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Fundamental vibration frequency prediction of historical masonry bridges

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Abstract. It is very common to find an empirical formulation in an earthquake design code to calculate fundamental vibration period of a structural system. Fundamental vibration period or frequency is a key parameter to provide adequate information pertinent to dynamic characteristics and performance assessment of a structure. This parameter enables to assess seismic demand of a structure. It is possible to find an empirical formulation related to reinforced concrete structures, masonry towers and slender masonry structures. Calculated natural vibration frequencies suggested by empirical formulation in the literatures has not suits in a high accuracy to the case of rest of the historical masonry bridges due to different construction techniques and wide variety of material properties. For the listed reasons, estimation of fundamental frequency gets harder. This paper aims to present an empirical formulation through Mean Square Error study to find ambient vibration frequency of historical masonry bridges by using a non-linear regression model. For this purpose, a series of data collected from literature especially focused on the finite element models of historical masonry bridges modelled in a full scale to get first global natural frequency, unit weight and elasticity modulus of used dominant material based on homogenization approach, length, height and width of the masonry bridge and main span length were considered to predict natural vibration frequency. An empirical formulation is proposed with 81% accuracy. Also, this study draw attention that this accuracy decreases to 35%, if the modulus of elasticity and unit weight are ignored.

Keywords: empirical formulation; fundamental frequency; finite element method; historical masonry bridges

1. Introduction

Fundamental vibration period of a structures is an indispensable parameter to calculate statically equivalent force that effect to a structure as a base shear (Guler *et al.* 2008). Therefore, fundamental frequency of a structure has an important role to estimate seismic vulnerability of a structure (Shakya *et al.* 2016). There are two widely used methods in seismic assessment of a structure; these are force-based analysis and displacement-based analysis. Displacement based analysis is new and more complex procedure than force-based analysis. Estimation of fundamental frequency of a structure is a key part of those two methods (Kocak and Yıldırım 2011, Kocak *et al.* 2017). However, there is a rare study on natural vibration period or frequency of historical masonry structure. Indeed, historical building or monumental stock is the most important patrimonial part of civilization where constructed. Thus, this cultural richness of the urban population should be protected through the centuries against external natural disasters. However, with developing numeric analysis tools, it is possible to estimate the global behavior of available historical structures before and after restoration situation under severe loads like earthquake. Structural characteristics of a historical monument mostly depend on the availability of local construction materials during the construction era (Dogangun and Sezen 2012). Historical characteristics of masonry structures are naturally complex and hard to assess their seismic performance due to intangible monumental value. it is difficult to determine the engineering properties of materials adapted to the historical structures due to the lack of experimental data and forbidden destructive test by authorities (Ural and Dogangun 2007). For this reason, indirect methods are developed to evaluate historical structures to reveal information related to available conditions. One of the most frequently used these indirect assessment methods is the monitoring technology. Applicability of monitoring allows for non-invasive assessment of monuments for both static and seismic actions (Wenzel and Kahle 1993). The estimation of basic dynamic characteristics, such as natural periods, mode shapes and damping by measuring environmentally-induced vibrations of historical masonry monuments is one of the techniques that become operative in the last years. A more permanent application of monitoring to monuments with Operational Modal Analysis (OMA) integrated with finite element method to evaluate failure and damage mechanism of a structure. This modelling and assessment method is indispensable to plan maintenance and structural rehabilitation. This issue also gains the interest of engineers (Spyrakos 2018). These are necessary to understand the damages and their causes and carry out a first interpretation of the phenomena (Binda and Tiraboschi 1999). Therefore, deep knowledge of engineering properties is unavailable. The non-destructive testing method was developed for this reason especially with OMA (Diaferio *et al.* 2018). This non-destructive technique is used for the finite element (FE) model to obtain a correct simulation of the real case (Onat *et al.* 2017). The key point is mostly composite geometry of historical masonry elements in terms of material property drives the need of simulation with a

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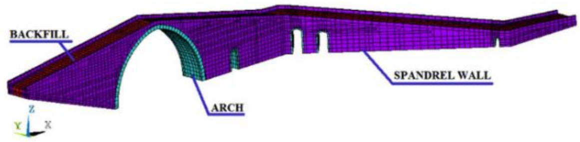
FE package to see the global behavior of investigated historic building (Mele *et al.* 2003). Even if, composite geometry and a wide variety of material differences of historical masonry structures, useful empirical formulations were reported by different authors related to historical structures such as Shakya *et al.* (2016). Shakya *et al.* studied on the empirical formulation of slender masonry structure and proposed an empirical equation. Shakya *et al.* (2016) focused especially on tower, minaret, chimney and Pagoda temple. Diaferio *et al.* (2018) proposed an empirical equation to predict the fundamental frequency of historical masonry towers. Diaferio *et al.* (2018) evaluated 24 different historical masonry towers constructed on different location of the territory. Diaferio *et al.* (2018) proposed two different formulation like bounded and isolated tower on the base of support condition. Ranieri and Fabbrocino (2011) and Faccio *et al.* (2011) suggested formulation to predict the natural vibration period depends on the height of the structure. In addition, useful formula was reported in the literature especially in Italy (NTC2008) and Spain (NCRS 2002) construction standard. In fore date of Turkish Seismic Code (TSC 1998), an empirical formulation is used to predict first natural vibration period for only reinforced concrete structures and steel structures. Mentioned TSC code is not suitable for historical masonry structures. However, the Italian code suggested empirical formulation depended on only height of the structure. Suggested formulation by Spanish seismic code includes height and minor base length of the building.

The presented paper intends to investigate and reveal an empirical formulation to provide fast evaluation of the modal characteristics of the historical masonry bridges. This goal expedites to get knowledge about historical masonry bridge and to reduce time and cost for cultural monument management. Maximum span length, height, length and number of arches are the typical parameters to describe a bridge according to Pérez-Gracia *et al.* (2011). In this context, maximum main arch span length, height of the bridge, length of the bridge, width of the bridge, unit weight and modulus of elasticity of dominant material was considered in this study. First natural vibration frequency of the historical masonry bridges was obtained from finite element model of the bridges. Because, the first natural vibration frequency obtained from finite element model is respectful also for other researchers such as Diaferio *et al.* (2018). One of the most important point related to finite element model is that considered numeric models have not included model structure error and the first global natural vibration frequency was considered. 81% accuracy was obtained with six variables. However, accuracy of the model was decreased to 35%, since modulus of elasticity and unit weight were ignored.

2. Historical masonry bridges

Passing over the streams by bridges were difficult in ancient times. Because, constructing masonry bearing elements was not easy while compared with contemporary techniques. Nearly all these type of historical masonry monuments are still in use. Therefore, load bearing capacities and seismic performances need to be assessed. For this reason, prediction

(a) Typical historical masonry bridge (Malabadi bridge)



(b) Longitudinal section drawing of Malabadi arch bridge

Fig. 1 Malabadi Historical Bridges (Karaton *et al*. 2017)

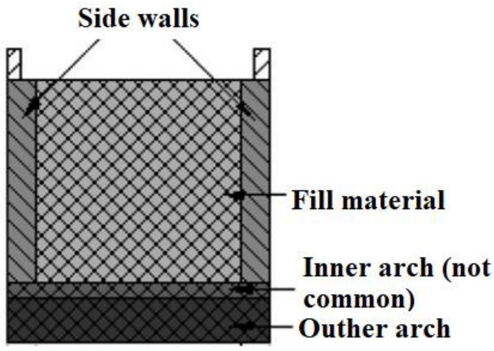
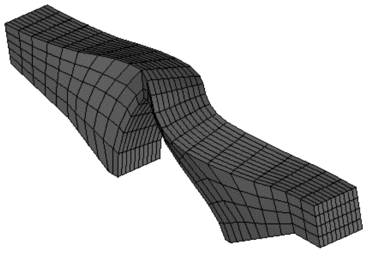
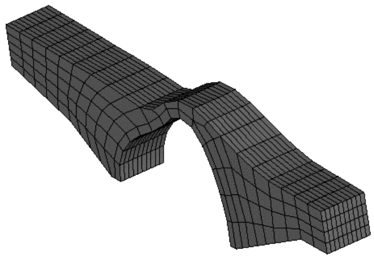


Fig. 2 Cross-section of a typical historical masonry arch bridge (Sevim *et al*. 2011)



(a) Transversal mod of historical masonry bridge



(b) Bending mod of historical masonry bridge

Fig. 3 Transversal and bending vibration direction of Tagar Bridge (Onat and Sayın 2015)

of natural vibration frequency with a robust empirical formulation come to an important role. An example of reported

Table 1 Properties of reported studies related to historical masonry bridges

Number	Name of Bridge	Authors	Material	Unit Weight (t/m^3)	E (GPA)	Width (m)	Main Arch Span (m)	Height (m)	Length (m)	First Natural FE Vibration Frequency (Hz)
1	Tarihî Kemer Köprülerin Sonlu Eleman Metoduyla Analizi (Coşandere)	Ural (2005)	Stone masonry	2.00	2.50	4.5	14	9	32	7.5
2	Limit analysis of a single span masonry bridge with unilateral frictional contact interfaces (Prestwood bridge)	Drosopoulos *et al.* (2006)	Stone masonry	2.00	15.00	3.8	6.55	1.81	10.99	8.09
3	Seismic assessment of masonry arch bridges (Marcello Pistoiese Bridge)	Pela *et al.* (2009)	Cut stone	2.20	5.50	5.8	21.5	23.25	72.5	4
4	Seismic assessment of masonry arch bridges (Cutigliano Bridge)	Pela *et al.* (2009)	Cut stone	2.20	5.50	7.4	16.5	16	82.5	6
5	Characterization of a Romanesque Bridge in Galicia (Spain) (Fillaboa Bridge)	Perez-Gracia *et al.* (2011)	Granite Ashlar	2.00	2.00	4.5	16.3	12.14	75	4.1
6	Finite element model calibration effects on the earthquake response of masonry arch bridges (Osmanlı)	Sevim *et al.* (2011)	Cut stone	1.40	2.50	3	25.2	13.47	51.7	3.84
7	Finite element model calibration effects on the earthquake response of masonry arch bridges (Şenyuva)	Sevim *et al.* (2011)	Cut stone	1.40	2.50	2.5	24.8	13.6	52.4	3.347
8	Assessment of nonlinear seismic performance of a restored historical arch bridge using ambient vibrations (Mikron)	Sevim *et al.* (2011)	Stone	1.40	2.50	3	19.49	11.6	33.8	5.415
9	Static and dynamic analysis of the old stone bridge in Mostar	Radnic *et al.* (2012)	Tenelija stone	1.98	2.30	3.95	28.62	57	40	11.43
10	The effect of arch geometry on the structural behavior of masonry bridges (Göderni Bridge, Diyarbakır)	Altunışık *et al.* (2015)	Yellowish colored cut stone	2.00	3.00	6.05	12	12	62	3.35
11	Updating Numerical Models of Masonry Arch Bridges by Operational Modal Analysis (Lazaro Bridge)	Costa *et al.* (2015)	Stone	2.60	35.00	3.3	7.5	4.48	28	7.7
12	Updating Numerical Models of Masonry Arch Bridges by Operational Modal Analysis (Lagoncinha Bridge)	Costa *et al.* (2015)	Stone	2.60	35.00	3.5	15.46	12.37	150	3.8
13	Updating Numerical Models of Masonry Arch Bridges by Operational Modal Analysis (Villa Fria Bridge)	Costa *et al.* (2015)	Granite Stone	2.60	22.00	6	5.8	7.55	60	6.8
14	Full 3D nonlinear time history analysis of dynamic soil-structure interaction for a historical masonry arch bridge (Mataracı Bridge, Trabzon)	Güllü and Jaf (2016)	Stone arch	2.37	2.50	6	16	9.5	50	3.5
15	Nonlinear seismic response of a masonry arch bridge (Nadir Bridge)	Sayın (2016)	Stone masonry	2.20	1.20	3.9	7.9	4.46	50.14	14.626
16	Nonlinear seismic performance of a 12th century historical masonry bridge under different earthquake levels (Malabadi Bridge)	Karaton *et al.* (2017)	Limestone	2.48	8.28	7.0	38.6	25.59	227.84	3.69
17	Tarihi Tağar Köprüsünün Doğrusal Olmayan Sismik Analizi (Tagar Bridge)	Onat and Sayın (2015)	Limestone	2.20	2.40	4.2	15.8	9.0	45.0	6.45
18	Seismic assessment of a historical masonry archbridge (Dutpınar Bridge)	Ozmen and Sayın (2018)	Stone masonry	2.4	2.4	3.7	11.05	7.33	19.78	10.07

typical historical masonry bridge and finite element simulation were presented in Fig. 1.

There are numerous historical masonry bridges since medieval times or earlier and has been used to transfer human and vehicle. The construction technology and type of material used for bridges depends on the availability of local materials. The construction technique related to bed width of river, altitude of passing river. Commonly composed of a main large span over the bed of the river and single or multiple arch. Historical masonry bridges are composed mainly of masonry materials as depicted in Fig. 2.

A typical historical masonry bridge composed of side walls, fill material and arch. Commonly side walls and arch composed of the same material. However, the density of fill material is relatively low while compared with the masonry unit. For this reason, the dominant material is naturally being masonry elements that affect natural vibration frequency. Oscillation of bridge was generally observed through either transversal or bending direction at the main arch span as seen in Fig. 3(a) and (b).

Many historical masonry bridges were investigated to assess their vulnerability against severe external condition especially against earthquake. For this assessment, finite element modelling with correct parameters is very important and indispensable. While performing this assessment, the reliability of the numeric model completely depends on the modelling without model structure error. The scientific papers that deal with the seismic assessment of historical masonry bridge listed in Table 1. According to this table, dimensions of bridges, dominant material type, engineering properties of dominant materials were listed.

Table 2 Proposed formulations in the literature

Eqn. Number	Reference	Proposed Equation	Explanation
1	NCSE (2002)	$\mathrm{f(H,L)} = \dfrac{\sqrt{\mathrm{L}}}{0.06\mathrm{H}\sqrt{\dfrac{\mathrm{H}}{2\mathrm{L}+\mathrm{H}}}}$	Spanish Standard for masonry structures
2	NTC08 (2008)	$\mathrm{f(H)} = \dfrac{1}{0.05 * \mathrm{H}^{\frac{3}{4}}}$	Italian technical standard for building
3	Shakya *et al.* (2016)	$\mathrm{f(H)} = \dfrac{1}{0.0151 * \mathrm{H}^{1.08}}$	Empirical law
4	Ranieri and Fabbrocino (2011)	$\mathrm{f(H)} = \dfrac{1}{0.01137 * \mathrm{H}^{1.138}}$	Empirical law
5	Faccio *et al.* (2011)	$\mathrm{f(H)} = \dfrac{1}{0.0187 * \mathrm{H}}$	Empirical law
6	Diaferio *et al.* (2018)	$\mathrm{f(H)} = 28.35 * \dfrac{1}{\mathrm{H}^{0.83}}$	Empirical law for bounded masonry tower
7	Diaferio *et al.* (2018)	$\mathrm{f(H)} = 135.343 * \dfrac{1}{\mathrm{H}^{1.32}}$	Empirical law for isolated masonry tower

Majority of the paper listed in Table 1, studied historical masonry bridges both experimentally and numerically. Experimental non-invasive methods can be listed as OMA, Schmidt hammer according to rebound number and Ultrasonic Pulse Velocity test methods as indicated by Karaton *et al.* (2017). Obtained material properties with these methods are classified in the literature also reliable,

Table 3 First natural vibration frequencies of historical masonry bridges calculated with finite element model and proposed formulations

Num	Name of Bridge	Authors	FE Vibration Frequency (Hz)	NCSE (2002)	NTC08 (2008)	Shakya *et al.* (2016)	Ranieri and Fabbrocino (2011)	Faccio *et al.* (2011)	Diaferio *et al.* (2018) Bounded	Diaferio *et al.* (2018) Isolated
1	Tarihî Kemer Köprülerin Sonlu Eleman Metoduyla Analizi (Coşandere)	Ural (2005)	7.5	5.56	3.85	6.17	7.22	5.94	4.58	7.44
2	Limit analysis of a single span masonry bridge with unilateral frictional contact interfaces (Prestwood bridge)	Drosopoulos *et al.* (2006)	8.09	40.93	12.82	34.89	44.77	29.54	17.33	61.84
3	Seismic assessment of masonry arch bridges (Marcello Pistoiese Bridge)	Pela *et al.* (2009)	4	2.11	1.89	2.21	2.45	2.30	2.08	2.13
4	Seismic assessment of masonry arch bridges (Cutigliano Bridge)	Pela *et al.* (2009)	6	3.93	2.50	3.32	3.75	3.34	2.84	3.48
5	Characterization of a Romanesque Bridge in Galicia (Spain) (Fillaboa Bridge)	Perez-Gracia *et al.* (2011)	4.1	3.84	3.08	4.47	5.13	4.40	3.57	5.01
6	Finite element model calibration effects on the earthquake response of masonry arch bridges (Osmanlı)	Sevim *et al.* (2011)	3.84	2.58	2.84	3.99	4.56	3.97	3.27	4.37
7	Finite element model calibration effects on the earthquake response of masonry arch bridges (Şenyuva)	Sevim *et al.* (2011)	3.347	2.27	2.82	3.95	4.51	3.93	3.25	4.32
8	Assessment of nonlinear seismic performance of a restored historical arch bridge using ambient vibrations (Mikron)	Sevim *et al.* (2011)	6.06	3.07	3.18	4.69	5.41	4.61	3.71	5.33
9	Static and dynamic analysis of the old stone bridge in Mostar	Radnic *et al.* (2012)	11.43	0.62	0.96	0.84	0.88	0.94	0.99	0.65
10	The effect of arch geometry on the structural behavior of masonry bridges (Göderni Bridge, Diyarbakır)	Altunışık *et al.* (2015)	3.35	4.84	3.10	4.52	5.20	4.46	3.60	5.09
11	Updating Numerical Models of Masonry Arch Bridges by Operational Modal Analysis (Lazaro Bridge)	Costa *et al.* (2015)	7.7	10.63	6.49	13.11	15.96	11.94	8.17	18.70
12	Updating Numerical Models of Masonry Arch Bridges by Operational Modal Analysis (Lagoncinha Bridge)	Costa *et al.* (2015)	3.8	3.15	3.03	4.38	5.02	4.32	3.51	4.89
13	Updating Numerical Models of Masonry Arch Bridges by Operational Modal Analysis (Villa Fria Bridge)	Costa *et al.* (2015)	6.8	8.70	4.39	7.46	8.81	7.08	5.29	9.39
14	Full 3D nonlinear time history analysis of dynamic soil-structure interaction for a historical masonry arch bridge (Mataracı Bridge, Trabzon)	Güllü and Jaf (2016)	3.5	6.46	3.70	5.82	6.79	5.63	4.38	6.93
15	Nonlinear seismic response of a masonry arch bridge (Nadir Bridge)	Sayın (2016)	14.626	12.24	6.52	13.17	16.04	11.99	8.20	18.81
16	Nonlinear seismic performance of a 12th century historical masonry bridge under different earthquake levels (Malabadi Bridge)	Karaton *et al.* (2017)	3.69	2.14	1.76	2.00	2.20	2.09	1.92	1.87
17	Tarihi Tağar Köprüsünün Doğrusal Olmayan Sismik Analizi (Tagar Bridge)	Onat and Sayın (2015)	6.45	5.27	3.85	6.17	7.21	5.94	4.58	7.44
18	Seismic assessment of a historical masonry archbridge (Dutpınar Bridge)	Ozmen and Sayın (2018)	10.07	6.20	4.49	7.70	9.11	7.29	5.43	9.76

since it is not possible to find accelerometer to perform OMA test to extract natural vibration frequency. On this basis, the availability of a finite element model of historical masonry bridge in a journal paper is a priority of this paper.

3. Formulation of natural vibration frequencies

In this section of the paper, adapted formulations to determine first natural vibration frequency were evaluated. Majority of these formulations depend on the height of the evaluated structure. These formulations are listed in Table 2.

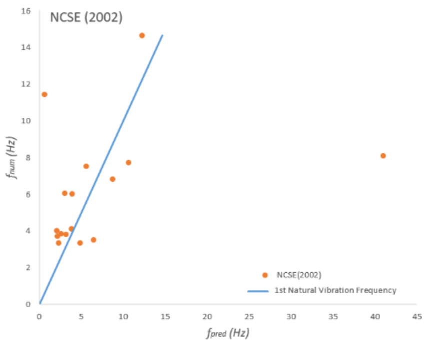
As seen from Table 2, proposed formulations for structures depend on the height of the structures except for the Spanish standard. The proposed formula by Spanish code use two different parameters full Height (H) of the structure and length of the structure (L) through oscillation direction or possible first vibration direction. There is an important point to draw attention related to historical masonry bridges, length of the bridges is more effective than height of the bridges. Moreover, first vibration frequency is inversely affected by complex material geometry of bridges, width and main span arch length; mass distribution and stiffness of the structure. The issue of this paper is related to prediction of first natural vibration frequency with considering unit weight and elastic modulus of dominant masonry material. In addition, width, main arch span length, height and length of the historical masonry bridge were also considered as tabulated in Table 1. Prediction of first natural vibration frequency distributed in a wide variety of scale as seen in Table 3 calculated by empirical formulation as indicated in Table 2.

Aforementioned data presented in Table 3 proved that natural vibration frequency of the historical masonry bridges need to be presented with a formulation consist of different parameters pertinent to characteristic definition of a historical bridge. Seven empirical formulation were evaluated and plotted in different graph to see the distribution of calculated natural first vibration frequency compared to first natural vibration frequency obtained from finite element model. These graphs can be seen in Fig. 4 below.

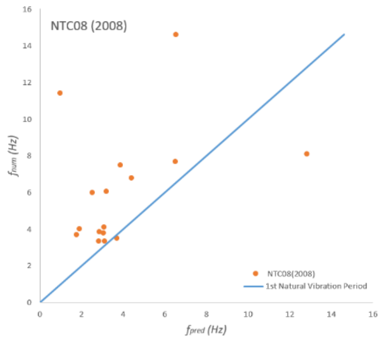
As seen in Fig. 4, there is a solid bisector line in all Fig.s presents 1st natural vibration frequency obtained from the finite element model. In addition, there are dots that represents calculated frequencies with suggested formulations.

4. Proposed equations, results and discussion

The available data presented in Table 1 analyzed to obtain a new formulation for robust estimation of natural vibration frequency of historical masonry bridges. For this purpose, five different new formulations were tried to develop with this study. However, one of them is useful for robust prediction of historical masonry bridge. Nonlinear regression analysis was performed to increase predictive performance of the suggested formulation. Hereby, proposed equations were derived on the base of an exponential law model as indicated in Equation 8 below.



(a)



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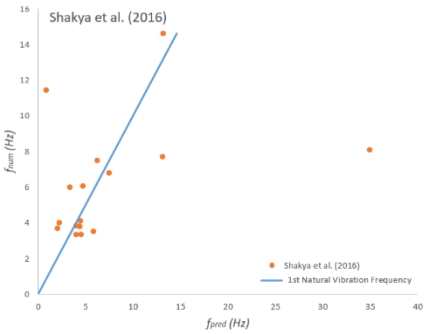

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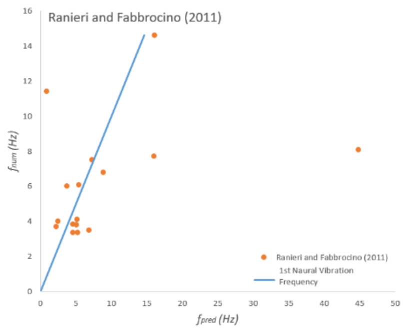

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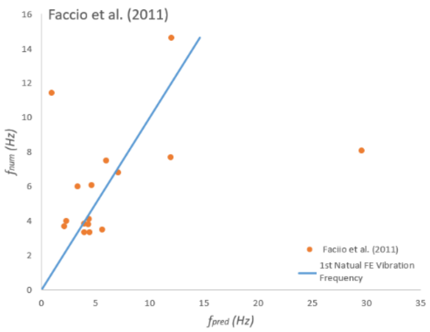

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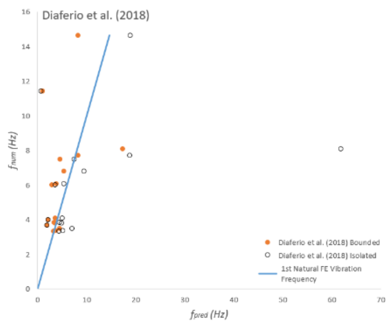

(f)

Fig. 4 Comparison between natural finite element vibra tion frequency and predicted natural frequencies

$$f_1 = x_1 * \frac{UW^{x_2} * MAS^{x_3} * H^{x_4}}{E^{x_5} * W^{x_6} * L^{x_7}} \tag{8}$$

Where f1 is the first natural vibration frequency (Unit in Hz), UW is the unit weight of the dominant material constructed generally side walls of the historical masonry bridge in t/m^3, MAS is the Main Arch Span length of the bridge in meter, H is the total height of the structure in meter, E is the modulus of elasticity of dominant material generally side walls of the historical masonry bridge in GPa, W is the width of the bridge in meter and L is the total length of the bridge in meter. Parameters that addressed in Equation 8 is the identified by minimizing the Mean Square Error (MSE) of the nonlinear regression. Hundreds of iterations performed and among iterated equations, five of them were mentioned. Whereas, one of them is the most useful among other equation was determined by maximizing accuracy of the coefficient “r” with the expression indicated in Eq. (9).

$$r^2 = 1 - \frac{\sum_{i=1}^{n}(f_{num,i} - f_{1,i})^2}{\sum_{j=1}^{n}\left(f_{num,j} - \frac{\sum_{i=1}^{n} f_{num,i}}{n}\right)^2} \tag{9}$$

Where the fnum,i is the ith numerical first natural frequency, n the number of data that belong to considered historical masonry bridge is equal to 18. On the base of power calculation that gives formulation by using the data tabulated in Table 1.

The first formulation was developed as a function of height, width and length of the historical masonry bridges as indicated in Eq. (10) below.

$$f_1 = 23.116 * \frac{(H)^{0.109}}{(W)^{0.01} * (L)^{0.398}} \tag{10}$$

The accuracy of the proposed law was determined with r2 value and this value is obtained 0.184. Visual performance of this equation with respect to numerical frequency can be seen in Fig. 5.

In Eq. (10), there is a wide distribution of predicted frequencies due to low accuracy. Only two of the historical masonry bridges were predicted these are Lazaro Bridge and Malabadi Brdige.

Only physical dimension is not enough for a high accuracy estimation, modulus of elasticity of dominant material was used also with retaining length, width and height of the historical masonry bridges. A second formulation was

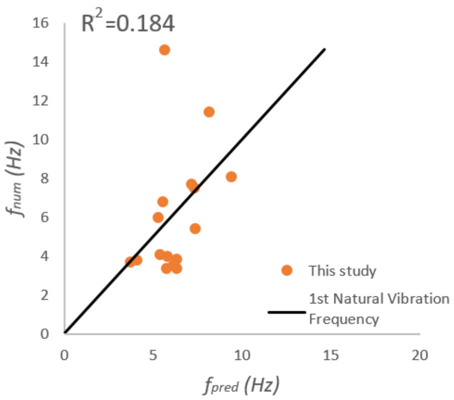

Fig. 5 Performance of proposed Eq. (10) for the prediction of natural vibration frequency

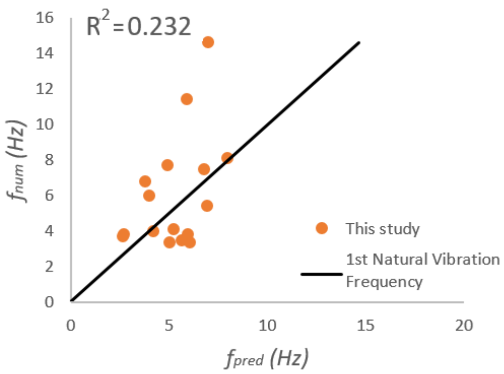

Fig. 6 Performance of proposed Eq. (11) for the prediction of natural vibration frequency

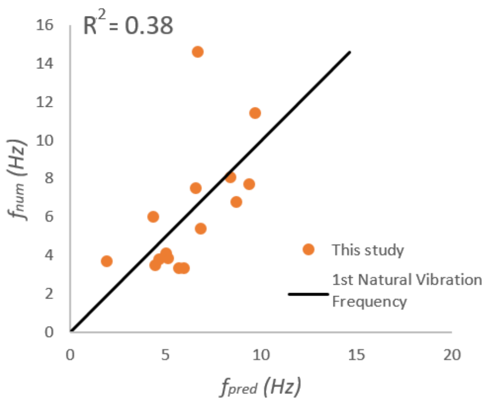

Fig. 7 Performance of proposed Eq. (12) for the prediction of natural vibration frequency

proposed as indicated in Eq. (11).

$$f_1 = 32.8 * \frac{(E)^{-0.165} * (H)^{-0.053}}{(W)^{0.12} * (L)^{0.327}} \tag{11}$$

Accuracy of proposed formulation by using in Eq. (11) with indicated numerical power indices is 23.2%. Prediction performance of Equation 11 can be seen in Fig. 6.

Even though, r2 value of proposed formulation was increased, only five of the bridges' natural vibration frequency were estimated with a high accuracy. It was realized that fundamental natural vibration frequency estimation was

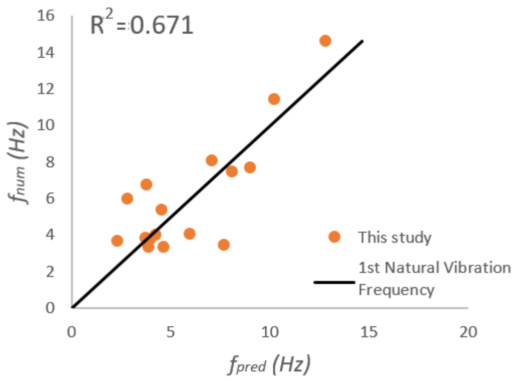

Fig. 8 Performance of proposed Eq. (13) for the prediction of natural vibration frequency

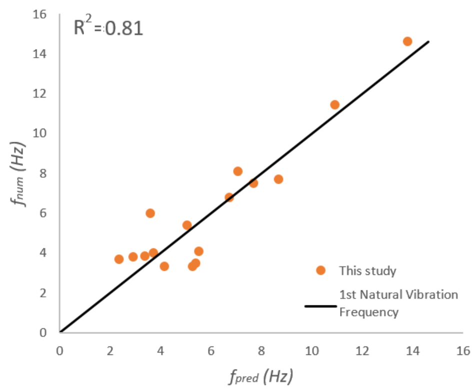

Fig. 9 Performance of proposed Eq. (14) for the prediction of natural vibration frequency

reached to average 0.232 r squared value with this proposed formulation. Moreover, this formulation predicts natural vibration frequency of the historical masonry bridges whom natural frequency is between 4 Hz and 7 Hz.

Modulus of elasticity was replaced with Main Arch Span length of historical masonry bridge in Eq. (12). This equation is also composed of only with physical descriptive parameters. These are length, height, with and main arch span length of bridges. r square value was increased to 0.38 with this formulation.

$$f_1 = 139.59 * \frac{(MAS)^{-0.85} * (H)^{0.546}}{(W)^{0.498} * (L)^{0.364}} \tag{12}$$

Visual performance of Eq. (12) is presented in Fig. 7.

In a proposed fourth equation main arch span length was ignored, modulus of elasticity and unit weight of dominant material was added in addition to length, width and height of the bridge. This formulation was presented in Eq. (13) with power indices.

$$f_1 = 41.388 * \frac{(UW)^{3.003} * (H)^{0.101}}{(E)^{0.401} * (W)^{1.216} * (L)^{0.502}} \tag{13}$$

r square value of Eq. (13) is 0.671. Performance distribution of this equation was plotted in Fig. 8.

Eq. (14) predicted especially the natural frequency of bridges with a high frequency over 8 Hz. These bridges are

Table 4 MSE values of proposed equations in the literature and this study

	NCSE (2002)	NTC08 (2008)	Ranieri and Fabbrocino (2011)	Faccio *et al.* (2011)	Shakya *et al.* (2016)	Diaferio *et al.* (2018) Bounded	Diaferio *et al.* (2018) Isolated	This Study Eq. (10)	This Study Eq. (11)	This Study Eq. (12)	This Study Eq. (13)	This Study Eq. (14)
MSE	77.93	15.41	97.47	38.59	55.34	16.74	198.88	8.22	8.49	6.21	3.3	1.95

Preswood Bridge, Mostar Bridge and Nadir Bridges.

On the basis of power calculation with natural vibration frequency, hereby a novel developed formulation was presented in Eq. (14) with a highest r square value that is 0.81. Eq. (14) comprises both physical parameters like width, length, height and main arch span length of the bridge and mechanical properties of dominant material properties like modulus of elasticity and unit weight of the material.

$$f_1 = 467.78 * \frac{(UW)^{1.724} * (MAS)^{-1.103} * (H)^{0.595}}{(E)^{0.38} * (W)^{1.08} * (L)^{0.5}} \tag{14}$$

After evaluation of five alternatives defined above, Eq. (14) was predicted natural vibration frequency of historical masonry bridges with a high r square value that is 0.81. Visual performance of Eq. (10) as illustrated in Fig. 9.

The plot of Fig. 9 indicates the performance of the proposed equation on the base of the data tabulated in Table 1. The correlations between the 1st natural FE frequencies and proposed equation showed 81% correlation. There are only three of data showed a discrepancy between the 1st natural FE frequency and proposed equations. The reason for them is the length of these historical bridges does not straight. After a certain length, the direction of bridge changes to different other direction as seen in Fig. 1(a) and Fig. 1(b). This direction deviation increases vibration frequency. For instance, the total length of the Malabadi bridge is 227.84 meters, since this direction deviation problem, the effective length of the bridge should be considered as 117.57 meters. This length was measured over main arch span and always first natural vibration oscillation occurs at this location. Considering the effective length of this bridge increased prediction of 1st natural vibration 40% by using Eq. (14).

To demonstrate accuracy of proposed equations with compared literature, mean square errors of proposed equations in literature and this study are listed in Table 7.

According to Table 4, proposed formulations gave rather higher MSE value, since first vibration frequency estimation equation depends on the only height of the structure. Only NCSE (2002) proposes an equation with two variables. These are height (H) and length (L) of the structure through oscillation direction. More than two variables were considered with this study to obtain robust estimation. Calculated MSE values with proposed equations in this study for Eqs. (10), (11), (12), (13) and (14) are 8.22, 8.49, 6.21, 3.3 and 1.95, respectively. This study provides researchers and practitioners a novel approach to consider more than one parameter for robust prediction of the first natural vibration frequency of historical masonry bridges.

5. Conclusions

In this research paper, the finite element model-based database was used to predict 1st natural vibration frequency of historical masonry bridges to propose fast assessment of modal characteristics such as natural frequencies. Data were collected from published literature of historical masonry bridges regarding finite element 1st natural frequency, geometrical dimensions and mechanical characteristics. After hundreds of iterations and constituting a suitable formulation, one of the them is proposed with 81% accuracy (r square) among five of presented. This proposed formulation was considered both physical and mechanical properties. Physical properties can be listed length, width, height and main arch span length of the bridge. Moreover, mechanical properties that considered to derive a formula are modulus of elasticity and density of dominant material. This developed formulation showed a certain deviation especially to the bridges whom length shows a direction deviation and whom arch system is constructed on high masonry column. Evaluation of a formulation only with physical dimensions of the bridges decreases accuracy of formulations to 18.4% and 38%depending on considered parameters.

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