limit ratio of width to thickness, the factored moment resistance of the composite section is calculated by assuming a linear stress distribution in the steel section at first yielding of the steel section and a plastic stress distribution in the concrete slab. The following constraints should also be applied:

$$\frac{M_{dl} + M_{sd}}{S_{3n}} + \frac{M_{ll}}{S_n} - \phi F_y \leq 0.0 \tag{30}$$

where M_{dl}, M_{sd} and M_{ll} are factored moments due to dead load, superimposed dead load and live load, respectively. S_n and S_{3n} are the composite section modulus with elastic modulus ratio n and $3n$, respectively.

In the negative moment region, the contribution of concrete to the moment resistance M_r is neglected, therefore,

$$M_r = \phi S F_y \tag{31}$$

where S=section modulus of the steel box girder.

The maximum deflection of the bridge deck under the live load should satisfy the deflection criterion suggested by AASHTO. The constraint imposed on deflection can be expressed as:

$$\frac{800\Delta_{max}}{L} - 1.0 \leq 0.0 \tag{32}$$

where Δ_{max} is the calculated maximum deflection due to live load. $L/800$ is the allowable deflection prescribed by AASHTO.

In addition to the constraints discussed above, constraints converted from the design criteria such as fatigue, interaction of compression and moment, shear force, and buckling should be imposed. Similar constraints should be imposed on the tower as well.

5. Design algorithm and application

Based on the foregoing method, a computer program has been developed which integrates the nonlinear structural analysis of a cable-stayed bridge with the nonlinear optimization technique. The overall flowchart of the program is shown in Fig. 5. An initial design is first estimated to start the program and the structural analysis is performed. If the initial design is feasible, the optimization procedure is then commenced. If the initial design is not feasible, one feasible design point is then determined by the above mentioned method. The procedure is repeated until the

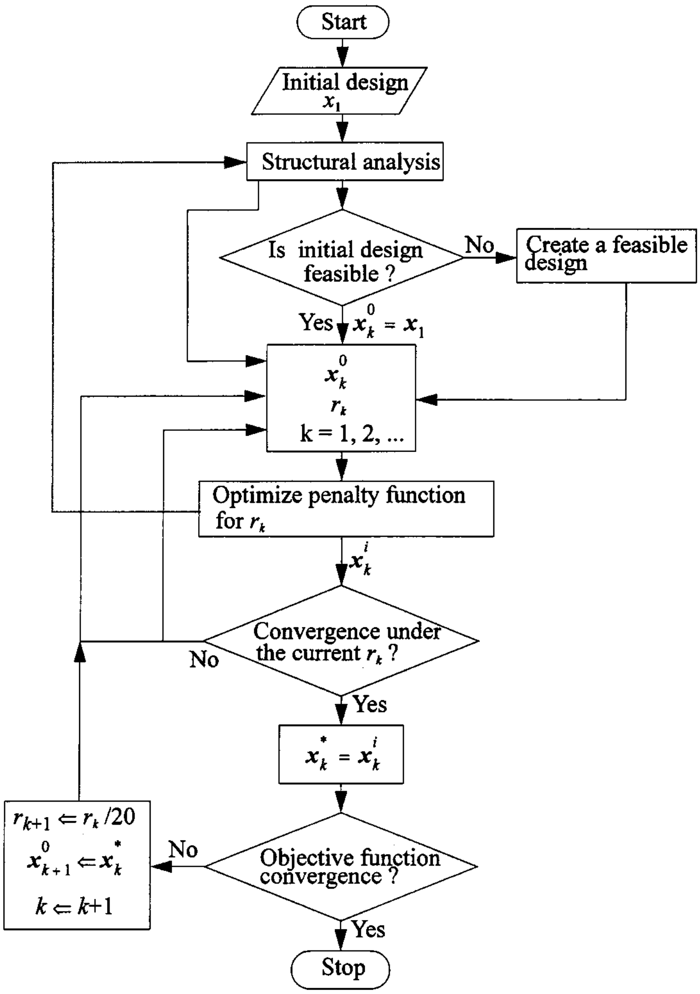


Fig. 5 Overall flowchart

Table 2 Optimum results

Design variables	Lower bound (m)	Upper bound (m)	Initial design (m)	Optimal design (m)
x_1	0.160	0.260	0.200	0.1600
x_2	1.000	7.500	1.500	1.0000
x_3	0.008	0.060	0.040	0.0317
x_4	0.800	5.000	2.500	1.9261
x_5	0.006	0.020	0.008	0.0087
x_6	0.008	0.080	0.022	0.0171
x_7	0.020	0.400	0.180	0.1482
x_8	0.006	0.040	0.020	0.0149
x_9	0.200	1.000	0.300	0.3999
x_{10}	0.020	0.400	0.210	0.1129
x_{11}	0.006	0.040	0.022	0.0114
x_{12}	0.500	5.000	1.000	1.5998
x_{13}	0.020	0.400	0.100	0.0468
x_{14}	0.006	0.040	0.012	0.0091
x_{15}	0.500	5.000	1.000	3.8521
x_{16}	1.500	6.000	3.500	3.6154
x_{17}	1.500	6.000	2.500	2.7964
x_{18}	0.000 (mm^2)	-	150000 (mm^2)	102300 (mm^2)
x_{19}	1000 (mm^2)	1000000 (mm^2)	30000 (mm^2)	9723.581 (mm^2)
x_{20}	1000 (mm^2)	1000000 (mm^2)	30000 (mm^2)	8762.250 (mm^2)
x_{21}	1000 (mm^2)	1000000 (mm^2)	30000 (mm^2)	11624.84 (mm^2)
x_{22}	1000 (mm^2)	1000000 (mm^2)	30000 (mm^2)	12983.38 (mm^2)
x_{23}	1000 (mm^2)	1000000 (mm^2)	30000 (mm^2)	3282.093 (mm^2)
x_{24}	1000 (mm^2)	1000000 (mm^2)	30000 (mm^2)	3312.603 (mm^2)
x_{25}	1000 (mm^2)	1000000 (mm^2)	30000 (mm^2)	3259.472 (mm^2)
x_{26}	1000 (mm^2)	1000000 (mm^2)	30000 (mm^2)	6105.155 (mm^2)
x_{27}	1000 (mm^2)	1000000 (mm^2)	30000 (mm^2)	13927.420 (mm^2)
x_{28}	1000 (mm^2)	1000000 (mm^2)	30000 (mm^2)	18893.420 (mm^2)
x_{29}	1000 (mm^2)	1000000 (mm^2)	30000 (mm^2)	16632.800 (mm^2)
x_{30}	1000 (mm^2)	1000000 (mm^2)	30000 (mm^2)	2742.956 (mm^2)
x_{31}	1000 (mm^2)	1000000 (mm^2)	30000 (mm^2)	3381.941 (mm^2)
x_{32}	1000 (mm^2)	1000000 (mm^2)	30000 (mm^2)	8761.671 (mm^2)
x_{33}	1000 (mm^2)	1000000 (mm^2)	30000 (mm^2)	9158.220 (mm^2)
x_{34}	1000 (mm^2)	1000000 (mm^2)	30000 (mm^2)	2873.630 (mm^2)
x_{35}	1000 (mm^2)	1000000 (mm^2)	30000 (mm^2)	3096.786 (mm^2)
x_{36}	1000 (mm^2)	1000000 (mm^2)	30000 (mm^2)	3641.061 (mm^2)
x_{37}	1000 (mm^2)	1000000 (mm^2)	30000 (mm^2)	3057.19 (mm^2)
x_{38}	1000 (mm^2)	1000000 (mm^2)	30000 (mm^2)	3053.26 (mm^2)
COST ($)	-	-	24,817,716	8,290,233

optimum design is achieved. All the sectional properties and the resistance capacities of a cable-stayed bridge are obtained by the program.

At each iteration of the optimization, the design variables are modified slightly. Consequently, the sectional properties of the bridge are changed. Theoretically, any modification in sectional properties will cause some changes in the design forces for a statically indeterminate structure. This means that a detailed finite-element structural analysis would be needed for each iteration. Some approximate concepts have been developed in order to reduce the number of structural analyses (Schmit and Farshi 1974). However, calculations indicate that small change in sectional properties does not result in large variations in the maximum moments, shears and axial forces in cable-stayed bridges. In other words, the critical section forces are not too sensitive to small changes in design variables. Therefore, instead of performing the time-consuming structural nonlinear analysis during each iteration, the cable-stayed bridge is analyzed only when the design variables have been changed by a relatively large amount from the previous structural analysis. The number of structural analyses can be reduced significantly in this way.

In order to test the validity and effectiveness of the algorithm, several practical-sized cable-stayed bridges with composite box girders have been designed and the optimum solutions are obtained using the developed program.

One of these designs is shown in Fig. 4. A cable-stayed bridge with side-spans of 130 m and a center span of 300 m is designed. Parameters used in the design are as follows: steel elastic modulus E_s=200 GPa, specified steel minimum yield point F_y=400 MPa, concrete elastic modulus E_c=27.4 GPa, specified concrete compressive strength f_c'=30 MPa. The unit prices are as follows: P_c=110 \$/m^3, P_s=2.1 \$/kg, P_{cl}=10 \$/kg, P_{re}=1.0 \$/kg, $P_{ls}=P_{ts1}=P_{ts2}$=6.6 \$/kg. P_f=27 \$/m^2.

The dead load applied to the structure includes the material self-weight, an estimated floor system of 10.5 kN/m, and an estimated superimposed dead load of 33.71 kN/m from asphalt paving, curb, paraphet and railing.

A uniformly distributed load of 9.4 kN/m is specified as live loading. Therefore, for 4 lanes, the live load applied to the bridge is 4×9.4 kN/m=37.6 kN/m.

The variables' lower bounds, upper bounds, initial design, along with their optimum results are listed in Table 2. The comparison of the initial design with the optimum design indicates that the optimum design is much cheaper than the initial design. The optimum design saves as much as 67% in price. The cost reduction history of the objective function with response factor r_k is shown in Fig. 6. It can be seen that most of the cost reduction takes place in 6 response surfaces. Fig. 7

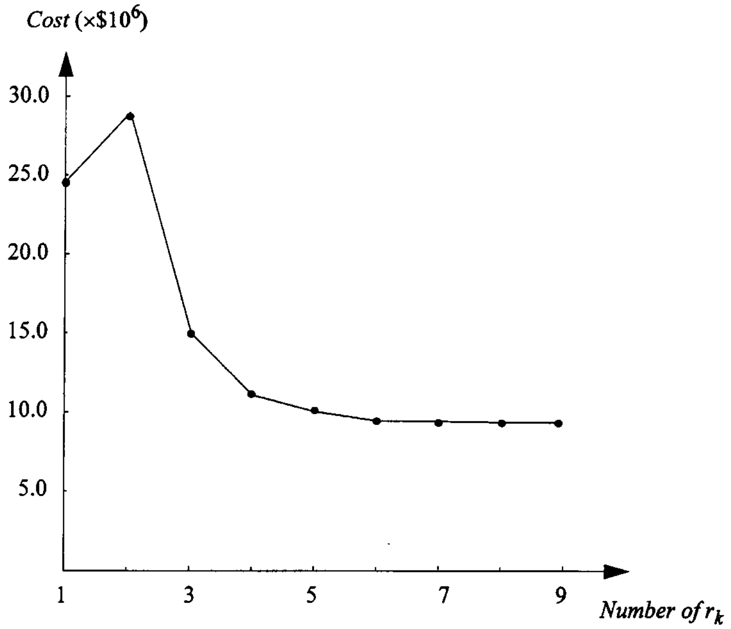


Fig. 6 Cost reduction history

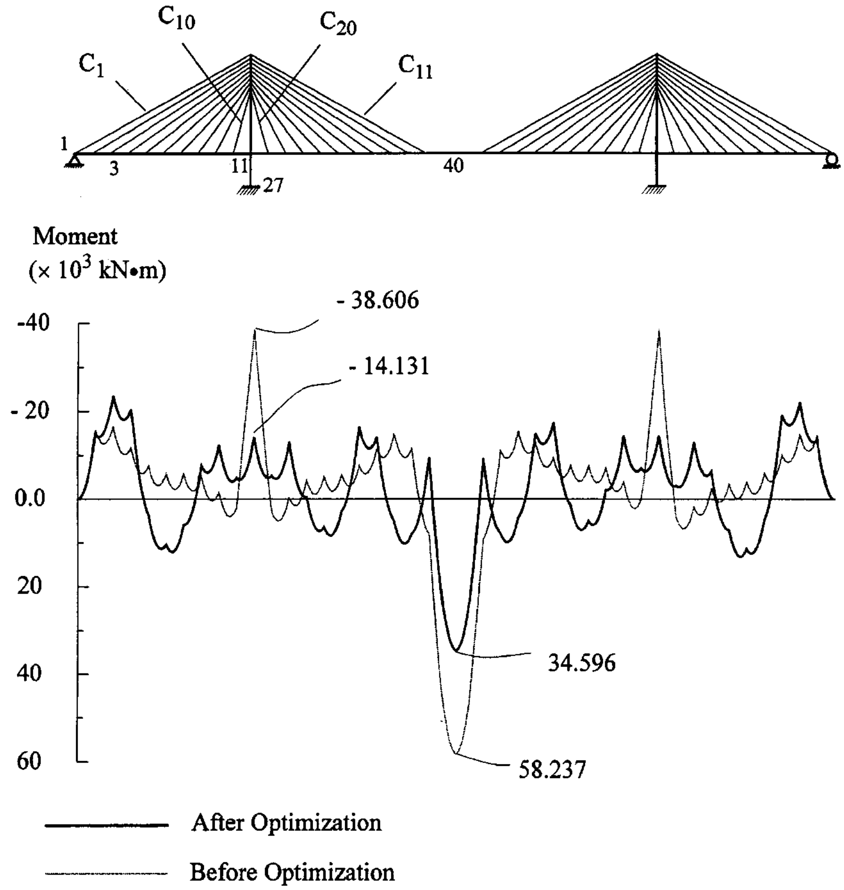


Fig. 7 Moment diagram of composite girder under dead load, superimposed dead load and live load

shows the girder's moment diagram before and after optimization. It is obvious that after optimization the girder's moment become more uniform along the bridge.

6. Conclusions

A procedure for the optimum design of cable-stayed bridge with composite box girder and concrete towers has been presented. The structural analysis and the optimization technique are integrated together. The finite element method is employed to perform the structural analysis and the nonlinearities of the cable-stayed bridges are taken into consideration. The problem is formulated as a general mathematical algorithm which uses the cost of a bridge as the objective function with bending forces, shear forces, fatigue, buckling and deflection as constraints. The design is based on the limit states design method. The design constraints are transferred from the design criteria of Canadian specifications. The optimization method used is the internal penalty function method.

The developed program has been applied to the design of several practice-sized cable-stayed bridges and its validity has been verified. It may be concluded that the proposed method provides an effective and efficient way of designing of cable-stayed bridges with composite box girders, offering saving in cost and design time. The proposed method is general and can be applied to the design of other types of cable-stayed bridges with minimal modification.

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Notations

The following symbols are used in this paper:

A = area of cross section of member
b_e = effective length of concrete slab
C' = compressive resistance of steel
C_e = Euler buckling force
E = modulus of elasticity
e = lever arm
F = objective function
F_u = ultimate strength of cables
F_y = specified minimum yield point of steel
g = inequality constraint
k = member stiffness matrix
L = length of member
l = horizontal projected length of cable
N = axial compression force in girder and tower, or number of stiffeners
M = moment
P = unit price
r_k = response factor
S = section modulus
T = cable tensile force
V = shear force
w = uniformly distributed cable weight
ρ = material density
Δ = deflection
ϕ = resistance factor

Subscripts

c = concrete
cl = cable
dd = dead load
eq = equivalent

f	= factor
ls	= longitudinal stiffener
ll	= live load
m	= member
n	= elastic modulus ratio of steel and concrete
re	= reinforcement
r	= resistance
s	= steel
sd	= superimposed dead load
ts	= transverse stiffener