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# Shear waves propagation in an initially stressed piezoelectric layer imperfectly bonded over a micropolar elastic half space

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**Abstract.** The present study investigates the propagation of shear waves in a composite structure comprised of imperfectly bonded piezoelectric layer with a micropolar half space. Piezoelectric layer is considered to be initially stressed. Micropolar theory of elasticity has been employed which is most suitable to explain the size effects on small length scale. The general dispersion equations for the existence of waves in the coupled structure are obtained analytically in the closed form. Some particular cases have been discussed and in one particular case the dispersion relation is in well agreement to the classical-Love wave equation. The effects of various parameters viz. initial stress, interfacial imperfection and micropolarity on the phase velocity are obtained for electrically open and mechanically free system. Numerical computations are carried out and results are depicted graphically to illustrate the utility of the problem. The phase velocity of the shear waves is found to be influenced by initial stress, interface imperfection and the presence of micropolarity in the elastic half space. The theoretical results obtained are useful for the design of high performance surface acoustic devices.

**Keywords:** shear wave; micropolar; piezoelectric; dispersion; phase velocity

## 1. Introduction

Due to the electromechanical property of piezoelectric materials, it has been widely used in modern smart materials devices like sensors, actuators and transducers. A Transducer which consists of thin piezoelectric layer coupled with an elastic substrate is more prevalent in signal processing, low energy consumption technologies. The micro-sensors constituted of piezoelectric composite which is based upon surface acoustic waves (SAW) have numerous applications in many fields due to their high sensitivity and enhanced electromechanical responses.

Shear waves propagation in the piezoelectric bi-material has received much attention due to various applications like non-destructive testing techniques to estimate the surface mechanical properties of solids or to achieve the time delay effects for acoustic applications. The phenomenon of wave propagation in piezoelectric materials is well known and has been explored extensively by many researchers. Bleustein (1968) derived the expression for velocity of new type surface waves in piezoelectric materials which has no counterpart in purely elastic homogeneous material. Mindlin (1952) and Tiersten (1963) worked on pure piezoelectric materials and studied about the thickness vibrations in these materials. Curtis and Redwood (1973) developed the general conditions for the existence of various modes for both Love waves and Bleustein-Gulyaev waves in a piezoelectric composite material. Wang *et al.* (2001) theoretically derived the dispersive characteristic and electric potential for love waves in the thickness direction of piezoelectric layer bonded onto a semi-infinite solid medium. Qian *et al.* (2004) and Qian *et al.* (2004) combined different media with piezoelectric material to study surface wave propagation. Liu and Wang (2005) discussed the propagation behavior of Love waves in a functionally graded piezoelectric layered structure. Qian *et al.* (2011) presented the study of transverse surface waves in piezoelectric material with multiple hard metal interlayers. Some theoretical aspects and derivations of constitutive equations for the piezoelectric materials in the context of theory of thermoelasticity can be found in the papers (El-Karamany and Ezzat 2005 and Ezzat *et al.* 2010). Marin (2008) proved the existence and uniqueness of the solution for the boundary value problem in the dipolar elastic structure and recently Marin and Öchsner (2018) obtained these results for an initial boundary value problem in the piezoelectric dipolar elastic solids.

In most of the investigations related to piezoelectric layered structure the interface between the materials is assumed to be perfectly bonded but this condition is rarely true in practical problems. Due to mismatch properties of materials, residual stress and interface imperfection may exist during the manufacture process of piezoelectric surface acoustic wave devices. Chen *et al.* (2008) analyzed the effect of imperfect bonded interface on the propagation

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of SH piezoelectric waves guided by a piezo-ceramics plate between two piezo-ceramic half-spaces. Qian *et al.* (2009) investigated the propagation of transverse surface waves in a piezoelectric material carrying a pre-stressed metal layer of finite thickness. Liu *et al.* (2010) investigated the dispersive behavior of shear surface waves in a piezoelectric- elastic structure with imperfect interface using shear-lag model. Son and Kang (2011) studied the effect of initial stress on the propagation behavior of shear waves in piezoelectric-elastic layer structure. Wang and Zhao (2013) considered an imperfect interface between the piezoelectric and elastic layer to study the influence of interfacial defects on the propagation of love waves. Kurt *et al.* (2016) investigated the influence of the initial stresses on Lamb wave propagation in a sandwich composite structure comprised of pre-stressed elastic and piezoelectric layers. Singh *et al.* (2017) investigated the effects of imperfectly bonded piezoelectric layer having irregularity with a fibre-reinforced half space. Chaudhary *et al.* (2018) recently studied the effects of initial stress on the propagation of SH waves in a rotating piezoelectric structure where the interface between the layer and the substrate is assumed to be imperfect.

The micropolar theory of elasticity is preferred for describing the behavior of complex media due to its ability to explain size effects on small length scale by taking into consideration the additional degree of freedoms. In this theory, each element or grain of microstructure is not only translated but also rotated about its center of gravity. The behavior of materials such as cellular solids, platelet composites, aluminium epoxy, bones, masonry, granular materials, polymers, crystals and many other have complex microstructure can be described by using micropolar theory of elasticity. The classical theory of elasticity is found to be inadequate to explain the behavior such materials as it considered the material to be continuum in the mathematical sense. In particular, the classical theory does not explain some discrepancies that take place in the case of elastic vibration of high frequency and small wavelength. Voigt (1887) tried to remove these shortcomings by introducing additional couple stress vector to describe the interaction between two particles in a body. Eringen and Suhubi (1964) initiated the general linear and nonlinear micropolar theory of elastic continua. Eringen (1966) generalized the classical theory of elasticity by considering three extra rotational degrees of freedom in addition to classical displacement degrees of freedom. Eringen (1999) proved the uniqueness theorems for micropolar elasticity and linear theory of micropolar thermoelasticity. El-Karamany and Ezzat (2009), Ezzat and Awad (2010), and El-Karamany and Ezzat (2013) derived the constitutive laws and proved the uniqueness theorem for linear micropolar theory of thermoelasticity using relaxation time, two temperature and phase lag models. The papers (Marin 2015 and Marin and Baleanu 2016) explored the behavior of micropolar media in the context of heat flux theory and theory of thermoelasticity without energy dissipation.

The phenomenon of wave propagation in micropolar solid is well-known due to their practical applications in the various fields of science and technology such as seismology, acoustics, aerospace and submarine structures. Eringen (1999) discussed about the existence of Rayleigh waves in homogeneous micropolar medium. Singh and Kumar (1998) evaluated the amplitude ratio for reflection and refraction of plane waves at an interface between a micropolar elastic and viscoelastic solid. Midya (2004) obtained the frequency equation for the propagation of Love waves in homogeneous micropolar isotropic elastic media consisting of a layer of finite thickness lying over a semi-infinite medium. Kumar and Deswal (2006) studied the problem of wave propagation in a micropolar media with voids. Ezzat *et al.* (2010) studied the thermoelastic plane waves in micropolar elastic solid taking into account the electro-magnetic and two temperatures effects. Kumar *et al.* (2014) studied the propagation of waves in micropolar generalized thermoelastic material bordered with half space or layer of inviscid liquid under stress-free boundary conditions in the context of Green and Lindsay (G-L) theory. Kaur *et al.* (2017) investigate the shear wave propagation in vertically heterogeneous viscoelastic layer over a micropolar elastic half-space. Recently Kundu *et al.* (2017) studied the propagation of Love waves in heterogeneous micropolar layer over an elastic inhomogeneous media.

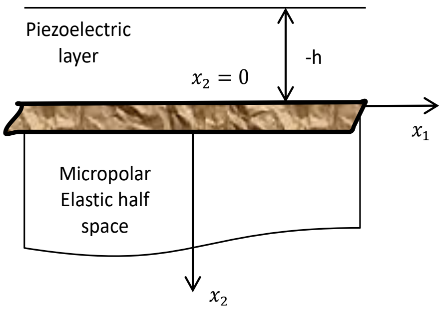

Fig. 1 Geometry of the problem

In the present article, the propagation of shear wave in an initially stressed piezoelectric layer bonded imperfectly over a homogeneous isotropic micropolar elastic half space has been studied. The dispersion relation of shear waves in closed form is obtained analytically. The effects of initial stress, interfacial defects and the micropolarity have been investigated. Numerical computation for phase velocity is carried out and the results are illustrated graphically.

## 2. Formulation of the problem

As shown in Fig. 1, we consider a layer of piezoelectric material of thickness h bonded imperfectly over a micropolar elastic half space. The rectangular Cartesian coordinate system is considered such that the piezoelectric material is polarized along $x_3$ direction perpendicular to $x_1 - x_2$ plane. It is assumed that the shear wave propagates in the $x_1$ direction and $x_2$-axis is positive vertically downward. For shear wave propagating in the $x_1 - x_2$ plane, the displacement components will be independent of $x_3$ coordinate.

### *2.1 Dynamics of piezoelectric layer*

Following Qian *et al.* (2004), the constitutive relations and field equations of piezoelectric medium with initial stress can be expressed as

$$\begin{cases}\sigma_{ij} = c_{ijkl}S_{kl} - e_{kij}E_k \\ D_j = e_{jkl}S_{kl} + \epsilon_{jk}E_k\end{cases}, \tag{1}$$

$$\begin{cases}\sigma_{ij,j} + (u_{i,k}\sigma_{kj}^0)_{,j} = \rho\ddot{u}_i \\ D_{i,i} + (u_{i,k}D_j^0)_{,i} = 0\end{cases}, \tag{2}$$

where $\sigma_{ij}$ is stress tensor, $\sigma_{kj}^0$ is initial stress tensor, $u_i$ and $D_j$ denote the components mechanical and electrical displacement, $D_j^0$ is the initial electrical displacement, ρ represents the density of piezoelectric material. In Eq. (1), $c_{ijkl}$ , $e_{kij}$ and $\epsilon_{jk}$ stand for elastic, piezoelectric and dielectric constants respectively. We have taken a homogeneous transversely isotropic piezoelectric medium with $x_3$ −axis as the symmetric axis of the material and for this type of material there are ten independent constants. Out of these ten constants five are elastic ($c_{11}$ , $c_{12}$ , $c_{13}$ , $c_{33}$ , $c_{44}$) three are piezoelectric ($e_{15}$ , $e_{31}$ , $e_{33}$) and two are dielectric ($\epsilon_{11}$ , $\epsilon_{33}$) constants.

The constitutive Eqs. (1) in terms of components can be written as

$$\begin{cases}\sigma_{11} = c_{11}S_{11} + c_{12}S_{22} + c_{13}S_{33} - e_{31}E_3 , \\ \sigma_{22} = c_{12}S_{11} + c_{11}S_{22} + c_{13}S_{33} - e_{31}E_3 , \\ \sigma_{33} = c_{13}S_{11} + c_{13}S_{22} + c_{33}S_{33} - e_{33}E_3 , \\ \sigma_{23} = c_{44}S_{23} - e_{15}E_2 , \\ \sigma_{31} = c_{44}S_{31} - e_{15}E_1 , \\ \sigma_{12} = \frac{1}{2}(c_{11} - c_{12})S_{12} , \\ D_1 = e_{15}S_{31} + \epsilon_{11}E_1 , \\ D_2 = e_{15}S_{23} + \epsilon_{11}E_2 , \\ D_3 = e_{31}S_{11} + e_{31}S_{22} + e_{33}S_{33} + \epsilon_{33}E_3 .\end{cases} \tag{3}$$

The strain tensor $S_{ij}$ is related to the mechanical displacement $u_i$ as

$$S_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \tag{4}$$

while the electric field $E_i$ is expressible in terms of electric potential $\varphi$ as

$$E_i = -\frac{\partial\varphi}{\partial x_i}, \tag{5}$$

Here, we are interested in shear wave propagation in the $x_1$-direction and causing displacement in $x_3$-direction. So, the components of displacement, electric potential in piezoelectric layer can be expressed as (Bleustein 1968).

$$\begin{gathered}u_1 = u_2 = 0, \\ u_3 = u_3\,(x_1, x_2, t), \\ \varphi = \varphi(x_1, x_2, t).\end{gathered} \tag{6}$$

Substituting the values of $S_{ij}$ and $E_i$ from Eqs. (4)-(5) in Eq. (1) and then making use of Eqs. (2)-(3) and Eq. (6), we get following equations for the propagation of shear waves in piezoelectric layer.

$$(c_{44} + \sigma_{11}^0)\frac{\partial^2 u_3}{\partial {x_1}^2} + c_{44}\frac{\partial^2 u_3}{\partial {x_2}^2} + e_{15}\left(\frac{\partial^2\varphi}{\partial {x_1}^2} + \frac{\partial^2\varphi}{\partial {x_2}^2}\right) = \rho\frac{\partial^2 u_3}{\partial t^2}, \tag{7}$$

$$e_{15}\left(\frac{\partial^2 u_3}{\partial {x_1}^2} + \frac{\partial^2 u_3}{\partial {x_2}^2}\right) - \epsilon_{11}\left(\frac{\partial^2\varphi}{\partial {x_1}^2} + \frac{\partial^2\varphi}{\partial {x_2}^2}\right) = 0 , \tag{8}$$

Substitution of Eq. (6) into Eqs. (3) yields

$$\begin{cases}\sigma_{23} = c_{44}\dfrac{\partial u_3}{\partial x_2} + e_{15}\dfrac{\partial\varphi}{\partial x_2}, \\ \sigma_{31} = c_{44}\dfrac{\partial u_3}{\partial x_1} + e_{15}\dfrac{\partial\varphi}{\partial x_1},\end{cases} \tag{9}$$

$$\begin{cases}D_1 = e_{15}\dfrac{\partial u_3}{\partial x_1} - \epsilon_{11}\dfrac{\partial\varphi}{\partial x_1}, \\ D_2 = e_{15}\dfrac{\partial u_3}{\partial x_2} - \epsilon_{11}\dfrac{\partial\varphi}{\partial x_2}.\end{cases} \tag{10}$$

In Eq. (6), the components of initial stress $\sigma_{22}^0$ and $\sigma_{12}^0$ are neglected as their magnitude is very small (Jin *et al.* 2011) as compared to that of $\sigma_{11}^0$.

The solution of Eqs. (7)-(8) for wave propagation in the $x_1$ direction can be taken as

$$\begin{cases}u_3 = u_3(x_2)e^{i(kx_1 - \omega t)}, \\ \varphi = \varphi(x_2)e^{i(kx_1 - \omega t)} .\end{cases} \tag{11}$$

where $k$ is the wave number and $\omega = kc$ is the circular frequency.

Substituting the values of $u_3$ and φ from Eq. (11) into Eqs. (7)-(8) yields the general solution as follows

$$\begin{gathered}u_3 = (A_1 cos\lambda_1 x_2 + A_2 sin\lambda_1 x_2)e^{i(kx_1 - \omega t)}, \\ \varphi = [B_1 e^{kx_2} + B_2 e^{-kx_2} + \tfrac{e_{15}}{\epsilon_{11}}(A_1 cos\lambda_1 x_2 + A_2 sin\lambda_1 x_2)]\, e^{i(kx_1 - \omega t)},\end{gathered} \tag{12}$$

where $\lambda_1 = k\sqrt{\left(\frac{c^2 - \frac{\sigma_{11}^0}{\rho}}{c_{sh}^2} - 1\right)}$ and $c_{sh} = \sqrt{\frac{c_{44}\epsilon_{11} + (e_{15})^2}{\rho\epsilon_{11}}}$ is the bulk shear wave velocity in the piezoelectric layer. $A_1$, $A_2$, $B_1$ and $B_2$ are arbitrary constants. Solution in Eq. (12) is obtained under the assumption that $c > c_{sh}$.

### *2.2 Dynamics of micropolar elastic half space*

Following Eringen (1966), the equation of motion and constitutive relation for a micropolar elastic space are given by

$$(\mu + \kappa)\nabla^2\vec{u} + (\lambda + \mu)\nabla(\nabla.\vec{u}) + \kappa(\nabla \times \vec{\phi}) = \rho^m\frac{\partial^2\vec{u}}{\partial t^2}, \tag{13}$$

$$(\alpha + \beta + \gamma)\,\nabla(\nabla.\vec{\phi}) - \gamma\nabla \times (\nabla \times \vec{\phi}) + \kappa(\nabla \times \vec{u}) - 2\kappa\vec{\phi} = \rho^m j\frac{\partial^2\vec{\phi}}{\partial t^2}, \tag{14}$$

$$\sigma_{ij}^{m} = \lambda u_{l,l}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) + \kappa\left(u_{j,i} - \epsilon_{ijl}\phi_l\right), \quad (15)$$

$$m_{ij} = \alpha\, \phi_{l,l}\delta_{ij} + \beta\, \phi_{i,j} + \gamma\, \phi_{j,i} \quad (16)$$

where $(i,j,l = 1,2,3)$, $\vec{u}$ is the displacement vector, $\rho^m$ is the density of the micropolar elastic material, $j$ is the microinertia, $\vec{\phi}$ is the microrotation vector, $\lambda, \mu$ are Lame's constants $\kappa, \alpha, \beta, \gamma$ are micropolar material constants. $\sigma_{ij}^{m}$ and $m_{ij}$ are the stress tensor and couple stress tensor respectively. $\epsilon_{ijl}$ is the permutation tensor and $\delta_{ij}$ is the kronecker delta.

For shear wave propagation in the lower micropolar half space let $\vec{u} = (u_1^m, u_2^m, u_3^m)$ be the components of displacement and $\vec{\phi} = (\phi_1, \phi_2, \phi_3)$ denotes the components of microrotation. As shear wave propagates in $x_1$-direction, causing displacement in $x_3$-direction so the components of displacement and microrotation can be expressed as

$$u_1^m = u_2^m = 0,$$

$$u_3^m = u_3^m(x_1, x_2, t), \quad (17)$$

$$\phi_1 = \phi_1(x_1, x_2, t), \quad \phi_2 = \phi_2(x_1, x_2, t), \quad \phi_3 = 0.$$

To decouple the Eqs. (13)-(14), let us introduce the potential functions $\psi$ and $\xi$ as

$$\phi_1 = \frac{\partial\psi}{\partial x_1} + \frac{\partial\xi}{\partial x_2}, \quad \phi_2 = \frac{\partial\psi}{\partial x_2} - \frac{\partial\xi}{\partial x_1}. \quad (18)$$

By Substituting Eqs. (17)-(18) into Eqs. (13)-(14), we obtain wave equations of shear waves in micropolar elastic half space as follows

$$\nabla^2 u_3^m - c_1\nabla^2\xi = \frac{1}{c_2^2}\frac{\partial^2 u_3^m}{\partial t^2}, \quad (19)$$

$$\nabla^2\psi - \frac{2c_5^2}{c_3^2 + c_4^2}\psi = \frac{1}{c_3^2 + c_4^2}\frac{\partial^2\psi}{\partial t^2}, \quad (20)$$

$$\nabla^2\xi - \frac{2c_5^2}{c_3^2}\xi + \frac{c_5^2}{c_3^2}u_3^m = \frac{1}{c_3^2}\frac{\partial^2\xi}{\partial t^2}, \quad (21)$$

where $c_1 = \frac{\kappa}{\mu+\kappa}$, $c_2 = \sqrt{\frac{\mu+\kappa}{\rho^m}}$, $c_3 = \sqrt{\frac{\gamma}{\rho^m j}}$, $c_4 = \sqrt{\frac{\alpha+\beta}{\rho^m j}}$, $c_5 = \sqrt{\frac{\kappa}{\rho^m j}}$.

The solutions of Eqs. (19)- (21) can be expressed as

$$(\psi, \xi, u_3^m)(x_1, x_2, t) = (\psi, \xi, u_3^m)(x_2)e^{i(kx_1 - \omega t)}. \quad (22)$$

Putting the values $\psi, \xi$ and $u_3^m$ from (22) in the Eqs. (19)-(21), we get

$$(D^2 - r^2)\psi(x_2) = 0, \quad (23)$$

$$(D^4 - PD^2 + Q)(\xi, u_3^m)(x_2) = 0, \quad (24)$$

where

$$D = \frac{d}{dx_2}, \quad r^2 = k^2 - \frac{\omega^2}{c_3^2+c_4^2} + \frac{2c_5^2}{c_3^2+c_4^2},$$

$$P = 2k^2 - \omega^2\left(\frac{1}{c_2^2} + \frac{1}{c_3^2}\right) + \frac{c_5^2}{c_3^2}(2 - c_1),$$

$$Q = \left(k^2 - \frac{\omega^2}{c_2^2}\right)\left(k^2 - \frac{\omega^2}{c_3^2} + \frac{2c_5^2}{c_3^2}\right) - \frac{c_1 c_5^2 k^2}{c_3^2}.$$

Using the radiation conditions $\psi(x_2)$, $\xi(x_2)$, $u_3^m(x_2) \to 0$ as $x_2 \to \infty$ on the general solutions of the Eqs. (23)-(24) yields

$$\psi = (De^{-rx_2})e^{i(kx_1 - \omega t)}, \quad (25)$$

$$\xi = (Ee^{-px_2} + Fe^{-qx_2})e^{i(kx_1 - \omega t)}, \quad (26)$$

$$u_3^m = (Es_1e^{-px_2} + Fs_2e^{-qx_2})e^{i(kx_1 - \omega t)}, \quad (27)$$

where

$$p, q = \sqrt{\frac{P \pm \sqrt{P^2 - 4Q}}{2}}, \quad p^2 + q^2 = P, \quad p^2q^2 = Q. \quad (28)$$

From Eqs. (18), (25) and (26), we obtain the components of microrotation as

$$\phi_1 = (ikDe^{-rx_2} - pEe^{-px_2} - qFe^{-qx_2})e^{i(kx_1 - \omega t)}, \quad (29)$$

$$\phi_2 = \left(-rDe^{-rx_2} - ik(Ee^{-px_2} + Fe^{-qx_2})\right)e^{i(kx_1 - \omega t)}. \quad (30)$$

The derivations in Eqs. (25)-(30) are obtained under the assumption that $c < c_2$ and $c < c_3$. The wave corresponding to $c > c_2$ represents refracted waves carrying energy away from the layer. These types of waves are not significant as they lose their energy very quickly.

## 3. Boundary conditions and dispersion relation

For the propagation of shear waves in piezoelectric layer with its surface bonded imperfectly with a micropolar half space, following boundary conditions should be satisfied.

(i) The upper most surface of piezoelectric layer is considered mechanically free and electrically open i.e., at $x_2 = -h$

$$\sigma_{23} = 0 \quad (31)$$

$$D_2 = 0 \quad (32)$$

(ii) As the common interface $x_2 = 0$ between the layer and half space is not perfectly bonded so there will a jump of mechanical displacement at this interface. To characterize the interface imperfection, linear spring model (Baltazar *et al.* 2003, Chen *et al.* 2004 and Chen *et al.* 2008) has been applied which can be expressed as

$$\chi\sigma_{23}^m = u_3^m(x_1, 0, t) - u_3(x_1, 0, t), \quad \sigma_{23} = \sigma_{23}^m, \quad (33)$$

where $\chi$ is the interfacial parameter describing the behavior of imperfect surface. When $\chi = 0$ then interface is perfectly bonded and $\chi \to \infty$ corresponds to the sliding interface.

(iii) At the common interface $x_2 = 0$ between piezoelectric layer and micropolar elastic half space, piezoelectric medium does not exhibit micropolar property so couple stress must vanish at common surface. Also, we considered piezoelectric layer to be electrically shorted at the common interface. From both these conditions, we obtain

$$m_{21} = 0, \qquad m_{22} = 0, \qquad \varphi = 0 \tag{34}$$

Using Eqs. (12), (15)-(16) and (25)-(27) in the boundary conditions (31)-(34), we obtain following equations with seven unknown constants $A_1, A_2, B_1, B_2, D, E, F$.

$$\begin{gathered}
\bar{c}_{44}\lambda_1(A_1 sin\lambda_1 h + A_2 cos\lambda_1 h) + e_{15}k\left(B_1 e^{-kh} - B_2 e^{kh}\right) = 0,\\
B_1 e^{-kh} - B_2 e^{kh} = 0,\\
A_1 = E(s_1 - \chi s_3) + F(s_2 - \chi s_4) - \chi i s_5 D,\\
\bar{c}_{44}\lambda_1 A_2 + e_{15}k(B_1 - B_2) = s_3 E + s_4 F + i s_5 D,\\
\bar{e}_{15}A_1 + B_1 + B_2 = 0,\\
i s_6 D + s_7 E + s_8 F = 0,\\
s_9 D + i s_{10} E + i s_{11} F = 0,
\end{gathered} \tag{35}$$

where, $\bar{e}_{15} = \frac{e_{15}}{\epsilon_{11}}$ , $s_1 = \frac{c_3^2}{c_5^2}\left(k^2 - \frac{\omega^2}{c_3^2} + \frac{2c_5^2}{c_3^2} - p^2\right)$,

$$s_2 = \frac{c_3^2}{c_5^2}\left(k^2 - \frac{\omega^2}{c_3^2} + \frac{2c_5^2}{c_3^2} - q^2\right),$$

$$s_3 = p[-\mu s_1 + \kappa(1 - s_1)], \quad s_4 = q[-\mu s_2 + \kappa(1 - s_2)],$$
$$s_5 = -k\kappa, \quad s_6 = -(\beta + \gamma)kr,$$

$$s_7 = \beta k^2 + \gamma p^2, \quad s_8 = \beta k^2 + \gamma q^2, \quad s_9 = -\alpha k^2 + (\alpha + \beta + \gamma)r^2, \quad s_{10} = (\beta + \gamma)kp,$$

$$s_{11} = (\beta + \gamma)kq,$$

$\bar{c}_{44} = c_{44} + \frac{e_{15}^2}{\epsilon_{11}}$ being the piezoelectrically stiffened elastic constant as defined by Curtis and Redwood (1973). For nontrivial solutions of system of system of Eqs. (35), the determinant of the coefficient matrix must be equal to zero, which leads to

$$\frac{ke_{15}^2 tanh(kh)}{\epsilon_{11}} + \frac{-s_5L_1 + s_3L_2 - s_4L_3}{\chi s_5 L_1 + L_2(s_1 - \chi s_3) - L_3(s_2 - \chi s_4)} = \bar{c}_{44}\ \lambda_1\ tan(\lambda_1 h). \tag{36}$$

where

$$L_1 = s_7 s_{11} - s_8 s_{10},$$
$$L_2 = s_6 s_{11} + s_8 s_9,$$
$$L_3 = s_6 s_{10} + s_7 s_9.$$

Eq. (36) is the dispersion equation of shear waves propagating in the initially stressed piezoelectric layer imperfectly bonded over a micropolar elastic half space.

## 4. Particular case

(i) On substituting $\chi = 0$ and $\sigma_{11}^0 = 0$ in the Eq. (36), we obtain the following dispersion relation of shear waves propagating in the piezoelectric layer perfectly bonded with a micropolar half space.

$$\frac{k\ e_{15}^2 tanh(kh)}{\epsilon_{11}} + \frac{-s_5L_1 + s_3L_2 - s_4L_3}{L_2 s_1 - L_3 s_2} = \bar{c}_{44}k\sqrt{\left(\frac{c^2}{c_{sh}^2} - 1\right)}\,tan\left(kh\sqrt{\frac{c^2}{c_{sh}^2} - 1}\right). \tag{37}$$

In the manufacturing process of composite structure, if there is no residual stress left in the piezoelectric layer and the contact between the layer and the half space is also very perfect, then this equation can be helpful to determine the phase velocity in the structure.

(ii) In the absence of micropolar constants i.e., $(\alpha, \beta, \gamma, j, \kappa \to 0)$ in Eq. (37) we get the following changed values of $c_2$ and $\frac{-s_5L_1+s_3L_2-s_4L_3}{L_2s_1-L_3s_2}$ as

$$c_2 = \sqrt{\frac{\mu+\kappa}{\rho^m}} \to \sqrt{\frac{\mu}{\rho^m}} = \beta_2(\text{say}) \ ,$$

and using Eq. (28), we get

$$\frac{-s_5L_1+s_3L_2-s_4L_3}{L_2s_1-L_3s_2} \to \mu k\sqrt{1 - \frac{c^2}{\beta_2^2}}\ .$$

Substituting the values of $c_2$ and $\frac{-s_5L_1+s_3L_2-s_4L_3}{L_2s_1-L_3s_2}$ in Eq. (37), we obtain

$$\frac{e_{15}^2\ tanh(kh)}{\epsilon_{11}} + \mu\sqrt{1 - \frac{c^2}{\beta_2^2}} = \bar{c}_{44}\sqrt{\frac{c^2}{c_{sh}^2} - 1}\ tan\left(kh\sqrt{\frac{c^2}{c_{sh}^2} - 1}\right). \tag{38}$$

Eq. (38) is the dispersion equation for the propagation of shear waves in piezoelectric layer without initial stress lying perfectly over an elastic half space.

(iii) In the absence of piezoelectric parameter $e_{15}$ , the piezoelectrically stiffened constant reduces to an elastic constant in the layer i.e., $\bar{c}_{44} = c_{44}$. Also, shear wave velocity in the piezoelectric layer becomes the shear wave velocity in the elastic layer i.e., $c_{sh} = \sqrt{\frac{c_{44}}{\rho}}$ and let us denote this velocity by $\beta_1$. On substituting the values of $e_{15}$ , $\bar{c}_{44}$ and $c_{sh} = \beta_1$ in Eq. (38), we obtain

$$c_{44}\sqrt{\frac{c^2}{\beta_1^2} - 1}\ tan\left(kh\sqrt{\frac{c^2}{\beta_1^2} - 1}\right) = \mu\sqrt{1 - \frac{c^2}{\beta_2^2}}. \tag{39}$$

Eq. (39) is the well-known classical equation given by Love (1920).

## 4. Numerical calculations and discussions

To study the behavior of shear wave based upon the dispersion relation (36) numerical computations have been carried out and results are depicted graphically. Values of relevant physical constants used in numerical calculations are listed below, where aluminium epoxy is taken as a

Table 1 The relevant material properties

| | $\rho^m(\mathrm{kg/m^3})$ | $\lambda(\mathrm{N/m^2})$ | $\mu(\mathrm{N/m^2})$ | $\kappa(\mathrm{N/m^2})$ |
|---|---|---|---|---|
| Aluminium Epoxy | $2.19\times10^3$ | $7.59\times10^{10}$ | $1.89\times10^{10}$ | $0.0149\times10^{10}$ |
| | $\alpha(\mathrm{N})$ | $\beta(\mathrm{N})$ | $\gamma(\mathrm{N})$ | $j(\mathrm{m^2})$ |
| | $0.01\times10^6$ | $0.015\times10^6$ | $0.268\times10^6$ | $0.196\times10^4$ |

Table 2 The relevant material properties

| | $\rho$ (kg/m$^3$) | $e_{15}(\mathrm{C/m^2})$ | $C_{44}(\mathrm{N/m^2})$ | $\epsilon_{11}(\mathrm{F/m})$ |
|---|---|---|---|---|
| PZT-4 | $7.5\times10^3$ | 12.7 | $2.56\times10^{10}$ | $6.46\times10^{-9}$ |

micropolar solid and PZT-4 is the piezoelectric material.

For micropolar elastic half space following Gauthier (1982), the material properties are tabulated in Table 1.

The material properties of piezoelectric material (PZT-4) (Wang and Zhao 2013) are given in Table 2.

The non-dimensional phase velocity $(c/c_{sh})$ has been plotted against non-dimensional wave number $(kh/2\pi)$. The non-dimensional interfacial imperfection is taken as $\delta=\chi C_{44}/h$. The thickness of layer is considered to be fixed at $h = 5m$ for numerical calculations.

Fig. 2 compares the non-dimensional phase velocity in the piezo-micropolar structure with the phase velocity of piezo-elastic structure for perfectly bonded interface. First two modes of phase velocity have been compared and the layer is considered to be without any initial stress. It is readily seen from the plot that the micropolarity has a significant effect on the first mode phase velocity. It is found that first mode phase velocity (curve-3) increases due the presence the micropolarity effect in piezo-elastic structure (curve-1). In second mode, the phase velocity dispersion curves of piezo-micropolar structure coincides with the piezo-elastic structure (curve-2 and curve-4), which means that micropolarity effect reduces on higher modes. For large wave number values all the velocity curves approaches the bulk shear wave velocity of piezoelectric material.

Fig. 3 represents the effects of interfacial imperfection parameter on the first mode phase velocity in piezo-micropolar and piezo-elastic structure. Curve-1 and curve-3 represents the phase velocity in piezo-micropolar case for two different values of imperfection parameter $\delta = 0.1$ and $\delta = 1$. It is observed that increasing imperfection parameter value decreases the phase velocity. Similar effect of imperfection interface is observed for piezo-elastic structure (curve-2, curve-4).

Figs. 4-5 shows the effects micropolar parameter $\kappa/\mu$ on the first mode of non-dimensional phase velocity in piezo-micropolar elastic structure for perfectly and imperfectly bonded interface. As clear from Fig. 4, first mode phase velocity increases with even small increment in the value of micropolar parameter. Further, with increase in wave number value, micropolarity effect decreases and all the phase velocity curves approaches the bulk shear wave velocity of piezoelectric material. Same behavior is noted in case of imperfect interface as shown in Fig. 5. Figs. 4-5 endorsed the fact that micropolarity plays important role in guiding the behavior of shear waves.

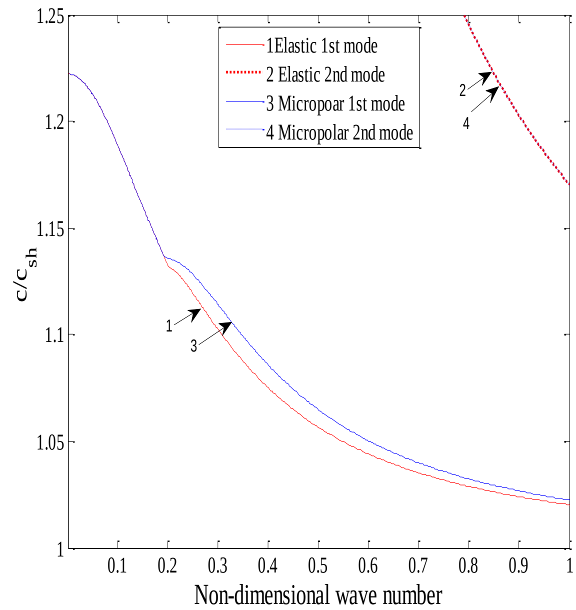


Fig. 2 Dispersion curves for Piezo-Micropolar and Piezo-Elastic structure for perfectly bonded interface ($\delta = 0$)

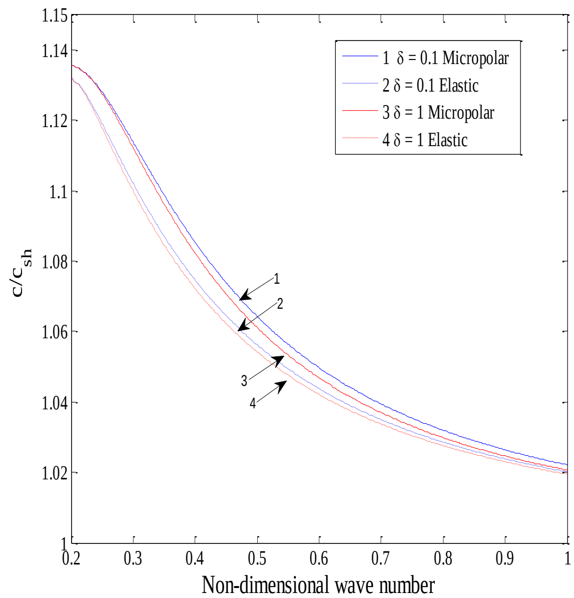


Fig. 3 Effect of imperfection parameter on the first mode phase velocity in Piezo-Micropolar and Piezo-Elastic structure ($\delta = 0.1$)

Figs. 6-7 are drawn to investigate the effects of initial stress on the phase velocity in Piezo-Micropolar structure in perfectly and imperfectly bonded interface. It is observed that the initial stress has negligible effect for $|\sigma_{11}^0| < 10^8$ Pa. On the other hand, when the initial stress is greater than certain value ($|\sigma_{11}^0| > 10^8$Pa), it has significant effects on the properties of shear wave propagation. The influence of initial stress is also dependent on the thickness of the layer. As shown in Figs. 6-7, it is found that the phase velocity of first and second mode increases with increase of initial tensile stress and it decreases with increase of initial compressive stress in the piezoelectric layer.

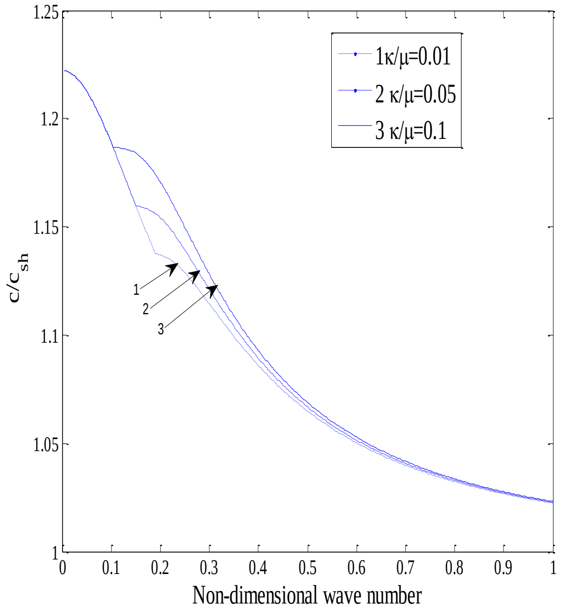


Fig. 4 Effect of micropolar parameter (κ/μ) on the first mode phase velocity in Piezo-Micropolar structure with perfect bonded interface ($\delta = 0$)

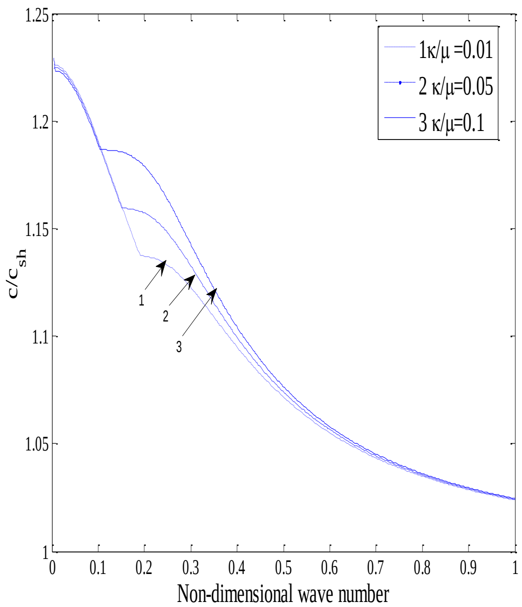


Fig. 5 Effect of micropolar parameter (κ/μ) on the first mode phase velocity in Piezo-Micropolar structure with imperfect bonded interface ($\delta = 0.1$)

## 5. Conclusions

This paper presents the study of SH wave propagation in an initially stressed piezoelectric layer which is imperfectly bonded over a micropolar elastic solid. A general dispersion equation of the wave is derived and the relation obtained as a particular case is in agreement with the classical equation given by Love. Numerical computations are performed and the results are depicted graphically to investigate the effects of various parameters such as micropolarity, interfacial imperfection defects and initial stress on the phase velocity

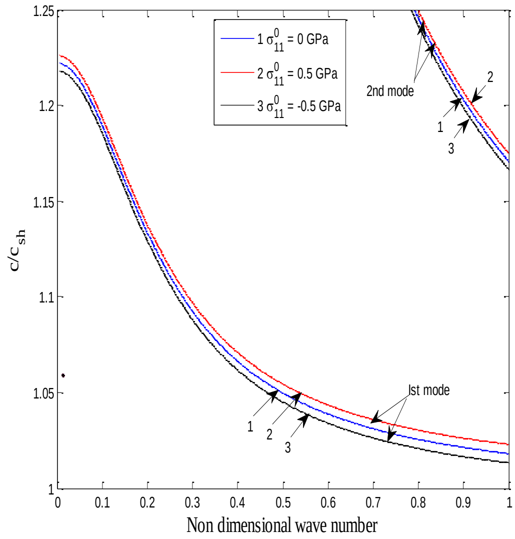


Fig. 6 Effect of initial stress on first and second mode phase velocity in Piezo-Micropolar structure with perfectly bonded interface ($\delta = 0$)

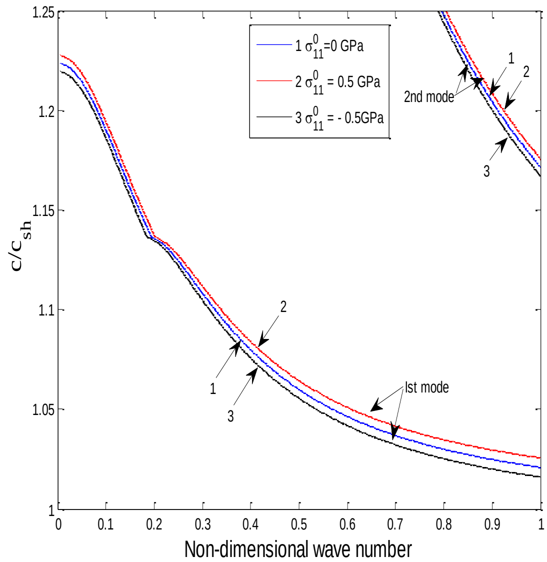


Fig. 7 Effect of initial stress on first and second mode phase velocity in Piezo-Micropolar structure with imperfectly bonded interface ($\delta = 0.1$)

in the Piezo-Micropolar structure. Following conclusion may be drawn from the present study.

• Shear waves exist in the composite structure consisting of an initially stressed piezoelectric layer imperfectly bonded over a micropolar elastic material and there is significant effect of micropolarity on the propagation of shear waves. The phase velocity of waves increases in a particular region due to micropolar nature of elastic half space and then vanishes with increasing wave number.

• The presence of interfacial imperfection lowers the phase velocity in the Piezo-Micropolar elastic as well as Piezo-Elastic composite structure. Depending upon the combination of wave number and the thickness of the layer the phase velocity of shear waves is substantially influenced by imperfection parameter.

• For a fixed thickness of the piezoelectric layer, an initial stress of $|\sigma_{11}^0| > 10^8$ Pa has significant effects on the phase velocity and this effect is negligible for $|\sigma_{11}^0| < 10^8$ Pa. For considered layer thickness, phase velocity increases with increase of initial tensile stress and decreases with increase of initial compressive stress in the layer.

The performance of surface acoustic wave sensors based on layered structure depends upon the combination of piezoelectric layer with suitable elastic material and in the present study a micropolar elastic material has been considered for this purpose. The results obtained in this model may provide a theoretical foundation for design and development of new piezoelectric sensors and surface acoustic devices.

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