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A novel refined plate theory for stability analysis of hybrid and symmetric S-FGM plates

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Abstract. In this paper, buckling analysis of hybrid functionally graded plates using a novel four variable refined plate theory is presented. In this theory the distribution of transverse shear deformation is parabolic across the thickness of the plate by satisfying the surface conditions. Therefore, it is unnecessary to use a shear correction factor. The variations of properties of the plate through the thickness are according to a symmetric sigmoid law (symmetric S-FGM). The principle virtual works is used herein to extract equilibrium equations. The analytical solution is determined using the Navier method for a simply supported rectangular plate subjected to axial forces. The precision of this theory is verified by comparing it with the various solutions available in the literature.

Keywords: refined plate theory; buckling analysis; symmetric S-FGM plate

1. Introduction

The functionally graded materials (FGM) are an exceptional class of composite in which the composition and structure gradually change, causing a corresponding change in the properties of the material. These types of materials have attracted much attention in recent years, caused of their advantages to reduce the disparity in material properties and reducing the thermal stresses Zhong and Yu (2007). The use of these material in numerous applications (Ahmed 2014, Zidi *et al.* 2014, 2017, Kar *et al.* 2016, Akavci 2016, Ahouel *et al.* 2016, Aldousari 2017, Karami *et al.* 2017, Bellifa *et al.* 2017a, Zine *et al.* 2018) such as aeronautic, civil engineering, nuclear and are also found in biomedical applications Baron and Naili (2008). For study of the behaviour of the FG plates under mechanical loading, many theories have been developed. The simplest is the Kirchoff-Love theory (Classical Plate Theory) generally used for thin plates which stipulates that the straight lines remaining straight and perpendicular to the mid-plane after deformation. Therefore, this theory neglects the transverse shears effects (Fourn *et al.* 2018, Shahsavari *et al.* 2017, Bellifa *et al.* 2017b, El-Haina *et al.* 2017, Klouche *et al.* 2017, Boukhari *et al.* 2016, Belkorissat *et al.* 2015, Tounsi *et al.* 2013). Several researchers used the CPT (Classical Plate Theory) for buckling analysis of functionally graded plates such as Feldman and Aboudi (1997), Javaheri and Eslami (2002), Abrate (2008). The buckling of functionally graded plates subjected to uniform compression was also examined by Mahdavian (2009) using the CPT and the Fourier solution. The free vibration of the FG plate resting on elastic foundations was studied by Chakraverty and Pradhan (2014). Since the CPT over predicts frequencies as well as buckling loads of moderately thick plate (Reddy 2004), Reissner (1945) and Mindlin (1951) have developed the first shear deformation plate theory (FSDT) which introduce the transverse shear effect through a linear distribution of displacements across the thickness of the plate. Several works have been presented using the FSDT to study the free vibration of composite and functionally graded plates (Whitney 1969, Reddy 1979, Praveen and Reddy 1998, Chen 2005, Kant and Swaminathan 2001, Hosseini-Hashemi *et al.* 2010). The bending of the plate under mechanical and thermal stresses was examined by Della Croce and Venini (2004) and the buckling of the plates was analyzed by Lanhe (2004) and Bouazza *et al.* (2010). Recently, Meksi *et al.* (2015), Mantari and Granados (2015), Bellifa *et al.* (2016), Hadji *et al.* (2016) proposed a new simple FSDT with only four variables for static and vibration analysis of functionally graded plates. Since FSDT considers a uniform distribution of transverse shear stresses across the thickness and predicting shear stresses at the top and bottom surface of the plate (Al-Basyouni *et al.* 2015, Bouderba *et al.* 2016, Shokravi 2017a, Youcef *et al.* 2018), it is necessary to use the shear correction factor. To avoid the use of this factor

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and to pass through the limitations of the CPT which neglect the shear effect and the FSDT which requires a shear correction factor, several high order shear deformation plate theories have been proposed such as Levinson (1980), Bhimaraddi and Stevens (1984), Reddy (1984), Kant and Pandya (1988), Shahrjerdi *et al.* (2011), Viswanathan *et al.* (2013), Nedri *et al.* (2014), Ait Amar Meziane *et al.* (2014). These theories (HSDTs) satisfies the condition of the nullity of transverse shear deformations and stresses at the top and bottom surface of the plate without introducing a shear correction factor. The high order shear deformation plate theory is widely used in many works such as Mantari and Guedes Soares (2012) for static analysis of composites, isotropic and sandwich plates, Saidi *et al.* (2016) for the vibration of the rectangular functionally graded plate on elastic foundation, Tounsi *et al.* (2016) for buckling analysis and the vibration of sandwich plate. Kolahchi and Moniri Bidgoli (2016) studied the dynamic instability of single-walled carbon nanotubes using size-dependent sinusoidal beam model. Arani and Kolahchi (2016) presented buckling analysis of embedded concrete columns armed with carbon nanotubes. Kolahchi *et al.* (2016a) employed differential cubature and quadrature-Bolotin methods for dynamic stability of embedded piezoelectric nanoplates based on visco-nonlocal-piezoelasticity theories. Bilouei *et al.* (2016) analyzed buckling response of concrete columns retrofitted with Nano-Fiber Reinforced Polymer (NFRP). Kolahchi *et al.* (2016b) presented dynamic stability analysis of temperature-dependent functionally graded CNT-reinforced visco-plates resting on orthotropic elastomeric medium. Kolahchi and Cheraghbak (2017) examined agglomeration effects on the dynamic buckling of viscoelastic microplates reinforced with SWCNTs using Bolotin method. Zamanian *et al.* (2017) investigated also agglomeration effects on the buckling behaviour of embedded concrete columns reinforced with SiO_2 nano-particles. Golabchi *et al.* (2018) presented vibration and instability analysis of pipes reinforced by SiO_2 nanoparticles considering agglomeration effects. Manypapers have been publishedbased on HSDT (Ahmed 2014, Bousahla *et al.* 2016, Meradjah *et al.* 2015, Attia *et al.* 2015, Ait Atmane *et al.* 2015, Merazi *et al.* 2015, Bakora and Tounsi 2015, Tebboune *et al.* 2015, Nguyen *et al.* 2015, Mahi *et al.* 2015, Chikh *et al.* 2016, Eltaher *et al.* 2016, Bourada *et al.* 2016, Bounouara *et al.* 2016, Mouaici *et al.* 2016, Beldjelili *et al.* 2016, Karami and Janghorban 2016, Madani *et al.* 2016, Kolahchi 2017, Kolahchi *et al.* 2017a, b, c, Khetir *et al.* 2017, Hajmohammad *et al.* 2017, Mouffoki *et al.* 2017, Sekkal *et al.* 2017a, Benadouda *et al.* 2017, Shokravi 2017b, c, d, Bouderba *et al.* 2013, Hajmohammad *et al.* 2018a, b, c, d, Karami *et al.* 2018a, b, c, Bouadi *et al.* 2018, Shahsavari *et al.* 2018a, b, Yazid *et al.* 2018, Kadari *et al.* 2018, Mokhtar *et al.* 2018). Recently, new development of advanced materials is developed for improving mechanical properties of structures made of these structures (Karami *et al.* 2017, Karami *et al.* 2018d, e, f, g, h, Heydari and Shariati 2018, Ferezghi *et al.* 2018, Khelifa *et al.* 2018).

In this paper, the study of the stability of ceramic-FGM-metal plates and symmetric S-FGM plates subjected to in-plane loads using a novel four variables refined plate theory is presented. By employing the Navier method, the closed-form solutions have been obtained to analyze the buckling behaviours of plates. The variation of the critical buckling load of the symmetric S-FGM and hybrid plates under the effects of the material index, the thickness of FGM layer, the geometric dimensions, modulus ratios and types of solicitations are investigated and discussed.

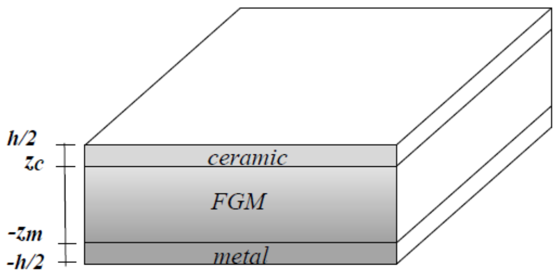

Fig. 1 geometry of hybrid functionally graded plate

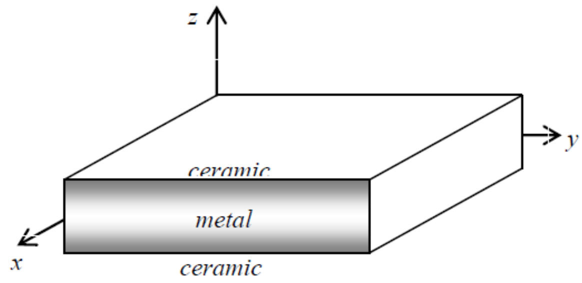

Fig. 2 Geometry of symmetric S-FGM plate

2. Theoretical formulation

2.1 Modeling of functionally graded material

In this work, two types of plate are used: the first one is hybrid (ceramic- FGM- metal) which resembles a laminated sandwich plate. The total thickness of plate (h) is composed of three layers ($h = t_C + t_{FGM} + t_M$), a ceramic top layer ($z_C \le t_C \le h/2$), a metal lower layer ($-h/2 \le t_M \le -z_M$) and intermediate functionally graded plate layer($-z_M \le t_{FGM} \le z_C$) as shown in Fig. 1.

The Young's modulus $E(z)$ of an FGM layer of the functionally graded plate using the Voigt model (Gibson *et al.* 1995, Abdelaziz *et al.* 2017, Ait Yahia *et al.* 2015) are assumed as

$$E(z) = E_C V_C + E_C(1 - V_C) \tag{1}$$

where E_c, E_M are the elastic modulus of ceramic and metal, respectively. $V_c(z)$ is the volume fraction of the ceramic and is expressed by a simple power low as follows

$$V_C(z) = \left(\frac{2z + h}{2h}\right)^P \tag{2}$$

By replacing Eq. (2) in Eq. (1), the Young modulus becomes

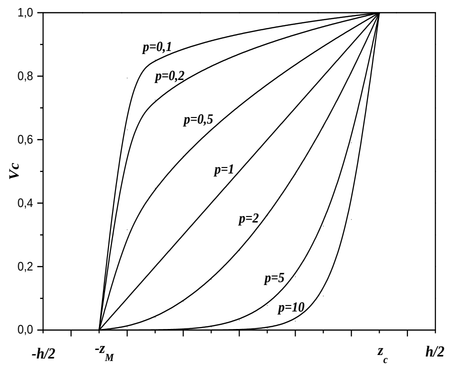

Fig. 3 The variation of ceramic volume fraction along the thickness of the functionally graded plate

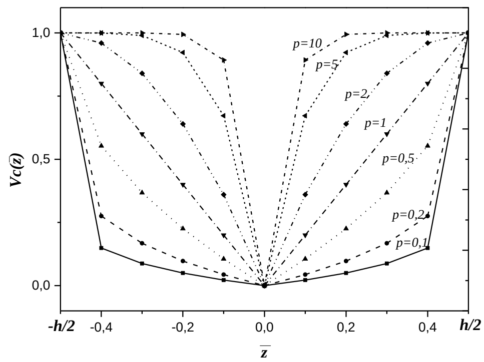

Fig. 4 The variation of ceramic volume fraction along the thickness of symmetric S-FGM plate

$$E(z) = (E_C - E_M)\left(\frac{2z+h}{2h}\right)^P + E_M \tag{3}$$

The second type of plate is symmetric S-FGM plate that consist of three layers fabricated with functionally graded ceramic and metal, the plate is symmetric with respect to median axis, the plate is metallic in the mid-plane, the upper and lower surfaces of the plate are made of ceramic (Fig. 2).

The Young's modulus (Eq. (1)) is always retained, but the analytical model of the volume fraction becomes

$$V_M(z) = \begin{cases} \left(\dfrac{2z+h}{h}\right)^P & for - h/2 \le z \le 0 \\ \left(\dfrac{-2z+h}{h}\right)^P & for \quad 0 \le z \le h/2 \end{cases} : V_C(z) = 1 - V_M(z) \tag{3}$$

The variation of ceramic volume fraction along the thickness of the hybrid FGM plate and Symmetric S-FGM plate are illustrated in Figs. 3 and 4, respectively.

2.2 The displacement base field

The field displacement of the new HSDT is given as follows (Fahsi *et al.* 2017, Menasria *et al.* 2017, Meksi *et al.* 2019)

$$u(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + k_1\, f(z)\int \theta(x,y,t)\,dx \tag{5a}$$

$$v(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + k_2\, f(z)\int \theta(x,y,t)\,dy \tag{5b}$$

$$w(x,y,z,t) = w_0(x,y,t) \tag{5c}$$

In this research, the proposed higher-order shear deformation plate theory (HSDT) is determined by considering

$$f(z) = z\left(\frac{5}{4} - \frac{5z^2}{3h^2}\right) \tag{6}$$

The non-zero strains associated with displacements in Eq. (5) are

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z)\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix}, \tag{7}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \tag{8a}$$

$$\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} k_1 \theta \\ k_2 \theta \\ k_1 \dfrac{\partial}{\partial y}\displaystyle\int \theta\, dx + k_2 \dfrac{\partial}{\partial x}\int \theta\, dy \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_2 \displaystyle\int \theta\, dy \\ k_1 \displaystyle\int \theta\, dx \end{Bmatrix},$$

and

$$g(z) = \frac{df(z)}{dz} \tag{8b}$$

The apparent integrals in the above expressions shall be resolved by a Navier type method and can be written as follows

$$\frac{\partial}{\partial y}\int \theta\, dx = A'\frac{\partial^2 \theta}{\partial x \partial y}, \quad \frac{\partial}{\partial x}\int \theta\, dy = B'\frac{\partial^2 \theta}{\partial x \partial y}, \quad \int \theta\, dx = A'\frac{\partial \theta}{\partial x}, \quad \int \theta\, dy = B'\frac{\partial \theta}{\partial y} \tag{9}$$

where

$$A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2}, k_1 = \alpha^2, k_2 = \beta^2 \tag{10}$$

and α, β are defined in expression (18).

2.3 Constitutive equations

For the functionally graded plate, the constitutive relations can be written as

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{xz}\end{Bmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & 0 & 0 & 0\\ Q_{12} & Q_{22} & 0 & 0 & 0\\ 0 & 0 & Q_{66} & 0 & 0\\ 0 & 0 & 0 & Q_{44} & 0\\ 0 & 0 & 0 & 0 & Q_{55}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} \tag{11}$$

where $(\sigma_x, \sigma_y, \tau_{xy}, \tau_{yz}, \tau_{xz})$, $(\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{yz}, \gamma_{xz})$ are stresses and strains components respectively and Q_{ij} are the reduced elastic constants in the material axes of the plate, and are expressed as

$$Q_{11}=Q_{22}=\frac{E(z)}{1-\nu^2},\ Q_{12}=\frac{\nu E(z)}{1-\nu^2}\ Q_{66}=G_{12},$$
$$Q_{44}=Q_{55}=Q_{66}=G(z)=\frac{E(z)}{2(1+\nu)} \tag{12}$$

where $E(z)$, $G(z)$ and ν are Young's modulus, shear modulus, and Poisson's ratios, respectively.

2.4 Governing equations

The principle of minimum total potential energy is used herein to derive the governing equations. The principle can be expressed in analytical form as (Attia *et al.* 2018, Kaci *et al.* 2018, Belabed *et al.* 2018, Besseghier *et al.* 2017, Hachemi *et al.* 2017, Houari *et al.* 2016, Mahi *et al.* 2015, Zemri *et al.* 2015)

$$\delta U+\delta V=0 \tag{13}$$

where δU and δV are the variation of strain energy and the variation of the external work, respectively.

The governing equations can be expressed in terms of displacements (δu_0, δv_0, δw_0, $\delta\theta$) and take the following form

$$\begin{aligned}&A_{11}d_{11}u_0+A_{66}d_{22}u_0+(A_{12}+A_{66})d_{12}v_0\\&-(B_{11}d_{111}w_0+(B_{12}+2B_{66})d_{122}w_0)\\&+(k_1A'+k_2B')B^s_{66}d_{122}\theta+(k_1B^s_{11}+k_2B^s_{12})d_1\theta=0,\end{aligned} \tag{14a}$$

$$\begin{aligned}&(A_{12}+A_{66})d_{12}u_0+A_{66}d_{22}v_0+A_{22}d_{22}v_0\\&-((B_{12}+2B_{66})d_{112}w_0+B_{22}d_{222}w_0)\\&+(k_1A'+k_2B')B^s_{66}d_{112}\theta+(k_2B^s_{22}+k_1B^s_{12})d_2\theta=0,\end{aligned} \tag{14b}$$

$$\begin{aligned}&(B_{11}d_{111}u_0+(B_{12}+2B_{66})d_{122}u_0)+\\&((B_{12}+2B_{66})d_{112}v_0+B_{22}d_{222}v_0)\\&-D_{11}d_{1111}w_0-2(D_{12}+2D_{66})d_{1122}w_0-D_{22}d_{2222}w_0\\&+(k_1D^s_{11}+k_2D^s_{12})d_{11}\theta+2(k_1A'+k_2B')D^s_{66}d_{1122}\theta\\&+(k_1D^s_{12}+k_2D^s_{22})d_{22}\theta\\&+N^0_x d_{11}w_0+2N^0_{xy}d_{12}w_0+N^0_y d_{22}w_0=0\end{aligned} \tag{14c}$$

$$\begin{aligned}&-((k_1A'+k_2B')B^s_{66}d_{122}u_0+(k_1B^s_{11}+k_2B^s_{12})d_1u_0)\\&-((k_1A'+k_2B')B^s_{66}d_{112}v_0+(k_2B^s_{22}+k_1B^s_{12})d_2v_0)\\&+(D^s_{11}+D^s_{12})d_{11}w_0+2(k_1A'+k_2B')D^s_{66}d_{1122}w_0\\&+(D^s_{12}+D^s_{22})d_{22}w_0-k_1^2H^s_{11}\theta-k_2^2H^s_{22}\theta-2k_1k_2H^s_{12}\theta\\&-(k_1A'+k_2B')^2H^s_{66}d_{1122}\theta+A^s_{44}(k_2B')^2d_{22}\theta\\&+A^s_{55}(k_1A')^2d_{11}\theta=0\end{aligned} \tag{14d}$$

Table 1 Properties of materials

Material	Young modulus (GPa)	Poison's ratio
Aluminium (Al)	70	0.3
Silicon carbide (Sic)	420	

where N^0_x, N^0_y, N^0_{xy} are axial pre-buckling forces and d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$d_{ij}=\frac{\partial^2}{\partial x_i\partial x_j},\quad d_{ijl}=\frac{\partial^3}{\partial x_i\partial x_j\partial x_l},$$
$$d_{ijlm}=\frac{\partial^4}{\partial x_i\partial x_j\partial x_l\partial x_m},\quad d_i=\frac{\partial}{\partial x_i},\ (i,j,l,m=1,2). \tag{15}$$

The stiffness components in the governing equations with material parameters of hybrid functionally graded plates are expressed as

$$\{A_{ij},B_{ij},D_{ij},B^s_{ij},D^s_{ij},H^s_{ij}\}=\int_{-h/2}^{h/2}Q_{ij}\{1,z,z^2,f(z),z\,f(z),f^2(z)\}dz,\quad i,j=1,2,6 \tag{16a}$$

$$A^s_{ij}=\int_{-h/2}^{h/2}Q_{ij}[g(z)]^2dz,\quad i,j=4,5 \tag{16b}$$

2.5 Closed-form solution

The critical buckling loads of simply supported hybrid FGM plate will be computed in this investigation by using Navier's procedure, the plate is subjected to an in-plane loading in two directions $N^0_x=\gamma_1N_{cr}$, $N^0_y=\gamma_2N_{cr}$, $N^0_{xy}=0$ (where γ_1 and γ_2 are non-dimensional load parameters).

The solution of the displacement variables satisfying the above boundary conditions based on double Fourier series can be expressed as follows

$$\begin{Bmatrix}u_0\\ v_0\\ w_0\\ \theta\end{Bmatrix}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix}U_{mn}cos(\alpha x)sin(\beta y)\\ V_{mn}sin(\alpha x)cos(\beta y)\\ W_{mn}sin(\alpha x)sin(\beta y)\\ X_{mn}sin(\alpha x)sin(\beta y)\end{Bmatrix} \tag{17}$$

with

$$\alpha=m\pi/a \text{ and } \beta=n\pi/b \tag{18}$$

and U_{mn}, V_{mn}, W_{mn}, X_{mn} are arbitrary parameters to be determined. Substituting Eq. (17) into Eq. (14), the closed-form solution of buckling load can be determined from

$$\begin{bmatrix}S_{11} & S_{12} & S_{13} & S_{14}\\ S_{12} & S_{22} & S_{23} & S_{24}\\ S_{13} & S_{23} & S_{33}+k & S_{34}\\ S_{14} & S_{24} & S_{34} & S_{44}\end{bmatrix}\begin{Bmatrix}U_{mn}\\ V_{mn}\\ W_{mn}\\ X_{mn}\end{Bmatrix}=\begin{Bmatrix}0\\0\\0\\0\end{Bmatrix} \tag{19}$$

Table 2 Comparison of critical buckling load N_{cr}(MN/m) of rectangular all FGM plate with (a/h=10)

(γ_1, γ_2)	b/a	Theory	p		
			0	1	2
(-1,0)	0.5	Present	2079.758	1028.449	780.228
		HSDT*	2080.010	1028.554	780.149
		HSDT**	2079,721	1028,412	780,097
	1	Present	1437.390	702.251	534.835
		HSDT*	1437.452	702.276	534.807
		HSDT**	1437,361	702,304	534,441
	1.5	Present	1527.994[a]	748.988[a]	569.825[a]
		HSDT*	1528.089[a]	749.027[a]	569.786[a]
		HSDT**	1527,903[a]	748,92[a]	569,751[a]
(-1,-1)	0.5	Present	1663.807	822.759	624.182
		HSDT*	1664.008	822.843	624.119
		HSDT**	1663,777	822,738	624,158
	1	Present	718.695	351.126	267.418
		HSDT*	718.726	351.138	267.403
		HSDT**	718,692	351,124	267,416
	1.5	Present	526.862	256.776	195.714
		HSDT*	526.878	256.782	195.706
		HSDT**	526,861	256,776	195,714
(-1,1)	0.5	Present	2773.011	1371.265	1040.304
		HSDT*	2773.347	1371.406	1040.199
		HSDT**	2772,98	1371,653	1040,519
	1	Present	2773.011[a]	1371.265[a]	1040.304[a]
		HSDT*	2773.347[a]	1371.406[a]	1040.199[a]
		HSDT**	2772,98[a]	1371,653[a]	1040,519[a]
	1.5	Present	2773.011[b]	1371.265[b]	1040.304[b]
		HSDT*	2773.347[b]	1371.406[b]	1040.199[b]
		HSDT**	2772,98[b]	1371,653[b]	1040,519[b]

[a]Mode for plate is (m, n) = (1, 2), *Fekrar *et al.* (2012)

where

$$
\begin{gathered}
S_{11} = -\left(A_{11}\alpha^2 + A_{66}\beta^2\right),\ S_{12} = -\alpha\beta\left(A_{12} + A_{66}\right) \\
S_{13} = \alpha\left(B_{11}\alpha^2 + B_{12}\beta^2 + 2B_{66}\beta^2\right) \\
S_{14} = \alpha\left(k_1 B^s_{11} + k_2 B^s_{12} - \left(k_1 A' + k_2 B'\right) B^s_{66}\beta^2\right), \\
S_{22} = -\left(A_{66}\alpha^2 + A_{22}\beta^2\right)\ S_{23} = \beta\left(B_{22}\beta^2 + B_{12}\alpha^2 + 2B_{66}\alpha^2\right) \\
S_{24} = \beta\left(k_2 B^s_{22} + k_1 B^s_{12} - \left(k_1 A' + k_2 B'\right) B^s_{66}\alpha^2\right), \\
S_{33} = -\left(D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4\right), \\
S_{34} = -\left(k_1 D^s_{11} + k_2 D^s_{12}\right)\alpha^2 + 2(k_1 A' + k_2 B')\, D^s_{66}\alpha^2\beta^2 \\
-\left(k_2 D^s_{22} + k_1 D^s_{12}\right)\beta^2 \\
S_{44} = -k_1\left(k_1 H^s_{11} + k_2 H^s_{12}\right) - (k_1 A' + k_2 B')^2\left(H^s_{66}\alpha^2\beta^2\right) \\
-k_2\left(k_1 H^s_{12} + k_2 H^s_{22}\right) - \left(k_1 A'\right)^2 A^s_{55}\alpha^2 - \left(k_2 B'\right)^2 A^s_{44}\beta^2 \\
k = N_{cr}\left(\gamma_1\alpha^2 + \gamma_2\beta^2\right)
\end{gathered} \tag{20}
$$

By applying the condensation approach to eliminate the in-plane displacements U_{mn} and V_{mn}, Eq. (19) can be rewritten as

$$
\begin{bmatrix} \bar{S}_{33} + k & \bar{S}_{34} \\ \bar{S}_{43} & \bar{S}_{44} \end{bmatrix} \begin{Bmatrix} W_{mn} \\ X_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{21}
$$

where

$$
\begin{aligned}
\bar{S}_{33} &= S_{33} - \frac{S_{13}(S_{13}S_{22} - S_{12}S_{23}) - S_{23}(S_{11}S_{23} - S_{12}S_{13})}{S_{11}S_{22} - S_{12}^2} \\
\bar{S}_{34} &= S_{34} - \frac{S_{14}(S_{13}S_{22} - S_{12}S_{23}) - S_{24}(S_{11}S_{23} - S_{12}S_{13})}{S_{11}S_{22} - S_{12}^2} \\
\bar{S}_{43} &= S_{34} - \frac{S_{13}(S_{14}S_{22} - S_{12}S_{24}) - S_{23}(S_{11}S_{24} - S_{12}S_{14})}{S_{11}S_{22} - S_{12}^2} \\
\bar{S}_{44} &= S_{44} - \frac{S_{14}(S_{14}S_{22} - S_{12}S_{24}) - S_{24}(S_{11}S_{24} - S_{12}S_{14})}{S_{11}S_{22} - S_{12}^2}
\end{aligned} \tag{22}
$$

Table 3 The effect of material index (p) and (t_{FGM}/h) on non-dimensional critical buckling load of square hybrid functionally graded plate with (a/h=10) under different loading conditions

(t_{FGM}/h)	(γ_1,γ_2)	Theory	p								
			0	0.1	0.2	0.5	1	2	5	10	∞
0	(-1,0)	Present	7.5618	7.5618	7.5618	7.5618	7.5618	7.5618	7.5618	7.5618	7.5618
		HSDT*	7.5632	7.5632	7.5632	7.5632	7.5632	7.5632	7.5632	7.5632	7.5632
	(-1,-1)	Present	.7809	3.7809	3.7809	3.7809	3.7809	3.7809	3.7809	3.7809	3.7809
		HSDT*	3.7816	3.7816	3.7816	3.7816	3.7816	3.7816	3.7816	3.7816	3.7816
	(-1,1)	Present	15.0339[a]	15.0339[a]	15.0339[a]	15.0339[a]	15.0339[a]	15.0339[a]	15.0339[a]	15.0339[a]	15.0339[a]
		HSDT*	15.0408[a]	15.0408[a]	15.0408[a]	15.0408[a]	15.0408[a]	15.0408[a]	15.0408[a]	15.0408[a]	15.0408[a]
0.2	(-1,0)	Present	8.5611	8.3968	8.2601	7.9635	7.6805	7.4249	7.216	7.1429	7.0767
		HSDT*	8.5641	8.3996	8.2626	7.9655	7.6818	7.42520	7.2150	7.1411	7.0740
	(-1,-1)	Present	4.2806	4.1984	4.1301	3.9818	3.8403	3.7124	3.608	3.5714	3.5384
		HSDT*	4.2820	4.2000	4.1313	3.9828	3.8409	3.7126	3.6075	3.5705	3.5370
	(-1,1)	Present	17.0813[a]	16.7430[a]	16.4611[a]	15.8469[a]	15.2552[a]	14.71[a]	14.2454[a]	14.072[a]	13.9013[a]
		HSDT*	17.095[a]	16.7562[a]	16.4733[a]	15.8565[a]	15.2614[a]	14.7118[a]	14.2411[a]	14.0641[a]	13.8888[a]
0.4	(-1,0)	Present	10.2427	9.8218	9.4722	8.7206	8.0265	7.4470	7.0535	6.9504	6.8801
		HSDT*	10.2454	9.8243	9.4746	8.7225	8.0275	7.4463	7.0493	6.9438	6.8707
	(-1,-1)	Present	5.1213	4.9109	4.7361	4.3603	4.0132	3.7235	3.5267	3.4752	3.4401
		HSDT*	5.1227	4.9122	4.7373	4.3613	4.0138	3.7231	3.5247	3.4719	3.4354
	(-1,1)	Present	20.3808[a]	19.536[a]	18.8332[a]	17.3160[a]	15.8980[a]	14.6780[a]	13.7699[a]	13.4792[a]	13.2168a
		HSDT*	20.3936[a]	19.5482[a]	18.8448[a]	17.3252[a]	15.903[a]	14.675[a]	13.7512[a]	13.4502[a]	13.1758[a]
0.6	(-1,0)	Present	12.7325	11.9330	11.2697	9.8518	8.5695	7.5563	6.9523	6.8137	6.6783
		HSDT*	12.7333	11.9341	11.2709	9.853	8.5702	7.5551	6.946	6.8039	6.6679
	(-1,-1)	Present	6.3663	5.9665	5.6348	4.9259	4.2847	3.7782	3.4762	3.4069	3.3391
		HSDT*	6.3667	5.9670	5.6354	4.9265	4.2851	3.7776	3.4730	3.4019	3.3340
	(-1,1)	Present	25.1446[a]	23.5741[a]	22.2687[a]	19.4666[a]	16.9027[a]	14.8089[a]	13.3907[a]	12.9505[a]	12.4865[a]
		HSDT*	25.1484[a]	23.5792[a]	22.2746[a]	19.47264[a]	16.906[a]	14.8037[a]	13.363[a]	12.9086[a]	12.4435[a]
0.8	(-1,0)	Present	16.1301	14.802	13.7008	11.3549	9.2604	7.6612	6.7618	6.5137	6.0209
		HSDT*	16.1289	14.8014	13.7007	11.3554	9.2608	7.6603	6.7565	6.5073	6.0209
	(-1,-1)	Present	8.0650	7.401	6.8504	5.6775	4.6302	3.8306	3.3809	3.2569	3.0105
		HSDT*	8.06443	7.4007	6.8503	5.6777	4.6304	3.8302	3.3782	3.2537	3.0105
	(-1,1)	Present	31.5034[a]	28.9501[a]	26.8273[a]	22.2815[a]	18.1746[a]	14.9426[a]	12.8795[a]	12.1886[a]	11.1667[a]
		HSDT*	31.4984[a]	28.948[a]	26.8271[a]	22.2839[a]	18.1768[a]	14.939[a]	12.857[a]	12.1622[a]	11.1671[a]
1	(-1,0)	Present	20.5341	18.4960	16.8069	13.2159	10.0322	7.6405	6.2504	5.6352	3.4224
		HSDT*	20.535	18.4968	16.8076	13.2165	10.0325	7.6401	6.2476	5.6339	3.4225
	(-1,-1)	Present	10.2671	9.2480	8.4034	6.608	5.0161	3.8203	3.1252	2.8176	1.7112
		HSDT*	10.2675	9.2484	8.4038	6.6082	5.0163	3.8201	3.1238	2.817	1.7112
	(-1,1)	Present	39.6144[a]	35.7697[a]	32.5716[a]	25.7292[a]	19.5895[a]	14.8615[a]	11.8698[a]	10.5713[a]	6.6024[a]
		HSDT*	39.6192[a]	35.7739[a]	32.5753[a]	25.7322[a]	19.5915[a]	14.86[a]	11.8583[a]	10.5663[a]	6.6032[a]

[a]Mode for plate is (m, n) = (1, 2), [b]Mode for plate is (m, n) = (1, 3), *Fekrar *et al.* (2012), **Bodaghi and Saidi (2010).

The system of homogeneous Eq. (21) has a nontrivial solution only for discrete values of the buckling load. For a nontrivial solution, the determinant of the coefficients (W_{mn}, X_{mn}) must equal zero

$$\begin{vmatrix} \bar{S}_{33}+k & \bar{S}_{34} \\ \bar{S}_{43} & \bar{S}_{44} \end{vmatrix} = 0 \tag{23}$$

The resulting equation may be solved for the buckling load. This gives the following expression for buckling load

$$k = \frac{\bar{S}_{34}\bar{S}_{43} - \bar{S}_{33}\bar{S}_{44}}{\bar{S}_{44}} \tag{24}$$

Table 4 The effect of material index (p) and side to thickness ratio (*a*/*h*) on non-dimensional critical buckling load of square hybrid functionally graded plate with (t_{FGM}/h=0.8) under different loading conditions

a/h	(γ_1,γ_2)	Theory	P								
			0	0.1	0.2	0.5	1	2	5	10	∞
10	(-1,0)	Present	16.1301	14.802	13.7008	11.3549	9.2604	7.6612	6.7618	6.5137	6.0209
		HSDT*	16.1289	14.8014	13.7007	11.3554	9.2608	7.6603	6.7565	6.5073	6.0209
	(-1,-1)	Present	8.0650	7.401	6.8504	5.6775	4.6302	3.8306	3.3809	3.2569	3.0105
		HSDT*	8.0644	7.4007	6.8503	5.6777	4.6304	3.8302	3.3782	3.2537	3.0105
	(-1,1)	Present	31.5034[a]	28.9501[a]	26.8273[a]	22.2815[a]	18.1746[a]	14.9426[a]	12.8795[a]	12.1886[a]	11.1667[a]
		HSDT*	31.4984[a]	28.9480[a]	26.8271[a]	22.2839[a]	18.1768[a]	14.9390[a]	1.857[a]	12.1622[a]	11.1671[a]
20	(-1,0)	Present	16.6871	15.3014	14.1540	11.7169	9.5546	7.9319	7.0951	6.9061	6.4175
		HSDT*	16.6868	15.3012	14.154	11.717	9.5547	7.9316	7.0936	6.9043	6.4175
	(-1,-1)	Present	8.3435	7.6507	7.0770	5.8584	4.7773	3.9659	3.5476	3.4531	3.2087
		HSDT*	8.3434	7.6506	7.0770	5.8585	4.7774	3.9658	3.5468	3.4521	3.2087
	(-1,1)	Present	34.1746[a]	31.3489[a]	29.0076[a]	24.0271[a]	19.5941[a]	16.2378[a]	14.4258[a]	13.9669[a]	12.9433[a]
		HSDT*	34.1729[a]	31.3482[a]	29.0074[a]	24.0277[a]	19.5946[a]	16.2366[a]	14.4185[a]	13.958[a]	12.9833[a]
30	(-1,0)	Present	16.7946	15.3976	14.2413	11.7865	9.6112	7.9841	7.1605	6.9841	6.4968
		HSDT*	16.7944	15.3975	14.2413	11.7865	9.6113	7.984	7.1599	6.9832	6.4968
	(-1,-1)	Present	8.3973	7.6988	7.1206	5.8932	4.8056	3.9921	3.5803	3.4920	3.2484
		HSDT*	8.397	7.6988	7.1206	5.8933	4.8056	3.9920	3.5799	3.4916	3.2484
	(-1,1)	Present	34.7204[a]	31.8380[a]	29.4514[a]	24.3813[a]	19.8821[a]	16.5031[a]	14.7545[a]	14.3557[a]	13.3372[a]
		HSDT*	34.7196[a]	31.8377[a]	29.4513[a]	24.3816[a]	19.8823[a]	16.5026[a]	14.7511[a]	14.3514[a]	13.3371[a]
40	(-1,0)	Present	16.8325	15.4316	14.2721	11.8110	9.6312	8.0026	7.1837	7.0118	6.5250
		HSDT*	16.8324	15.4315	14.2721	11.8111	9.6312	8.0025	7.1833	7.0113	6.5250
	(-1,-1)	Present	8.4162	7.7158	7.1361	5.9055	4.8156	4.0013	3.5919	3.5059	3.2625
		HSDT*	8.4162	7.7158	7.1360	5.9055	4.8156	4.0013	3.5917	3.5057	3.2625
	(-1,1)	Present	34.9156[a]	32.0129[a]	29.6100[a]	24.5078[a]	19.9849[a]	16.5981[a]	14.8732[a]	14.497[a]	13.4808[a]
		HSDT*	34.9151[a]	32.0127[a]	29.61[a]	24.508[a]	19.985[a]	16.5978[a]	14.8713[a]	14.4945[a]	13.4808[a]
50	(-1,0)	Present	16.8501	15.4473	14.2864	11.8224	9.6405	8.0112	7.1945	7.0247	6.5382
		HSDT*	16.8501	15.4473	14.2864	11.8225	9.6405	8.0111	7.1943	7.0244	6.5381
	(-1,-1)	Present	8.4251	7.7237	7.1432	5.9112	4.8202	4.0056	3.5973	3.5124	3.2691
		HSDT*	8.425	7.7237	7.1432	5.9112	4.8202	4.0056	3.5971	3.5122	3.2691
	(-1,1)	Present	35.0067[a]	32.0945[a]	29.6840[a]	24.5668[a]	20.0329[a]	16.6424[a]	14.9288[a]	14.5633[a]	13.5483[a]
		HSDT*	35.0064[a]	32.0943[a]	29.684[a]	24.5669[a]	20.033[a]	16.6422[a]	14.9275[a]	14.5618[a]	13.5408[a]

[a]Mode for plate is (m, n) = (1, 2), *Fekrar *et al*. (2012).

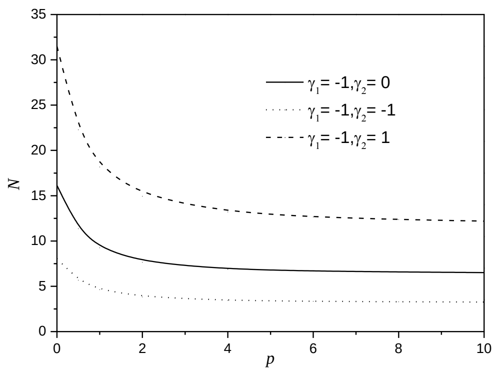

Fig. 5 The non-dimensional critical buckling load of square hybrid functionally graded plate versus the power of FGM for three different types of loading with *a*/*h*=10 and t_{FGM}/h=0.8

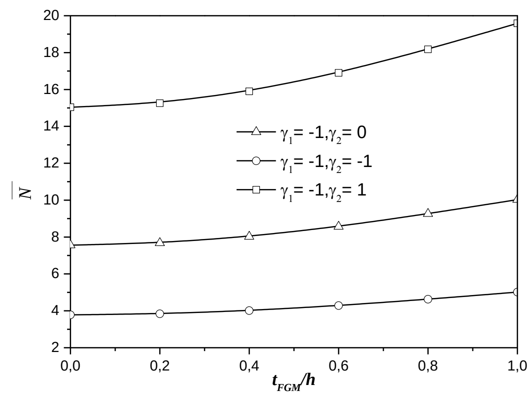

Fig. 6 The non-dimensional critical buckling load ($\overline{N}$) of square hybrid functionally graded plate versus t_{FGM}/h=0.8 with *a*/*h*=10 and p=1

Table 5 The variation of the critical buckling load N_{cr}(MN/m) of symmetric S-FGM plate as function of the geometry (a/h) and dimension (b/a) ratios

(γ_1, γ_2)	b/a	a/h	p = 0	0.1	0.2	0.5	1	2	5	10
(-1,0)	0.5	10	346.626	596.729	787.223	1168.493	1503.366	1780.511	1977.262	2034.639
		5	1013.749	1596.665	2024.656	2909.524	3776.341	4628.765	5424.033	5745.541
	1	10	239.565	424.258	567.430	852.237	1092.888	1278.46	1394.162	1421.39
		5	826.349	1389.326	1812.725	2665.856	3438.599	4112.494	4634.953	4806.456
	1.5	10	254.666[a]	447.471[a]	596.165[a]	892.397[a]	1145.46[a]	1344.668[a]	1473.860[a]	1506.517[a]
		5	837.080[a]	1378.468[a]	1781.791[a]	2600.223[a]	3360.999[a]	4052.681[a]	4625.764[a]	4829.492[a]
(-1,-1)	0.5	10	277.301	477.383	629.779	934.795	1202.692	1424.409	1581.81	1627.711
		5	810.999	1277.332	1619.725	2327.619	3021.073	3703.012	4339.226	4596.433
	1	10	119.782	212.129	283.715	426.118	546.444	639.23	697.081	710.695
		5	413.175	694.663	906.362	1332.928	1719.299	2056.247	2317.477	2403.228
	1.5	10	87.810	156.422	209.82	315.936	404.860	472.349	513.127	522.145
		5	314.414	537.750	707.231	1047.024	1348.055	1600.960	1785.207	1840.936
(-1,1)	0.5	10	462.169	795.638	1049.631	1557.991	2004.487	2374.015	2636.35	2712.852
		5	1351.665	2128.887	2699.542	3879.365	5035.121	6171.687	7232.043	7660.722
	1	10	462.169[a]	795.638[a]	1049.631[a]	1557.991[a]	2004.487[a]	2374.015[a]	2636.35[a]	2712.852[a]
		5	1351.665[a]	2128.887[a]	2699.542[a]	3879.365[a]	5035.121[a]	6171.687[a]	7232.043[a]	7660.722[a]
	1.5	10	462.169[b]	795.638[b]	1049.631[b]	1557.991[b]	2004.487[b]	2374.014[b]	2636.35[b]	2712.852[b]
		5	1351.665[b]	2128.887[b]	2699.542[b]	3879.365[b]	5035.121[b]	6171.687[b]	7232.043[b]	7660.722[b]

[a]Mode for plate is (m, n) = (1, 2), [b]Mode for plate is (m, n) = (1, 3).

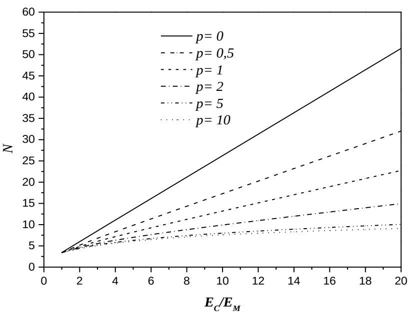

Fig. 7 The effect of modulus ratio and the power of FGM (p=1) on non-dimensional critical buckling load ($\overline{N}$) of square hybrid plate (a/h=10) under uni-axial compression along the x-axis ($\gamma_1 = -1, \gamma_2 = 0$) with t_{FGM}/h=0.8

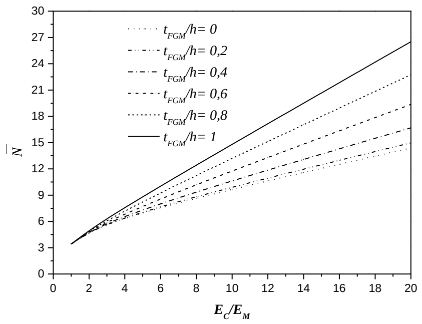

Fig. 8 The effect of modulus ratio and the FGM layer thickness (t_{FGM}/h) on non-dimensional critical buckling load ($\overline{N}$) of square hybrid plate (a/h=10) under uni-axial compression along the x-axis ($\gamma_1 = -1, \gamma_2 = 0$) with p=1

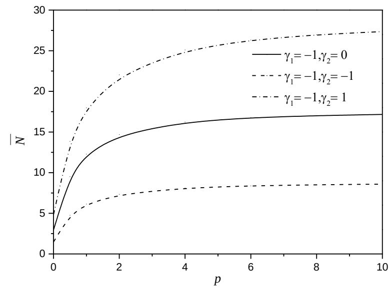

Fig. 9 The non-dimensional critical buckling load of square hybrid functionally graded plate versus the Symmetric S-FGM for three different types of loading with a/h=5

By employing the Eq. (23), the following expression for critical buckling load is determined

$$N_{cr} = \frac{1}{\left(\gamma_1 \alpha^2 + \gamma_2 \beta^2\right)} \frac{\bar{S}_{34}\bar{S}_{43} - \bar{S}_{33}\bar{S}_{44}}{\bar{S}_{44}} \tag{25}$$

3. Numerical results and discussions

In this part, the buckling study of hybrid functionally graded plate (ceramic-FGM-metal) is presented, for convenience, the following non-dimensional buckling load is utilized

$$\overline{N} = \frac{N_{cr} a^2}{E_2 h^3} \tag{26}$$

Table 6 The effect of fraction index (p) and geometry ratio (a/h) on the non-dimensional critical buckling load of square symmetric S-FGM plate under different types loading conditions

a/h	(γ_1,γ_2)	P							
		0	0.1	0.2	0.5	1	2	5	10
5	(-1,0)	2.9512	4.9619	6.4740	9.5209	12.2807	14.6875	16.5534	17.1659
	(-1,-1)	1.4756	2.4809	3.2370	4.7605	6.1404	7.3437	8.2767	8.583
	(-1,1)	4.8274[a]	7.6032[a]	9.6412[a]	13.8549[a]	17.9826[a]	22.0417[a]	25.8287[a]	27.3597[a]
10	(-1,0)	3.4224	6.0608	8.1061	12.1748	15.6127	18.2637	19.9166	20.3056
	(-1,-1)	1.7112	3.0304	4.0531	6.0874	7.8063	9.1319	9.9583	10.1528
	(-1,1)	6.6024[a]	11.3663[a]	14.9947[a]	22.2570[a]	28.6355[a]	33.9145[a]	37.6621[a]	38.7550[a]
20	(-1,0)	3.565	6.4174	8.6537	8.6539	16.7536	19.4521	20.9859	21.2815
	(-1,-1)	1.7825	3.2087	4.3268	6.5454	8.3768	9.7261	10.493	10.6407
	(-1,1)	7.2754[a]	12.9874[a]	17.4393[a]	26.2834[a]	33.6725[a]	39.2479[a]	42.5773[a]	43.2957[a]
50	(-1,0)	3.6071	6.525	8.8207	13.3729	17.104	19.8134	21.3065	21.572
	(-1,-1)	1.8036	3.2625	4.4103	6.6865	8.552	9.9067	10.6532	10.786
	(-1,1)	7.4895[a]	13.5289[a]	18.2756[a]	27.6898[a]	35.4216[a]	41.0601[a]	44.1955[a]	44.7670[a]
100	(-1,0)	3.6132	6.5407	8.8451	13.4142	17.1552	19.8662	21.3531	21.6141
	(-1,-1)	1.8066	3.2703	4.4225	6.7071	8.5776	9.9331	10.6765	10.8071
	(-1,1)	7.5211[a]	13.6100[a]	18.4018[a]	27.9032[a]	35.6866[a]	41.3329[a]	44.437[a]	44.9855[a]

[a]Mode for plate is (m, n) = (1, 2).

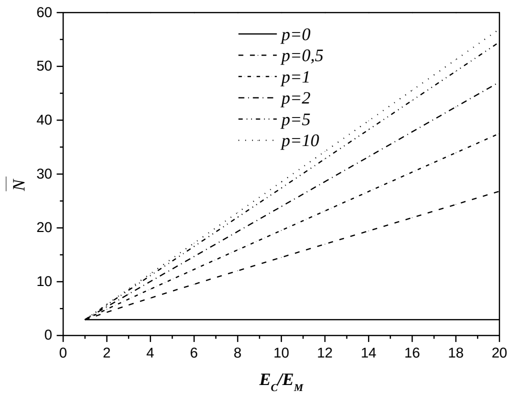

Fig. 10 The effect of modulus ratio on non-dimensional critical buckling load ($\overline{N}$) of square Symmetric S-FGM plate (a/h=5) under uni-axial compression along the x-axis ($\gamma_1=-1,\gamma_2=0$)

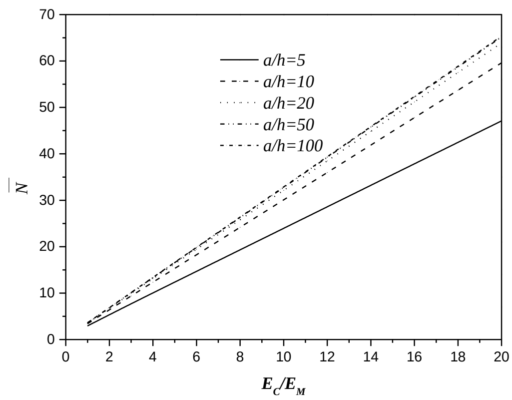

Fig. 11 The effect of side-to thickness and modulus ratio on non-dimensional critical buckling load of Symmetric S-FGM plate under uni-axial compression along the x-axis ($\gamma_1=-1,\gamma_2=0$) with p =2

where a is the length of the square plate and h is the thickness of the plate.

Table 2 presents the critical buckling loads N_{cr}(MN/m) of all FGM plate (h=t_{FGM}), as a function of the dimension ratio (b/a) and the materiel index (p). A comparison is made between the results obtained by the present model and those found by Bodaghi and Saidi (2010) using the Levy solution and those obtained by Fekrar *et al.* (2012) based on high order shear deformation plate theory with four variable. The obtained results are in good agreement with the models already developed by Bodaghi and Saidi (2010) and Fekrar *et al*. (2012).

In Tables 3 and 4, we have presented the non-dimensional values of the critical buckling load of hybrid square plate subjected to axial forces (uni-axial compression, bi-axial compression, compression along the x-axis and a tension along the y-axis) as function of the material index (p), the results are compared with those obtained by Fekrar *et al.* (2012). It should be noted that a good concordance is confirmed with the results of Fekrar *et al.* (2012) and this for the different values of the FGM layer thickness (t_{FGM}) and the geometry ratio (a/h).

Fig. 5 illustrate the effect of materiel index (p) on the non-dimensional critical buckling load under the differentload types with (a/h=10) and (t_{FGM}/h=0.8) it can be seen that the no-dimensional values of the critical buckling load are in inverse relation with the materiel index (p).

The variation of the non-dimensional critical buckling load as function of FGM layer thickness (t_{FGM}) is shown in Fig. 6. It can be observed that the critical buckling load ($\overline{N}$

) increases with increasing of the plate core thickness (t_{FGM}), and it can be seen that the largest values of the non-dimensional critical buckling load ($\overline{N}$) are obtained for bi-axial loading with compression along x-axis and tension along y-axis.

The effects of the modulus ratio (E_c/E_m) and the variation of the FGM layer thickness (t_{FGM}/h) on the non-dimensional critical buckling load ($\overline{N}$) of hybrid square plate are shown in Figs. 7 and 8, respectively. It can be observed that the critical buckling load ($\overline{N}$) increases with the increase of the FGM layer thickness (t_{FGM}/h) and modulus ratio (E_c/E_m).

This second part is devoted to the study of the stability of the rectangular plate symmetric S-FGM. The variation of the critical buckling load N_{cr}(MN/m) of symmetric S-FGM plate as a function of the geometry ratio (a/h) and dimension ratio (b/a) is presented in Table 5 for the different loading types (uni-axial, bi-axial), it can be observed the critical buckling load N_{cr}(MN/m) is in direct correlation relation with the fraction index (p), it should be noted that the lowest values of the critical buckling load N_{cr}(MN/m) are obtained for the square plate.

The effect of fraction index (p) and the geometry ratio (a/h) on the non-dimensional critical buckling load of square symmetric S-FGM plate is shown in Table 6, it canbe seen that the critical buckling load ($\overline{N}$) increases with the increase of the geometry ratio and the largest values are obtained for a most important material index (p).

Fig. 9 shows the variation of the non-dimensional critical buckling load of the square symmetric S-FGM plate for three types of loading under the effect of power (p) with geometry ratio (a/h=5), it should be noted that the non-dimensional critical buckling load ($\overline{N}$) is in direct correlation relation with the material index (p), the lowest values of the critical buckling load ($\overline{N}$) are obtained for bi-axial compression loading, on the other hand a bi-axial loading with compression along the x-axis and tension along y-axis gives the largest values of the critical load ($\overline{N}$).

The effects of the modulus and the geometry ratios on the variation of the non-dimensional critical buckling load ($\overline{N}$) are shown in the Figs. 10 and 11, the plate are subjected to normal compressive forces along the x-axis ($\gamma_1 = -1, \gamma_2 = 0$), we observe that the non-dimensional critical buckling load ($\overline{N}$) increases with increasing modulus ratio (E_c/E_m) and geometry (a/h) ratio.

4. Conclusions

In this research work, buckling analysis of thick symmetric S-FGM and hybrid plates has been presented, based on a novel four variables refined plate theory. Governing equations are obtained from the principle of virtual works. Closed-form solutions are obtained for simply supported functionally graded plates. The accuracy of the developed model has been checked for stability of functionally graded plates. Other mathematical modelling and numerical methods (Rehab *et al.* 2018, Henderson *et al.* 2018, Wang *et al.* 2018, Cherif *et al.* 2018) can be used in future to investigate this type of problem applied to FGM structures. Finally, an improvement of present approach will be considered in the future study to account for the thickness stretching effect by using quasi-3D shear deformation models (Belabed *et al.* 2014, Bousahla *et al.* 2014, Hebali *et al.* 2014, Bourada *et al.* 2015, Hamidi *et al.* 2015, Larbi Chaht *et al.* 2015, Bennoun *et al.* 2016, Draiche *et al.* 2016, Benahmed *et al.* 2017, Bouafia *et al.* 2017, Sekkal *et al.* 2017b, Bouhadra *et al.* 2017, Karami *et al.* 2018i, j, Shahsavari *et al.* 2018c, d, Abualnour *et al.* 2018, Younsi *et al.* 2018, Benchohra *et al.* 2018, Zaoui *et al.* 2019).

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