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Modal analysis of FG sandwich doubly curved shell structure

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Abstract. The modal frequency responses of functionally graded (FG) sandwich doubly curved shell panels are investigated using a higher-order finite element formulation. The system of equations of the panel structure derived using Hamilton's principle for the evaluation of natural frequencies. The present shell panel model is discretised using the isoparametric Lagrangian element (nine nodes and nine degrees of freedom per node). An in-house MATLAB code is prepared using higher-order kinematics in association with the finite element scheme for the calculation of modal values. The stability of the opted numerical vibration frequency solutions for the various shell geometries i.e., single and doubly curved FG sandwich structure are proven via the convergence test. Further, close conformance of the finite element frequency solutions for the FG sandwich structures is found when compared with the published theoretical predictions (numerical, analytical and 3D elasticity solutions). Subsequently, appropriate numerical examples are solved pertaining to various design factors (curvature ratio, core-face thickness ratio, aspect ratio, support conditions, power-law index and sandwich symmetry type) those have the significant influence on the free vibration modal data of the FG sandwich curved structure.

Keywords: functionally graded sandwich curved panel; HSDT; free vibration; FEM; MATLAB

1. Introduction

Functionally graded materials (FGMs) belong to a class of advanced engineered materials those are characterized by smooth and continuous variation in properties as the dimension varies. These are made up of ceramics and metals and are able resist high temperature gradient while maintaining structural integrity. Thus, they are preferred over conventional monolithic as well as laminated composite materials as the structure/structural components for extremely higher temperature environments. Explicitly, the delamination concern in case of laminated composite shell panels owing to the abrupt change in material properties between the interfaces of different layers is mitigated by the use of FGMs in which the microstructure is varied from one material to another material with a specific gradient by changing the volume fraction of constituent materials along thickness of panel.

FG Sandwich structures are considered to be the most functional forms of the composite structures developed in recent years that has conquered a wide acceptance in the weight sensitive and high-performance engineering applications due to its low specific weight along-side excellent flexural, vibrational and fatigue characteristics (Vinson 2001). As a consequence, extensive work has been carried out to study the static and dynamic characteristics of FG sandwich shell panels numerically as well as experimentally (Jha *et al.* 2013, Bousahla *et al.* 2016, Boukhari *et al.* 2016). We note, that the flexural and free vibration responses of FG sandwich panels have been studied from time to time by utilizing several kinematic models (Thai *et al.* 2015, Belabed *et al.* 2014, Thai *et al.* 2014, Natarajan *et al.* 2012, Mantari *et al.* 2011, Bouderba *et al.* 2016, El-Haina *et al.* 2017, Bellifa *et al.* 2016) aiming at attaining the exact flexure of the structure. Lok, et al. (2001) proposed the closed-form solutions to the forced and free vibration responses of orthotropic sandwich panels with truss core. A similar work implementing finite element method (FEM) for the evaluation of the elastic constants of sandwich structures with various types of core has also been reported (Cheng *et al.* 2006). Khare and his colleagues (Khare *et al.* 2004, Garg *et al.* 2006) used a higher-order shear-deformation theory (HST) based FE approach to study the vibration characteristics of composite and sandwich curved laminates. Zenkour (2005a, b) utilized a sinusoidal shear deformation theory and presented analytical solutions to the bending, buckling and free vibration responses of FG sandwich plates. 3D elasticity solutions to the flexural (Kashtalyan and Menshykova 2009), free vibration responses (Li, *et al.* 2008) of FG

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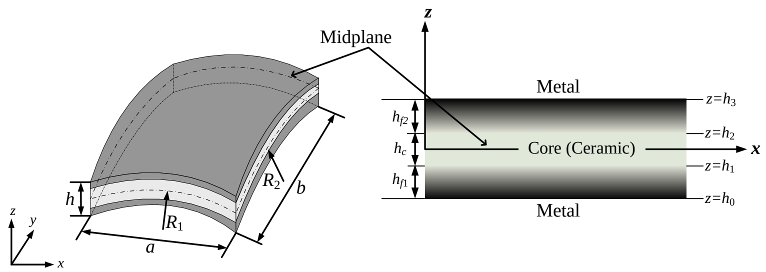

Fig. 1 Geometry and material variation in FG sandwich curved panels with ceramic core

sandwich plates with homogenous/FG core and cylindrical sandwich panel with FG core (Alibeigloo and Liew 2014) have also been reported in open literature. A hyperbolic shear deformation theory is proposed (Meiche *et al.* 2011, Abdelaziz *et al.* 2017) to investigate the buckling and vibration frequency parameters of the laminated sandwich plates. Further, the theory is adopted by Neves *et al.* (2012) including the zig-zag and warping effects for analysis of the flexural responses of FG sandwich panels. Additionally, four-unknown shear and normal deformation theories have also been implemented to examine the bending (Zenkour 2013) and vibration responses (Hadji, *et al.* 2011) of FG sandwich plates. Further, a higher-order equivalent single layer theory is proposed by Tornabene *et al.* (2014) to compute the frequencies of the doubly-curved FG sandwich shell panels. Moreover, quasi-3D theories have been employed for studying the bending, buckling and vibration responses of FG plates (Neves *et al.* 2013) and FG sandwich beams (Osofero *et al.* 2016). Alipour and Shariyat (2012) performed the axisymmetric bending and stress analysis of circular FG sandwich plates subjected to transversely distributed load via an elasticity-equilibrium-based zigzag theory. Additionally, the HST has also been extended to study the free vibration behaviour of sandwich plates with CNT reinforced composite face-sheets (Natarajan, *et al.* 2014). Studies investigating the bending, vibration an buckling responses of FG sandwich shell panels exposed to thermal environment (Zenkour and Alghamdi 2010) in the framework of higher-order (Houari *et al.* 2013, Bousahla *et al.* 2014, Ait Yahia *et al.* 2015, Sekkal *et al.* 2017, Menasria *et al.* 2017, Karami *et al.* 2018, Zine *et al.* 2018) hyperbolic (Kettaf *et al.* 2013, Hebali *et al.* 2014, Mahi *et al.* 2015, Zidi *et al.* 2017, Belabed *et al.* 2018,), refined (Tounsi *et al.* 2013, Meziane *et al.* 2014, Bennoun *et al.* 2016, Draiche *et al.* 2016, Bellifa *et al.* 2017, Attia *et al.* 2018, Fourn *et al.* 2018) shear deformation theories are also found in open literature. Past researchers also performed the nonlinear bending, vibration, and post-buckling analyses of sandwich panels with FG sheets exposed to elevated thermal loading (Wang and Shen 2011). We also note the inclusion of stretching effect (Bourada *et al.* 2015, Chaht *et al.* 2015, Hamidi *et al.* 2015, Abualnour *et al.* 2018, Bouhadra *et al.* 2018, Younsi *et al.* 2018, Bouafia *et al.* 2017, Zaoui *et al.* 2019) in such analysis in an intention to accurately estimate the original flexure of the structure. Kolahchi and his colleague (Kolahchi *et al.* 2016, Arani *et al.* 2016, Arani *et al.* 2015, Hajmohammad *et al.* 2017, Hosseini *et al.* 2018) studied the responses of FG structure reinforced with CNTs from time to time with the aid of numerical as well as analytical models based on different existing and refined higher-order theories.

We note that the analytical/numerical flexural and free-vibration responses of composite and FG sandwich structure have got a lot of attention in past and though various mid-plane kinematic theories have been employed for analyses purpose, the accuracy of the approach is ominously reliant on the choice of element/shape function. However, majority of the studies focused on flat panels only and the studies highlighting the vibration characteristics of FG sandwich shell panel structures with curvature on both sides (spherical, elliptical and hyperboloid) are scarce. Additionally, no numerical study on the free vibration characteristics of FG sandwich single/doubly curved panels implementing HST with FEM has been reported till date. In this paper, the free vibration characteristics of curved FG shell panel structures are addressed using FEM and Reddy's HST with nine-degrees of freedom. A nine-noded quadrilateral isoparametric element is utilized for discretization purpose. The convergence behaviour as well as the validity of the proposed scheme with those of the other 2D analytical/numerical models and 3D elasticity solutions is established by considering various parameters. Subsequently, numerical results are provided to investigate the influence of various design factors (curvature ratio, thickness ratio, aspect ratio, power-law index, support conditions and the FG sandwich symmetry type) on the natural frequency of FG sandwich curved panel structures followed by few useful concluding remarks.

2. Problem formulation

A general mathematical formulation for obtaining the flexural responses of doubly curved functionally graded sandwich shell panels is derived. The core is considered to be purely ceramic whereas the face sheets have material graded (from ceramic to metal) functionally along the thickness direction. The material properties (young's modulus, density and Poisson's ratio) vary as per the following relation (Zenkour 2005a)

$$P(z) = P_m + (P_c - P_m)V_f^{(n)} \tag{1}$$

where, P_m and P_c are the material properties of metal and ceramic, respectively, $V_f^{(n)}$ is the volume fraction of the ceramic (n=1,2,3) that varies through the thickness following power-law (Zenkour 2005a)

$$\left.\begin{aligned} V_f^{(1)} &= \left(\frac{z-h_0}{h_1-h_0}\right)^k, z\in[h_0,h_1]; V_f^{(2)}=1, z\in[h_1,h_2] \\ V_f^{(3)} &= \left(\frac{z-h_3}{h_2-h_3}\right)^k, z\in[h_2,h_3] \end{aligned}\right\} \tag{2}$$

where, the thickness coordinate (z) levels h_0, h_1, h_2 and h_3 are defined in Fig. 1 which illustrates the geometry of the shell panels.

In the present analysis, curved panels with dimensions ($a\times b\times h$) m^{3}and having a rectangular base (projection of the curved panel would be a rectangle) are considered. The thickness of the core is denoted as "h_c" whereas the thickness of the bottom and top face sheets are denoted as "h_{f1}" and "h_{f2}", respectively such that $h=h_c+h_{f1}+h_{f2}$, $h_c=h_2-h_1$, $h_{f1}=h_3-h_2$ and $h_{f2}=h_1-h_0$. The principal radius of curvature along x and y direction is R_1 and R_2, respectively. The panel geometries are defined as: cylindrical ($R_1=R$, $R_2=\infty$), spherical ($R_1=R$, $R_2=R$), elliptical ($R_1=R$, $R_2=2R$), hyperboloid ($R_1=R$, $R_2=-R$) and flat ($R_1=R_2=\infty$) on the basis of curvature, where $R$is a constant. The displacement field (p, q and r) of the FG shell panel *i.e.*, the displacements of a point along the x, y and z coordinates based on the HST kinematic relation is expressed as (Kant *et al.* 2002)

$$\left.\begin{aligned} p(x,y,z) &= p_0(x,y)+zp_1(x,y)+z^2p_2(x,y)+z^3p_3(x,y) \\ q(x,y,z) &= q_0(x,y)+zq_1(x,y)+z^2q_2(x,y)+z^3q_3(x,y) \\ r(x,y,z) &= r_0(x,y) \end{aligned}\right\} \tag{3}$$

where, p_0, q_0 and r_0 are the mid-plane displacements of a point with respect to corresponding coordinates. p_1 and q_1 are the rotations of transverse normal about the y- and x-axes, respectively and p_2, q_2, p_3 and q_3 are the higher-order functions defined in the mid-plane of the shell.

Now, the strain displacement field can be expressed as

$$\begin{aligned} \{\varepsilon\} &= \{\varepsilon_{xx} \quad \varepsilon_{yy} \quad \gamma_{xy} \quad \gamma_{xz} \quad \gamma_{yz}\} \\ &= \left\{\begin{aligned} &\left(\frac{\partial p}{\partial x}+\frac{r}{R_1}\right) \quad \left(\frac{\partial q}{\partial y}+\frac{r}{R_2}\right) \quad \left(\frac{\partial p}{\partial y}+\frac{\partial q}{\partial x}+\frac{2r}{R_{12}}\right)\cdots \\ &\cdots\left(\frac{\partial p}{\partial z}+\frac{\partial r}{\partial x}-\frac{p}{R_1}\right) \quad \left(\frac{\partial q}{\partial z}+\frac{\partial r}{\partial y}-\frac{q}{R_2}\right) \end{aligned}\right\} \end{aligned} \tag{4}$$

By substituting Eq. (3) in Eq. (4), the strain vector can further be written as

$$\varepsilon = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \\ \varepsilon_{xz}^0 \\ \varepsilon_{yz}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x^1 \\ k_y^1 \\ k_{xy}^1 \\ k_{xz}^1 \\ k_{yz}^1 \end{Bmatrix} + z^2\begin{Bmatrix} k_x^2 \\ k_y^2 \\ k_{xy}^2 \\ k_{xz}^2 \\ k_{yz}^2 \end{Bmatrix} + z^3\begin{Bmatrix} k_x^3 \\ k_y^3 \\ k_{xy}^3 \\ k_{xz}^3 \\ k_{yz}^3 \end{Bmatrix} \tag{5}$$

The Eq. (5) can further be expressed in terms of the function of thickness coordinate [T] as

$$\{\varepsilon\} = [T]\{\hat{\varepsilon}\} \tag{6}$$

where,

$$\{\hat{\varepsilon}\} = \{\varepsilon_x^0 \ \varepsilon_y^0 \ \varepsilon_{xy}^0 \ \varepsilon_{xz}^0 \ \varepsilon_{yz}^0 \ k_x^1 \ k_y^1 \ k_{xy}^1 \ k_{xz}^1 \ k_{yz}^1 \ k_x^2 \ k_y^2 \ k_{xy}^2 \ k_{xz}^2 \ k_{yz}^2 \ k_x^3 \ k_y^3 \ k_{xy}^3 \ k_{xz}^3 \ k_{yz}^3\}^T$$

is the mid-plane strain vector.

The present shell panel model is discretised using a nine noded quadrilateral Lagrangian isoparametric element with nine degrees of freedom associated with each node.

The elemental displacement vector is given as

$$\{\lambda_0\} = \sum_{i=1}^{9} N_i\{\lambda_{0_i}\} \tag{7}$$

where, $\{\lambda_{0_i}\} = \{p_{0_i} \ q_{0_i} \ r_{0_i} \ p_{1_i} \ q_{1_i} \ p_{2_i} \ q_{2_i} \ p_{3_i} \ q_{3_i}\}^T$ is the nodal displacement vector at node i. N_i is the shape function for the i^{th} node and the details can be seen from the source (Cook *et al.* 2000).

The mid-plane strain vector as expressed in Eq. (6) can be rewritten in terms of nodal displacement vectors as

$$\{\hat{\varepsilon}\} = [B]\{\lambda_{0_i}\} \tag{8}$$

where, [B] is the product form of differential operators and the shape functions in the strain terms. Thus, the stress-strain relationship for the FG shell panel is given by

$$\{\sigma\} = [\bar{Q}]\{\varepsilon\} \tag{9}$$

where, $\{\sigma\} = \{\sigma_{xx} \ \sigma_{yy} \ \tau_{xy} \ \tau_{xz} \ \tau_{yz}\}^T$ is stress vector and $[\bar{Q}]$ is the reduced stiffness matrix.

Now, the global displacement field vector as in Eq. (3) can also be expressed as

$$\{\delta_*\} = \{p \quad q \quad r\}^T = [\mathrm{f}]\{\lambda_0\} \tag{10}$$

where, $\{\delta_*\}$ is the global displacement vector, $\{\lambda_0\} = \{p_0 \ q_0 \ r_0 \ p_1 \ q_1 \ p_2 \ q_2 \ p_3 \ q_3\}^T$ is the displacement vector at any point in the mid-plane and [f] is the function of thickness coordinate.

Thus, the kinetic energy (KE) of the vibrating doubly curved FG sandwich shell panel is given by

$$KE = \frac{1}{2}\int \rho\{\dot{\delta}_*\}^T\{\dot{\delta}_*\}\,dV \tag{11}$$

where, ρ, $\{\dot{\delta}_*\}$ are the mass density and first-order derivative of the global displacement vector with respect to time, respectively. Substituting Eq. (10) in Eq. (11), the expression for kinetic energy becomes

$$\left.\begin{aligned} KE &= \frac{1}{2}\int_A\left(\int_{-h/2}^{h/2}\{\dot{\lambda}_0\}^T[\mathrm{f}]^T\rho[\mathrm{f}]\{\dot{\lambda}_0\}\,dz\right)dA \\ &= \frac{1}{2}\int_A\{\dot{\lambda}_0\}^T[m]\{\dot{\lambda}_0\}\,dA \end{aligned}\right\} \tag{12}$$

where, $[m] = \int_{-h/2}^{h/2}[\mathrm{f}]^T\rho[\mathrm{f}]\,dz$ is the elemental inertia matrix.

The elemental form of kinetic energy of FG shell panels may be expressed as

$$KE_e = \frac{1}{2}\{\dot{\lambda}_{0i}\}^T[M_e]\{\dot{\lambda}_{0i}\}\, dA \tag{13}$$

where, $[M_e] = \int_{-1}^{1}\int_{-1}^{1}[N]^T[m][N]|J|\,d\xi d\eta$ represent the elemental mass matrix at node i.

The strain energy of the curved shell panel can be expressed as

$$U = \frac{1}{2}\int_v \{\varepsilon\}^T\{\sigma\}dV \tag{14}$$

Substituting strains and stresses from Eq. (6) and Eq. (9), Eq. (14) is conceded as

$$U = \frac{1}{2}\int_A\left(\{\hat{\varepsilon}\}^T[D]\{\hat{\varepsilon}\}\right)dA \tag{15}$$

where, $[D] = \int_{-h/2}^{+h/2}[T]^T\left[\bar{Q}\right][T]dz$.

The elemental form of strain energy (as given by Eq. (15)) can be rearranged by substituting Eq. (8) in it to have the following form

$$U_e = \frac{1}{2}\int_A\left(\left\{\lambda_{0_i}\right\}^T[B]^T[D][B]\left\{\lambda_{0_i}\right\}\right)dA \tag{16}$$

Eq. (16) can further be expressed as

$$U_e = \frac{1}{2}\left\{\lambda_{0_i}\right\}^T[K_e]\left\{\lambda_{0_i}\right\} \tag{17}$$

where, $[K_e] = \int_{-1}^{1}\int_{-1}^{1}[B]^T[D][B]|J|\,d\xi d\eta$ represents the elemental stiffness matrix.

The governing differential equation for the freely vibrating composite sandwich curved shell panel is obtained by using the Hamilton's principle which is designated as

$$\delta\int_{t_1}^{t_2}(KE - U)dt = 0 \tag{18}$$

On substituting the expressions for kinetic energy, KE and strain energy, U from Eqs. (13) and (17), the elemental form of governing equation can be represented as

$$[K_e]\{\lambda\} + [M_e]\{\ddot{\lambda}\} = 0 \tag{19}$$

The global form of the equilibrium equation for free vibration analysis is obtained by assembling the elemental matrices and conceded as

$$[K]\{\lambda\} + [M]\{\ddot{\lambda}\} = 0 \tag{20}$$

The eigenvalue form of Eq. (20) is expressed as

$$\left\{[K] - \omega^2[M]\right\}\Delta = 0 \tag{21}$$

Table 1 Configuration in different FG sandwich panel symmetries

Symmetry	h_0	h_1	h_2	h_3
1-2-2	-h/2	-3h/10	h/10	h/2
2-1-3	-h/2	-h/6	0	h/2
1-1-3	-h/2	-3h/10	-h/10	h/2
2-1-4	-h/2	-3h/14	-h/14	h/2
4-1-3	-h/2	*0*	h/8	h/2
3-1-4	-h/2	*-3h/8*	*0*	h/2

where, ω is the eigenvalue (natural frequency) and Δ is the corresponding eigenvector. Eq. (21) is solved to obtain the free vibration responses of the system.

3. Numerical results and discussion

The proposed HST based FE scheme is now exploited to evaluate the free vibration characteristics of FG sandwich structures. The natural frequencies of curved higher-order FG sandwich shell panels are computed by means of personalized MATLAB computer code prepared for the implementation of the present model. The cylindrical, spherical, hyperboloid, elliptical and flat shell panel geometries are considered for the current study. The modal analysis is conducted by considering two combinations of metal and ceramic materials namely, Aluminum/Alumina [Al/Al_2O_3] and Aluminum/Zirconia [Al/ZrO_2]. The material properties of the metal and the ceramic in the aforementioned combinations are (Aluminum: E=70 GPa, ρ=2700 kg/m^3, ν=0.3, Alumina: E=380 GPa, ρ=3800 kg/m^3, ν=0.3, Zirconia: E=151 GPa, ρ=3000 kg/m^3, ν=0.3). The symmetry of FG panels is defined in terms of the ratio of face and core thickness and represented as $h_{f1} - h_c - h_{f2}$. The core-face thickness ratio (*CFR*) is defined as the ratio of thickness of core to the thickness of face ($CFR=h_c/h_f$, where, $h_{f1}= h_{f2}= h_f$). The panels are assumed to have the following properties throughout, unless specified otherwise: h=0.005m, a/b=1 and power-law index, k=2. The Aluminum/Alumina (Al/Al_2O_3) material properties are utilized, if not stated explicitly. The various symmetry schemes of the panels considered in the present analysis are summarized in Table 1. The variation of volume fraction along the panel thickness for various power-law indexes are also portrayed in Fig. 2. The fundamental frequencies are expressed in non-dimensional form by using the equation $\varpi = \omega a^2\left(\rho_f / E_{2,f}\right)/h$. The model is first tested for stability and accuracy via suitable convergence and validation of the results. Thereafter, several numerical examples are solved to highlight the influence of various structural parameters on the free vibration characteristics of FG sandwich curved shell panels.

The solutions are computed using different sets of support conditions in the combination of clamped (C), simply-supported (S) and free (F) supports to avoid rigid body motion and to reduce the number of unknowns. The

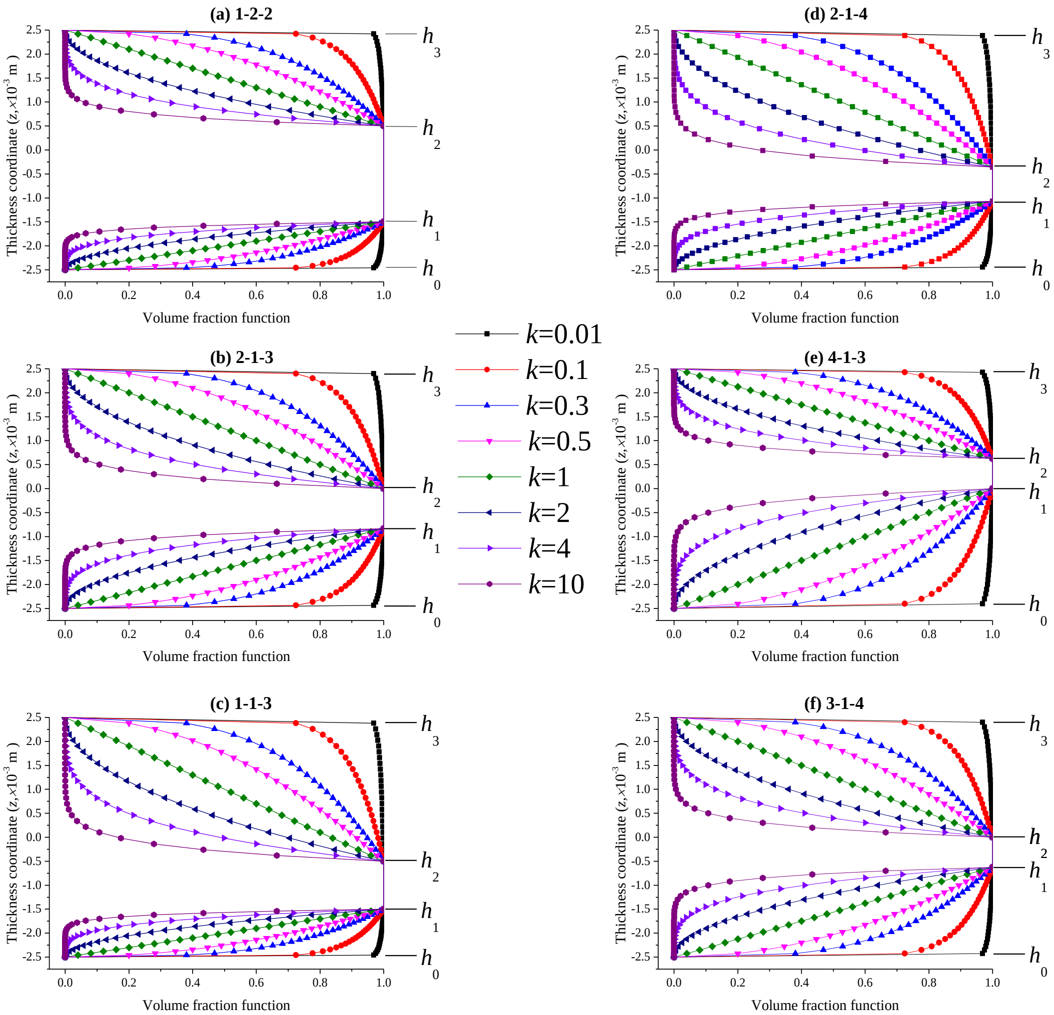

Fig. 2 Variation of volume fraction along the panel thickness for various power-law indexes: (a) 1-2-2, (b) 2-1-3, (c) 1-1-3, (d) 2-1-4, (e) 4-1-3, (f) 3-1-4 FG sandwich panel

restricted field of variables at the panel edges corresponding to each condition is given as:

Simply-supported (S)

$$x=0,\ a \quad q_0 = r_0 = q_1 = q_2 = q_3 = 0$$

$$y=0,\ b \quad p_0 = r_0 = p_1 = p_2 = p_3 = 0$$

Clamped (C)

$$x=0,\ a;\ y=0,\ b\ p_0 = q_0 = r_0 = p_1 = q_1 = p_2 = q_2 = p_3 = q_3 = 0$$

Free (F)

$$x=0,\ a;\ y=0,\ b\ p_0 = q_0 = r_0 = p_1 = q_1 = p_2 = q_2 = p_3 = q_3 \neq 0$$

Based on this definition, the combinations such as: (a) All sides simply supported [SSSS], (b) All sides clamped [CCCC], (c) Two opposite sides simply supported and others free [SFSF], (d) Two opposite sides simply supported and others clamped [SCSC], (e) Two opposite sides clamped and others free [CFCF] and (f) Cantilever (one side clamped, others free) [CFFF] condition has been attained. The corresponding boundary conditions have been mentioned wherever they are utilized throughout the present analysis.

3.1 Model convergence and validity assessment

As a very first step, the convergence behaviour of the present structural model has been studied. The nondimensional fundamental frequency is computed for different mesh sizes by considering Aluminum-Alumina (Al/Al_2O_3) and Aluminum-Zirconia (Al/ZrO_2) FG sandwich spherical and cylindrical shell panels under diverse support conditions ([SSSS], [CCCC] and [SCSC]). The physical property for the Al/Al_2O_3 sandwich panels is: h=0.005 m, a/b=1, a/h=20, R/a=5, CFR=3 and k=5, whereas the corresponding properties for the Al/ZrO_2 panels is: h=0.005 m, a/b=1, a/h=80, R/a=20, CFR=25 and k=10. Fig. 3(a) and (b) depicts the variation of nondimensional fundamental frequency with increasing mesh size for Al/Al_2O_3 and Al/ZrO_2 sandwich panels, respectively. It is evident that the present results converge well with mesh refinement not only for both of the materials considered but also under different end conditions as well. Inferentially, a (6×6) mesh is utilized for the computation purpose throughout the present analysis.

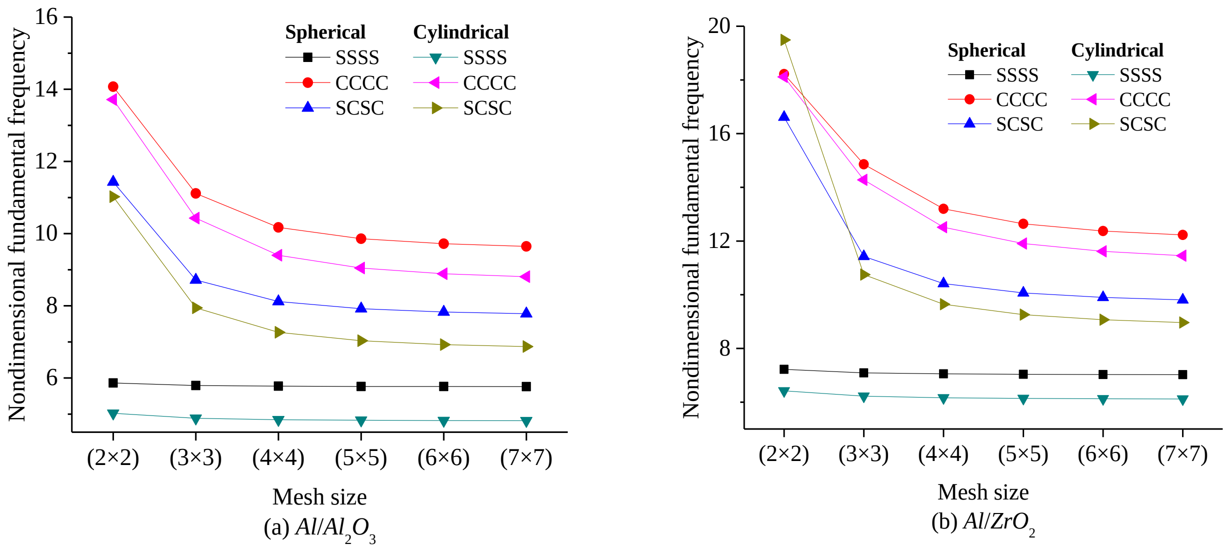

Fig. 3 Convergence of nondimensional fundamental frequency of spherical and cylindrical panels: (a) Al/Al_2O_3, (b) Al/ZrO_2

Table 2 Comparison of nondimensional fundamental frequencies of FG sandwich flat panels with different power-law indexes

k	Theory	1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1
0.5	CPT (Zenkour 2005b)	1.47157	1.51242	1.54264	1.54903	1.58374	1.60722
	FSDT (Zenkour 2005b)	1.44168	1.48159	1.51035	1.51695	1.55001	1.57274
	TSDT (Zenkour 2005b)	1.44424	1.48408	1.51253	1.51922	1.55199	1.57451
	SSDT (Zenkour 2005b)	1.44436	1.48418	1.51258	1.51927	1.55202	1.57450
	3D (Zenkour 2005b)	1.44614	1.48608	1.50841	1.52131	1.54926	1.57668
	Present (HST)	1.4442	1.4840	1.50624	1.5191	1.5470	1.5743
1	CPT (Zenkour 2005b)	1.26238	1.32023	1.3715	1.37521	1.43247	1.46497
	FSDT (Zenkour 2005b)	1.24031	1.29729	1.34637	1.35072	1.40555	1.43722
	TSDT (Zenkour 2005b)	1.24320	1.30011	1.34888	1.35333	1.40789	1.43934
	SSDT (Zenkour 2005b)	1.24335	1.30023	1.34894	1.35339	1.40792	1.43931
	3D (Zenkour 2005b)	1.24470	1.30181	1.33511	1.35523	1.39763	1.44137
	Present (HST)	1.2433	1.3003	1.3335	1.3535	1.3958	1.4394
5	CPT (Zenkour 2005b)	0.95844	0.99190	1.08797	1.05565	1.16195	1.18867
	FSDT (Zenkour 2005b)	0.94256	0.97870	1.07156	1.04183	1.14467	1.17159
	TSDT (Zenkour 2005b)	0.94598	0.98184	1.07432	1.04466	1.14731	1.17397
	SSDT (Zenkour 2005b)	0.94630	0.98207	1.07445	1.04481	1.14741	1.17399
	3D (Zenkour 2005b)	0.94476	0.98103	1.02942	1.04532	1.10983	1.17567
	Present (HST)	0.9454	0.9815	1.0298	1.0448	1.1090	1.1745
10	CPT (Zenkour 2005b)	0.94321	0.95244	1.05185	1.00524	1.11883	1.13614
	FSDT (Zenkour 2005b)	0.92508	0.93962	1.03580	0.99256	1.10261	1.12067
	TSDT (Zenkour 2005b)	0.92839	0.94297	1.03862	0.99551	1.10533	1.12314
	SSDT (Zenkour 2005b)	0.92875	0.94332	1.04558	0.99519	1.04154	1.13460
	3D (Zenkour 2005b)	0.92727	0.94078	0.98929	0.99523	1.06104	1.12466
	Present (HST)	0.9278	0.9423	0.9908	0.9954	1.0608	1.1237

Subsequently, the validity of the natural frequencies obtained via the current HST based FE approach is examined by comparing them with the analytical/numerical/3D elasticity solution values for flat panel cases (flat panels are considered to be the simplest form of shell panels) reported in open literature. Firstly, the nondimensional fundamental frequencies of simply supported FG sandwich flat panels (h=0.1 m, a/b=1, a/h=10, k=0.5, 1, 5 and 10) as considered by Zenkour (Zenkour 2005b) are reproduced for different symmetries

Table 3 Comparison of nondimensional fundamental frequencies of FG sandwich flat panels with different thickness ratios

a/h	Theory	1-1-1				2-2-1		
		0	0.5	1	5	0.5	1	5
5	HST13 Natarajan and Manickam (2012)	1.6774	1.4219	1.2778	0.9986	1.4455	1.3144	1.0565
	HST11 Natarajan and Manickam (2012)	1.6774	1.4219	1.2778	0.9988	1.4455	1.3144	1.0566
	HST9 Natarajan and Manickam (2012)	1.6774	1.4152	1.2714	0.9937	1.4387	1.3078	1.0510
	FSDT Natarajan and Manickam (2012)	1.6689	1.4076	1.2628	0.9860	1.4320	1.3002	1.0444
	3D (Li *et al*. 2008)	1.6771	1.4218	1.2777	0.9980	1.4454	1.3143	1.0561
	Present (HST)	1.6680	1.4135	1.2699	0.9927	1.4009	1.2500	0.9736
10	HST13 Natarajan and Manickam (2012)	1.8269	1.5214	1.3553	1.0455	1.5494	1.3977	1.1100
	HST11 Natarajan and Manickam (2012)	1.8269	1.5214	1.3553	1.0456	1.5494	1.3977	1.1100
	HST9 Natarajan and Manickam (2012)	1.8245	1.5193	1.3553	1.0441	1.5472	1.3957	1.1084
	FSDT Natarajan and Manickam (2012)	1.8242	1.5168	1.3506	1.0418	1.5451	1.3932	1.1064
	3D (Li *et al*. 2008)	1.8268	1.5213	1.3552	1.0453	1.5493	1.3976	1.1098
	Present (HST)	1.8237	1.5191	1.3535	1.0448	1.5062	1.3335	1.0298
100	HST13 Natarajan and Manickam (2012)	1.8884	1.5605	1.3852	1.0631	1.5904	1.4300	1.1303
	HST11 Natarajan and Manickam (2012)	1.8884	1.5605	1.3852	1.0631	1.5904	1.4300	1.1303
	HST9 Natarajan and Manickam (2012)	1.8883	1.5605	1.3851	1.0631	1.5904	1.4300	1.1302
	FSDT Natarajan and Manickam (2012)	1.8883	1.5605	1.3851	1.0631	1.5904	1.4299	1.1302
	3D (Li *et al*. 2008)	1.8883	1.5605	1.3851	1.0631	1.5903	1.4299	1.1302
	Present (HST)	1.8985	1.5691	1.3928	1.0691	1.5561	1.3727	1.0561

(1-0-1, 2-1-2, 2-1-1, 1-1-1, 2-2-1 and 1-2-1) using the present scheme and listed in Table 2 alongside the reference values.

Further, to justify the capability of the present scheme to accurately capture the influences of physical conditions such as material property gradient and the stress variation along the thickness and material discontinuity at the core-face interface on the free vibration responses simply supported square FGM sandwich plates with homogeneous core as considered by Natarajan and Manickam (2012) are analyzed. The fundamental frequency parameters are computed by using the geometry, material properties and support conditions analogous to that of the reference (Natarajan and Manickam 2012) and presented in Table 3.

The results in both the comparison studies clearly show the close conformance of the current values with the reference data. However, it is noted that the present values are marginally smaller while compared to the higher-order solutions reported by Zenkour (2005b) and Natarajan and Manickam (2012). However, the present results are showing higher values in comparison to the 3D elasticity (Li *et al*. 2008) solutions for each type of parameters i.e., the thickness ratio and power-law index values.

3.2 Parametric study

In this section, the parametric studies have been carried out to investigate the impact of several design parameters such as curvature ratio, thickness ratio, aspect ratio, power-law index, support conditions and the FG sandwich symmetry type on the natural frequency of FG sandwich curved shell panel. Several numerical examples are solved with the aid of the present higher-order FE model and the typical results are shown and discussed in detail.

Firstly, the influence of curvature ratio (R/a=1, 2, 5, 10, 25) on the frequency response of symmetric FG sandwich curved panels under [SCSC] support conditions is investigated. The nondimensional fundamental frequencies are computed for increasing *CFR* and illustrated in Fig. 4(a) and (b) for spherical, cylindrical, and elliptical, hyperboloid, geometry, respectively. It can clearly be observed that the frequency parameters decrease with increasing curvature ratio whereas increase with increasing *CFR* for all of the geometries considered. However, the decrease in the natural frequency is relatively larger at lower curvature ratios in comparison to that at higher curvature ratios. It is important to mark that for the present configuration the frequency values for the spherical and hyperboloid shell panels are higher than the corresponding

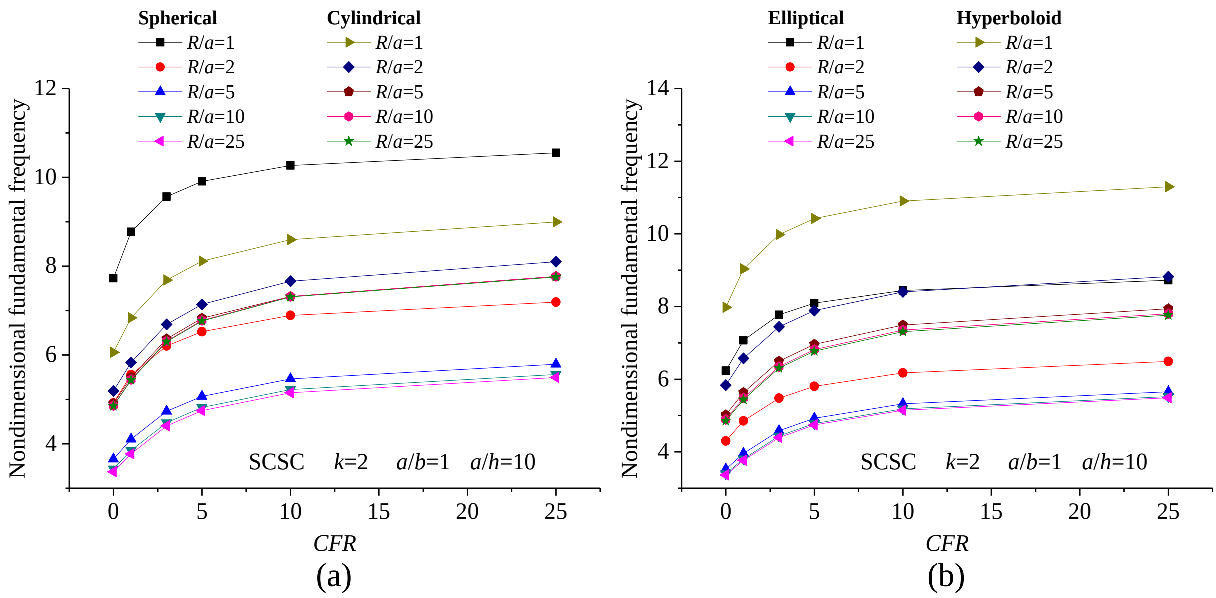

Fig. 4 Variation of nondimensional fundamental frequency with curvature ratio: (a) Spherical and Cylindrical, (b) Elliptical and Hyperboloid FG sandwich shell panels

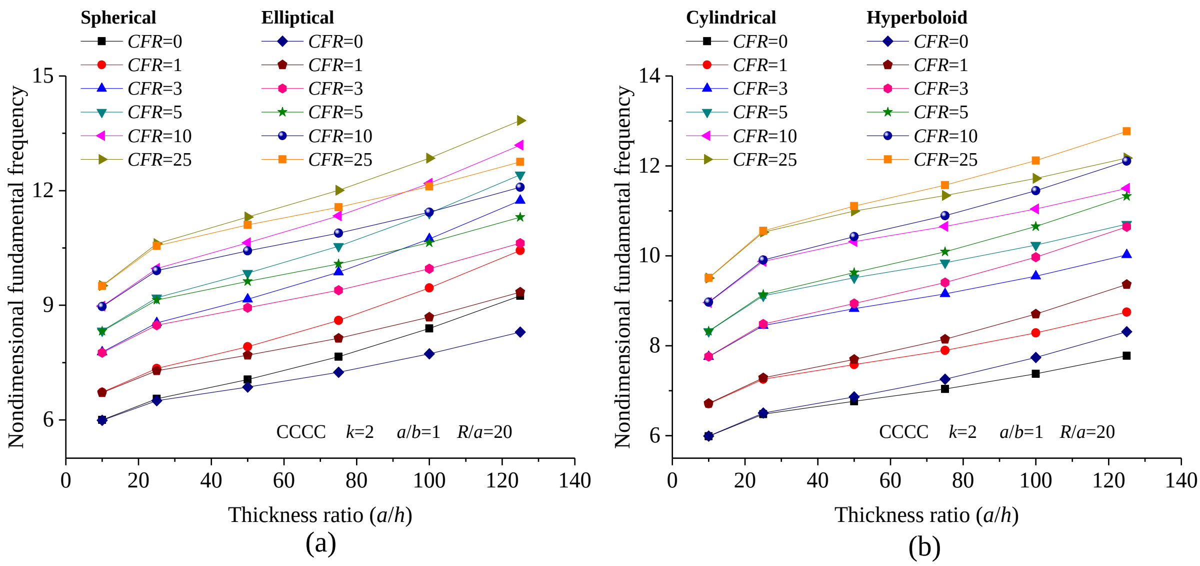

Fig. 5 Variation of nondimensional fundamental frequency with core-face thickness ratio (CFR): (a) Spherical and Cylindrical, (b) Elliptical and Hyperboloid FG sandwich shell panels

values for the cylindrical elliptical panels, respectively and hyperboloid panels exhibit the highest frequency. This is attributed to the presence of positive and negative curvatures in the longitudinal and the transverse directions, respectively that makes the panel stiffer in comparison to the other geometries.

The clamped FG sandwich square shell panels (R/a=20) are considered for studying the influence of *CFR* on fundamental frequency responses. The thickness of core and face is varied keeping the overall thickness (h) of the panels as constant so as to attain the *CFR* values as 0, 1, 3, 5, 10 and 25. The nondimensional fundamental frequency values are calculated using the above *CFR* values for all types of geometries and depicted in Fig. 5. With increasing *CFR* values the core becomes thicker thereby increasing the stiffness of the panels and as a result of that the natural frequency increases. Also, it is evident that the influence of *CFR* on the vibration responses for a particular value of thickness ratio is more pronounced as the panels tend to become thin. In consistence with the results in the preceding subsection, the hyperboloid shell panels are stiffer compared to other shell geometries and the same is evident from the higher frequency values as illustrated in Fig. 5(b).

Fig. 6 shows the variation in nondimensional fundamental frequency values of FG sandwich curved shell panels under [CFCF] support condition with increasing aspect ratio (a/b). In this example, "a" is varied by keeping "b" constant so as to have the values of aspect ratios (a/b)

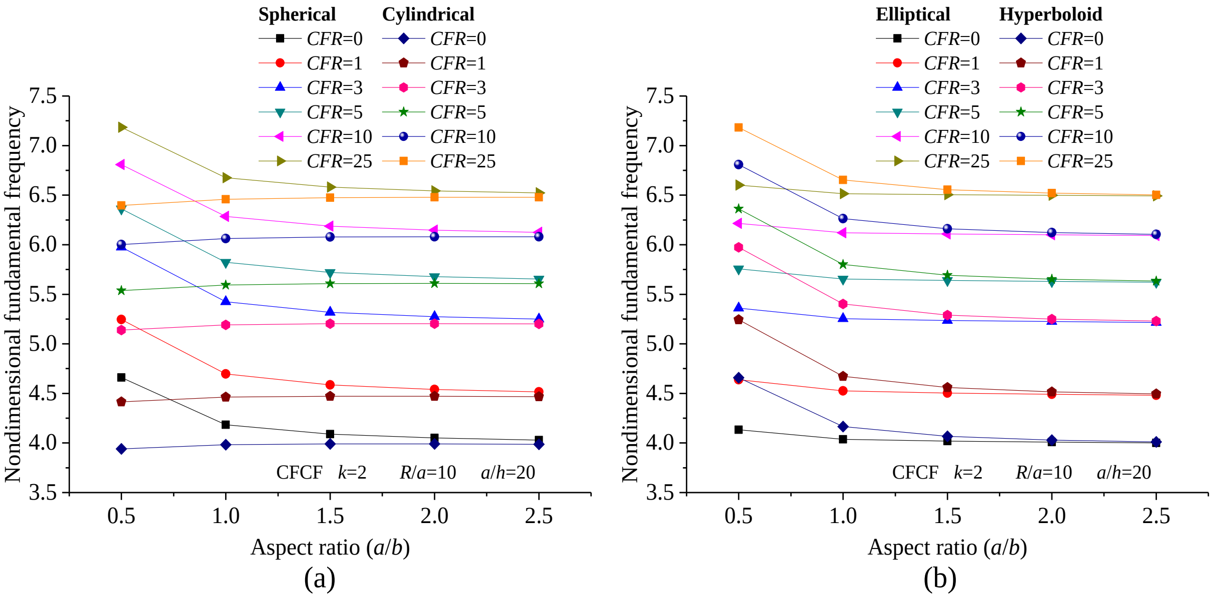

Fig. 6 Variation of nondimensional fundamental frequency with aspect ratio (a/b): (a) Spherical and Cylindrical, (b) Elliptical and Hyperboloid FG sandwich shell panels

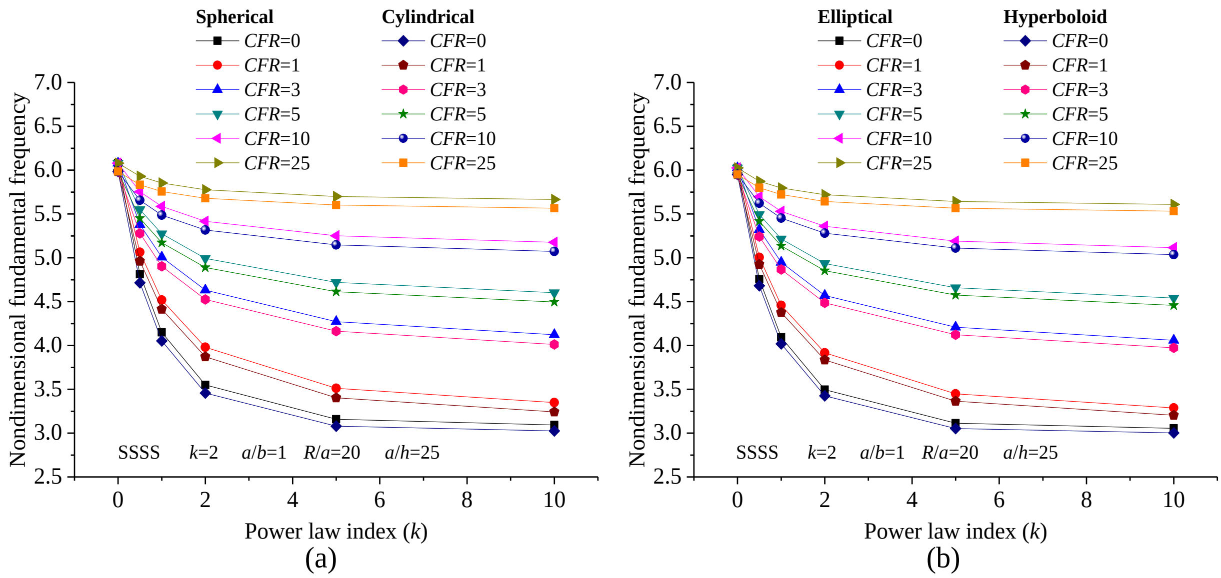

Fig. 7 Variation of nondimensional fundamental frequency with power-law index (k): (a) Spherical and Cylindrical, (b) Elliptical and Hyperboloid FG sandwich shell panels

as a/b=0.5, 1, 1.5, 2 and 2.5. It is evident that the fundamental frequency parameter follows a decreasing trend with increasing aspect ratio and the spherical as well as the hyperboloid shell panels are more susceptible to variation in aspect ratio in contrast to other geometries for which the frequency remains near constant over the range of aspect ratios values considered.

Simply supported curved FG sandwich square shell panels (a/h=25 and R/a=20) are now examined to bring out the influence of power-law index on their free vibration response. The power-law index is varied as k=0, 0.5, 1, 2, 5 and 10 and the frequency values are calculated for diverse *CFR* values (*CFR*=0, 1, 3, 5, 10 and 25) corresponding to each value of k. It can be observed from the results presented in Fig. 7 that the fundamental frequency decreases monotonously with increasing power-law index. However, the decrease in frequency for all of the considered geometries is much significant for lower *CFR* values. Further, in the present case the difference in frequency values for spherical and cylindrical panels as compared to the elliptical and hyperboloid panels respectively is insignificant.

In order to investigate the effect of various support conditions ([SSSS], [CCCC], [SCSC], [CFCF] and

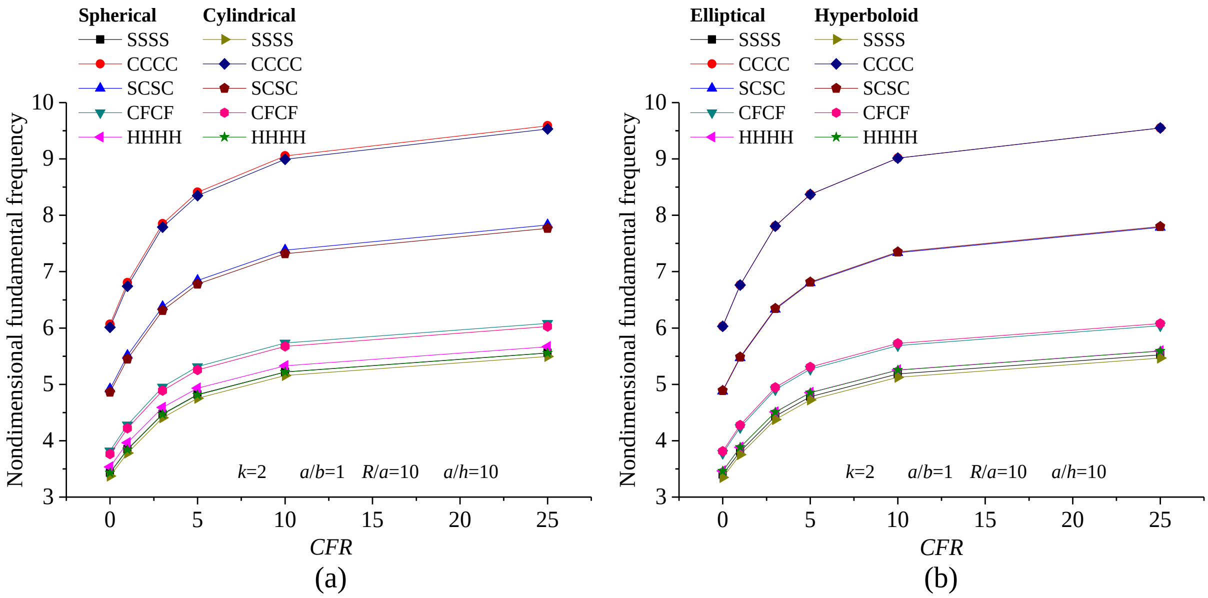

Fig. 8 Variation of nondimensional fundamental frequency with support conditions: (a) Spherical and Cylindrical, (b) Elliptical and Hyperboloid FG sandwich shell panels

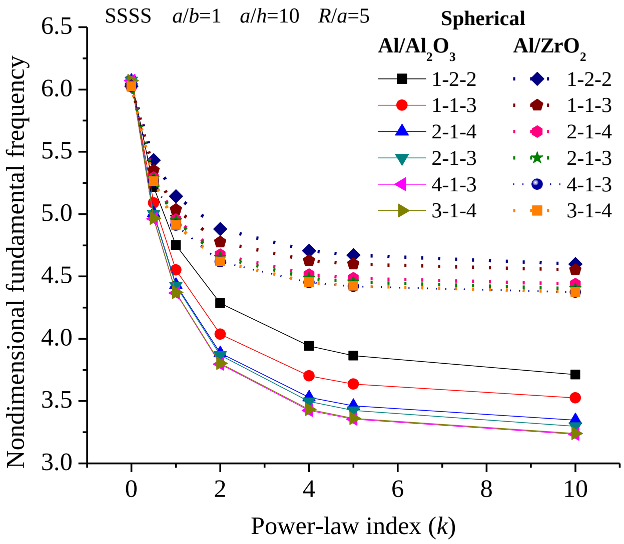

Fig. 9 Variation of nondimensional fundamental frequency of spherical shell panels with sandwich symmetry

[HHHH]) on the free vibration responses, FG sandwich curved square shell panels (R/a=10 and a/h=10) are analyzed. It is observed that the increasing number of constraints has a stiffening influence on the panels and the same is evident from the plot of nondimensional fundamental frequency values with respect to diverse *CFR* values as shown in Fig. 8. The panels exhibit highest and lowest frequency values corresponding to [CCCC] and [SSSS] support condition, respectively irrespective of their geometries. Also, the fundamental frequency increases with increasing *CFR* for all the geometries and under every support conditions. Interestingly, the [SCSC] and [CCCC] case have identical behavior in case of elliptical and hyperboloid geometries corresponding to all *CFR* values.

Finally, the influence of sandwich symmetry type on the free vibration responses of simply supported FG sandwich spherical square shell panels is studied. The Al/Al_2O_3 and Al/ZrO_2 panels with a/h=10 and R/a=5 are considered. The 1-2-2, 1-1-3, 2-1-4, 2-1-3, 4-1-3 and 3-1-4 schemes are considered. The nondimensional fundamental frequencies are computed for increasing power-law index (k) and plotted in Fig. 9. The thickness of the core relative to the thickness of the face sheets is observed to have a direct influence on the frequencies. The 1-2-2 scheme has the thickest core and so has the highest fundamental frequency of all the schemes for both of the material combinations. The 4-1-3 and 3-1-4 schemes exhibit identical behavior specifically for lower k values. It is worthy to note that, the Al/ZrO_2 panels are stiffer in comparison to the Al/Al_2O_3 panels and exhibit higher values of frequency corresponding to all of the power-law index values.

4. Conclusions

In this article, the free vibration responses of FG sandwich doubly curved shell panels are investigated numerically in the framework of the HST based mid-plane kinematics relation with nine degrees of freedom via own FE code developed in MATLAB environment. The core layer is made of a fully ceramic layer whereas the face layers are considered to be an isotropic material with material properties varying smoothly in the thickness direction only. The convergence and validation study confirmed the competency of the present formulation for reliable estimation of the natural frequencies of FG sandwich shell panels. From the results obtained in the parametric study it is revealed that the fundamental frequencies decrease with increasing curvature ratio whereas increase with increasing *CFR* values. In similar line, the sandwich symmetry type with the thickest core exhibits highest frequency. Moreover, irrespective of the panel geometries, the simply supported and clamped support condition leads to least and value of frequencies, respectively.

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