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Multi-objective optimization of double wishbone suspension of a kinestatic vehicle model for handling and stability improvement

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Abstract. One of the important problems in the vehicle design is vehicle handling and stability. Effective parameters which should be considered in the vehicle handling and stability are roll angle, camber angle and scrub radius. In this paper, a planar vehicle model is considered that two right and left suspensions are double wishbone suspension system. For a better analysis of the suspension geometry, a kinestatic model of vehicle is considered which instantaneous kinematic and statics relations are analyzed simultaneously. In this model, suspension geometry is considered completely. In order to optimum design of double wishbones suspension system, a multi-objective genetic algorithm is applied. Three important parameters of suspension including roll angle, camber angle and scrub radius are taken into account as objective functions. Coordinates of suspension hard points are design variables of optimization which optimum values of them, corresponding to each optimum point, are obtained in the optimization process. Pareto solutions for three objective functions are derived. There are important optimum points in these Pareto solutions which each point represents an optimum status in the model. In other words, corresponding to any optimal point, a specific geometric position is determined for the suspension hard points. Each of the obtained points in the Pareto optimization can be selected for a special design purpose by designer to create an optimum condition in the vehicle handling and stability.

Keywords: multi-objective optimization; double wishbone suspension; kinestatic analysis; vehicle handling and stability

1. Introduction

In order to study the effect of suspension system on the dynamic behaviour of vehicle, there are many vehicle models such as kinematic model. This model considers the kinematic relation between links and coordinates of hard points. In the kinematic model, the geometry of suspension is described completely and details of suspension connection to vehicle chassis and body is shown. By simulation of this model, the control and stability of vehicle can be assured. Fallah *et al.* (2009) considered a two-degree(two-DOF) freedom kinematic model of Macpherson strut suspension system in conjunction with sprung mass. They assumed a nonlinear quarter model of vehicle for improving the ride quality and evaluating the kinematic parameters. The stiffness of a suspension system was provided by Zhao *et al.* (2012) using the bushings and the stiffness of the wheel center control from the suspension's elasto-kinematic (e-k) specification. Tang *et al.* (2018) studied the design of an integrated suspension tilting mechanism for narrow tilting vehicles. One of the new kinematic models is based on kinematic-static analysis of suspension system (Kim *et al.* 2013). In kinematic-static analysis, using Jacobin method, instantaneous kinematic and static relations (kinestatic) of half vehicle model are derived. In this method, the kinematic relations of planar quarter vehicle model are first derived and then using two instantaneous rotation centers of a pair of quarter vehicle models as virtual joints, the kinestatic relations of half vehicle model are obtained.

For optimization of suspension system, there are many optimization methods such as genetic algorithm (GA). In the optimization process, one or more mechanical or geometrical constants of the suspension system are considered as design variables and one or more dynamic parameters of vehicle model are assumed as objective functions to be optimized. A nonlinear quarter car suspension-seat-driver model was implemented for optimum design by Nagarkar *et al.* (2016). Sert and Boyraz (2017) presented a method for systematic investigations on static and dynamic roll behavior and improvement to the stability dynamics based on increasing roll stiffness of the suspension. The suspension parameters of a vehicle model were estimated by Papaioannou and Koulocheris (2018) using multi-objective optimization procedures with genetic algorithm in order to overcome the well-known conflict of ride comfort and road holding. Li *et al.* (2018b) considered double-wishbone independent suspension with two unequal-length arms and analyzed the sensitivities of front wheel alignment parameters using the space analytic geometry method with insight module in ADAMS software. They applied genetic algorithm to calculate the coordinate

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values of hard points. The geometry of the suspension system can change values of roll center height and camber angle significantly and affect the ride characteristics and vehicle handling. Therefore, geometrical characteristics such as coordinates of hard points as design variables and change of dynamic parameters such as roll and camber angles as objective functions can be considered in the optimization process. Kaleibar *et al.* (2013) optimized the geometrical parameters of double wishbone (DW) suspension using genetic algorithm to improve comfort, handling and stability of the vehicle. Wang *et al.* (2011) applied a kinematic simulation for double wishbone suspension system and optimized the sideways displacement of wheel by determining important parameters. Yang *et al.* (2012) improved the ride comfort, handling and stability of the vehicle by modifying hard point parameters of the double wishbone suspension. In optimization problems which several parameters are considered as objective functions, the multi-objective optimization algorithm such as multi-objective genetic algorithm can be used. Sancibrian *et al.* (2010) considered three-DOF kinematic model of double wishbone suspension system and optimized it using multi-objective optimization algorithm. For the design of double wishbone suspension system, Arikere *et al.* (2010) used Non-dominated sorting genetic algorithm II (NSGA-II) to optimize the camber and toe parameters as objective functions. Cheng and Lin (2014) obtained an initial design for double wishbone suspension system and optimized it applying particle swarm optimization method. Su *et al.* (2018) investigated the kinematic characteristic analysis and optimization design of a minivan MacPherson-strut suspension system. They optimized and designed the objective function with neighborhood cultivation genetic algorithm (NCGA). Li *et al.* (2018a) proposed a sparse response surface (SRS) method to optimize a double wishbone suspension model constructed from a few non-adaptive sampling points and they optimized front wheel positioning parameters.

In the present research, our main contribution as follows: a kinestatic model of vehicle with planar double wishbone suspension system is investigated and is optimized by multi-objective genetic algorithm to improve handling and stability of vehicle. The optimum coordinates for connection points of upper control arm, lower control arm and strut to the vehicle chassis and body are obtained. The significant parameters in the vehicle handling and stability including body roll angle change, camber angle change after rolling of the body and scrub radius after rolling of the body are minimized in the optimization process as objective functions. Using kinestatic relations of half vehicle model, vehicle roll conditions under centripetal force are considered. Then, body roll angle change, camber angle change after rolling of the body and scrub radius after rolling of the body are optimized by applying NSGA-II to improve the vehicle handling and stability. The optimum coordinates for connection points of upper control arm, lower control arm and strut to the vehicle chassis and body are obtained and Pareto fronts related to trade-off points for pairs of objective functions are plotted. Finally, the effect of optimized parameters in the kinestatic model are studied.

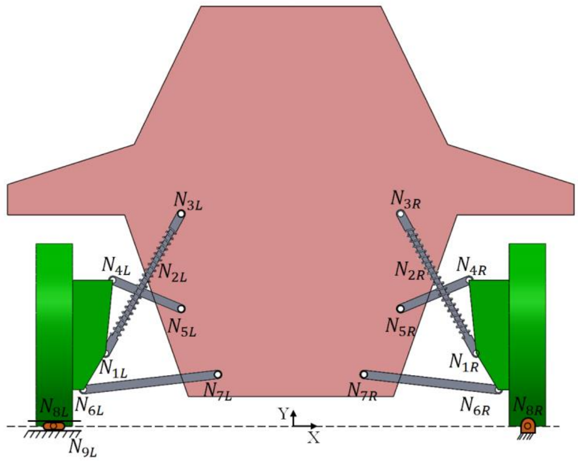

Fig. 1 Half vehicle model

2. Kinestatic analysis and optimization of half vehicle model

In this study, a half vehicle model with a pair of double wishbone suspension mechanisms are considered as shown in Fig. 1. This half vehicle kinematic model is a two degree of freedom parallel mechanism (Kim *et al*. 2013). As shown in Fig. 1, all joints are revolute joint except joints N_{2L}, N_{2R} and N_{9L}, which are prismatic joint. The contact between the left wheel and ground is modeled as serially connected revolute joint (N_{8L}) and prismatic joint (N_{9L}) and the contact between the right wheel and ground is considered by revolute joint (N_{8R}). In this model, only the left wheel has sliding motion along the x-axis.

The inverse static relation between the small change $\delta\hat{\boldsymbol{w}}$ in the external wrench and small changes $\delta\boldsymbol{\tau}$ in the spring forces can be obtained as (Kim *et al*. 2013)

$$\delta\boldsymbol{\tau} = \boldsymbol{J}_q \begin{bmatrix} \dfrac{1}{\hat{\boldsymbol{r}}_{CL}^T\ \hat{\boldsymbol{R}}_{CL}} & 0 \\ 0 & \dfrac{1}{\hat{\boldsymbol{r}}_{CR}^T\ \hat{\boldsymbol{R}}_{CR}} \end{bmatrix} \boldsymbol{J}_{rp}^T \delta\hat{\boldsymbol{w}} \tag{1}$$

Also, the forward kinematic relation of half vehicle model between the small displacement twist $\delta\hat{\boldsymbol{D}}$ and small displacement of springs $\delta\boldsymbol{l}$ is given by

$$\delta\hat{\boldsymbol{D}} = \boldsymbol{J}_{rp} \begin{bmatrix} \dfrac{1}{\hat{\boldsymbol{r}}_{CL}^T\ \hat{\boldsymbol{R}}_{CL}} & 0 \\ 0 & \dfrac{1}{\hat{\boldsymbol{r}}_{CR}^T\ \hat{\boldsymbol{R}}_{CR}} \end{bmatrix} \boldsymbol{J}_q\, \delta\boldsymbol{l} \tag{2}$$

Where $\boldsymbol{J}_q$ and $\boldsymbol{J}_{rp}$ are Jacobian and reciprocal Jacobian of half vehicle model respectively, and $\boldsymbol{J}_{rp}$ expressed by

$$\boldsymbol{J}_{rp} = \left\lfloor \hat{\boldsymbol{R}}_{CL} \quad \hat{\boldsymbol{R}}_{CR} \right\rfloor \tag{3}$$

Where $\hat{\boldsymbol{R}}_{CL}$ and $\hat{\boldsymbol{R}}_{CR}$ are column vector of the reciprocal Jacobian and $\hat{\boldsymbol{r}}_{CL}$ and $\hat{\boldsymbol{r}}_{CR}$ are row vector of the Jacobian respectively. On the other hand

Table 1 Physical parameters of the vehicle model (Kim *et al*. 2013)

Weight	Lateral acceleration	Suspension spring constant
10000 N	0.3 g	30 N/mm

Table 2 Hard points coordinates of the vehicle model (Kim *et al*. 2013)

Point	Coordinates (mm)		Point	Coordinates (mm)	
	X	Y		X	Y
N_{1L}	-750	300	N_{1R}	750	300
N_{3L}	-450	900	N_{3R}	450	900
N_{4L}	-720	600	N_{4R}	720	600
N_{5L}	-450	510	N_{5R}	450	510
N_{6L}	-840	150	N_{6R}	840	150
N_{7L}	-300	240	N_{7R}	300	240
N_{8L}	-960	0	N_{8R}	960	0
MC*	0	900			

*MC: Mass centre point

$$\delta\hat{\mathbf{D}} \equiv \delta\phi[y \quad -x \quad 1]^T \tag{4}$$

Where $\delta\phi$ is body roll angle change and (x,y) is coordinates of instantaneous body roll centre.

The vehicle is assumed to undergo a cornering motion and lateral acceleration is applied to vehicle body, which causes the rolling of vehicle body. The physical parameters and hard points coordinate of the vehicle model are given in Tables 1 and 2 respectively (Kim *et al*. 2013).

A single-objective optimization can be formulated as follows

$$\min\ f(\boldsymbol{x}), \boldsymbol{x} \in \boldsymbol{S} \tag{5}$$

Where f is a scalar function and $\boldsymbol{S}$ is the set of constraints that can be defined as

$$\boldsymbol{S} = \{\boldsymbol{x} \in \boldsymbol{R}^m : h(\boldsymbol{x}) = 0, g(\boldsymbol{x}) \geq 0\} \tag{6}$$

Where $h(\boldsymbol{x})$ and $g(\boldsymbol{x})$ are the constraint functions.

In the problems that need to several objective functions are optimized, the multi-objective optimization problems (MOOP) are applied. In multi-objective optimization, finding a dominant optimal solution is often not possible. A solution may be optimal for an objective function but not optimal for another. In this case, the two objective functions are conflicting. Multi-objective optimization can be described in mathematical form as follows

$$\min\left[f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ldots, f_n(\boldsymbol{x})\right], \boldsymbol{x} \in \boldsymbol{S} \tag{7}$$

Where f_i (i=1,2,…,n and $n > 1$)are objective functions and $\boldsymbol{S}$ is the set of constraints defined above. The space that the objective vector belongs is called the objective space. The scalar concept of "optimality" is not directly applicable to multi-objective optimization, but the concept of "Pareto optimality" is introduced. Basically, in multi-objective optimization, a vector $\boldsymbol{x}^* \in \boldsymbol{S}$ is said to be Pareto optimal, if all other vectors $\boldsymbol{x} \in \boldsymbol{S}$ have a higher value for at the least one of the objective functions f_i, or have the same value for all objective function.

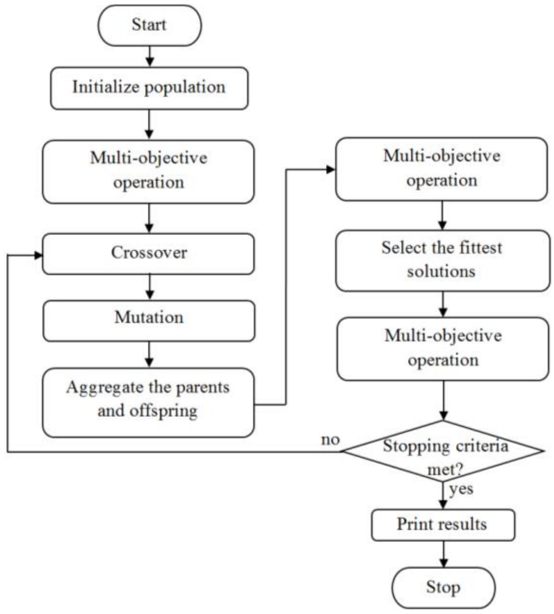

Fig. 2 Main steps of NSGA-II

The advantage of the multi-objective genetic algorithm method with respect to other optimization methods is to maintaining a diverse population. In the optimization process of multi-objective genetic algorithm, the diverse population is maintained to obtain solutions uniformly distributed over the Pareto front. Without using preventive proceeding, the population tends to form relatively few clusters, which this phenomenon is called genetic drift. In the multi-objective genetic algorithm to prevent genetic drift, several approaches such as fitness sharing, crowding distance and cell-based density have been devised. One of the non-elitism based algorithms of multi-objective optimization is the Non-dominated sorting genetic algorithm (NSGA). The difference between NSGA and the simple genetic algorithm is in the implementation of selection operator. The crossover and mutation operators are executed as usual. By developing a diversity-preserving mechanism and matching it with the search algorithm, different Pareto optimal solutions are found.

NSGA-II is an improved version of NSGA which solved computational complexity and non-elitism. This algorithm by using elitism, creates a diverse Pareto optimal front. Low computational complexity and elitism are main features of NSGA-II. The elitism is accomplished by storing all non-dominated solutions founded from the initial population so far. The main steps of NSGA-II is shown in Fig. 2.

3. Parameters of vehicle handling and optimization problem

The characteristics of the vehicle which have significant influence on the vehicle handling and stability, are the roll resistance of the vehicle, wheel inclination and Kingpin inclination. Therefore, in the subject of vehicle handling and stability, it is necessary to pay special attention to the parameters such as body roll angle, camber angle and scrub radius. In the present research, for kinestatic model of vehicle under lateral acceleration, the body roll angle change (RA), camber angle change (CA) of right tire after rolling of body and scrub radius (SR) of right tire after rolling of body, is considered as objective functions. The values of the objective functions are extracted after solving the first iteration of the algorithm of kinestatic analysis. Therefore, the objective functions in Eq. (7) are defined as follows:

RA: minimization of absolute value of body roll angle change

$$f_1(\boldsymbol{x}) = f_{RA}(\boldsymbol{x}) = \delta\phi \tag{8}$$

CA: minimization of absolute value of camber angle change of right tire after rolling of the body

$$f_2(\boldsymbol{x}) = f_{CA}(\boldsymbol{x}) = |\delta\gamma| \tag{9}$$

SR: minimization of absolute value of scrub radius of right tire after rolling of the body

$$f_3(\boldsymbol{x}) = f_{SR}(\boldsymbol{x}) = |SR_{final}| \tag{10}$$

Study shows that two objective functions cannot be minimized simultaneously, in other words, the mentioned objective functions are conflicting. Therefore, multi-objective optimization is used to optimize the objective functions simultaneously and create compromises between them. In Pareto solutions of multi-objective optimization, optimum points are obtained which provide good compromises between the optimal values of objective functions.

In kinestatic model of vehicle, effective parameters which affect on characteristics of vehicle handling and stability are hard point coordinates of suspension. Therefore, the coordinates of hard points of suspension can be considered as design variables. In the present research, coordinates of connection points of strut (N_3), upper and lower control arm (N_5 and N_7) to chassis and body, are considered as design variables of MOOP. Some of design variables have inverse effect on different objective functions, and therefore improvement in an objective function has a worsen effect on another. As mentioned, to solve this problem and create compromises between conflicting objective functions, the multi-objective optimization is applied. In the present research, to create these compromises and solve multi-objective optimization problem, NSGA-II is used. Table 3 presents the design variables and their corresponding lower and upper ranges. The constraints in multi-objective optimization problem are variable bound constraints which defined in Table 3.

4. Analysis and results

To improve the vehicle handling and stability of the half vehicle kinematic model shown in Fig. 1, the NSGA-II is applied. To optimize the vehicle handling and stability, the

Table 3 Design variable ranges

Design variables	Lower value(mm)	Upper value(mm)
$x_{3R} = (-x_{3L})$	400	700
$x_{5R} = (-x_{5L})$	337	562
$x_{7R} = (-x_{7L})$	225	375
$y_{3R} = (y_{3L})$	600	900
$y_{5R} = (y_{5L})$	375	580
$y_{7R} = (y_{7L})$	180	375

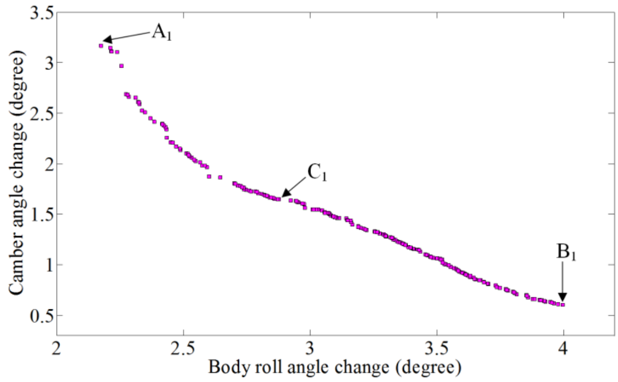

Fig. 3 Pareto front for body roll angle change and camber angle change

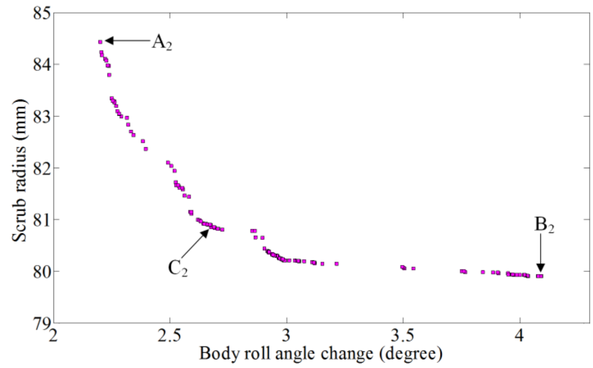

Fig. 4 Pareto front for body roll angle change and scrub radius

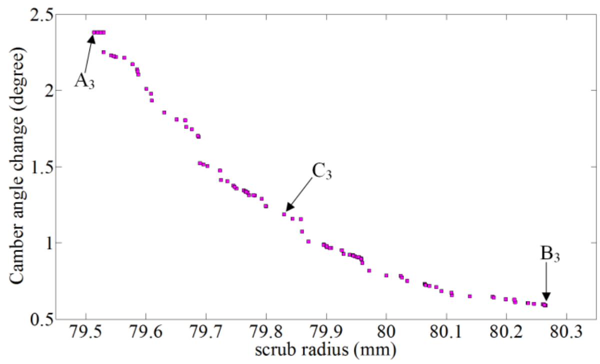

Fig. 5 Pareto front for scrub radius and camber angle change

various pairs of objective functions are considered in the optimization process. The pairs of objective functions which are separately optimized are (RA, CA), (RA, SR) and (SR, CA). In optimization algorithm a population of 200 individuals with a crossover probability of 0.7 and a mutation probability of 0.3 are utilized. The Pareto fronts obtained for each pair of selected objective functions are shown in Figs. 3-5.

The Pareto front of the body roll angle change and camber angle change of right tire after rolling of the body is shown in Fig. 3, that different non-dominated optimum points relative to the conflicting objective functions are displayed. Optimization solutions are appeared as trade-off optimum points in Pareto fronts. In this figure, the points A_1 and B_1 stand for the best body roll angle change and camber angle change of right tire after rolling of the body, respectively. The obtained points on Pareto front are Non-dominated, and each point can be considered as an optimal condition of the vehicle model for design. According to the Pareto front, the confliction of the objective functions to each other is clear, and obtaining a better value for an objective function causes a worse value for another.

There are important optimal points in Pareto fronts which demonstrate the superiority of Pareto optimization method to other multi-objective optimization methods. In Fig. 3, point C_1 represents one of the important points in vehicle optimal design. The optimum point C_1 exhibits a small increase in camber angle change relative to point B_1 (design with least camber angle change), while the body roll angle change is improved considerably. Finding trade-off design point C_1 among the various optimum solutions is an important feature that highlights the significance of Pareto optimization method used in this study.

The Pareto fronts, containing the non-dominated optimum points for other objective function pairs, is shown in Figs. 4 and 5. In Fig. 4, the design points A_2 and B_2 stand for the best body roll angle change and best scrub radius of right tire after rolling of the body, respectively, and the point C_2 is one of the important trade-off points in the optimum solutions. Also, in Fig. 5, the design points A_3 and B_3 stand for the best scrub radius of right tire after rolling of the body and best camber angle change of right tire after rolling of the body, respectively, and the point C_3 is one of the important trade-off points in the corresponding optimum solutions. The derived optimum design points and the values of the corresponding objective functions and the design variables related to these optimum points are listed in Table 4, that F_A and F_B are the values of the first and second objective functions corresponding to these points, respectively.

With regard to the population of 200 individuals in the optimization algorithm, it can be seen that each of the points on the Pareto fronts represents several optimum points obtained in the optimization results which coincide at a geometrically point due to the closeness of value of their objective functions. The repeatability of the solutions and the swarm of the points in each position of the Pareto front show the stability of the applied optimization method and singularities of the obtained results.

In order to compare the optimum points obtained in the present research with the initial configuration of the vehicle model, the value of roll angle change, camber angle change of right tire and final scrub radius of right tire after solving the last iteration of the algorithm of kinestatic analysis, for some optimum points, is given in Table 5. Regarding the values listed in the table, using the design points obtained in the present research, the value of the body roll angle change, camber angle change after rolling of the body and scrub radius after rolling of the body are improved considerably. It should be noted that the value of the body

Table 4 The values of objective functions and design variables of the optimum points

	F_A	F_B	x_{3R}	x_{5R}	x_{7R}	y_{3R}	y_{5R}	y_{7R}
A_1	2.18	3.17	684	458	365	792	546	363
B_1	4.00	0.60	633	508	232	833	386	184
C_1	2.88	1.65	696	407	277	783	517	262
A_2	2.20	84.4	670	344	373	840	544	354
B_2	4.09	79.9	480	537	232	762	507	187
C_2	2.68	80.9	673	390	334	791	573	264
A_3	79.6	2.38	444	554	294	624	523	187
B_3	80.3	0.59	592	531	257	750	388	183
C_3	79.8	1.19	462	551	276	725	482	184

Table 5 The results of kinestatic model analysis corresponding to initial configuration and optimum points

	$\delta\phi$	$\delta\gamma_R$	SR_R
IC*	3.65	1.90	-80.9
C_1	2.76	1.62	-80.8
C_2	2.59	1.81	-80.8
C_3	4.25	1.17	-79.8
A_1	1.85	3.21	-85.5
A_2	1.92	2.90	-84.4

*IC: Initial configuration

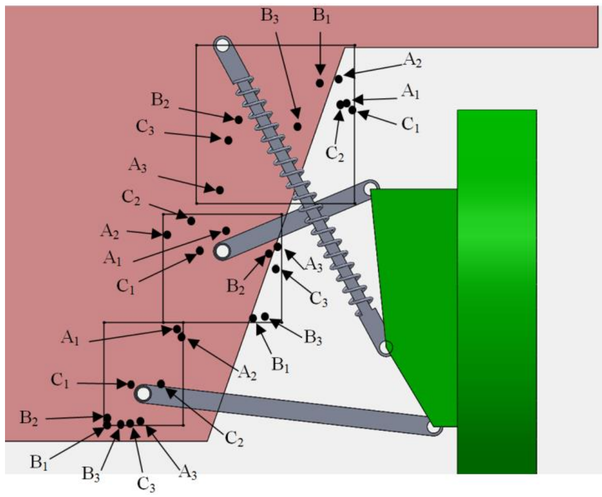

Fig. 6 Geometrical ranges of connection points considered in optimization process and position of optimum points

roll angle change corresponding to the optimum points A_1 and A_2 relative to their respective values in the initial configuration of the vehicle model, are improved about 49% and 47%, respectively.

The geometrical ranges of connection points considered in optimization process and position of derived optimum points on suspension system is shown in Fig. 6. According to this figure, in considered geometrical range for coordinates of connection points, the following results are obtained:

1. To reduce the body roll angle change, points N_3 and N_7 must be transmitted to top and outward of the vehicle and point N_5 to top of the vehicle.

2. To reduce the camber angle change after rolling of the body, point N_5 must be transmitted to bottom and outward of the vehicle and point N_7 to bottom and inward of the vehicle.

3. To reduce the scrub radius after rolling of the body, point N_5 must be transmitted to top and outward of the vehicle and point N_7 to bottom and inward of the vehicle.

5. Conclusions

A multi-objective genetic algorithm was employed to optimize the kinestatic vehicle model. Several objective functions that are in conflict with each other, such as body roll angle change, camber angle change after rolling of the body and scrub radius after rolling of the body were selected. By the multi-objective optimizing of the vehicle model, the optimum coordinates for connection points of strut, upper and lower control arm to chassis and body were derived. The comparison of the obtained optimum points with the initial configuration of the vehicle model, a significant improvement in the parameters which considered as objective functions was observed. By applying the derived optimum points, a high level of vehicle handling and stability is achieved.

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