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Prediction of compressive strength of GGBS based concrete using RVM

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Abstract. Ground granulated blast furnace slag (GGBS) is a by product obtained from iron and steel industries, useful in the design and development of high quality cement paste/mortar and concrete. This paper investigates the applicability of relevance vector machine (RVM) based regression model to predict the compressive strength of various GGBS based concrete mixes. Compressive strength data for various GGBS based concrete mixes has been obtained by considering the effect of water binder ratio and steel fibres. RVM is a machine learning technique which employs Bayesian inference to obtain parsimonious solutions for regression and classification. The RVM is an extension of support vector machine which couples probabilistic classification and regression. RVM is established based on a Bayesian formulation of a linear model with an appropriate prior that results in a sparse representation. Compressive strength model has been developed by using MATLAB software for training and prediction. About 70% of the data has been used for development of RVM model and 30% of the data is used for validation. The predicted compressive strength for GGBS based concrete mixes is found to be in very good agreement with those of the corresponding experimental observations.

Keywords: relevance vector machine; GGBS, concrete; compressive strength; variance

1. Introduction

Concrete made by using Portland cement (PC) is the most widely used material in the construction sector accounting for about 30% of all materials on the planet and 70% of all materials in the built environment. The production of cement clinker is highly expensive and harmful in view of ecologically and environmentally due to hazardous emissions of CO_2, NO_x and SO_x, which are the significant contributors to the “greenhouse gas (GHG) effect”. To reduce greenhouse effect, various supplementary cementitious materials, such as ground granulated blast furnace slag (GGBS), fly ash and silica fume are commonly used in concrete. These supplementary cementitious materials not only reduce the greenhouse effect but also improve durability, reduce porosity and improve the interface with the aggregate. While using the supplementary cementitious materials certain aspects such as economics (lower cement requirement), energy, and environmental considerations are to be studied for better engineering and performance properties. The lower cement requirement for preparation of concrete leads to a reduction of CO_2 (Badogiannis *et al.* 2004, Roy *et al.* 2001, Ferraris *et al.* 2001; Chan and Wu, 2000). The inclusion of mineral admixture such as GGBS has been recognized to improve certain concrete properties. In comparison to Ordinary Portland Cement (OPC), the production of GGBS requires less energy and it produces less greenhouse gases. GGBS is a by-product in the manufacture of pig iron and the amounts of iron and slag obtained are of the same order. The slag contains lime, silica, and alumina, the same oxides that required for Portland cement, but in different proportion (Sha and Pereira 2001, Domone and Soutsos 1995).

The composition of blast-furnace slag is typically, silicon, calcium, aluminum, magnesium, and oxygen, hence a GGBS-blended concrete is a more environmentally friendly concrete compared to an OPC concrete. A GGBS-blended concrete paste found to improve fluidity and reduction of bleeding (Gao *et al.* 2005). It is well documented on the aspects related to mechanical properties of GGBS-blended concrete (Gopalkrishnan *et al.* 2001, Rols *et al.* 1999, Megat *et al.* 1999, Binici *et al.* 2007). Vejmelkova *et al.* (2009) mentioned that GGBS-blended concrete with 10% replacement of cement by GGBS (specific surface area of 360 m^2/kg) exhibited a 20% reduction in the open porosity measured by water vacuum saturation method and mercury intrusion porosimetry compared to OPC concrete. Several investigations were carried out on the effect of GGBS in numerous ways, namely, workability of concrete (Megat *et al.* 2011), compressive strength (Guneyisi and Gesoglu 2008, Oner and Akyuz 2007, Erdogan *et al.* 2016, Chidiac and Panesar 2008, Yazici 2007), durability properties (Teng *et al.* 2013 Chidiac and Panesar 2008).

In view of difficulty in conducting experiments everal times and to reduce time and effort, analytical models will be useful to predict the mechanical properties. There are several advanced statistical models, namely, Artificial

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Neural Network, Gaussian regression process, least squares support vector machine, relevance vector machine, extreme learning machine and multivariate adaptive regression splines to predict the response of the structural components or mechanical and durability properties of concrete mixes (Yuvaraj *et al.* 2013a, 2013b, 2014a, 2014b, Shantaram *et al.* 2014, Vishal *et al.* 2014, Susom Dutta *et al.* 2017, Jaideep Kaur *et al.* 2017, Erdem 2017, Engin *et al.* 2015, Mansouri *et al.* 2016). In the present investigation, it is proposed to employ relevance vector machine to predict the compressive strength of various GGBS based concrete mixes.

Relevance vector machine was initially proposed by Tipping (2001) which was developed based on support vector machine (SVM). In SVM, the target function minimizes a measure of error on the training set data and simultaneously maximizes the 'margin' between the two classes (in the feature space implicitly defined by the kernel). This found to be an effective mechanism to avoid over fitting (Tipping 2001). Though there are good predictions obtained from SVM, it was found that there are several limitations and demerits (Tipping 2000, Caesarendra *et al.* 2009). RVM is a special case of a sparse kernel model, which consists of a Bayesian treatment of a generalized linear model of identical functional form as in the case of support vector machine (SVM). RVM differs from SVM in the case of solution, which is based on probabilistic interpretation of its output (Wei *et al.* 2005). RVM evades the complexity by producing simple models that have both a structure and a parameterization process together in relation to the data type. RVM is a probabilistic based approach, introduces a prior over the model weights governed by a set of hyperparameters associated with each weight, whose most probable values are interactively estimated from the data. The important feature of RVM is that it requires less kernel functions. RVM based regression and classifications are popular in many fields (Dawei *et al.* 2002, Wei *et al.* 2005, Sarat Kumar Das and Pijush Samui 2008, Achmad *et al.* 2009, Xiaodong Wanga *et al.* 2009, Kefei Liu and Zhisheng Xu 2011, Yuvaraj *et al.* 2014b). From the above literature, it was found that RVM based models for prediction of data in the field of structural engineering is limited. Authors were carried out several experimental studies to evaluate various mechanical properties of different GGBS based concrete mixes. In the present study, compressive strength values for various GGBS based mixes are predicted by developing a regression model based on relevance vector machine approach.

2. Compressive strength of various CGBS based concrete mixes

For various GGBS based concrete mixes, compressive strength data obtained from experiments for 28 days, 90 days and 180 days are compiled and the data is presented in Table 1(b). Compressive strength is compiled against water to binder ratio, % of cement replacement by GGBS, % of steel fibres and cement quantity. The compressive strength of various mixes was obtained by testing a cube of

Table 1(a) Physical and Chemical Properties of Cement and GGBS

Parameter	Cement	GGBS
Specific gravity (g/cm^3)	3.13	2.88
Specific surface (cm^2/g)	3513	4250
Fineness (retained on 90μm sieve)	9%	---
Fineness (retained on 45μm sieve)	-----	4%
Consistency	28%	
Initial setting time (min)	100	
Final setting time (min)	312	
Soundness test/expansion of cement (mm)	1	
SiO2	20.48	32.51
Al2O3	5.44	14.37
Fe2O3	2.71	0.15
CaO	64.30	43.98
Na2O	0.21	0.31
MgO	1.72	5.17
K2O	0.42	0.26
SO3	1.94	3.03

Table 1(b) Compressive strength of various GGBS based concrete mixes

GGBS concrete mix input				Output		
% of cement replacement by GGBS	% steel fibres	Water to binder ratio	Cement quantity kg/m^3	Compressive strength, MPa 28 days	90 days	180 days
0		0.4	350	44.5	46.32	48.46
20		0.4	350	52.66	60.56	62.14
40		0.4	350	50.33	59.39	61.86
60		0.4	350	46.33	57.68	60.69
80		0.4	350	37.16	47.75	49.65
0		0.5	350	49.33	51.55	53.77
20		0.5	350	50.16	57.58	58.94
40		0.5	350	49.66	59.1	61.08
60		0.5	350	46.81	57.62	60.38
80		0.5	350	25.5	32.64	34.17
0	0.5	0.5	350	53.28		
20	0.5		350	55.63		
40	0.5		350	54.74		
60	0.5		350	50.52		
80	0.5		350	27.85		
0		0.3	400	69.6	72.61	75.52
20		0.3	400	74.26	86.88	87.26
40		0.3	400	71.42	84.96	87.85
50		0.3	400	68.42	83.26	86.89
60		0.3	400	65.46	81.56	85.10
80		0.3	400	64.00	80.32	83.20
0		0.4	400	45.30	47.22	49.23
20		0.4	400	48.20	55.04	56.35
40		0.4	400	47.66	56.72	58.48
50		0.4	400	43.58	52.95	54.04
60		0.4	400	40.96	50.79	53.25
80		0.4	400	38.62	49.05	51.56

Table 1(b) Continued

0		0.5	400	42.86	44.83	46.76
20		0.5	400	44.50	50.73	52.29
40		0.5	400	43.00	50.91	52.68
50		0.5	400	38.48	46.91	49.25
60		0.5	400	37.00	45.88	48.29
80		0.5	400	33.40	42.42	44.42
0		0.5	400	45.98		
20	0.5		400	48.65		
40	0.5		400	46.87		
50	0.5		400	43.21		
60	0.5		400	40.11		
80	0.5		400	35.24		
0		0.3	450	71.61	75.19	77.67
20		0.3	450	77.00	88.17	89.32
40		0.3	450	74.65	87.71	91.07
50		0.3	450	69.01	84.19	85.57
60		0.3	450	65.56	81.62	83.92
80		0.3	450	59.52	71.42	76.78
0		0.4	450	46.60	48.88	50.65
20		0.4	450	54.62	61.72	63.91
40		0.4	450	49.12	58.45	60.42
50		0.4	450	45.20	55.14	57.86
60		0.4	450	43.50	53.94	56.12
80		0.4	450	37.56	47.70	48.83
0	0.5	0.4	450	49.56		
20	0.5		450	56.92		
40	0.5		450	53.28		
50	0.5		450	47.34		
60	0.5		450	45.04		
80	0.5		450	40.56		
0		0.5	450	43.83	45.96	47.39
20		0.5	450	45.62	52.01	53.38
40		0.5	450	44.36	52.34	54.56
50		0.5	450	42.12	51.39	53.07
60		0.5	450	39.10	48.09	50.44
80		0.5	450	34.50	43.82	45.89
0		0.3	500	76.28	80.25	82.96
20		0.3	500	81.82	92.46	96.55
40		0.3	500	77.80	92.19	94.92
50		0.3	500	69.25	88.29	88.64
60		0.3	500	66.80	82.83	86.84
80		0.3	500	60.24	76.50	80.12
0	0.5	0.3	500	79.42		
20	0.5		500	87.69		
40	0.5		500	83.37		
50	0.5		500	71.13		
60	0.5		500	69.09		
80	0.5		500	64.07		

Table 1(b) Continued

0		0.4	500	51.33	53.67	55.93
20		0.4	500	58.92	66.58	68.94
40		0.4	500	52.62	62.35	64.66
50		0.4	500	48.63	59.33	61.03
60		0.4	500	45.26	56.30	58.82
80		0.4	500	41.42	52.60	55.38
0		0.5	500	49.30	51.56	53.15
20		0.5	500	51.60	56.28	60.77
40		0.5	500	50.61	60.00	62.05
50		0.5	500	47.95	58.93	61.18
60		0.5	500	42.05	52.35	54.45

size 100 mm. The properties of steel fibres are: (i) hooked end steel fiber (ii) fiber length = 30 mm (iii) fiber diameter = 0.5 mm (iv) aspect ratio = 60 (v) tensile strength = 1100 MPa (vi) Young's modulus = 2×10^5 MPa (v) density = 7800 kg/m^3. The physical and chemical properties of cement and GGBS are shown in Table 1(a).

3. Relevance vector machine

This section provides a brief description about RVM. Full details about model can be found in Tipping (2000, 2001). RVM is an extension of support vector machine which employs Bayesian model and kernel function (Tipping 2001). The key feature of RVM is that it offers a generalized performance and the inferred predictors are exceedingly sparse wherein they contain relatively few "relevance vectors".

As mentioned earlier, RVM starts with the base of linear models, i.e., the function of y(x) to be predicted at some arbitrary point x given a set of (typically noisy) measurements of the function $t=(t_1, y, t_N)$ and with some training points $x=(x_1, y, x_N)$

$$t_i = y(x_i) + \varepsilon_i \tag{1}$$

Where, ε_i is the noise component of the measurement with mean 0 and variance σ^2. With a linear model assumption, the unknown function y(x) can be written as a linear combination of some known basis function i.e.,

$$y(x) = \sum_{i=1}^{M} w_i \varphi_i(x) \tag{2}$$

where, $w_i=(w_1,\ldots,w_M)$ = a vector consisting of the linear combination weights

y(x) = the output which is a linearly-weighted sum of M, generally nonlinear and fixed basis functions $\varphi_i(x) = (\varphi_1(x), \varphi_2(x), \ldots\ldots\ldots\ldots, \varphi_M(x))^T$.

The details of analysis of function as shown in equation 2 are available in Tipping (2001). To arrive at good predictions, during the development of model, the majority of parameters are default set to zero (Tipping 2000, 2001).

$$t = \Phi w + \varepsilon \tag{3}$$

where, Φ = NxM design matrix, whose i^{th} column is formed with the values of basis function $\Phi_i(x)$ at all the training points

$\varepsilon_i = (\varepsilon_1,\ldots, \varepsilon_N)$, the noise vector.

As a supervised learning, RVM starts with a set of data input $\{x_n\}_n^N = 1$ and their corresponding target vector $\{t_n\}_n^N = 1$.

The basic aim of the 'training' set is to learn a model of the dependency of the target vectors on the inputs to make accurate prediction of t for previously unseen value of x.

For the case of SVM, the prediction is made by assuming the function of the form given below

$$y(x) = \sum_{i=1}^{N} w_i K(x, x_i) + w_0 \tag{4}$$

where, $w_i = w_1, w_2,\ldots, w_N$, weight vector
$K(x, x_i)$ = a kernel function and w_0 is the bias

Radial basis kernel function is used in the present study and the related equation is given below

$$K(x_i, x) = \exp\left\{-\frac{(x_i - x)^T (x_i - x)}{2\sigma^2}\right\} \tag{5}$$

where, x_i and x = the training and test patterns, respectively. d = a dimension of the input vector, σ = width of the basis function.

For a given input dataset, it is assumed as $\{x_n, t_n\}_n^N = 1$. Further, it is assumed that p (t|x) is Gaussian N (t|y(x), σ^2). The mean of this distribution for a given x is modelled by y(x) as mentioned in Eq. (4).The likelihood of dataset can be expressed as

$$p(t|w,\sigma^2) = (2\pi\sigma^2)^{-N/2} \exp \left\{-\frac{1}{2\sigma^2}\|t - \Phi w\|^2\right\} \tag{6}$$

Where, $t_i = (t_1\ldots, t_N)^T$, $\omega_i = (\omega_0,\ldots,\omega_N)$ and

$$\Phi^T = \begin{bmatrix} 1 & K(x_1,x_1) & K(x_1,x_2) & \cdots & K(x_1,x_n) \\ 1 & K(x_2,x_1) & K(x_2,x_2) & \cdots & K(x_2,x_n) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & K(x_n,x_1) & K(x_n,x_2) & \cdots & K(x_n,x_n) \end{bmatrix}$$

Where, $K(x_i, x_n)$ is the kernel function

New higher-level parameters are generally preferred to constrain an explicit zero-mean Gaussian prior probability distribution to the weights

$$p(w|\alpha) = \prod_{i=0}^{N} N(w_i|0, \alpha_i^{-1}) \tag{7a}$$

where α is a vector of (N+1) hyperparameters, controls the deviation of weight (Caesarendra 2010). By using Bayes' rule, the posterior all unknowns can be computed, given the defined non-informative prior-distributions. To complete the specification of the prior-distribution, hyperpriors are to be defined for α and noise variance σ^2. These quantities are typical scale parameters and suitable prior are Gamma Distributions (Tipping 2000)

$$p(\alpha) = \prod_{i=0}^{N} \text{Gamma}(\alpha_i|a, b), \tag{7b}$$

$$p(\beta) = \prod_{i=0}^{N} \text{Gamma}(\beta|c, d) \tag{7c}$$

Where, $\beta = \sigma^{-2}$.

Hence, for α and σ, the distribution is gamma distribution and for w, it is normal distribution and after the prior-distributions, Bayes rule is followed.

$$p(w, \alpha, \sigma^2|t) = \frac{p(t|w, \alpha, \sigma^2)p(w, \alpha, \sigma^2)}{p(t)} \tag{8a}$$

The predictive distribution for a new test point (X_*) corresponding to target (t_*) is given below

$$p(t_*|t) = \int p(t_*|w, \alpha, \sigma^2)p(w, \alpha, \sigma^2|t)dw\, d\alpha\, d\sigma^2 \tag{8b}$$

The above equation is solved by decomposition of posterior, which is given below

$$p(w, \alpha, \sigma^2|t) = p(w|t, \alpha, \sigma^2)p(\alpha, \sigma^2|t) \tag{9}$$

The posterior distribution has been analysed by considering the appropriate weights due to the property of normalization integral is convolution of gaussians (Tipping 2000). Accordingly, the equation 9 has been modified as

$$p(w|t, \alpha, \sigma^2) = \frac{p(t|w, \sigma^2)p(w, \alpha)}{p(t|\alpha, \sigma^2)} \tag{10}$$

By using the Bayes rule, the above equation can be modified as

$$p(w|t,\alpha,\sigma^2) = (2\pi)^{-(N+1)/2}|\Sigma|^{-1/2} \exp\left\{-\frac{1}{2}(w-\mu)^T \Sigma^{-1}(w-\mu)\right\} \tag{11}$$

The solution for the above equation is given below.

$$\Sigma = (\sigma^{-2}\Phi^T\Phi + A)^{-1} \tag{12}$$

$$\mu = \sigma^{-2}\Sigma\Phi^T t \tag{13}$$

Where, ∑ = covariance, μ = mean, A = (α_0, $\alpha_{1\ldots}$ α_N).

Maximization of

$$p\left(\alpha,\alpha_{\in_n}^2\middle|y\right)\alpha p\left(y\middle|\alpha,\alpha_{\in_n}^2\right)$$
$$p(\alpha)p\left(\alpha_{\in_n}^2\right)$$

with respect to a α and σ^2.
yields a search for the hyperparameters posterior

For the case of uniform hyperpriors, maximization is to be done for the term $p\left(y\middle|\alpha,\alpha_{\in_n}^2\right)$, as described below.

$$p\left(y\middle|\alpha,\alpha_{\in_n}^2\right)=\int p\left(y\middle|w,\alpha_{\in_n}^2\right)$$
$$p\left(w\middle|\alpha\right)dw$$

$$=(2\pi)^{-1/2}\left|\alpha_{\in_n}^2 I+\Phi A^{-1}\Phi^T\right|^{1/2} \tag{14}$$
$$\times\exp\left\{-\frac{1}{2}y^T\left(\alpha_{\in_n}^2 I+\Phi A^{-1}\Phi^T\right|\right)^{-1}y\right\}$$

Determination of hyperparameters can be done by using an iterative formula, namely, a gradient ascent on the objective function (Tipping 2000, Ghosh and Mujumdar 2008). Predictions for a new data are then made by performing integration of the weights to arrive at the marginal likelihood for the hyperparameters. The predictions are made based on the posterior distribution over the weights, conditioned on the maximized most probable values of α and $\sigma_{\in_n}^2, \alpha_{MP}$ and σ_{MP}^2 respectively.

$$p\left(y_*\middle|y,\alpha_{MP},\sigma_{MP}^2\right)= \int p\left(y_*\middle|w,\sigma_{MP}^2\right)p\left(w\middle|y,\alpha_{MP},\middle|\sigma_{MP}^2\right)dw \tag{15}$$

This can readily be evaluated as

$$p\left(y_*\middle|y,\alpha_{MP},\sigma_{MP}^2\right)=N\left(y_*\middle|t_*,\sigma_*^2\right) \tag{16}$$

$$t_*=\mu^T\Phi(x_*) \tag{17}$$

With

$$\sigma_*^2=\sigma_{MP}^2+\Phi(x_*)^T\sum\Phi(x_*) \tag{18}$$

the result of the optimization involved in RVM (i.e., max of $p\left(y\middle|\alpha,\sigma_{\in_n}^2\right)$), is that many of α tend to infinity such that 'w' will have only a few nonzero weights that can be considered as relevant vectors (Ghosh and Mujumdar, 2008). The relevant vectors (RVs) can be viewed as counterparts of support vectors (SVs) in SVM. Hence, the developed model contains the benefits of SVM (sparsity and generalization) and in addition, provides estimates of uncertainty bounds in the predictions (Ghosh and Mujumdar 2008).

3.1 RVM Based analysis

For prediction of the compressive strength of various

Table 2 Training data set of various GGBS based concrete mixes

GGBS concrete mix input				Output		
% of cement replacement by GGBS	% steel fibres	Water to binder ratio	Cement quantity kg/m^3	Compressive strength, MPa 28 days	90 days	180 days
0		0.4	350	44.5	46.32	48.46
20		0.4	350	52.66	60.56	62.14
60		0.4	350	46.33	57.68	60.69
0		0.5	350	49.33	51.55	53.77
20		0.5	350	50.16	57.58	58.94
40		0.5	350	49.66	59.1	61.08
80		0.5	350	25.5	32.64	34.17
0	0.5	0.5	350	53.28		
20	0.5	0.5	350	55.63		
60	0.5	0.5	350	50.52		
80	0.5	0.5	350	27.85		
0		0.3	400	69.6	72.61	75.52
40		0.3	400	71.42	84.96	87.85
80		0.3	400	64.00	80.32	83.20
0		0.4	400	45.30	47.22	49.23
20		0.4	400	48.20	55.04	56.35
40		0.4	400	47.66	56.72	58.48
80		0.4	400	38.62	49.05	51.56
0		0.5	400	42.86	44.83	46.76
50		0.5	400	38.48	46.91	49.25
80		0.5	400	33.40	42.42	44.42
0		0.5	400	45.98		
20	0.5	0.5	400	48.65		
40	0.5	0.5	400	46.87		
80	0.5	0.5	400	35.24		
0		0.3	450	71.61	75.19	77.67
20		0.3	450	77.00	88.17	89.32
50		0.3	450	69.01	84.19	85.57
80		0.3	450	59.52	71.42	76.78
0		0.4	450	46.60	48.88	50.65
20		0.4	450	54.62	61.72	63.91
50		0.4	450	45.20	55.14	57.86
80		0.4	450	37.56	47.70	48.83
0	0.5	0.4	450	49.56		
20	0.5	0.4	450	56.92		
40	0.5	0.4	450	53.28		
50	0.5	0.4	450	47.34		
80	0.5	0.4	450	40.56		
0		0.5	450	43.83	45.96	47.39
20		0.5	450	45.62	52.01	53.38
50		0.5	450	42.12	51.39	53.07
60		0.5	450	39.10	48.09	50.44
80		0.5	450	34.50	43.82	45.89
0		0.3	500	76.28	80.25	82.96

Table 2 Continued

40		0.3	500	77.80	92.19	94.92
50		0.3	500	69.25	88.29	88.64
60		0.3	500	66.80	82.83	86.84
0	0.5	0.3	500	79.42		
20	0.5	0.3	500	87.69		
40	0.5	0.3	500	83.37		
60	0.5	0.3	500	69.09		
80	0.5	0.3	500	64.07		
0		0.4	500	51.33	53.67	55.93
20		0.4	500	58.92	66.58	68.94
50		0.4	500	48.63	59.33	61.03
80		0.4	500	41.42	52.60	55.38
0		0.5	500	49.30	51.56	53.15
40		0.5	500	50.61	60.00	62.05
80		0.5	500	38.56	49.67	51.28

GGBS based concrete mixes, RVM model has been developed. From the experimental studies (Table 2), it can be noted that the compressive strength is influenced by the water binder ratio and water cement ratio. These two parameters from the input vector and it can also be noted that the input vector has different quantitative limit as shown in Table 2. Hence, a normalization of the data has been performed before presenting the input patterns to statistical machine learning algorithm. MALAB software has been used for development of model. Thus, equation 19 has been used for the linear normalization of the data to the data values between 0 and 1.

$$x_i^n = \frac{x_i^a - x_i^{\min}}{x_i^{\max} - x_i^{\min}} \tag{19}$$

where, x_i^a and x_i^n are i[th] components of the input vector before and after normalization, respectively, $x_i^{\max}$ and $x_i^{\min}$ are the maximum and minimum values of all the components of the input vector before normalization.

3.1.1 Development of RVM model

Compressive strength data of about 86 for various GGBS based concrete mixes were tabulated for development of model. About 70 % of data set is used for the development of RVM model and about 30% of the data set is used for testing and verification of the developed model. Testing and verification of the model is done by comparing the experimental compressive strength with the predicted compressive strength by using the developed RVM model. From the literature, it was observed that the important aspect of development of RVM model is the selection of kernel width which was determined by using post modelling analysis (Wahyu *et al.* 2010). Post-

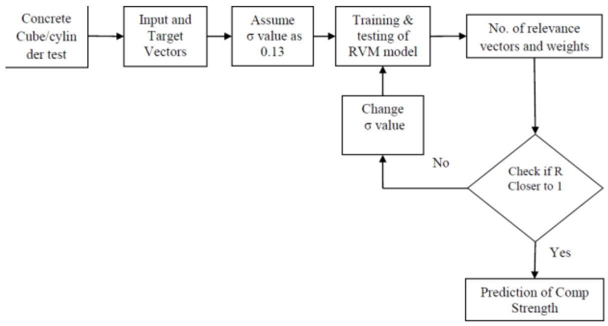

Fig. 1 Schematic diagram-development of RVM models

Table 3 Performance of developed RVM models

Parameters	Coefficient of correlation (R)		width (σ)	No. of RVs used out of total 59 dataset	No of RVs (% of training data set)
	Training	Testing			
Model I (Comp. Strength at 28 days)	0.994	0.992	0.13	36	62.06
Model I (Comp. Strength at 90 days)	0.984	0.982	0.13	28	56.06
Model I (Comp. Strength at 180 days)	0.996	0.990	0.13	28	56.06

Table 4(a) Values of weights (w_i) for RVM models (for compressive strength at 28 days)

i =1,2…59	w_i	i =1,2…59	w_i
1	0	30	0.02
2	0.062	31	0.001
3	0	32	0.013
4	0	33	0.0512
5	0	34	0.0612
6	0.035	35	0.0045
7	0	36	0.0185
8	0.125	37	0.02
9	0	38	0.051
10	0.170	39	0.0782
11	0.131	40	0.03
12	0.058	41	0.012
13	0.0543	42	0.0234
14	0.095	43	0.034
15	0.121	44	0.098
16	0.106	45	0.0421
17	0.112	46	0.002
18	0	47	0
19	0.101	48	0.003
20	0.0321	49	0.005
21	0.012	50	0.002
22	0.02	51	0.031
23	0.063	52	0.046
24	0.012	53	0.012
25	0.02	54	0.06
26	0	55	0.04
27	0.1331	56	0.0233
28	0.0134	57	0.014
29	0.0083	58	0.0231
		59	0.0543

Table 4(b) Values of weights (w_i) for RVM models (for compressive strength at 90 days)

i =1,2…59	w_i	i =1,2…59	w_i
1	0	30	0.013
2	0.032	31	0.0011
3	0	32	0
4	0	33	0
5	0	34	0
6	0.024	35	0
7	0	36	0
8	0	37	0.01
9	0	38	0.012
10	0.112	39	0.0122
11	0.151	40	0.014
12	0.023	41	0.01
13	0.0543	42	0.0112
14	0.002	43	0.012
15	0.001	44	0.043
16	0.121	45	0.0311
17	0.102	46	0
18	0.03	47	0
19	0.01	48	0
20	0.0141	49	0
21	0	50	0
22	0	51	0.021
23	0	52	0.032
24	0	53	0.011
25	0.013	54	0.012
26	0.012	55	0.024
27	0.1031	56	0.012
28	0.0121	57	0.0132
29	0.0021	58	0.0121
		59	0.010

Table 4(c) Values of weights (w_i) for RVM models (for compressive strength at 180 days)

i =1,2…59	w_i	i =1,2…59	w_i
1	0	30	0.031
2	0.045	31	0.011
3	0	32	0
4	0	33	0
5	0	34	0
6	0.0025	35	0
7	0	36	0
8	0	37	0.021
9	0	38	0.054
10	0.023	39	0.022
11	0.0131	40	0.067
12	0.005	41	0.023
13	0.012	42	0.046
14	0.043	43	0.076
15	0.02	44	0.054
16	0.093	45	0.0211
17	0.032	46	0
18	0	47	0
19	0.02	48	0
20	0.0321	49	0
21	0	50	0
22	0	51	0.087
23	0	52	0.082
24	0	53	0.087
25	0.0013	54	0.06
26	0	55	0.04
27	0.0161	56	0.0203
28	0.005	57	0.0104
29	0.0062	58	0.0201
		59	0.005

modelling analysis of the training and testing R values is associated with the number of relevance vectors (NRV) involved in the model and their corresponding weights & variation in the kernel width. The value of σ is assumed initially as 0.13 and for the assumed valued of σ, the model has been developed. Fig. 1 shows the schematic diagram of RVM model. The developed model gives the NRVs used and their corresponding weights (w_i). The quality of the developed model is evaluated based on the coefficient of correlation (R), which is given below.

$$R = \frac{\sum_{i=1}^{n}\left(E_{ai}-\overline{E}_a\right)\left(E_{pi}-\overline{E}_p\right)}{\sqrt{\sum_{i=1}^{n}\left(E_{ai}-\overline{E}_a\right)}\sqrt{\sum_{i=1}^{n}\left(E_{pi}-\overline{E}_p\right)}} \tag{20}$$

where, E_{ai} and E_{pi} are the actual and predicted values, respectively.

$\overline{E}_a$ and $\overline{E}_p$ are mean of actual and predicted E values corresponding to n patterns. In each iteration, R value is computed and the model is finalized when the R value is closer to one.

Table 4 shows the weights for RVM model for compressive strength at 28 days, 90 days and 180 days respectively.

It is observed that the testing R value achieved its maximum at kernel widths shown in Table 3 for the corresponding models, involving minimum number of relevance vectors. The training and testing R values obtained for models are presented in Table 3.

By using Eqs. (14), (15) and Table 4 with w_o as zero, the following equations have been obtained from the developed RVM model.

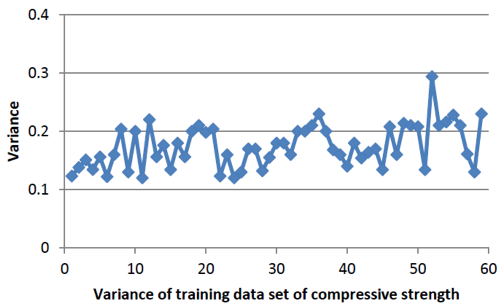

Fig. 2 Variance of training data set for compressive strength at 28 days

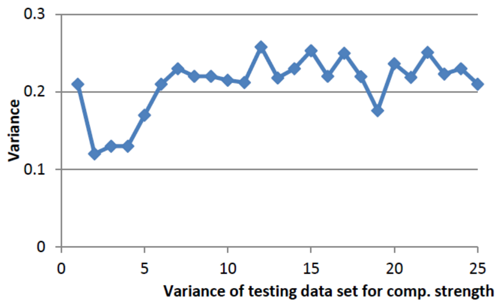

Fig. 3 Variance of testing data set for compressive strength at 28 days

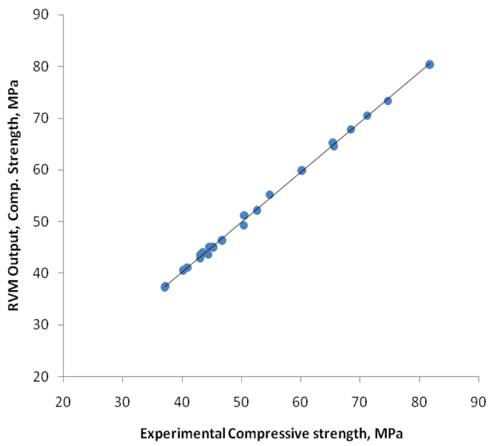

Fig. 4(a) Predicted and experimental compressive strength for 28 days

$$y = Comp\,\mathrm{Str\ at\ 28\ days} = \sum_{i=1}^{59} w_i \exp\left\{ -\frac{(x_i - x)^T (x_i - x)}{0.034} \right\} \quad (21)$$

$$y = Comp\,\mathrm{Str\ at\ 90\ days} = \sum_{i=1}^{59} w_i \exp\left\{ -\frac{(x_i - x)^T (x_i - x)}{0.034} \right\} \quad (22)$$

$$y = Comp\,\mathrm{Str\ at\ 180\ days} = \sum_{i=1}^{59} w_i \exp\left\{ -\frac{(x_i - x)^T (x_i - x)}{0.034} \right\} \quad (23)$$

Table 5 Predicted and experimental compressive strength

GGBS concrete mix input				Output					
% of cement replace-ment by GGBS	% steel fibres	Water to binder ratio	Cement qty. kg/m^3	Compressive strength, MPa					
				28 days	28 days	90 days	90 days	180 days	180 days
				Exptl.	RVM	Exptl.	RVM	Exptl.	RVM
40		0.4	350	50.33	49.32	59.39	59.32	61.86	60.54
80		0.4	350	37.16	37.56	47.75	46.98	49.65	48.92
60		0.5	350	46.81	46.21	57.62	56.87	60.38	60.87
40	0.5		350	54.74	55.21				
60	0.5		350	50.52	51.21				
50		0.3	400	68.42	67.87	83.26	81.32	86.89	85.32
60		0.3	400	65.46	65.12	81.56	80.43	85.10	84.43
50		0.4	400	43.58	43.87	52.95	52.54	54.04	53.76
60		0.4	400	40.96	41.12	50.79	51.21	53.25	53.98
20		0.5	400	44.50	45.11	50.73	51.03	52.29	51.54
40		0.5	400	43.00	42.83	50.91	50.76	52.68	52.03
60		0.5	400	37.00	37.15	45.88	46.31	48.29	49.05
50	0.5		400	43.21	43.65				
60	0.5		400	40.11	40.43				
40		0.3	450	74.65	73.32	87.71	85.32	91.07	89.43
60		0.3	450	65.56	64.62	81.62	80.21	83.92	81.98
60		0.4	450	43.50	43.89	53.94	52.78	56.12	54.21
60	0.5		450	45.04	45.13				
40		0.5	450	44.36	43.67	52.34	51.45	54.56	53.42
20		0.3	500	81.82	80.23	92.46	90.54	96.55	93.43
80		0.3	500	60.24	59.76	76.50	75.21	80.12	79.31
50	0.5		500	71.13	70.56				
40		0.4	500	52.62	52.01	62.35	63.21	64.66	65.43
60		0.4	500	45.26	44.87	56.30	56.98	58.82	59.32
20		0.5	500	51.60	50.98	56.28	56.89	60.77	61.32
50		0.5	500	47.95	47.32	58.93	59.32	61.18	62.34
60		0.5	500	42.05	52.35	54.45			

The values of weights, w_i for all the training data sets are available in Table 4 (a-c).

Variance for training and testing data set for the typical developed model are plotted and shown in Figs. 2 and 3.

The developed RVM model has been verified with the remaining 27 data sets and the results are shown in Table 5.

The normalized output vector obtained from the RVM model is converted back to original value by using the equation below.

$$x_i^a = x_i^n \left(x_i^{\max} - x_i^{\min} \right) + x_i^{\min} \quad (24)$$

where, x_i^n is the normalized result obtained after the test for the ith component.

x_i^a is the actual result obtained for ith componenet, and $x_i^{\max}$ and $x_i^{\min}$ are the maximum and minimum values of all the components of the corresponding input

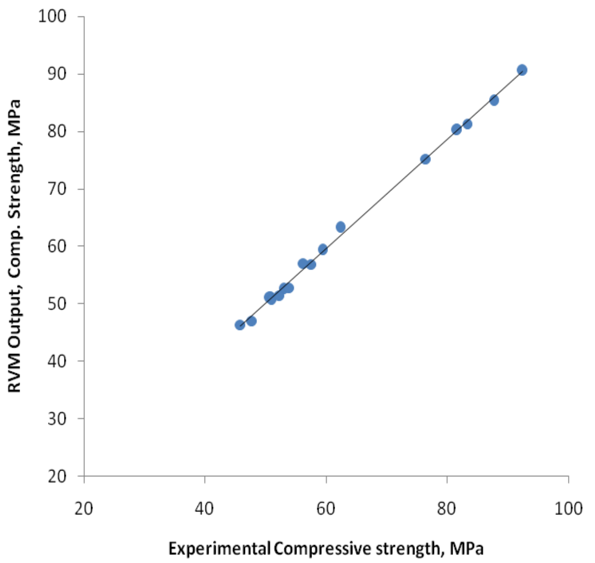

Fig. 4(b) Predicted and experimental compressive strength for 90 days

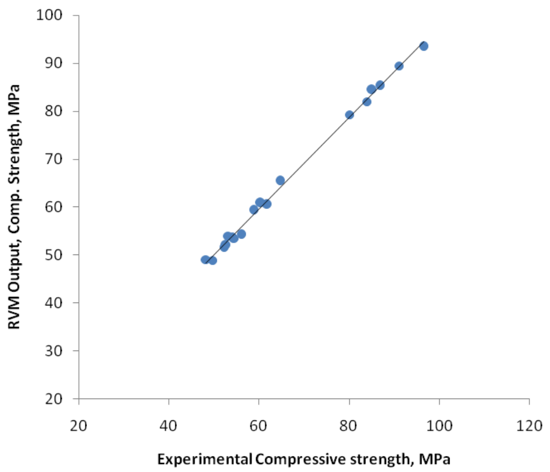

Fig. 4(c) Predicted and experimental compressive strength for 180 days

vector before the normalization.

From Table 5, it can be observed that the predicted compressive strength for 28th, 90th and 180th day is in very good agreement with the corresponding experimental observations. Fig. 4 shows the comparison plot of predicted and the corresponding experimental compressive strength. From Table 4 and Fig. 4, it can be concluded that the developed model is robust and reliable.

4. Conclusions

Relevance vector machine, one of the advanced statistical models was developed to predict a compressive strength for various GGBS based concrete mixes. The input parameters are Cement quantity, % of cement replacement by GGBS, % of steel fibres, water to binder ratio. Compressive strength data obtained from experiments for various GGBS mixes has been consolidated to develop and validate the model. Models were developed to predict the compressive strength at 28th, 90th and 180th days. MATLAB software has been used for training and prediction of compressive strength. About 70% of the data has been used for development of model and 30% of the data is used validation. The predicted compressive strength for GGBS mixes is found to be in very good agreement with those of the corresponding experimental observations. The developed equations for prediction of compressive strength can be used for all practical purposes. The R value for the developed model is found to be closer to 1 indicating better predictability of the models. From the overall study, it can be concluded that the developed RVM model is found to be robust and reliable.

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