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Vibration analysis of different material distributions of functionally graded microbeam

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Abstract. In the current research paper, a quasi-3D beam theory is developed for free vibration analysis of functionally graded microbeams. The volume fractions of metal and ceramic are assumed to be distributed through a beam thickness by three functions, power function, symmetric power function and sigmoid law distribution. The modified coupled stress theory is used to incorporate size dependency of micobeam. The equation of motion is derived by using Hamilton's principle, however, Navier type solution method is used to obtain frequencies. Numerical results show the effects of the function distribution, power index and material scale parameter on fundamental frequencies of microbeams. This model provides designers with guidance to select the proper distributions and functions.

Keywords: vibration; microbeam; law distribution; quasi-3D theory; functionally graded material

1. Introduction

Microbeams are important micro-scale structures that have been widely used in micro and nanotechnology industries such as microelectromechanical and biomechanical devices, micro sensors and actuators, and atomic force microscopes. The design and optimization of microbeams are extensively investigated in the literature. Since the classical continuum theory is powerless sincapting size effect, researchers have developed theories to study the size-dependencies of microstructures reasonably as nonlocal elasticity theory (Eringen 1972), strain gradient theory (Arefi *et al.* 2018, Karami *et al.* 2017, 2018c, Karami *et al.* 2018b), micropolar elasticity (Nowacki 1986) and modified couple stress theory (Yang *et al.* 2002) on which the current study is based, relates the couple stress tensor to the symmetric rotational gradient with only one material length scale parameter is used in the constitutive equations. The nonlocal continuum theory founded by Eringen (1972), assumes that the stress state at a given reference point is considered to be function of the strain states of all points in the body. therefore, it should mention some pioneer work based on the nonlocal continuum theory (Amnieh *et al.* 2018, Arani and Kolahchi 2016a, Hajmohammad *et al.* 2018b, Babak *et al.* 2016, Kolahchi 2017a, Mehdi *et al.* 2017, Kolahchi *et al.* 2017c, Bounouara *et al.* 2016, Mokhtar *et al.* 2018, Yazid *et al.* 2018). Kolahchi (2017e) studied the visco-nonlocal-refined Zigzag theories for dynamic buckling of laminated nanoplates.

New type of composite developed recently, named functionally graded material (FGM), has high potential to use as a structural material (Bennoun *et al.* 2016, Bouderba *et al.* 2013, Boukhari *et al.* 2016, Bousahla *et al.* 2016, El-Haina *et al.* 2017, Tounsi *et al.* 2013, Yahia *et al.* 2015). Recently, the application of FG materials has broadly been spread in nano-composite (Guessas *et al.* 2018, Hajmohammad *et al.* 2017, Hamid *et al.* 2016, Maryam Shokravi 2017a, Kolahchi *et al.* 2016d, Kolahchi *et al.* 2017b, Khetir *et al.* 2017). Shokravi (2017a) analyzed the buckling of embedded laminated plates with CNT-reinforced composite layers using FSDT theory and DQM method. Utilising the advantage of the modified couple stress theory, the size-dependent behaviors of FG microbeams and nanobeams has been study by many researchers (Trinh *et al*. 2016, Al-Basyouni *et al*. 2015, Bensattalah *et al.* 2016, Bouazza *et al.* 2014, Bouazza *et al.* 2015, Rakrak *et al.* 2016, Zidour *et al.* 2015, Mahmoud et al 2014, Ahouel *et al.* 2016, Bellifa *et al.* 2017b, Bouafia *et al.* 2017, Cherif *et al.* 2018, LarbiChaht *et al.* 2015, Mouffoki *et al.* 2017, Youcef *et al.* 2018, Zemri *et al.* 2015). A large number of documents discussing the size effect of the FG microbeams have been published based on modified couple stress theory.

An Euler-Bernoulli beam model for free vibration and

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buckling analysis was proposed by Kong *et al.* (2008). Asghari *et al.* (2010a, 2010b, 2011) studies static and vibration analysis of functionally graded Euler-Bernoulli and Timoshenko microbeam models. On the basis of the modified couple stress theory. Reddy (2011) has developed nonlocal models for bending, free vibration and buckling of functionally graded beam according to Euler-Bernoulli and Timoshenko beam theories. Static and dynamic analysis of third-order shear deformation functionally graded microbeams by Salamat-Talab *et al.* (2012). Reddy and Arbind (2012); developed algebraic relationships between the bending solutions of Timoshenko beam theory (TBT) and homogeneous Bernoulli-Euler beams for microstructure dependent FGM beams. Euler-Bernoulli and Timoshenko models have been widely used in the last years.

Since the shear deformation effect is more pronounced in advanced structures, shear deformation theories such as first-order shear deformation theory (FSDT) and higher-order shear deformation theories (HSDTs) (Abdelaziz *et al.* 2017, Belabed *et al.* 2014, Belabed *et al.* 2018, Bouadi *et al.* 2018, Bouhadra *et al.* 2018, Bousahla *et al.* 2014, Chikh *et al.* 2017, Mahi *et al.* 2015, Menasria *et al.* 2017, Zidi *et al.* 2017, Zine *et al.* 2018). These theories should be used to predict the static, buckling and vibration (Kolahchi *et al.* 2016c, Kolahchi and Cheraghbak 2017b, Shokravi 2017b). in the last two decades, a considerable research reports on the nanoparticles reinforced polymer (Golabchi *et al.* 2018, Hajmohammad *et al.* 2018a, Bakhadda *et al.* 2018, Besseghier *et al.* 2017, Karami *et al.* 2018a) and concrete (Hajmohammad *et al.* 2018c) investigated that they have good properties to produce high multifunctional composites for various potential applications. Maryam Shokravi (2017b) has considered nanocomposites beams made from concrete reinforced by silica nanoparticles. Zarei *et al.* (2017) stressed of emphasize on the Seismic response of underwater fluid-conveying concrete pipes reinforced with SiO_2 nanoparticles and fiber reinforced polymer (FRP) layer.

Şimşek and Reddy (2013) presented a unified higher-order beam theory for an FGM micro-beam embedded in elastic Pasternak medium. Dehrouyeh-Semnani and Nikkhah-Bahrami (2014) investigated the influence of size-dependent shear deformation on mechanical behavior of microstructures dependentbeam based on modified couple stress theory. Tounsi *et al.* (2015) and Hanifi *et al.* (2017) used a modified couple stress theory and neutral surface position to investigate the bending and dynamic behaviors of functionally graded microbeams. By using modified couple stress-theory, Thai *et al.* (2015) studied the static, vibration and buckling behaviors of FG sandwich beams without a shear correction factor. Using quasi-3D theories, a considerable research investigates the behaviors of functionally graded and composite plates (Abualnour *et al.* 2018, Benchohra *et al.* 2018, Hebali *et al.* 2014, Younsi *et al.* 2018). Trinh *et al.* (2016) investigates the behaviors of functionally graded (FG) microbeams using various shear deformation theories based on the modified couple stress theory.

Based on the frame work of the modified couple stress theory and Hamilton's principle, Trinh *et al.* (2017), studied the free vibration behavior of bi-dimensional functionally graded microbeams using a quasi-3D theory under arbitrary boundary conditions. Fang *et al.* (2018) developed a size-dependent three-dimensional dynamic model of rotating FGM micro-beams. Li *et al.* (2018), focuses on the buckling behaviors of a micro-scaled bi-directional functionally graded (FG) beam based on a generalized differential quadrature method (GDQM).

The classical beam theory (CBT) or Euler-Bernoulli beam model is the well-known one and is appropriate only for thin beams because it assumes that planes initially normal to the mid plane remain plane and normal after deformation, The CBT neglects the effects of transverse shear deformation. In order to take into account the shear deformations, the Timoshenko or the First-order Beam Theory (FBT) which is appropriate for thick beams is introduced. However, this theory is limited in use because it assumes a constant transverse shear deformation through the thickness of the beam. Therefore, a shear correction factor is required to appropriately represent the strain energy of shear deformation. To overcome this limitation, several higher order shear deformation theories (HSDTs) have been proposed (Bouderba *et al.* 2016, Bourada *et al.* 2015, Kaci *et al.* 2018), third-order deformation theory (TDT), sinusoidal deformation theory (SDT) (Bourada *et al.* 2019, Houari *et al.* 2016), exponential deformation theory (EDT), hyperbolic deformation theory (HDT) and refined deformation theory (Attia *et al.* 2018, Beldjelili *et al.* 2016, Belkorissat *et al.* 2015, Bellifa *et al.* 2017a, Fourn *et al.* 2018, Meziane *et al.* 2014, Zidi *et al.* 2014). They all neglect the thickness stretching by considering the transverse displacement independent of the thickness coordinate. For this reason, other HSDTs that include stretching effect, called quasi-3D theories, have been developed (Draiche *et al.* 2016, Hamidi *et al.* 2015). Those effects become important for very thick beams.

In short, most analyses of microbeams use the power law distribution and Mori-Tanaka scheme to calculate the effective material properties of FG microbeam. To the best of our knowledge, microbeam vibration of symmetric power function and sigmoid function are not yet studied in literature.

2. Functionally graded materials

Consider a FG microbeam with rectangular cross-section $b \times h$ and length ℓ, Fig. 1, which is made of metal and ceramic. The material properties such as Young's modulus E, density ρ and Poisson's ratio v are assumed to vary through the beam's depth continuously.

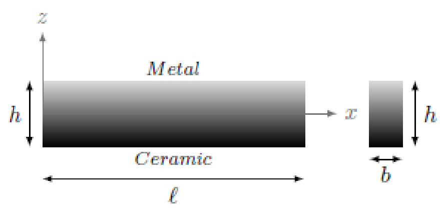


Fig. 1 Geometry and coordinate of a FG microbeam

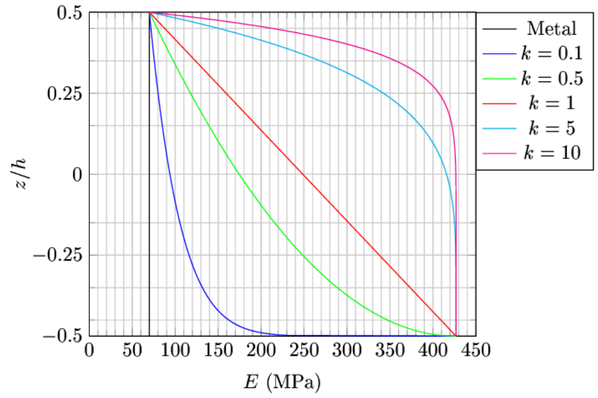


Fig. 2 Power-Law function (P-FGM)

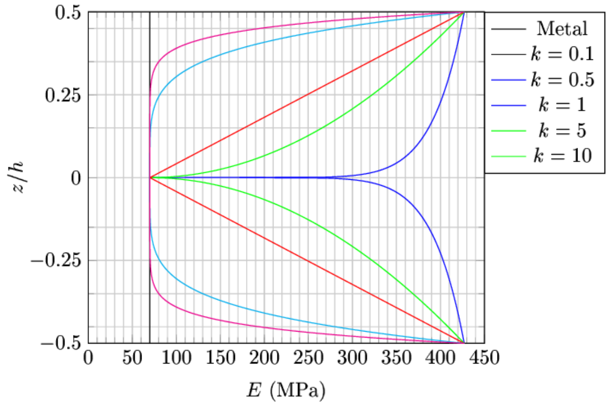


Fig. 3 Symmetric power-law function (SP-FGM)

2.1 Spatial material graduation functions

In the current analysis, three functions are assumed to describe the spatial distribution of materials through the thickness direction. The first is the power law function P-FGM, (Kolahchi *et al.* 2015, Bennai *et al.* 2015, Ranjan Kar *et al.* 2016, Tlidji *et al.* 2014), which is described by

$$P_e = P_m V_m + P_c V_c \tag{1}$$

P_m and P_c are the material properties of metal and ceramic, and V_m and V_c represent the volume fraction of metal and ceramic, which are assumed to be

$$V_c = \left(\frac{1}{2} + \frac{z}{h}\right)^k \tag{2a}$$

$$V_m = 1 - V_c \tag{2b}$$

Where k is the power-law index.

The modified symmetric power-law function S-P-FGM, Aldousari (2017), has the following form

$$P_e = P_c + (P_m + P_c)\left(\frac{-2z}{h}\right)^k \quad \left(\frac{-h}{2} \le z \le 0\right) \tag{3a}$$

$$P_e = P_c + (P_m + P_c)\left(\frac{2z}{h}\right)^k \quad \left(0 \le z \le \frac{h}{2}\right) \tag{3b}$$

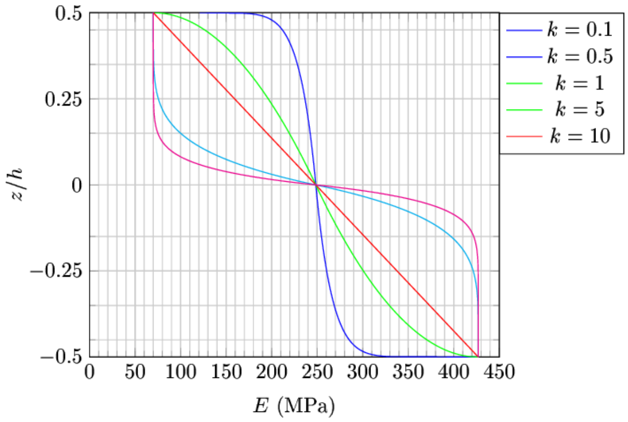


Fig. 4 Sigmoid function S-FGM.

The third function used in this study is the Sigmoid function S-FGM (Aldousari 2017, Bouguenina *et al.* 2015). This function is depicted by

$$P_e = P_m + \frac{1}{2}(P_m - P_c)\left(1 + \frac{2z}{h}\right)^k \quad \left(\frac{-h}{2} \le z \le 0\right) \tag{4a}$$

$$P_e = P_c - \frac{1}{2}(P_m - P_c)\left(1 - \frac{2z}{h}\right)^k \quad \left(0 \le z \le \frac{h}{2}\right) \tag{4b}$$

The distribution of Young's modulus through the beam thickness for P-FGM, SP-FGM and sigmoidal distribution is presented in Figs. 2, 3 and 4, respectively.

2.2 Constitutive equations

The linear stress-strain relations are expressed by (Trinh *et al.* 2016)

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{13}} & 0 \\ \overline{Q_{13}} & \overline{Q_{13}} & 0 \\ 0 & 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{yz} \end{Bmatrix} \tag{5}$$

With

$$\overline{Q_{11}} = \frac{E(z)}{1-\nu^2}, \quad \overline{Q_{13}} = \frac{\nu E(z)}{1-\nu^2} \quad \text{and} \quad Q_{55} = \frac{E(z)}{2(1+\nu)}$$

3. Governing equations of motion

In the modified couple stress theory, (Rahmani *et al.* 2018, Kolahchi and Bidgoli 2016d), the virtual strain energy is expressed in terms of both strain tensor and curvature tensor as

$$\delta U = \int_v \left(\sigma_{ij}\delta\varepsilon_{ij} + m_{ij}\delta\chi_{ij}\right) dv \quad i, j = x, y, z \tag{6}$$

σ_{ij} and ε_{ij} are the components of the stress tensor and strain tensor m_{ij} and χ_{ij} denote deviatoric part of the couple stress tensor, and symmetric curvature tensor, which are defined as

$$\chi_{ij} = \frac{1}{2}\left(\theta_{i,j} + \theta_{j,i}\right), \quad m_{ij} = 2Gl^2\chi_{ij} \tag{7}$$

G is the shear modulus; l is the material length scale parameter; and θ_i are the components of the rotation vector related to the displacement field as

$$\theta_x = \frac{1}{2}\left(\frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3}\right) \tag{8a}$$

$$\theta_y = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) \tag{8b}$$

$$\theta_z = \frac{1}{2}\left(\frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2}\right) \tag{8c}$$

According to the quasi-3D beam theory (Bennai *et al.* 2015), the displacement field is given by

$$u_1(x,z,t) = u(x,t) - z\frac{dw_b(x,t)}{dx} - f(z)\frac{dw_s(x,t)}{dx} \tag{9a}$$

$$u_2(x,z,t) = 0 \tag{9b}$$

$$u_3(x,z,t) = w_b(x,t) + w_s(x,t) + g(z)w_z(x,t) \tag{9c}$$

$u(x,t)$, $w_b(x,t)$, $w_s(x,t)$ and $w_z(x,t)$ are four unknown displacements of midplane of the beam. The thickness stretching effect in quasi-3D theories is taken into account by adding the component $g(z)w_z(x,t)$ in Eq. (9c). While $f(z)$ and $g(z)$ represent functions determining the distribution of the transverse shear and normal stresses along the thickness of the beam. In this study, the shape function is chosen based on the hyperbolic function proposed by Soldatos (HBT) (Soldatos 1992) and EBT by (Karama *et al.* 2003).

The nonzero components of the strain and the curvature tensors can be obtained as

$$\varepsilon_x = \frac{\partial u_1}{\partial x} = u' - zw_b'' - fw_s'' \tag{10a}$$

$$\gamma_{xz} = \frac{\partial u_3}{\partial x} + \frac{\partial u_1}{\partial z} = g\left(w_s' - w_z'\right) \tag{10b}$$

$$\varepsilon_z = \frac{\partial u_3}{\partial z} = g' w_z \tag{10c}$$

$$\chi_{xy} = \frac{1}{2}\frac{\partial \theta_y}{\partial x} = -\frac{1}{2}\left(w_b'' + w_s''\right) + \frac{g}{4}\left(w_s'' - w_z''\right) \tag{10d}$$

$$\chi_{yz} = \frac{1}{2}\frac{\partial \theta_y}{\partial z} = \frac{g'}{4}\left(w_s' - w_z'\right) \tag{10e}$$

Table 1 Dimensionless fundamental frequency of P-FGM microbeams ℓ/h=5

h/l	Theory	k=0	0.5	1	10
1	Classical Beam Theory (CBT)	16.0020	13.5770	12.1927	8.1401
	First-order Beam Theory (FBT)	14.7917	12.5885	11.3293	7.4837
	Exponential Beam Theory (EBT)	15.7266	13.3456	12.0034	8.0348
	hyperbolic beam theory (HBT)	15.7140	13.3316	11.9948	8.0431
	Third-order Beam Theory (TBT)*	15.7140	13.3318	11.9948	8.0425
	Sinusoidal Beam Theory (SBT)*	15.7174	13.3364	11.9971	8.0375
	Quasi-3DExponential Beam Theory (Quasi-3D EBT)	15.6441	13.2825	11.9571	7.9857
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	15.6248	13.2625	11.9444	7.9976
	Quasi-3D Third-order Beam Theory (Quasi-3D TBT) *	15.6249	13.2627	11.9444	7.9967
	Quasi-3D Sinusoidal Beam Theory (Quasi-3D SBT) *	15.6304	13.2692	11.9477	7.9887
2	Classical Beam Theory (CBT)	9.7649	8.1817	7.2974	5.1338
	First-order Beam Theory (FBT)	9.3153	7.8316	6.9992	4.8579
	Exponential Beam Theory (EBT)	9.5237	7.9931	7.1410	4.9945
	hyperbolic beam theory (HBT)	9.5175	7.9866	7.1369	5.0032
	Third-order Beam Theory (TBT)*	9.5175	7.9867	7.1369	5.0026
	Sinusoidal Beam Theory (SBT)*	9.5191	7.9888	7.1380	4.9975
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	9.5030	7.9883	7.1489	4.9894
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	9.4917	7.9776	7.1420	4.9987
	Quasi-3D Third-order Beam Theory (Quasi-3D TBT) *	9.4917	7.9776	7.1420	4.9979
	Quasi-3D Sinusoidal Beam Theory (Quasi-3D SBT) *	9.4950	7.9809	7.1435	4.9913
4	Classical Beam Theory (CBT)	7.4281	6.1304	5.4202	4.0457
	First-order Beam Theory (FBT)	7.1237	5.9008	5.2281	3.8445
	Exponential Beam Theory (EBT)	7.1785	5.9436	5.2631	3.8592
	hyperbolic beam theory (HBT)	7.1753	5.9407	5.2614	3.8649
	Third-order Beam Theory (TBT)*	7.1753	5.9407	5.2614	3.8645
	Sinusoidal Beam Theory (SBT)*	7.1761	5.9416	5.2619	3.8610
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	7.1798	5.9624	5.2963	3.8781
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	7.1713	5.9547	5.2914	3.8839
	Quasi-3D Third-order Beam Theory (Quasi-3D TBT) *	7.1713	5.9547	5.2913	3.8833
	Quasi-3D Sinusoidal Beam Theory (Quasi-3D SBT) *	7.1738	5.9568	5.2922	3.8786
8	Classical Beam Theory (CBT)	6.7181	5.4993	4.8382	3.7243
	First-order Beam Theory (FBT)	6.4448	5.2952	4.6687	3.5393
	Exponential Beam Theory (EBT)	6.4603	5.3089	4.6776	3.5133
	hyperbolic beam theory (HBT)	6.4583	5.3073	4.6764	3.5159
	Third-order Beam Theory (TBT)*	6.4583	5.3073	4.6764	3.5157
	Sinusoidal Beam Theory (SBT)*	6.4588	5.3078	4.6767	3.5139
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	6.4692	5.3365	4.7202	3.5420
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	6.4615	5.3296	4.7160	3.5448
	Quasi-3D Third-order Beam Theory (Quasi-3D TBT) *	6.4615	5.3296	4.7159	3.5444

Table 1 Continued

8	Quasi-3D Sinusoidal Beam Theory (Quasi-3D SBT) *	6.4638	5.3314	4.7166	3.5413
l=0	Classical Beam Theory (CBT)	6.4657	5.2736	4.6294	3.6115
	First-order Beam Theory (FBT)	6.2021	5.0775	4.4667	3.4317
	Exponential Beam Theory (EBT)	6.2041	5.0813	4.4666	3.3900
	hyperbolic beam theory (HBT)	6.2025	5.0801	4.4657	3.3910
	Third-order Beam Theory (TBT)*	6.2025	5.0801	4.4657	3.3909
	Sinusoidal Beam Theory (SBT)*	6.2029	5.0804	4.4659	3.3900
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	6.2159	5.1122	4.5132	3.4227
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	6.2085	5.1057	4.5092	3.4240
	Quasi-3D Third-order Beam Theory (Quasi-3D TBT) *	6.2085	5.1057	4.5091	3.4237
	Quasi-3D Sinusoidal Beam Theory (Quasi-3D SBT) *	6.2107	5.1073	4.5097	3.4214

*Trinh *et al.* (2016)

Hamilton's principle (Kolahchi 2016a) is used here to derive the equations of motion. The principle can be stated in analytical form as

$$\delta\int_{t_1}^{t_2}(U-K)dt=0 \tag{11}$$

t is the time; U is the strain energy; and K is the kinetic energy.

The governing equations of motion are obtained as

$$\delta u: N_x^{'}=I_0\ddot{u}-I_1\ddot{w}_b^{'}-I_3\ddot{w}_s^{'} \tag{12a}$$

$$\delta w_b: M_x^{b''}+R_{xy}''=I_1\ddot{u}'+I_0(\ddot{w}_b+\ddot{w}_s)-I_2\ddot{w}_b''-I_4\ddot{w}_s''+I_6\ddot{w}_z \tag{12b}$$

$$\delta w_s: M_x^{s''}+Q_{xz}^{'}+R_{xy}''-\frac{S_{xy}^{''}}{2}-\frac{T_{yz}^{'}}{2}=I_3\ddot{u}'+I_0(\ddot{w}_b+\ddot{w}_s)-I_4\ddot{w}''_b-I_5\ddot{w}''_s+I_6\ddot{w}_z \tag{12c}$$

$$\delta w_z: Q_{xz}^{'}-R_z+\frac{S_{xy}^{''}}{2}+\frac{T_{yz}^{'}}{2}=I_6(\ddot{w}_b+\ddot{w}_s)+I_7\ddot{w}_z \tag{12d}$$

The stress and moment resultants are given by

$$N_x=\int_A\sigma_x dA=Au^{'}-Bw_b''-B_s w_s''+Xw_z \tag{13a}$$

$$M_x^b=\int_A z\sigma_x dA=Bu^{'}-Dw_b''-D_s w_s''+Yw_z \tag{13b}$$

$$M_x^s=\int_A f\sigma_x dA=B_s u^{'}-D_s w_b''-Hw_s''+Y_s w_z \tag{13c}$$

$$Q_{xz}=\int_A g\sigma_{xz} dA=A_s(w_s'+w_z') \tag{13d}$$

Table 2 Dimensionless fundamental frequencies of P-FGM microbeams ℓ/h=10

h/l	Theory	k=0	0.5	1	10
1	Classical Beam Theory (CBT)	16.1966	13.7529	12.3671	8.2646
	First-order Beam Theory (FBT)	15.8337	13.4558	12.1057	8.0624
	Exponential Beam Theory (EBT)	16.1178	13.6863	12.3118	8.2332
	Hyperbolic beam theory (HBT)	16.1144	13.6824	12.3095	8.2359
	Third-order Beam Theory (TBT)*	16.1144	13.6824	12.3095	8.2357
	Sinusoidal Beam Theory (SBT)*	16.1152	13.6837	12.3100	8.2341
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	16.1012	13.6796	12.3140	8.2323
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	16.0945	13.6727	12.3100	8.2366
	Quasi-3D Third-order Beam Theory (Quasi-3D TBT) *	16.0945	13.6728	12.3100	8.2363
	Quasi-3D Sinusoidal Beam Theory (Quasi-3D SBT) *	16.0963	13.6747	12.3107	8.2330
2	Classical Beam Theory (CBT)	9.8837	8.2867	7.3994	5.2101
	First-order Beam Theory (FBT)	9.7550	8.1863	7.3134	5.1293
	Exponential Beam Theory (EBT)	9.8157	8.2333	7.3548	5.1697
	Hyperbolic beam theory (HBT)	9.8140	8.2315	7.3537	5.1724
	Third-order Beam Theory (TBT)*	9.8140	8.2316	7.3536	5.1723
	Sinusoidal Beam Theory (SBT)*	9.8144	8.2321	7.3539	5.1707
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	9.8129	8.2446	7.3779	5.1851
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	9.8072	8.2393	7.3748	5.1893
	Quasi-3D Third-order Beam Theory (Quasi-3D TBT) *	9.8072	8.2393	7.3747	5.1889
	Quasi-3D Sinusoidal Beam Theory (Quasi-3D SBT) *	9.8087	8.2406	7.3751	5.1855
4	Classical Beam Theory (CBT)	7.5185	6.2089	5.4955	4.1055
	First-order Beam Theory (FBT)	7.4332	6.1445	5.4415	4.0479
	Exponential Beam Theory (EBT)	7.4488	6.1567	5.4515	4.0517
	Hyperbolic beam theory (HBT)	7.4479	6.1559	5.4510	4.0536
	Third-order Beam Theory (TBT)*	7.4479	6.1559	5.4510	4.0534
	Sinusoidal Beam Theory (SBT)*	7.4481	6.1561	5.4511	4.0523
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	7.4527	6.1784	5.4886	4.0781
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	7.4468	6.1734	5.4857	4.0814
	Quasi-3D Third-order Beam Theory (Quasi-3D TBT) *	7.4468	6.1733	5.4856	4.0810
	Quasi-3D Sinusoidal Beam Theory (Quasi-3D SBT) *	7.4484	6.1745	5.4857	4.0779
8	Classical Beam Theory (CBT)	6.7998	5.5696	4.9054	3.7792
	First-order Beam Theory (FBT)	6.7238	5.5129	4.8581	3.7266
	Exponential Beam Theory (EBT)	6.7281	5.5167	4.8606	3.7184
	Hyperbolic beam theory (HBT)	6.7276	5.5163	4.8603	3.7193
	Third-order Beam Theory (TBT)*	6.7276	5.5163	4.8603	3.7192
	Sinusoidal Beam Theory (SBT)*	6.7277	5.5164	4.8604	3.7187
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	6.7345	5.5429	4.9038	3.7494
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	6.7285	5.5376	4.9008	3.7520
	Quasi-3D Third-order Beam Theory (Quasi-3D TBT) *	6.7285	5.5376	4.9007	3.7516

Table 2 Continued

8	Quasi-3D Sinusoidal Beam Theory (Quasi-3D SBT) *	6.7301	5.5387	4.9008	3.7488
l=0	Classical Beam Theory (CBT)	6.5444	5.3410	4.6937	3.6647
	First-order Beam Theory (FBT)	6.4713	5.2867	4.6484	3.6138
	Exponential Beam Theory (EBT)	6.4718	5.2877	4.6484	3.6010
	Hyperbolic beam theory (HBT)	6.4713	5.2874	4.6481	3.6013
	Third-order Beam Theory (TBT)*	6.4713	5.2874	4.6481	3.6013
	Sinusoidal Beam Theory (SBT)*	6.4714	5.2875	4.6482	3.6010
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	6.4791	5.3158	4.6940	3.6338
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	6.4730	5.3104	4.6911	3.6359
	Quasi-3D Third-order Beam Theory (Quasi-3D TBT) *	6.4731	5.3102	4.6909	3.6356
	Quasi-3D Sinusoidal Beam Theory (Quasi-3D SBT) *	6.4747	5.3113	4.6910	3.6330

*Trinh *et al.* (2016)

$$R_z = \int_A g'\sigma_z dA = Xu' - Yw_b'' - Y_s w_s'' + zw_z \tag{13e}$$

$$R_{xy} = \int_A m_{xy} dA = -A_n(w_b'' + w_s'') + \frac{B_n}{2}(w_s'' - w_z'') \tag{13f}$$

$$S_{xy} = \int_A g m_{xy} dA = -B_n(w_b'' + w_s'') + \frac{D_n}{2}(w_s'' - w_z'') \tag{13g}$$

$$T_{yz} = \int_A g' m_{yz} dA = \frac{H_n}{2}(w_s' - w_z') \tag{13h}$$

The governing equations of motion of EBT and HBT are obtained by neglecting the shape function $g(z)$ in Eq. (9), as

$$\delta u : N_x' = I_0 \ddot{u} - I_1 \ddot{w}_b' - I_3 \ddot{w}_s' \tag{14a}$$

$$\delta w_b : M_x^{b''} + R_{xy}'' = I_1 \ddot{u}' + I_0(\ddot{w}_b + \ddot{w}_s) - I_2 \ddot{w}_b'' - I_4 \ddot{w}_s'' \tag{14b}$$

$$\delta w_s : M_x^{s''} + Q_{xz}' + R_{xy}'' - \frac{S_{xy}''}{2} - \frac{T_{yz}'}{2} = I_3 \ddot{u}' + I_0(\ddot{w}_b + \ddot{w}_s) - I_4 \ddot{w}''_b - I_5 \ddot{w}''_s \tag{14c}$$

The stress and moment resultants are

$$N_x = \int_A \sigma_x dA = Au' - Bw_b'' - B_s w_s'' \tag{15a}$$

$$M_x^b = \int_A z\sigma_x dA = Bu' - Dw_b'' - D_s w_s'' \tag{15b}$$

$$M_x^s = \int_A f\sigma_x dA = B_s u' - D_s w_b'' - Hw_s'' \tag{15c}$$

$$Q_{xz} = \int_A g\sigma_{xz} dA = A_s w_s' \tag{15d}$$

$$R_{xy} = \int_A m_{xy} dA = -A_n(w_b'' + w_s'') + \frac{B_n}{2} w_s'' \tag{15e}$$

$$S_{xy} = \int_A g m_{xy} dA = -B_n(w_b'' + w_s'') + \frac{D_n}{2} w_s'' \tag{15f}$$

$$T_{yz} = \int_A g' m_{yz} dA = \frac{H_n}{2} w_s' \tag{15g}$$

By considering the shape functions $f(z)=0$, $g(z)=1$, the governing equations of motion of FBT.

$$\delta u : N_x' = I_0 \ddot{u} - I_1 \ddot{w}_b' \tag{16a}$$

$$\delta w_b : M_x^{b''} + R_{xy}'' = I_1 \ddot{u}' + I_0(\ddot{w}_b + \ddot{w}_s) - I_2 \ddot{w}_b'' \tag{16b}$$

$$\delta w_s : Q_{xz}' + \frac{R_{xy}''}{2} = I_0(\ddot{w}_b + \ddot{w}_s) \tag{16c}$$

In this case, the stress and moment resultants are

$$N_x = \int_A \sigma_x dA = Au' - Bw_b'' \tag{17a}$$

$$M_x^b = \int_A z\sigma_x dA = Bu' - Dw_b'' \tag{17b}$$

$$Q_{xz} = \int_A g\sigma_{xz} dA = A_s w_s' \tag{17c}$$

$$R_{xy} = \int_A m_{xy} dA = -A_n w_b'' - \frac{A_n}{2} w_s'' \tag{17d}$$

The governing equations of motion of CBT can be obtained by neglecting shear component $w_s=0$ and considering the shape functions as $f(z)=z$, $g(z)=0$.

$$\delta u : N_x' = I_0 \ddot{u} - I_1 \ddot{w}_b' \tag{18a}$$

$$\delta w_b : M_x^{b''} + R_{xy}'' = I_1 \ddot{u}' + I_0 \ddot{w}_b - I_2 \ddot{w}_b'' \tag{18b}$$

The stress and moment resultants of CBT are

$$N_x = \int_A \sigma_x dA = Au' - Bw_b'' \tag{19a}$$

$$M_x^b = \int_A z\sigma_x dA = Bu' - Dw_b'' \tag{19b}$$

$$R_{xy} = \int_A m_{xy} dA = -A_n w_b'' \tag{19c}$$

The various stiffness parameters are defined as follows

$$(A, B, B_s, D) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (1, z, f, z^2)\overline{Q_{11}} b dz \tag{20a}$$

Table 3 Dimensionless fundamental frequencies of SP-FGM microbeams ℓ/h=5

h/l	Theory	k=0	0.5	1	10
1	Classical Beam Theory (CBT)	16.0020	13.9694	12.7708	8.3964
	First-order Beam Theory (FBT)	14.7917	12.8199	11.6672	7.6264
	Exponential Beam Theory (EBT)	15.7266	13.7610	12.5933	8.2750
	Hyperbolic beam theory (HBT)	15.7140	13.7573	12.5955	8.1042
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	15.6441	13.6665	12.4939	8.1918
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	15.6248	13.6610	12.4980	8.2166
2	Classical Beam Theory (CBT)	9.7649	8.8086	8.1996	5.5026
	First-order Beam Theory (FBT)	9.3153	8.3275	7.7085	5.1367
	Exponential Beam Theory (EBT)	9.5237	8.5537	7.9401	5.3172
	Hyperbolic beam theory (HBT)	9.5175	8.5565	7.9507	5.5212
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	9.5030	8.5203	7.8998	5.2828
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	9.4917	8.5207	7.9099	5.3020
4	Classical Beam Theory (CBT)	7.4281	6.9429	6.5778	4.4970
	First-order Beam Theory (FBT)	7.1237	6.5928	6.2084	4.2125
	Exponential Beam Theory (EBT)	7.1753	6.6249	6.2250	4.0598
	Hyperbolic beam theory (HBT)	7.1785	6.6173	6.2080	4.2339
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	7.1798	6.6118	6.1985	4.2258
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	7.1713	6.6165	6.2140	4.2342
8	Classical Beam Theory (CBT)	6.7181	6.3919	6.1054	4.2082
	First-order Beam Theory (FBT)	6.4448	6.0705	5.7629	3.9418
	Exponential Beam Theory (EBT)	6.4603	6.0300	5.6773	3.9062
	Hyperbolic beam theory (HBT)	6.4583	6.0396	5.6964	4.2277
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	6.4692	6.0353	5.6804	3.9087
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	6.4615	6.0418	5.6979	3.9080
l=0	Classical Beam Theory (CBT)	6.4657	6.1986	5.9407	4.1081
	First-order Beam Theory (FBT)	6.2021	5.8862	5.6066	3.8475
	Exponential Beam Theory (EBT)	6.2041	5.8213	5.4874	3.7896
	Hyperbolic beam theory (HBT)	6.2025	5.8316	5.5071	3.5160
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	6.2159	5.8307	5.4955	3.7964
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	6.2085	5.8380	5.5136	3.7909

$$(D_s, H, Z) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (zf, f^2, g'^2)\overline{Q_{11}} b dz \tag{20b}$$

$$A_s = \int_{-\frac{h}{2}}^{\frac{h}{2}} g^2 \overline{Q_{55}} b dz \tag{20c}$$

$$(X, Y, Y_s) = \int_{-\frac{h}{2}}^{\frac{h}{2}} g'(1, z, f)\overline{Q_{13}} b dz \tag{20d}$$

$$(A_n, B_n) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (1, g)\frac{l^2 E(z)}{2(1+\nu)} dz \tag{20e}$$

$$(D_n, H_n) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (g^2, g'^2)\frac{l^2 E(z)}{2(1+\nu)} dz \tag{20f}$$

The mass parameters are defined by

$$(I_0, I_1, I_2) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho(1, z, z^2) b dz \tag{21a}$$

$$(I_3, I_4, I_5) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho(f, zf, f^2) b dz \tag{21b}$$

$$(I_6, I_7) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho(g, g^2) b dz \tag{21c}$$

4. Analytical solutions

The equations of motion are solved using the Navier solutions for simply supported Microbeams. The variables u, w_b, w_s and w_z can be written by assuming the following forms

$$u(x,t) = \sum_{n=1}^{\infty} U_n \cos \alpha x e^{iwt} \tag{22a}$$

$$w_b(x,t) = \sum_{n=1}^{\infty} W_{bn} \sin \alpha x e^{iwt} \tag{22b}$$

$$w_s(x,t) = \sum_{n=1}^{\infty} W_{sn} \sin \alpha x e^{iwt} \tag{22c}$$

$$w_z(x,t) = \sum_{n=1}^{\infty} W_{zn} \sin \alpha x e^{iwt} \tag{22d}$$

5. Numerical results and discussion

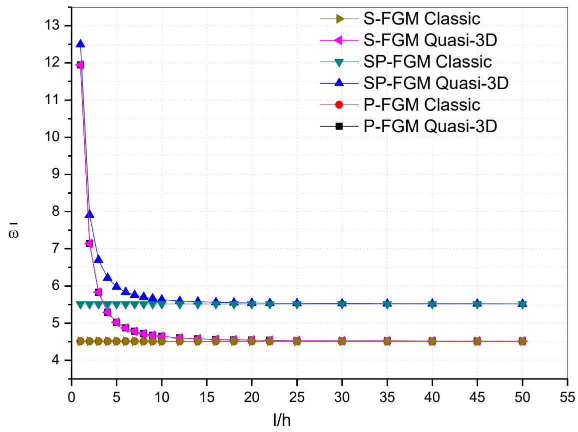
The aim of this analysis is showing the accuracy of the developed formulation. We validate by comparing the computed natural frequencies with respect to reference

Table 4 Dimensionless fundamental frequencies of SP-FGM microbeams ℓ/h=10

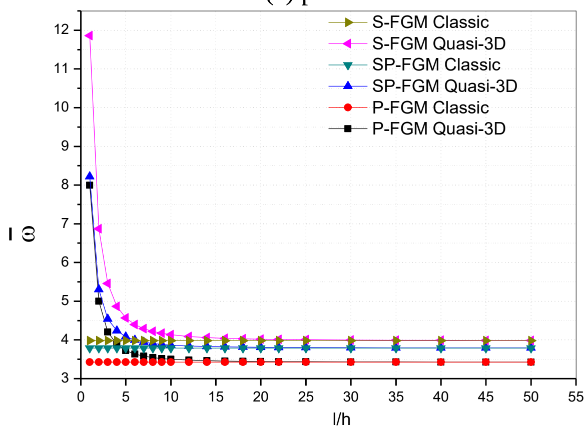
h/*l*	Theory	*k*=0	0.5	1	10
1	Classical Beam Theory (CBT)	16.1966	14.1436	12.9314	8.5006
	First-order Beam Theory (FBT)	15.8337	13.7955	12.5954	8.2649
	Exponential Beam Theory (EBT)	16.1178	14.0835	12.8799	8.4653
	Hyperbolic beam theory (HBT)	16.1144	14.0827	12.8809	8.4710
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	16.1012	14.0630	12.8578	8.4463
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	16.0945	14.0612	12.8593	8.4542
2	Classical Beam Theory (CBT)	9.8837	8.9184	8.3027	5.5709
	First-order Beam Theory (FBT)	9.7550	8.7794	8.1599	5.4639
	Exponential Beam Theory (EBT)	9.8157	8.84580	8.2283	5.5176
	Hyperbolic beam theory (HBT)	9.8140	8.8469	8.2318	5.5237
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	9.81289	8.8384	8.2185	5.5097
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	9.8072	8.8378	8.2219	5.5165
4	Classical Beam Theory (CBT)	7.5185	7.0294	6.6605	4.5528
	First-order Beam Theory (FBT)	7.4332	6.9303	6.5552	4.4712
	Exponential Beam Theory (EBT)	7.4488	6.9369	6.5541	4.4768
	Hyperbolic beam theory (HBT)	7.4479	6.9393	6.5595	4.4801
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	7.4527	6.9372	6.5524	4.4762
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	7.4468	6.9375	6.5575	4.4800
8	Classical Beam Theory (CBT)	6.7998	6.4716	6.1822	4.2604
	First-order Beam Theory (FBT)	6.7238	6.3812	6.0852	4.1845
	Exponential Beam Theory (EBT)	6.7281	6.3687	6.0584	4.1728
	Hyperbolic beam theory (HBT)	6.7276	6.3717	6.0645	4.1731
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	6.7345	6.3722	6.0606	4.1755
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	6.7285	6.3729	6.0662	4.1765
l=0	Classical Beam Theory (CBT)	6.5444	6.2759	6.0154	4.1591
	First-order Beam Theory (FBT)	6.4713	6.1882	5.9210	4.0849
	Exponential Beam Theory (EBT)	6.4718	6.1686	5.8841	4.0665
	Hyperbolic beam theory (HBT)	6.4713	6.1718	5.8904	3.6325
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	6.4791	6.1734	5.8878	4.0706
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	6.4730	6.1743	5.8937	4.0700

solutions available in the literature. A fully simply supported FG Microbeams composed of *Al*/*Sic*, E_m=70 Gpa, ρ_m=2702 kg/m^3, v_m=0.3 and E_c=427 GPa, ρ_m=3100 kg/m^3, v_c=0.17 with two slenderness ratios (ℓ/h=5,10) are considered. The materiel proprieties are estimated by three rules of mixture (P-FGM, S-FGM and SP-FGM). The length scale parameter is assumed to be constant l=15 μm, Thai *et al.* (2015). The natural frequencies are normalized by

$$\overline{w} = w\frac{\ell}{h}\sqrt{\frac{\rho_m}{E_m}} \tag{23}$$



(a) p1



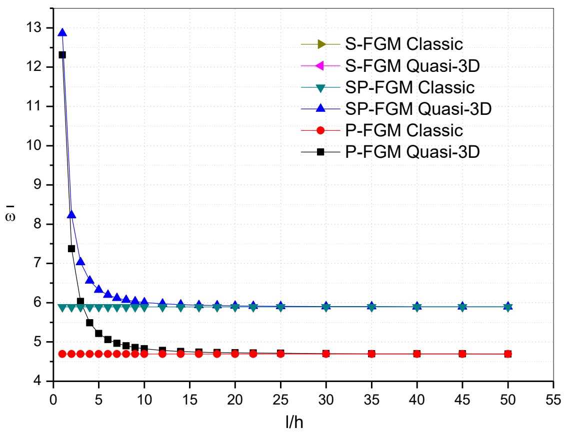
(b) p10

Fig. 5 Dimensionless fundamental frequencies of SiC/Al microbeams (ℓ/h=5)

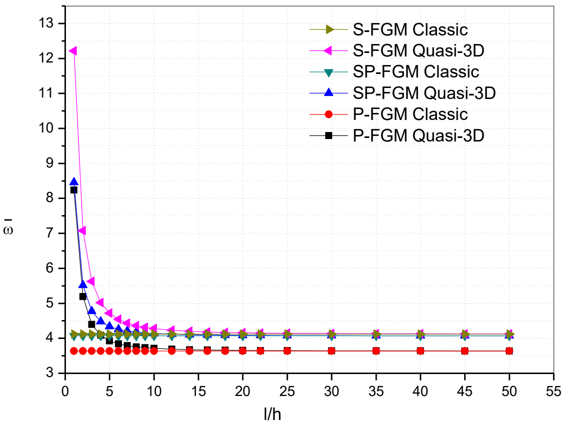
The fundamental frequencies of P-FGM microbeams are presented in Tables 1, 2 with varying material scale parameter and material distribution for two slenderness ratios respectively. It is obvious that the results are in excellent agreement with those generated by Trinh *et al.* (2016) for SBT and Quasi-3D SBT, the slight difference is due to the beam theory used. It appears that, increase in material distribution tends to decrease the frequency at the same material scale parameter. The frequencies are higher when the size effect is very strong and the increase in length scale parameter leads to decrease the natural frequency. As observed by Trinh *et al.* (2016), the frequencies computed by EBT and HBT are slightly higher than those form quasi-3D theories. And the results of the EBT, HBT and quasi-3D theories are between those of CBT and FBT.

In Tables 3, 4, for respectively, the variation of natural frequencies of SP-FGM microbeams are illustrated, the same effect is noted for SP-FGM as P-FGM microbeams. At the same value of material distribution and material scale parameter, the natural frequencies for SP-FGM are higher than P-FGM. This is due to the distribution of ceramics phase in SP-FGM is less than the distribution in P-FGM.

It is also observed that for a sigmoid distribution, the natural frequencies decreased as the material parameter distribution *k* increasing and length scale parameter decreasing, as presented in Tables 4, 5. It showed that for



(a) p=1



(b) p=10

Fig. 6 Dimensionless fundamental frequencies of SiC/Al microbeams (ℓ/h=10)

k=1 S-FGM and P-FGM generate the same dimensionless fundamental frequency. By varying the material distribution index from 0 to 10, the dimensionless fundamental frequency is smoothly reduced for S-FGM distribution law. However, the reduction in frequencies for P-FGM and SP-FGM laws distribution is very important.

Figs. 5 and 6 present the variation of dimensionless fundamental frequency versus material length scale parameter for the three distribution lows, for k=0 and k=10, is given for HBT quasi-3D and (ℓ/h=5) and 10, respectively. The size effects in frequencies are very significant when h/l<5, but become insignificant for h/l<10.

6. Conclusions

Vibration analysis of an FG simply supported Microbeam modeled according to quasi-3D theory. The volume fractions of metal and ceramic are assumed to be distributed through a beam thickness by three functions, which are, power function, symmetric power function, and sigmoid function. The equations of motion are derived according to Hamilton's principle.

The results are validated compared to previous studies. Numerical results show significant effects of the function distribution, the power index and the material scale parameter on the fundamental frequencies.

Table 5 Dimensionless fundamental frequencies of S-FGM microbeams ℓ/h=5

h/l	Theory	k=0	0.5	1	10
1	Classical Beam Theory (CBT)	12.3381	12.2675	12.1927	12.0503
	First-order Beam Theory (FBT)	11.3867	11.3598	11.3293	11.2845
	Exponential Beam Theory (EBT)	12.1200	12.0638	12.0034	11.8916
	Hyperbolic beam theory (HBT)	12.1102	12.055	11.9948	11.8825
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	12.0428	12.0024	11.9571	11.8745
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	12.0252	11.9884	11.9444	11.8589
2	Classical Beam Theory (CBT)	7.5859	7.4459	7.2974	6.9575
	First-order Beam Theory (FBT)	7.2223	7.1150	6.9993	6.7298
	Exponential Beam Theory (EBT)	7.3896	7.2692	7.1410	6.8467
	Hyperbolic beam theory (HBT)	7.3847	7.2649	7.1369	6.8422
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	7.3702	7.2639	7.1489	6.8789
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	7.3582	7.2556	7.1420	6.8689
4	Classical Beam Theory (CBT)	5.8203	5.6274	5.4202	4.9152
	First-order Beam Theory (FBT)	5.5697	5.4063	5.2281	4.7848
	Exponential Beam Theory (EBT)	5.6142	5.4455	5.2634	4.8163
	Hyperbolic beam theory (HBT)	5.6116	5.4434	5.2615	4.8136
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	5.6180	5.4641	5.2963	4.8764
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	5.6075	5.4577	5.2914	4.8674
8	Classical Beam Theory (CBT)	5.2875	5.0718	4.8382	4.2538
	First-order Beam Theory (FBT)	5.0612	4.8742	4.6687	4.1434
	Exponential Beam Theory (EBT)	5.0739	4.8841	4.6776	4.1560
	Hyperbolic beam theory (HBT)	5.0722	4.8828	4.6764	4.1539
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	5.0860	4.9116	4.7202	4.2291
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	5.0758	4.9058	4.7160	4.2200
l=0	Classical Beam Theory (CBT)	5.0988	4.8739	4.6294	4.0108
	First-order Beam Theory (FBT)	4.8801	4.6835	4.4667	3.9063
	Exponential Beam Theory (EBT)	4.8817	4.6835	4.4666	3.9127
	Hyperbolic beam theory (HBT)	4.8804	4.6824	4.4657	3.9109
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	4.8970	4.7143	4.5132	3.9915
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	4.8869	4.7087	4.5092	3.9823

Table 6 Dimensionless fundamental frequencies of S-FGM microbeams ℓ/h=10

h/l	Theory	k=0	0.5	1	10
1	Classical Beam Theory (CBT)	12.4882	12.4297	12.3671	12.2527
	First-order Beam Theory (FBT)	12.2025	12.1560	12.1057	12.0182
	Exponential Beam Theory (EBT)	12.4258	12.3708	12.3118	12.2050
	Hyperbolic beam theory (HBT)	12.4231	12.3684	12.3095	12.2024
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	12.4100	12.3644	12.3140	12.2222
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	12.4027	12.3596	12.3100	12.2161

Table 6 Continued

2	Classical Beam Theory (CBT)	7.6782	7.5431	7.3994	7.0694
	First-order Beam Theory (FBT)	7.2223	7.4480	7.3134	7.0032
	Exponential Beam Theory (EBT)	7.6228	7.4930	7.3548	7.0372
	Hyperbolic beam theory (HBT)	7.6214	7.4918	7.3537	7.0359
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	7.6217	7.5044	7.3779	7.0806
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	7.6142	7.5003	7.3748	7.0746
4	Classical Beam Theory (CBT)	5.8911	5.7007	5.4955	4.9935
	First-order Beam Theory (FBT)	5.8207	5.6385	5.4415	4.9566
	Exponential Beam Theory (EBT)	5.8334	5.6497	5.4515	4.9655
	Hyperbolic beam theory (HBT)	5.8327	5.6491	5.4510	4.9648
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	5.8400	5.6718	5.4886	5.0307
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	5.8316	5.6676	5.4857	5.0237
8	Classical Beam Theory (CBT)	5.3518	5.1378	4.9054	4.3214
	First-order Beam Theory (FBT)	5.2887	5.0827	4.8581	4.2906
	Exponential Beam Theory (EBT)	5.2923	5.0855	4.8606	4.2941
	Hyperbolic beam theory (HBT)	5.2918	5.0852	4.8603	4.2935
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	5.3017	5.1119	4.9038	4.3703
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	5.2929	5.1076	4.9008	4.3625
l=0	Classical Beam Theory (CBT)	5.1608	4.9373	4.6937	4.0745
	First-order Beam Theory (FBT)	5.1000	4.8844	4.6484	4.0455
	Exponential Beam Theory (EBT)	5.1004	4.8843	4.6484	4.0472
	Hyperbolic beam theory (HBT)	5.1001	4.8840	4.6481	4.0096
	Quasi-3D Exponential Beam Theory (Quasi-3D EBT)	5.1110	4.9125	4.6941	4.1283
	Quasi-3D hyperbolic beam theory (Quasi-3D HBT)	5.1019	4.9081	4.6911	4.1202

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