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A novel refined shear deformation theory for the buckling analysis of thick isotropic plates

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Abstract. In present study, a novel refined hyperbolic shear deformation theory is proposed for the buckling analysis of thick isotropic plates. The new displacement field is constructed with only two unknowns, as against three or more in other higher order shear deformation theories. However, the hyperbolic sine function is assigned according to the shearing stress distribution across the plate thickness, and satisfies the zero traction boundary conditions on the top and bottom surfaces of the plate without using any shear correction factors. The equations of motion associated with the present theory are obtained using the principle of virtual work. The analytical solution of the buckling of simply supported plates subjected to uniaxial and biaxial loading conditions was obtained using the Navier method. The critical buckling load results for thick isotropic square plates are compared with various available results in the literature given by other theories. From the present analysis, it can be concluded that the proposed theory is accurate and efficient in predicting the buckling response of isotropic plates.

Keywords: buckling analysis; isotropic plates; new displacement field; Navier method; analytical modeling

1. Introduction

Typically, the Buckling is a phenomenon of elastic instability of a plate which is subjected on their edges to external in-plane compressive and shear loads acting strictly in the middle plane of the plate, it becomes unstable and begins to buckle. The magnitude of the compressive axial load at which the plate passes into unstable is termed the critical buckling load. If the load is increased beyond this critical buckling load, it results in a large deflection and the plate seeks another equilibrium configuration. Hence, the load at which a plate becomes unstable is of practical importance in design. However, a buckling analysis of plates has been an important part of research in the area of solid mechanics for a long time. The major fields of applications of supported plates and design of steel structures as structural members include aeronautical, automotives, marine, civil and mechanical engineering structures.

The increasing importance of plates in these engineering applications has led to predict the buckling behaviour of isotropic, orthotropic and laminated composite plate, so that a variety of plate theories have been developed based on considering the transverse shear deformation effect. The buckling problem for a simply supported plate subjected to the direct, constant compressive forces acting in one and two directions was first solved by Bryan (1981) using the energy method. The buckling behaviour of a rectangular plate with varying edge conditions under uniform compressive in-plane load was studied by Timoshenko and Gere (1961). Their work confirmed that when a simply supported plate buckles elastically, the out-of-plane displacement profile forms sinusoidal waves along the length and width of the plate. It should be noted that the classical plate theory (CPT), which is based on the Kirchhoff hypothesis, ignores the transverse shear deformation and gives good results only for the buckling of thin plates, but not be suitable for moderately thick or thick plates in which the transverse shear deformation effects are more significant. Yet, for the vast majority of experiments have shown that Kirchhoff's classical plate theory underestimates deflections and overestimates natural frequencies and buckling loads for moderately thick plates. The first order shear deformation plate theory (FSDT) developed by Reissner (1945) and Mindlin (1951) includes the effect of transverse shear deformation by the way of linear variation of in-plane displacements through the thickness but does not satisfy shear stress-free conditions at top and bottom surfaces of the plate, which requires the addition of a shear correction factor in order to rectify the unrealistic variation of the shear strain-stress across the thickness, hence, this factor depend not only on material

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and geometric parameters but also on the loading and boundary conditions. To overcome the limitations of CPT and FSDT, many polynomial and non-polynomial higher order shear deformation theories (HSDTs) have been developed in the past few decades to prevent the use of the shear correction factor and have a better representation of the bending, buckling and vibration analysis of isotropic and advanced composite plates (Tounsi *et al.* 2013, Zidi *et al.* 2014, Mahi *et al.* 2015, Attia *et al.* 2015, Bounouara *et al.* 2016, Bellifa *et al.* 2016, Beldjelili *et al.* 2016, Boukhari *et al.* 2016, Bousahla *et al.* 2016, Klouche *et al.* 2017, Benadouda *et al.* 2017, Menasria *et al.* 2017, Besseghier *et al.* 2017, Attia *et al.* 2018, Belabed *et al.* 2018, Bouadi *et al.* 2018, Bouhadra *et al.* 2018, Bourada *et al.* 2018, Bourada *et al.* 2019). Reddy and Phan (1985) used a higher-order shear deformation theory to determine the natural frequencies and buckling loads of simply supported plates, in which a parabolic distribution of the transverse shear strains through the thickness of the plate is accounted and stress-free boundary conditions are satisfied. Shufrin and Eisenberger (2005) have applied a first order shear deformation plate theory and the higher order shear deformation plate theory of Reddy in order to calculate the natural frequencies and buckling loads of thickelastic rectangular plates with various combinations of boundary conditions. The numerical results for buckling analysis of a simply supported isotropic and orthotropic rectangular plates subjected to in-plane loading has been obtained by Kim *et al.* (2009) using the two variable refined plate theory and the Navier method. Sayyad and Ghugal (2012) used a displacement based an exponential shear deformation theory (ESDT) for the buckling analysis of thick isotropic square plates with all simply supported edges subjected to uniaxial and biaxial in-plane loads. Analytical solutions for bending, buckling, and vibration analysis of thick rectangular plates with two opposite edges simply supported and the other two edges having arbitrary boundary conditions are presented by Thai and Choi (2013) using two variable refined plate theory. Grover *et al.* (2013) proposed a new inverse hyperbolic shear deformation theory for static and buckling analysis of laminated composite and sandwich plates, which gives non-linear distribution of transverse shear stresses and also satisfies the zero tangential traction boundary conditions on the surface of the plate. Ait Amar Meziane *et al.* (2014) developed an efficient and simple refined shear deformation theory for the bucking and free vibration analysis of exponentially graded material sandwich plates resting on two-parameter elastic foundations under various boundary conditions. Nguyen *et al.* (2015) presented a refined higher-order shear deformation theory for bending, vibration and buckling of functionally graded material sandwich plates using a hyperbolic shape function. Sayyad *et al.* (2016) applied a simple trigonometric shear deformation theory for the bending, buckling and free vibration responses of cross-ply laminated composite plates. This theory involves four unknown variables and four governing differential equations. Bourada *et al.* (2016) proposed a novel four variable refined plate theory for the buckling analysis of isotropic and orthotropic rectangular plates under the axial loading. Using the previous theory, Hebali *et al.* (2016) studied the bending, buckling, and vibration responses of functionally graded plates. Tounsi *et al.* (2016) presented a new three unknowns non-polynomial shear deformation theory for the buckling and vibration analysis of FGM sandwich plates.

Recently, Meksi *et al.* (2019) developed a new simple higher order shear deformation theory for the bending, buckling and free vibration of FG sandwich plates. Based on the visco-nonlocal-refined Zigzag theories, dynamic buckling responses was examined by Kolahchi *et al.* (2017) for a sandwich nanoplate subjected to harmonic compressive load and resting on visco-Pasternak's foundation. Chikh *et al.* (2017) presented a simplified higher order shear deformation theory (HSDT) with four unknowns for thermal buckling analysis of simply supported isotropic, orthotropic and cross-ply laminated plates under uniform temperature rise. El-Haina *et al.* (2017) studied the thermal buckling response of FG sandwich plates by using a novel and simple higher shear deformation theory in which a trigonometric variation of transverse shear stress is considered. Hajmohammada *et al.* (2018a) presented a dynamic buckling analysis of Multiphase nanocomposite viscoelastic laminated conical shells subjected to magneto-hygrothermal loads. It is noted that recently, different authors developed new types of HSDTs for investigating mechanical behavior of structures (Ahmed 2014, Akavci and Tanrikulu 2015, Kar and Panda 2016a, Aldousari 2017, Bellifa *et al.* 2017a, b, Zine *et al.* 2017, Fakhar and Kolahchi 2018, Hajmohammada *et al.* 2018b, Karami *et al.* 2018).

Also, some essential theories related to the mechanical behavior of advanced composite structures like, the first shear deformation theory and the higher-order shear deformation theory with or without zig-zag function, nonlocal theories, Visco-nonlocal-piezoelectricity theory, visco-nonlocal-refined zigzag theory and some innovative studies are given in references (Kolahchi and Bidgoli 2016, Kolahchi *et al.* 2016a, b, Kolahchi 2017, Kolahchi *et al.* 2017a, b, c, Katariya *et al.* 2017a, Hajmohammad *et al.* 2017, Hajmohammad *et al.* 2018c, d) which are applied on the advanced composite structures.

The objective of this study is to propose a novel refined shear deformation theory for the buckling analysis of thick isotropic plates. The proposed theory is based on a new displacement field with only two unknowns by considering undetermined integral terms, which is even less than the other Higher order shear deformation theories, the hyperbolic shape function is attributed according to the non-linear distribution of shear stress through the thickness of the plate, and satisfies the zero traction boundary conditions without using shear correction factors. The utilization of the integral term in the proposed kinematic led to a reducing in the number of variables and governing equations. Hence, the proposed theory has only four unknowns and four governing equations. The plate governing equations and its boundary conditions are derived by utilizing the principle of virtual works. Navier-type analytical solution is obtained for simply supported isotropic plates subjected to the uniaxial and biaxial loading conditions. Numerical results of

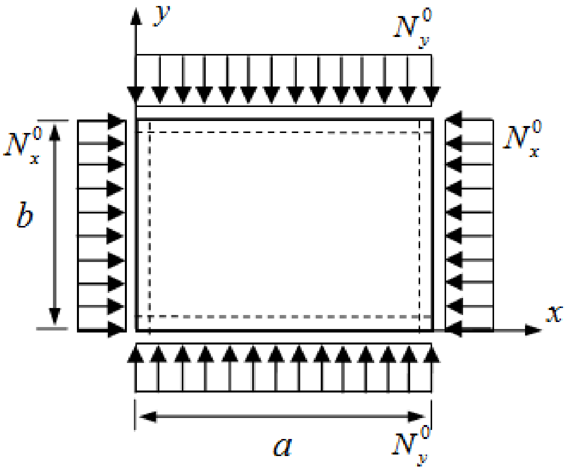

Fig. 1 Rectangular plate subjected to in-plane loads

the critical buckling load for thick to thin isotropic square plates are presented and compared with other shear deformation theories to demonstrate the validity and efficiency of the present theory.

2. Theoretical formulation

2.1 Isotropic plate under consideration

Consider an elastic isotropic rectangular plate of the length a, width b and a constant thickness h in z-direction as shown in Fig. 1. The plate is simply supported on all four edges and subjected to various in-plane distributed loads (N_x^0, N_y^0). It is noted that the in-plane shear forces are not included $(N_{xy}^0 = N_{yx}^0 = 0)$. The plate under consideration occupies the region $0 \le x \le a$, $0 \le y \le b$, $-h/2 \le z \le h/2$ in Cartesian coordinate system.

2.2 Kinematic and constitutive relations

In this present study, the conventional higher order shear deformation theory presented by Sayyad and Ghugal (2012) is modified by introducing some simplifying suppositions in order to reduce the number of unknown variables. The displacement field of the existing HSDT is defined by

$$\begin{aligned} u(x,y,z) &= -z\frac{\partial w_0}{\partial x} + f(z)\,\phi(x,y) \\ v(x,y,z) &= -z\frac{\partial w_0}{\partial y} + f(z)\,\psi(x,y) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{1}$$

where u, v and w denote the displacement components along the x, y and z coordinate directions, respectively, ϕ and ψ represents the rotations about the y and x axes, whereas $f(z)$ denote a shape function determining the distribution of the transverse shear strains and the stresses through the thickness of the plate. By employing that $\phi = \int \theta(x,y)dx$ and $\psi = \int \theta(x,y)dy$, the new displacement field of the proposed hyperbolic shear deformation theory (HySDT) can be expressed only with two unknowns in the form as

$$\begin{aligned} u(x,y,z) &= -z\frac{\partial w_0}{\partial x} + k_1 f(z)\int \theta(x,y)dx \\ v(x,y,z) &= -z\frac{\partial w_0}{\partial y} + k_2 f(z)\int \theta(x,y)dy \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{2}$$

where w_0 and θ are two unknowns displacement functions of middle surface of the isotropic plate. The constants k_1 and k_2 depends on the geometry. In this work, the present theory is obtained by putting

$$f(z) = h\sinh\left(\frac{z}{h}\right) - z\cosh\left(\frac{1}{2}\right) \tag{3}$$

The non-linear von Karman strains related to displacement field in Eq. (2) are

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \end{Bmatrix} = \begin{Bmatrix} \dfrac{1}{2}\left(\dfrac{\partial w_0}{\partial x}\right)^2 \\ \dfrac{1}{2}\left(\dfrac{\partial w_0}{\partial y}\right)^2 \\ \dfrac{\partial w_0}{\partial x}\dfrac{\partial w_0}{\partial y} \end{Bmatrix} + z\begin{Bmatrix} \varepsilon_x^1 \\ \varepsilon_y^1 \\ \gamma_{xy}^1 \end{Bmatrix} + f(z)\begin{Bmatrix} \varepsilon_x^2 \\ \varepsilon_y^2 \\ \gamma_{xy}^2 \end{Bmatrix} \tag{4a}$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial v}{\partial z} + \dfrac{\partial w}{\partial y} \\ \dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial x} \end{Bmatrix} = g(z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \tag{4b}$$

where

$$\begin{Bmatrix} \varepsilon_x^1 \\ \varepsilon_y^1 \\ \gamma_{xy}^1 \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} \varepsilon_x^2 \\ \varepsilon_y^2 \\ \gamma_{xy}^2 \end{Bmatrix} = \begin{Bmatrix} k_1\theta \\ k_2\theta \\ k_1\dfrac{\partial}{\partial y}\int \theta dx + k_2\dfrac{\partial}{\partial x}\int \theta dy \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_2\int \theta dy \\ k_1\int \theta dx \end{Bmatrix} \tag{5}$$

$$g(z) = \frac{df(z)}{dz} \tag{6}$$

The integrals adopted in the previous relations shall be resolved by a Navier solution and can be determined by

$$\begin{aligned} &\frac{\partial}{\partial y}\int \theta dx = A'\frac{\partial^2\theta}{\partial x \partial y}, \quad \frac{\partial}{\partial x}\int \theta dy = B'\frac{\partial^2\theta}{\partial x \partial y}, \\ &\int \theta dx = A'\frac{\partial \theta}{\partial x}, \quad \int \theta dy = B'\frac{\partial \theta}{\partial y} \end{aligned} \tag{7}$$

where A' and B' are defined according to the type of solution employed, in this case via Navier. Thus, the parameters A' and B' are expressed by

$$A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2}, \quad k_1 = \alpha^2, \quad k_2 = \beta^2 \tag{8}$$

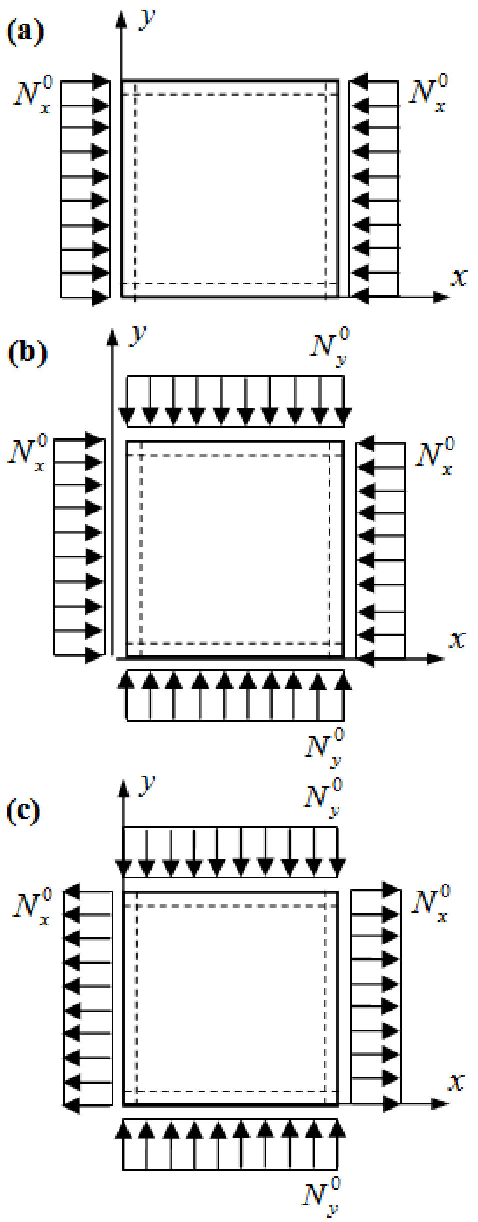

Fig. 2 The loading conditions of isotropic square plate for (a) uniaxial compression, (b) biaxial compression and (c) tension in the x direction and compression in the y direction

where the parameters α and β are defined as

$$\alpha = \frac{m\pi}{a}, \quad \beta = \frac{n\pi}{b} \tag{9}$$

The stress-strain relationships accounting for transversal shear deformation in the isotropic plate coordinates, can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 & 0 & 0 \\ \nu & 1 & 0 & 0 & 0 \\ 0 & 0 & (1-\nu)/2 & 0 & 0 \\ 0 & 0 & 0 & (1-\nu)/2 & 0 \\ 0 & 0 & 0 & 0 & (1-\nu)/2 \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{10}$$

where E and ν are Young's modulus and Poisson's ratio, respectively.

2.3 Governing equations

In the proposed theory, the principle of virtual work is used to obtain the governing equations and boundary conditions for the isotropic plate under consideration. The principle can be stated in analytical form as

$$\begin{aligned} &\int_{-h/2}^{h/2} \int_A \left(\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \tau_{xy} \delta\gamma_{xy} + \tau_{yz} \delta\gamma_{yz} + \tau_{xz} \delta\gamma_{xz}\right) dA dz \\ &- \int_A q \delta w_0 \, dA \\ &+ \int_A \left(N_x^0 \frac{\partial w_0}{\partial x} \frac{\partial \delta w_0}{\partial x} + N_y^0 \frac{\partial w_0}{\partial y} \frac{\partial \delta w_0}{\partial y} + 2N_{xy}^0 \frac{\partial w_0}{\partial x} \frac{\partial \delta w_0}{\partial y} \right) dA = 0 \end{aligned} \tag{11}$$

Where δ is a variational operator, A is the top surfaceof the plate, q and (N_x^0, N_y^0, N_{xy}^0) are transverse and in-plane distributed loads, respectively. By substituting the expressions for virtual strains given in Eq. (4) into Eq. (11), the principle of virtual work can be rewritten as

$$\begin{aligned} &\int_A \Big\{ M_x^b \delta\varepsilon_x^1 + M_y^b \delta\varepsilon_y^1 + M_{xy}^b \delta\gamma_{xy}^1 + M_x^s \delta\varepsilon_x^2 + M_y^s \delta\varepsilon_y^2 + M_{xy}^s \delta\gamma_{xy}^2 + S_{yz}^s \delta\gamma_{yz}^0 + S_{xz}^s \delta\gamma_{xz}^0 \\ &- q\delta w_0 + N_x^0 \frac{\partial w_0}{\partial x} \frac{\partial \delta w_0}{\partial x} + N_y^0 \frac{\partial w_0}{\partial y} \frac{\partial \delta w_0}{\partial y} + 2N_{xy}^0 \frac{\partial w_0}{\partial x} \frac{\partial \delta w_0}{\partial y} \Big\} dA = 0 \end{aligned} \tag{12}$$

where M^b, M^s and S^s are the stress resultants defined by the following integrations

$$\begin{aligned} &\begin{Bmatrix} M_x^b, & M_y^b, & M_{xy}^b \\ M_x^s, & M_y^s, & M_{xy}^s \end{Bmatrix} = \int_{-h/2}^{h/2} \left(\sigma_x, \sigma_y, \tau_{xy}\right) \begin{Bmatrix} z \\ f(z) \end{Bmatrix} dz, \\ &\left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} \left(\tau_{xz}, \tau_{yz}\right) g(z) \, dz \end{aligned} \tag{13}$$

Substituting stress-strain relations from Eq. (10) into the Eq. (13), the stress resultants are obtained in terms of strains as following form

$$\begin{aligned} M_x^b &= D_{11}\varepsilon_x^1 + D_{12}\varepsilon_y^1 + F_{11}\varepsilon_x^2 + F_{12}\varepsilon_y^2, \\ M_y^b &= D_{12}\varepsilon_x^1 + D_{22}\varepsilon_y^1 + F_{12}\varepsilon_x^2 + F_{22}\varepsilon_y^2, \\ M_{xy}^b &= D_{66}\gamma_{xy}^1 + F_{66}\gamma_{xy}^2, \\ M_x^s &= F_{11}\varepsilon_x^1 + F_{12}\varepsilon_y^1 + H_{11}\varepsilon_x^2 + H_{12}\varepsilon_y^2, \\ M_y^s &= F_{12}\varepsilon_x^1 + F_{22}\varepsilon_y^1 + H_{12}\varepsilon_x^2 + H_{22}\varepsilon_y^2, \\ M_{xy}^s &= F_{66}\gamma_{xy}^1 + H_{66}\gamma_{xy}^2, \\ S_{yz}^s &= A_{44}^s\gamma_{yz}^0, \\ S_{xz}^s &= A_{55}^s\gamma_{xz}^0 \end{aligned} \tag{14}$$

where D_{ij}, F_{ij}, H_{ij} and A_{ij}^s are the plate stiffness coefficients given by

$$\begin{Bmatrix} D_{11}, & F_{11}, & H_{11} \\ D_{12}, & F_{12}, & H_{12} \\ D_{66}, & F_{66}, & H_{66} \end{Bmatrix} = \int_{-h/2}^{h/2} \frac{E}{1-\nu^2} \left(z^2, z\, f(z), [f(z)]^2\right) \begin{Bmatrix} 1 \\ \nu \\ (1-\nu)/2 \end{Bmatrix} dz \tag{15a}$$

$$\left(D_{22}, F_{22}, H_{22}\right) = \left(D_{11}, F_{11}, H_{11}\right) \tag{15b}$$

$$A_{44}^{s} = A_{55}^{s} = \int_{-h/2}^{h/2} \frac{E}{2(1+\nu)} \left[g(z)\right]^2 dz \tag{15c}$$

Substituting strain-displacement and stress-strain relations from Eq. (5) and (10) of the proposed theory into Eq. (12) and integrating by parts and collecting the coefficients of δw_0 and $\delta\theta$, the governing differential equations in terms of stress resultants are obtained as follows

$$\delta w_0 : \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + N_x^0 \frac{\partial^2 w_0}{\partial x^2} + N_y^0 \frac{\partial^2 w_0}{\partial y^2} + 2N_{xy}^0 \frac{\partial^2 w_0}{\partial x \partial y} + q = 0 \tag{16a}$$

$$\delta \theta : -k_1 A' \frac{\partial^2 M_x^s}{\partial x^2} - (k_1 A' + k_2 B') \frac{\partial^2 M_{xy}^s}{\partial x \partial y} - k_2 B' \frac{\partial^2 M_y^s}{\partial y^2} + k_1 A' \frac{\partial S_{xz}^s}{\partial x} + k_2 B' \frac{\partial S_{yz}^s}{\partial y} = 0 \tag{16b}$$

Using Eqs. (5) and (14), the governing differential equations Eq. (16) based on the present shear deformation theory can be rewritten in terms of displacement variables (w_0, θ) as

$$\begin{aligned} & k_1 A' F_{11} \frac{\partial^4 w_0}{\partial x^4} + k_1 A' (F_{12} + 2F_{66}) \frac{\partial^4 w_0}{\partial x^2 \partial y^2} + k_2 B' (F_{12} + 2F_{66}) \frac{\partial^4 w_0}{\partial x^2 \partial y^2} + k_2 B' F_{22} \frac{\partial^4 w_0}{\partial y^4} \\ & - (k_1 A')^2 H_{11} \frac{\partial^4 \theta}{\partial x^4} - 2 k_1 A' k_2 B' (H_{12} + H_{66}) \frac{\partial^4 \theta}{\partial x^2 \partial y^2} - \left((k_1 A')^2 + (k_2 B')^2\right) H_{66} \frac{\partial^4 \theta}{\partial x^2 \partial y^2} \\ & - (k_2 B')^2 H_{22} \frac{\partial^4 \theta}{\partial y^4} + (k_2 B')^2 As_{44} \frac{\partial^2 \theta}{\partial y^2} + (k_1 A')^2 As_{55} \frac{\partial^2 \theta}{\partial x^2} = 0 \end{aligned} \tag{17a}$$

$$\begin{aligned} & D_{11} \frac{\partial^4 w_0}{\partial x^4} + 2(D_{12} + 2D_{66}) \frac{\partial^4 w_0}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 w_0}{\partial y^4} - k_1 A' F_{11} \frac{\partial^4 \theta}{\partial x^4} \\ & - k_1 A' (F_{12} + 2F_{66}) \frac{\partial^4 \theta}{\partial x^2 \partial y^2} - k_2 B' (F_{12} + 2F_{66}) \frac{\partial^4 \theta}{\partial x^2 \partial y^2} - k_2 B' F_{22} \frac{\partial^4 \theta}{\partial y^4} = \\ & N_x^0 \frac{\partial^2 w_0}{\partial x^2} + N_y^0 \frac{\partial^2 w_0}{\partial y^2} + 2N_{xy}^0 \frac{\partial^2 w_0}{\partial x \partial y} + q \end{aligned} \tag{17b}$$

2.4 Buckling analysis of isotropic plates using Navier solution

Buckling analysis of simply supported isotropic square plate is obtained using Navier solution procedure. The plate is subjected to in-plane distributed loads $(N_x^0 = \gamma_1 N_0, N_y^0 = \gamma_2 N_0, N_{xy}^0 = 0)$, as shown in Fig. 2. However, in case of static buckling problem, all other loads acting on plate are assumed to be zero (q=0). The simply supported boundary conditions on all four edges of the square plate can be expressed as
at edges (x=0, a)

$$w_0 = M_x^b = M_x^s = \theta = 0 \tag{18a}$$

at edges (y=0, b)

$$w_0 = M_y^b = M_y^s = \theta = 0 \tag{18b}$$

Based on this procedure, the solution of the displacement variables satisfying the boundary conditions given by Eq. (18), and can be expressed in the double-Fourier sine series as

Based on this procedure, the solution of the displacement variables satisfying the boundary conditions given by Eq. (18), and can be expressed in the double-Fourier sine series as

$$\begin{Bmatrix} w_0 \\ \theta \end{Bmatrix} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \begin{Bmatrix} W_{mn} \sin(\alpha\, x) \sin(\beta\, y) \\ \Phi_{mn} \sin(\alpha\, x) \sin(\beta\, y) \end{Bmatrix} \tag{19}$$

where W_{mn} and Φ_{mn} are unknown coefficients, so the parameters α and β are already defined in Eq. (9). Substitution of this solution of Eq. (19) into the governing equations Eq. (17), the critical buckling loads of isotropic plates can be obtained from the following matrix form

$$\left\{ \begin{bmatrix} K_{11} & K_{12} \\ K_{12} & K_{22} \end{bmatrix} - N_0 \begin{bmatrix} N_{11} & 0 \\ 0 & 0 \end{bmatrix} \right\} \begin{Bmatrix} w_0 \\ \theta \end{Bmatrix} = 0 \tag{20}$$

where

$$\begin{aligned} & K_{11} = -2\alpha^2 \beta^2 (D_{12} + 2D_{66}) - \alpha^4 D_{11} - \beta^4 D_{22}, \\ & K_{12} = k_1 A' \alpha^2 \left(\alpha^2 F_{11} + \beta^2 F_{12}\right) + k_2 B' \beta^2 \left(\alpha^2 F_{12} + \beta^2 F_{22}\right) \\ & + 2\alpha^2 \beta^2 (k_1 A' + k_2 B') F_{66}, \\ & K_{22} = -k_1 A' \alpha^2 \left(k_1 A' \alpha^2 H_{11} + k_2 B' \beta^2 H_{12}\right) \\ & - k_2 B' \beta^2 \left(k_1 A' \alpha^2 H_{12} + k_2 B' \beta^2 H_{22}\right) \\ & - \alpha^2 \beta^2 (k_1 A' + k_2 B')(k_1 A' + k_2 B') H_{66} \\ & - (k_2 B')^2 \beta^2 A_{44}^s - (k_1 A')^2 \alpha^2 A_{55}^s \\ & N_{11} = \gamma_1 \alpha^2 + \gamma_2 \beta^2 \end{aligned} \tag{21}$$

3. Numerical results and discussions

To prove the validity and efficiency of the proposed hyperbolic shear deformation plate theory applied for the buckling analysis of isotropic plates subjected to in-plane loading conditions, the results obtained for critical buckling load are compared and discussed with those obtained by the classical plate theory (CPT), FSDT of Mindlin (1951), HSDT of Reddy (1984) and the exponential shear deformation theory (ESDT) developed by Sayyad and Ghugal (2012). Since the exact elasticity solution for plate buckling analysis is not available in the literature. The description of various plate theories used in this study are listed in Table 1.

The following material properties are used to obtain the numerical results.

Material 1: steel plate

$$E = 210\,GPa, \quad \nu = 0.3 \tag{22}$$

Material 2: aluminum plate

$$E = 70\,GPa, \quad \nu = 0.33 \tag{23}$$

For convenience, the following non-dimensional critical buckling load is used in presenting the numerical results

Table 1 Displacement models

Model	Theory	Unknowns
CPT	Classical plate theory	3
FSDT	First-order shear deformation theory (Mindlin 1951)	5
HSDT	Higher-order shear deformation theory (Reddy 1984)	5
ESDT	Exponential shear deformation theory (Sayyad and Ghugal 2012)	3
Present	Hyperbolic shear deformation theory	2

Table 2 Comparison of non-dimensional critical buckling load $\overline{N}_{cr}$ of square plates subjected to uniaxial compression ($\gamma_1 = -1, \gamma_2 = 0$, material 1)

Mode	Theory	a/h				
		5	10	20	50	100
(1, 1)	Present (HySDT)	2.9512	3.4224	3.5649	3.6071	3.6132
	Sayyad and Ghugal (ESDT) [(a)]	2.9603	3.4242	3.5654	3.6072	3.6132
	Reddy (HSDT) [(a)]	2.9512	3.4224	3.5649	3.6068	3.6130
	Mindlin (FSDT) [(a)]	2.9498	3.4222	3.5649	3.6071	3.6130
	Kirchhoff (CPT)[(a)]	3.6152	3.6152	3.6152	3.6152	3.6152

[(a)] Results taken from reference Ghugal and Sayyad (2012)

Table 3 Comparison of non-dimensional critical buckling load $\overline{N}_{cr}$ of square plates subjected to biaxial compression ($\gamma_1 = -1, \gamma_2 = -1$, material 1)

Mode	Theory	a/h				
		5	10	20	50	100
(1, 1)	Present (HySDT)	1.4756	1.7112	1.7825	1.8035	1.8066
	Sayyad and Ghugal (ESDT) [(a)]	1.4802	1.7121	1.7827	1.8038	1.8065
	Reddy (HSDT) [(a)]	1.4756	1.7112	1.7825	1.8034	1.8065
	Mindlin (FSDT) [(a)]	1.4749	1.7111	1.7825	1.8035	1.8065
	Kirchhoff (CPT)[(a)]	1.8076	1.8076	1.8076	1.8076	1.8076

[(a)] Results taken from reference Ghugal and Sayyad (2012)

$$\overline{N}_{cr} = \frac{N_0 a^2}{E h^3} \tag{24}$$

In the first section, the results of the non-dimensional critical buckling load of simply supported steel plates subjected to the uniaxial and biaxial loading conditions for different values of side-to-thickness ratio (*a/h*=5, 10, 20, 50, 100) are given in Tables 2 to 4. The theoretical values of critical buckling load were also plotted in Figs. 3 through6according to the variation of ratio (*a/h*). In order to verify the accuracy of the present theory in predicting the mechanical buckling behaviour, another comparison is carried out for the aluminum plates subjected to in-plane loads, as presented in Tables 5 to 7. The mode for the plate considered in this analysis is (1, 1) when the plate is subjected to uniaxial or biaxial compressions (see Figs. 2(a), 2(b)), and (1, 2) when the plate is subjected to tension in *x*-direction and compression in *y*-direction (see Fig. 2(c)).

Table 4 Comparison of non-dimensional critical buckling load $\overline{N}_{cr}$ of square plates subjected to biaxial compression ($\gamma_1 = 1, \gamma_2 = -1$, material 1)

Mode	Theory	a/h				
		5	10	20	50	100
(1, 2)	Present (HySDT)	4.8272	6.6024	7.2754	7.4895	7.5212
	Sayyad and Ghugal (ESDT) [(a)]	4.8798	6.6133	7.2777	7.4898	7.5212
	Reddy (HSDT) [(a)]	4.8274	6.6024	7.2754	7.4893	7.5201
	Mindlin (FSDT) [(a)]	4.8158	6.6010	7.2753	7.4895	7.5211
	Kirchhoff (CPT)[(a)]	7.5317	7.5317	7.5317	7.5317	7.5317

[(a)] Results taken from reference Ghugal and Sayyad (2012)

Table 5 Comparison of non-dimensional critical buckling load $\overline{N}_{cr}$ of square plates subjected to uniaxial compression ($\gamma_1 = -1, \gamma_2 = 0$, material 2)

Mode	Theory	a/h				
		5	10	20	50	100
(1, 1)	Present (HySDT)	2.9893	3.4866	3.6383	3.6832	3.6897
	Sayyad and Ghugal (ESDT) [(a)]	2.9991	3.4886	3.6388	3.6833	3.6898
	Reddy (HSDT) [(a)]	2.9893	3.4866	3.6383	3.6833	3.6896
	Mindlin (FSDT) [(a)]	2.9877	3.4865	3.6383	3.6832	3.6900
	Kirchhoff (CPT)[(a)]	3.6919	3.6919	3.6919	3.6919	3.6919

[(a)]Results taken from reference Ghugal and Sayyad (2012)

Table 6 Comparison of non-dimensional critical buckling load $\overline{N}_{cr}$ of square plates subjected to biaxial compression ($\gamma_1 = -1, \gamma_2 = -1$, material 2)

Mode	Theory	a/h				
		5	10	20	50	100
(1, 1)	Present (HySDT)	1.4947	1.7433	1.8192	1.8416	1.8448
	Sayyad and Ghugal (ESDT) [(a)]	1.4995	1.7443	1.8194	1.8416	1.8449
	Reddy (HSDT) [(a)]	1.4947	1.7433	1.8192	1.8416	1.8448
	Mindlin (FSDT) [(a)]	1.4939	1.7433	1.8192	1.8415	1.8450
	Kirchhoff (CPT)[(a)]	1.8459	1.8459	1.8459	1.8459	1.8459

[(a)] Results taken from reference Ghugal and Sayyad (2012)

According to the analytical solutions provided in Tables 2 to 7, it can be observed that, the non-dimensional critical buckling load obtained by present theory (HySDT) and Reddy's theory (HSDT) is in good agreement with each other for all loading cases ranging from thick to thin plates, on the other hand, it should be noted that the numerical results of present theory are even better than those reported by Sayyad and Ghugal (2012) based on ESDT, especially for the case of very thick square plates with (*a/h*=5). From these

Table 7 Comparison of non-dimensional critical buckling load $\overline{N}_{cr}$ of square plates subjected to biaxial compression ($\gamma_1 = 1, \gamma_2 = -1$, material 2)

Mode	Theory	*a/h*				
		5	10	20	50	100
(1, 2)	Present (HySDT)	4.8521	6.7054	7.4184	7.6464	7.6802
	Sayyad and Ghugal (ESDT) [(a)]	4.9083	6.7172	7.4208	7.6468	7.6803
	Reddy (HSDT) [(a)]	4.8523	6.7055	7.4184	7.6465	7.6804
	Mindlin (FSDT) [(a)]	4.8398	6.7040	7.4183	7.6465	7.6810
	Kirchhoff (CPT)[(a)]	7.6915	7.6915	7.6915	7.6915	7.6915

[(a)] Results taken from reference Ghugal and Sayyad (2012)

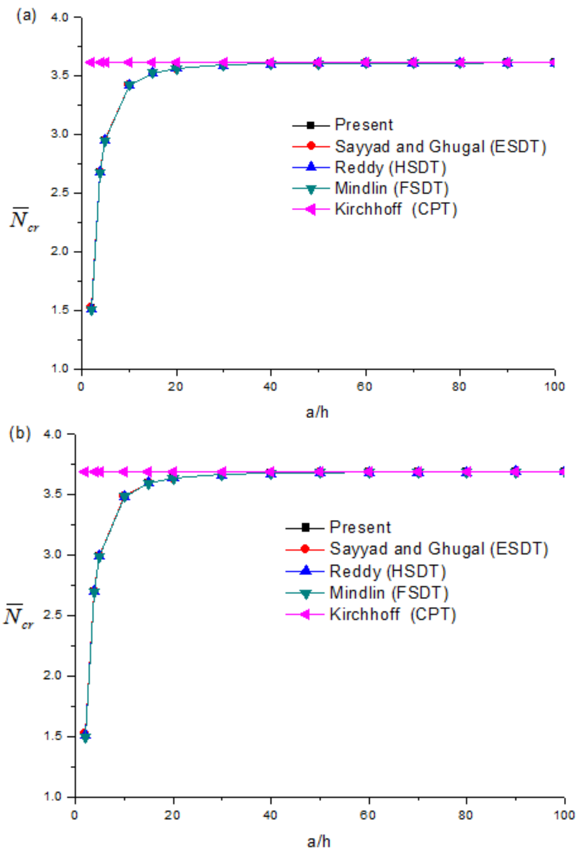

Fig. 3 The effect of side-to-thickness ratio on the critical buckling load of square plates subjected to uniaxial compression. (a) material 1 and (b) material 2

figures and tables the effect of the ratio (*a/h*) may be analyzed too. It can be seen that the value of critical buckling load obtained using various shear deformation plate theories (i.e., HySDT, ESDT, HSDT, and FSDT) is increased with increase in ratio (*a/h*), whereas the CPT overestimates the critical buckling loads for all side-to-thickness ratio due to neglect of the transverse shear deformation effect. However, the comparison of Tables 3 and 6 with Tables 4 and 7 shows that the critical buckling load for the plate under biaxial compression, is less than the corresponding values for the plate subjected to tension in *x*-direction and compression in *y*-direction and a good agreement has been achieved between HySDT and HSDT solutions.

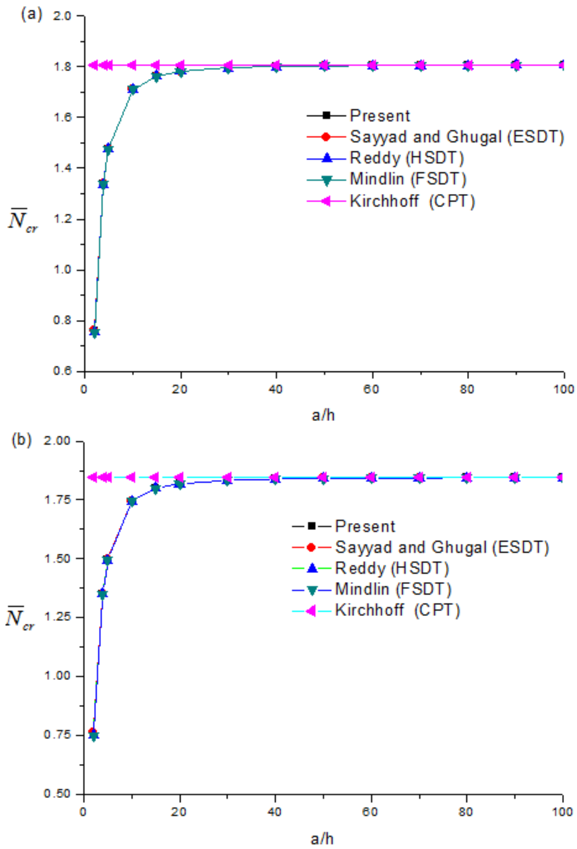

Fig. 4 The effect of side-to-thickness ratio on the critical buckling load of square plates subjected to biaxial compression. (a) material 1 and (b) material 2

It is evident from Figs. 3 through 5, that the results obtained by the different theories are more or less identical for the higher value of ratio (*a/h*).

4. Conclusions

In this work, the buckling behaviour of isotropic plates subjected to the uniaxial and biaxial loading conditions is studied based on the novel refined hyperbolic shear deformation theory, in which the displacement field contains a smaller number of unknowns with an undetermined integral term. The proposed theory satisfies the shear stress-free boundary conditions on the top and bottom surfaces of the plate, without using any shear correction factors. The governing differential equations and boundary conditions of simply supported square plates are derived by utilizing the principle of virtual work and solved using Navier's solution method. The numerical results of the critical buckling load for isotropic plates are verified by comparing them with various available results in the literature. Lastly, it can be said that the novel refined shear deformation theory with only two unknowns is not only more accurate but also simple than the conventional higher order shear deformation theory in predicting the buckling

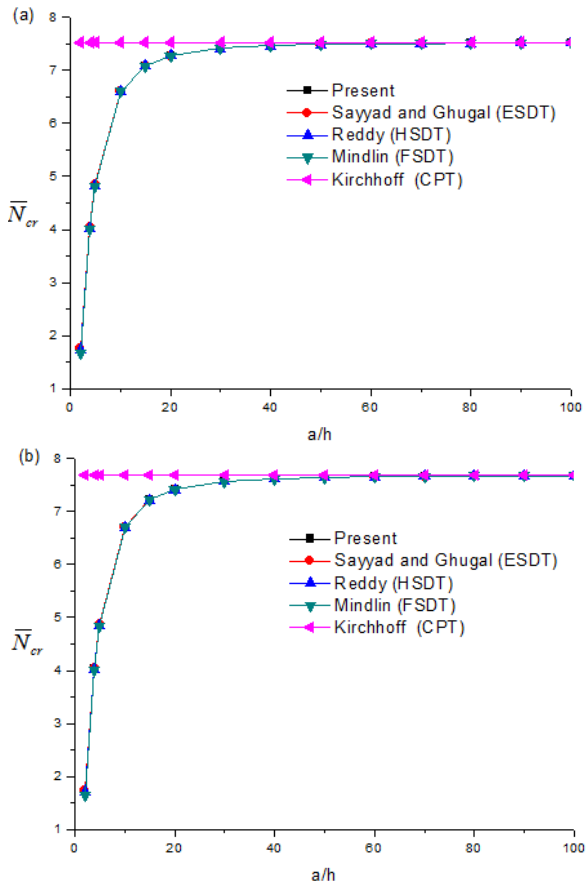

Fig. 5 The effect of side-to-thickness ratio on the critical buckling load of square plates subjected to tension in the *x*-direction and compression in the *y*-direction. (a) material 1 and (b) material 2

response of thick isotropic plates. Finally, the formulation lend sit self particularly well to study several problems related to the mechanical behaviour of concrete structures retrofitted with nano-fiber (Arani and Kolahchi 2016, Bilouei *et al.* 2016, Zamanian *et al.* 2017, Bakhadda *et al.* 2018), also by using various shear deformation theories with and without stretching effect to predict the static, mechanical buckling, thermal buckling and free vibration behavior of multilayered structures (Panda and Singh 2009, Panda and Singh 2010, Bousahla *et al.* 2014, Hebali *et al.* 2014, Belabed *et al.* 2014, Ait Yahia *et al.* 2015, Zemri *et al.* 2015, Bourada *et al.* 2015, Larbi Chaht *et al.* 2015, Hamidi *et al.* 2015, Bennoun *et al.* 2016, Katariya and Panda 2016, Draiche *et al.* 2016, Kar *et al.* 2016, Ahouel *et al.* 2016, Kar and Panda 2016b, Houari *et al.* 2016, Kar *et al.* 2017, Katariya *et al.* 2017b, Fahsi *et al.* 2017, Hachemi *et al.* 2017, Zidi *et al.* 2017, Abdelaziz *et al.* 2017, Sekkal *et al.* 2017a, b, Abualnour *et al.* 2018, Benchohra *et al.* 2018, Draiche *et al.* 2016, Fourn *et al.* 2018, Kaci *et al.* 2018, Karami *et al.* 2018a, b, Zaoui *et al.* 2018, Younsi *et al.* 2018, Karami *et al.* 2019), which will be considered in the near future. The present computations also provide a solid benchmark for verification of finite element and other numerical simulations of nanostructures (Khetir *et al.* 2017, Mokhtar *et al.* 2017, Mouffoki *et al.* 2017, Yazid *et al.* 2018, Youcef *et al.* 2017, Mehar *et al.* 2018, Cherif *et al.* 2018, Kadari *et al.* 2018, Mahmoudi *et al.* 2019).

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