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Thermomechanical interactions in transversely isotropic thick circular plate with axisymmetric heat supply

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Abstract. The present investigation has focus on the study of deformation due to thermomechanical sources in a thick circular plate. The thick circular plate is homogeneous, transversely isotropic with two temperatures and without energy dissipation. The upper and lower surfaces of the thick circular plate are traction free. The Laplace and Hankel transform has been used for finding the general solution to the field equations. The analytical expressions of stresses, conductive temperature and displacement components are computed in the transformed domain. However, the resulting quantities are obtained in the physical domain by using numerical inversion technique. Numerically simulated results are illustrated graphically. The effects of two temperatures by considering different values of temperature parameters are shown on the various components. Some particular cases are also figured out from the present investigation.

Keywords: Laplace and Hankel transform; two temperature; thermoelastic; thick circular plate; transversely isotropic

1. Introduction

Thermoelasticity is a branch of applied mechanics, which deals with the effect of heat on the solid elastic bodies giving rise to the deformation and stresses in the body. The change in temperature of a real homogeneous body may occur due to non-uniform heating of a body, body force, the motion of a body or the external loading of a body. The materials (natural and synthetic) in which the properties like thermal conductivity vary with orientation are called anisotropic materials. Transversely isotropic medium is a special kind of anisotropic medium. Transversely isotropic media are those in which there is one axis of elastic and thermal symmetry. If this axis is taken as the X_3 axis, then in any plane perpendicular to this axis (in other words in plane parallel to the X_1X_2-plane), the elastic and thermal behaviour is isotropic, whereas in the X_3 direction these properties are different. The number of independent constants for transversely isotropic medium in the (fourth-rank) elasticity tensor is reduced to 5. The hexagonal crystals, like Cadmium and Zinc, are transversely isotropic.

The theory of thermoelasticity deals with the prediction of thermomechanical behaviour of the elastic solids. It represents an overview of both the theory of heat conduction and theory of elasticity in solids. Temperature changes cause thermal effects on materials like thermal stress, strain, and deformation. The study can be useful to the design of structure or machines in engineering applications. The proposed model for studying the variations in the transversely isotropic thick circular plates can be useful in numerous applications in engineering discipline such as nuclear reactor design, geothermal engineering, advanced aircraft structure design, industrial engineering, submarine structures, high-energy particle accelerators, and many emerging technologies.

Chen *et al.* (1968a, 1968b, 1969) formulated a two temperature thermoelasticity of deformable bodies for the conduction of heat depending on two types of temperatures, the thermodynamic temperature T, and the conductive temperature φ. For the cases which are time-independent, the difference between T and φ is proportional to supply of heat, and in the absence of heat supply, these temperatures are equal. For the problems which depends on time, the difference in two temperatures is non-zero and do not depend on heat supply. When the two-temperature factor is zero, then $\varphi=T$ and the coupled thermoelasticity can be derived from the two-temperature theory.

Marin (1997) had proved the Cesaro means of the kinetic and strain energies of dipolar bodies with finite energy. Marin (1998) investigated and solved the initial-boundary value problem without recourse either to an energy conservation law or to any boundedness assumptions on the thermoelastic coefficients in thermoelastic bodies with voids. Marin and Stan (2013) studied the micro stretch elastic bodies using Lesan and Quintanilla of dipolar bodies with stretch. Marin *et al.* (2013) constructed a new theory of thermoelasticity by considering heat conduction in deformable bodies depending upon two temperatures. Ezzat *et al.* (2016) built a model of two-temperature thermoelasticity theory with time-delay and Kernel function. Marin *et al.* (2017) studied the GN-thermoelastic theory for a dipolar body using mixed

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initial BVP and proved a result of Hölder's-type stability. Ezzat *et al*. (2017) developed a unified mathematical fractional model of two-temperature phase-lag Green-Naghdi thermoelasticity theories based on two-temperature. Ezzat *et al.* (2012), Ezzat *et al.* (2015), Ezzat and El-Bary (2016), Ezzat and El-Bary (2017) presented a new mathematical models of two-temperature electro-thermo viscoelasticity theory in the perspective of heat conduction and provided applications of this model to different problems like concrete problems, a thermal shock problem and a problem for a half-space exposed to ramp-type heating respectively.

Tripathi *et al.* (2015) presented the effect of axisymmetric heat supply for diffusion in an infinite as well as finitely thick thermoelastic copper plate with one relaxation time. Tripathi *et al.* (2016) presented the thermoelastic diffusion interactions in a thick circular copper material plate. Kant and Mukhopadhyay (2017) studied the thermoelastic effect of axisymmetric temperature distribution applied inside at the lower and upper surfaces of an infinitely extended thick plate. Kumar *et al*. (2016a, 2016b) depicted the effect of time and thermal, diffusion phase lags for an axisymmetric heat supply in a ring and thick circular plate respectively. Moreover, Kumar *et al.* (2017) investigated the homogeneous isotropic thermoelastic thick circular plate with dual phase lag and two temperature. Kumar *et al.* (2017) investigated the Rayleigh waves in a homogeneous transversely isotropic magnetothermoelastic with two temperature, in the presence of Hall current and rotation. Shahani and Torki (2018) investigated the thermoelasticity problem in a thick-walled orthotropic hollow cylinder by applying time-dependent thermal and mechanical boundary conditions on the inner and the outer surfaces of the cylinder. Despite of this several researchers worked on different theory of thermoelasticity as Akbaş (2017), Ozdemir (2018), Taleb *et al.* (2018), Houari *et al.* (2018), Heydari (2018), Liu *et al.* (2019).

In this paper, we have attempted to study the deformation in transversely isotropic thick circular plate due to thermal and mechanical sources. The Laplace and Hankel transform has been used for finding the general solution to the field equations. The analytical expressions of stresses, conductive temperature, displacement components are computed in transformed domain. However, the resulting quantities are obtained in the physical domain by using numerical inversion technique.

2. Basic equations

Following Chandrasekharaiah (1998), Youssef (2011) and Green and Naghdi (1992), the constitutive relations and field equations for an anisotropic thermoelastic medium with GN theory of type-II in absence of body forces and heat sources are

$$t_{ij} = C_{ijkl}e_{kl} - \beta_{ij}T, \tag{1}$$

$$C_{ijkl}e_{kl,j} - \beta_{ij}T_{,j} = \rho\, \ddot{u}_i \tag{2}$$

$$K_{ij}\varphi_{,ij} = \beta_{ij}T_0\ddot{e}_{ij} + \rho C_E\ddot{T} \tag{3}$$

where

$$T = \varphi - a_{ij}\varphi_{,ij}, \tag{4}$$

$$\beta_{ij} = C_{ijkl}\alpha_{ij} \tag{5}$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right).\ i = 1,2,3 \tag{6}$$

Here C_{ijkl} are elastic parameters and having symmetry $(C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk})$. The basis of these symmetries of C_{ijkl} is due to

i. The stress tensor is symmetric, which is only possible if $(C_{ijkl} = C_{jikl})$

ii. If a strain energy density exists for the material, the elastic stiffness tensor must satisfy $C_{ijkl} = C_{klij}$

iii. From stress tensor and elastic stiffness tensor symmetries infer $(C_{ijkl} = C_{ijlk})$ and $C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk}$

β_{ij} is the thermal tensor, T is the thermodynamic temperature, T_0 is the reference temperature, t_{ij} are the components of stress tensor, e_{ij} are the components of strain tensor, u_i are the displacement components, ρ is the density, C_E is the specific heat, K_{ij} is the materialistic constant, a_{ij} are the two temperature parameters, α_{ij} is the coefficient of linear thermal expansion.

3. Formulation of the problem

Consider a transversely isotropic thick circular plate of thickness 2b occupying the space D defined by $0 \le r \le \infty, -b \le z \le b$. Let the plate be subjected to axisymmetric heat supply on the radial and the axial direction of the cylindrical co-ordinate system. The initial temperature in the thick circular plate is given by a constant temperature T_0 and heat flux $g_0F(r,z)$ is prescribed on the upper and lower surfaces. We take a cylindrical polar co-ordinate system (r,θ,z) with symmetry about Z-axis. As the problem considered is plane axisymmetric, the field component $(v = 0)$, and $(u, w, and\ \varphi)$ are independent of θ. We restrict our analysis to two-dimension problem with $\vec{u} = (u, 0, w)$, also we use the appropriate transformation following Slaughter (2002) on the set of Eqs. (1)-(3) to derive the equations for transversely isotropic thermoelastic solid with two temperatures and without energy dissipation, to obtain

$$C_{11}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{1}{r}u\right) + C_{13}\left(\frac{\partial^2 w}{\partial r\partial z}\right) + C_{44}\frac{\partial^2 u}{\partial z^2} + C_{44}\left(\frac{\partial^2 w}{\partial r\partial z}\right) - \beta_1\frac{\partial}{\partial r}\left\{\varphi - a_1\left(\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r}\right) - a_3\frac{\partial^2\varphi}{\partial z^2}\right\} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{7}$$

$$(C_{11} + C_{44})\left(\frac{\partial^2 u}{\partial r\partial z} + \frac{1}{r}\frac{\partial u}{\partial z}\right) + C_{44}\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) + C_{33}\frac{\partial^2 w}{\partial z^2} - \beta_3\frac{\partial}{\partial z}\left\{\varphi - a_1\left(\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r}\right) - a_3\frac{\partial^2\varphi}{\partial z^2}\right\} = \rho\frac{\partial^2 w}{\partial t^2} \tag{8}$$

$$
\begin{aligned}
K_1\left(\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}\right)+K_3\frac{\partial^2\varphi}{\partial z^2}
&= T_0\frac{\partial^2}{\partial t^2}\left(\beta_1\frac{\partial u}{\partial r}+\beta_3\frac{\partial w}{\partial z}\right)\\
&\quad+\rho C_E\frac{\partial^2}{\partial t^2}\left\{\varphi-a_1\left(\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}\right)\right.\\
&\quad\left.-a_3\frac{\partial^2\varphi}{\partial z^2}\right\}.
\end{aligned}
\tag{9}
$$

Constitutive relations are

$$
\begin{aligned}
t_{rr} &= c_{11}e_{rr}+c_{12}e_{\theta\theta}+c_{13}e_{zz}-\beta_1 T,\\
t_{zr} &= 2c_{44}e_{rz},\\
t_{zz} &= c_{13}e_{rr}+c_{13}e_{\theta\theta}+c_{33}e_{zz}-\beta_3 T,\\
t_{\theta\theta} &= c_{12}e_{rr}+c_{11}e_{\theta\theta}+c_{13}e_{zz}-\beta_3 T,
\end{aligned}
\tag{10}
$$

where

$$
\begin{gathered}
e_{rz}=\frac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right),\\
e_{rr}=\frac{\partial u}{\partial r},\\
e_{\theta\theta}=\frac{u}{r},\\
e_{zz}=\frac{\partial w}{\partial z},\\
T=\varphi-a_1\left(\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}\right)-a_3\frac{\partial^2\varphi}{\partial z^2},\\
\beta_{ij}=\beta_i\delta_{ij}, \qquad K_{ij}=K_i\delta_{ij},\\
\beta_1=(c_{11}+c_{12})\alpha_1+c_{13}\alpha_3,\\
\beta_3=2c_{13}\alpha_1+c_{33}\alpha_3.
\end{gathered}
$$

To facilitate the solution, the following dimensionless quantities are introduced

$$
r'=\frac{r}{L},\quad z'=\frac{z}{L},\quad t'=\frac{c_1}{L}t,\quad u'=\frac{\rho c_1^2}{L\beta_1 T_0}u,\quad w'=\frac{\rho c_1^2}{L\beta_1 T_0}w, T'=\frac{T}{T_0}, t'_{zr}=\frac{t_{zr}}{\beta_1 T_0}, t'_{zz}=\frac{t_{zz}}{\beta_1 T_0}, \varphi'=\frac{\varphi}{T_0}, a'_1=\frac{a_1}{L^2}, a'_3=\frac{a_3}{L^2}. \tag{11}
$$

Using the dimensionless quantities defined by (11) in Eqs. (7)-(9) and after that suppressing the primes and applying the Laplace and Hankel transforms defined by

$$
f^*(r,z,s)=\int_0^\infty f(r,z,t)e^{-st}dt \tag{12}
$$

$$
\tilde{f}(\xi,z,s)=\int_0^\infty f^*(r,z,s)rJ_n(r\xi)dr \tag{13}
$$

on the resulting quantities, we obtain

$$
(-\xi^2-s^2+\delta_2D^2)\tilde{u}+(1-\xi)\delta_1D\widetilde{w}+\big((1-\xi)(1-a_3D^2)+a_1\xi^3\big)=0, \tag{14}
$$

$$
\begin{aligned}
-\delta_1D\tilde{u}+(\delta_3D^2-\xi^2-s^2)\widetilde{w}
&-\left(\frac{\beta_3}{\beta_1}[(1-a_3D^2)D+\xi^2a_1]\right)\tilde{\varphi}\\
&=0,
\end{aligned}
\tag{15}
$$

$$
\delta_4s^2(1-\xi)\tilde{u}-\delta_5s^2D\widetilde{w}+\left(-\delta_6s^2(1+\xi^2a_1)+\xi^2-D^2\left(\frac{K_3}{K_1}-a_3\delta_6s^2\right)\right)\tilde{\varphi}=0, \tag{16}
$$

where

$$
\begin{gathered}
\delta_1=\frac{c_{13}+c_{44}}{c_{11}},\qquad \delta_2=\frac{c_{44}}{c_{11}},\qquad \delta_3=\frac{c_{33}}{c_{11}},\\
\delta_4=\frac{\beta_1^2T_0}{K_1\rho},\quad \delta_5=-\frac{\beta_1\beta_3T_0}{K_1\rho},\\
\delta_6=\frac{\rho C_E C_1^2}{K_1}
\end{gathered}
$$

and

$$
\begin{aligned}
\widetilde{t_{zz}} &= \sum A_i(\xi,s)\eta_i cosh(q_iz)\\
&\quad+\sum \mu_i A_i(\xi,s)\sinh(q_iz),
\end{aligned}
\tag{17}
$$

$$
\widetilde{t_{rz}}=\sum A_i(\xi,s)M_i cosh(q_iz), \tag{18}
$$

$$
\begin{aligned}
t_{rr} &= \sum A_i(\xi,s)R_i cosh(q_iz)\\
&\quad+\sum S_i A_i(\xi,s)\sinh(q_iz).
\end{aligned}
\tag{19}
$$

where

$$
\begin{gathered}
\eta_i=\frac{C_{13}}{C_{11}}(2-\xi)-\frac{\beta_3}{\beta_1}\zeta_4 l_i+\frac{\beta_3}{\beta_1}a_3 l_i q_i^2,\\
R_i=(C_{11}+C_{12})\xi-l_i(1+a_1\xi^2)+a_3q_i^2,\\
S_i=C_{13}d_iq_i,\\
\mu_i=\frac{C_{33}}{C_{11}}d_iq_i,\\
M_i=\delta_2D+(1-\xi)l_i, i=1,2,3.
\end{gathered}
$$

The non-trivial solution of (14)-(16) by eliminating $\tilde{u}$, $\widetilde{w}$, and $\tilde{\varphi}$ yields

$$
AD^6+BD^4+CD^2+E=0, \tag{20}
$$

where

$$
\begin{gathered}
A=\delta_2\delta_3\zeta_5+a_3\delta_2\zeta_3,\\
B=\delta_2\delta_3(\zeta_2+\zeta_5-\zeta_4)-\delta_1\delta_5s^2\zeta_8+\zeta_{10}\zeta_5+\frac{\beta_3}{\beta_1}\zeta_6\zeta_8\delta_1,\\
C=\delta_3\zeta_9\zeta_2+\delta_3\zeta_2\zeta_5-\zeta_3\zeta_4\zeta_9+\delta_2\zeta_1\zeta_2+\delta_1\delta_5s^2\zeta_7+\\
\delta_1\zeta_7\delta_5s^2+\zeta_6\zeta_1\zeta_8-\zeta_6\zeta_7\delta_3+\delta_3\zeta_6\zeta_8+\zeta_{10}\zeta_2-\delta_1(1-\\
\xi)\frac{\beta_3}{\beta_1}\zeta_6\zeta_4,\\
E=\zeta_2\zeta_1\zeta_9-\zeta_6\zeta_7\zeta_1.
\end{gathered}
$$

The solutions of the Eq. (20) can be written in the form

$$
\tilde{u}=\sum A_i(\xi,s)cosh(q_iz), \tag{21}
$$

$$
\widetilde{w}=\sum d_iA_i(\xi,s)cosh(q_iz), \tag{22}
$$

$$
\tilde{\varphi}=\sum l_iA_i(\xi,s)cosh(q_iz), \tag{23}
$$

where $A_{i,}\ i=1,2,3$ being arbitrary constants, $\pm q_i(i=1,2,3)$ are the roots of the Eq. (20) and d_i and l_i are given by

$$d_i = \frac{\delta_2\zeta_5 q_i^4 + (\zeta_8\zeta_6 + \zeta_5\zeta_9 + \delta_2\zeta_2)q_i^2 + \zeta_2\zeta_9 - \zeta_6\zeta_7}{(a_3\zeta_3 + \delta_3\zeta_5)q_i^4 + (\delta_3\zeta_2 + \zeta_5\zeta_1 - \zeta_3\zeta_4)q_i^2 + \zeta_1\zeta_2} \quad (24)$$

$$l_i = \frac{\delta_2\delta_3 q_i^4 + (\zeta_9\delta_3 + \delta_2\zeta_1 + \zeta_{10})q_i^2 + \zeta_1\zeta_9}{(a_3\zeta_3 + \delta_3\zeta_5)q_i^4 + (\delta_3\zeta_2 + \zeta_5\zeta_1 - \zeta_3\zeta_4)q_i^2 + \zeta_1\zeta_2} \quad (25)$$

and

$$\zeta_1 = \delta_2\xi^2 - s^2,$$
$$\zeta_2 = \delta_6 s^2(1 + a_1\xi^2) + \xi^2$$
$$\zeta_3 = \delta_5 s^2 \frac{\beta_3}{\beta_1},$$
$$\zeta_4 = (1 + a_1\xi^2),$$
$$\zeta_5 = -a_3 s^2\delta_6 - \frac{K_3}{K_1},$$
$$\zeta_6 = \delta_4 s^2(1 - \xi),$$
$$\zeta_7 = 1 - \xi + a_1\xi^3,$$
$$\zeta_8 = a_3(1 - \xi)$$
$$\zeta_9 = -\xi^2 - s^2,$$
$$\zeta_{10} = (1 - \xi)\delta_1^2.$$

4. Boundary conditions

We consider a cubical thermal source and normal force following Kumar *et al.* (2016) of unit magnitude along with vanishing of tangential stress components at the stress free surface at z = ±b. Mathematically, these can be written as

$$\frac{\partial\varphi}{\partial z} = \pm g_o F(r,z), \quad (26)$$

$$t_{zz}(r,z,t) = f(r,t) \quad (27)$$

$$t_{rz}(r,z,t) = 0 \quad (28)$$

Using the dimensionless quantities defined by (11) on the Eqs. (26)-(28) and taking Hankel and Laplace transform of the resulting equations and then using (17)-(19) and (21)-(23) yields

$$\sum A_i\, l_i q_i \sinh(q_i z) = \pm g_o \tilde{F}(\xi,z), \quad (29)$$

$$\sum A_i(\xi,s)\eta_i cosh(q_i z) + \sum \mu_i A_i(\xi,s)\sinh(q_i z) = \tilde{f}(\xi,s), \quad (30)$$

$$\sum A_i(\xi,s)(\delta_2 q_i \sinh(q_i z) + (1-\xi)l_i cosh(q_i z)) = 0. \quad (31)$$

Solving the Eqs. (21)-(23) with the aid of (29)-(31) and also solving (17)-(19), we obtain

$$\tilde{u} = \frac{\tilde{f}(\xi,s)}{\Delta}\{-\chi_1\vartheta_1 + \chi_2\vartheta_2 - \chi_3\vartheta_3\} + \frac{g_o\tilde{F}(\xi,z)}{\Delta}\{\chi_4\vartheta_1 - \chi_5\vartheta_2 + \chi_6\vartheta_3\}, \quad (32)$$

$$\tilde{w} = \frac{\tilde{f}(\xi,s)}{\Delta}\{-\chi_1 d_1\vartheta_1 + \chi_2 d_2\vartheta_2 - \chi_3 d_3\vartheta_3\} + \frac{g_o\tilde{F}(\xi,z)}{\Delta}\{\chi_4 d_1\vartheta_1 - \chi_5 d_2\vartheta_2 + \chi_6 d_3\vartheta_3\}, \quad (33)$$

$$\left(-\delta_6 s^2(1+\xi^2 a_1) + \xi^2 - D^2\left(\frac{K_3}{K_1} - a_3\delta_6 s^2\right)\right)\tilde{\varphi} = 0,$$

$$\tilde{\varphi} = \frac{\tilde{f}(\xi,s)}{\Delta}\{-\chi_1 l_1\vartheta_1 + \chi_2 l_2\vartheta_2 - \chi_3 l_3\vartheta_3\} + \frac{g_o\tilde{F}(\xi,z)}{\Delta}\{\chi_4 l_1\vartheta_1 - \chi_5 l_2\vartheta_2 + \chi_6 l_3\vartheta_3\}, \quad (34)$$

$$\widetilde{t_{zz}} = \frac{\tilde{f}(\xi,s)}{\Delta}\{-\chi_1(\eta_1\vartheta_1 + \mu_1\theta_1) + \chi_2(\eta_2\vartheta_2 + \mu_2\theta_2) - \chi_3(\eta_3\vartheta_3 + \mu_3\theta_3)\} + \frac{g_o\tilde{F}(\xi,z)}{\Delta}\{\chi_4(\eta_1\vartheta_1 + \mu_1\theta_1) - \chi_5(\eta_2\vartheta_2 + \mu_2\theta_2) + \chi_6(\eta_3\vartheta_3 + \mu_3\theta_3)\}, \quad (35)$$

$$\widetilde{t_{zr}} = \frac{\tilde{f}(\xi,s)}{\Delta}\{-\chi_1(\varsigma_1)\vartheta_1 + \delta_2 q_1\theta_1) + \chi_2(\varsigma_2)\vartheta_2 + \delta_2 q_2\theta_2) - \chi_3(\varsigma_3)\vartheta_3 + \delta_2 q_3\theta_3)\} + \frac{g_o\tilde{F}(\xi,z)}{\Delta}\{\chi_4(\varsigma_1)\vartheta_1 + \delta_2 q_1\theta_1) - \chi_5(\varsigma_2\vartheta_2 + \delta_2 q_2\theta_2) + \chi_6\varsigma_3 + \delta_2 q_3\theta_3)\} \quad (36)$$

$$\widetilde{t_{rr}} = \frac{\tilde{f}(\xi,s)}{\Delta}\{-\chi_1(R_1\vartheta_1 + S_1\theta_1) + \chi_2(R_2\vartheta_2 + S_2\theta_2) - \chi_3(R_3\vartheta_3 + S_3\theta_3)\} + \frac{g_o\tilde{F}(\xi,z)}{\Delta}\{\chi_4(R_1\vartheta_1 + S_1\theta_1) - \chi_5(R_2\vartheta_2 + S_2\theta_2) + \chi_6((R_3\vartheta_3 + S_3\theta_3)\} \quad (37)$$

where

$$G_i = l_i q_i\theta_i,$$
$$G_{i+3} = \eta_i\vartheta_i + \mu_i\theta_i,$$
$$G_{i+6} = \delta_2 q_i\theta_i + \varsigma_i\vartheta_i,\ i = 1,2,3.$$
$$\Delta = G_1\chi_4 - G_2\chi_5 + G_3\chi_6,$$
$$\Delta_1 = -\tilde{f}(\xi,s)\chi_1 + g_o\tilde{F}(\xi,z)\chi_4,$$
$$\Delta_2 = \tilde{f}(\xi,s)\chi_2 - g_o\tilde{F}(\xi,z)\chi_5,$$
$$\Delta_3 = -\tilde{f}(\xi,s)\chi_3 + g_o\tilde{F}(\xi,z)\chi_6,$$
$$\chi_1 = [G_2G_9 - G_8G_3],$$
$$\chi_2 = [G_1G_9 - G_7G_3],$$
$$\chi_3 = [G_1G_8 - G_2G_7],$$
$$\chi_4 = [G_5G_9 - G_8G_6],$$
$$\chi_5 = [G_4G_9 - G_6G_7],$$
$$\chi_6 = [G_4G_8 - G_5G_7],$$
$$\vartheta_i = \cosh(q_i z),$$
$$\theta_i = \sinh(q_i z),$$
$$\varsigma_i = (1-\xi)l_i, i = 1,2,3$$

5. Applications

As an application of the problem, we take the source functions $F(\mathrm{r},\mathrm{z})$, which decays exponentially as moving away from the centre of the thick circular plate in the radial direction and symmetrically increases along the axial directions is specified by

$$F(\mathrm{r},\mathrm{z}) = \mathrm{z}^2 \mathrm{e}^{-\omega \mathrm{r}}, \tag{38}$$

$$f(r,t) = \frac{1}{2\pi r}\delta(ct - r), \tag{39}$$

where δ (ct − r) is the Dirac delta function

Applying Laplace and Hankel Transform, on Eqs. (38)-(39), gives

$$\tilde{F}(\xi, z) = \frac{z^2 \omega}{(\xi^2 + \omega^2)^{\frac{3}{2}}} \tag{40}$$

$$\tilde{f}(\xi, s) = \frac{1}{2\pi\sqrt{\xi^2 + \frac{s}{c^2}^2}} \tag{41}$$

6. Inversion of the transforms

To find the solution of the problem in physical domain following Sharma *et al.* (2015), we must invert the transforms in Eqs. (32)-(37) These equations are functions of z, the parameters of Laplace and Hankel transforms s and ξ, respectively, and hence are of the form $\tilde{f}(\xi, z, s)$. To get the function $f(r, z, t)$ in the physical domain, first we invert the Hankel transform using

$$f^*(r, z, s) = \int_0^\infty \xi \tilde{f}(\xi, z, s) J_n(\xi r) d\xi \tag{42}$$

Now for the fixed values of the $\xi, z, and\, r$ in the expression above can be considered as the Laplace transform of $g^*(s)$ of $g(t)$. Following Honig and Hirdes (1984), the Laplace transform function $\hat{g}(s)$ can be inverted.

The last step is to calculate the integral in Eq. (42). The method for evaluating this integral is described in Press *et al.* (1986), which uses Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

7. Numerical results and discussion

In order to illustrate our theoretical results in the proceeding section and to show the effect of two temperature, we now present some numerical results. Copper material is chosen for the purpose of numerical calculation, which is transversely isotropic. The physical data for copper material, which is transversely isotropic, is

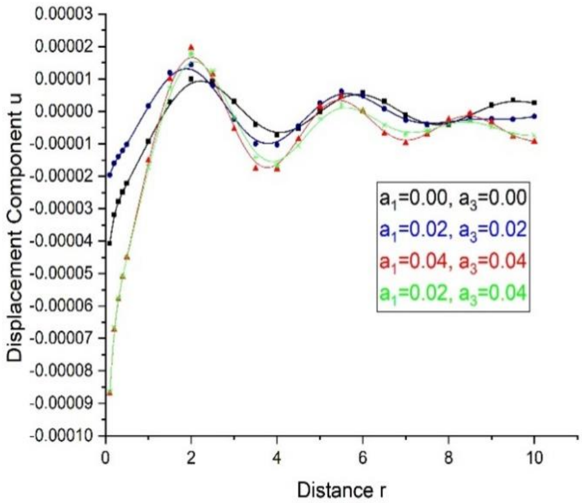

Fig. 1 Demonstrates the variations of displacement component u with distance r

taken from (Dhaliwal and Singh 1980) is given by

$$c_{11} = 18.87 \times 10^{10} Kgm^{-1}s^{-2},$$
$$c_{12} = 8.76 \times 10^{10} Kgm^{-1}s^{-2},$$
$$c_{13} = 8.00 \times 10^{10} Kgm^{-1}s^{-2},$$
$$c_{33} = 17.2 \times 10^{10} Kgm^{-1}s^{-2},$$
$$c_{44} = 5.06 \times 10^{10} Kgm^{-1}s^{-2},$$
$$C_E = 0.6331 \times 10^3 jKg^{-1}K^{-1},$$
$$\alpha_1 = 2.98 \times 10^{-5} K^{-1},$$
$$\alpha_3 = 2.4 \times 10^{-5} K^{-1},$$
$$\rho = 8.954 \times 10^3 Kgm^{-3}$$
$$K_1 = 0.433 \times 10^3 Wm^{-1}K^{-1},$$
$$K_3 = 0.450 \times 10^3 Wm^{-1}K^{-1}.$$

The values of normal force stress t_{zz}, tangential stress t_{zr}, radial stress t_{rr} and conductive temperature φ for a transversely isotropic thermoelastic solid with two temperature is presented graphically to show the impact of two temperature.

i. The solid black line with centre symbol square corresponds to without two temperature parameter i.e., for $a_1 = 0.00, a_3 = 0.00$,

ii. The solid blue line with centre symbol circle corresponds to two temperature parameter $a_1 = 0.02, a_3 = 0.02$,

iii. The solid red line with centre symbol triangle corresponds to two temperature parameter $a_1 = 0.04, a_3 = 0.04$,

iv. The solid green line with centre symbol star corresponds to two temperature parameter $a_1 = 0.02, a_3 = 0.04$.

Fig. 1 shows the variations of displacement component u with distance r. In the initial range of distance r, there is a sharp increase in the value of displacement component for the curves when the two temperatures are i.e., $a_1 = 0.00 = a_3, a_1 = 0.02 = a_3$, $a_1 = 0.04 = a_3$, and $a_1 = 0.02, a_3 = 0.04$. However, away from source applied, it follows oscillatory behaviour near the zero value. We can see that the two temperature have significant effect on the displacement component in all the cases as there are more

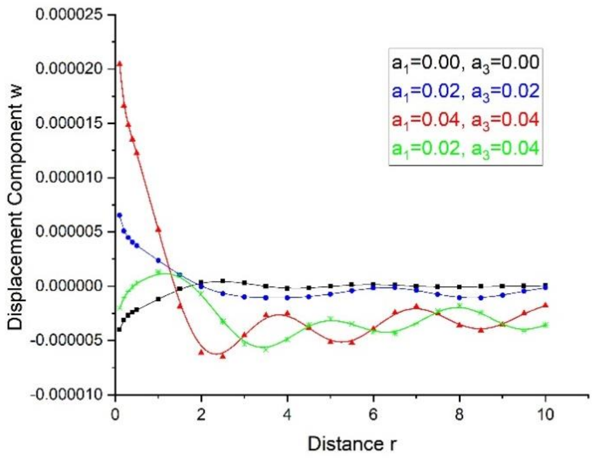

Fig. 2 illustrates the variations of displacement component w with distance r

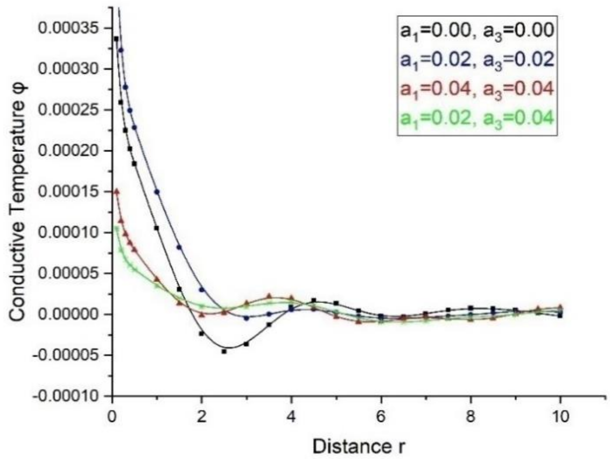

Fig. 3 depicts the behavior of conductive temperature φ with distance r

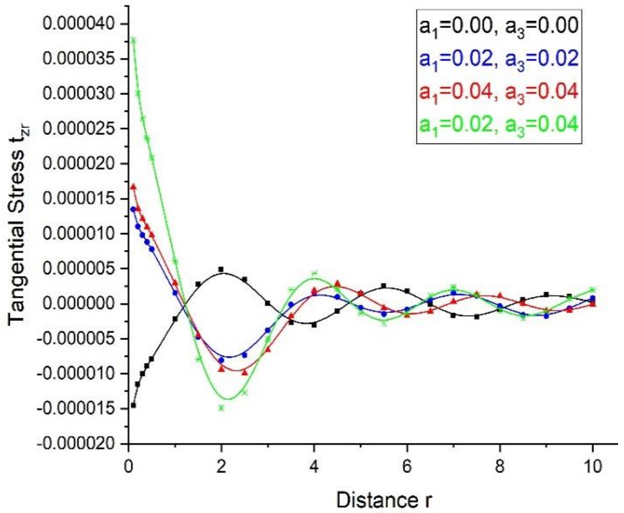

Fig. 4 Shows the variations of tangential stress t_{zr} with distance r

variations in u in case of when both temperature are non-zero and equal as compared to when both temperature are zero.

Fig. 2 shows the variations of displacement component w with distance r. In the initial range of distance r, there is a increase in the value of displacement component for $a_1 = 0.00 = a_3$ and $a_1 = 0.02, a_3 = 0.04$.

However, there is a sharp decrease in the value of displacement component for the curves when the two temperatures are i.e., $a_1 = 0.02 = a_3$, and $a_1 = 0.04 = a_3$, but again away from source applied, it follows opposite oscillatory behaviour.

Fig. 3 demonstrates the variations of conductive temperature φ with distance r.In the initial range of distance r, there is a sharp decrease in the value of conductive temperature for the curves when the two temperatures are i.e., $a_1 = 0.00 = a_3$, $a_1 = 0.02 = a_3$, $a_1 = 0.04 = a_3$, and $a_1 = 0.02, a_3 = 0.04$, but away from source applied, it follows oscillatory behaviour. We can see that the two temperature have significant effect on the conductive temperature in all the cases as there are more variations in φ in case of when the two temperatures are i.e., $a_1 = 0.00 = a_3$. However, for two temperatures, there is a sharp decrease in the range $0 \le r \le 3$ but pattern is oscillatory in the rest of the range. In case of when both temperature are non-zero and equal, oscillations are of lesser magnitude than in case of when both temperature are zero. Two-temperature effect is more prominent in the range $0 \le r \le 3$ for all the curves and curves are close to each other in the remaining range with minor difference in the magnitude.

Fig. 4 illustrates the variations of tangential stress t_{zr} with distance r. In the initial range of r, there is a sharp decrease in the values of t_{zr} for all the curves when the two temperatures are i.e., $a_1 = 0.02 = a_3$ and $a_1 = 0.02, a_3 = 0.02$ and sharp increase in the values of t_{zr} for $a_1 = 0.00 = a_3$ but away from source applied, it follows oscillatory behaviour. However, for $a_1 = 0.00 = a_3$ it follows opposite oscillatory behaviour. It is evident from Fig. 4 that near the point of application of source there is increase in the values when both temperature are non-zero and equal and has small variation near the zero value in the remaining range. However, for $a_1 = 0.02$, $a_3 = 0.04$, there is a sharp decrease in the range $0 \le r \le 2$ but pattern is oscillatory near the zero value in the rest of the range. In case of $a_1 = 0.02, a_3 = 0.04$ oscillations are of greater magnitude than in case of $a_1 = 0.00 = a_3$.

Fig. 5 shows the variations of normal stress $t_{zz}$with distance r. In the initial range of r, there is a sharp decrease in the value of normal stress for the curves i.e., $a_1 = 0.02 = a_3$, $a_1 = 0.00 = a_3$, $a_1 = 0.04 = a_3$ and a sharp increase in the value of normal stress for the curve when two temperature i.e., $a_1 = 0.02$, $a_3 = 0.04$ but away from source applied, it follows oscillatory behaviour. We can see that the two temperature have significant effect on the normal stress in all the cases as there are less variations in t_{zz} in case of $a_1 = 0.00 = a_3$. However, for when two temperature $a_1 = 0.02$, $a_3 = 0.04$ and $a_1 = 0.04 = a_3$, there is a sharp decrease in the range $0 \le r \le 3$ but pattern is oscillatory near the zero value in the rest of the range.

Fig. 6 shows the variations of radial stress $t_{rr}$with

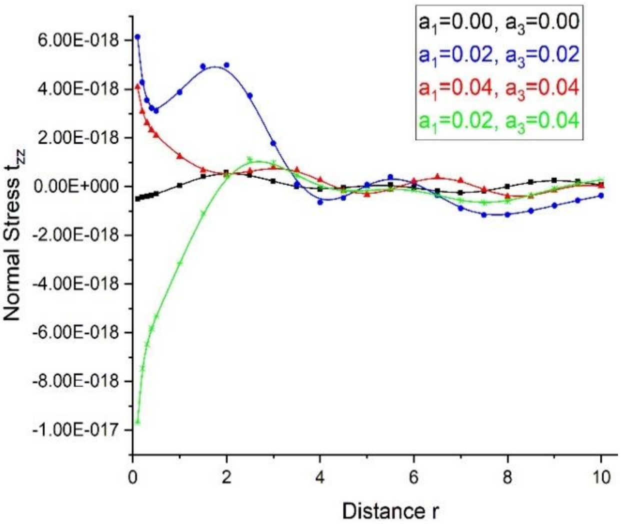

Fig. 5 Shows the variations of normal stress t_{zz} with distance r

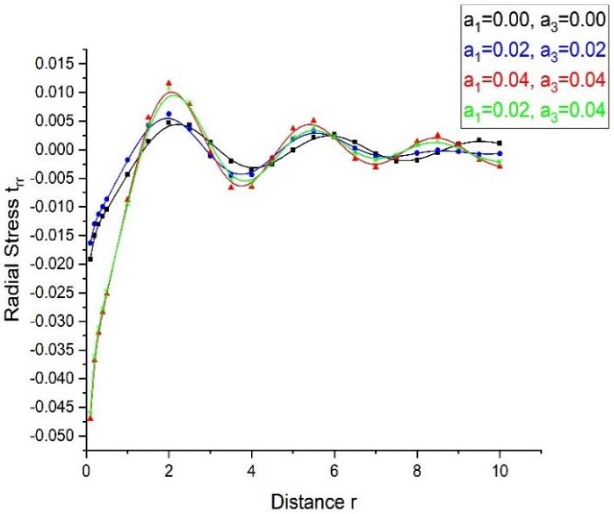

Fig. 6 Shows the variations of radial stress t_{rr} with distance r

distance r. In the initial range of distance r, there is a sharp increase in the value of radial stress for the curves when the two temperatures are i.e., $a_1 = 0.00 = a_3, a_1 = 0.02 = a_3$, $a_1 = 0.04 = a_3$, and $a_1 = 0.02,\ a_3 = 0.04$. However, away from source applied, it follows oscillatory behaviour.

8. Conclusions

From the figures, it is clear that there is a significant impact on the deformation of various components of stresses, displacement, conductive temperature, and temperature change in the thick circular plate while comparing the effect of two temperatures. The effect of two temperature has played an important part in the deformation of thick circular plate with two temperature and without energy dissipation. As distance r diverse from the point of application of the source, the components of normal stress, tangential stress and conductive temperature follow an oscillatory pattern. Much variations in amplitude and behaviour are observed while studying the effect of two-temperature. It is observed that as the disturbances travel through different constituents of the medium, the variations of normal stress t_{zz}, tangential stress t_{zr} and conductive temperature φ, suffer sudden changes resulting in an inconsistent non- uniform pattern of curves. The trend of curves exhibits the properties of two temperature of the medium and satisfies the requisite condition of the problem. The results of this problem are very useful in the two dimensional problem of dynamic response of the transversely isotropic thermoelastic solid with and without two temperature which has various geophysical and industrial applications and beneficial to dissect the deformation field such as geothermal engineering; advanced aircraft structure design, thermal power plants, composite engineering, geology, high-energy particle accelerators, and many emerging technologies.

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