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A new Bayesian approach to derive Paris' law parameters from S-N curve data

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Abstract. The determination of Paris' law parameters based on crack growth experiments is an important procedure of fatigue life assessment. However, it is a challenging task because it involves various sources of uncertainty. This paper proposes a novel probabilistic method, termed the S-N Paris law (SNPL) method, to quantify the uncertainties underlying the Paris' law parameters, by finding the best estimates of their statistical parameters from the S-N curve data using a Bayesian approach. Through a series of steps, the SNPL method determines the statistical parameters (e.g., mean and standard deviation) of the Paris' law parameters that will maximize the likelihood of observing the given S-N data. Because the SNPL method is based on a Bayesian approach, the prior statistical parameters can be updated when additional S-N test data are available. Thus, information on the Paris' law parameters can be obtained with greater reliability. The proposed method is tested by applying it to S-N curves of 40H steel and 20G steel, and the corresponding analysis results are in good agreement with the experimental observations.

Keywords: Bayesian approach; fatigue crack growth; Paris' law; statistical parameter; S-N curve

1. Introduction

Numerous studies have been conducted to investigate the material properties related to fatigue crack propagation as well as predicting fatigue life of structures. However, it still remains a challenging task because fatigue crack propagation involves various sources of uncertainty, and fatigue life assessment continues to be a hot spot in structural engineering research because of its importance to understanding fatigue mechanisms in detail and thus preventing damage in various areas of engineering (Byers *et al*. 1997). For example, the impact of fatigue can be observed in steel bridges owing to repetitive vehicle loading (Lee and Cho 2016), in aircraft wings owing to service loads (Millwater and Wieland 2010), in offshore oil or gas platforms owing to oceanic waves or currents (Karamchandani *et al*. 1992), and in offshore wind turbines owing to aerodynamic loads (Dong *et al*. 2012). These examples illustrate the significance of understanding fatigue as well as its impact on various engineering fields and applications. Indeed, damage due to fatigue could lead to catastrophic consequences that may result in fatalities or severe economic losses. According to a study conducted by the American Society of Civil Engineers (ASCE) Committee on fatigue and fracture reliability, 80%-90% of failures in steel structures were found to be related to fatigue and fracture (ASCE 1982). Therefore, it is necessary to develop advanced methodologies to improve the reliability of structures by conducting probabilistic studies and reliability assessments.

The approaches for fatigue life assessment can be categorized into two groups. The methods in the first category are relying on the S-N curve, which has been reported in a number of published works (Suresh 1998, Stephens *et al*. 2000, Sonsino 2007, Dong 2015, Pradan *et al*. 2017, Qian *et al*. 2014, Keating and Fisher 1986). The S-N curves are established based on the stress-life method, which is developed to a "safe-life" approach to design against fatigue. These approaches employing the S-N curve has been shown to be practical and effective, and they have been applied to various structural problems.

The approaches in the second category are adopting models based on fatigue crack propagation and linear elastic fracture mechanics such as Paris' law (Paris and Erdogan 1963), which accounts for the crack propagation rate. Development of Paris' law was an important breakthrough, because this law facilities the characterization of fatigue crack growth and enables the rigorous assessment of service life or inspection intervals of structures, especially while they are in use. Thus, Paris' law has been applied to various steel structures including bridges, ship structures, offshore platfoms, and wind turbine blades (Lee and Song 2014, Zhao *et al*. 1994, Soares and Garbatov 1996, Sørensen 2009, Karamchandani *et al*. 1992, Moan and Song 2000).
In these previous studies, one of the important procedures

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of applying the Paris' law is the determination of the Paris' law parameters. Experiments for determining these parameters involve considerable uncertainty in terms of measuring the crack growth rate and stress intensity factor (Virkler *et al*. 1979). Because of the uncertainties involved in fatigue crack growth and experimental processes, the Paris' law parameters need to be represented using a probabilistic distribution and the related statistical parameters rather than using deterministic values.

This paper proposes a new probabilistic method, termed the S-N Paris law (SNPL) method, which probabilistically determines the statistical parameters (e.g., mean and standard deviation) of the Paris' law parameters from the S-N curve data by using a Bayesian approach. In the proposed method, stress level and fatigue life obtained from the S-N data are provided as inputs. Based on this data, limit-state functions are constructed and then used to construct an objective function representing the likelihood of observing the given S-N data. Through a series of steps and by using a Bayesian approach, the SNPL method quantifies the underlying uncertainties in the Paris' law material parameters, by finding the best estimates of their statistical parameters, which will maximize the likelihood of observing the given S-N data. Thus, the SNPL method allows to obtain the statistical parameters of the Paris' law parameters from the S-N curve data, which involve many uncertainties, as stated previously. In addition, the SNPL method quantifies the uncertainty in transition crack length from crack initiation to crack propagation by finding the best statistical parameter(s). Another advantage of the SNPL method is the use of a Bayesian approach, by which the prior statistical parameters of the Paris' law parameters can be updated when additional S-N data are available. Thus, information on the Paris' law parameters can be obtained with greater reliability.

2. Two classical approaches for fatigue analysis

There have been numerous studies on fatigue failure in metallic structures (Schütz 1996, Suresh 1998, Paris and Erdogan 1963), and one of the most well-known and widely-used approaches to understand the fatigue behavior is the stress-life approach, which gives the relationship between the stress range and the number of cycles until failure. This relationship can be established experimentally and is represented in the form of an S-N curve, which is a log-log scale plot of the constant stress amplitude, *S*, versus the number of cycles until failure, *N*, for given material and detail. The curves are developed by conducting constant amplitude load tests at several load levels in the finite life region, where the number of cycles until failure at each load level is noted. A typical S-N curve is shown in Fig. 1.

Another important concept based on the S-N curve is the fatigue limit (denoted by the horizontal line in Fig. 1). It means, if the stress range less than the fatigue limit is applied, no failure will occur, which indicates that the life of the specimen or structure can be infinite.

The stress-life (S-N curve) method is mainly applied during the design stage or during the preliminary evaluation of fatigue life. Although this method is developed to a

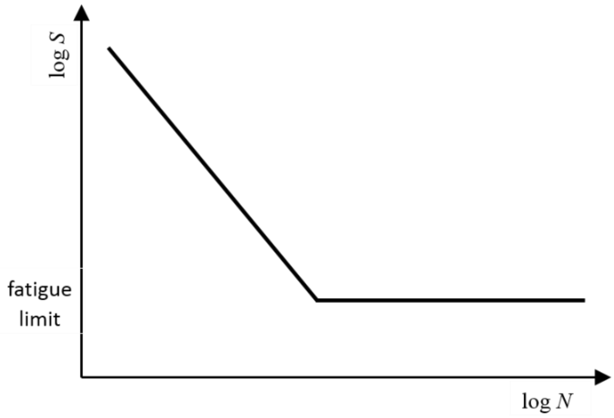

Fig. 1 Typical S-N curve

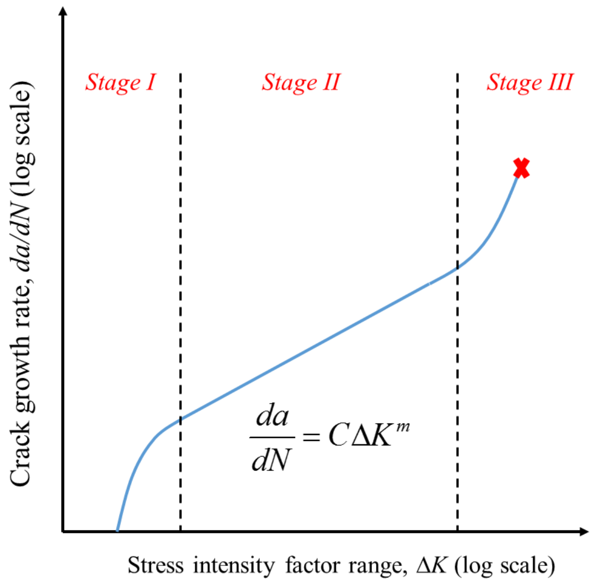

Fig. 2 Relation between crack growth rate and stress intensity factor

"safe-life" approach for structural design against fatigue, it does not provide any information about crack initiation or propagation, especially when the target structure is in use.

Meanwhile, fracture mechanics theory deals with the study of crack propagation. Paris and Erdogan (1963) developed a relationship between the crack propagation rate (*da/dN)* and the range of stress intensity factor (ΔK). This relationship is known as Paris' law given as

$$\frac{da}{dN} = C\left(\Delta K\right)^m \tag{1}$$

where a is the crack length, N is the number of loading applications, and C and m are material parameters. The range of the stress intensity factor can be estimated using Newman's approximation (Newman 1998) as

$$\Delta K = \Delta S \cdot Y\left(a\right)\sqrt{\pi a} \tag{2}$$

where ΔS is the range of the far-field stress (i.e., nominal stress) and *Y(a)* is the geometry function. Although nominal stress is applied to this study, other types of stress such as hot-spot stress and notch stress (Oh *et al*. 2014) can also be applied if the stress intensity factor can be expressed by a function of the stress as in Eq. (2). A schematic log-log plot of the typical relationship between the crack growth rate and the range of stress intensity factor is shown in Fig. 2.

In Fig. 2, *Stage I* is referred to as the crack initiation stage, which involves crack nucleation and short crack propagation. *Stage II* is referred to as the crack propagation stage, which involves propagation of long cracks; it is the stage where Paris' law is applicable. *Stage III* is the fracture stage, in which the failure occurs due to extremely high crack propagation rate, and the number of load cycles in this stage is relatively small.

Paris' law has been applied to many structural problems. However, it is also known that it has a few limitations. For example, it is not applicable to the crack initiation stage wherein the crack growth rate is not proportional to stress intensity factor range (Dowling *et al*. 2009).

Although both the S-N curve and Paris' law are effective bases for fatigue analysis, the use of Paris' law can be often more effective, especially when the target structure is in use. It is because Paris' law can characterize crack growth and facilitate the assessment of the remaining service life or the decision-making on the optimal inspection intervals under definite loading conditions and service environments (Lee and Song 2014, Zhao *et al*. 1994, Soares and Garbatov 1996, Sørensen 2009). By substituting Eq. (2) into Eq. (1), one can obtain

$$\frac{1}{\left[Y(a)\sqrt{\pi a}\right]^m}da = C\cdot\Delta S^m dN \tag{3}$$

By integrating Eq. (3) from the transition (or initial) crack length (a_0) through the critical crack length (a_c), one can solve for the total number of loading applications required for the failure N_P as follows

$$N_P = \int_{a_0}^{a_c} \frac{1}{C\left(\Delta S\cdot Y(a)\sqrt{\pi a}\right)^m}da \tag{4}$$

For a structure subjected to a cyclic load with frequency ν_0, the time duration until crack failure T (i.e., fatigue life) can be described as

$$T = \frac{N_P}{\nu_0} = \frac{1}{\nu_0}\int_{a_0}^{a_c} \frac{1}{C\left(\Delta S\cdot Y(a)\sqrt{\pi a}\right)^m}da \tag{5}$$

Eq. (5) or a similar equation derived from Paris' law can be useful in many structural problems, such as evaluating the fatigue life and determining the inspection intervals of various structures such as bridges, ship structures, aircraft, wind turbine blades, and offshore platforms (Lee and Song 2014, Lee and Song 2012, Zhao *et al*. 1994, Soares and Garbatov 1996, Sørensen 2009, Karamchandani *et al*. 1992, Moan and Song 2000).

However, a fatigue lifetime estimated using Paris' law is sensitive to material parameters (i.e., C and m). Particularly, m is a very important parameter, because it is a power term in Paris' law and thus becomes to have a dominant effect on the fatigue lifetime.

Another advantage of approaches based on Paris' law is its capability of updating the fatigue failure risk after observing various crack inspection results of the structure (Lee and Song 2014, Lee *et al*. 2017). This feature is beneficial in various structural problems, because the risk of fatigue failure can be updated according to Paris' law and inspections can hence be scheduled in an optimized manner.

Although Paris' law is sufficiently powerful to predict crack growth, it involves many uncertainties. These uncertainties arise from determination of the material parameters C and m as well as from the initial crack length. The initial crack length can be considered as the transition crack length, a_0, from *Stage I* to *Stage II*. While the material parameters can be determined experimentally through fatigue crack growth rate tests, the transition crack length is often challenging to obtain from such experiments. One of the uncertainties involved in obtaining deterministic values of C and m is the material inhomogeneity, which leads to variation in the crack growth rate each time the experiment is carried out (Virkler *et al*. 1979). Other uncertainties arise from measurement of the crack growth rate and stress intensity factor during the experiment (Virkler *et al*. 1979). The uncertainties involved in fatigue become more evident from observations of the scatter in the S-N data. Therefore, probability distributions can be used to describe these uncertainties (Wirsching 1983). It can be seen that for the same range of applied stress, the specimen fails after different numbers of cycles. A statistical and probabilistic approach is used to account for these uncertainties. Because S-N curve experiments have been conducted for most materials, a vast amount of S-N curve data is available, which helps to reduce the uncertainty in using S-N curves.

3. SNPL method

To derive the statistical parameters of the Paris' law parameters from the S-N curve data, this paper proposes a new probabilistic method, termed the S-N Paris law (SNPL) method (Ramachandra Prabhu and Lee 2017). The proposed method consists of four steps, which are explained below in detail.

Step 1: Divide the S-N curve data into two cases

During the S-N test, different ranges of stress are applied to the specimen, and the number of cycles until failure is noted. The basic procedure is to start testing the specimen at a higher stress amplitude, where failure is expected in a relatively small number of cycles. The applied stress is then decreased for each succeeding specimen until one or two specimens do not fail within the threshold value of the number of cycles. This threshold, where the fatigue test is terminated, is called the runout number (N_R). The runout number is influenced by the fatigue limit of the material, type of loading, and shape of the specimen (Weibull 2013). The fatigue limit is the maximum stress amplitude which the material does not fail. Therefore, the stress amplitude beyond the fatigue limit is ignored in the proposed method. Because the runout number represents this information, it is used to divide the S-N data into two cases, namely failure and non-failure cases. If the specimen reaches the runout number, it is considered as a non-failure case; otherwise, it is considered as a failure case. In general, the runout number is set to one million cycles or two million cycles (Bannantine *et al*. 1990), and in this study, the failure and non-failure cases are described as follows:

a. A non-failure case is a case in which the specimen does not fail within the runout number of cycles ($N_{SN} > N_R$);
b. A failure case is a case in which the specimen fails within the runout number of cycles ($N_{SN} \leq N_R$).

In the proposed method, for a failure case, the number of load cycles obtained from the S-N curve, N_{SN}, is used, while runout number, N_R, is introduced for a non-failure case.

Step 2: Calculate the number of cycles for crack propagation, N_P

When it is assumed that the number of load cycles for ultimate fracture stage (i.e., *Stage III* in Fig. 3) is relatively small, the number of cycles obtained from the S-N data can be divided into the number of cycles for crack initiation and the number of cycles for crack propagation.

Since Paris' law (in Eq. (1)) and the total number of loading applications until failure (in Eq. (4)) are applicable only to the *Stage II*, the number of cycles for crack propagation has to be calculated from the total number of cycles to failure obtained from the S-N data. This can be done either by using the empirical technique proposed by Manson (1966) or by using the acoustic emission method proposed by Singh (2002). The empirical technique proposed by Manson is used in this study. Based on a large number of test results obtained by conducting a two-load-level test, Manson proposed an equation for initiation and propagation lives that would best fit his data. Manson's equation for calculating the number of cycles for a failure case, N_{PF}, is given by

$$N_{PF} = 14 \times N_{SN}^{0.6} \tag{6}$$

Likewise, the number of cycles for a non-failure case, N_{PNF}, can be calculated as

$$N_{PNF} = 14 \times N_{R}^{0.6} \tag{7}$$

It should be noted that the parameters in the Manson's equation could be different from material types. The parameters for the structural steel are shown in the equation. The parameters in the equation were determined by fatigue testing from limited types of materials. Thus, if the values of the parameters are known for a particular material, the parameters could be changed.

Step 3: Construct the likelihood function for both cases

The concept of the limit-state function, which has evolved from structural reliability theories, is used in this step. Lee and Cho (2016) and Lee and Song (2014) derived a series of formulations for estimating the structural risk of fatigue-induced failure. These formulations are introduced to derive limit-state functions in the present study. With regard to structural reliability, in general, the limit-state function is the criterion that determines when the load effect exceeds the resistance of a structure. The mathematical representation of the limit-state function is given by

$$g(\mathbf{X}) = R(\mathbf{X}) - S(\mathbf{X}) < 0 \tag{8}$$

where g is the limit-state function, R is the resistance of the structure, S is the load effect, and $\mathbf{X}$ is the vector of random variables.

Based on this definition of the limit-state function, the probability of fatigue failure P_f is calculated as

$$P_f = P\left(R(\mathbf{X}) - S(\mathbf{X}) < 0\right) \tag{9}$$

This concept of constructing a limit-state function and finding the probability of fatigue failure is used in the SNPL method for two cases. The first case is called the inequality case, which represents non-failure cases, and the second case is called the equality case, which represents failure cases.

Case 1: Inequality case

When failure is not observed until a certain number of load applications in an S-N test, it is considered as the inequality case. The case means that not as many load cycles as required for failure were not given in the S-N experiment, and it is called the *inequality case* because an inequality sign is included in the limit-state function g_{NF} which is given as

$$g_{NF}\left(\mathbf{X}\right) = N_{PNF} - N_P\left(\mathbf{X}\right) < 0 \tag{10}$$

Based on the limit-state function given by Eq. (10), the probability of the non-failure case is calculated as

$$P_{NF} = P\left(N_{PNF} - N_P\left(\mathbf{X}\right) < 0\right) \tag{11}$$

It should be noted that N_{PNF} and N_P can be expressed by Eqs. (7) and (4), respectively, and the probability P_{NF} can be calculated using conventional reliability analysis methods, such as the first order reliability method (FORM) (Lee and Song 2014, Lee *et al*. 2008, Kang *et al*. 2012).

Case 2: Equality case

When failure is observed after a certain number of load applications in an S-N test, it is considered as the *equality case*, and the limit-state function is given as

$$g_F\left(\mathbf{X}\right) = N_{PF} - N_P\left(\mathbf{X}\right) = 0 \tag{12}$$

Based on the limit-state function given by Eq. (12), the probability of the failure case is expressed by

$$P_F = P\left(N_{PF} - N_P\left(\mathbf{X}\right) = 0\right) \tag{13}$$

Unlike Eq. (11), Eq. (13) represents an equality condition, and thus the probability is mathematically zero. Hence, to proceed with this term, an alternative formulation technique of taking its derivative (Lee and Song 2014, Straub 2011) is adopted as follows

$$P_F = \left[\frac{\partial}{\partial \theta} P\left[\left(g_F\left(\mathbf{X}\right) + \theta \right) \leq 0 \right] \right]_{\theta=0} \tag{14}$$

In Eq. (14), an infinitesimal quantity θ is added to the limit-state function to eliminate the equality sign. Thus, one can establish inequality information, which will give a non-zero probability. Hence, Eq. (14) can be rewritten as

$$P_F = \left[\frac{\partial}{\partial \theta} P\left[\left(N_{PF} - N_P\left(\mathbf{X}\right) + \theta \right) \leq 0 \right] \right]_{\theta=0} \tag{15}$$

Note that P_F in Eq. (15) can be calculated by taking the numerical differentiation of the probability results obtained from conventional reliability analysis methods, such as the first order reliability method (FORM) (Lee and Song 2014). If there are multiple inequality and equality cases, the final probability P_{final} is the product of individual probabilities $P_{NF,i}$ and $P_{F,j}$:

$$P_{final} = \prod_{i=1}^{N_{NF}} P_{NF,i} \times \prod_{j=1}^{N_F} P_{F,j} \tag{16}$$

where N_{NF} and N_F are the number of inequality cases (i.e., non-failure cases) and the number of equality cases (i.e., failure cases), respectively.

Step 4: Optimization

The SNPL method attempts to find the best statistical parameters of the Paris' law parameters through an optimization process, which maximizes the probability that the given S-N test results are observed (i.e., P_{final} in Eq. (16)). It means P_{final} is introduced as the objective function of the optimization. To reduce computational errors during the optimization process, the natural logarithm of P_{final} is introduced as the objective function

$$L\left(\mathbf{X}|\boldsymbol{\theta}\right) = \sum_{i=1}^{N_{NF}} \ln P_{NF,i}\left(\mathbf{X}|\boldsymbol{\theta}\right) + \sum_{j=1}^{N_F} \ln P_{F,j}\left(\mathbf{X}|\boldsymbol{\theta}\right) \tag{17}$$

where $\boldsymbol{\theta}$ is the vector of statistical parameters of the Paris' law parameters.

To find the best estimates of the statistical parameters of the transition crack length a_0 and material parameters C and m, optimization based on based on Eq. (17) is performed as follows

$$L_{\max}\left(\mathbf{X}|\boldsymbol{\theta}\right) = \max \left\{ \sum_{i=1}^{N_{nofail}} \ln P_{nofail,i}\left(\mathbf{X}|\boldsymbol{\theta}\right) + \sum_{j=1}^{N_{fail}} \ln P_{fail,j}\left(\mathbf{X}|\boldsymbol{\theta}\right) \right\} \tag{18}$$

In this study, the Nelder-Mead simplex algorithm from MATLAB's Optimization Toolbox is used as the optimization solver. It determines the best statistical measures by satisfying Eq. (18).

Fig. 3 shows the flowchart of the proposed SNPL method. A significant advantage of the SNPL method is that the current statistical parameters can be updated when additional S-N data becomes available, and this is because the SNPL method is adopting a Bayesian approach. Steps 1-4 can be repeated by incorporating the new S-N data, which helps to further reduce the uncertainties and thus provides more reliable data for the Paris' law parameters.

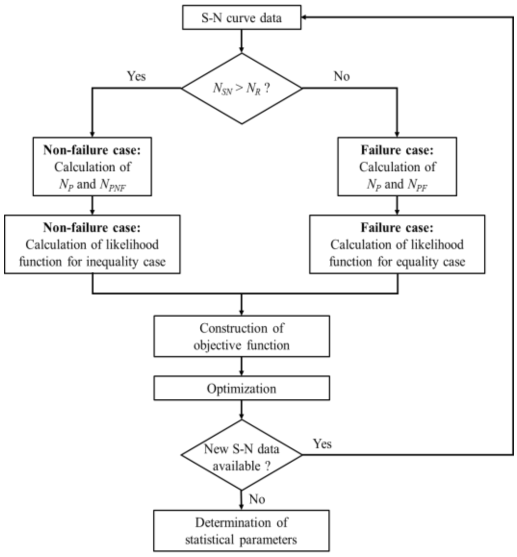

Fig. 3 Flowchart of SNPL method

Table 1 Material properties and approximate Paris' law material parameter values of 40H and 20G steel

Material	Yield strength S_y (MPa)	Ultimate strength S_u (MPa)	C (m/(cycle·MPa·m$^{0.5}$))	m
40H steel	780	980	3.96×10^{-12}	2.97
20G steel	280	460	2×10^{-11}	3

4. Application of SNPL method

The proposed SNPL method was tested by applying it to 40H steel and 20G steel, whose Paris' law material parameters have already been determined experimentally. The Paris' law parameters of these materials were derived using the SNPL method, and the results were compared with the known experimental values. Because the S-N data for these materials were not available, synthetic S-N data were generated using a method adopted in previous studies (Juvinall and Marshek 2006, Lee *et al*. 2011, VDME 2003, Ramachandra Prabhu and Lee 2017). This method gives an empirical relationship between the ultimate tensile strength and the fatigue limit over one million cycles. The material properties of 40H steel and 20G steel, along with the approximate Paris' law material parameter values, are summarized in Table 1 (Szata and Lesiuk 2009).

4.1 Generation of S-N curve data for 40H steel and 20G steel

The synthetic S-N data were generated as recommended by (Juvinall and Marshek 2006) for ductile materials. The

Table 2 Stress amplitudes for 10^3 and 10^6 cycles

Material	Ultimate Strength S_u (MPa)	Stress amplitude for 10^3 cycles S_c (MPa)	Stress amplitude for 10^6 cycles, S_n (MPa)
40H steel	980	735	397
20G steel	460	345	186

Table 3 S-N curve data for 40H steel and 20G steel

40H steel		20G steel	
Stress amplitude (MPa)	Number of cycles	Stress amplitude (MPa)	Number of cycles
735	1×10^3	345	1×10^3
636.68	5×10^3	298.95	5×10^3
598.51	1×10^4	281.02	1×10^4
518.47	5×10^4	243.44	5×10^4
487.38	1×10^5	228.84	1×10^5
422.2	5×10^5	198.24	5×10^5
396.9	1×10^6	186.36	1×10^6

following factors were considered for both 40H steel and 20G steel.

• The specimen is axially loaded for stress ratio (R) of -1
• The specimen has a commercially polished surface
• Cross section diameter < 50.8 mm (i.e., 2 inch)
• Gradient correction factor, C_G, = 0.9; temperature correction factor, C_T, = 1.0; reliability correction factor, C_R, = 1.0; and load correction factor, C_L, = 1.0
• Surface factor, C_S, = 0.9 since S_u < 1100 MPa

Based on the factors mentioned above, the 10^3-cycle stress amplitude for an axially loaded ductile material was calculated as (Bannantine 1990)

$$S_c = 0.75 S_u \tag{19}$$

The 10^6-cycle stress amplitude for an axially loaded ductile material was calculated as (Bannantine 1990)

$$S_n = S_n' C_L C_G C_S C_T C_R \tag{20}$$

where S_n' is given by

$$S_n' = 0.5 S_u \tag{21}$$

Based on these calculations, the following values were obtained for 40H steel and 20G steel (see Table 2).

Based on the values listed in Table 2, a straight line was drawn between S_c and S_n to obtain the S-N curve. Based on the equation of the line, the stress amplitude corresponding to various numbers of cycles until failure were obtained (see Table 3).

For a set of experimental S-N data, as described in Sec. 2, there is scatter in the number of cycles until failure for the same stress amplitude in general, which involves uncertainty in fatigue. To introduce this uncertainty into the current S-N data, 10 samples were generated for each stress amplitude with a c.o.v. of 0.3, following a lognormal distribution (Wirsching 1983). It was observed in a preliminary analysis that at least 50 samples are required to

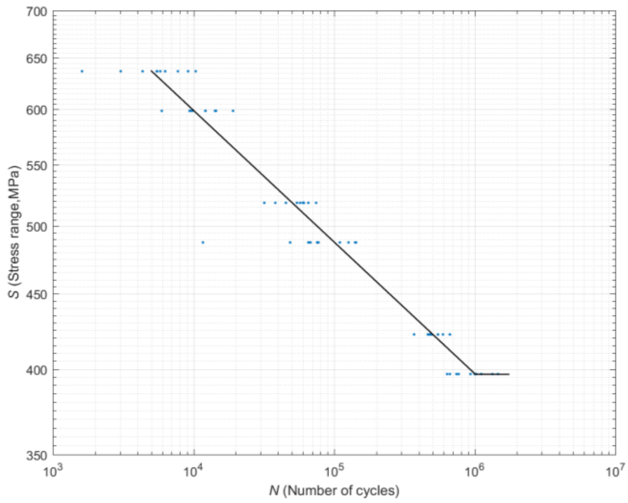

Fig. 4 S-N curve for 40H steel

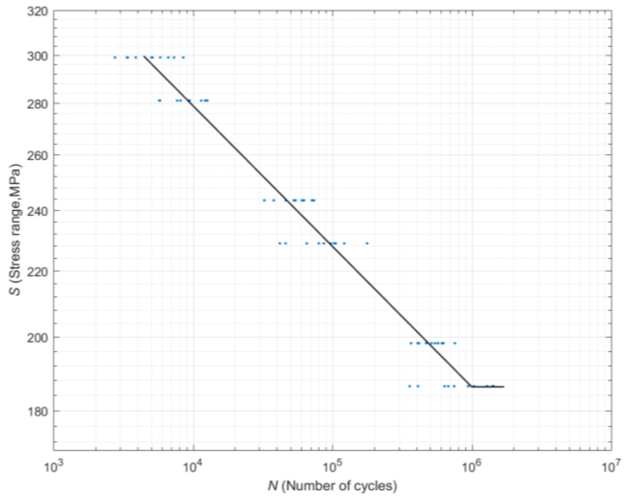

Fig. 5 S-N curve for 20G steel

provide a reasonable estimation of the Paris' law parameters, so a total of 70 samples were used for each of 40H steel and 20G steel.

Based on these S-N data, S-N curves were plotted for 40H steel and 20G steel, as shown in Figs. 4 and 5, respectively. The SNPL method is applied to these S-N curves.

4.2 Analysis results

Through the analysis employing the SNPL method, the statistical parameters of the Paris' law material parameters (C and m) and the transition crack length (a_0) are determined. In addition, the proposed method determines the correlation between C and m. In the analysis, C and m were considered to be lognormally distributed, and the transition crack length was considered to be exponentially distributed, following previous studies (Lee and Song 2012, 2014). In the calculations, the width of the specimen (W) was taken as 30 mm, and the critical crack length was taken

Table 4 Results from SNPL method

Material	C (lognormal distribution)		m (lognormal distribution)		Correlation coefficient between C and m	a_0 (exponential distribution)
	Mean (m/(cycle·MPa·m$^{0.5}$))	c.o.v.	Mean	c.o.v.		Mean (mm)
40H steel	4.46×10^{-12}	0.57	3.05	0.04	−0.9	1.18×10^{-1}
20G steel	2.42×10^{-11}	0.49	3.14	0.06	−0.87	1.92×10^{-1}

as 15 mm. An edge crack was assumed and the following crack geometry function $Y(a)$ (Tada *et al.* 2000) was selected

$$Y(a)=\frac{\left\{0.752+2.02\frac{a}{W}+0.37\left[1-\sin\left(\frac{\pi a}{2W}\right)\right]^3\right\}}{\cos\left(\frac{\pi a}{2W}\right)} \times\sqrt{\frac{2W}{\pi a}\tan\left(\frac{\pi a}{2W}\right)} \tag{22}$$

The SNPL method was tested with the generated sets for 20G and 40H steels. Each of the steps explained in Sec. 3 was carried out. The first step in the SNPL method is to divide the S-N data into failure and non-failure cases. The runout number was selected as 1 million. Based on the S-N data for both 40H steel and 20G steel, 56 samples were failure cases and 4 samples were non-failure cases. The next step is to divide the number of cycles from the S-N data into the number of cycles for crack initiation and the number of cycles for crack propagation based on Manson's equation. Then, step 3 was carried out based on the calculations for the equality and inequality cases as described in Sec. 3 by inputting various values for C, m, and a_0. Lastly, optimization (step 4) was carried out based on the best outputs. The best results of the statistical measures for C, m, and a_0 were obtained after optimization, which maximized the likelihood of observing the given S-N data. The analysis results are summarized in Table 4.

The SNPL method results were compared with the experimental results shown in Table 1, and it is observed that the results in Table 4 showed good agreement with the experimental results. In particular, the errors of m which is an exponential term in Paris' law (i.e., Eq. (1)) and thus has a dominant effect on crack growth rate and fatigue life were estimated to be 2.69% and 4.67% for 40H steel and 20H steel, respectively.

In addition, it has been reported that there is a strong negative correlation between C and m of metals (Lee and Song 2012), which can also be found from the result (i.e., −0.9 for 40H steel and −0.87 for 20G steel). Moreover, the SNPL method provides the c.o.v.s of the Paris law parameters which are not obtainable from the S-N curve. Thus, it can be inferred that the SNPL method predicts the values of the Paris' law material parameters accurately when S-N data are available, and the results can be compared with the values provided by several engineering standards such as BS 7910 (British Standard Institution 2015).

The SNPL method can also quantify the uncertainty in the transition crack length, which is otherwise an unknown parameter based on crack propagation data. As shown in Table 4, the mean values of the initial crack length for 40H steel and 20G steel are estimated to be 0.118 mm and 0.192 mm, respectively, in the application example. Although there was no experimental results to compare, it was reported in several previous studies that the initial crack length of steel were estimated to be 0.1-0.2 mm (Karamchandani *et al.* 1992, Moan and Song 2000, Lee and Song 2011, Sova *et al.* 1976, McCarver and Ritchie 1982). Thus, it can be inferred that the initial crack lengths obtained from the proposed method are in a reasonable range.

4. Conclusions

A novel probabilistic method, termed the SNPL method, was developed to derive Paris' law parameters from S-N curve data. Based on a Bayesian approach, the SNPL method considers the uncertainties in the S-N data to provide meaningful derivations of the statistical parameters of the Paris' law parameters and the transition crack length. The validation results showed that the statistical parameters of the Paris' law parameters and the transition crack length derived by the SNPL method from the S-N curve data are in good agreement with the actual experimental results. In addition, when additional S-N data are available, the statistical parameters of the Paris' law parameters and the transition crack length can be updated. Thus, the uncertainties can be reduced further and results that are more accurate can be obtained. In summary, the proposed method is a useful approach to get the statistical information on crack growth rate when S-N data are available.

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