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Weighted sum multi-objective optimization of skew composite laminates

Kanak Kalita*[1], Uvaraja Ragavendran[2], Manickam Ramachandran[3] and Akash Kumar Bhoi[4]

[1]*Department of Aerospace Engineering and Applied Mechanics, Indian Institute of Engineering, Science and Technology, Howrah, West Bengal 711103, India*
[2]*Department of Electronics and Telecommunication Engineering, MPSTME SVKM'S Narsee Monjee Institute of Management Studies, Shirpur, Maharashtra 425405, India*
[3]*Department of Mechanical Engineering, MPSTME SVKM'S Narsee Monjee Institute of Management Studies, Shirpur, Maharashtra 425405, India*
[4]*Department of Electrical & Electronics Engineering, Sikkim Manipal Institute of Technology, Sikkim Manipal University, Majhitar, Sikkim 737136, India*

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Abstract. Optimizing composite structures to exploit their maximum potential is a realistic application with promising returns. In this research, simultaneous maximization of the fundamental frequency and frequency separation between the first two modes by optimizing the fiber angles is considered. A high-fidelity design optimization methodology is developed by combining the high-accuracy of finite element method with iterative improvement capability of metaheuristic algorithms. Three powerful nature-inspired optimization algorithms viz. a genetic algorithm (GA), a particle swarm optimization (PSO) variant and a cuckoo search (CS) variant are used. Advanced memetic features are incorporated in the PSO and CS to form their respective variants-RPSOLC (repulsive particle swarm optimization with local search and chaotic perturbation) and CHP (co-evolutionary host-parasite). A comprehensive set of benchmark solutions on several new problems are reported. Statistical tests and comprehensive assessment of the predicted results show CHP comprehensively outperforms RPSOLC and GA, while RPSOLC has a little superiority over GA. Extensive simulations show that the on repeated trials of the same experiment, CHP has very low variability. About 50% fewer variations are seen in RPSOLC as compared to GA on repeated trials.

Keywords: cuckoo search; finite element method; genetic algorithm; multiobjective optimization; particle swarm optimization

1. Introduction

Skew composite laminates find wide application in aircraft, marine, civil and mechanical engineering industry. Skew laminates serve various functional, structural or aesthetic requirements in these sectors. Some common uses are wings, tails, fins of swept-wing aircraft, missiles, ship hulls, skew bridge decks etc. (Lee and Park 2009, Vosoughi *et al*. 2018). From a designer's perspective, their dynamic response is of great interest. It is often desired that such structural components are not in resonance with the external excitation frequencies. Resonance may be avoided by designing the structures such that they operate well beyond the range of external sources. However, for any particular application the geometric dimensions like length, width, thickness, skewness etc. are not independently alterable. Changing one such geometric parameter would mean significant changes in the design and therefore, would require not only a revision of the composite structure but of its associated components as well. In contrast, changing the fiber angles to alter the frequency parameters is not associated with such complications. For any pre-decided thickness, material and number of plies, the fiber angles can be independently altered without disturbing the physical design. This is why studies concerning optimal layup angles to maximize fundamental frequency (Ameri *et al*. 2012, Apalak *et al.* 2014) or frequency separation (Farshi and Rabiei 2007, Duffy and Adali 1991) has been so popular.

The optimal fiber angle combination problem is an NP-hard problem. Over time, both deterministic and stochastic search algorithms have been used to tackle this combinatorial optimization problem. Genetic algorithm (GA) (Ameri *et al*. 2012, Apalak *et al*. 2014) ant colony optimization (ACO) (Koide *et al.* 2013, Koide and Luersen 2013), PSO (Bargh and Sadr 2012, Ghashochi-Bargh and Sadr 2013), artificial bee colony (ABC) (Apalak *et al*. 2014) are the some of the most commonly used stochastic search algorithms in composite laminate optimization.

Le Riche and Haftka were among the first researchers to apply GA to composite laminate optimization problems. They used GA for buckling load maximization (Le Riche and Haftka 1993), thickness minimization (Le Riche and Haftka 1995), laminate optimization (Grosset *et al*. 2006), etc. However, there are only a few comparative studies on the performance of different stochastic search algorithms in composite laminate frequency parameter optimization. Apalak *et al.* (2014) used an ABC algorithm and a GA to maximize the fundamental frequency of composite plates using fiber angles as a design variable. They observed that despite the ABC algorithm having a simpler structure than

*Corresponding author
E-mail: kanakkalita02@gmail.com

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GA, was as much effective. Ameri *et al*. (2012) used a hybrid Nelder-Mead algorithm and a GA to find optimal fiber angles to maximize fundamental frequency. They observed that the hybrid Nelder-Mead algorithm was faster and more accurate than the GA. However, it is hard to state whether the superior performance of the Nelder-Mead algorithm was genuinely due to algorithmic superiority or because the authors chose to incorporate the design variables as continuous in Nelder-Mead algorithm whereas in GA they considered discrete values. Similarly, Koide *et al*. (2013) used an ACO algorithm to maximize the fundamental frequency in cylindrical shells and compared their optimal solutions with GA solutions from literature. The ACO predictions were at par with the GA counterparts. Tabakov and Moyo (2017) compared the performance of GA, PSO and Big Bang-Big Crunch algorithm while considering a burst pressure maximization problem in a composite cylinder. Hemmatian *et al*. (2014) used an imperialist competitive algorithm (ICA) along with GA and ACO to simultaneously optimize the weight and cost of a rectangular composite plate. They reported that in terms of objective function magnitude and constraint accuracy, ICA outperformed GA and ACO.

Another category in which the literature lacks is in the application of optimization algorithms to multi-criteria design optimization of composite laminates. Primarily, the multi-criteria design optimization results may be broadly expressed in two forms-a set of not-dominated optimal solution points called Pareto front or a unique solution corresponding to any particular pre-decided decision criteria. Any solution is said to be Pareto optimal if one of its objectives can be improved only by worsening at least one of its other objectives. However, in practice, usually, only one solution is required (Jakob and Blume 2014) and thus, the search for Pareto optimal fronts in high-fidelity applications involving finite element simulations would lead to huge computation cost. Nevertheless, the importance of presenting the decision maker with a host of non-dominated solutions to choose from and thus, taking a better-informed decision cannot be denied. Correia *et al.* (2017) considered stacking sequence as the design variables while maximizing frequency parameters and minimizing strain energy in composite plates with piezoelectric layers. Vo-Duy *et al*. (2017) used non-dominated sorting genetic algorithm II (NSGA-II) in conjunction with finite element method to minimize weight and maximize the frequency of composite plates. Ghasemi and Hajmohammad (2017) used a similar NSGA-II based strategy to minimize the cost of composite shells while increasing its buckling strength. While Correia *et al*. (2017), Vo-Duy *et al*. (2017), and Ghasemi and Hajmohammad (2017) made use of Pareto fronts, other like Abachizadeh and Tahani (2009), Sudhagar *et al*. (2017), Hemmatian *et al.* (2014) and Topal (2009) have used weighted sum approach to report unique multiobjective solution points.

In this article, several problems on multiobjective optimization of skew composite laminates are solved using three different nature-inspired optimization algorithms. A genetic algorithm is selected as the first optimization algorithm due to its wide popularity. GA, since its inception in the 1970s, has been virtually applied to all classes of optimization problems with significant success. Thus, it is an interesting exercise to see how well the other two algorithms fare against GA. Particle swarm optimization is another popular bio-inspired swarm intelligence-based optimization algorithm that has gained significant popularity due to its easy to implement structure. According to Zhang *et al.* (2015), PSO is the most widely used swarm intelligence-based optimization algorithm. Thus, in this research, the standard PSO is upgraded to form a robust high-fidelity optimization tool by incorporating certain advanced memetic features in it. In addition to these two algorithms, a cuckoo search algorithm is also in the current work. Cuckoo search is a recent but powerful addition to the nature-inspired optimization family. So far, it has shown significant potential in tackling several NP-hard problems. Due to the relatively less amount of work done on it so far, tremendous potential exists to improve it further to form more powerful variants. Thus, in this research, the standard cuckoo search is significantly improved by incorporating certain new features to the basic algorithm design. In this research, these three metaheuristic algorithms are combined with a first order shear deformation theory based finite element model to form a high-fidelity optimization tool for frequency parameter optimization of laminated composites. The applicability of the weighted sum approach to predict optimal designs that satisfy multiple objectives is shown.

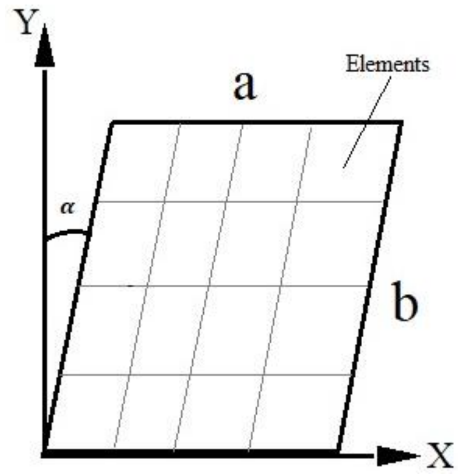

Fig. 1 Schematic of the skew plate with finite element mesh

Algorithm 1:
begin
Objective function, y=f(x)
Initiate generation, t=0
Initiate a random population P_t by encoding
Calculate fitness of all individuals in the population using FEM
do
{
Create new population C_t from P_t by crossover
Modify population C_t by mutation
Evaluate population C_t by decoding
Calculate fitness of all individuals in the population using FEM
Select P_{t+1} from C_t by selection
t=t+1
}
until *($t=t_{max}$) or (stop criterion)*
report the best individual
end

Fig. 2 Algorithm for FE-GA

2. Problem description

A symmetric layered composite with dimensions (a × b × h) having n layers is considered in this study (Fig. 1). The plate is skewed at an angle α. Material is considered as a Graphite-epoxy composite (Jones, 1975) $E_1 = 138\,GPa$, $E_2 = 8.96\,GPa$, $G_{12} = 7.1\,GPa$, $\upsilon_{12} = 0.3$. The thickness of the plate is considered to be moderate h/a=0.01. The objective is to simultaneously maximize fundamental frequency ($\boldsymbol{\lambda_1}$) and frequency separation between the first two modes ($\boldsymbol{\lambda_{21}}$) based on the optimal ply-angle arrangements for a given set of geometric parameters and boundary conditions.

The problems are formulated as unconstrained optimization problems. Each multiobjective problem is converted to an equivalent single-objective problem by applying the concept of weighted-sum. Weighted-sum approach allows aggregation of different optimization criteria to a single quality value (Jakob and Blume 2014, Abachizadeh and Tahani 2009). Since λ_1 and λ_{21} have different scales, first, they must be normalized.

$$\lambda_{1_{norm}} = \frac{\lambda_1}{\lambda_{1_{max}}} \; and \; \lambda_{21_{norm}} = \frac{\lambda_{21}}{\lambda_{21_{max}}} \tag{1}$$

Where, $\lambda_{1_{max}}$ and $\lambda_{21_{max}}$ are the global maxima for fundamental frequency and frequency separation reported in Kalita (2018).

The unconstrained optimization problem may be stated as,

$$\text{Maximize } \lambda_{1-21} = w_1 \cdot \lambda_{1_{norm}} + w_2 \cdot \lambda_{21_{norm}} \tag{2}$$

with the limits, $-90^o \leq \theta_j \leq 90^o$

where, θ_j is the ply-angle of the j[th] ply among total n plies. w_1 and w_2 are the weights assigned to objective 1 and objective 2 respectively. The weights can be anything provided that the sum of weights equals unity. The individual weight values for each objective is decided by the designer depending on the priorities of the target output responses. In this work, equal weights are imposed on λ_1 and λ_{21} i.e. $w_1 = w_2 = 0.5$.

3. Methodology

In the current work, a finite element (FE) formulation is used to simulate the natural frequencies. The FE formulation uses a nine-node isoparametric plate bending element. Rotary inertia and shear deformation are included by considering first order shear deformation theory (FSDT). The finite element formulation used in the present work has been extensively used, discussed and validated by the author(s) in their previous papers (Kalita and Haldar 2017, Kalita *et al*. 2016, 2018).

3.1 Genetic algorithm

Genetic Algorithm (GA) is a population-based evolutionary algorithm. It is based on the principle of natural selection, which states that biological evolution is a continuous process (Kalita *et al*. 2018). Any typical GA contains four main characteristics, (1) a population of candidate solutions, (2) a means of calculating the goodness of solutions, (3) a method of combining certain parts of 'good' solutions to form, in general, 'better' solutions, (4) a way to introduce some random diversity into the solutions.

```
Algorithm 2:
begin
    Objective function, y=f(x)
    Initiate generation, t=0
    Initiate a random swarm S_t of 'n' particles
        do
        {
            Update inertia weight
            do for all particles
                {
                Select a random particle (say k)
                Calculate fitness using FEM
                If current fitness > pBest in history
                    {
                    Set current fitness = pBest
                    }
                Initiate local search
                  {
                Search for best in neighbourhood
                If best in neighbourhood fitness > pBest in history
                    {
                    Set best in neighbourhood = pBest
                    Update best in neighbourhood position as current position
                    }
                  }
                Select a random particle (say m)
                Update velocity of particle k using eqn. (5)
                Update position of particle k using eqn. (4)
                }
            Select a random particle (say l)
            If particle l is caught in local optima
                {
                Insert chaotic perturbation
                }
        }
        until (t=t_max) or (stop criterion)
    report the best individual
end
```

Fig. 3 Algorithm for FE-RPSOLC

GA initiates by assuming a set of candidate solutions, called population. Using its inherent rule-based mechanism, the GA updates the population, trying to make the population 'fitter' as generations progress. In each generation, the GA uses the current generation population to create the next generation population. The updating of the population is done by means of selection, crossover and mutation. Selection is the mechanism by which the GA selects certain members, called parents, to undergo crossover. Parts of these parents are randomly recombined to form children that make up the next generation. Random changes are made in certain individuals to form new individuals by means of mutation. This process continues until the termination criteria are met. In this case, the algorithm terminates when the total predetermined number of finite-element iterations is reached, and it reports back the best solution encountered among all the generation. The algorithm of the FE-GA is illustrated in Fig. 2.

3.2 Repulsive particle swarm optimization with local search and chaotic perturbation

Much like GA, particle swarm optimization (PSO) too is a population-based algorithm. It is based on the flocking behavior of birds or swarm behavior of insects. The standard particle swarm optimization (SPSO) algorithm

```
Algorithm 3:
begin
    Objective function, y=f(x)
    Initiate generation, t=0
    Initiate a random population of 'n_h' hosts and 'n_p' parasites
    Calculate fitness of all individuals in the population using FEM
      do
       {
       Update detection probability, P_t^det as per eqn. 8
       do for all parasites
           {
           Select a random parasite (say m) with fitness f(p_t^m)
           Update its position via Lévy flight as per eqn. 7
               {
               if post-flight f(p_t^m) >pre-flight f(p_t^m)
               else maintain pre-flight position
               }
           Select a random uninvaded host nest (say l) with fitness f(h_t^l)
           if f(p_t^m)>f(h_t^l) and parasite egg not detected
           Replace l by new solution
           end if
           }
               do for all uninvaded hosts
               {
               Select a random uninvaded host nest (say k) with fitness f(h_t^k)
               Update its position via Lévy flight as per eqn. 9
                   {
                   if post-flight f(h_t^k) >pre-flight f(h_t^k)
                   else maintain pre-flight position
                   }
               }
       }
       Select best n_p parasites to enter the next generation
    until (t=t_max) or (stop criterion)
  Report the best individual
end
```

Fig. 4 Algorithm for FE-CHP

starts by assuming a swarm, S_t of 'n' particles. Every particle that makes up the swarm has access to some information. Firstly, they know the current value of their solution and their current position which is a solution to the problem, the algorithm is trying to solve. Each particle tracks its personal best value it has attained ('*pBest*') and the position that was achieved. Each particle also has access to the global best solution value ('*gBest*') and the positions at which this was discovered. Lastly, a particle is aware of its current velocity, i.e., how fast its position is changing. Any k^{th} particle continuously updates its velocity and position as follows

$$v_{t+1}{}^{k} = \omega . v_t{}^{k} + c_1 . r_1 . (pBest^k - x_t{}^{k}) + c_2 . r_2 . (gBest - x_t{}^{k}) \tag{3}$$

$$x_{t+1}{}^{k} = x_t{}^{k} + v_{t+1}{}^{k} \tag{4}$$

Where, subscripts t and $(t+1)$ represent the current and the next iteration, r_1, r_2 generates random numbers between 0 to 1, c_1 and c_2 are the cognitive and social parameters respectively. v and x represent velocity and position respectively. ω is inertia weight, which controls the influence of the last velocity on the current velocity.

Repulsive particle swarm optimization (RPSO), developed by Urfalıoglu in (2004) is a PSO variant in which the velocity of the particle is updated as,

$$v_{t+1}{}^{k} = \omega . v_t{}^{k} + \alpha . r_1 . (pBest^k - x_t{}^{k}) + \omega . \beta . r_2 . (pBest^m - x_t{}^{k}) + \omega . \gamma . r_3 . v_t{}^{r} \tag{5}$$

Table 1 Non-dimensional fundamental frequencies $\boldsymbol{\lambda = \omega a^2/h\sqrt{\rho/E_2}}$ for simply supported cross-ply ($0^0/90^0$) square laminates. [E_1/E_2 = 40, G_{12} = G_{13}= $0.6E_2$, G_{23} = $0.5E_2$, ν_{12}= 0.25]

Source	h/a				
	0.25	0.1	0.05	0.02	0.01
Current	8.0350	10.4730	11.0780	11.2700	11.3000
RPT (Thai and Kim 2010)	8.2651	10.548	11.0997	11.2742	11.2999
(Error %)	(2.78)	(0.71)	(0.20)	(0.04)	(0.00)
TSDT (Reddy 1997)	8.3546	10.568	11.1052	11.2751	11.3002
(Error %)	(3.83)	(0.90)	(0.24)	(0.05)	(0.00)
FSDT (Whitney and Pagano 1970)	8.0349	10.4731	11.0779	11.2705	11.2990
(Error %)	(0.00)	(0.00)	(0.00)	(0.00)	(-0.01)

Here, α, β, γ are constants, $pBest^k$ is the personal best position of the k^th particle whereas $pBest^m$ is the personal best position of a randomly chosen m^th particle from the swarm population. $v_t{}^{r}$ is a random velocity component.

In the current work, the traditional RPSO is further modified by augmenting its local search capability as per the suggestions of Mishra *et al.* (2010). Instead of making complicated changes to the existing velocity updating scheme of RPSO, a separate independent module is built-in to help the particles in local search. Each particle can visit its surrounding and search for a better solution. The domain of its search is controlled by a local search. This local search has no preference for gradients in any direction and resembles closely to tunneling (Santos *et al.* 2010). This added exploration capability makes the modified RPSO more realistic. Another important feature, called chaotic perturbation (r_{chaos}) is also included. Whenever a particle gets trapped in local optima, a chaotic perturbation is included by multiplying it to the velocity component in particle position updating equation reported in eqn. 4, such that

$$x_{t+1}{}^{k} = x_t{}^{k} + v_{t+1}{}^{k} . (1 + r_{chaos}) \tag{6}$$

The modified RPSO is henceforth called as Repulsive Particle Swarm Optimization with local search and chaotic perturbation (RPSOLC). The algorithm of the FE-RPSOLC is illustrated in Fig. 3.

3.3 Co-evolutionary host-parasite optimization

Cuckoo search algorithm (CS), is an algorithmic implementation of the parasitic behavior of cuckoos in laying eggs in crow nests. In the current work, the traditional cuckoo search is endowed with better memetic attributes. The traditional CS does not present a strategy for the crows to regenerate their nests and thus, there is no scope of coevolution. This is countered in the present work by allowing crows to take Lévy flights. Secondly, in the traditional cuckoo search, the detection probability is pre-decided (Mishra 2013), which in real life scenario is not necessarily true. In fact, wherever there is cuckoo and crow interaction the detection rate would increase. This is modeled in the current work by using the Gompertz function, a sigmoid curve to model the detection parameter. The modified cuckoo search, originally proposed by Mishra (2013) is called Co-evolutionary host-parasite (CHP).

Table 2 Optimal layer orientation for 8-layered symmetric $[\theta_1/\theta_2/\theta_3/\theta_4]_s$ rhombic composite plate (a/b=1) for max. λ_{1-21}

BC	FE-GA				FE-RPSOLC				FE-CHP			
	$\alpha = 30^o$		$\alpha = 60^o$		$\alpha = 30^o$		$\alpha = 60^o$		$\alpha = 30^o$		$\alpha = 60^o$	
	Optimal layup	λ_{1-21}	Optimal layup	λ_{1-21}	Optimal layup	λ_{1-21}	Optimal layup	λ_{1-21}	Optimal layup	λ_{1-21}	Optimal layup	λ_{1-21}
SSSS	[30/-65/-55/-40]s	0.8971	[65/-403]s	0.8803	[30/-603]s	0.8975	[70/-403]s	0.8840	[30/-603]s	0.8975	[-40/75/70/-55]s	0.8812
SSSF	[20/-65/-80/20]s	0.8499	[-20/-903]s	0.9500	[-70/102/-70]s	0.8329	[-25/85/-90/-85]s	0.9530	[15/-75/-60/-55]s	0.8513	[-25/85/-902]s	0.9531
SSSC	[-20/802/-90]s	0.9117	[-55/802/-65]s	0.9255	[-20/80/752]s	0.9118	[-50/85/752]s	0.9245	[-20/75/-90/40]s	0.9111	[-50/85/80/85]s	0.9246
SSFF	[30/-55/-65/30]s	0.9406	[20/-80/∓5]s	0.9813	[30/-602/25]s	0.9418	[15/-75/152]s	0.9961	[30/-602/25]s	0.9418	[15/-70/15/0]s	0.9920
SSCF	[20/-653]s	0.7957	[-65/-552/-65]s	0.9638	[20/-65/-80/-65]s	0.7947	[-604]s	0.9646	[20/-75/-65/-45]s	0.7950	[-55/-65/90/-80]s	0.9588
SSCC	[80/-203]s	0.9545	[-90/-552/-80]s	0.9931	[70/-253]s	0.9547	[-65/75/-60/80]s	0.9907	[-20/802/65]s	0.9540	[-60/802/-25]s	0.9907
SFCF	[-304]s	0.8218	[-40/-552/20]s	0.8608	[-304]s	0.8218	[-50/-30/50/-10]s	0.8618	[-35/-30/-15/-30]s	0.8131	[-45/-25/55/30]s	0.8598
SCSF	[-80/53]s	0.8968	[-20/802/-30]s	0.9789	[10/-90/-60/-65]s	0.8895	[-20/802/-20]s	0.9792	[15/-75/-60/-55]s	0.8973	[-20/802/65]s	0.9781
SCSC	[-20/80/-5/80]s	0.8327	[-552/80/90]s	0.8852	[-15/80/-20/80]s	0.8331	[-50/90/-50/-55]s	0.8858	[-20/65/-80/-30]s	0.8307	[-50/-65/75/-55]s	0.8847
SCFF	[30/-55/5/-55]s	0.9282	[20/5/-802]s	0.9753	[25/-602/20]s	0.9339	[15/-80/102]s	0.9828	[25/-602/20]s	0.9339	[15/-80/102]s	0.9828
SCCF	[-80/20/52]s	0.8436	[-80/-40/-55/-65]s	0.9648	[-80/15/102]s	0.8467	[-80/-40/-55/-65]s	0.9648	[-80/15/102]s	0.8467	[-75/-35/-75/-70]s	0.9661
CSCF	[-65/303]s	0.8219	[-40/-80/-55/-40]s	0.9860	[-65/302/25]s	0.8220	[-40/-80/-502]s	0.9862	[-65/302/25]s	0.8220	[-40/-80/-502]s	0.9862
CFFF	[20/-403]s	0.8146	[-40/202/5]s	0.9041	[25/-453]s	0.8212	[-45/15/20/15]s	0.9079	[25/-453]s	0.8212	[-45/15/20/15]s	0.9079
CFCF	[30/-65/302]s	0.7300	[5/802/5]s	0.7786	[(30/-70) 2]s	0.7359	[10/70/15/10]s	0.7820	[(30/-70) 2]s	0.7359	[10/752/10]s	0.7991
CCFF	[20/65/-402]s	0.9634	[20/5/-80/-65]s	0.9936	[30/-45/80/45]s	0.9622	[152/-752]s	0.9981	[30/-45/70/55]s	0.9638	[152/-752]s	0.9981
CCCS	[80/-203]s	0.8994	[-804]s	0.9592	[80/-203]s	0.8994	[-804]s	0.9592	[80/-203]s	0.8994	[-804]s	0.9592
CCCF	[-80/202/-80]s	0.8080	[-40/-80/-30/-65]s	0.9862	[-75/25/20/-75]s	0.8106	[-35/-80/-65/-40]s	0.9872	[-75/25/20/-75]s	0.8106	[-35/-80/-70/-40]s	0.9873
CCCC	[-20/803]s	0.9500	[-80/-65/-802]s	0.9944	[80/-20/-30/75]s	0.9439	[-754]s	0.9970	[-20/802/85]s	0.9500	[-754]s	0.9970

CHP is initialized by assuming a population of 'n_h' hosts and 'n_p' parasites each distributed randomly in the search space. The fitness of each host and parasite is evaluated. The fitness of k^{th} host in the host population n_h and the m^{th} parasite in the parasite population n_p is expressed as $f(h_t{}^k)\ and\ f(p_t{}^m)$. The suffix t represents the generation or iteration counter. Each parasite than takes a Lévy flight and tries to update its position, which is given for the m^{th} parasite as,

$$p_{t+1}{}^m = p_t{}^m + [\alpha(r_1 - 0.5). Levy(\beta)]. [h_t - p_t{}^m] \tag{7}$$

Where $\alpha = 0.0001 + r_2{}^2; \beta = \frac{3}{2}$

If the parasite's post-flight fitness is worse than its pre-flight fitness, then it does not attempt to update its position. However, if its post-flight fitness is better than the pre-flight fitness, it randomly chooses a host nest that has not been invaded yet and lays an egg provided that the host egg is inferior to the parasite egg. But the parasite eggs may be detected with a probability, $P_t{}^{det}$ and destroyed by the host. At each next iteration (t+1), $P_{t+1}{}^{det}$ increase as per the Gompertz growth curve rule as

$$P_{t+1}{}^{det} = P_{max.}{}^{det}. e^{-2.e^{-\left(1+\ln\left(1+P_t{}^{det}\right)\right)^{-1}}} \tag{8}$$

However, if undetected, the parasite egg hatches and joins the parasite population. But only the best n_p parasites enter the next generation. This is like elitism used in genetic algorithms.

Like the parasites, after every iteration, each uninvaded host also takes a Lévy flight to update its position which is given for the k^{th} host as,

$$h_{t+1}{}^k = h_t{}^k + [\omega(r_3 - 0.5). Levy(\gamma)]. [p_t - h_t{}^k] \tag{9}$$

Where $\omega = 0.0001 + r_4{}^2; \gamma = \frac{5}{3}$

If a better post-flight fitness is found, the host updates itself otherwise maintains its pre-flight position. This continues until a pre-specified tolerance level or the maximum number of generations is not reached. The algorithm of the modified CS i.e., FE-CHP is illustrated in Fig. .

4. Results and discussion

In this section, the finite element coupled metaheuristic approaches discussed in section 3 are used to solve certain composite laminate weighted sum multiobjective optimization problems. Due to the paucity of space, validation of the finite element formulation is reported very briefly. Table 1 shows the comparison of the current FSDT based FE simulation results along with solutions obtained using RPT (Thai and Kim 2010), TSDT (Reddy 1997) and Pagano's FSDT (Whitney and Pagano 1970). It is clear that for the range of thickness (i.e., h/a=0.01) considered in this study, the current FSDT is as accurate as RPT and TSDT. Thus, for this study, the current FSDT based FE formulation can be confidently used without worrying about the inherent limitations of FSDT.

Table 3 Optimal layer orientation for 8-layered symmetric $[\theta_1/\theta_2/\theta_3/\theta_4]_s$ skew composite plate (a/b=2) for max. λ_{1-21}

BC	FE-GA				FE-RPSOLC				FE-CHP			
	$\alpha = 30^o$		$\alpha = 60^o$		$\alpha = 30^o$		$\alpha = 60^o$		$\alpha = 30^o$		$\alpha = 60^o$	
	Optimal layup	λ_{1-21}	Optimal layup	λ_{1-21}	Optimal layup	λ_{1-21}	Optimal layup	λ_{1-21}	Optimal layup	λ_{1-21}	Optimal layup	λ_{1-21}
SSSS	[-40/403]s	0.8260	[∓40/±40]s	0.8021	[-45/403]s	0.8267	[∓45/±45]s	0.8128	[-40/402/-55]s	0.8253	[-45/452/-45]s	0.8128
SSSF	[5/-55/202]s	0.9432	[-30/55/-20/-5]s	0.9835	[10/-55/52]s	0.9447	[25/60/-30/-20]s	0.8018	[10/-55/52]s	0.9447	[-25/60/-30/-20]s	0.9857
SSSC	[-40/403]s	0.7885	[∓40/±40]s	0.7643	[-35/40/35/30]s	0.7858	[(-35/40) 2]s	0.7591	[-40/35/40/35]s	0.7890	[-40/402/-55]s	0.7638
SSFF	[30/-552/30]s	0.9785	[20/-55/52]s	0.9216	[30/-502/30]s	0.9807	[15/-55/202]s	0.9232	[30/-50/-60/-55]s	0.9774	[15/-55/202]s	0.9232
SSCF	[-5/80/-302]s	0.9296	[-40/55/-402]s	0.9591	[-10/-90/±15]s	0.9302	[-40/60/-40/-30]s	0.9597	[-10/-90/±15]s	0.9302	[-35/60/-55/25]s	0.9565
SSCC	[-30/40/-302]s	0.7812	[∓40/-30/-40]s	0.7585	[∓35/±35]s	0.7840	[∓35/-352]s	0.7531	[∓35/-30/35]s	0.7794	[∓35/-352]s	0.7531
SFCF	[(20/-55)2]s	0.7990	[-5/-65/-55/5]s	0.9243	[(20/-55)2]s	0.7990	[0/-602/-10]s	0.9239	[(20/-55)2]s	0.7990	[0/-602/-10]s	0.9239
SCSF	[5/-65/-5/-20]s	0.8764	[(-30/55)2]s	0.8672	[0/-65/10/5]s	0.8767	[-35/552/-30]s	0.8687	[0/-65/10/50]s	0.8758	[-30/60/-15/-55]s	0.8662
SCSC	[-30/303]s	0.7251	[-904]s	0.5607	[∓35/302]s	0.7277	[-30/35/-352]s	0.7240	[-40/35/40/0]s	0.7232	[-30/35/-352]s	0.7240
SCFF	[30/-653]s	0.8242	[80/20/5/80]s	0.8854	[30/-653]s	0.8242	[75/15/80/15]s	0.8867	[30/-602/50]s	0.8222	[(75/15) 2]s	0.8867
SCCF	[-20/65/-52]s	0.9692	[-40/55/-40/-20]s	0.8915	[-15/65/-152]s	0.9698	[-40/55/-35/-25]s	0.8916	[-10/-80/0/-10]s	0.9668	[-35/60/-55/25]s	0.8892
CSCF	[-5/80/-55/-5]s	0.9056	[-40/55/-302]s	0.9662	[-10/85/45/-15]s	0.9056	[-35/55/-40/-35]s	0.9668	[-10/80/45/75]s	0.9032	[-35/55/-40/-35]s	0.9668
CFFF	[20/-40/∓30]s	0.8406	[5/-40/30/20]s	0.9126	[20/-35/-40/30]s	0.8411	[10/-40/20/-35]s	0.9169	[20/-35/-20/30]s	0.8366	[10/-40/20/-35]s	0.9169
CFCF	[30/-552/20]s	0.7692	[-65/302/-55]s	0.7869	[30/-552/25]s	0.7692	[-60/30/±45]s	0.7790	[30/-602/50]s	0.7647	[-60/252/-60]s	0.7902
CCFF	[30/-652/20]s	0.7896	[80/20/5/80]s	0.8884	[25/-60/25/-65]s	0.7912	[75/15/80/15]s	0.8903	[25/-65/-55/60]s	0.7889	[75/15/80/15]s	0.8903
CCCS	[-30/40/-302]s	0.7894	[∓40/-402]s	0.7816	[-35/45/-352]s	0.7917	[∓45/-452]s	0.7879	[-30/40/-40/-70]s	0.7870	[∓45/-452]s	0.7879
CCCF	[-5/80/-20/-5]s	0.9697	[-30/55/-402]s	0.8897	[-10/70/-102]s	0.9745	[-35/50/-35/-30]s	0.8908	[-15/70/0/65]s	0.9683	[-35/50/-35/-30]s	0.8908
CCCC	[-30/40/-302]s	0.7421	[∓40/-402]s	0.7820	[-25/35/-252]s	0.7403	[∓40/-40/-45]s	0.7821	[∓35/-30/35]s	0.7375	[∓40/-40/-45]s	0.7821

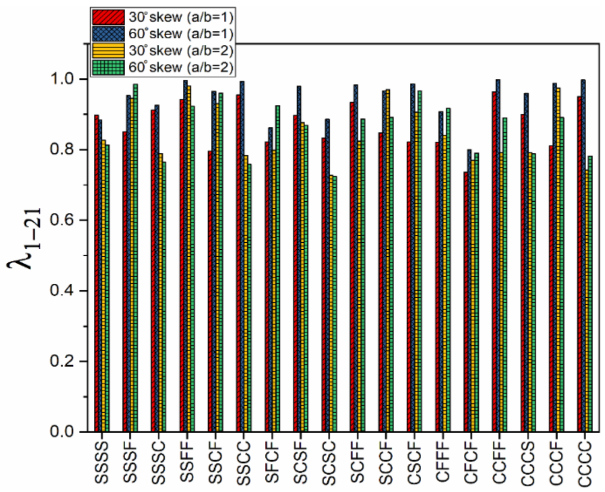

Fig. 5 Variation of maxima with respect to different boundary condition, skew angle and aspect ratio for 8-ply symmetric laminate

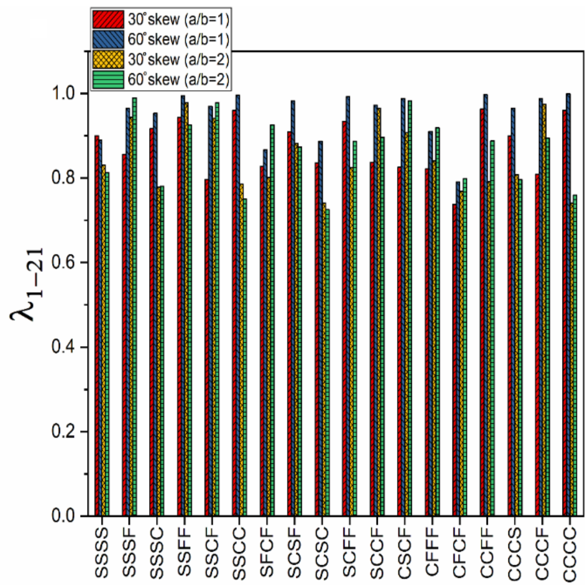

Fig. 6 Variation of maxima with respect to different boundary condition, skew angle and aspect ratio for 12-ply symmetric laminate

The FE-GA approach used in this work was validated against published works of Apalak *et al.* (2014) and Narita (2003) in a very recent work by the author(s) (Kalita *et al.* 2018). It was shown that the current FE-GA is capable of producing optimal results at par with the artificial bee colony (Apalak *et al.* 2014) and Ritz-based layerwise method (Narita 2003).

In the current work, the maximum function evaluations (i.e., total FE iterations) for a particular trial is fixed at 50,000, based on author(s) previous study (Kalita *et al.* 2018). A mesh size of 4×4 elements is used in each FE iteration. Based on (Kalita *et al.* 2018), the various GA tuning parameters are set as-population size 500, generation 100, crossover and mutation probabilities as 0.85 and 0.1 respectively. For adjusting the tuning parameters of the RPSOLC algorithm, recommendations of Santos *et al.* (2011), Santos *et al.* (2010) are followed. Thus, in this

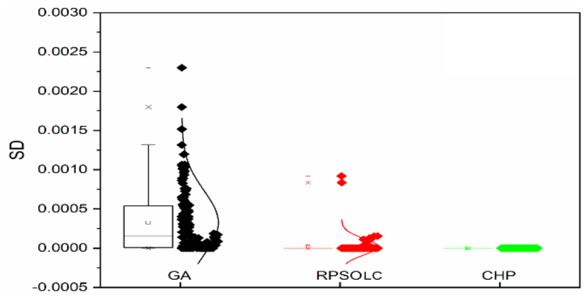

Fig. 7 Stochastic performance of the three metaheuristics

work, a swarm of 50 particles is allowed to iterate for 100 generations with local search for each particle as 10. In the current work, for the CHP algorithm, $n_h = n_p = 50$ is used based on Gandomi *et al.* (2013)'s suggestion. The maximum detection probability i.e., $P_{max.}{}^{det}$ in eqn. 8 is set as 0.7.

4.1 Problems on skew plates

Independently combining the finite element model with each of the three metaheuristic algorithms discussed in section 3, high-fidelity design optimization approaches are developed. Several sparsely solved examples from the literature are solved using these approaches to depict the efficacy of the developed optimization routines. Maximized $\boldsymbol{\lambda_{1-21}}$ (i.e., simultaneously maximized $\boldsymbol{\lambda_1}$ and $\boldsymbol{\lambda_{21}}$) for rhombic 8-ply symmetric composite laminate are reported in Table 2. Similar results for 8-ply skew (a/b=2) symmetric composite laminates are reported Table 3. Simultaneously maximized λ_1 and λ_{21} for 12-ply skew symmetric composite laminates for a/b=1 and a/b=2 at different skew angles are reported in Tables 4-5 respectively.

The weighted sum multiobjective optimization index (λ_{1-21}) indicates a scaled function and as such the ideal case value would be 1, provided that the global optima values of the single objective optimization are provided to the algorithm initially. Because the weighted sum multiobjective optimization index is essentially a unification of two scaled single objectives, a conclusive trend with respect to variation in geometric parameters like aspect ratio, skew angle, number of plies is not seen. The variation of global maxima with respect to various boundary conditions, skew angle and aspect ratio is shown in Figs. 5 and 6 for 8-ply and 12-ply symmetric laminate respectively. It is seen that in general, the weighted sum multiobjective frequency parameter increases with an increase in skew angle. It is also observed that the weighted sum multiobjective optimization index decreases with an increase in aspect ratio (a/b).

4.2 Performance comparison of the metaheuristics

In total, 144 new problems (8 geometric configurations with 18 boundary conditions each) are reported in this article. Among the 144 individual cases reported in Tables 2-5, FE-CHP successfully located the maxima in approximately 102 (~71%) cases, while FE-RPSOLC and FE-GA were successful on 77 (~53.5%) and 18 (~12.5%)

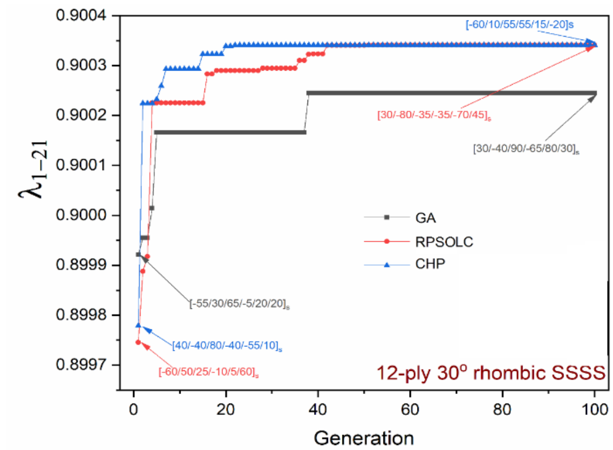

Fig. 8 Iterative improvement of feasible solutions towards optimality in a typical case

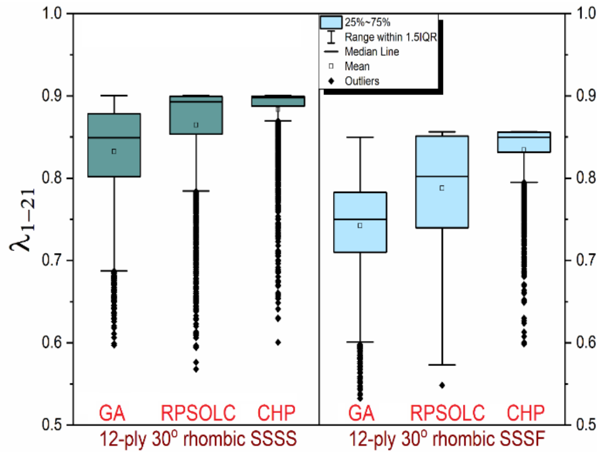

Fig. 9 Distribution of 50,000 function evaluations by GA, RPSOLC and CHP in two typical cases

cases respectively. However, in most cases, even when FE-CHP could not locate the maxima in absolute terms, it was seen to be in the near-maxima; zone. FE-GA and FE-RPSOLC, on the other hand, seem to have landed in the pit of local optima, at least one time each-Table 3, skew 60˚, SCSC and SSSF respectively.

Each trial was repeated 10 times to account for the variability in the prediction of the stochastic algorithms. Naturally, it is desirable to have a metaheuristic algorithm that could predict the maxima 10 out 10 times. Though only the absolute maximum among the 10 trials in each case is reported in Tables 2-5, the standard deviations for each of the 144 cases were recorded. Fig. 7 shows the spread of standard deviation for each of the three metaheuristics in all the 144 cases. It is seen that the variability is the optimal predictions is most in FE-GA and least in FE-CHP. This means that on an average, FE-CHP is a more stable and robust approach as compared to FE-GA and FE-RPSOLC. FE-CHP is successful in locating the global optima repeatedly on multiple trials. Thus, from Fig. 7, in terms of precision the three approaches may be ranked as FE-CHP > FE-RPSOLC > FE-GA. Additionally, it should be noted that, in general, the variability of FE-GA is less than 0.00125, with outliers ranging as high as 0.00225. Thus, it

Table 4 Optimal layer orientation for 12-layered symmetric skew composite plate (a/b=1) for max. λ_{1-21}

BC	FE-GA[1]		FE-RPSOLC				FE-CHP			
	λ_{1-21}		$\alpha = 30^o$		$\alpha = 60^o$		$\alpha = 30^o$		$\alpha = 60^o$	
	$\alpha = 30^o$	$\alpha = 60^o$	Optimal layup	λ_{1-21}	Optimal layup	λ_{1-21}	Optimal layup	λ_{1-21}	Optimal layup	λ_{1-21}
SSSS	0.9003	0.8862	$[30/-80/-35_2/-70/45]s$	0.9003	$[70/-40_2/70/-40_2]s$	0.8898	$[-60/10/55_2/15/-20]s$	0.9003	$[70/-40_2/70/-40/-45]s$	0.8899
SSSF	0.8515	0.9627	$[15/-80/-75/20/25/0]s$	0.8530	$[85/-25_3/-80/-25]s$	0.9645	$[-75/15_2/20/15/-70]s$	0.8562	$[-25/85/-90/-25_3]s$	0.9649
SSSC	0.9167	0.9513	$[80/-20_2/-15/-25/70]s$	0.9169	$[-50/75/-50/80_2/-90]s$	0.9529	$[80/-20_2/-15/-20/80]s$	0.9169	$[-45/80_2/-50_2/-80]s$	0.9529
SSFF	0.9406	0.9859	$[30/-60/30/-65/25_2]s$	0.9421	$[10/-75/25/20/30/15]s$	0.9879	$[(30/-60)_2/25_2]s$	0.9428	$[15_2/-75_2/15_2]s$	0.9947
SSCF	0.7938	0.9683	$[15/-75/-85/35/-60/20]s$	0.7859	$[-60_3/-65/-60_2]s$	0.9691	$[-70/20_3/-70_2]s$	0.7965	$[-60_2/-65/-60_2]s$	0.9691
SSCC	0.9600	0.9958	$[80/-20/-15/75/0/-50]s$	0.9589	$[85/-60_3/-80/-75]s$	0.9958	$[80/-20_2/75/-20/-25]s$	0.9602	$[-55/85_2/-65/-85/-70]s$	0.9959
SFCF	0.8267	0.8666	$[-30_6]s$	0.8275	$[-50/-40/-45/40_2/-5]s$	0.8659	$[-30_6]s$	0.8275	$[-50/-45/35/-45_2/-20]s$	0.8656
SCSF	0.9051	0.9817	$[-75/15_3/-10/-65]s$	0.9047	$[-20/90/-15/80/-25/-20]s$	0.9750	$[10/-80_2/15_2/-80]s$	0.9094	$[80/-20_5]s$	0.9821
SCSC	0.8350	0.8865	$[-15/80/-20_2/80_2]s$	0.8359	$[-55_2/80/-50/-55/-65]s$	0.8868	$[-15/80/-20_2/80_2]s$	0.8359	$[-55_2/80/-50/-55/-65]s$	0.8868
SCFF	0.9317	0.9868	$[25/-60/25/-55/20/25]s$	0.9346	$[15_2/-80/10/-75/5]s$	0.9925	$[25/-60/25/-55/20/25]s$	0.9346	$[15_2/-80/10/-75/50]s$	0.9916
SCCF	0.8322	0.9710	$[20/-80/-65/5/45/-25]s$	0.8275	$[-75_2/-30/-35/-80/-65]s$	0.9725	$[15/-80/-75_2/15_2]s$	0.8367	$[-75_2/-30/-35/-75/-70]s$	0.9725
CSCF	0.8249	0.9874	$[-65/30_2/-65_2/60]s$	0.8246	$[-45_2/90/-50_2/-45]s$	0.9882	$[-65/30_2/-65_2/25]s$	0.8252	$[-45_2/90/-50_2/-45]s$	0.9882
CFFF	0.8176	0.9060	$[25/-45_2/25_2/15]s$	0.8219	$[-45/15_2/-45_2/20]s$	0.9098	$[25/-45_2/25_3]s$	0.8220	$[15/-45_3/20/15]s$	0.9099
CFCF	0.7292	0.7707	$[30/-70/25/30/35/25]s$	0.7339	$[15/70/75/15/25/55]s$	0.7719	$[30/(-75/30)_2/25]s$	0.7376	$[15/75/15_2/75/10]s$	0.7906
CCFF	0.9620	0.9902	$[20/50/-55/-65/-20/55]s$	0.9617	$[15_2/-75/15/10/25]s$	0.9970	$[20/50/-65/-30/35_2]s$	0.9633	$[15_2/-75/15_3]s$	0.9972
CCCS	0.8987	0.9646	$[80/-25/75/-20/-10/-5]s$	0.8982	$[-80/-85/-75/-80/-85/-75]s$	0.9648	$[-20/80_2/75/80/-20]s$	0.8998	$[-80_6]s$	0.9652
CCCF	0.8053	0.9875	$[-75/25/-75/20_2/-70]s$	0.8086	$[-75/-30/-35/-40/-50_2]s$	0.9879	$[-75/25/-75/20_2/-70]s$	0.8086	$[-75/-30/-35_2/-70/-50]s$	0.9879
CCCC	0.9599	0.9966	$[-20/80_2/-20/75/85]s$	0.9600	$[-75_6]s$	0.9992	$[-20/80_2/-20/80/85]s$	0.9600	$[-75_6]s$	0.9992

is imperative to point out that FE-GA would serve as a reliable solver-optimizer for most practical purpose problems where near optimal solutions are equally acceptable.

The convergence and iterative improvement capability of the three metaheuristics are depicted in Fig. 8 for a typical case. In this example, FE-GA converged to a sub-optimal solution. However, it is important to state here that FE-GA on repeated trials (using 10 different random seeds) was indeed able to locate the maxima as seen in Table 4. Though both FE-RPSOLC and FE-CHP were able to locate the maxima, the convergence of FE-CHP was much faster. It is also worth mentioning here that despite converging to the same global maxima in terms of magnitude FE-RPSOLC and FE-CHP in Fig. 8 predicted different sets of optimal fiber angles. This further reinforces the belief that the function domain is highly multimodal and likely to contain multiple global maxima. In fact, baring a few out of 144 cases (Tables 2-5), in cases where the three metaheuristic algorithms predicted the same maxima, in general, non-identical optimal fiber-angle combinations, were reported. Another interesting stochastic iterative improvement trait that is seen in Fig. 8 is that despite FE-GA being at a better initial solution as compared to FE-RPSOLC and FE-CHP, it converged to a sub-optimal solution. The superior search capability of FE-RPSOLC and FE-CHP could be perhaps due to the advanced memetic attributes introduced in them.

To further understand the superiority of FE-RPSOLC and FE-CHP, Box plots containing the information regarding 50,000 function evaluations (FE iterations) for two typical cases are reported in Fig. 9. The total spread length (on objective function scale) of all the 50,000 evaluations for the three metaheuristics is similar. However, the nature of spread i.e. the accumulation of evaluated functions on the objective function scale shows stark differences. While in the case of FE-GA the evaluated functions are seen to be uniformly distributed across the spread length, in FE-CHP there is a significant accumulation above the mean value with a light tail. This signifies that the FE-GA takes a lot of function evaluations to reach an optimum value. Thus, for this same problem, if the maximum number of function evaluations is decreased (say to 10,000 iterations), it is quite possible that FE-GA could fail miserably in locating the absolute maxima. In other words, the iterative improvement capability of FE-GA is somewhat sluggish. Contrarily, for FE-CHP the region above 3rd quartile (i.e., ~75%) of the Box plots have negligible spread length and lie above the mean value of 50,000 evaluations. This signifies that FE-CHP is quickly able to locate the zone of global optima. It is continuously able to keep track of best solutions and thus, effectively and quickly able to locate the global optima, thereby imparting it a very fast convergence rate. FE-RPSOLC shows somewhat similar spread and evaluated function accumulation like FE-GA, though it appears to be marginally better.

[1]Due to paucity of space, FE-GA predicted optimal layups are not included in Tables 3 and 4. Moreover, in general, the predicted maxima of FE-RPSOLC and FE-CHP are better than that of FE-GA.

Table 5 Optimal layer orientation for 12-layered symmetric skew composite plate (a/b=2) for max. λ_{1-21}

BC	FE-GA		FE-RPSOLC				FE-CHP			
	λ_{1-21}		$\alpha = 30^o$		$\alpha = 60^o$		$\alpha = 30^o$		$\alpha = 60^o$	
	$\alpha = 30^o$	$\alpha = 60^o$	Optimal layup	λ_{1-21}	Optimal layup	λ_{1-21}	Optimal layup	λ_{1-21}	Optimal layup	λ_{1-21}
SSSS	0.8287	0.8022	[40/-45/-40/-50/40_2]s	0.8298	[∓45/$(-40/45)_2$]s	0.8112	[40/-45_2/40_2]s	0.8307	[$(-45/45)_2$/±45]s	0.8129
SSSF	0.9430	0.9879	[5/10/-60_2/10/0]s	0.9439	[-25/60/-10/-30/-15/-20]s	0.9865	[5/10/-60_2/10/0]s	0.9439	[-25/60/-25_3/-10]s	0.9896
SSSC	0.7773	0.7785	[-45/35/40/-40_2/20]s	0.7773	[$(-40/40)_2$/±40]s	0.7804	[-40/35_2/-40_2/40]s	0.7789	[$(-40/40)_2$/±40]s	0.7804
SSFF	0.9760	0.9224	[30/-50/25/-50_2/35]s	0.9770	[15/20/-60/15/-55/15]s	0.9254	[30/-50/30/-50_2/30]s	0.9783	[15/20/-60/15/-55/15]s	0.9254
SSCF	0.9413	0.9778	[5/-30/±75/-15/10]s	0.9413	[-40/65/-35_3/-20]s	0.9780	[5/-30/±75/-15/10]s	0.9413	[-40/65/-35_3/-20]s	0.9780
SSCC	0.7812	0.7369	[-35/40/-30/35/-40/-50]s	0.7843	[30/-35_2/-30/35/-25]s	0.7320	[-35/40/∓35/-35_2]s	0.7854	[∓35/-35_2/±35]s	0.7502
SFCF	0.8008	0.9247	[20/15/-60/-55/-50/30]s	0.7964	[0/-60/-5/-65/0/-55]s	0.9258	[20/-55/20_2/-55/15]s	0.8011	[0/-60/-50/-65/0/-55]s	0.9040
SCSF	0.8815	0.8726	[0_2/-70/0/-75/-5]s	0.8824	[-35/55/-30/60/-30/-20]s	0.8734	[0_2/-70/0/-75/-50]s	0.8823	[-35/55/-30/60/-30/-20]s	0.8734
SCSC	0.7390	0.7136	[-35/25/±35/25/-40]s	0.7376	[$(∓35)_2$/-35_2]s	0.7252	[-35/30_2/-35/30/-35]s	0.7406	[$(∓35)_2$/-35_2]s	0.7252
SCFF	0.8246	0.8849	[-65/30_3/-65/45]s	0.8249	[75/15/75_2/10/75]s	0.8861	[-65/30_3/-65/30]s	0.8250	[80/15/75/15/75_2]s	0.8871
SCCF	0.9639	0.8965	[-10/-15/85/55/-15/-10]s	0.9646	[-35/-30/65/35/-25_2]s	0.8954	[-10/-15/85/55/-15/-10]s	0.9646	[-35/-30/65/35/-25_2]s	0.8954
CSCF	0.9063	0.9816	[-5/85/-10/75/-10/0]s	0.9067	[-40/-35/55_2/-15/-70]s	0.9804	[-5/85/-10/75/-10/0]s	0.9067	[-35/60/-35_2/-30_2]s	0.9822
CFFF	0.8397	0.9174	[20/-40/25/-35_2/20]s	0.8412	[10/-40/15/-30/20/10]s	0.9188	[20/-40/25/-35_2/20]s	0.8412	[10/-40/15/-30/20/10]s	0.9188
CFCF	0.7680	0.7977	[-55/30_3/25/20]s	0.7685	[$(-65/30)_2$/-65/-70]s	0.7984	[-55/30_2/25_3]s	0.7686	[$(-65/30)_2$/-65/-60]s	0.7987
CCFF	0.7906	0.8853	[25/-60/25/-65/25_2]s	0.7920	[$(80/15)_2$/80/45]s	0.8872	[25/-60/25/-65/25_2]s	0.7920	[$(80/15)_2$/75_2]s	0.8874
CCCS	0.8056	0.7936	[-25/-30/50/40/-35/20]s	0.8033	[∓40/-40/-35/-45/-55]s	0.7949	[-30/45/-30_4]s	0.8082	[∓40/-40_3/-45]s	0.7962
CCCF	0.9741	0.8927	[-10/85/-5/∓10/-25]s	0.9711	[-30/50/-25/-35/-25/-65]s	0.8939	[-5_2/±80/-20_2]s	0.9741	[-30/50/-30_2/-25_2]s	0.8948
CCCC	0.7410	0.7596	[-25/35/-25_4]s	0.7397	[-35_2/45/-40/-35_2]s	0.7578	[-25/35/-25_4]s	0.7397	[-35_2/40/-35_2/-40]s	0.7564

Table 6 Paired sample t-Test between FE-GA, FE-RPSOLC and FE-CHP (Case Table 5, SCFF)

Optimization method	N	Mean	SD	SE	Median	t-value	p-value
FE-GA	10	0.8838	0.0011	0.0004	0.8843	-6.6074	4.92E-05
FE-RPSOLC	10	0.8861	0.0000	0.0000	0.8861		
Estimate for average difference: -0.00232							
H_o: mean (FE-GA) >= mean (FE-RPSOLC) ***H_a: mean (FE-GA) < mean (FE-RPSOLC)***							
FE-GA	10	0.8838	0.0011	0.0004	0.8843	-9.2478	3.42E-06
FE-CHP	10	0.8871	0.0000	0.0000	0.8871		
Estimate for average difference: -0.00325							
H_o: mean (FE-GA) >= mean (FE-CHP) ***H_a: mean (FE-GA) < mean (FE-CHP)***							
FE-RPSOLC	10	0.8861	0.0000	0.0000	0.8861	--	--
FE-CHP	10	0.8871	0.0000	0.0000	0.8871		
Estimate for average difference: -0.00093							
H_o: mean (FE-RPSOLC) >= mean (FE-CHP) ***H_a: mean (FE-RPSOLC) < mean (FE-CHP)***							

Additionally, two-sample paired t-tests between FE-GA, FE-RPSOLC and FE-CHP are carried out (Table 6). In all the three *t*-tests, the *p*-value is very close to 0. Thus, it is safe to reject the null hypothesis (H_o). At 5% significance level, the optimization performance of FE-CHP is significantly superior to FE-GA and FE-RPSOLC. Moreover, there is 95% confidence that the FE-RPSOLC could outperform FE-GA.

5. Conclusions

In this article, three nature-inspired optimization algorithms-genetic algorithm (GA), repulsive particle swarm optimization with local search and chaotic perturbation (RPSOLC) and co-evolutionary host-parasite (CHP) are separately combined with a nine-node isoparametric finite element formulation (FE) to design optimized skew laminates. The rotary inertia and shear deformation are accounted for by considering first-order shear deformation theory. Using fiber-angles as the design variables, skew laminates are optimized such that the fundamental frequency and frequency separation between the first two modes is simultaneously maximized. Several examples of skew plates with different boundary conditions, geometry and number of layers are used to validate the accuracy of the FE-GA, FE-RPSOLC and FE-CHP. It is found that all the three approaches FE-GA, FE-RPSOLC and FE-CHP have high potential in finding the optimal stacking sequence of composite laminates. FE-CHP comprehensively outperformed the other two approaches. It showed quick convergence and low dispersion of the evaluated function population, making it ideal for application in highly computationally expensive structural optimization problems. FE-CHP successful located the maxima in about 71% of the total 144 examples, whereas FE-RPSO and FE-GA managed to do so in about 53% and 12.5% examples respectively. In the remaining 29% of the trials where FE-CHP could not locate the absolute maxima, it very marginally missed the maxima (which was found by either FE-GA or FE-RPSO). FE-RPSOLC performed a little better than FE-GA and could also serve as a robust tool to maximize the frequency parameters of composite plates by suggesting suitable ply orientations. Further, it should be pointed out that both FE-CHP and FE-RPSOLC employ a fewer number of self-tuning parameters than FE-GA making them much simpler to operate. Thus, the FE-CHP algorithm is recommended for carrying out high-fidelity optimization of laminated composites. It can lead to tremendous saving in computation effort while achieving high functionality.

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