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Vibration response and wave propagation in FG plates resting on elastic foundations using HSDT

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Abstract. This paper presents an analytical study of wave propagation in simply supported graduated functional plates resting on a two-parameter elastic foundation (Pasternak model) using a new theory of high order shear strain. Unlike other higher order theories, the number of unknowns and governing equations of the present theory is only four unknown displacement functions, which is even lower than the theory of first order shear deformation (FSDT). Unlike other elements, the present work includes a new field of motion, which introduces indeterminate integral variables. The properties of the materials are assumed to be ordered in the thickness direction according to the two power law distributions in terms of volume fractions of the constituents. The wave propagation equations in FG plates are derived using the principle of virtual displacements. The analytical dispersion relation of the FG plate is obtained by solving an eigenvalue problem. Numerical examples selected from the literature are illustrated. A good agreement is obtained between the numerical results of the current theory and those of reference. A parametric study is presented to examine the effect of material gradation, thickness ratio and elastic foundation on the free vibration and phase velocity of the FG plate.

Keywords: FG plate; shear deformation theory; free vibration; wave propagation; phase velocity; elastic foundations

1. Introduction

Functionally Graded Materials (FGMs) are heterogeneous composite materials for which material properties, such as Young's modulus, density, and fish coefficient continuously vary, giving a considerable advantage over homogeneous and laminated materials in maintaining the integrity of the FGM structure can also be defined as a composite in which the properties of the material gradually vary in a certain direction as a function of the coordinates of the position to obtain the desired strength and rigidity. Currently, FGM materials are increasingly used in the energy, aeronautics, aerospace, electronics, automotive and chemical industries (Ait Atmane *et al.* 2010, Ahmed 2014, Zemri *et al.* 2015, Mahi *et al.* 2015, Akavci 2016, Benbakhti *et al.* 2016, Bounouara *et al.* 2016, Janghorban 2016, Aldousari 2017, Bellifa *et al.* 2017a, Zidi *et al.* 2017, Mouffoki *et al.* 2017, Benadouda *et al.* 2017, Ahouel *et al.* 2016, Selmi and Bisharat 2018, Shahsavari *et al.* 2018, Younsi *et al.* 2018, Karami *et al.* 2019). Therefore, knowledge of the characteristics of FGM plates is of great practical importance for structural design. To this end, numerous studies have been carried out in order to understand the dynamic behavior of structures made by this type of material. It is clear from the literature that the behavior of elastic-based FGM structures has attracted the intention of many researchers. Describe the interactions of the structure and its foundation in an appropriate very where possible, scientist has proposed different types of foundation models (Kerr 1964). The simplest model for the elastic foundation is the Winkler model, which considers the foundation as a series of separate springs without coupling effects, which has the disadvantage of a discontinuous deflection on the interacting surface of the plate (Benferhat *et al.* 2016). Pasternak (1954) corrects the Winkler model by introducing a shear layer as a parameter. From that moment, the Pasternak model has been widely used to describe the mechanical behavior of structure-foundation interactions. Reddy (2000) is one of the first to analyzed the static behavior of FGM rectangular plates based on his plate theory. Cheng and Batra (2000) have found correspondence between eigenvalues of membranes and functionally graded simply supported polygonal plate. The same membrane analogy was later applied to FGM plate and shell analysis based on a third order theory of plates by Reddy (2002). Vel and Batra (2004) presented a three-dimensional exact solution for the vibration of functionally graded rectangular plates. By using first order and higher shear deformation theories, Shufrin and Eisenberger (2005) studied the free vibration and stability of FG deformable plates. Zenkour (2006) presented a generalized shear deformation theory in which function across the thickness. Woo *et al.* (2006)

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studied the non-linear free vibration behavior of plates made of FGMs using the Von Karman theory for large transverse deflection. Matsunaga (2008) analyzed the free vibration and stability of FG plates based on a two-dimensional higher-order theory. Zhao *et al.* (2009) used the element-free kp-Ritz method to investigate the free vibration behavior of FG plates. Later, Ait Atmane *et al.* (2010) and Benachour *et al.* (2011) proposed some new shape functions. Hosseini-Hashemi *et al.* (2011) proposed an exact analytical solution for transverse vibration investigation of Lévy-type rectangular plates. In addition, Neves *et al.* (2012a, 2012b, 2013c) proposed a sinusoidal and a hybrid type quasi-3D hyperbolic shear deformation theories to study bending, free vibration and buckling responses of FG plates. Based on the Reddy’s third order shear deformation plate model, A new higher-order shear deformation theory for static analysis of laminated composite and sandwich plates was established by Mantari *et al.* (2012). Sobhy (2013) studied the vibration and buckling behavior of exponentially graded material sandwich plate resting on elastic foundations under various boundary conditions. The first-order shear deformation theory (FSDT), including the effects of transverse shear deformation, was employed by some researches to analyze buckling behavior of moderately thick FGM plates (Yaghoobi and Yaghoobi 2013). In the same way, Bouremana *et al.* (2013) developed a new first shear deformation beam theory based on neutral surface position for FG beams. An efficient and simple higher order shear and normal deformation theory for static and free vibration of functionally graded plates was developed by Belabed *et al.* (2014). Recently, by employing a novel four variables refined plate theory against five in case of other shear deformation theories, some studies investigated a series of buckling, bending and vibration response of FG plate/beam and laminated plate supported by elastic foundation (Fekrar *et al.* 2012, Bouderba *et al.* 2013, Kettaf *et al.* 2013, Tounsi *et al.* 2013, Nedri *et al.* 2014, Zidi *et al.* 2014, Attia *et al.* 2015 and 2018, Beldjelili *et al.* 2016, Bousahla *et al.* 2016, Laoufi *et al.* 2016, Khetir *et al.* 2017, Fahsi *et al.* 2017, Zine *et al.* 2018, Chikh *et al.* 2017, Sekkal *et al.* 2017a, Besseghier *et al.* 2017, Menasria *et al.* 2017). Bousahla *et al.* (2014) presented a novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates. Chakraverty and Pradhan (2014) studied the free vibration of exponential functionally graded rectangular plates in thermal environment with general boundary conditions. Hebali *et al.* (2014) developed a new quasi-3D hyperbolic shear deformation theory for the bending and free vibration behavior of FG plate. A quasi-3D theory, which incorporates both shear deformation and thickness stretching effects, supposes that the in plane and out-plane displacements are a higher-order variation within the thickness. A new simple shear and normal deformations theory was developed by Bourada *et al.* (2015) for the analysis of the behavior of functionally graded beams. Recently, Bennai *et al.* (2015) proposed a novel higher-order shear and normal deformation theory for the study of vibration and stability for FG sandwich beams. Ait Atmane *et al.* (2015) examined dynamics of FG porous beams with different beams theories. Belkorissat *et al.* (2015) studied the dynamic properties of FG nanoscale plates using a novel nonlocal refined four variable theory. Kolahchi and Moniri Bidgoli (2016) presented a model for the dynamic instability of embedded single-walled carbon nanotubes using a sinusoidal shear deformation. Tounsi *et al.* (2016) proposed a new 3-unknowns non-polynomial plate theory for buckling and vibration of FG sandwich plate. Kolahchi *et al.* (2016) studied the dynamic stability response of embedded piezoelectric nanoplates made of polyvinylidene fluoride (PVDF) based on visco-nonlocal-piezo elasticity theories. Bouderba *et al.* (2016) studied the thermal stability of FG sandwich plates using a simple shear deformation theory. Kolahchi *et al.* (2016) is investigated nonlinear dynamic stability analysis of embedded temperature-dependent viscoelastic plates reinforced by single-walled carbon nanotubes. Bellifa *et al.* (2016) presented static bending and dynamic analysis of FG plates using a simple shear deformation theory and the concept the neutral surface position. The buckling of straight concrete columns armed with single-walled carbon nanotubes or Nano-Fiber Reinforced Polymer (NFRP) was studied by Arani and Kolahchi (2016) and Safari Bilouei *et al.* (2016). Houari *et al.* (2016) presented a new simple three-unknown sinusoidal shear deformation theory for FG plates. Madani *et al.* (2016) presented the Vibration analysis of embedded functionally graded (FG)-carbon nanotubes (CNT)-reinforced piezoelectric cylindrical shell subjected to uniform and non-uniform temperature distributions. Draiche *et al.* (2016) used a refined theory with stretching effect for the flexure analysis of laminated composite plates. Al Jahwari and Naguib (2016) investigated FG porous plates with different plate theories and cellular distribution model. Hajmohammad *et al.* (2017) studied the dynamic buckling behavior of a sandwich plate composed of laminated viscoelastic nanocomposite layers integrated with viscoelastic piezoelectric layers. Kolahchi and Cheraghbak (2017) studied nonlocal dynamic buckling analysis of embedded microplates reinforced by single-walled carbon nanotubes using Bolotin method. Ait Atmane *et al.* (2017) is study the effect of stretching the thickness and porosity on the mechanical response of a FG beam resting on elastic foundations. Kolahchi (2017) presented a comparative study on the bending, vibration, and buckling of viscoelastic sandwich nano-plates using on various nonlocal theories employing DC, HDQ and DQ methods. Kolahchi *et al.* (2017) presented optimization of embedded piezoelectric sandwich nanocomposite plates for dynamic buckling analysis based on Grey Wolf algorithm. Kolahchi *et al.* (2017) investigated dynamic buckling of sandwich nano plate (SNP) subjected to harmonic compressive load based on nonlocal elasticity theory. Shokravi (2017) studied the effect agglomeration on vibration analysis of silica nanoparticles-reinforced concrete beams. Shokravi (2017) studied buckling analysis of embedded laminated plates with nanocomposite layers. The same author Shokravi (2017) examined dynamic pull-in and pullout analysis of viscoelastic nanoplate switch under electrostatic and intermolecular Casimir forces. Shokravi (2017) presented reddy plate theory for buckling of sandwich plates with FG-CNT-reinforced layers resting on orthotropic elastic medium was studied. Zamanian *et al.* (2017) investigated nonlinear buckling of embedded straight concrete columns reinforced with silicon dioxide (SiO2) nanoparticles. Zarei *et al.* (2017) investigated the seismic response of the fluid-conveying concrete pipes reinforced with SiO2 nanoparticles and fiber reinforced polymer (FRP) layer. Amnieh *et al.* (2018) presented the dynamic analysis of non-homogeneous concrete block resting on soil foundation subjected to blast load with a studied experimentally and theoretically. Golabchi *et al.* (2018) examined the agglomeration effects on vibration and buckling analysis of pipes reinforced by SiO2 nanoparticles. Hajmohammad *et al.* (2018) presented the smart control and vibration analysis of laminated sandwich truncated conical shells with piezoelectric layers as sensor and actuator. The same author presented a numerical work on the dynamic response of cylindrical shells submerged in an incompressible fluid subjected to earthquake, thermal and moisture loads (Hajmohammad *et al.* 2018). A study of the seismic response of underwater fluid-conveying concrete pipes reinforced with nano-fiber reinforced polymer layer during the earthquake in Kobe was presented by Hajmohammad *et al.* (2018). A shear deformation theory with four variables was used by Yousfi *et al.* (2018) for the analysis of the vibratory behavior of porous FGM plates. Shahsavari *et al.* (2018) presented a novel quasi-3D hyperbolic theory for free vibration of FG plates with porosities resting on Winkler/Pasternak/Kerr foundation. Mokhtar *et al.* (2018) employed a novel shear deformation theory for buckling analysis of single layer graphene sheet based on nonlocal elasticity theory. Wave propagation is an important dynamic characteristic of functionally graduated structures, for which reason it is necessary to study this characteristic in this type of high-frequency structures in order to be able to use them in different domains. The study of wave propagation in FG plates has also received a lot of attention from various researchers. Han *et al.* (2000a, 2001b) used in an analytical-numerical method to study transient waves in a functionally graduated cylinder. Han *et al.* (2002) also proposed a numerical method for studying the transient wave in FG plates excited by impact loads. Boukhari *et al.* (2016) introduced an efficient shear deformation theory for wave propagation of functionally graded material plates. Kolahchi *et al.* (2017) studied a refined zigzag theory for wave propagation of embedded viscoelastic FG-CNT-reinforced sandwich plates integrated with sensor and actuator. Sharma *et al.* (2017) studied vibroacoustic behavior of shear deformable laminated composite flat panel using BEM and higher order shear deformation theory. An analytical study of wave propagation and free vibration of FG porous beams by a new high order theory of four variables has been proposed by Ayache *et al.* (2018). Fourn *et al.* (2018) developed a new, refined theory of four-variable plates for analyzing the wave propagation of FGM plates.

As the review of the above literature shows, most research work on the study of wave propagation in FG structures is limited to plates without elastic foundations. In this work, we study free vibration and phase velocity characteristics in functionally graduated (FG) plates resting on elastic bases with simply supported edges. The base is described by Pasternak's two-parameter model. The proposed higher shear strain theory (HSDT) has a new displacement field that includes indeterminate integrated terms and contains only four unknowns. The equations governing wave propagation in the functionally graded plate are derived using the Hamilton principle. Analytical dispersion relations of the functionally graded plate are obtained by solving a eigenvalue problem. The accuracy of the current model is checked against the results calculated with those of the literature. The influence of geometric parameters and elastic foundation on the frequency and phase velocity of wave propagation in FG plates are clearly discussed.

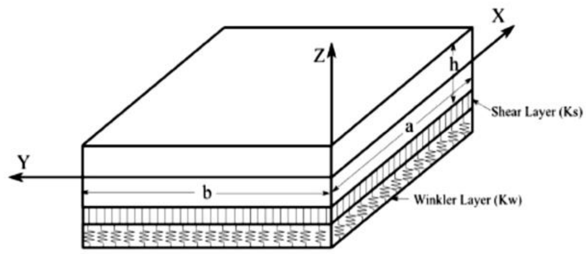

Fig. 1 Geometric configuration of FGMs plate with elastic foundation

2. Problem formulation

Consider a solid rectangular plate of length a, width b and thickness h made of FGM with the coordinate system as shown in Fig. 1. It is assumed to be rested on a Winkler-Pasternak type elastic foundation with the Winkler stiffness of *kw* and shear stiffness of *kp*.

The ceramic is at the top surface (z =+ h/2) of the plate, and metal is at the bottom surface (z =- h/2).

The material characteristics of this plate change across the plate thickness with different power law distributions of the volume fractions of the constituents of the two materials as:

i. Power law distribution

$$P(z) = P_m + \left(P_c - P_m\right)\left(\frac{1}{2} + \frac{z}{h}\right)^p \tag{1}$$

ii. Exponential law distribution

$$p(z) = p_m e^{p\left(\frac{1}{2}+\frac{z}{h}\right)} \tag{2}$$

Where is the power law index, which takes values greater than or equal to zero. Where denotes the effective material characteristic such as Young's modulus and mass density subscripts and denote the metallic and ceramic components, respectively, and is the power law exponent. The value of equal to zero indicates a fully ceramic plate, whereas infinite represents a fully metallic plate. Since the influences of the variation of Poisson's ratio on the behavior of FG, plates are very small (Yang *et al.* 2005, Kitipornchai *et al.* 2006), it is supposed to be constant for convenience.

i. Power law distribution

$$E(z) = E_m + (E_c - E_m)\left(\frac{1}{2} + \frac{z}{h}\right)^p, \qquad p(z) = p_m + (p_c - p_m)\left(\frac{1}{2} + \frac{z}{h}\right)^p, \tag{3a}$$

ii. Exponential law distribution

$$E(z) = E_m e^{p\left(\frac{1}{2}+\frac{z}{h}\right)}, \quad p(z) = p_m e^{p\left(\frac{1}{2}+\frac{z}{h}\right)}, \tag{3b}$$

3. Mathematical formulation

3.1 Theoretical formulation

In this article, further simplifying supposition are made to the conventional HSDT so that the number of unknowns is reduced. The displacement field of the conventional HSDT is given by (Bouchafa *et al.* 2015, Baseri *et al.* 2016)

$$u(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + f(z)\phi_x(x,y,t) \tag{4a}$$

$$v(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + f(z)\phi_y(x,y,t) \tag{4b}$$

$$w(x,y,z,t) = w_0(x,y,t) \tag{4c}$$

Where u_0, v_0, w_0, φ_x, φ_y are five unknown displacements of the mid-plane of the plate, $f(z)$ denotes shape function representing the variation of the transverse shear strains and stresses within the thickness. By considering that $\varphi_x = \int \theta(x,y)dx$ and $\varphi_y = \int \theta(x,y)dy$, the displacement field of the present model can be expressed in a simpler form as (Bourada *et al.* 2016, El-Haina *et al.* 2017, Bourada *et al.* 2018, Meksi *et al.* 2019)

$$u(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + k_1 f(z)\int \theta(x,y,t)dx \tag{5a}$$

$$v(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + k_2 f(z)\int \theta(x,y,t)dy \tag{5b}$$

$$w(x,y,z,t) = w_0(x,y,t) \tag{5c}$$

The shape function $f(z)$ is chosen to satisfy the boundary conditions without stress on the upper and lower surfaces of the plate. Therefore, a shear correction factor is not necessary. In this study, this shape function is chosen based on the higher-order shear deformation plate theory (HSDT) of Reissner (1945). This equation is expressed as

$$f(z) = \left(\frac{5}{4}z - \frac{5z^3}{3h^2}\right) \tag{6}$$

It can be seen that the displacement field in Eq. (5) introduces only four unknowns (u_0, v_0, w_0 and θ). The nonzero strains associated with the displacement field in Eq. (5) are

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z)\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \tag{7}$$

Where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w}{\partial^2 x} \\ -\dfrac{\partial^2 w}{\partial^2 y} \\ -2\dfrac{\partial^2 w}{\partial^2 x\,\partial^2 y} \end{Bmatrix}, \quad \text{and}$$

$$\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} k_1\theta \\ k_2\theta \\ k_1\dfrac{\partial}{\partial y}\int \theta dx + k_2\dfrac{\partial}{\partial x}\int \theta dy \end{Bmatrix}, \tag{8}$$

$$\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_2\int \theta dy \\ k_1\int \theta dx \end{Bmatrix}, \quad g(z) = \frac{\partial f(z)}{\partial z}$$

And the integrals defined in the above equations shall be resolved by a Navier type method and can be written as follows

$$\frac{\partial}{\partial y}\int \theta\, dx = A'\frac{\partial^2\theta}{\partial x\,\partial y}, \frac{\partial}{\partial x}\int \theta\, dy = B'\frac{\partial^2\theta}{\partial x\,\partial y}$$
$$\int \theta\, dx = A'\frac{\partial\theta}{\partial x}, \int \theta\, dy = B'\frac{\partial\theta}{\partial y} \tag{9}$$

Where the coefficients A' and B' are expressed according to the type of solution used, in this case via Navier. Therefore, A', B', k_1 and k_2 are expressed as follows

$$A' = -\frac{1}{\kappa_1^2}, B' = -\frac{1}{\kappa_2^2}, k_1 = \kappa_1^2, k_2 = \kappa_2^2 \tag{10}$$

Where κ_1 and κ_2 are the wave numbers of wave propagation along x-axis and y-axis directions respectively.

For elastic and isotropic FGMs, the constitutive relations can be expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{21} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \\ \gamma_{xy} \\ \gamma_{xy} \end{Bmatrix} \tag{11}$$

where (σ_x, σ_y, τ_{xy}, τ_{yz}, τ_{xz}) and (ε_x, ε_y, γ_{xy}, γ_{yz}, γ_{xz}) are the stress and strain components, respectively. Using the material properties defined in Eqs. (1)-(2), stiffness Coefficients, C_{ij}, can be given as

$$
\begin{aligned}
&Q_{11} = \frac{E(z)}{1-\nu^2}, \quad Q_{11} = Q_{22} \\
&Q_{12} = \frac{\nu E(z)}{1-\nu^2}, \quad Q_{12} = Q_{12} \\
&Q_{44} = \frac{E(z)}{2(1+\nu)}, \quad Q_{44} = Q_{55} = Q_{66}
\end{aligned}
\tag{12}
$$

3.2 Equations of motion

Using Hamilton's energy principle we derive the equation of motion of the FG plate (Hachemi *et al.* 2017, Bellifa *et al.* 2017b, Klouche *et al.* 2017, Kaci *et al.* 2018, Belabed *et al.* 2018, Youcef *et al.* 2018, Cherif *et al.* 2018, Bouadi *et al.* 2018, Yazid *et al.* 2018, Bakhadda *et al.* 2018, Kadari *et al.* 2018, Karami *et al.* 2018a, Bourada *et al.* 2019)

$$
\int_0^t (U + U_{ef} - K)\, dt = 0 \tag{13}
$$

Where:

δU: Variation of energy deformation of the FG plate, U_{ef}: Variation of the energy deformation of the elastic medium, δK: Variation of kinetic energy.

The Variation of energy deformation of the FG plate is given by

$$
\begin{aligned}
\delta U &= \int_V (\sigma_{xx}\,\delta\varepsilon_{xx} + \sigma_{yy}\,\delta\varepsilon_{yy} + \tau_{xy}\,\delta\tau_{xy} \\
&\quad + \tau_{yz}\,\delta\tau_{yz} + \tau_{xz}\,\delta\tau_{xz})\, dV \\
&= \int_A (N_x\,\delta\varepsilon_x^0 + N_y\,\delta\varepsilon_y^0 + N_{xy}\,\delta\varepsilon_{xy}^0 + M_x^b\,\delta k_x^b + M_y^b\,\delta k_y^b \\
&\quad + M_{xy}^b\,\delta k_{xy}^b + M_x^s\,\delta k_x^s + M_y^s\,\delta k_y^s + M_{xy}^s\,\delta k_{xy}^s \\
&\quad + R_{yz}^s\,\delta R_{yz}^s + R_{xz}^s\,\delta R_{xz}^s)\, dA
\end{aligned}
\tag{14}
$$

Where A is the top surface and the stress resultants N, M, and R are defined by

$$
\begin{aligned}
&\left(N_i, M_i^b, M_i^s\right) = \int_{-h/2}^{h/2} (1, z, f)\,\sigma_i\, dz, \quad (i = x, y, xy) \\
&\left(R_{xz}^s, R_{yz}^s\right) = \int_{-h/2}^{h/2} (\tau_{xz}, \tau_{yz})\, g\; dz
\end{aligned}
\tag{15}
$$

The Variation of the deformation energy of the elastic medium (Pasternak foundation) can be expressed as

$$
\delta U_{ef} = \int_A \left(K_w w\,\delta w - K_p\left(\frac{d^2 w}{dx^2} + \frac{d^2 w}{dy^2}\right)\delta w\right) dA \tag{16}
$$

With K_w and K_p are the transverse and shear stiffness coefficients for the elastic medium respectively. If the foundation is modelled as the linear Winkler foundation, the coefficient kp in Eq. (16) is zero.

The variation of kinetic energy of the plate can be expressed as

$$
\delta K = \int_V (\dot{u}\,\delta\dot{u} + \dot{v}\,\delta\dot{v} + \dot{w}\,\delta\dot{w})\, dV
$$

$$
= \int_V \left\{
\begin{aligned}
&I_0\left(\dot{u}_0\delta\dot{u}_0 + \dot{v}_0\delta\dot{v}_0 + \dot{w}_0\delta\dot{w}_0\right) \\
&-I_1\left(\dot{u}_0\frac{\partial\delta\dot{w}_0}{\partial x} + \frac{\partial\dot{w}_0}{\partial x}\delta\dot{u}_0 + \dot{v}_0\frac{\partial\delta\dot{w}_0}{\partial y} + \frac{\partial w_0}{\partial y}\delta\dot{v}_0\right) + \\
&J_1\left(\begin{aligned}&\left(k_1A'\right)\left(\dot{u}_0\frac{\partial\delta\dot{\theta}}{\partial x} + \frac{\partial\dot{\theta}}{\partial x}\delta\dot{u}_0\right) \\ &+\left(k_2B'\right)\left(\dot{v}_0\frac{\partial\delta\dot{\theta}}{\partial y} + \frac{\partial\dot{\theta}}{\partial y}\delta\dot{v}_0\right)\end{aligned}\right) \\
&+I_2\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\dot{w}_0}{\partial x} + \frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\dot{w}_0}{\partial y}\right) \\
&+K_2\left(\left(k_1A'\right)^2\left(\frac{\partial\dot{\theta}}{\partial x}\frac{\partial\delta\dot{\theta}}{\partial x}\right) + \left(k_2B'\right)^2\left(\frac{\partial\dot{\theta}}{\partial y}\frac{\partial\delta\dot{\theta}}{\partial y}\right)\right) \\
&-J_2\left(\begin{aligned}&\left(k_1A'\right)\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\dot{\theta}}{\partial x} + \frac{\partial\delta\dot{w}_0}{\partial x}\frac{\partial\dot{\theta}}{\partial x}\right) \\ &+\left(k_2B'\right)\left(\frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\dot{\theta}}{\partial y} + \frac{\partial\delta\dot{w}_0}{\partial y}\frac{\partial\dot{\theta}}{\partial y}\right)\end{aligned}\right)
\end{aligned}
\right\} dV
\tag{17}
$$

Where (.) indicates the differentiation with respect to the time variable t and (I_i, J_i, K_i) are mass inertias expressed by

$$
\begin{aligned}
&\left(I_0, I_1, I_2\right) = \int_{-h/2}^{h/2} (1, z, z^2)\, p(z)\, dz \\
&\left(J_1, J_2, K_2\right) = \int_{-h/2}^{h/2} (f, zf, f^2)\, p(z)\, dz
\end{aligned}
\tag{18}
$$

By substituting Eqs. (14), (16) and (17) into Eq. (13), the following can be derived

$$
\begin{aligned}
\delta u_0:\;& \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0\ddot{u}_0 - I_1\frac{\partial\ddot{w}_0}{\partial x} + J_1\, k_1A'\frac{\partial\ddot{\theta}}{\partial x} \\
\delta v_0:\;& \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = I_0\ddot{v}_0 - I_1\frac{\partial\ddot{w}_0}{\partial y} + J_1\, k_2B'\frac{\partial\ddot{\theta}}{\partial y} \\
\delta w_0:\;& \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x\,\partial y} + \frac{\partial^2 M_y^b}{\partial y^2} = I_0\ddot{w}_0 \\
& + I_1\left(\frac{\partial\ddot{u}_0}{\partial x} + \frac{\partial\ddot{v}_0}{\partial y}\right) - I_2\nabla^2\ddot{w}_0 + J_2\left(k_1A'\frac{\partial^2\ddot{\theta}}{\partial x^2} + k_2B'\frac{\partial^2\ddot{\theta}}{\partial y^2}\right) \\
\delta\theta:\;& -k_1 M_x^s - k_2 M_y^s - (k_1A' + k_2B')\frac{\partial^2 M_{xy}^s}{\partial x\,\partial y} + \\
& k_1A'\frac{\partial R_{xz}^s}{\partial x} + k_2B'\frac{\partial R_{yz}^s}{\partial y} = -J_1\left(k_1A'\frac{\partial\ddot{u}_0}{\partial x} + k_2B'\frac{\partial\ddot{v}_0}{\partial y}\right) \\
& - K_2\left(\left(k_1A'\right)^2\frac{\partial^2\ddot{\theta}}{\partial x^2} + \left(k_2B'\right)^2\frac{\partial^2\ddot{\theta}}{\partial y^2}\right) \\
& + J_2\left(k_1A'\frac{\partial^2\ddot{w}}{\partial x^2} + k_2B'\frac{\partial^2\ddot{w}}{\partial y^2}\right)
\end{aligned}
\tag{19}
$$

Substituting Eq. (8) into Eq. (11) and the subsequent results into Eqs. (15), the stress resultants are obtained in terms of strains as following compact form

$$
\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix}, \quad S = A^s\,\gamma \tag{20}
$$

In which

$$N=\left\{N_x,\ N_y,\ N_{xy}\right\}^t,\quad M^b=\left\{M_x^b\ \ M_y^b\ \ M_{xy}^b\right\}^t,$$
$$M^s=\left\{M_x^s\ \ M_y^s\ \ M_{xy}^s\right\}^t$$
$$and$$
$$\varepsilon=\left\{\varepsilon_x^0,\ \varepsilon_x^0,\ \varepsilon_{xy}^0\right\}^t,\quad k^b=\left\{k_x^b\ \ k_y^b\ \ k_{xy}^b\right\}^t,$$
$$k^s=\left\{k_x^s\ \ k_y^s\ \ k_{xy}^s\right\}^t \tag{21a}$$

$$A=\begin{bmatrix}A_{11} & A_{12} & 0\\ A_{21} & A_{22} & 0\\ 0 & 0 & A_{66}\end{bmatrix},\quad B=\begin{bmatrix}B_{11} & B_{12} & 0\\ B_{21} & B_{22} & 0\\ 0 & 0 & B_{66}\end{bmatrix},$$
$$D=\begin{bmatrix}D_{11} & D_{12} & 0\\ D_{21} & D_{22} & 0\\ 0 & 0 & D_{66}\end{bmatrix} \tag{21b}$$

$$B^s=\begin{bmatrix}B_{11}^s & B_{12}^s & 0\\ B_{21}^s & B_{22}^s & 0\\ 0 & 0 & B_{66}^s\end{bmatrix},\quad D^s=\begin{bmatrix}D_{11}^s & D_{12}^s & 0\\ D_{21}^s & B_{22}^s & 0\\ 0 & 0 & D_{66}^s\end{bmatrix},$$
$$H^s=\begin{bmatrix}H_{11}^s & H_{12}^s & 0\\ H_{21}^s & H_{22}^s & 0\\ 0 & 0 & H_{66}^s\end{bmatrix} \tag{21c}$$

$$S=\left\{R_{xz}^s,\ R_{yz}^s\right\},\quad \gamma=\left\{\gamma_{xz}^0\ \ \gamma_{yz}^0\right\}\quad A^s=\begin{pmatrix}A_{44}^s & 0\\ 0 & A_{55}^s\end{pmatrix} \tag{21d}$$

In addition, stiffness component are given as follows

$$\begin{Bmatrix}A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s\\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s\\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s\end{Bmatrix}=$$
$$\int_{-h/2}^{h/2}Q_{11}\left(1,\quad z,\quad z^2,\quad f(z),\quad zf(z),\quad f(z)^2\right)dz \tag{22a}$$

$$\left(A_{22}\quad B_{22}\quad D_{22}\quad B_{22}^s\quad D_{22}^s\quad H_{22}^s\right)=$$
$$\left(A_{11}\quad B_{11}\quad D_{11}\quad B_{11}^s\quad D_{11}^s\quad H_{11}^s\right) \tag{22b}$$

$$A_{44}^s=A_{55}^s=\int_{-h/2}^{h/2}Q_{44}[g(z)]^2dz \tag{22c}$$

Introducing Eq. (20) into Eq. (19), the equations of motion can be expressed in terms of displacements (u_0, v_0, w_0, θ) and the appropriate equations take the form

$$\begin{aligned}&A_{11}d_{11}u_0+A_{66}d_{22}v_0+\left(A_{11}+A_{66}\right)d_{12}v_0-B_{11}d_{11}w_0\\&-\left(B_{12}+2B_{66}\right)d_{122}w_0+\left(B_{66}^S\left(k_1A'+k_2B'\right)\right)d_{122}\theta\\&+\left(B_{11}^Sk_1+B_{12}^Sk_2\right)d_1\theta=I_0\ddot{u}_0-I_1d_1\ddot{w}_0+J_1k_1A'd_1\ddot{\theta}\end{aligned} \tag{23a}$$

$$\begin{aligned}&A_{22}d_{22}v_0+A_{66}d_{11}v_0+\left(A_{12}+A_{66}\right)d_{12}u_0-B_{22}d_{222}w_0\\&-\left(B_{12}+2B_{66}\right)d_{112}w_0+\left(B_{66}^s\left(k_1A'+k_2B'\right)\right)d_{112}\theta\\&+\left(B_{22}^sk_2+B_{12}^sk_1\right)d_2\theta=I_0\ddot{v}_0-I_1d_2\ddot{w}_0+J_1k_2B'd_2\ddot{\theta}\end{aligned} \tag{23b}$$

$$\begin{aligned}&B_{11}d_{111}u_0+\left(B_{12}+2B_{66}\right)d_{122}u_0+\left(B_{12}+2B_{66}\right)d_{112}v_0\\&+B_{22}d_{222}v_0-D_{11}d_{222}w_0-2\left(D_{12}+2D_{66}\right)d_{1122}w_0\\&-D_{22}d_{2222}w_0+\left(D_{11}^sk_1+D_{12}^sk_2\right)d_{11}\theta\\&+2\left(D_{66}^s\left(k_1A'+k_2B'\right)\right)d_{1122}\theta+\left(D_{12}^sk_1+D_{22}^sk_2\right)d_{22}\theta=I_0\ddot{w}_0\\&+I_1\left(d_1\ddot{u}_0+d_2\ddot{v}_0\right)-I_2\left(d_{11}\ddot{w}_0+d_{22}\ddot{w}_0\right)\\&+J_2\left(A'k_1d_{11}\ddot{\theta}+B'k_2d_{22}\ddot{\theta}\right)\end{aligned} \tag{23c}$$

$$\begin{aligned}&\left(-B_{11}^sk_1+B_{12}^sk_2\right)d_1u_0-\left(B_{66}^s\left(A'k_1+B'k_2\right)\right)d_{122}u_0\\&-\left(B_{66}^s\left(A'k_1+B'k_2\right)\right)d_{112}v_0-\left(B_{12}^sk_1+B_{22}^sk_2\right)d_2v_0\\&-\left(D_{11}^sk_1+D_{12}^sk_2\right)d_{11}w_0+2\left(D_{66}^s\left(A'k_1+B'k_2\right)\right)d_{1122}w_0\\&+\left(D_{12}^sk_1+D_{22}^sk_2\right)d_{22}w_0-H_{11}^sk_1^2\theta-H_{12}^sk_2^2\theta\\&-2H_{12}^sk_1k_2\theta-\left(H_{66}^s\left(A'k_1+B'k_2\right)^2\right)d_{1122}\theta\\&+A_{44}^s\left(B'k_2\right)^2d_{22}\theta+A_{55}^s\left(A'k_1\right)^2d_{11}\theta=\\&-J_1\left(A'k_1d_1\ddot{u}_0+B'k_2d_2\ddot{v}_0\right)+J_2\left(A'k_1d_{11}\ddot{w}_0+B'k_2d_{22}\ddot{w}_0\right)\\&-K_2\left(\left(A'k_1\right)^2d_{11}\ddot{\theta}+\left(B'k_2\right)^2d_{22}\ddot{\theta}\right)\end{aligned} \tag{23d}$$

Where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$\begin{aligned}&d_{ij}=\frac{\partial^2}{\partial x_i\partial x_j},d_{ijl}=\frac{\partial^3}{\partial x_i\partial x_j\partial x_l},\\&d_{ijlm}=\frac{\partial^4}{\partial x_i\partial x_j\partial x_l\partial x_m},d_i=\frac{\partial}{\partial x_i},(i,j,l,m=1,2).\end{aligned} \tag{24}$$

3.3 Dispersion relations

The solution for equations of motion are assumed to resolve using dispersion relations that describe propagation wave in plane x-y

$$\begin{Bmatrix}u_0(x,y,t)\\ v_0(x,y,t)\\ w_0(x,y,t)\\ \theta(x,y,t)\end{Bmatrix}=\begin{Bmatrix}U\exp\left[i\left(\kappa_1x+\kappa_2y-\omega t\right)\right]\\ V\exp\left[i\left(\kappa_1x+\kappa_2y-\omega t\right)\right]\\ W\exp\left[i\left(\kappa_1x+\kappa_2y-\omega t\right)\right]\\ X\exp\left[i\left(\kappa_1x+\kappa_2y-\omega t\right)\right]\end{Bmatrix} \tag{25}$$

where U, V, W and X are the coefficients of the wave amplitude, κ_1 and κ_2 are the wave numbers of wave propagation along x-axis and y-axis directions respectively, ω is the frequency, $\sqrt{i}=-1$ the imaginary unit.

Substituting Eq. (25) into Eq. (23), the following problem is obtained

$$\left(\begin{bmatrix}S_{11} & S_{12} & S_{13} & S_{14}\\ S_{12} & S_{22} & S_{23} & S_{24}\\ S_{13} & S_{23} & S_{33} & S_{34}\\ S_{14} & S_{24} & S_{34} & S_{44}\end{bmatrix}-\omega^2\begin{bmatrix}m_{11} & 0 & m_{13} & m_{14}\\ 0 & m_{22} & m_{23} & m_{24}\\ m_{31} & m_{32} & m_{33} & m_{34}\\ m_{41} & m_{41} & m_{43} & m_{44}\end{bmatrix}\right)\begin{Bmatrix}U\\ V\\ W\\ X\end{Bmatrix}=\begin{Bmatrix}0\\ 0\\ 0\\ 0\end{Bmatrix} \tag{26}$$

Where

$$
\begin{aligned}
&m_{11}=-I_0,\quad m_{13}=i\ \kappa_1 I_1,\quad m_{14}=-i\ J_1 k_1 A^{'}\kappa_1,\\
&m_{22}=-I_0,\quad m_{23}=i\ \kappa_2 I_1,\quad m_{24}=-i\ J_1 k_2 B^{'}\kappa_2,\\
&m_{31}=-i\ I_1\kappa_1,\quad m_{32}=-i\ \kappa_2 I_1,\quad m_{33}=-\left(I_0+I_2\left(\kappa_1^2+\kappa_2^2\right)\right),\\
&m_{34}=J_2\left(A^{'}k_1\kappa_1^2+B^{'}k_2\kappa_2^2\right),\quad m_{41}=i\ J_1 k_1 A^{'}\kappa_1,\\
&m_{42}=i\ J_1 k_2 B^{'}\kappa_2,\quad m_{34}=J_2\left(A^{'}k_1\kappa_1^2+B^{'}k_2\kappa_2^2\right),\\
&m_{44}=-K_2\left(\left(A^{'}k_1\right)^2\kappa_1^2+\left(B^{'}k_2\right)^2\kappa_2^2\right).
\end{aligned}
\tag{27a}
$$

$$
\begin{aligned}
&S_{11}=-\left(A_{11}\kappa_1^2+A_{66}\kappa_2^2\right),\\
&S_{12}=-\kappa_1\kappa_2\left(A_{12}+A_{66}\right),\\
&S_{13}=i\,\kappa_1(B_{11}\kappa_1^2+B_{12}\kappa_2^2+2B_{66}\kappa_2^2),\\
&S_{14}=i\,\kappa_1\left(k_1B_{11}^s+k_2B_{12}^s-\left(k_1A^{'}+k_2B^{'}\right)B_{66}^s\kappa_2^2\right),\\
&S_{12}=-\kappa_1\kappa_2\left(A_{12}+A_{66}\right),\\
&S_{22}=-\left(A_{22}\kappa_2^2+A_{66}\kappa_1^2\right),\\
&S_{23}=i\,\kappa_2(B_{22}\kappa_2^2+B_{12}\kappa_1^2+2B_{66}\kappa_1^2),\\
&S_{24}=i\,\kappa_2\left(k_2B_{22}^s+k_1B_{12}^s-\left(k_1A^{'}+k_2B^{'}\right)B_{66}^s\kappa_1^2\right),\\
&S_{31}=-\kappa_1 i\,(B_{11}\kappa_1^2+B_{12}\kappa_2^2+2B_{66}\kappa_2^2),\\
&S_{32}=-\kappa_2 i\,(B_{22}\kappa_2^2+B_{12}\kappa_1^2+2B_{66}\kappa_1^2),\\
&S_{33}=-\left(D_{11}\kappa_1^4+2\left(D_{12}+2D_{66}\right)\kappa_1^2\kappa_2^2+D_{22}\kappa_2^4\right)-K_W\\
&\qquad -K_P(\kappa_1^2+\kappa_2^2)\\
&S_{34}=-k_1\left(D_{11}^s\kappa_1^2+D_{12}^s\kappa_2^2\right)+2\left(k_1A^{'}+k_2B^{'}\right)D_{66}^s\kappa_1^2\kappa_2^2\\
&\qquad -k_2\left(D_{22}^s\kappa_2^2+D_{12}^s\kappa_1^2\right)\\
&S_{14}=-\kappa_1 i\left(k_1B_{11}^s+k_2B_{12}^s-\left(k_1A^{'}+k_2B^{'}\right)B_{66}^s\kappa_2^2\right),\\
&S_{42}=-i\,\kappa_2\left(k_2B_{22}^s+k_1B_{12}^s-\left(k_1A^{'}+k_2B^{'}\right)B_{66}^s\kappa_1^2\right),\\
&S_{34}=-k_1\left(D_{11}^s\kappa_1^2+D_{12}^s\kappa_2^2\right)+2\left(k_1A^{'}+k_2B^{'}\right)D_{66}^s\kappa_1^2\kappa_2^2\\
&\qquad -k_2\left(D_{22}^s\kappa_2^2+D_{12}^s\kappa_1^2\right)\\
&S_{44}=-k_1\left(H_{11}^sk_1+H_{12}^sk_2\right)-\left(k_1A^{'}+k_2B^{'}\right)^2H_{66}^s\kappa_1^2\kappa_2^2\\
&\qquad -k_2\left(H_{12}^sk_1+H_{22}^sk_2\right)-\left(k_1A^{'}\right)^2A_{55}^s\kappa_1^2-\left(k_2B^{'}\right)^2A_{44}^s\kappa_2^2
\end{aligned}
\tag{27b}
$$

The dispersion relations of wave propagation in the functionally graded beam are given by

$$\left|\left[K\right]-\omega^2\left[M\right]\right|=0 \tag{28}$$

The roots of Eq. (28) can be defined by

$$\omega_1=W_1(\kappa)\,,\ \omega_2=W_2(\kappa)\,,\ \omega_3=W_3(\kappa)\ \text{and}\ \ \omega_4=W_4(\kappa) \tag{29}$$

Its roots correspond to the wave modes M_1, M_2, M_3 and M_4 respectively. The wave modes M_1 and M_4 correspond to the flexural wave, the wave mode M_2 and M_3 corresponds to the extensional wave. The phase velocity of wave propagation in the FG plate can be obtained by

$$C_i=\frac{W_i(\kappa)}{\kappa},\quad (i=1,2,3,4) \tag{30}$$

4. Numerical results

In this work, the study of wave propagation and free vibrations in FG plates based on elastic foundation by a new theory of high order shear strain is proposed for investigation. The dispersion solutions for determining the phase velocities and frequencies of the FG plates are presented by solving eigenvalue equations. The Poisson's ratio is fixed at ν=0.3. Comparisons are made with the solutions available in the literature. To verify the accuracy of this analysis, some numerical examples are solved. The properties of the materials used in this study are as follows(Yahia *et al.* 2015, Shahsavari *et al.* 2018).

Ceramic (Alumina, Al_2O_3): E_c = 380 GPa, ν = 0.3 and ρ_c = 3800 kg/m^3.

Ceramic (Si_3N_4): E_c = 348.43 GPa, ν = 0.3 and ρ_c = 2370 kg/m^3.

Metal (Aluminium, Al) E_m = 70 GPa, ν = 0.3 and ρ_m = 2702 kg/ m^3.

Metal (SUS304) E_m = 201.04 GPa, ν = 0.3 and ρ_m =8166 kg/ m^3.

These properties change through the thickness of the plate according to the power law. The upper surface of FGM plate is rich in ceramic, while the lower surface of the FGM plate is rich in metal. The thickness of the functionally graded plate is taken h=0.2 and 0.1 m. In order to verify the effectiveness of the current theory in the study of wave propagation and free vibration of FG plates, numerical applications are presented and discussed. The study based on the proposed model is executed using the MAPLEprogram.

For convenience, the following expressions to compute the non-dimensional natural frequencies and foundation parameters were used

$$\varpi=\omega h\sqrt{\rho_m/E_m}\,,\quad \overline{K}_w=\frac{K_w a^4}{D_c}\,,$$

$$\overline{K}_p=\frac{K_p a^4}{D_c}\,,\ D_c=\frac{\left(E_m h^3\right)}{\left(12\left(1-\upsilon_c^2\right)\right)}$$

4.1 Comparison of natural frequencies of P-FGM plates

In order to study the effectiveness of the current theory in predicting the free vibration response of functionally graduated plates (P-FGM, Al/Al2O3) based on elastic foundations, the non-dimensionalfrequencies are computed and compared to those available to the literature.

In this part, various numerical examples are described, discussed and compared with other existing theories such as the theory of hyperbolic shear of quasi-3D shear presented by Benahmed *et al.* (2017), the third order shear of flat theory (TSDT) proposed by Baferani *et al.* (2011) and the quasi-3D hyperbolic theory developed by Shahsavari *et al.* (2017).

In Table 1, the natural frequencies of the square FG plates (Al/Al2O3) for different values of the power law p are compared with those of the quasi-3D of Benahmed *et al.* (2017), the third order shear deformation flat theory (TSDT) of Baferani *et al.* (2011) and quasi-3D of Shahsavari *et al.* (2017). Four thickness ratios (h/a) are

Table 1 Non-dimensional fundamental frequencies $\varpi = \omega h \sqrt{\rho_m / E_m}$ of square FG plates resting on Winkler-Pasternak foundations

Kw	*Kp*	*h/a*	Model	p = 0	0.5	1	2	5
0	0	0.05	(Benahmed *et al.* 2017)	0.0291	–	0.0226	0.0207	–
			(Baferani *et al.* 2011)	0.029	0.0249	0.0227	0.0209	0.0197
			(Shahsavari *et al.* 2018)	0.0291	0.0248	0.0226	0.0206	0.0195
			Present	0.0291	0.0246	0.0222	0.0202	0.0191
		0.2	(Benahmed *et al.* 2017)	0.4174	–	0.3264	0.2965	–
			(Baferani *et al.* 2011)	0.4154	0.3606	0.3299	0.3016	0.2765
			(Shahsavari *et al.* 2018)	0.4168	0.3586	0.326	0.2961	0.2722
			Present	0.4150	0.3551	0.3205	0.2892	0.2667
100	0	0.05	(Benahmed *et al.* 2017)	0.0298	–	0.0236	0.0218	–
			(Baferani *et al.* 2011)	0.0298	0.0258	0.0238	0.0221	0.021
			(Shahsavari *et al.* 2018)	0.0298	0.0257	0.0236	0.0218	0.0208
			Present	0.0299	0.0257	0.0234	0.0215	0.0215
		0.2	(Benahmed *et al.* 2017)	0.4286	–	0.3431	0.3158	–
			(Baferani *et al.* 2011)	0.4273	0.3758	0.3476	0.3219	0.2999
			(Shahsavari *et al.* 2018)	0.4284	0.3734	0.3431	0.3159	0.295
			Present	0.4269	0.3702	0.3381	0.3097	0.2901
100	100	0.05	(Benahmed *et al.* 2017)	0.0411	–	0.0386	0.0383	–
			(Baferani *et al.* 2011)	0.0411	0.0395	0.0388	0.0386	0.0388
			(Shahsavari *et al.* 2018)	0.0411	0.0393	0.0386	0.0383	0.0385
			Present	0.0411	0.0392	0.0384	0.0381	0.0383
		0.2	(Benahmed *et al.* 2017)	0.6089	–	0.5794	0.5752	–
			(Baferani *et al.* 2011)	0.6162	0.6026	0.5978	0.597	0.5993
			(Shahsavari *et al.* 2018)	0.6137	0.594	0.5856	0.5815	0.5843
			Present	0.6156	0.5950	0.5852	0.5800	0.5834

considered. From the results of the non-dimensional fundamental frequency presented in this table, we notice that the latter are almost identical to those obtained by the other theories of the literature for all ranges the values of the thickness ratio. In addition, it can be seen that the thickness ratio and the volume fraction index have a dominant role on the vibratory behavior of the FG plates rested on an elastic foundation. The results presented in this table also show that the frequencies of the P-FGM plate increase when the basic parameters increase.

In addition, it is noticed that the fundamental frequency without dimension increases when the foundation parameters increase. Compared with the effect of the Winkler parameter, it can be seen that the vibration responses of FG plates are more affected by the Pasternak foundation parameter than the Winkler parameter.

4.2 Parametric study of P-FGM plates

To further illustrate the precision of the theory proposed in this work for a wide range of thickness ratios (a/h), different gradient values of power law index (p), different values of the number of waves and different cases of foundation parameters (kw, kp), a variation analysis of the wave propagation frequency and phase velocity values calculated by the current theory for P-FGM (Si3N4/SUS304) plates have been presented in this section.

In Fig. 2, we examine the influence of the presence of a base of resilient foundation on the fundamental frequency and the rate of the phase of the P-FGM plates. Several values of the index of the power law (p) are used.

From the curves shown in Fig. 2, we see that the frequency of a plate resting on an elastic foundation is a little high compared to the other that does not take an elastic base. It is also noted that the fundamental frequency and the speed of the phase decrease with increasing values of the volume fraction index of the constituents of the material (P) and increase with the increase of the number of waves (κ).

Fig. 3 illustrates the curves of the variation of the fundamental frequency and the phase velocity of the different plates FG with kw = 1000, kp = 100 and p = 2. It can be seen that the thickness of the plate has an effect on the wave propagation frequency in the FG plate for large wave numbers (κ). In contrast, the frequencies are reduced

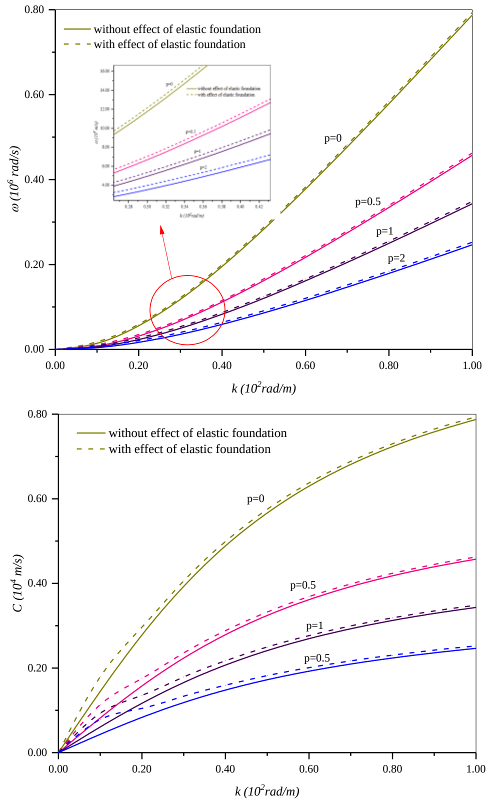

Fig. 2 Variation of frequency and phase velocity of different FG plates as a function of wave number (kw = 1000, kp = 100)

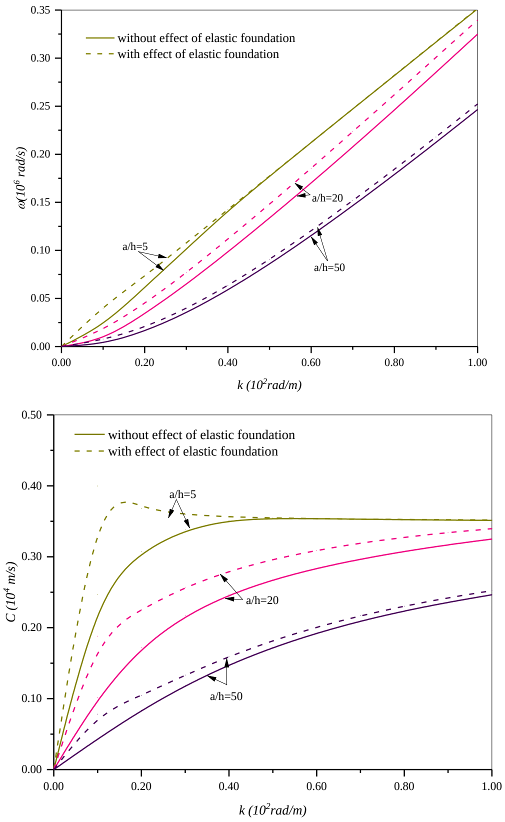

Fig. 3 Variation of frequency and phase velocity of FG plates as a function of wave number. (kw=1000, kp=100, p=2)

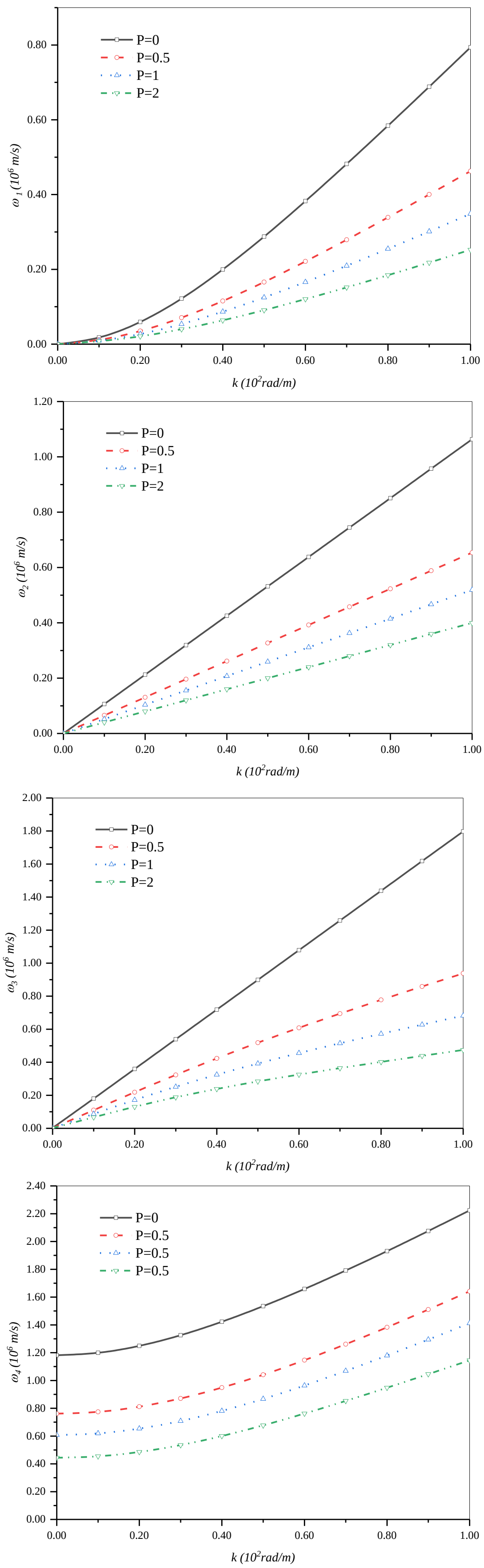

Fig. 4 The dispersion curves of the different plates FG rest on elastic foundation. (kw = 1000, kp = 100)

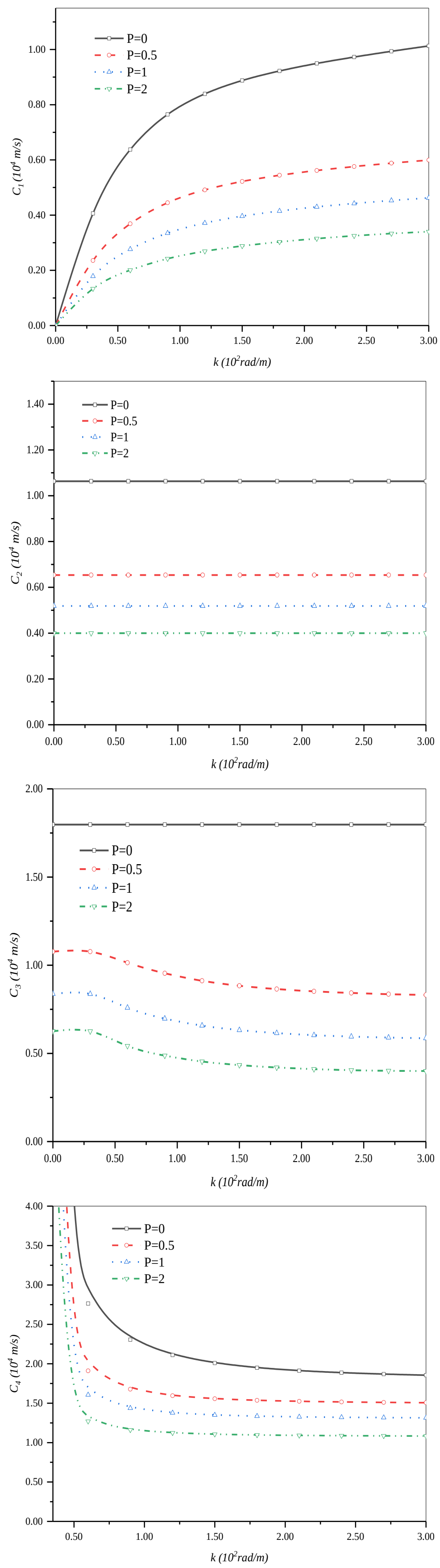

Fig. 5 The phase velocity curves of different functionally graded plates rest on elastic foundation. (kw = 1000, kp = 100)

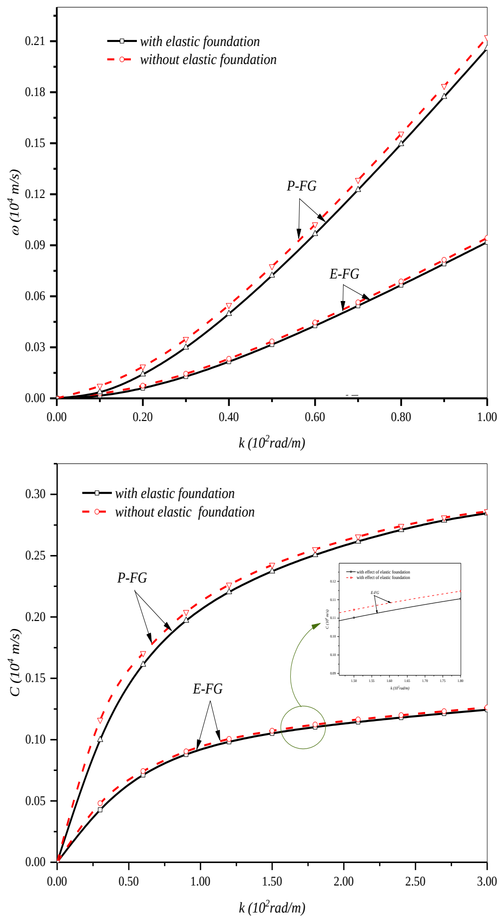

Fig. 6 Comparisons of frequencies and phase velocity of P-FGM and E-FGM plates (a/h=5, p=3)

when the thickness decreases. For the curves of the phase velocity, we see that the latter decreases with the decrease in the thickness of the plate. It should also be noted that for large values of the wavenumber, the phase velocities converge whatever the thickness.

In Fig. 4, the variation of the natural frequency of the different plates FG as a function of the number of waves presented. It can be noted that the propagation frequency of the waves in the plate FG increases with the decrease of the index of the power law, whatever the wave mode. In addition, the propagation frequency of the wave becomes maximum in the homogeneous plate (p = 0).

The variation of the phase velocity of the different plates FG according to the number of waves is represented in Fig. 5. From the curves shown in this figure, the similarities can be identified in the evolution of the parameters of the speed of propagation of FG plates.

It may be noted that the phase velocity of wave propagation in the plate FG increases when the index of the power law p decreases for the same wave number k. The phase velocity of the second and third waveform of the plate (p = 0) is constant, but the latter decreases when the index of the power law p becomes different from zero (p ≠ 0), this decrease becomes more visible in the fourth mode. In addition, for the homogeneous plate (p = 0), the phase velocity takes the maximum among those of all the other compositions.

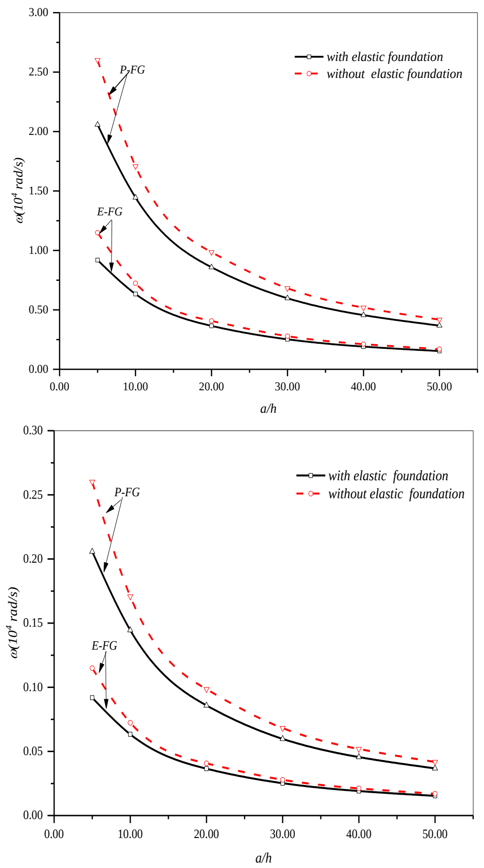

Fig. 7 Comparisons of frequencies and phase velocity of P-FGM and E-FGM plates (κ =10, p =3)

3.3 Comparative parametric study of P-FGM and E-FGM plates

The effect of the foundation on free vibration and wave propagation in thick P-FGM and E-FGM plates is shown in Fig. 6 (a/h = 5 and P = 3).

In Fig. 7, a comparison of the variation of the dispersion and the phase velocity of the plates P-FGM and E-FGM as a function of the thickness ratio (a/h) was presented. The number of waves and the index of power law are taken here equal, respectively, to 10 and 3. The results presented in both Figs. 6 and 7 show that the frequency and phase velocity parameters of the P-FGM plates are larger than those of the E-FGM plate. With this, it is to ensure a regular distribution of properties of the material along the thickness.

5. Conclusions

The present work focuses on the analysis of dynamic response and wave propagation in FG plates based on an elastic basis using a theory of high order shear strain with an integral displacement field. The main advantage of the proposed theory over existing higher order shear deformation theories is that the current theory involves fewer unknowns as well as the effect of the elastic foundation has been taken into account. The cost of calculation can be reduced. The equations of motion are obtained according to Hamilton's principle. These equations are solved using the dispersion relation, and then the fundamental frequencies and phase velocities are found by solving the eigenvalue problem. The results obtained were compared to those reported by other literature theories. The non-dimensional frequencies obtained are compared with others and a very good agreement has been found, which proves the precision of the proposed theory. A parametric study was carried out which made it possible to highlight the various factors influencing the vibratory behavior and wave propagation in FG plates. It is indicated that the responses to wave propagation in FG plates are affected by various parameters such as elastic foundation constants, gradient index, and thickness-to-length ratio. An improvement of present formulation will be considered in the future work to consider the thickness stretching effect by using quasi-3D shear deformation models (Hamidi *et al.* 2015, Larbi Chaht *et al.* 2015, Bennoun *et al.* 2016, Sekkal *et al.* 2017b, Bouafia *et al.* 2017, Abualnour *et al.* 2018, Bouhadra *et al.* 2018, Benchohra *et al.* 2018, Karami *et al.* 2018b, Mahmoudi *et al.* 2019, Zaoui *et al.* 2019).

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