Structural Engineering and Mechanics, *Vol. 68, No. 6 (2018) 639-646*
DOI: https://doi.org/10.12989/sem.2018.68.6.639

Dynamic analysis of laminated composite skew plates with cut-out

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(Received January 1, 2018, Revised October 9, 2018, Accepted November 1, 2018)

Abstract. The aim of the present paper deals with free vibration analysis of laminated composite skew plates with single and multiple cut-outs. For complete understanding of the dynamic behavior of laminated skew plates with cut-out a numerical analysis has been carried out by developing a computer code in FOTRAN. Special attention is drawn on the formulation of mass matrix by considering effect of rotary inertia. The results obtained by the finite element formulation using nine noded isoparametric plate bending element are validated by comparing the results from relevant published literature. Few new results on laminated skew plates with cut-out have been presented.

Keywords: skew laminate; multiple cut-out; modal analysis; rotary inertia; FSDT

1. Introduction

Since last few decades composite materials are considered as primary structural components in complicated structures due to their superior properties like relative lightness with respect to strength, long fatigue life etc. In particular, skew laminated plates with single or multiple cut-out are employed in several complicated engineering structures like hulls of different category of ships, parallelogram slabs in buildings, complex alignment problems in bridge, swept wings of aero planes, etc. The key factors which affect the dynamic behaviour of laminated skew plates are the skew angle and cut-outs. So, it is important to study on dynamic behavior of skew laminated composite plates with varying skew angle and cut-outs to design optimum skew laminates with cut-out.

The attention of researchers has been extended over last three decades on dynamic analysis of composite plates and the effect of cut-out on it. Various numerical investigations have been carried out on vibration of simple composite plates with cut-out and skew composite plates using different techniques till date. Kapania and Singhvi (1992) made a study on static and dynamic analyses of laminated skew plates with arbitrary edge conditions using Rayleigh-Ritz method. Han and Dickinson (1997) introduced a Ritz approach for the dynamic analysis of symmetrically laminated thin skew plates. Sivakumar *et al*. (1998) carried out an optimum design of laminated composite plates with elliptical cut-outs using genetic algorithm. The first order shear deformation theory was applied for the study of the free vibration response. Liew *et al*. (2003) studied free vibration behaviour of moderately thick symmetrically laminated composite plates by adopting FSDT in moving least squares differential quadrature method. Shi *et al*. (2004) presented free vibration analysis of arbitrarily laminated plate with all four edges clamped using Galerkin method. Park *et al*. (2009) made a free vibration analysis of composite skew plates with cut-out and de-lamination around cutout based on HSDT. Lee (2010) carried out a finite element dynamic stability analysis of laminated composite skew structures subjected to in-plane pulsating forces based on higher-order shear deformation theory. Murthy *et al*. (2013) investigated free vibration analysis of a thick FRP skew laminated composite plate with a circular cutout and clamped boundary conditions with the help of ANSYS software package. Wang *et al*. (2014) introduced a new version of the differential quadrature method to analyse dynamic behaviour of skew plates. Dalir and Shooshtari (2015) presented an exact mathematical solution for free vibration of thick laminated plates. Abdelali *et al*. (2015) took a fully clamped symmetrically laminated composite skew plate under consideration to make a study on geometrically non-linear free vibration. Fantuzzi and Tornabene (2016) presented a finite element method and a strong form collocation method for analysing laminated composite plates to predict the direction of propagation of fracture in specimen. Dimitri *et al*. (2017) proposed an application of the level set method combined with the numerically extended finite element method (XFEM). Tornabene *et al.* (2017) introduced Cosserat theory of elasticity to model -structured materials and structures. An advanced strong form pseudo-spectral method is used to deal with composite structures having holes and discontinuities. Mandal *et al.* (2017) made an experimental and numerical analysis of a composite skew plate with a cut-out at centre.

It is clear from the extensive literature review that literature dealing with free vibration response of skew plates and specifically, laminated composite skew plates

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http://www.techno-press.com/journals/sem&subpage=7 ISSN: 1225-4568 (Print), 1598-6217 (Online)

with cut-out is rare. To the best of the authors' knowledge the dynamic analysis of composite skew plates with multiple cut-out is not reported in the published literature till date. The laminated composite skew plates with multiple cut-outs are unavoidable structural components in several practical applications where vibration is an inevitable occurrence. The vibration characteristics get affected significantly due to the simultaneous change of mass and stiffness due to skew angle and cut-outs. Hence detailed free vibration study on the variation of natural frequencies and mode shapes needs special attention. Furthermore, rotary inertia has a considerable effect on the natural frequencies of skewed laminated plates which has been considered in the previous investigations. Therefore, the present study has attempted to develop a numerical modal analysis considering rotary inertial contribution in the mass matrix to obtain natural frequencies and mode shapes of laminated skew composite plates with varying skew angle and cut-outs. Another objective of this study is to find out the effect on dynamic behavior of skew plate when there are multiple numbers of cut-outs instead of single large size cut-out. For this purpose, a finite element based formulation with a lumped mass model is developed in FOTRAN software package.

2. Formulation

2.1 Finite element formulation for Modal analysis

The present finite element formulation is based on first order shear deformation theory. The nine noded isoparametric plate bending element having five degrees of freedom viz. three translations and two rotations per node has been used for the present formulation. A computer code has been developed using FOTRAN.

The nodal displacements at any node 'r' of the plate element can be expressed as

$$\{\delta_r\}^T = \{u_r \quad v_r \quad w_r \quad \theta_{xr} \quad \theta_{xr}\} \tag{1}$$

where,

$$u = \sum_{r=1}^{9} N_r\, u_r, v = \sum_{r=1}^{9} N_r\, v_r\,, w = \sum_{r=1}^{9} N_r\, w_r\,,$$
$$\theta_x = \sum_{r=1}^{9} N_r\, \theta_{xr}, \qquad \theta_y = \sum_{r=1}^{9} N_r\, \theta_{yr} \tag{2}$$

u and *v* are in-plane displacements, *w* is transverse displacement and θ_x & θ_y are total rotations in bending about *Y* and *X* axis respectively. Lagrangian interpolation function is used to develop the shape functions N_r.

As bending rotations are considered as independent field variables, the effect of shear deformation may be expressed as

$$\begin{Bmatrix} \emptyset_x \\ \emptyset_y \end{Bmatrix} = \begin{bmatrix} \theta_x - \dfrac{\partial w}{\partial x} \\ \theta y - \dfrac{\partial w}{\partial y} \end{bmatrix} \tag{3}$$

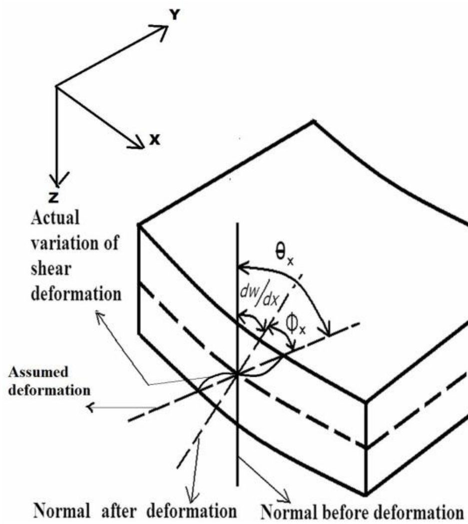

Fig. 1 Deformation of plate panel

where, $\emptyset_x$ and $\emptyset_y$ are the average shear rotation over the entire plate thickness and θ_x and θ_y are the total rotations in bending (Fig. 1).

The generalized stress-strain relationship with respect to its reference plane may be expressed as

$$\{\sigma\} = [D]\{\varepsilon\} \tag{4}$$

Where, $\{\sigma\}$ is the stress resultants vector of stress resultants and it may be expressed as

$$\{\sigma\}^T = \{N_x \quad N_y \quad N_{xy} \quad M_x \quad M_y\, M_{xy}\, Q_x\, Q_y\} \tag{5}$$

$$\{\varepsilon\}^T = \left[\frac{\partial u}{\partial x}\ \frac{\partial v}{\partial y}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) - \frac{\partial \theta_x}{\partial x} - \frac{\partial \theta_y}{\partial y}\left(\frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x}\right) - \emptyset_x - \emptyset_y\right] \tag{6}$$

and [D] is the rigidity matrix of the laminate containing the extensional (A_{ij}), coupling (B_{ij}), bending (D_{ij}) and shear (A_{lk}) stiffness matrices (Reddy 1997) and expressed as

$$[\,D] = \begin{bmatrix} [A_{ij}]_{3\times3} & [B_{ij}]_{3\times3} & [0]_{3\times2} \\ [B_{ij}]_{3\times3} & [D_{ij}]_{3\times3} & [0]_{3\times2} \\ [0]_{2\times3} & [0]_{2\times3} & [A_{lk}]_{3\times3} \end{bmatrix} \tag{7}$$

where

$$A_{ij} = \sum_{k=1}^{n}(Q_{ij})(z_K - z_{K-1}),$$

$$B_{ij} = \sum_{k=1}^{n}(Q_{ij})(\,{z_k}^2 - {z_{k-1}}^2)$$

$$D_{ij} = \sum_{k=1}^{n}(Q_{ij})(\,{z_k}^3 - {z_{k-1}}^3)$$

z_k is the distance of k_{th} layer from reference surface.

The stiffness matrix $[K]_e$ of an element derived using virtual work method may be expressed as

$$[K]_e = \int_{-1}^{+1}\int_{-1}^{+1}[B]^T[D][B]\,|J|\,d\xi\, d\eta \tag{8}$$

where, [B] relates strain-displacement matrix and $\left|J\right|$ is the determinant of Jacobian matrix.

Using the concept of consistent mass matrix, a lumped mass matrix has been derived. While ρ is the density of the material and h is the thickness of the laminate. The lumped mass matrix may be expressed as

$$[M]=\rho h\int_{-1}^{+1}\int_{-1}^{+1}\begin{pmatrix}[N_u]^T[N_u]+[N_v]^T[N_v]+[N_w]^T[N_w]\\ +\frac{h^2}{12}[N_{\theta x}]^T[N_{\theta x}]+\frac{h^2}{12}[N_{\theta y}]^T[N_{yx}]\end{pmatrix}|J|d\xi d\eta \tag{9}$$

where

$$[N_u]=[[N_r][N_0][N_0][N_0][N_0]]$$
$$[N_v]=[[N_0][N_r][N_0][N_0][N_0]]$$
$$[N_w]=[[N_0][N_0][N_r][N_0][N_0]]$$
$$[N_{\theta_x}]=[[N_0][N_0][N_0][N_r][N_0]]$$
$$[N_{\theta_y}]=[[N_0][N_0][N_0][N_0][N_r]]$$

N_r is the corresponding interpolation function and $[N_0]$ is a null matrix.

In the consistent mass matrix formulation inclusion of the concept of rotary inertia is not possible. So, at the time of formation of formulation of lump mass matrix we take rotary inertia under consideration which is depicted by last two terms in Eq. (9). Contribution of rotary inertia becomes significant only for thicker laminates.

First two terms of mass matrix in Eq. (9) are associated with in-plane movements of mass and third term indicates transverse movement of mass (which is usually found to contribute the major inertia).

Two types of proportionate mass lumping schemes have been used in the present analysis. For first lumping scheme, 9NEWORI (mass lumping without rotary inertia) only the effect of in-plane and transverse movements of mass has been considered and may be expressed as

$$m_{ii}^{wl}=\frac{m_{ii}}{\sum m_{ii}}m_e \text{ (i=1,2,3,6,7,8,11,12,13,16,17,18, 21,22,23,26,27,28,31,32,33,36,37,38,41,42,43)} \tag{10}$$

where m_{ii}^{wl} are the ith diagonal elements corresponding to u, v, w of the proposed lump mass matrix, m_{ii} is the ith diagonal element of the consistent mass matrix and m_e is actual mass of element.

In the second lumping scheme, 9NEWRI (mass lumping with rotary inertia) the effect of rotary inertia has also been taken into account and may be expressed as.

$$m_{ii}^{wl}=\frac{m_{ii}}{\sum m_{ii}}m_e \text{ (i=1,2,3,6,7,8,11,12,13,16,17,18, 21,22,23,26,27,28,31,32,33,36,37,38,41,42,43)} \tag{11}$$

$$m_{ii}^{\theta xl}=\frac{h^2}{12}\frac{m_{ii}}{\sum m_{ii}}m_e \text{ (i=4, 9, 14, 19, 24, 29, 34, 39, 44)} \tag{12}$$

$$m_{ii}^{\theta xl}=\frac{h^2}{12}\frac{m_{ii}}{\sum m_{ii}}m_e \text{ (i=5, 10, 15, 20, 25, 30, 35, 40, 45)} \tag{13}$$

To form the global stiffness matrix $[K_0]$ and mass matrix $[M_0]$ element stiffness and mass matrices have been assembled together.

The equation of motion may be expressed as

Table 1 Frequency parameter $\lambda = (wa^2/h)\sqrt{(\rho/E_2)}$ of clamped angle ply laminates with varying thickness ratio (h/a)

Boundary Condition	h/a	Lamination	Mode	Shi *et al.* (2004)	9NEWRI	9NEWORI
Four Edges Fixed	0.1	15°/-15°/15°/-15°	1	18.085	18.068	18.122
			2	23.719	23.683	23.869
			3	32.634	32.551	32.947
			4	35.321	35.303	35.413
		45°/-45°/45°/-45°	1	18.189	18.171	18.205
			2	30.78	30.759	30.856
			3	30.78	30.759	30.856
			4	41.855	41.834	41.931
	0.01	15°/-15°/15°/-15°	1	31.309	31.759	31.762
			2	40.886	41.106	41.116
			3	57.18	57.196	57.221
			4	80.162	80.061	80.120
		45°/-45°/45°/-45°	1	29.961	30.243	30.246
			2	59.821	60.316	60.330
			3	59.821	60.317	60.330
			4	94.702	95.565	95.597

$$[K_0]\{\delta\}=\omega^2[M_0]\{\delta\} \tag{14}$$

Eq. (14) has been solved by using the simultaneous iterative technique following Corr and Jenings (1976) to obtain the natural frequencies of first few modes.

3. Results and discussion

There are two part of this section. In the first part, the present finite element formulation is validated by comparing the results with several benchmark problems. Some new results are also reported on skew composite plates with single and multiple cut-out at different positions.

3.1 Validation of the present finite element formulation

3.1.1 Square anti-symmetric angle ply laminates

For this study anti-symmetric angle ply square thin (h/a = 0.01) and moderately thick (h/a=0.1) laminates fixed along all the four edges have been taken under consideration. The relative material properties are $E_1/E_2=25$, $G_{12}=G_{23}=0.5E_2$, $G_{23}=0.2E_2$, $\gamma_{12}=0.25$. First four natural frequencies obtained by the present finite element formulation with both lumping scheme (9NEWRI, 9NEWORI) have been presented in Table 1 and compared with the results of Shi *et al.* (2004) which are in very good agreement. Table 1 shows that the natural frequencies obtained by 9NEWRI are slightly closer to the published results than those obtained by 9NEWORI. The differences between the natural frequencies obtained by these two mass lumping schemes are less for lower thickness ratio of the

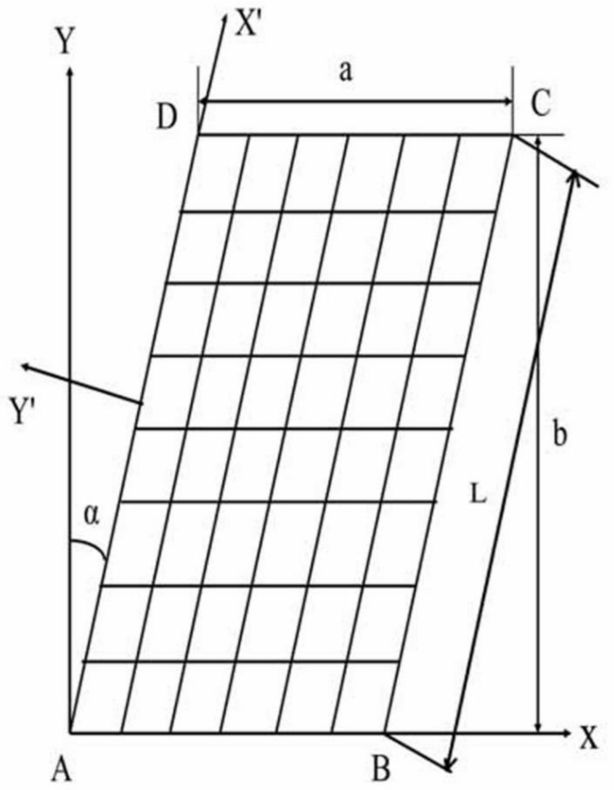

Fig. 2 Skew plate model

Table 2 The fundamental frequencies of the simply supported and clamped skew laminates with varying skew angles ($\lambda = (\omega a^2/\pi^2 h)\sqrt{(\rho /E_2)}$, a/h=10

Boundary condition	Lamination	References	Skew angle (α)			
			75°	60°	45°	30°
Simply Supported	90°/0°/90°/0°/90°	9NEWRI	9.143	4.521	2.896	2.090
		Liew *et al.* (2003)	9.1315	4.499	2.881	2.095
	45°/-45°/45°/-45°/45°	9NEWRI	8.610	3.894	2.492	2.003
		Liew *et al.* (2003)	8.602	3.974	2.519	2.004
Clamped	90°/0°/90°/0°/90°	9NEWRI	9.318	4.913	3.449	2.775
		Liew *et al.* (2003)	9.350	4.943	3.471	2.790
	45°/-45°/45°/-45°/45°	9NEWRI	9.250	4.815	3.332	2.649
		Liew *et al.* (2003)	9.307	4.857	3.358	2.661

plates. So, it can be concluded that mass lumping scheme with rotary inertia is more appropriate for thick plates as well as thin plates but 9NEWORI is preferable only for thin plates. Since mass lumping scheme 9NEWRI is suitable for both thick and thin laminates, the subsequent examples have been analyzed by using 9NEWRI. All the results converge at mesh division (16×16).

3.1.2 Simply supported and clamped symmetric cross ply and angle ply skew laminates

Skew laminates with varying skew angle as shown in (Fig. 2) with simply supported and clamped boundary condition have been analyzed. To express the degrees of freedom of the nodes on inclined (to the global axis system (X-Y)) sides (BC and AD) along x/-y/ necessary

Table 3 Frequency parameter $\lambda = (wa^2/h)\sqrt{(\rho /E_2)}$ of simply supported anti-symmetric angle ply laminates with different cutout size

References	Fundamental Frequency			
h/a	0.05		0.1	
Cut out size	0.2a × 0.2a	0.4a × 0.2a	0.2a × 0.2a	0.4a × 0.2a
9NEWRI	14.866	14.259	13.321	12.883
Sivakumar *et al.* (1999)	14.344	14.211	13.191	13.027

Table 4 Frequency parameter $\lambda = ((wa^2/h)\sqrt{(\rho /E_2)})/\pi^2$ of simply supported symmetric angle ply laminates with skew angle

Skew angle	Support condition	Fiber orientation (in degree)	mode1	mode2	mode3
30	Simply supported	15/-15/15/-15/15	1.632	2.434	3.537
		30/-30/30/-30/30	1.794	2.966	4.029
		45/-45/45/-45/45	1.801	3.282	3.953
		60/-60/60/-60/60	1.741	3.057	3.514
		75/-75/75/-75/75	1.682	2.588	3.654
	Fixed supported	15/-15/15/-15/15	2.182	2.890	4.000
		30/-30/30/-30/30	2.342	3.427	4.404
		45/-45/45/-45/45	2.450	3.777	4.408
		60/-60/60/-60/60	2.395	3.576	4.381
		75/-75/75/-75/75	2.257	3.112	4.072
45	Simply supported	15/-15/15/-15/15	1.719	2.481	3.478
		30/-30/30/-30/30	1.856	2.835	3.197
		45/-45/45/-45/45	1.933	2.677	3.157
		60/-60/60/-60/60	1.973	3.052	3.199
		75/-75/75/-75/75	1.998	2.993	3.266
	Fixed supported	15/-15/15/-15/15	2.385	3.012	3.986
		30/-30/30/-30/30	2.647	3.499	4.690
		45/-45/45/-45/45	2.821	3.800	4.956
		60/-60/60/-60/60	2.792	3.725	4.698
		75/-75/75/-75/75	2.619	3.423	4.200

transformation has been made. The relative material properties used in the present analysis are $E_1/E_2=40$, $G_{12}=G_{13}=0.6E_2$, $G_{23}=0.5E_2$, $\gamma_{12}=0.25$. Fundamental frequencies obtained by the present finite element formulation are shown in Table 2 and compared with those obtained from Liew *et al.* (2003). The results are in very good agreement which indicates the validity of the present formulation for skew plates. Table 2 also shows that the fundamental frequency decreases with the decrease in skew angle. In other word increment of skew angle increases the fundamental frequency aspect ratio remaining same.

3.1.3 Simply supported anti-symmetric angle ply laminates with cut-out

Simply supported anti-symmetric angle ply laminates [45°/-45°] with different sizes of cut-outs at the center of the laminate have been analyzed. Material properties are $E_1/E_2=40$, $G_{12}=G_{13}=0.6E_2$, $G_{23}=0.5E_2$, $\gamma_{12}=0.25$, $\rho=1500$

kg/m³. The fundamental frequency obtained by the present finite element formulation (9NEWRI) along with the finite element solutions of Sivakumar *et al.* (1999) are shown and compared in Table 3. The present results show very good agreement with published results. It is found that the fundamental frequency decreases with the increase of cut-out size in one direction of the plates studied here. This comparison also shows the effectiveness of the present formulation for laminates with cut-out.

3.2 Parametric study

3.2.1 Simply supported and clamped symmetric cross ply and angle ply skew laminates with different fiber orientation

After validation of the present formulation and effect of rotary inertia few new examples have been presented. First a symmetric cross ply and angle ply skew laminates with different skew angle (Fig. 2) have been analyzed. The relative material properties used in the present analysis are E1/E2 = 40, G12 = G13 = 0.6E2, G23 = 0.5E2, γ12 = 0.25. In this study, various fiber orientation has been taken under consideration. The laminate has its all outer edges clamped and simply supported. Its side-to-thickness ratio (a/h) is kept 10. For all the cases mass of the plate remain same as the vertical distance of two parallel horizontal edges (b (Fig. 2)) remain same but aspect ratio ($^{a}/_{L}$) change with skew angle. It can be observed from results provided in Table 4 that with the change of the angle between fibers of alternative layers modal frequencies gradually change. It is clearly noticed that fundamental frequency initially increases up to a certain limit and then starts decreasing with the increment of the angle between the fibers of alternative layers. It is observed that the frequencies are maximum when fiber orientation is 45°/-45°/45°/-45°/45° for any skew angle. Table 4 also shows that the fundamental frequency increases with the increase in skew angle when mass of the plate remain same.

3.2.2 Clamped symmetric cross ply and angle ply skew laminates with different fiber orientation and cut-out at centre

Skew composite plate fixed along all the four edges with different skew angle and fiber orientation has been analyzed when a cut-out is present at the centre of the plate. Edges of the cut-out are parallel to the edges of the plate. The relative material properties used in the present analysis are E_1/E_2= 40, G_{12}=G_{13}=0.6E_2, G_{23}=0.5E_2, γ12=0.25. Its thickness ratio (h/a) is kept 0.1. Here also mass of the skew plates (without cut-out) remain same as the vertical distance of two parallel horizontal edges (b) remains same but aspect ratio ($^{a}/_{L}$) changes with the change in skew angle. From the results reported in Table 5 it is observed that natural frequencies increase with the increment of the cut-out size for skew fixed supported composite plate. Reduction of mass becomes the governing factor for these cases. It is also observed that there is no any significant effect of the orientation of fibers (fiber orientation parallel to the cut-out edge) on the dynamic behavior of skew plate in presence of cut-out.

Table 5 Frequency parameter $\lambda = ((wa^2/h)\sqrt{(\rho /E_2)})/\pi^2$ of fixed symmetric angle ply laminates with cut-out at centre

Support condition	Skew angle	Fiber orientation (in degrees)	Cut-out size	mode1	mode2	mode3
Fixed	45	15/-15/15/-15/15	0.1ax0.1L	2.392	2.995	3.995
			0.2ax0.2L	2.500	2.948	3.928
			0.3ax0.3L	2.728	2.940	3.761
			0.4ax0.4L	2.997	3.039	4.115
		30/-30/30/-30/30	0.1ax0.1L	2.638	3.464	4.687
			0.2ax0.2L	2.747	3.356	4.163
			0.3ax0.3L	3.008	3.293	4.011
			0.4ax0.4L	3.352	3.406	4.390
		45/-45/45/-45/45	0.1ax0.1L	2.800	3.750	4.938
			0.2ax0.2L	2.917	3.594	4.208
			0.3ax0.3L	3.210	3.518	4.109
			0.4ax0.4L	3.631	3.700	4.516
		60/-60/60/-60/60	0.1ax0.1L	2.767	3.684	4.689
			0.2ax0.2L	2.896	3.542	3.963
			0.3ax0.3L	3.222	3.506	3.920
			0.4ax0.4L	3.715	3.783	4.363
		75/-75/75/-75/75	0.1ax0.1L	2.603	3.399	4.202
			0.2ax0.2L	2.747	3.323	3.582
			0.3ax0.3L	3.096	3.347	3.593
			0.4ax0.4L	3.634	3.707	4.049
Fixed	30	15/-15/15/-15/15	0.1ax0.1L	2.196	2.875	4.002
			0.2ax0.2L	2.300	2.828	3.683
			0.3ax0.3L	2.519	2.812	3.514
			0.4ax0.4L	2.815	2.907	3.769
		30/-30/30/-30/30	0.1ax0.1L	2.338	3.385	4.247
			0.2ax0.2L	2.423	3.253	3.813
			0.3ax0.3L	2.656	3.157	3.618
			0.4ax0.4L	3.035	3.243	3.839
		45/-45/45/-45/45	0.1ax0.1L	2.433	3.713	4.242
			0.2ax0.2L	2.516	3.520	3.796
			0.3ax0.3L	2.765	3.395	3.596
			0.4ax0.4L	3.213	3.538	3.792
		60/-60/60/-60/60	0.1ax0.1L	2.390	3.539	4.163
			0.2ax0.2L	2.490	3.428	3.624
			0.3ax0.3L	2.754	3.362	3.439
			0.4ax0.4L	3.212	3.533	3.678
		75/-75/75/-75/75	0.1ax0.1L	2.264	3.095	4.038
			0.2ax0.2L	2.377	3.031	3.445
			0.3ax0.3L	2.634	2.982	3.322
			0.4ax0.4L	3.030	3.145	3.619

3.2.3 Simply supported symmetric cross ply and angle ply skew laminates with different fiber orientation and cut-out at centre

In this section similar type of skew composite plate used in the previous study (3.2.2) with simply supported

Table 6 Frequency parameter λ= ((wa²/h)√(ρ /E₂))/π² of simply supported symmetric angle ply laminates with cut-out at centre

Support condition	Skew angle	Fiber orientation (in degree)	Cut-out size	mode1	mode2	mode3
Simply supported	45	15/-15/15/-15/15	0.1ax0.1L	1.676	2.467	3.482
			0.2ax0.2L	1.629	2.412	3.354
			0.3ax0.3L	1.634	2.349	2.806
			0.4ax0.4L	1.705	2.329	2.545
		30/-30/30/-30/30	0.1ax0.1L	1.776	2.803	3.244
			0.2ax0.2L	1.688	2.680	3.373
			0.3ax0.3L	1.662	2.513	2.774
			0.4ax0.4L	1.714	2.339	2.527
		45/-45/45/-45/45	0.1ax0.1L	1.801	2.710	3.099
			0.2ax0.2L	1.654	2.795	2.864
			0.3ax0.3L	1.575	2.476	2.784
			0.4ax0.4L	1.562	2.125	2.595
		60/-60/60/-60/60	0.1ax0.1L	1.824	3.081	3.149
			0.2ax0.2L	1.664	2.882	3.140
			0.3ax0.3L	1.590	2.342	2.827
			0.4ax0.4L	1.589	2.034	2.543
		75/-75/75/-75/75	0.1ax0.1L	1.851	2.968	3.285
			0.2ax0.2L	1.690	2.753	2.902
			0.3ax0.3L	1.605	2.165	2.737
			0.4ax0.4L	1.592	1.900	2.007
Simply supported	30	15/-15/15/-15/15	0.1ax0.1L	1.601	2.421	3.538
			0.2ax0.2L	1.570	2.372	3.258
			0.3ax0.3L	1.586	2.311	2.798
			0.4ax0.4L	1.665	2.289	2.574
		30/-30/30/-30/30	0.1ax0.1L	1.760	2.933	3.869
			0.2ax0.2L	1.744	2.815	3.365
			0.3ax0.3L	1.800	2.682	2.937
			0.4ax0.4L	1.942	2.642	2.753
		45/-45/45/-45/45	0.1ax0.1L	1.748	3.222	3.785
			0.2ax0.2L	1.705	3.018	3.265
			0.3ax0.3L	1.730	2.785	2.811
			0.4ax0.4L	1.829	2.526	2.712
		60/-60/60/-60/60	0.1ax0.1L	1.692	3.019	3.530
			0.2ax0.2L	1.664	2.882	3.140
			0.3ax0.3L	1.654	2.593	2.807
			0.4ax0.4L	1.706	2.338	2.730
		75/-75/75/-75/75	0.1ax0.1L	1.614	2.572	3.651
			0.2ax0.2L	1.529	2.483	3.002
			0.3ax0.3L	1.492	2.272	2.562
			0.4ax0.4L	1.517	2.050	2.450

boundary condition has been taken under consideration. Thickness ratio (h/a) is kept 0.1. For simply supported boundary condition in most of the cases natural frequencies decrease with the increment of the cut-out size but for some cases natural frequencies decrease first (with respect to the skew plate without cut-out) then start increasing.

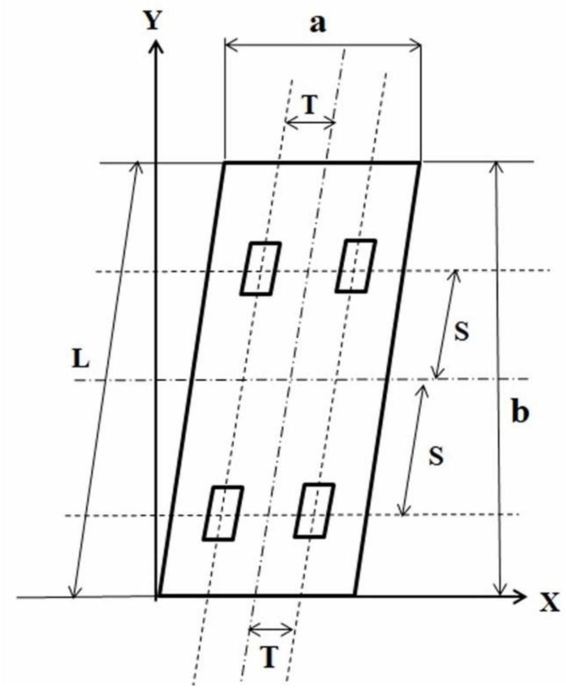

Fig. 3 Skew plate model with multiple cut-out

3.2.4 Clamped symmetric skew laminates with different fiber orientation and multiple cut-outs at variable distance from centre of plate

In this investigation, fixed supported skew plates with different skew angles, fiber orientations and multiple cut-outs at different positions have been taken under consideration. Mass of the plates (without cut-out) remain same for all skew angle as the vertical distance of two parallel horizontal edges (b) remain same (Fig. 3) but aspect ratio ($a/_L$) change with change of skew angle. Relative material properties of the composite plate are E_1/E_2= 40, G_{12}=G_{13}=0.6E_2, G_{23}=0.5E_2, γ_{12}=0.25. There are four eccentric cut-outs with size of cut-out to the edge of plate ratios are 0.1 and 0.2 which is equivalent to cut-out area are respectively (0.2×0.2) and (0.4×0.4). Edges of the cut-outs are parallel to the edges of the plate. Comparing Tables 5 and 7, it is observed that if multiple cut-outs are provided in skew plate instead of a single concentric cut-out with equivalent cut-out area, natural frequency decreases. Another observation from Table 7 is that when position of the cut-outs (distance from centre of plate) changes, natural frequencies also change.

3.2.5 Simply supported symmetric skew laminates with different fiber orientation and multiple cut-outs at variable distance from centre of plate

In this section similar type of simply supported skew composite plate (example 3.2.2) is analyzed. The mass of the plate (without cut-out) remains constant for all skew angles as the vertical distance of two parallel horizontal edges (b) remains same (Fig. 3) when aspect ratio gradually changes with skew angle. Comparing the Tables 6 and 8, it can be clearly observed that natural frequencies change if

Table 7 Frequency parameter λ= $((wa^2/h)\sqrt{(\rho/E_2)})/\pi^2$ of fixed symmetric angle ply laminates with cut-outs (h/a= 0.1)

Support condition	Skew angle	Fiber orientation(in degree)	no of cut-out	Cut-out size	Distance of Cut-out centers from centre of plate (T,S)	mode1	mode2	mode3
Fixed supported	45	15/-15/15/-15/15	4	0.1ax0.1L	0.3a, 0.3L	2.303	2.884	3.883
					0.2a, 0.2L	2.304	2.844	3.800
		30/-30/30/-30/30	4	0.1ax0.1L	0.3a, 0.3L	2.528	3.351	4.591
					0.2a, 0.2L	2.537	3.337	4.543
		45/-45/45/-45/45	4	0.1ax0.1L	0.3a, 0.3L	2.663	3.705	4.872
					0.2a, 0.2L	2.654	3.687	4.835
		60/-60/60/-60/60	4	0.1ax0.1L	0.3a, 0.3L	2.589	3.669	4.617
					0.2a, 0.2L	2.551	3.638	4.547
		75/-75/75/-75/75	4	0.1ax0.1L	0.3a, 0.3L	2.406	3.376	4.177
					0.2a, 0.2L	2.319	3.348	4.084
Fixed supported	30	15/-15/15/-15/15	4	0.1ax0.1L	0.3a, 0.3L	2.133	2.813	3.896
					0.2a, 0.2L	2.120	2.764	3.853
		30/-30/30/-30/30	4	0.1ax0.1L	0.3a, 0.3L	2.272	3.307	4.373
					0.2a, 0.2L	2.269	3.304	4.316
		45/-45/45/-45/45	4	0.1ax0.1L	0.3a, 0.3L	2.375	3.710	4.288
					0.2a, 0.2L	2.367	3.711	4.222
		60/-60/60/-60/60	4	0.1ax0.1L	0.3a, 0.3L	2.332	3.503	4.313
					0.2a, 0.2L	2.302	3.483	4.196
		75/-75/75/-75/75	4	0.1ax0.1L	0.3a, 0.3L	2.190	3.042	4.097
					0.2a, 0.2L	2.158	3.006	4.034
Fixed supported	45	15/-15/15/-15/15	4	0.2ax0.2L	0.3a, 0.3L	2.039	2.558	3.567
					0.2a, 0.2L	2.078	2.458	3.506
		30/-30/30/-30/30	4	0.2ax0.2L	0.3a, 0.3L	2.172	3.055	4.253
					0.2a, 0.2L	2.239	2.961	4.263
		45/-45/45/-45/45	4	0.2ax0.2L	0.3a, 0.3L	2.182	3.538	4.719
					0.2a, 0.2L	2.220	3.392	4.520
		60/-60/60/-60/60	4	0.2ax0.2L	0.3a, 0.3L	1.982	3.434	4.596
					0.2a, 0.2L	1.964	3.369	4.241
		75/-75/75/-75/75	4	0.2ax0.2L	0.3a, 0.3L	1.736	3.113	4.219
					0.2a, 0.2L	1.648	2.998	3.803
Fixed supported	30	15/-15/15/-15/15	4	0.2ax0.2L	0.3a, 0.3L	1.957	2.587	3.609
					0.2a, 0.2L	1.956	2.453	3.595
		30/-30/30/-30/30	4	0.2ax0.2L	0.3a, 0.3L	2.039	3.041	4.234
					0.2a, 0.2L	2.082	2.988	4.097
		45/-45/45/-45/45	4	0.2ax0.2L	0.3a, 0.3L	2.117	3.612	4.066
					0.2a, 0.2L	2.143	3.530	3.775
		60/-60/60/-60/60	4	0.2ax0.2L	0.3a, 0.3L	2.080	3.347	4.278
					0.2a, 0.2L	2.036	3.143	3.839
		75/-75/75/-75/75	4	0.2ax0.2L	0.3a, 0.3L	1.914	2.905	4.031
					0.2a, 0.2L	1.860	2.694	3.876

multiple cut-outs in skew plate are replaced by a single concentric cut-out with equivalent cut-out area. Table 8 also indicates that the natural frequencies change due to the change of position of the cut-outs (distance from centre of

Table 8 Frequency parameter λ= $((wa^2/h)\sqrt{(\rho/E_2)})/\pi^2$ of simply supported symmetric angle ply laminates with cut-outs,(h/a= 0.1)

Support condition	Skew angle	Fiber orientation(in degree)	no of cut-out	Cut-out size	Distance of Cut-out centers from centre of plate (T,S)	mode1	mode2	mode3
Simply supported	45	15/-15/15/-15/15	4	0.1ax0.1L	0.3a, 0.3L	1.602	2.362	3.330
					0.2a, 0.2L	1.625	2.322	3.086
		30/-30/30/-30/30	4	0.1ax0.1L	0.3a, 0.3L	1.667	2.638	3.076
					0.2a, 0.2L	1.748	2.638	2.991
		0/45/0/45/0	4	0.1ax0.1L	0.3a, 0.3L	1.537	2.365	2.380
					0.2a, 0.2L	1.580	2.262	2.370
		45/-45/45/-45/45	4	0.1ax0.1L	0.3a, 0.3L	1.656	2.592	2.901
					0.2a, 0.2L	1.782	2.581	2.969
		60/-60/60/-60/60	4	0.1ax0.1L	0.3a, 0.3L	1.657	2.969	3.001
					0.2a, 0.2L	1.780	2.826	3.051
		75/-75/75/-75/75	4	0.1ax0.1L	0.3a, 0.3L	1.709	2.736	3.054
					0.2a, 0.2L	1.793	2.754	2.850
Simply supported	30	15/-15/15/-15/15	4	0.1ax0.1L	0.3a, 0.3L	1.542	2.346	3.419
					0.2a, 0.2L	1.548	2.303	3.379
		0/30/0/30/0	4	0.1ax0.1L	0.3a, 0.3L	1.483	2.181	3.367
					0.2a, 0.2L	1.507	2.161	3.264
		30/-30/30/-30/30	4	0.1ax0.1L	0.3a, 0.3L	1.676	2.824	3.850
					0.2a, 0.2L	1.677	2.790	3.906
		45/-45/45/-45/45	4	0.1ax0.1L	0.3a, 0.3L	1.651	3.092	3.660
					0.2a, 0.2L	1.691	3.105	3.745
		60/-60/60/-60/60	4	0.1ax0.1L	0.3a, 0.3L	1.601	2.845	3.400
					0.2a, 0.2L	1.643	2.882	3.235
		75/-75/75/-75/75	4	0.1ax0.1L	0.3a, 0.3L	1.515	2.408	3.550
					0.2a, 0.2L	1.583	2.427	3.383
Simply supported	45	15/-15/15/-15/15	4	0.2ax0.2L	0.3a, 0.3L	1.411	2.161	2.398
					0.2a, 0.2L	1.393	1.916	1.957
		30/-30/30/-30/30	4	0.2ax0.2L	0.3a, 0.3L	1.388	2.423	2.459
					0.2a, 0.2L	1.454	2.104	2.229
		0/45/0/45/0	4	0.2ax0.2L	0.3a, 0.3L	1.391	2.197	2.224
					0.2a, 0.2L	1.434	1.773	2.140
		45/-45/45/-45/45	4	0.2ax0.2L	0.3a, 0.3L	1.284	2.057	2.635
					0.2a, 0.2L	1.439	1.757	2.617
		60/-60/60/-60/60	4	0.2ax0.2L	0.3a, 0.3L	1.185	2.185	2.573
					0.2a, 0.2L	1.342	1.639	2.695
		75/-75/75/-75/75	4	0.2ax0.2L	0.3a, 0.3L	1.127	2.007	2.207
					0.2a, 0.2L	1.332	1.378	2.438
Simply supported	30	15/-15/15/-15/15	4	0.2ax0.2L	0.3a, 0.3L	1.395	2.183	3.022
					0.2a, 0.2L	1.349	1.982	2.373
		0/30/0/30/0	4	0.2ax0.2L	0.3a, 0.3L	1.366	2.065	2.883
					0.2a, 0.2L	1.364	1.931	2.315
		30/-30/30/-30/30	4	0.2ax0.2L	0.3a, 0.3L	1.463	2.565	3.494
					0.2a, 0.2L	1.388	2.365	2.794
		45/-45/45/-45/45	4	0.2ax0.2L	0.3a, 0.3L	1.410	2.809	3.160
					0.2a, 0.2L	1.420	2.677	2.775
		60/-60/60/-60/60	4	0.2ax0.2L	0.3a, 0.3L	1.367	2.437	2.824
					0.2a, 0.2L	1.421	2.091	2.558
		75/-75/75/-75/75	4	0.2ax0.2L	0.3a, 0.3L	1.224	2.124	2.880
					0.2a, 0.2L	1.330	2.046	2.082

plate). For both cases total area of cut-out remains same, hence it is clear from these observations that the position of the cut-outs affects the stiffness of the plate and it becomes the governing factor.

5. Conclusions

In the present study, numerical investigations have been carried out on the laminated carbon-epoxy skew plates with single and multiple cut-outs. The finite element formulation is validated by solving several numerical examples and compared with those available in the published literature.

• It is also observed from the present numerical analysis that there is a significant effect of rotary inertia on the free vibration of thick laminated plates.

• It is recommended that the mass lumping scheme with rotary inertia is suitable for both thick and thin plates whereas the mass lumping scheme without rotary inertia is suitable for thin plates only.

• The study reveals that for skew composite plate, the fundamental frequency increases with the increase in skew angle.

• It is also observed that for fixed supported skew composite plate with single cut-out at center fundamental frequencies increase with the increment of cut-out size while for simply supported laminates natural frequencies decrease in most of the cases with the increment of the cut-out size but for some cases fundamental frequency decreases initially (with respect to the skew plate without cut-out) then starts increasing.

• For fixed supported skew composite plate when multiple cut-outs are provided instead of single cut-out at centre, fundamental frequency decreases. But for simply supported conditions, the fundamental frequency is higher for single cut-out in some cases and larger frequencies obtained for multiple cut-out for some other cases.

• For multiple cutout, the positions of the cut-outs (distance from centre of plate) alter natural frequencies.

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