Structural Engineering and Mechanics, *Vol. 69, No. 3 (2019) 257-268*
DOI: https://doi.org/10.12989/sem.2019.69.3.257

A new method for infill equivalent strut width

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(Received October 18, 2017, Revised September 10, 2018, Accepted September 12, 2018)

Abstract. Infills are as important members in structural design as beams, columns and braces. They have significant effect on structural behavior**.** Because of lots of variables in infills like material non-linear behavior, the interaction between frames and infill, etc., the infills performance during an earthquake is complicated, so have led designers do not consider the effect of infills in designing the structure. However, the experimental studies revealed that the infills have the remarkable effect on structure behavior. As if these effects ignored, it might occur soft-story phenomena, torsion or short-column effects on the structures. One simple and appropriate method for considering the infills effects in analyzing, is replacing the infills with diagonal compression strut with the same performance of real infill, instead of designing the whole infill. Because of too many uncertainties, codes and researchers gave many expressions that were not as the same as the others. The major intent of this paper is calculation the width of this diagonal strut, which has the most characteristics of infill. This paper by comprehensive on different parameters like the modulus of young or moment of inertia of columns presents a new formula for achieving the equivalent strut width. In fact, this new formula is extracted from about 60 FEM analyses models. It can be said that this formula is very efficient and accurate in estimating the equivalent strut width, considering the large number of effective parameters relative to similar relationships provided by other researchers. In most cases, the results are so close to the values obtained by the FEM**.** In this formula, the effect of out of plane buckling is neglected and this formula is used just in steel structures. Also, the thickness of infill panel, and the lateral force applied to frame are constant. In addition, this new formula is just for modeling the lateral stiffness. Obtaining the nearest response in analyzing is important to the designers, so this new formula can help them to reach more accurate response among a lot of experimental equations proposed by researchers.

Keywords: infill; equivalent strut width; Finite Element Method; macro mode

1. Introduction

Masonry infill walls as one of the important elements are common in the construction industry. Masonry infills have high strength and stiffness. In addition, they have strong positive and negative effects during an earthquake. The most important of negative effects are (Tabeshpour *et al.* 2012):

a) soft story failure
b) torsion
c) short column

These effects occur when the infill panel is not separated enough from the frame. Tabeshpour and Noorifard (2016) indicated the most appropriate method for calculating the story stiffness. In addition, Noorifard *et al.* (2015) have studied the effects of infill panels on torsion and soft story.

In the many national codes, the influence of infills are not considered in designing and frames are considered just as bare frames. Almost all researchers who have been introduce in this section disagree with this approach because they believed that stiffness and strength of infills could help the main structure's members and could reduce the size of the main structure's members. Infills have a high modulus of elasticity and shear stress, when infill is connected to the main structure directly, the most portion of lateral load is for the infill. Therefore, the frame has a little portion of lateral force and it should design for gravity load and a little lateral load. Therefore, the infills can reduce the member size if structure.

Modeling of infill walls because of lots of complex uncertainties, do not have a similar response. These uncertainties include the large number of possible failure modes, the interaction between mortar and masonry, the interaction between frame and infill, properties of masonry stuff etc. However, there are two general categories for modeling infill walls:

a) Micro modeling
b) Macro modeling.

The first category is based on Finite Element Method (FEM) in which common methods used in theory of elasticity and plasticity. Micro modeling is a complex method of analysis and it is always done by using finite element method. The benefits of using finite element approach is that, all possible modes of failure i.e., all the local effects are discussed in detail but its use is limited due to the greater computational effort and time-requirement.(Catherin and Jayalekshmi 2013).

There are two scenarios for micro modeling: first,

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http://www.techno-press.com/journals/sem&subpage=7
ISSN: 1225-4568 (Print), 1598-6217 (Online)

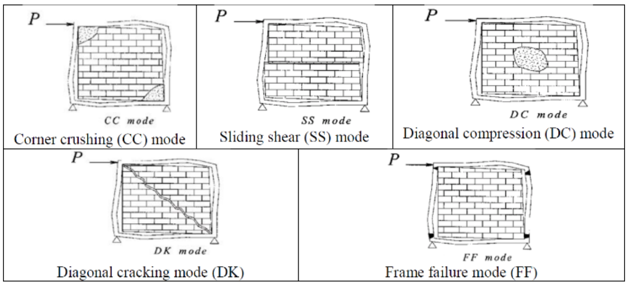


Fig. 1 Various modes of failure (EL-Dalkhani 2003)

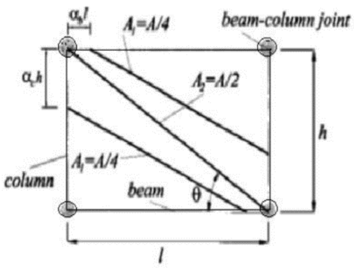


Fig. 2 Replace masonry infilled steel frame with three diagonal strut (El-Dakhakhni 2003)

modeling masonry infill with mortar and considering the effect of the mortar on the stiffness and strength. Second, assumed that the mortar strength is more than infill masonry, so the infill designed integrated.

In the second category instead of complex modeling of infill, it can be done simply with one or more structural elements. In other words, the masonry infill is replaced by an equivalent pin-jointed diagonal strut system. This simplification, because of shortening the design processes has always been accepted.

One of the best methods for macro modeling is modeling of diagonal strut instead of infill that called equivalent strut. In the early 1960s, Polyakov (1960) as reported by Klinger and Bertero (1976) and Mallick and Severn (1967), conducted one of the first analytical studies based on elastic theory. Polyakov proposed this method for each panel as an equivalent diagonal bracing. Holmes (1961) subsequently adopted this suggestion, and he was the first in modeling infill by using one pin-jointed diagonal strut and equation for the equivalent strut. He assumed that the width of the diagonal strut is third of the diagonal length

$$a = \frac{d}{3} \tag{1}$$

One year after Holmes, Stafford Smith (1962) relying on test done on steel frames, he found that the ratio of a/d_{inf} varied from 0.1 to 0.25, so he proposed the following equation. Paulay *et al.* (1992) offered a conservative value useful for designing similar to the Smith equation. They believed that a high value of equivalent strut width will result in stiff structures, and so it might have potentially higher seismic response

$$a = 0.25d \tag{2}$$

Stafford Smith and Carter (1969) saw that an equivalent strut is too simple and the width of strut should be modified, so they proposed a theoretical relation for the width of the diagonal strut based on the relative stiffness of infill and frame. They proposed a parameter expressing the relative stiffness of the infill to the frame, named λ_h

$$\lambda_h = \sqrt[4]{\frac{E_{inf} t_{inf} \sin 2\theta}{4 E_c I_c h_{inf}}} \tag{3}$$

$$a = 0.58\left(\frac{1}{h_{inf}}\right)^{-0.445} \left(\lambda_h h\right)^{0.335 d_{inf} \left(\frac{1}{h_{inf}}\right)^{0.064}} \tag{4}$$

Mainstone (1971) proposed an equation for determining the equivalent strut width based on experimental works

$$a = 0.16\left(\lambda_h h\right)^{-0.3} d \tag{5}$$

Hendry (1981) believed that the equivalent strut width is half the width proposed by Smith (1962) (Eq. (2)).

Te-Chang and Kwok-Hung (1984) adopted values for θ between 25° and 50° (typical for practical engineering purposes). They proposed a semi-empirical expression for calculating the equivalent strut width

$$\frac{a}{d} = \frac{0.95 \sin 2\theta}{2\sqrt{\lambda_h h}} \tag{6}$$

Mainstone and Weeks (1972) and Mainstone (1974) both on the basis of experimental and analytical data produce an empirical equation for the calculation of equivalent strut width, given by

$$\frac{a}{d} = 0.175\left(\lambda_h h\right)^{-0.4} \tag{7}$$

Decanini and Fantin (1987) proposed two set of equations considering cracked panel and uncracked panel

$$a = \left(\frac{0.748}{\lambda_h h} + 0.085\right) d \quad \text{if} \quad \lambda_h \le 7.85 \tag{8a}$$

Uncracked Panel

$$a = \left(\frac{0.393}{\lambda_h h} + 0.130\right) d \quad \text{if} \quad \lambda_h > 7.85$$

(8a)

$$a = \left(\frac{0.707}{\lambda_h h} + 0.010\right) d \quad \text{if} \quad \lambda_h \le 7.85$$

Cracked Panel

$$a = \left(\frac{0.470}{\lambda_h h} + 0.040\right) d \quad \text{if} \quad \lambda_h > 7.85 \tag{8b}$$

Bazan and Meli (1980), based on FEM analysis for one-bay and one-story infilled frame, proposed an empirical expression to calculate the equivalent strut width

$$a = (0.35 + 0.22\beta)h \tag{9}$$

$$\beta = \frac{E_c A_c}{G_{\inf} A_{\inf}} \tag{10}$$

They produced a diagram predicting the width of the equivalent strut for the case of failure on the diagonal of infill panel. Tassios (1984) proposed a simple representation of the results of this diagram, given by

$$\frac{a}{d} \cong 0.20 \sin\theta \sqrt{\frac{E_c A_c}{G_{\inf} A_{\inf}}} \quad \text{if } 1 < \frac{E_c A_c}{G_{\inf} A_{\inf}} < 5 \tag{11}$$

Durrani and Lou (1994), on the basis of empirical fitting of FEM analysis results and comparison with other models, produced the empirical expression for calculating the equivalent strut

$$\frac{a}{d} = \gamma \sin 2\theta \tag{12}$$

$$\gamma = 0.32\sqrt{\sin 2\theta}\left(\frac{h^4 E_{\inf} t_{\inf}}{m E_c I_c h_{\inf}}\right)^{-0.1} \tag{13}$$

$$m = 6\left(1 + \frac{6 E_b I_b h}{\pi E_c I_c L}\right) \tag{14}$$

This formula was adopted by Perera (2005).

There are some extensive researches on modeling and testing infilled frames too, like the reports of Cavaleri *et al.* (2005), they determined the ideal cross-section of equivalent strut under the cyclic lateral load, and Madan *et al.* (1997), based on the macro models investigated the frames with diagonal strut with hysteretic force-deformation rule. In addition, there are more researches to show the strong interaction between an infill panel and an infilled frame like the research from Žarnić *et al.* (2001), they experiment a two buildings which are scaled on the shaking table. They believed in the model responses showed that buildings designed according to Eurocodes are able to sustain relatively high dynamic excitations due to significant level of structural overstrength.

A numerical parametric study is presented in this study so as to addresses issues that not covered by codes and existing literature. In addition, the new empirical formula is proposed to estimate the strut width that easier and more comprehensive by considering more parameters which are effective on frames and infills instead of limited parameters which used in codes and formulas from other researchers.

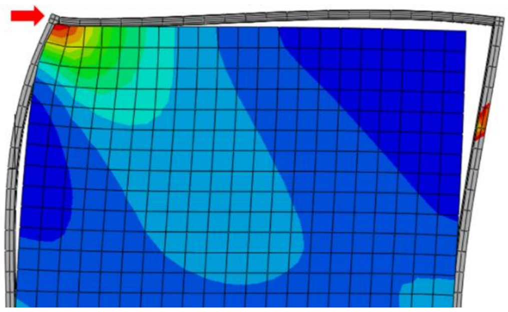
2. Modelling of masonry infill

2.1 General

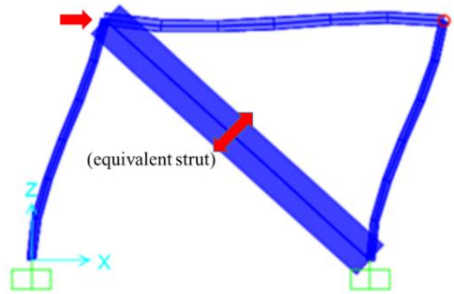
Finite element method (FEM) is used to determine the effect of the various situation of infills on frames stiffness and displacement (Micro Modelling). FEM is one of the most important methods of discrete analysis and has been found suitable for solution of problems. The benefits of using finite element approach are that all possible modes of failure i.e., all the local effects are discussed in detail but it's limited due to the greater computational effort and time requirement. In addition, this method is complex too, so it is not appropriate for using in general for every structure. In this concept assumed that infill and frame were connected together, and failure modes are not considered, also the masonry infill will separate from the frame when the lateral force is applied. In addition, since the strength of mortar, which is used between masonry elements (bricks), is more than masonry stuff, so it is assumed that the infill is integrated. These FEM analyses are conducted in Abaqus software and the element types are all shell. For FEM analyzing the elements should be divide into smaller pieces known as mesh. In these modeling mesh types is quad and structured.

In this process after analyzing the various infilled frames, efforts have been made the stress path on the infill, which is shown in stress contours in Fig. 3, modeled by a strut. The width of strut is the same by the width on a stress contours when about 80% of total stress covered by that width. The strut thickness is the same with infill thickness and the width of strut is equal to the width of stress path on contours. Actually, the stiffness of equivalent strut should be same as the stiffness of infill.

After analyzing infilled frames by FEM, the purpose of this paper is to propose a new simple formula according to FEM analyses for determining the more accurate width of equivalent strut. For this purpose, totally about 60 models were analyzed with different properties.



(a) Stress path on stress contours in FEM analyzing



(b) Simulation of stress path with equivalent strut

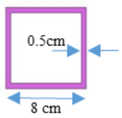
Fig. 3 Comparison between FEM analyzing and simplifying by equivalent strut

(a) 300×300 cm

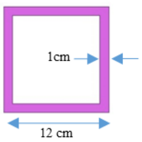
(b) 300×415 cm

(c) 300×540 cm

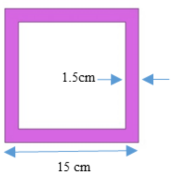
Fig. 4 Three frames are considered for this research



(a) Box 8×0.5



(b) Box 12×1



(c) Box 15×1.5

Fig. 5 Columns section

2.2 Development of the model

In this research, three steel frame with integrated infill panel by dimensions of 300×300 cm, 300×415 cm and 300×540 cm are considered.

The section of column's frame is box and beam's frame is I-shaped profile. The interaction between infill and frame is defined by contact element and the friction coefficient is 0.7, so the infill panel and frame can slip together easily, and infill panel in tension stresses will detach from the frame. The base connection of frame is continuous, and the infill panel is pinned.

For studying the stiffness effect, some parameters for simplifying the calculation were considered constant like the lateral force and the thickness of infill. In addition, in this research, the effect of out-of-plane buckling is neglected, because the load in these tests are applied just in direction of the infill length. The lateral load, which is applied to the column of these frames; in the center of the connection with the beam is 1500 kg and this load is just in one direction without any moving back and forth. The thickness of infill panel is considered 15 cm.

Another important parameter for reaching various cases of study is column. Three columns by dimensions of Box 8×0.5 cm, Box 12×1 cm and Box 15×1.5 cm are considered (Fig. 5), also another parameter is considered as the ratio of the plastic moment of column ($M_{p\text{-}c}$) to the plastic moment of beam ($M_{p\text{-}b}$). Three values of 0.8, 1.25 and 2 are proposed for this ratio and can get the beams section from these values.

Table 1 Defining the beams section from α_m

Column Sections (cm)	$\alpha_m = \frac{M_{p-c}}{M_{p-b}}$	Beam Section
Box 8×0.5	0.8	IPE 120
	1.25	IPE 100
	2	IPE 80
Box 12×1	0.8	IPE 200
	1.25	IPE 180
	2	IPE 140
Box 15×1.5	0.8	IPE 270
	1.25	IPE 240
	2	IPE 200

In this paper, the section of beams is selected from dividing $M_{p\text{-}c}$ to α_m. For example, a beam section when α_m=0.8 and the column is Box 8×0.5 is obtained from below

$$M_{p-c} = \frac{bh^2}{4} - (b-2t)\left(\frac{h}{2}-t\right)^2 = 42.25 \qquad \text{where:}$$

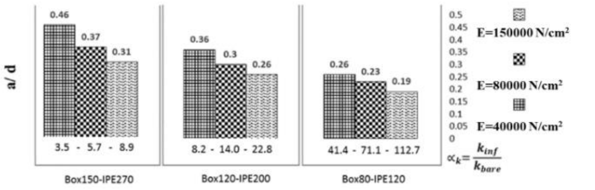
b=width, h=height, t= thickness

$$\alpha_m = \frac{M_{p-c}}{M_{p-b}} \rightarrow M_{p-b} = \frac{M_{p-c}}{\alpha_m} = \frac{42.25}{0.8} \rightarrow M_{p-b} = 52.8$$

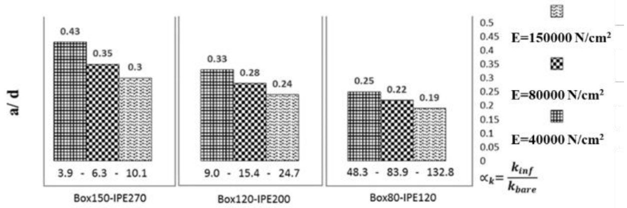
The nearest standard beam section for this $M_{p\text{-}b}$ is IPE 120. The remaining beam sections are selected like example respectively.

The last parameter, which is used in the new formula of the equivalent strut, is Young's modulus. There are a lot of infill masonry with a various modulus of elasticity, some of them are strong like concrete infills and some of them are weak like some light materials, but all walls, which researchers considered as infills, have Young's modulus of above 40000 N/cm^2. Hence, in this research three types of infill with the modulus of elasticity of 40000 N/cm^2, 80000 N/cm^2 and 150000 N/cm^2 have been considered.

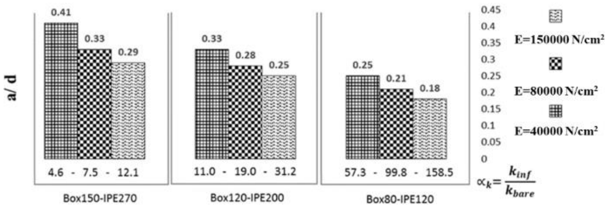
In summary, the parameters, which are used in various types in this research for obtaining the formula of the width of equivalent strut, are:



(a) α_m=0.8, L=415 cm



(b) α_m=1.25, L=415 cm



(c) α_m=2, L=415 cm

Fig. 6 the effect of α_k on a/d when α_m and β are constant

1- Moment inertia of column (I_c)
2- Moment inertia of beam (I_b)
3- The ratio of infill young modulus to column young modulus (α_E=E_{inf}/E_c)
4- The ratio of infilled frame stiffness to frame stiffness (α_k=$k_{\text{infilled}}/k_{frame}$)
5- Ratio of Length of frame to height of frame (β=L/h)
6- Ratio of plastic moment of column to plastic of moment beam (α_m=$M_{p\text{-}c}/M_{p\text{-}b}$)
7- Ratio of height of infill panel to thickness of infill panel (α_s=$h_{\text{inf}}/t_{\text{inf}}$)
8- Modulus of infill elasticity (E_{inf})

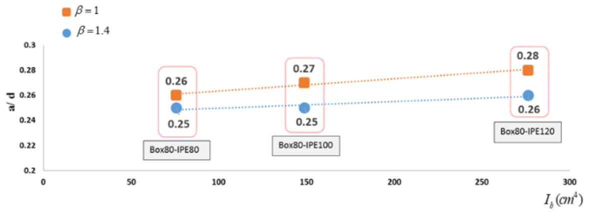
3. Defining the equivalent strut

The most important parameter in defining the equivalent strut is strut width (a). Researchers provide some recommendations for strut width, which depends on parameters like λ_k mentioned in the introduction and the ratio of the equivalent strut width to diameter (a/d). By changing one parameter, which mentioned in development section and keeping other parameters constant, the effect of parameters on the equivalent strut width is determined.

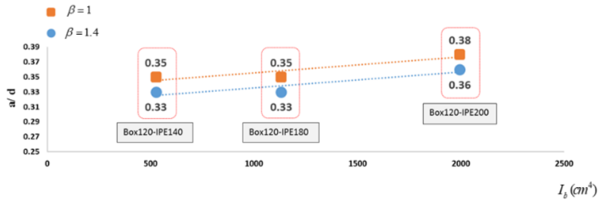
3.1 Effect of α_k on equivalent strut

By using FEM analyses, it is clear in Fig. 6 that by increasing α_k, the value of (a/d) decrease.

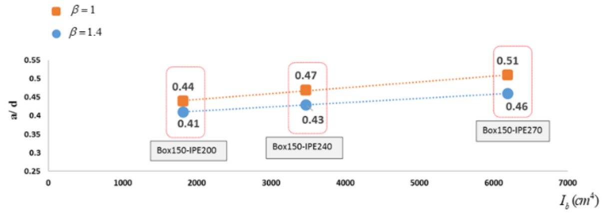
As the graphs shows, by increasing α_k the value of (a/d) decreased, also by increasing elastic modulus, the α_k value decreased. In addition, from the Fig. 6, the effect of α_m to α_k



(a) α_E=1.9 E−03, Box8×0.5



(b) α_E=1.9 E−03, Box12×1



(c) α_E=1.9 E−03, Box15×1.5

Fig. 7 the effect of I_b on a/d when α_E and column's section are constant

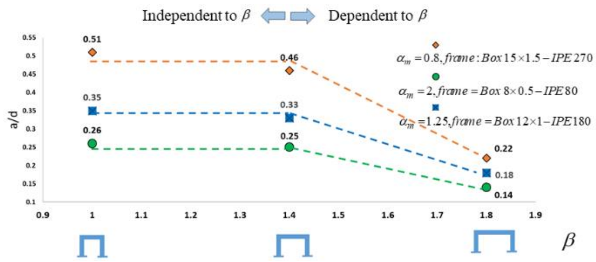


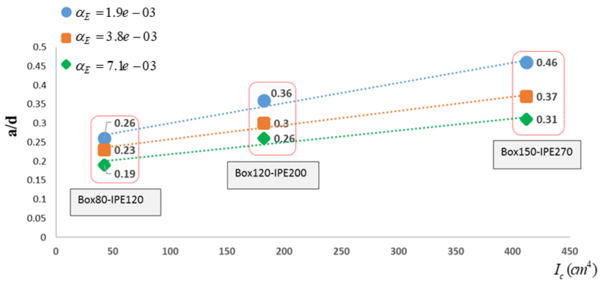
Fig. 8 the effect of β on a/d

change is negligible. However, in low level of α_k, this effect is noticeable.

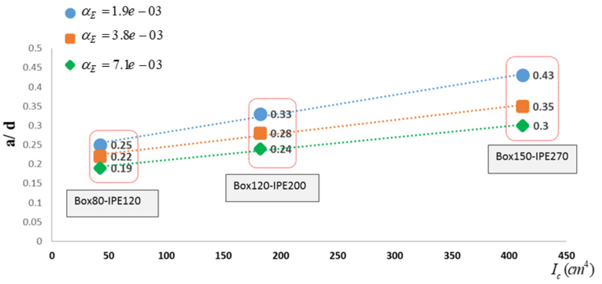
According to the materials which are used as infills, and theory of structures, it can be said that, approximately the ratio of α_k does not exceed more than 30 in the real world. It means that the results by values of α_k more than 30 in Fig. 6 are not practical, like frame with box 80 mm for columns and IPE 120 for beam in Fig. 6(c). The range of α_k for this frame is about 57 to 160, and that is not practical.

3.2 Effect of I_b on equivalent strut

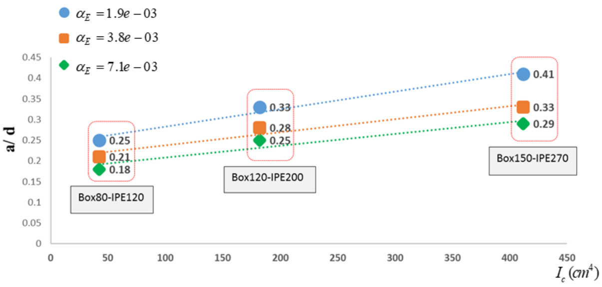
The next parameter is the moment inertia of frame's beam (I_b). According to the figures achieved in the following and from FEM analyses, it can be understood that by increasing I_b (in another word by increasing the beam section), the a/d ratio very slightly increased, so it can be neglected the I_b effect and did not include it in the new formula as an effective character.



(a) β=1.4, α_m=0.8



(b) β=1.4, α_m=1.25



(c) β=1.4, α_m=2

Fig. 9 the effect of I_c on a/d when α_m and β are constant

3.3 Effect of β on equivalent strut

β in this research means the ratio of length of the infill panel to the height of infill panel (L/h). It is obvious that by increasing the length of infill panel, the stiffness of infilled frame increases too. However, the expected width of equivalent strut decreased. Accordingly, the FEM analyses showed this fact not far-fetched and according to the Fig. 8, it is understood that by changing β from 1.4 to 1.8, the a/d value decreased significantly.

By increasing the infill length to specified value (1<β<1.4), almost can be said that the a/d value will not change much.

3.4 Effect of I_c on equivalent strut

Another parameter names I_c seems to be important and effective in determining the equivalent strut width as the FEMA used this parameter in equivalent strut formula (FEMA 2000).

As shown from Fig. 9 and from the FEM analyses, by increasing the I_c, the value of a/d increased too much. It can be said by increasing I_c, the stiffness of frame increased, and by increasing the stiffness of frame the α_k decreased and from the Fig. 6, it is understood that, by decreasing the α_k, the a/d increase.

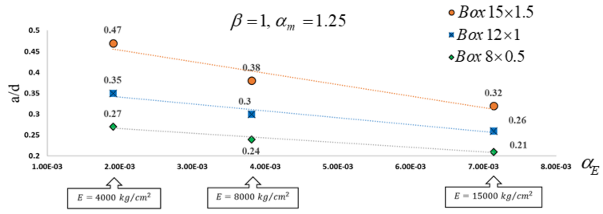
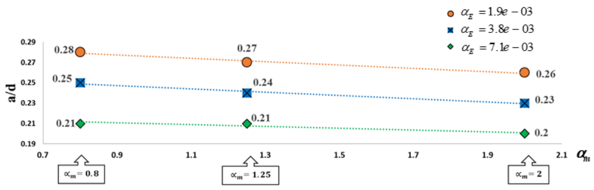
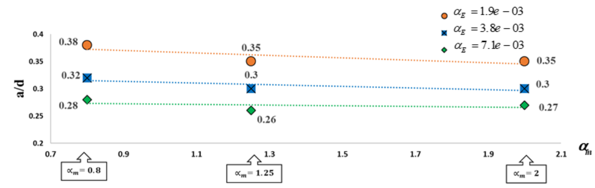


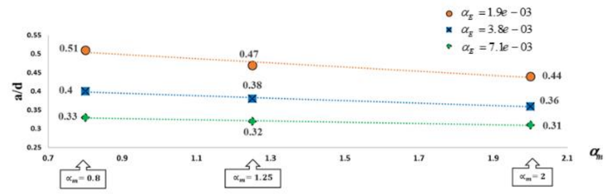
Fig. 10 the effect of α_E on $\dfrac{a}{d}$ when α_m and β are constant



(a) β=1, $Box8{\times}0.5$



(b) β=1, $Box12{\times}1$



(c) β=1, $Box15{\times}1.5$

Fig. 11 the effect of α_m on a/d when column's section and β are constant

3.5 Effect of α_E on equivalent strut

The modulus of elasticity has a significant impact in determining the equivalent strut width. By increasing the E_{inf}, the stiffness of infill panel increased and the a/d value decreased. In addition, according to the Fig. 10, by analyzing three different value of E_{inf} which are 4000 kg/cm^2, 8000 kg/cm^2 and 15000 kg/cm^2. As Fig. 10 shows, the more the modulus of elasticity, the less the a/d ratio.

3.6 Effect of α_m on equivalent strut

The ratio of the plastic moment of the column to the plastic moment of the beam could be important, and it can put the effect on the width of equivalent strut. This ratio can change the stress path, hence can change the equivalent strut path, for example from beam-to-beam into column-to-column.

For determining the effect of α_m on equivalent strut width, three types of α_m were considered. In Fig. 11, it is

Table 2 The results of models in FEM analysis

#	Frame		α_m	αE	α_k	$\beta = \frac{L}{h}$	αs	Infilled Frame disp. (cm)	bare frame disp. (cm)	Infilled Panel Stiffness (N/cm)	Actual a /d	New Formula	Deviation
	Column	Beam											
1	Box150	IPE270	0.8	1.90E-03	3	1	22.8	0.1117	0.3248	134288	0.51	0.47	0.036
2	Box150	IPE240	1.25	1.90E-03	3	1	23.0	0.1239	0.3972	121065	0.47	0.49	0.022
3	Box150	IPE200	2	1.90E-03	4	1	23.3	0.1314	0.4705	114155	0.44	0.49	0.046
4	Box150	IPE270	0.8	3.81E-03	5	1	22.8	0.0711	0.3248	210970	0.4	0.38	0.024
5	Box150	IPE240	1.25	3.81E-03	5	1	23.0	0.0779	0.3972	192555	0.38	0.39	0.012
6	Box150	IPE200	2	3.81E-03	6	1	23.3	0.0811	0.4705	184957	0.36	0.39	0.026
7	Box120	IPE200	0.8	1.90E-03	6	1	23.3	0.1545	0.9981	97087	0.38	0.38	0.002
8	Box120	IPE180	1.25	1.90E-03	7	1	23.5	0.1646	1.1759	91130	0.35	0.38	0.026
9	Box150	IPE270	0.8	7.14E-03	7	1	22.8	0.0451	0.3248	332594	0.33	0.30	0.027
10	Box150	IPE240	1.25	7.14E-03	8	1	23.0	0.0491	0.3972	305499	0.32	0.32	0.003
11	Box120	IPE140	2	1.90E-03	8	1	23.8	0.1713	1.4368	87566	0.35	0.46	0.105
12	Box150	IPE200	2	7.14E-03	9	1	23.3	0.0504	0.4705	297619	0.31	0.31	0.001
13	Box120	IPE200	0.8	3.81E-03	11	1	23.3	0.091	0.9981	164835	0.32	0.30	0.021
14	Box120	IPE180	1.25	3.81E-03	12	1	23.5	0.0968	1.1759	154959	0.3	0.30	0.003
15	Box120	IPE140	2	3.81E-03	14	1	23.8	0.0995	1.4368	150754	0.3	0.38	0.077
16	Box120	IPE200	0.8	7.14E-03	18	1	23.3	0.0556	0.9981	269784	0.28	0.24	0.039
17	Box120	IPE180	1.25	7.14E-03	20	1	23.5	0.0592	1.1759	253378	0.26	0.24	0.021
18	Box120	IPE140	2	7.14E-03	24	1	23.8	0.0601	1.4368	249584	0.27	0.31	0.045
19	Box80	IPE120	0.8	1.90E-03	32	1	24.0	0.2146	6.9319	69897	0.28	0.31	0.027
20	Box80	IPE100	1.25	1.90E-03	37	1	24.2	0.2225	8.1689	67416	0.27	0.31	0.035
21	Box80	IPE80	2	1.90E-03	43	1	24.3	0.2278	9.8862	65847	0.26	0.30	0.043
22	Box80	IPE120	0.8	3.81E-03	56	1	24.0	0.1229	6.9319	122050	0.25	0.25	0
23	Box80	IPE100	1.25	3.81E-03	64	1	24.2	0.1271	8.1689	118017	0.24	0.25	0.008
24	Box80	IPE80	2	3.81E-03	76	1	24.3	0.1297	9.8862	115652	0.23	0.24	0.015
25	Box80	IPE120	0.8	7.14E-03	91	1	24.0	0.0761	6.9319	197109	0.21	0.21	0.005
26	Box80	IPE100	1.25	7.14E-03	107	1	24.2	0.0763	8.1689	196592	0.21	0.20	0.007
27	Box80	IPE80	2	7.14E-03	124	1	24.3	0.0797	9.8862	188206	0.2	0.20	0
28	Box150	IPE270	0.8	1.90E-03	4	1.4	22.8	0.096	0.34	156250	0.46	0.48	0.02
29	Box150	IPE240	1.25	1.90E-03	4	1.4	23.0	0.107	0.413	140187	0.43	0.47	0.04
30	Box150	IPE200	2	1.90E-03	5	1.4	23.3	0.112	0.51	133929	0.41	0.46	0.053
31	Box150	IPE270	0.8	3.81E-03	6	1.4	22.8	0.06	0.34	250000	0.37	0.38	0.011
32	Box150	IPE240	1.25	3.81E-03	6	1.4	23.0	0.066	0.413	227273	0.35	0.37	0.022
33	Box150	IPE200	2	3.81E-03	8	1.4	23.3	0.068	0.51	220588	0.33	0.37	0.036
34	Box120	IPE200	0.8	1.90E-03	8	1.4	23.3	0.128	1.051	117188	0.36	0.36	0
35	Box150	IPE270	0.8	7.14E-03	9	1.4	22.8	0.038	0.34	394737	0.31	0.31	0.003
36	Box120	IPE180	1.25	1.90E-03	9	1.4	23.5	0.137	1.233	109489	0.33	0.36	0.027
37	Box150	IPE240	1.25	7.14E-03	10	1.4	23.0	0.041	0.413	365854	0.3	0.30	0
38	Box120	IPE140	2	1.90E-03	11	1.4	23.8	0.142	1.56	105634	0.33	0.39	0.059
39	Box150	IPE200	2	7.14E-03	12	1.4	23.3	0.042	0.51	357143	0.29	0.29	0.005
40	Box120	IPE200	0.8	3.81E-03	14	1.4	23.3	0.075	1.051	200000	0.3	0.28	0.017
41	Box120	IPE180	1.25	3.81E-03	15	1.4	23.5	0.08	1.233	187500	0.28	0.28	0.001
42	Box120	IPE140	2	3.81E-03	19	1.4	23.8	0.082	1.56	182927	0.28	0.31	0.028
43	Box120	IPE200	0.8	7.14E-03	23	1.4	23.3	0.046	1.051	326087	0.26	0.23	0.032
44	Box120	IPE180	1.25	7.14E-03	25	1.4	23.5	0.05	1.233	300000	0.24	0.23	0.014
45	Box120	IPE140	2	7.14E-03	31	1.4	23.8	0.05	1.56	300000	0.25	0.25	0.002

Table 2 Continued

46	Box80	IPE120	0.8	1.90E-03	41	1.4	24.0	0.177	7.323	84746	0.26	0.24	0.016
47	Box80	IPE100	1.25	1.90E-03	48	1.4	24.2	0.184	8.896	81522	0.25	0.24	0.01
48	Box80	IPE80	2	1.90E-03	57	1.4	24.3	0.188	10.775	79787	0.25	0.24	0.014
49	Box80	IPE120	0.8	3.81E-03	71	1.4	24.0	0.103	7.323	145631	0.23	0.19	0.038
50	Box80	IPE100	1.25	3.81E-03	84	1.4	24.2	0.106	8.896	141509	0.22	0.19	0.031
51	Box80	IPE80	2	3.81E-03	100	1.4	24.3	0.108	10.775	138889	0.21	0.19	0.024
52	Box80	IPE120	0.8	7.14E-03	113	1.4	24.0	0.065	7.323	230769	0.19	0.15	0.035
53	Box80	IPE100	1.25	7.14E-03	133	1.4	24.2	0.067	8.896	223881	0.19	0.15	0.038
54	Box80	IPE80	2	7.14E-03	158	1.4	24.3	0.068	10.775	220588	0.18	0.15	0.031
55	Box80	IPE80	2	1.90E-03	56	1.8	24.3	0.1752	9.8862	85616	0.14	0.13	0.015
56	Box120	IPE180	1.25	1.90E-03	9	1.8	23.5	0.1269	1.1759	118203	0.18	0.21	0.029
57	Box150	IPE270	0.8	1.90E-03	4	1.8	22.8	0.0905	0.3248	165746	0.22	0.29	0.066

seen that the slope of the graph is too low and this means that the effect of this parameter on equivalent strut width is negligible. In addition, the calculation of α_m for achieving the equivalent strut is time-consuming and complex, so α_m is not recommended for taking account into new formula.

3.7 Defining the new formula

According to the effect of parameters, which are listed above, on the equivalent strut width, it is obvious that parameters like α_k, α_E, I_c and β are important, effective and should be considered in the new formula. Hence, parameters like I_b and α_m due to the little effect on the equivalent strut width were not considered in new formula. In addition, in this paper, the effect of infill panel thickness ($t_{\inf}$) on equivalent strut width unmentioned, and the thickness of infill panel for all models were constant and is about 15 cm.

The proposed equivalent strut width (a) formula for Steel frames, which was obtained from the multiple regression analyses is as follows

$$\frac{a}{d} = 3.58\left(\frac{\alpha_s I_c}{\alpha_E}\right)^{0.25} \frac{\alpha_k^{-0.1}\alpha_G}{h} \tag{15}$$

where I_c is in cm^4, h is in cm and α_G is derived from Eq. (17) in below

$$\alpha_G = \begin{cases} For \quad \beta \le 1.4 \quad , \quad \alpha_G = 1 \\ \\ For \quad \beta > 1.4 \quad , \quad \alpha_G = 1.18\beta - 1.6 \end{cases} \tag{16}$$

This new formula in compared with other equations, which recommended by several codes and other researchers is more accurate and simple to calculate the equivalent width for considering the infill panel in structural design.

3.8 How to use the new formula

For calculating this new formula, first, the amount of a/d is assumed about 0.05. Second, the α_k value is calculated. By putting this value in the new equation, the new amount of a/d will be achieved, and this new amount of a/d is similar to the actual a/d, which is derived from FEM analyses with an insignificant deviation.

$$\text{Assumption} \quad \left(\frac{a}{d}\right)_1 = 0.05 \tag{17}$$

For calculating α_k we should first calculate k_{inf} and k_f by using these below equations from theory of structures

$$k_{inf} = \left(\frac{a}{d}\right)_1 t_{inf} E_{inf} \cos^2\theta \rightarrow k_{inf} = 0.05 t_{inf} E_{inf} \cos^2\theta \tag{18}$$

$$k_f = \frac{6E_c I_c}{h}\left(\frac{4\left(1+\frac{LI_c}{6hI_b}\right)}{4\left(1+\frac{LI_c}{6hI_b}\right)-3}\right) \tag{19}$$

$$\alpha_k = \frac{k_{infilled}}{k_{frame}} = \frac{k_{inf} + k_f}{k_f} \tag{20}$$

This new amount of α_k is a parameter of the new formula, so the new amount of a/d is calculated. This new a/d is more exact than all other formulas presented up to now.

4. Numerical validation of the new formula

In this section, about 57 models with various properties are built in FEM software (Abaqus) to demonstrate the validation of the new formula. These models are different with each other in some characteristics like length of beam, elastic modulus and etc., which are mentioned in section 3. As the table shows, the new formula is so close to the FEM analyses, and the average of deviation and standard deviation is about 0.024 and 0.031 respectively. This standard deviation presents that the results of this new formula is the same with numerical analyses which

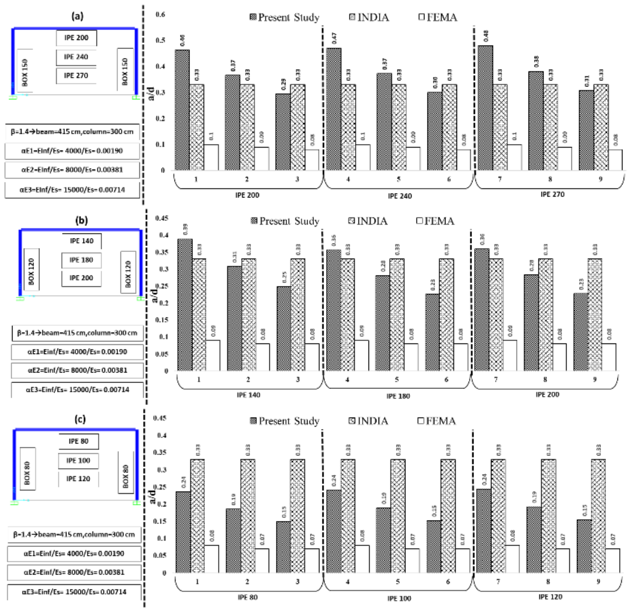


Fig. 13 Comparative of new formula result with FEMA and India by 27 models and β=1.4

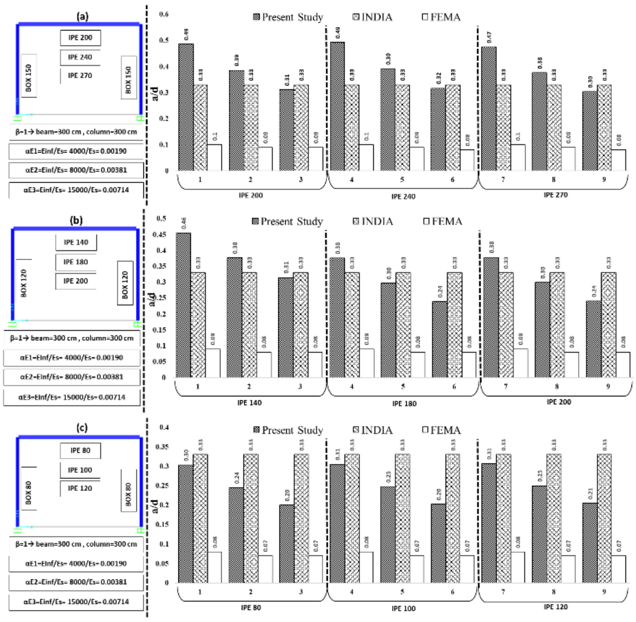


Fig. 12 Comparative of new formula result with FEMA and India by 27 models and β=1

calculated by FEM.

5. Comparative study of the new formula with the equations in FEMA and Indian code

For calculating the equivalent strut width, there are many equations in several codes, like FEMA, India and New Zealand, etc. In Indian code (Standard 1893), the amount of equivalent strut width (a) is one-third of the diagonal length of infill panel that is relatively a high value. This equation first offered by Holmes in 1961 (Holmes 1961) (Eq. (1)), and it has many defects without considering

Table 3 The properties of all conditions, which used in Figs. 12 and 13

#num	β	Einf (N/cm^2)	Beam Sec.	Col Sec.	Graph name		#num	β	Einf (N/cm^2)	Beam Sec.	Col Sec.	Graph name	
1	1	40000					28	1.4	40000				
2	1	80000	IPE200				29	1.4	80000	IPE200			
3	1	150000					30	1.4	150000				
4	1	40000					31	1.4	40000				
5	1	80000	IPE240	box150	Graph a		32	1.4	80000	IPE240	box150	Graph a	
6	1	150000					33	1.4	150000				
7	1	40000					34	1.4	40000				
8	1	80000	IPE270				35	1.4	80000	IPE270			
9	1	150000					36	1.4	150000				
10	1	40000					37	1.4	40000				
11	1	80000	IPE140				38	1.4	80000	IPE140			
12	1	150000					39	1.4	150000				
13	1	40000					40	1.4	40000				
14	1	80000	IPE180	box120	Graph b	Fig. 10	41	1.4	80000	IPE180	box120	Graph b	Fig. 11
15	1	150000					42	1.4	150000				
16	1	40000					43	1.4	40000				
17	1	80000	IPE200				44	1.4	80000	IPE200			
18	1	150000					45	1.4	150000				
19	1	40000					46	1.4	40000				
2	1	80000	IPE80				47	1.4	80000	IPE80			
21	1	150000					48	1.4	150000				
22	1	40000					49	1.4	40000				
23	1	80000	IPE100	box80	Graph c		50	1.4	80000	IPE100	box80	Graph c	
24	1	150000					51	1.4	150000				
25	1	40000					52	1.4	40000				
26	1	80000	IPE120				53	1.4	80000	IPE120			
27	1	150000					54	1.4	150000				

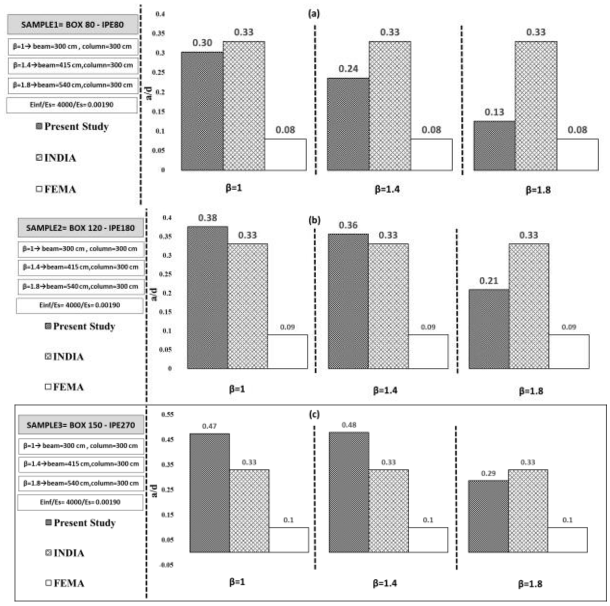


Fig. 14 Comparing new formula results, FEMA, India and FEM results with different β

any important parameters. The results of this equation, sometimes are more than real.

A relatively complex formula has been presented in FEMA (FEMA 2000) for infill stiffness effect containing many parameters. The results of this equation are low values and FEMA gives lower bound of the equivalent strut

$$a = 0.175\left(\lambda_h h\right)^{-0.4} d \qquad (21)$$

Some researchers provided equations to achieve the equivalent strut width considering certain assumption. In this part, the results of the new formula are compared with the results of FEMA and Indian code.

In Fig. 12, the amount of β for all three graphs is equal to 1 (h=L=300 cm) and in Fig. 13, the amount of β is equal to 1.4 (h=300 cm, L=415 cm).

In each graph, the section of columns was constant and every graph contain three types of beam section. For example, for graph (a) in Fig. 12, three frames with (Box 150) for columns and (IPE200, IPE 240 and IPE 270) for beams were considered. Then each of these frames, filled with three types of infill panel with various modulus of elasticity (40000, 80000 and 150000 N/cm^2).

Based on the Figs. 12 and 13 it can be said that:

• In general, it can be said, by increasing α_E, the value of a/d decreased. For example, in graph (a), when beam is IPE200, the results of a/d for three types of modulus of elasticity α_{E1}, α_{E2} and α_{E3} are 0.44, 0.36 and 0.31 respectively. All graphs followed this kind of reduction too.

• In general, by increasing the value of β the results of FEM decreased.

• All the results for Indian code were equal and constant 0.33. The results of Indian code is far from FEM results, except those that are in graph (a) when α_E is equal to the 0.00381 (α_{E2}) and 0.00714 (α_{E3}). In addition, in graph (b) in Fig. 13, the results of FEM analyses in α_{E1} and α_{E2} conditions are close to the Indian code. However, when the frame got weaker, the results of FEM got lower while the Indian code were constant, so in these conditions the results of Indian code are not good match with FEM results, graph (c) in Fig. 13.

• According to the FEMA results, it can be said that in every condition, the value of a/d are between 0.07 to 0.1 and these values are too lower than FEM results (the results of FEMA are about one-third or one-quarter of FEM results).

• The results of new formula, in every condition mentioned above were too close to the FEM results, except when frame is (Box 120 – IPE140) in α_{E1} condition. The match in the results between FEM and new formula, indicate the high accuracy of this new formula than FEMA and Indian code.

• The standard deviation of 54 models, that had been examined is less than 9%.

In Fig. 14, three types of frames were considered that each of these frames are examined in three condition of β (β=1, 1.4 and 1.8). According to above graphs, it can be said that by increasing the value of β, the value of a/d decreased, especially when β changed from 1.4 to 1.8. For all condition of Fig. 14 the results of FEMA in example 1 is constant 0.08, in example 2 is 0.09 and in example 3 is 0.1. The meaning of these results is that FEMA expression is independent from β. In Indian code, where β is greater than 1.4 (in this sample, β=1.8), the results of FEM are less than the results of Indian code. From Fig. 14, it can be said that the results of new formula in compared with FEMA and Indian code is more accurate and reliable.

Table 4 Geometrical parameters of frame members

Frame Element	Transverse section dimensions (m)	Transverse section area (m^2)	Moment of inertia (m4)
Beam	bgxhg=0.5x0.25	Ag=0.125	Ig=10.4x10-3
Column	bsxhs=0.5x0.5	As=0.25	Is=2.6x10-3

Table 5 The properties of the materials

Materials	Modulus of elasticity (kN/m^2)	Poisson coefficient
Concrete-C20/25	Eb=30x106	0.2
Masonry	Ez=4.5x106	0.19

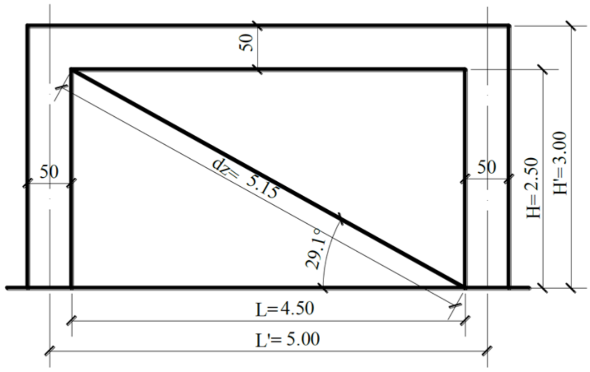


Fig. 15 Masonry infilled reinforced concrete frame (Samoilă 2012)

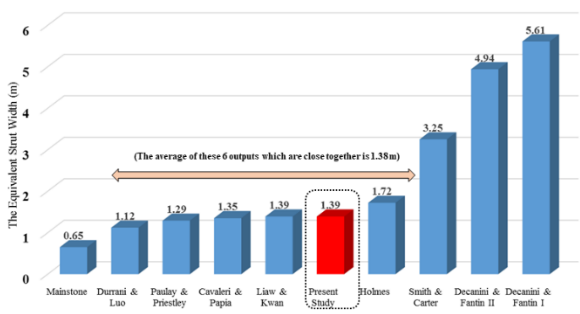


Fig. 16 Comparative of different expression for calculating the equivalent strut width with new formula

6. Additional comparison

Samoilă (2012) had done a comparative study of different expressions for calculating the equivalent strut width and revealed that the Paulay and Priestley equation is the most suitable choice because of approximate average value (value among those studied). In that paper, Samoila proposed a infilled frame model with concrete structure, single story and single bay. The properties of the model are shown as below.

In the following model, t=25 cm is the thickness of the masonry wall and θ=29.10 is the angle of inclination of the equivalent diagonal strut with the horizontal.

By using the above values in the new formula, the result of equivalent strut width gets the amount 1.39 m. According to the result of new formula, it can be understood that the new result is approximate average of other equations

From Fig. 16, it can be understood that the new formula is the most suitable, because the average of the six outputs which are close together in range of 1.12 to 1.72 in the above graph is 1.38 m, and this average is near to the result of the present study. The three results from Smith, Decanini in the graph is not suitable and they are conservative. In addition, the result of new formula is near to the Paulay and Priestly equation which was Samoila proposed too.

7. Conclusions

This paper presents a simple method of macro modeling of infilled frames due to the complicated behavior of non-linear of infill panels. In this work, about sixty infilled frame have been analyzed. In each model, the properties of a particular parameter have been changed to analyze the effects on infilled frame. Hereby and based on FEM analyses, the following conclusions are achieved:

-By increasing α_k the a/d value decrease significantly.

-By increasing I_b the a/d value increase too slightly that can be neglected.

-By increasing β the a/d value decrease significantly when β is greater than 1.4.

-By increasing I_c the a/d value increase too.

-By increasing α_E the a/d value decrease.

-By increasing α_m the a/d value decrease too slightly that can be neglected.

According to the above study, the new formula for determining the equivalent strut width has been created from multiple regression. Results of the new formula in comparison with Indian code and FEMA code is so close to the results of FEM analyses. The results of FEMA are conservative, and its values vary from 0.07 to 0.1. The expression of Indian code is too simple and the results for these 60 models in Indian code is the same about 0.33 for all models. In addition, in comparison, this new formula with other expressions, which proposed by other researchers, can say that the new formula is the best choice for calculating the equivalent strut width.

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Appendix A

a	equivalent strut width (*cm*)
d	diagonal length of infill panel (*cm*)
λ_h	Coefficient used to determine equivalent width of infill strut
t_{int}	thickness of infill panel (*cm*)
h_{inf}	Height of infill panel (*cm*)
h	Height of frame (*cm*)
L	Length of frame (*cm*)
θ	angle made by the strut with the horizontal (deg)
E_{inf}	Elastic modulus of infill panel (kg/cm^2)
E_c	Elastic modulus of column (kg/cm^2)
E_b	Elastic modulus of beam (kg/cm^2)
k_{infilled}	Infilled frame stiffness (kg/cm)
k_{frame}	Frame stiffness (kg/cm)
I_c	Moment of inertia of column (cm^4)
I_b	Moment of inertia of beam (cm^4)
A_c	The gross area of the column
$A_{\mathrm{inf}}=L_{\mathrm{inf}}t_{\mathrm{inf}}$	The area of the infill panel in the horizontal plane
G_{inf}	The shear modulus of the infill
$M_{p\text{-}c}$	Plastic moment of column
$M_{p\text{-}b}$	Plastic moment of beam
β	The ratio of length of infilled frame to height of infilled frame ($\frac{L}{h}$)