Structural Engineering and Mechanics, *Vol. 68, No. 4 (2018) 423-442*
DOI: https://doi.org/10.12989/sem.2018.68.4.423

In-plane response of masonry infilled RC framed structures: A probabilistic macromodeling approach

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(Received June 1, 2018, Revised September 22, 2018, Accepted September 29, 2018)

Abstract. In this paper, masonry infilled reinforced concrete (RC) frames are analyzed through a probabilistic approach. A macro-modeling technique, based on an equivalent diagonal pin-jointed strut, has been resorted to for modelling the stiffening contribution of the masonry panels. Since it is quite difficult to decide which mechanical characteristics to assume for the diagonal struts in such simplified model, the strut width is here considered as a random variable, whose stochastic characterization stems from a wide set of empirical expressions proposed in the literature. The stochastic analysis of the masonry infilled RC frame is conducted via the Probabilistic Transformation Method by employing a set of space transformation laws of random vectors to determine the probability density function (PDF) of the system response in a direct manner. The knowledge of the PDF of a set of response indicators, including displacements, bending moments, shear forces, interstory drifts, opens an interesting discussion about the influence of the uncertainty of the masonry infills and the resulting implications in a design process.

Keywords: RC frames; masonry infills; macro-modeling approaches; probability transformation method; probability density function; probability-based design

1. Introduction

Typically, reinforced concrete (RC) framed structures are infilled with non-structural panels in order to separate the internal building space from the external environment. The infill panels are, in the majority of cases, made by masonry, whose structural behavior is extremely complex, being affected by a number of uncertain parameters such as the mechanical characteristics of the raw materials (clay, concrete), the mortar thickness and quality, the brick geometry and arrangement, the relative stiffness of the frame and of the infill panel, as well as the actual workmanship expertise, to name just a few. Although they are considered as non-structural components in the structural calculation, masonry infills do modify the stiffness, strength and ductility response scenarios of the overall RC frame. Therefore, neglecting their presence in structural analysis and design of masonry infilled RC frames may lead to inaccurate predictions and wrong design conclusions. Additionally, the actual behavior is further complicated by the presence of irregularities in the distribution of infills in plan and elevation of the building, and the resulting overall interaction between infill walls and surrounding frame (Asteris *et al.* 2015a, Khoshnoud and Marsono 2016, Asteris *et al.* 2017a). This interaction may or may not be beneficial from a design viewpoint, for instance an irregular distribution of infills may produce torsional behaviors along with triggering undesired phenomena of soft stories, as demonstrated by observations after catastrophic earthquakes, see the emblematic examples illustrated in Fig. 1. Additionally, in common practice infill walls include openings (e.g., doors, windows) and this further complicates the determination of the mechanical response of masonry infilled frames (Asteris *et al.* 2011, Asteris *et al.* 2016a).

The above sources of uncertainty and irregularity, and the heterogeneous nature of the masonry panels make the related modeling task a rather intricate process. Indeed, there is lack of repeatability of results, even when carrying out experiments under macroscopically identical geometrical and mechanical conditions. In this regard, experiments have been conducted on infill walls since the late 50s, see e.g., (Benjamin and Williams 1957, Benjamin and Williams 1958, Matthies *et al.* 1997, Zarnic and Tomazevic 1985) for some landmark contributions involving monotonic loading, and (Esteva 1966, Chandrasekaran and Chandra 1970, Klinger and Bertero 1976, Valiasis and Stylianidis 1989, Mehrabi *et al.* 1994, Dawe *et al.* 1989, Dolce *et al.* 2005) for cycling loading, harmonic excitations and shake-table tests. The mentioned papers were mostly focused on the in-plane behavior of the masonry infills, whereas a wealth of literature also exists for the out-of-plane behavior, see e.g., (Angel *et al.* 1994, Felice and Giannini 2001, Flanagan and Bennett 1999, Pasca *et al.* 2017) for just a few examples. The failure mode of a masonry infill subject to horizontal (seismically-induced) loads may range from compression failure of diagonal strut (also referred to as "corner crushing"), which

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http://www.techno-press.com/journals/semandsubpage=7 ISSN: 1225-4568 (Print), 1598-6217 (Online)

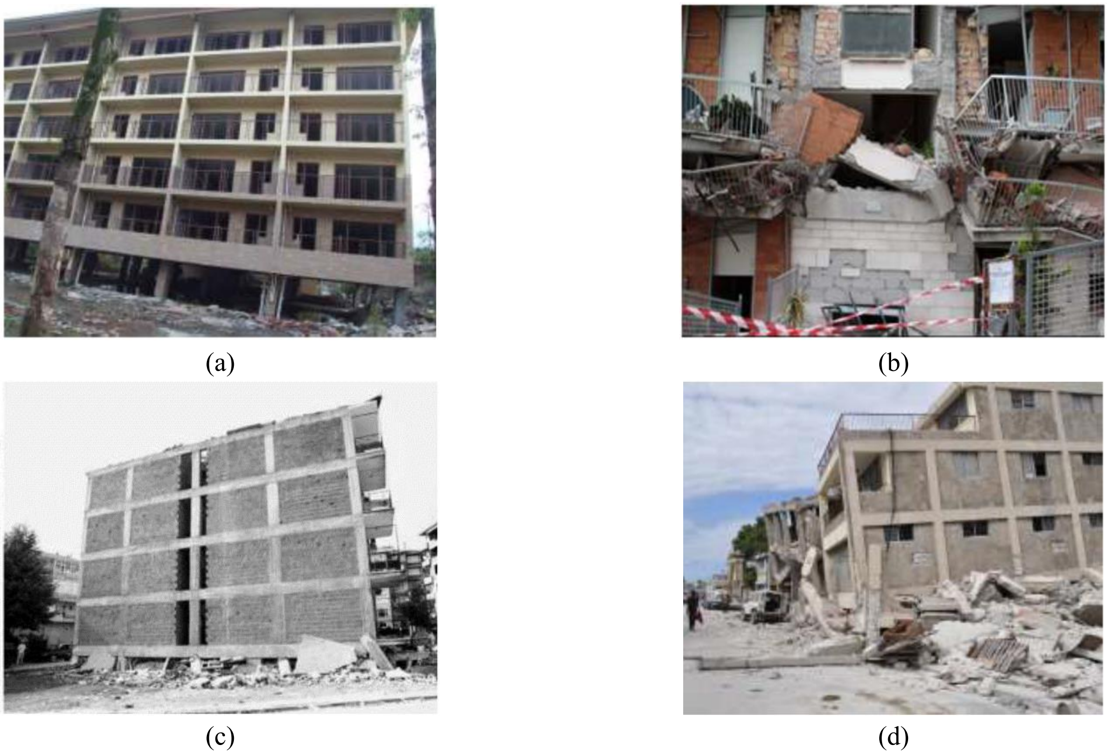


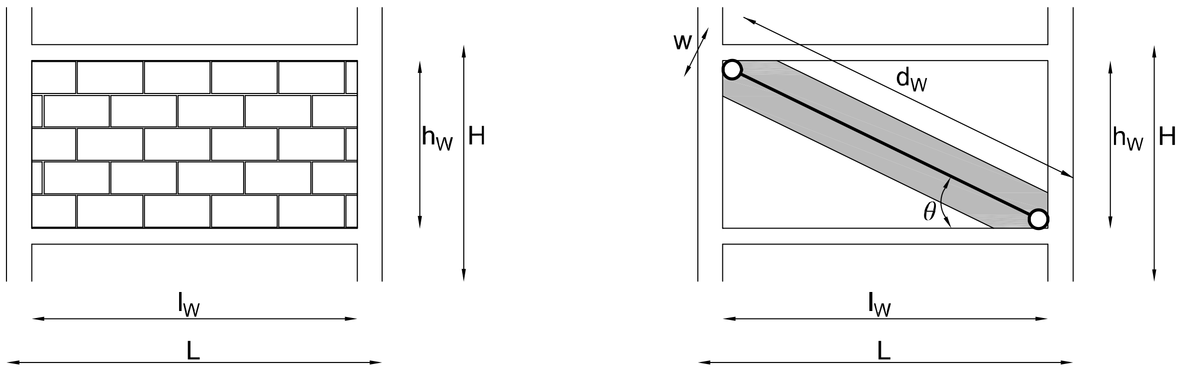
Fig. 1 Examples of poor seismic performance of RC frames with masonry infill walls: first-story damage in 2008 Wenchuan earthquake (China, Mw = 8.0) (a), intermediate story collapses due to infill failure in 1999 L'Aquila earthquake (Italy, Mw = 6.3) (b), first-two-story collapse in 1999 Kocaeli (Turkey, Mw = 7.4) (c) and in 2010 Haiti earthquake (Mw = 7.0) (d)

is quite frequent and typically occurs for low-compressive-strength infill materials, damage in the frame members (also referred to as frame failure mode), which originates from a damage mechanism of the column due to the forces transferred from the infill wall to the surrounding frame (it generally takes place for masonry infills having high compressive strength), diagonal cracking failure mode, sliding shear or out-of-plane failure whereby damage accumulates in the central zone of the infill panel due to the arching mechanism (Kheirollahi 2013).

The aforementioned references, although limited to just a few contributions, together with the variety of failure modes mentioned before give the general idea of the complexity in modeling the masonry infill behavior while properly accounting for its stiffening contribution and for the interaction with the surrounding frame in a RC framed building. Several researchers investigated this subject with a variety of numerical techniques, with different underlying theoretical bases and applicable to different scales of observations. Typical modeling approaches include micro-models (Döven and Kafkas 2017), macro-models or homogeneization models (Milani and Benasciutti 2010). Some literature surveys can be found in a couple of papers by Asteris and co-workers (Asteris *et al.* 2011, Asteris *et al.* 2013, Asteris *et al.* 2015b, Asteris *et al.* 2017b), and a quite recent overview of linear and nonlinear, micro-modelling, meso-modelling and macro-modelling approaches has been presented in (Tarque *et al.* 2015) and references therein. Extensive and in-depth state-of-the-art reports can be found in the following works (Amanat and Hoque 2006, Crowley and Pinho 2004, Crowley and Pinho 2006, Asteris *et al.* 2016b, Asteris 2016).

In line with this research field and considering the experimentally observed uncertain nature of the masonry behavior along with the dissemination of a large number of predictive expressions, the aim of this paper is to propose a probabilistic approach for the analysis of masonry infilled RC frames. This probabilistic framework is particularly motivated by the scatter of experimental results on which most of the predictive (simplified) expressions are based, which may induce doubts about which mechanical characteristics to assume for the diagonal struts in a simplified model.

The presence of a variety of empirical expressions in the literature makes it difficult to decide which is the most suitable one for design purposes, even more complicated by the uncertain nature of the masonry behavior. In this paper, we face the problem of determining the structural response of masonry infilled reinforced concrete (RC) frames from a probabilistic perspective, by assuming the diagonal strut (modeling the masonry panel) as a random variable. In this way, the suitability of the different expressions proposed in the literature may be assessed in terms of how closely they predict the mean values (or some characteristic values) of the relevant probability density function. Additionally, the probabilistic framework allows the evaluation of the implications of the uncertain nature of the masonry infills in a set of response indicators. This is important to asses to what extent the randomness of the masonry infills propagates in the structural response. To the authors' best knowledge, probabilistic approaches to this problem are very few (Erdolen and Doran 2012) and usually rely on Monte-Carlo-



(a) Schematic representation of the masonry infill (b) Conventions used for the macro-modeling approach

Fig. 2 Schematic representation of the masonry infill (a) and conventions used for the macro-modeling approach (b)

based sampling techniques involving repetitive operations and computational effort, especially for structures with many degrees of freedom (DOFs). On the contrary, the focus of this paper is on a more effective numerical procedure through which the probabilistic characterization of the response of a masonry infilled RC frame can be identified in a direct manner. The advantage of this procedure is that no sampling operation is required (unlike Monte Carlo method or other techniques from the literature). This implies great computational efficiency in comparison with other methods. Another strength of this procedure is its simplicity when handling uncertain parameters like the ones involved in the equivalent diagonal truss elements associated with the macro-modeling assumption of the masonry infills, as will be in-depth clarified in the following parts of the paper.

The outline of the paper is as follows. After this introductory section, in section 2 an overview of empirical/analytical expressions from the literature is presented for the macro-modeling of the masonry infills in RC frames. These expressions are useful to introduce the probabilistic framework adopted in this paper, as explained in section 3. Three different probabilistic approaches are then presented in section 4, including a novel method combining two earlier procedures from the literature, which is here for the first time applied to masonry infilled RC frames. Some simple numerical examples are illustrated in section 5 in order to present which kind of results the proposed procedure can offer, and which design implications the uncertain nature of the masonry infills may have, depending on the modeling assumptions and on the probabilistic parameters adopted. Finally, section 6 summarizes the main findings of this research work.

2. Macro-modelling approach and overview of expressions

The modeling of the masonry infills and of their stiffening contribution to the surrounding RC frame has been a topic of great interest for decades. Since the Polyakov's work in 1960 (Polyakov 1960), the most simplified way to account for the masonry panel has been to introduce an equivalent diagonal strut element that incorporates the stiffening contribution of the masonry infill. In this simplified manner, underlying a macro-modeling approach, the geometrical characteristics of the equivalent diagonal strut are chosen such that they reflect the geometrical and mechanical properties of the actual masonry panel. A schematic representation of this macro-modeling approach is reported in Fig. 2, wherein the main geometric characteristics of both the masonry panel and the equivalent diagonal strut are illustrated.

For monotonic loading only one strut is introduced in the compression direction, whereas for more general cyclic loading a couple of struts along the two main diagonals would be necessary. The former assumption is adopted in this paper, as we will restrict our attention to monotonic loading conditions. There exist more complicated macro-modeling layouts that involve more than a single diagonal strut element to represent the masonry panel behavior and to properly account for the interaction between the strut and the shear response of the column (Crisafulli 1997). Moreover, concentric and eccentric struts have also been investigated (Al-Chaar 2002), and it has demonstrated that a series of off-diagonal strut elements are more appropriate to capture the local effects arising from the interaction between masonry panel and surrounding frame (Crisafulli 1997). Furthermore, linear elastic and nonlinear hysteretic constitutive models can be adopted to represent the stress-strain relationship of the equivalent strut, for example incorporating nonlinear fiber elements (Crisafulli *et al.* 2000).

All these complex models strive for describing the actual behavior of the masonry panel with increasing accuracy. Considering all the uncertainties involved in the correct modeling and in the determination of the most appropriate parameters that reflect the actual, experimentally observed, masonry panel response, a variety of empirical expressions have been proposed in the literature so far. Due to the stochastic nature of the masonry behavior and the large scatter of the corresponding experimental results, it is quite difficult to decide which mechanical characteristics to assume for the diagonal struts in a simplified model. Even within the simplest framework of modelling the masonry panel via a single linear-elastic diagonal strut element, there exist a variety of formulae proposed by different authors in the last few decades. The validity of these expressions is limited to the specific geometric and mechanical properties of the masonry panel on which these formulae were calibrated.

Table 1 Expressions for calculation of the w/d_w ratio considered in the proposed probabilistic study (after Tarque *et al.* 2015)

Authors (year)	w/d_w expression	Note
Holmes (1961)	$w/d_w = 1/3$	valid for $\lambda_h < 2$ (see Eq. (1))
Stafford Smith (1967)	$0.10 < w/d_w < 0.25$	the value graphically depends on λ_h
Mainstone (1971)	$w/d_w = 0.16\lambda_h^{-0.3}$	λ_h is computed through Eq. (1)
Mainstone (1974)	$w/d_w = 0.17\lambda_h^{-0.4}$	adopted by FEMA-274 and FEMA-306
Bazan and Meli (1980)	$w = (0.35 + 0.022\beta)h_w$	β computed via Eq. (2)
Hendry (1981)	$w = \frac{1}{2}\sqrt{z_b^2 + z_c^2}$	z_b and z_c computed via Eqs. (3)
Tassios (1984)	$w/d_w = 0.20\beta\sin\theta$	valid for $1 \le \beta \le 5$
Liauw and Kwan (1984)	$w/d_w = \frac{0.95\sin 2\theta}{2\sqrt{\lambda_h}}$	valid for $25° \le \theta \le 50°$
Decanini and Fantin (1987)	$w/d_w = 0.010 + \frac{0.707}{\lambda_h}$ $w/d_w = 0.040 + \frac{0.470}{\lambda_h}$	for $\lambda_h \le 7.85$ for $\lambda_h > 7.85$
Paulay and Priestley (1992)	$w/d_w = 1/4$	valid for $\lambda_h < 4$
Durrani and Luo (1994)	$w/d_w = \gamma\sin 2\theta$	$\gamma = 0.32\sqrt{\sin 2\theta}\left(\frac{H^4 E_w t_w}{m E_c I_c h_w}\right)^{-0.1}$ $m = 6\left(1 + \frac{6}{\pi}\frac{E_b I_b H}{E_c I_c L}\right)$
Flanagan and Bennet (1999)	$w = \frac{\pi}{C\lambda_h\cos\theta}$	C is an empirical value dependent on the in-plane drift displacement
Cavaleri *et al.* (2005) and Amato *et al.* (2008)	$w/d_w = \frac{k}{z}\frac{c}{\left(\lambda^*\right)^{\delta}}$	c and δ are functions of the Poisson's ratio, k is a function of the vertical load and z is a geometric parameter

The aim of this paper is to present a probabilistic approach for the determination of the in-plane response of masonry infilled RC framed structures. Attention is paid to the simplest modeling assumption of a single linear-elastic diagonal strut element, but the generality of the proposed probabilistic approach is not confined to such assumption and extension to multiple struts would be possible, although the generalization to nonlinear behavior seems to be not straightforward. The cross area of the strut is generally computed as the product of the panel thickness t_w and an equivalent width w. The latter parameter w is here assumed as a random variable to take into consideration the uncertain nature of the masonry panel. The development and dissemination of a large number of formulae for w makes it difficult to make a reliable choice of the diagonal strut properties. As an emblematic example, the stiffening contribution arising from a macro-modeling of the masonry panel is reported in Fig. 3 (in terms of the w/d_w ratio) for a variety of empirical expressions, thus highlighting the variability of different formulations proposed in the literature. Consequently, the probabilistic characterization of w, discussed in the next section, will be based upon an ensemble of empirical expressions. The expressions considered in this study are all listed in Table 1, where the following positions have been considered

$$\lambda_h = \sqrt[4]{\frac{E_w t_w \sin 2\theta}{4 E_c I_c h_w}}H \tag{1}$$

with E_w and E_c the Young's modulus of masonry and reinforced concrete, respectively, I_c the second moment of the cross-sectional area of the column,

$$\beta = \frac{E_c A_c}{G_w A_w} \tag{2}$$

where $A_c = b_c h_c$ is the column gross area and $A_w = t_w l_w$ is the area of the masonry panel in the horizontal plane, while G_w is the shear modulus of the masonry. The b value in Eq. (2) must satisfy the following constraints: $0.9 \le \beta \le 11$ and $0.75 \le l_w / h_w \le 2.5$. Furthermore, the relative stiffness of beam and column λ_b and λ_c, respectively, and the related contact lengths z_b and z_c of the Hendry expression (Hendry 1981) are defined as

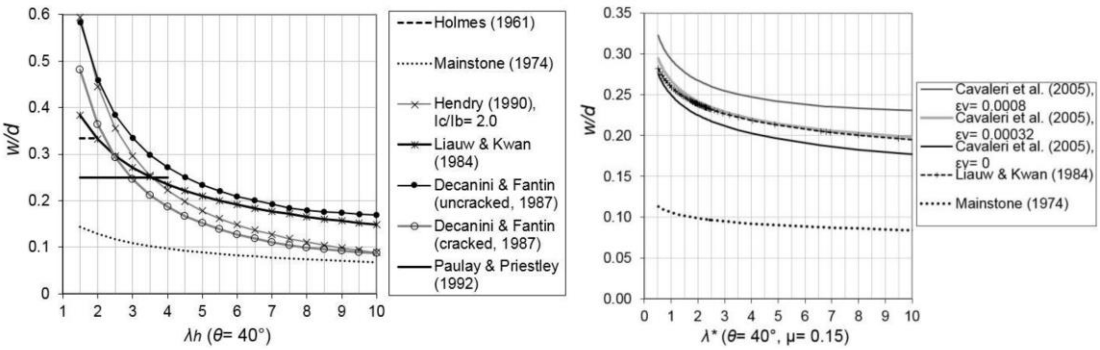


Fig. 3 Variability of the stiffening contribution (*w/d*) with respect to the geometrical and mechanical parameters of the masonry infills (λ_h and λ^*) – after Tarque *et al.* (2015)

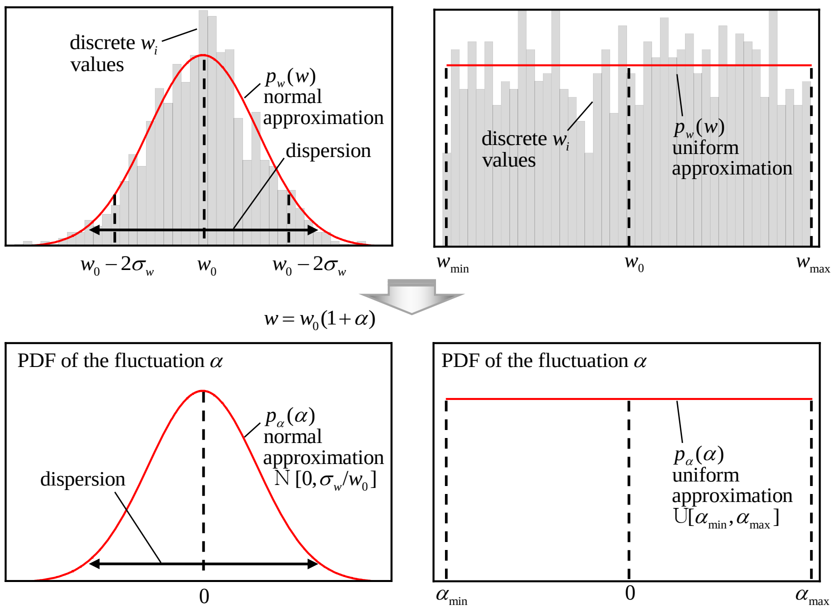


Fig. 4 PDF of the w and of the α variable in the normal (left) and uniform (right) approximation

$$\lambda_b = \sqrt[4]{\frac{E_w t_w \sin 2\theta}{4E_c I_b h_w}}; \quad \lambda_c = \sqrt[4]{\frac{E_w t_w \sin 2\theta}{4E_c I_c h_w}};$$
$$z_{b,c} = \frac{\pi}{2\lambda_b}; \quad z_c = \frac{\pi}{2\lambda_c} \tag{3}$$

with I_b denoting the second moment of the cross-sectional area of the beam. Finally, in the expression by Cavaleri *et al.* (2005) and Amato *et al.* (2008) the parameters are

$$c = 0.249 - 0.0116\,\nu_d + 0.567\,\nu_d^2;$$
$$\delta = 0.146 - 0.0073\,\nu_d + 0.126\,\nu_d^2$$
$$k = \frac{1}{0.75 + 0.25\dfrac{L}{H}}\left(1 + (18\lambda^* + 200)\,\varepsilon_v\right),\ \varepsilon_v = \frac{F_v}{2A_c E_c} \tag{4}$$
$$\lambda^* = \frac{E_d}{E_c}\frac{t_w (H - h_b/2)}{A_c}\left(\frac{(H - h_b/2)^2}{L^2} + \frac{A_c}{4A_b}\frac{1}{(H - h_b/2)}\right)$$

The expression conditions listed in the right column of Table 1 represent some upper bound and lower bound values of the analytical expressions. In case of a given threshold being exceeded, the limit value is assumed as an admissible range.

3. Probability characterization of masonry infills

The analysis performed in this paper is limited to static loading conditions and serviceability limit states, whereby the behavior of the masonry infills may be assumed as linear-elastic. In other words, no significant damage is expected to occur in the masonry infills. The uncertain mechanical behavior of the masonry infills and the related effects on RC framed structures has given rise to the dissemination of a large number of studies. In the framework of macro-modeling approaches, many empirical expressions have been proposed, as overviewed in the previous section. However, the validity of these empirical formulae is strictly related to the assumptions made for their development, and to the set of geometrical and mechanical properties considered for the validation of the corresponding models. As already said, it is quite difficult to decide which mechanical characteristics to assume for the diagonal struts in a simplified model. Therefore, in this research work we attempt to evaluate the effects of the masonry infills uncertainty on the structural response of RC frames. To this aim, the strut width is here considered as a random variable, whose stochastic properties stems from the above set of empirical expressions.

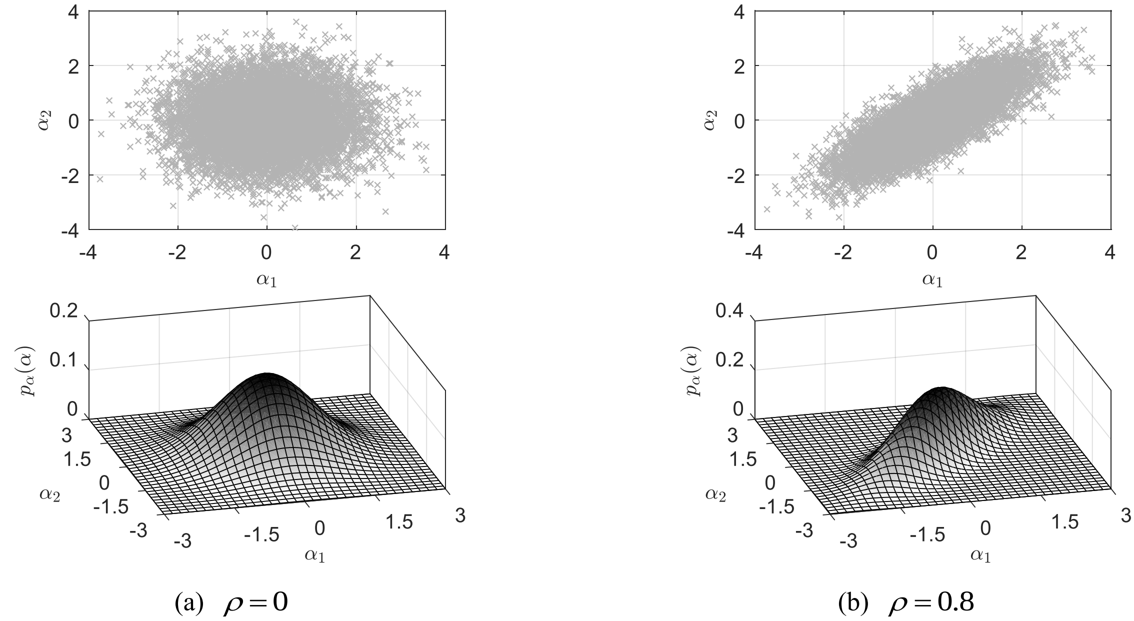


Fig. 5 Comparison of $\boldsymbol{\alpha}=[\alpha_1,\alpha_2]^T$ vector of fluctuations for uncorrelated ((a), $\rho=0$) and correlated assumptions ((b), $\rho=0.8$)

In particular, for given geometrical and mechanical properties of an assigned masonry infilled RC frame, all the parameters and coefficients entering the expressions reported in Table 1 are known. Therefore, a set of w_i values can be derived by applying the different formulae. At this stage, from this discrete set of values a probabilistic characterization of the strut width w can be extrapolated in the form of a PDF $p_w(w)$. The characteristics of such $p_w(w)$ are thus related to a set of empirical or macro-modeling-based approaches proposed in the literature by different authors. The mean value of the probability distribution $p_w(w)$ is denoted as w_0. It is meant that such w_0 value is related to the whole set of expressions, and takes into account the possible circumstance that different formulae may give rise to similar w_i values. This circumstance indicates a higher probability of occurrence of a specific interval within the present stochastic framework. Moreover, the dispersion of all the w_i values may be associated with the variance of the corresponding distribution $p_w(w)$.

In the spirit of perturbation approach of stochastic analysis, the strut width w can be modelled as a one-dimensional random variable with constant (deterministic) mean value w_0 and fluctuation α according to the expression

$$w = w_0(1+\alpha) \tag{5}$$

In so doing, instead of treating the strut width itself as a random variable, the basic random variables of this problem are represented by the zero-mean fluctuations α of the strut width with respect to its mean value w_0. In Fig. 4, two possible representations of the probabilistic characterization of the α variable are illustrated. From the discrete set of w_i values, arising from the group of expressions reported above, it is possible to extrapolate a best-fitting PDF representation (here presented in the form of either a normal PDF or a uniform PDF) from which the mean value w_0 and the dispersion characteristics can be identified. By application of relation (5) the probabilistic characterization of the zero-mean fluctuation α is straightforward, which is described in the bottom part of Fig. 4. From a probabilistic point of view, while the uniform distribution implies that all the w_i values have equal probability of occurrence between the minimum and maximum w_i values (denoted as $w_{\min}$ and $w_{\max}$, respectively), the normal distribution takes into account the concentration of the w_i values around the mean value w_0 (which is the most likely value, meaning that the majority of the above empirical expressions lead to values around such w_0).

Additionally, in the case of the normal distribution the statistical values $w_0 \equiv \mu_w$ and σ_w are directly estimated from the discrete set of the w_i values, whereas for the uniform distribution two steps arise: 1) estimation of the best-fitting uniform PDF $p_w(w)$ from the discrete set of the w_i values, 2) evaluation of the statistical moments, including mean and standard deviation $m_w \equiv w_0$ and s_w, respectively, and determination of the boundaries $\alpha_{\min}$ and $\alpha_{\max}$ of the zero-mean fluctuation PDF $p_\alpha(\alpha)$ as $\pm s_w/m_w$. This conversion between the $p_w(w)$ distribution and the $p_\alpha(\alpha)$ is necessary to obtain a zero-mean PDF for the fluctuations, which is consistent with Eq. (5). Furthermore, it is worth noting that in a real masonry infilled

RC frame there are more than just one equivalent diagonal strut element due to the presence of several masonry infills. From a probabilistic point of view, it is therefore necessary to introduce a zero-mean multivariate normal distribution defined by a covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{\alpha}}$ involving cross-correlation terms between the various struts $\boldsymbol{\alpha}=[\alpha_1,\ldots,\alpha_m]^T$. It is reasonably expected that two adjacent masonry infills are more correlated than two farther ones, which suggests to introduce a correlation function ρ dependent upon the distance between the centroid of each diagonal strut element d, i.e., $\rho=\rho(d)$. For the simplest scenario of just two masonry infills, in Fig. 5 the two-dimensional PDF of the $\boldsymbol{\alpha}=[\alpha_1,\alpha_2]^T$ vector of fluctuations is sketched in the two cases of uncorrelated and correlated fluctuations, in the latter case assuming a correlation factor $\rho=0.8$.

4. Probability-based modelling techniques

The topic dealt with in this paper is part of a broader and more general class of problems, widely investigated in the relevant literature, which are related to the structural analysis of systems with uncertain parameters (Schueller *et al.* 1987, Schueller *et al.* 2009, Mehrabi *et al.* 1994, Liu *et al.* 1987, De Domenico *et al.* 2018a, b). In particular, the uncertain variables can be of geometrical or mechanical nature. In the present paper, the stochastic modeling of the masonry equivalent strut width implicitly incorporates both the uncertainty in the mechanical parameters of the masonry and the uncertainty of the geometric definition of the equivalent truss member.

In the context of probability-based approaches, the simplest method is to perform simulations via the Monte Carlo method. This method has unique advantages when dealing with nonlinear systems (Roberts and Spanos 1990), and complicated input-output relationships. However, the computational effort related to the Monte Carlo method can be disproportionately large, requiring thousands of simulations to identify the statistics of the response with good accuracy. This may be an issue, especially for systems having several DOFs. Another class of approaches is the one based on the perturbation method, which may be effective if the level of uncertainty that is present in the system is reasonably low. Alternative methods are those relying on the projection approaches, e.g., the Karhunen-Loève expansion (Ghanem and Spanos 1991) or the Galerkin projection scheme and the Wiener integral representation, which are based on the projection of the solution on a complete stochastic basis (Ghanem and Kruger 1996).

In this paper, the structural response of masonry infilled RC framed structures is investigated via a handy probabilistic method of analysis that combines the "approximated deformation principal modes" (APDM) method (Falsone and Settineri 2014) with the "probabilistic transformation method" (PTM) (Falsone and Settineri 2013a, Falsone and Settineri 2013b). This combined method, first time applied to masonry infilled RC frames, leads to the determination of the PDF response directly, i.e., without requiring any sampling technique.

4.1 Approximated deformation principal modes (ADPM) method

The strut widths are modeled as random variables having mean values w_{0i} and fluctuations α_i, with $i=1,\ldots,m$ and m is the total number of the masonry panels. We suppose that the considered structure has been discretized according to usual finite element techniques and is subject to a set of applied forces (e.g., the dead loads, the variable actions, the seismic lateral loads or wind loads) that are collected in the $n\times 1$ (deterministic) vector $\mathbf{F}$, with n being the number of DOFs of the discretized system. The equilibrium equation of the structure reads

$$\mathbf{K}(\boldsymbol{\alpha})\mathbf{u}(\boldsymbol{\alpha})=\mathbf{F} \tag{6}$$

in which $\boldsymbol{\alpha}=[\alpha_1,\alpha_2,\ldots,\alpha_m]^T$. Probabilistic characterization of this vector, in the form of the joint PDF $p_{\boldsymbol{\alpha}}(\boldsymbol{\alpha})$, is assumed to be known. In Eq. (6), $\mathbf{K}(\boldsymbol{\alpha})$ is the $n\times n$ structural stiffness matrix that implicitly depends on the vector of uncertain strut width $\boldsymbol{\alpha}$, and $\mathbf{u}(\boldsymbol{\alpha})$ is the vector of the structural displacements that is obviously affected by the uncertain parameters, besides of the force vector $\mathbf{F}$. In the spirit of a first-order perturbation approach, the stiffness matrix can be expressed as

$$\mathbf{K}(\boldsymbol{\alpha})= \mathbf{K}_0+\sum_{i=1}^{m}\mathbf{K}_i\alpha_i;\quad \mathbf{K}_i=\left.\frac{\partial\mathbf{K}(\boldsymbol{\alpha})}{\partial\alpha_i}\right|_{\boldsymbol{\alpha}=\mathbf{0}} \tag{7}$$

in which $\mathbf{K}_0$ is the stiffness matrix obtained by setting $\alpha_i=0$, and $\mathbf{K}_i$ are deterministic matrices corresponding to a single contribution of the $\boldsymbol{\alpha}$ vector. The APDM approximate the vector response $\mathbf{u}(\boldsymbol{\alpha})$ of the system governed by Eq. (6), in the following form

$$\mathbf{u}(\boldsymbol{\alpha})\approx\mathbf{u}_0+\Delta\mathbf{u}(\boldsymbol{\alpha});\qquad \Delta\mathbf{u}(\boldsymbol{\alpha})=\sum_{i}^{m}\mathbf{u}_i(\alpha_i) \tag{8}$$

where $\mathbf{u}_0$ is the deterministic response of the system without uncertainties, i.e., obtained by setting $\alpha_i=0$ with $i=1,2,\ldots,m$, while $\Delta\mathbf{u}(\boldsymbol{\alpha})$ is the vector of fluctuations, computed as the sum of $\mathbf{u}_i(\alpha_i)$ representing the stochastic response displacements in which only $\alpha_i\neq 0$ and $\alpha_j=0$ with $j\neq i$ and $j=1,2,\ldots,m$. Therefore, $\mathbf{u}_i(\alpha_i)$ is the vector of the displacements for a scenario in which only α_i affects the probabilistic response of the system. As such, this vector can be computed as follows

$$\mathbf{u}_i(\alpha_i)=(\mathbf{K}_0+\alpha_i\mathbf{K}_i)^{-1}(-\alpha_i\mathbf{K}_i\mathbf{u}_0) \tag{9}$$

A computational strategy for finding an explicit relationship between α_i and $\mathbf{u}_i$ was proposed in (Falsone and Impollonia 2002), in which after computing the eigenproperties of the $\mathbf{K}_0^{-1}\mathbf{K}_i$ matrix, the following relationship of the $\mathbf{u}_i(\alpha_i)$ vector was obtained

$$\mathbf{u}_i(\alpha_i) = -\alpha_i \boldsymbol{\Phi}_i [\mathbf{I}_n + \alpha_i \boldsymbol{\Lambda}_i]^{-1} \boldsymbol{\Lambda}_i \boldsymbol{\Phi}_i^T \mathbf{F} \tag{10}$$

where $\boldsymbol{\Lambda}_i$ and $\boldsymbol{\Phi}_i$ are the eigenvalue and eigenvector matrices of the matrix $\mathbf{K}_0^{-1}\mathbf{K}_i$, while $\mathbf{I}_n$ is the identity matrix of order n. In this way, the matrix entering the square brackets is diagonal, which simplifies the calculation of its inverse, as will be seen next. Moreover, the number of non-zero eigenvalues of $\mathbf{K}_0^{-1}\mathbf{K}_i$, i.e., the non-zero elements of $\boldsymbol{\Lambda}_i$, is equal to the number of the structural principal modes directly affected by α_i. We here emphasize that the number of non-zero eigenvalues q is far lower than the number of DOFs of the system n, making the evaluation of $\mathbf{u}_i(\alpha_i)$ computationally simpler.

According to the macro-modelling assumptions outlined in Section 2, the single uncertain masonry infill is treated as an equivalent diagonal pin-jointed strut within a more complex RC framed structure. For simplicity, this diagonal strut is discretized as a single truss element for each masonry infill. Therefore, the single random variable α_i, representing the uncertain strut width fluctuation, influences only such single finite element. It is well-known that for a truss finite element the number of principal deformation modes $q=1$. As a result, the evaluation of $\mathbf{u}_i(\alpha_i)$ in Eq. (10) can be readily performed according to

$$u_{i_j}(\alpha_i) = -\Phi_{i_{jk}} \frac{\alpha_i \lambda_i q_{0,i_k}}{1+\alpha_i \lambda_i} = \frac{a_{i_j}\alpha_i}{1+b_{i_j}\alpha_i} \tag{11}$$

$u_{i_j}(\alpha_i)$ being the j^{th} element of $\mathbf{u}_i(\alpha_i)$, λ_i is the unique non-zero term of the eigenvalue matrix $\boldsymbol{\Lambda}_i$, $\Phi_{i_{jk}}$ the $(j,k)^{\text{th}}$ element of the eigenvector matrix $\boldsymbol{\Phi}_i$, and $\mathbf{q}_{0,i} = \boldsymbol{\Phi}_i^{-1}\mathbf{u}_0$. The quantities a_{i_j} and b_{i_j} appearing in the last term of Eq. (11) can be obtained once that the eigenvalue problem of $\mathbf{K}_0^{-1}\mathbf{K}_i$ is solved. Alternatively, a more direct evaluation of a_{i_j} and b_{i_j} can be obtained by solving twice Eq. (9) in which two arbitrary and different deterministic values of α_i are assumed. The j^{th} component of the $\mathbf{u}(\boldsymbol{\alpha})$ vector in (8) is given by

$$u_j(\boldsymbol{\alpha}) = u_{0_j} + \Delta u_j(\boldsymbol{\alpha}) = u_{0_j} + \sum_{i=1}^{m} u_{i_j}(\alpha_i) = u_{0_j} + \mathbf{1}^T \mathbf{u}_j(\boldsymbol{\alpha}) \tag{12}$$

where $\mathbf{1}^T$ is a $1\times m$ vector having all components equal to one and $\mathbf{u}_j(\boldsymbol{\alpha})$ is a $m\times 1$ vector comprising the $u_{i_j}(\alpha_i)$ terms whose summation is denoted by $\Delta u_{i_j}(\boldsymbol{\alpha})$. The evaluation of the stochastic response vector is achieved. This has several advantages in terms of computational effort as compared to the Eq. (6), since the stochastic response is computed without requiring the inversion of $\mathbf{K}(\boldsymbol{\alpha})$ explicitly. However, the evaluation of $\mathbf{u}(\boldsymbol{\alpha})$ still implies Monte Carlo simulations, as samplings of α_i from the assumed $p_{\boldsymbol{\alpha}}(\boldsymbol{\alpha})$ are necessary to calculate the $u_{i_j}(\alpha_i)$ terms in Eq. (11). Therefore, the PDF of the response cannot be obtained directly, but is approximated as the number of sampling increases, which is a shortcoming of any Monte Carlo-based technique. In the next section an alternative method, called PTM, will be presented. We prove that this method, when combined with the APDM method presented above, leads to the determination of the PDF of the system response directly, without implying any sampling operations.

4.2 Probabilistic transformation method (PTM)

The PTM is based on the probabilistic approach of the space transformation laws of random vectors. Let us consider the m-dimensional random vector $\boldsymbol{\alpha}$ (input uncertain variables, defined as the strut width fluctuations in this case), whose joint PDF $p_{\boldsymbol{\alpha}}(\boldsymbol{\alpha})$ is known, and let $\mathbf{h}(\cdot)$ be a m-dimensional invertible application with $\mathbf{h}^{-1}(\cdot) = \mathbf{f}(\cdot)$, such that one can write

$$\mathbf{u} = \mathbf{h}(\boldsymbol{\alpha}); \qquad \boldsymbol{\alpha} = \mathbf{f}(\mathbf{u}) \tag{13}$$

It is well known that once the direct and inverse relationships in (13) are defined, the joint PDF of the random vector $\mathbf{u}$ (output variables, representing the unknown system displacements), that is $p_{\mathbf{u}}(\mathbf{u})$, can easily be obtained through the following expression (Papoulis and Pillai 2002)

$$p_{\mathbf{u}}(\mathbf{u}) = \left|\det[\mathbf{J}_{\mathbf{f}}(\mathbf{u})]\right| p_{\boldsymbol{\alpha}}(\mathbf{f}(\mathbf{u})) \tag{14}$$

where $\mathbf{J}_{\mathbf{f}}(\mathbf{u})$ is the Jacobian matrix associated to the transformation law given in Eq. $(13)_2$

$$\mathbf{J}_{\mathbf{f}}(\mathbf{u}) = \nabla_{\mathbf{u}}^T \otimes \mathbf{f}(\mathbf{u}) \tag{15}$$

$\nabla_{\mathbf{u}}^T$ being the m^{th} order row-vector operator collecting all the partial derivatives with respect to the components u_i of $\mathbf{u}$ and the symbol $\otimes$ indicating the Kronecker tensor product (Graham 1981). Expression (14) gives a direct deterministic relationship between the joint PDFs of the system response $\mathbf{u}$ and that of the uncertain parameters $\boldsymbol{\alpha}$ that are functions of multidimensional variables. In other words, the PDF of the output variables $p_{\mathbf{u}}(\mathbf{u})$ can be computed once the PDF of the input variables $p_{\boldsymbol{\alpha}}(\boldsymbol{\alpha})$ is known and the transformation law is defined.

Let u_j be the single component of the output random vector $\mathbf{u}$ defined by scalar transformation $u_j = h_j(\boldsymbol{\alpha})$. In (Falsone and Settineri 2013a, b) it was shown that the PDF of u_j, by using the properties of the Dirac delta function, can be reduced as follows

$$p_{u_j}(u_j) = \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} p_{\boldsymbol{\alpha}}(\mathbf{y})\, \delta(u_j - h_j(\mathbf{y}))\, \mathrm{d}y_1 \cdots \mathrm{d}y_m \tag{16}$$

In an analogous way, the joint PDF of two components of the output random vector $\mathbf{u}$ can be obtained as

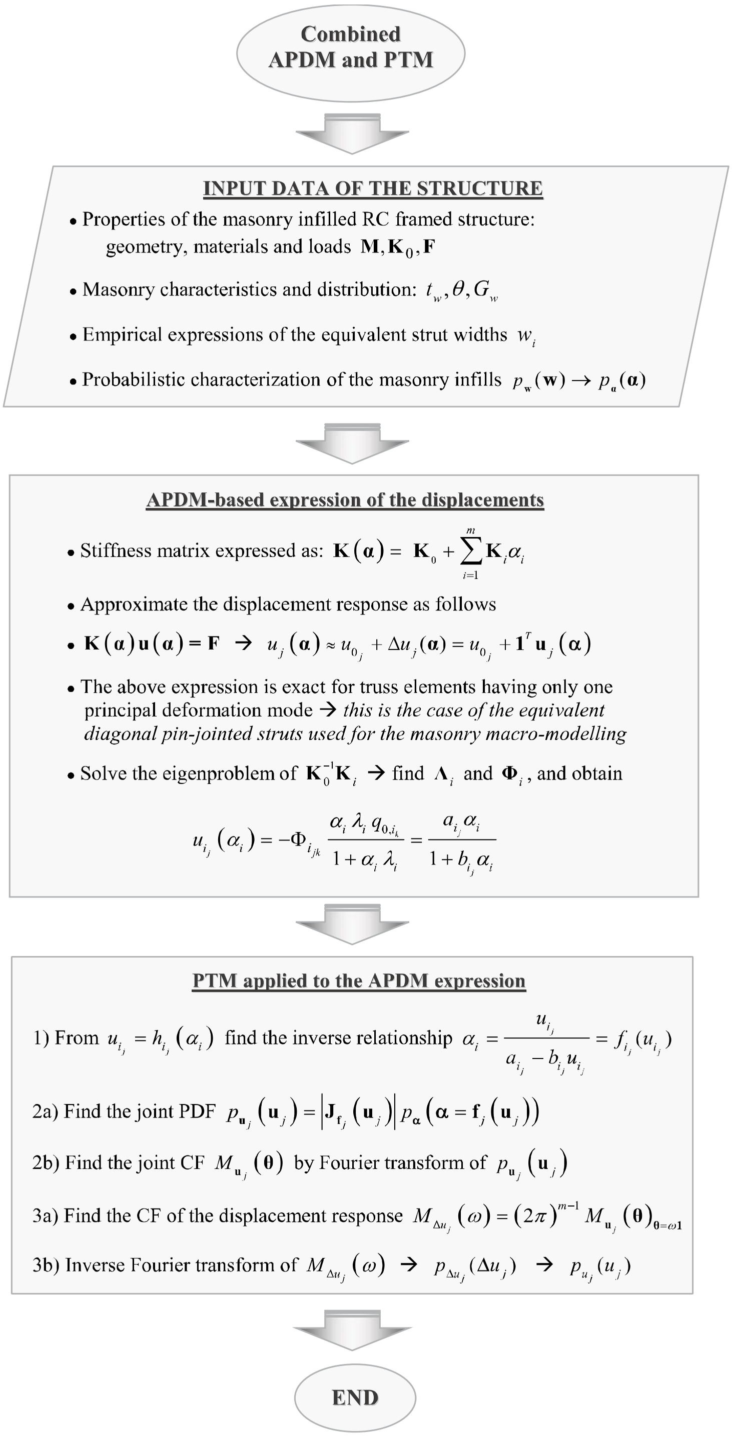


Fig. 6 Flow chart of the proposed probabilistic procedure

$$p_{u_j u_k}(u_j, u_k) = \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} p_{\boldsymbol{\alpha}}(\mathbf{y})\, \delta(u_j - h_j(\mathbf{y}))\, \delta(u_k - h_k(\mathbf{y}))\, \mathrm{d}\, y_1 \cdots \mathrm{d}\, y_m. \tag{17}$$

When $h_j(\boldsymbol{\alpha})$ is a linear combination of the components of $\boldsymbol{\alpha}$, that is, $h_j(\boldsymbol{\alpha}) = \mathbf{h}_j^T \boldsymbol{\alpha}$, a more direct calculation of the marginal PDF of the system response can be obtained by using the characteristic function (CF).

Although this is not the case of the $h_j(\boldsymbol{\alpha})$ function defined in Eqs. (11) and (12), it will be later demonstrated that there exists a mathematical manipulation through which this relation holds. By applying the Fourier transform to both sides of Eq. (16) we obtain

$$M_{u_j}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} p_{u_j}(u_j) \exp(-\omega u_j)\, du_j = \frac{1}{2\pi} \times \int_{-\infty}^{\infty} \left[\int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} p_{\boldsymbol{\alpha}}(\mathbf{y})\, \delta(u_j - \mathbf{h}_j^T \mathbf{y})\, \mathrm{d}y_1 \cdots \mathrm{d}y_m \right] \mathrm{e}^{-\omega u_j}\, du_j. \tag{18}$$

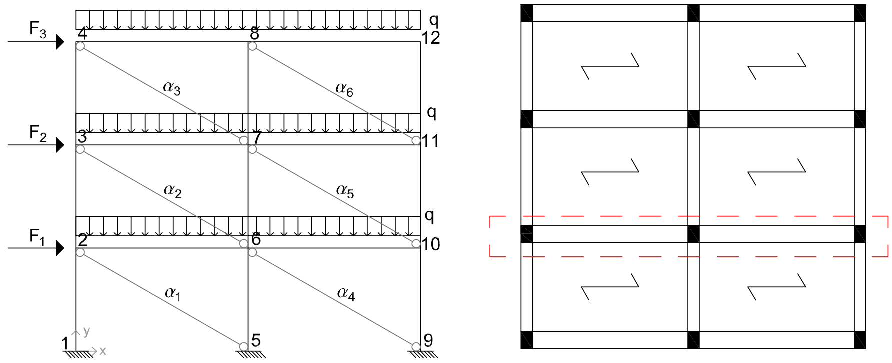


Fig. 7 Sketch of the RC frame analyzed in the numerical example

Table 2 Distributed load per unit area applied to the one-way floor slab for every level of the building

Level	$G_1\,[\mathrm{kN/m^2}] + G_2\,[\mathrm{kN/m^2}]$	$Q\,[\mathrm{kN/m^2}]$	ψ_2 [a]
Floor 1,2,3	8.0	2.0	0.3

[a] combination coefficient for the quasi-permanent value of variable action [16]

Once again, by exploiting the properties of the Dirac delta function, Eq. (18) becomes

$$M_{u_j}(\omega) = (2\pi)^{m-1} M_{\boldsymbol{\alpha}}(\boldsymbol{\theta})_{\boldsymbol{\theta}=\omega\mathbf{h}_j} \tag{19}$$

that directly relates the characteristic functions of $\boldsymbol{\alpha}$ and of u_j, thus avoiding any type of integration. If the joint probability of the two variables u_j and u_k is required, exploiting similar procedures, it is easy to show that the following relationship holds

$$M_{u_j u_k}(\omega_j, \omega_k) = (2\pi)^{m-2} M_{\boldsymbol{\alpha}}(\boldsymbol{\theta})_{\boldsymbol{\theta}=\omega_j\mathbf{h}_j+\omega_k\mathbf{h}_k}. \tag{20}$$

From the CF of the system response, the PDF of the displacements can be easily obtained by applying the inverse Fourier transform to Eqs. (19) and (20).

4.3 Matching the APDM method and the PTM

In this subsection, we demonstrate that it is possible to couple the APDM method and the PTM in order to determine the direct probabilistic characterization of $\Delta u_j(\boldsymbol{\alpha})$, introduced into Eq. (12) that, in turn, leads to the PDF, $p_{u_j}(u_j)$, of the j^{th} component of the displacement response. Starting from the j^{th} element of $\mathbf{u}_i(\alpha_i)$, that is $u_{i_j} = h_{i_j}(\alpha_i)$ whose expression is given into Eq. (11), if the application of the PTM is considered, then the evaluation of the inverse relationship $\alpha_i = h_{i_j}^{-1}(u_{i_j}) = f_{i_j}(u_{i_j})$ is necessary. It can be easily seen that it has the following expression

$$\alpha_i = \frac{u_{i_j}}{a_{i_j} - b_{i_j} u_{i_j}} = f_{i_j}(u_{i_j}). \tag{21}$$

The application of the PTM to the vector $\mathbf{u}_j$ leads to the following joint PDF

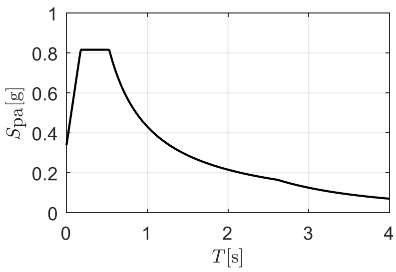


Fig. 8 Elastic response spectrum of the installation site of the building

$$p_{\mathbf{u}_j}(\mathbf{u}_j) = \left|\mathbf{J}_{\mathbf{f}_j}(\mathbf{u}_j)\right| p_{\boldsymbol{\alpha}}\left(\mathbf{f}_j(\mathbf{u}_j)\right). \tag{22}$$

Since the inverse functions $f_{i_j}(\cdot)$ in Eq. (21) depend on the response parameter u_{i_j} only, the Jacobian $\mathbf{J}_{\mathbf{f}_j}(\cdot)$ is a diagonal matrix and its determinant can be simply evaluated as

$$\left|\mathbf{J}_{\mathbf{f}}(\mathbf{u}_j)\right| = \prod_{i=1}^{m} \left| \frac{df_{i_j}(u_{i_j})}{du_{i_j}} \right| \tag{23}$$

The Eq. (22) allows the evaluation of the joint PDF of the vector $\mathbf{u}_j$. Nevertheless, if one wants to characterize the response component $u_j(\boldsymbol{\alpha})$, and, as consequence, the random part $\Delta u_j(\boldsymbol{\alpha})$, then, from Eq. (12), the PTM must be applied to the following relationship

$$\Delta u_j(\boldsymbol{\alpha}) = \mathbf{1}^T \mathbf{u}_j. \tag{24}$$

Considering the previous remarks about the advantageous application of the CF when a linear input-output relation is involved, it is convenient to exploit this strategy and compute the CF of $\Delta u_j(\boldsymbol{\alpha})$ as follows

$$M_{\Delta u_j}(\omega)=(2\pi)^{m-1}M_{\mathbf{u}_j}(\boldsymbol{\theta})_{\boldsymbol{\theta}=\omega\mathbf{1}} \tag{25}$$

that is nothing but Eq. (19) with the vector $\mathbf{1}$ playing the role of the $\mathbf{h}$ function in the input-output relationship. Once $M_{\Delta u_j}(\omega)$ is calculated, the inverse Fourier transform is applied to determine $p_{\Delta u_j}(\Delta u_j)$ and, thus, the probabilistic characterization of $p_{u_j}(u_j)$ is definitely achieved. Extension of this procedure to the determination of the joint PDF of two response components $p_{u_j u_k}(u_j,u_k)$ is not difficult and the details can be found in (Falsone and Laudani 2018).

In order to summarize and clarify the main steps of the proposed algorithm, a schematic flow-chart has been constructed and reported in Fig. 6. In this flow-chart it is clearly shown how the two probabilistic methods (APDM and PTM) are linked together to derive the PDF of the system response directly, resorting to the use of the CF as explained above.

5. Numerical examples

The proposed probabilistic procedure is here applied to compute the PDF of the response of masonry infilled RC frames in which the equivalent diagonal pin-jointed struts are assumed uncertain. For the sake of simplicity, reference is made to a planar frame of a regular RC structure. The elevation and plan views of the RC structure is shown in Fig. 7. The following data are assumed as known (deterministically) input parameters: bay width equal to $6.0\,\mathrm{m}$, inter-story height $3.2\,\mathrm{m}$, column sections $40\times 60\,\mathrm{cm}$, beam sections $30\times 50\,\mathrm{cm}$, concrete having Young's modulus $E_c=30\,\mathrm{GPa}$, which is typical of ordinary concrete structural elements (Pisano *et al.* 2013a, 2013b, 2014, 2015, De Domenico *et al.* 2014a, 2014b, De Domenico 2015), masonry with mean Young's modulus $E_w=5\,\mathrm{GPa}$, mean Poisson's ratio in the diagonal direction $\nu_d=0.25$, and thickness $t_w=40\,\mathrm{cm}$. The loads acting on the planar frame are calculated based on the loads per unit area reported in Table 2.

The seismic analysis is performed by means of an equivalent static lateral force procedure, with a distribution of horizontal forces detected by the response spectrum of the installation site, whose shape is reported in Fig. 8. The installation site is placed in Messina, Italy, and the peak ground acceleration (PGA) is $\ddot{u}_{g0}=0.254\,g$ with g denoting the gravity acceleration. The first (fundamental) period of vibration is calculated according to the simplified formula for reinforced concrete building (D.M. LL. PP. 2008)

$$T_1=C\,H^{3/4}=0.075(9.6)^{3/4}\cong 0.41\mathrm{s} \tag{26}$$

The lateral (equivalent) seismic forces acting along the building height are distributed according to the fundamental mode of vibration of the building (D.M. LL. PP. 2008), and are scaled to the spectral acceleration of the elastic response

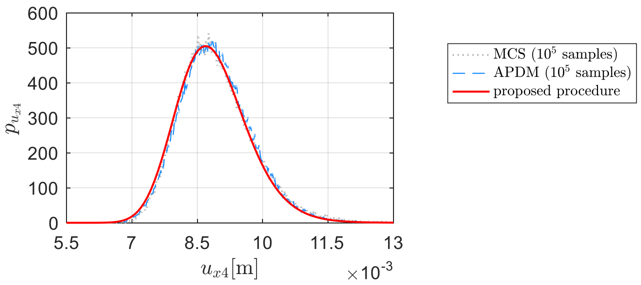


Fig. 9 PDF of the last-floor displacement (node 4) for normal distribution assumption and uncorrelated fluctuations: Comparison between the three probabilistic methods

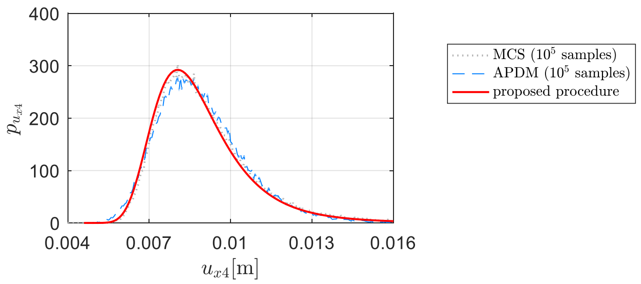


Fig. 10 PDF of the last-floor displacement (node 4) for normal distribution assumption and correlated fluctuations between the struts: Comparison between the three probabilistic methods

spectrum at the first mode of vibration $S_{pa}(T_1)$

$$F_i=F_h\cdot\frac{z_i\cdot W_i}{\sum_j z_j\cdot W_j} \tag{27}$$

with $F_h=S_{pa}(T_1)\cdot\lambda\cdot W_{tot}/\mathrm{g}$, $\lambda=0.85$ (D.M. LL. PP. 2008), and $W_{tot}=\sum_j W_j$ is the total weight of the building.

Adopting the elastic- (rather than the design-) response spectrum for computing the seismic force distribution is consistent with the assumption of a linear-elastic behavior of the masonry infilled RC frame as a whole, i.e., only the elastic response is investigated in this paper. Investigating the post-elastic behavior of the structure would imply a modification of the proposed procedure to incorporate a nonlinear constitutive behavior of the diagonal struts (Crisafulli *et al.* 2000) and, consequently, would justify the adoption of a behavior factor greater than one to describe the energy dissipation mechanisms occurring in the structure. The main aim of this paper is the probabilistic characterization of the in-plane elastic response of the masonry infilled RC frame, while the analysis of this post-elastic behavior is left for future research.

The results presented in this Section aim to highlight the influence of the uncertain characteristics of the masonry infills on a few response indicators of the RC frame. To this aim, the PDFs of several response indicators have been computed through the combined APDM and PTM methods of analysis described in Section 4.3. To validate the proposed probabilistic procedure, comparison against Monte Carlo simulation results (obtained with thousands of samples and,

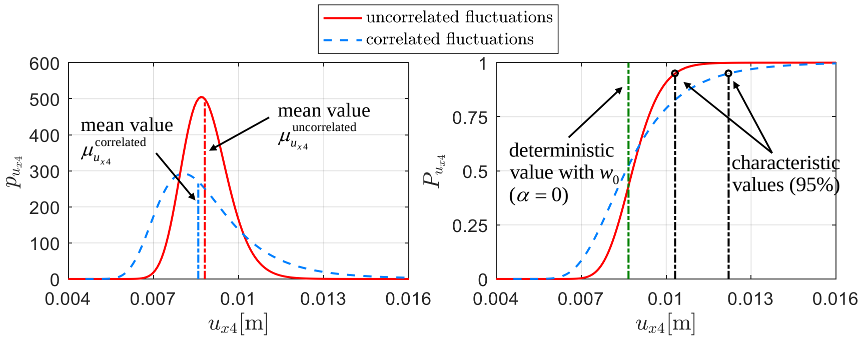


Fig. 11 PDF (left) and CDF (right) of the last-floor displacement (node 4) for normal distribution in the two hypotheses of uncorrelated and correlated fluctuations $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_6]^T$

consequently, requiring much higher computational effort) is made for just a few selected quantities. Moreover, to demonstrate the consistency of the results of the combined APDM+PTM with the APDM method proposed by Falsone and Impollonia (Falsone and Impollonia 2002), described above in Section 4.1, in addition to the Monte Carlo technique we also compare the results with the procedure presented in (Falsone and Impollonia 2002). In this way, the improvements and computational savings achieved by the proposed probabilistic procedure can be assessed in comparison with two alternative probabilistic techniques.

Considering the geometric and mechanical properties of the analyzed masonry infilled RC frame, application of the empirical expressions reported in Table 1 along with the assumptions outlined in Section 3 provides the probabilistic characterization of the equivalent strut widths in terms of a physically-based (in the spirit of a macro-modelling approach) joint PDF $p_{\boldsymbol{\alpha}}(\boldsymbol{\alpha})$ of the fluctuations $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_6]^T$. The adjective "physically-based" refers to the fact that the input PDF $p_{\boldsymbol{\alpha}}(\boldsymbol{\alpha})$ derives from some empirical (macro-modelling) expressions proposed in the literature. Therefore, the expressions reported in Table 1 form the basis of the probabilistic characterization of the fluctuations in this probabilistic study.

Subsequently, the procedures described in Section 4 are applied to obtain a probabilistic characterization of the system response in terms of a variety of response indicators. As an example, in Fig. 9 the PDF of the last-floor displacement (corresponding to node 4, i.e., u_{x4}) is shown. In this first case, we assumed that the $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_6]^T$ fluctuation variables are probabilistically described by a zero-mean multivariate normal distribution that best fits the empirical values. Moreover, the $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_6]^T$ fluctuations are also assumed as uncorrelated random variables in this first example. By inspection of Fig. 9 it is noted that the proposed probabilistic procedure is able to provide the PDF of the displacement response, which is only approximately described by the other two techniques (MCS and APDM) depending on the number of samples utilized. The first consideration is about the mean value of the top-story displacement, that is, $\mu_{u_{x4}} = 8.87 \times 10^{-3}\,\text{m}$. The displacement response spectrum corresponding to the first natural period of the frame without diagonal pin-jointed struts ($T_1 = 0.41\,\text{s}$) is

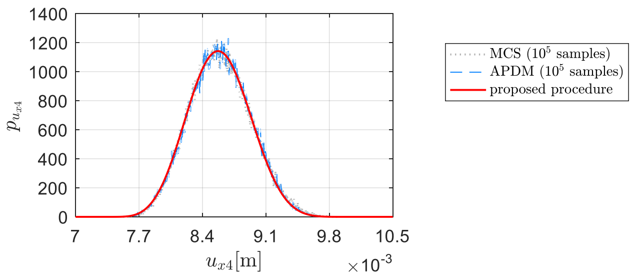


Fig. 12 PDF of the last-floor displacement (node 4) for uniform distribution assumption of the fluctuations: Comparison between the three probabilistic methods

$u_{x4,\text{spectrum}} = 3.24 \times 10^{-2}\,\text{m}$.

Obviously, the introduction of the struts leads to a significantly reduce, of almost four times, the displacement due to a stiffening contribution that reduces the first natural period accordingly. Furthermore, the deterministic value of the displacement computed adopting the largest stiffness of the diagonal struts among the empirical expressions reported in Table 1 is $u_{x4,\min} = 6.80 \times 10^{-3}\,\text{m}$, while that corresponding to the minimum strut width (related to the minimum stiffness) is $u_{x4,\max} = 1.21 \times 10^{-2}\,\text{m}$. These two values represent more or less the boundaries of the PDF reported in Fig. 8.

Next, in order to take into account the correlation that may exist between the mechanical properties of masonry infills that are close to each other, a correlation function has been introduced. According to Section 3, this correlation function depends upon the distance between the centroid of each diagonal strut element, i.e., $\rho = \rho(d)$, in particular the following exponential decaying function has been assumed

$$\rho = \exp\left(-\frac{d}{\lambda}\right) \tag{28}$$

where $d = \left\| \mathbf{x}_i - \mathbf{x}_j \right\|$ is the Euclidean distance between the centroid of the strut i and j, while $\lambda = 15\,\text{m}$ is an arbitrary correlation length that is here chosen such that two adjacent masonry infills have a correlation equal to 0.8. In Fig. 10 the PDF of the top-story displacement (node 4) is illustrated for normal distribution assumption of the fluctuations, but assuming the correlation function given in Eq. (28). It is noted that the introducing cross-correlation terms within the covariance matrix

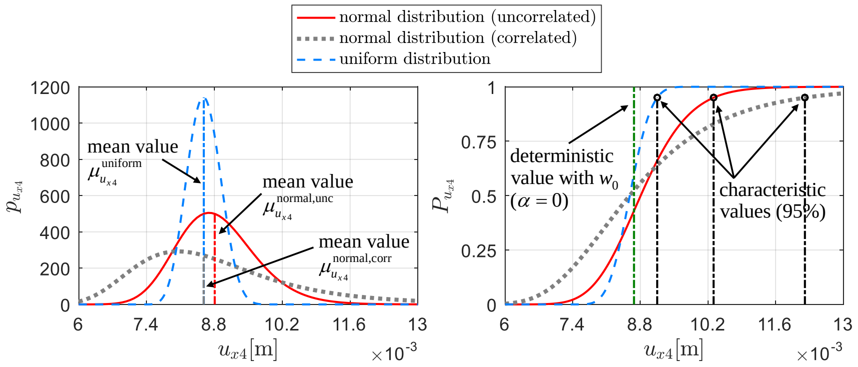


Fig. 13 PDF (left) and CDF (right) of the last-floor displacement (node 4) for normal (uncorrelated and correlated) and uniform distribution assumption

of the fluctuation $\mathbf{\Sigma}_{\boldsymbol{\alpha}}$ requires a larger number of samples to approximate the PDF given by the proposed probabilistic procedure. Indeed, for the given number of samples adopted also for the previous case (10^5 samples), little deviations are observed by comparing the PDF with the APDM-based approximated one. The PDF and cumulative distribution function (CDF) of the last-floor displacement in the two cases of uncorrelated and correlated fluctuations $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_6]^T$ are depicted in Fig. 11. It is noted that the two assumptions not only lead to a slightly different mean value μ_{ux4} (which is lower in the correlated case), but also produce remarkable differences in terms of characteristic values (95th percentile of the distribution). In particular, such characteristic value is about 20% higher in the correlated case due to the different shape of the distribution. Also, in the more realistic scenario of correlated fluctuations the dispersion of the displacement values is found to be higher than in the case of uncorrelated fluctuations. Furthermore, it is also noticed that the deterministic value of the last-floor displacement computed assuming that no fluctuations are present in the masonry panels (i.e., $w = w_0$ and $a = 0$ for all the equivalent diagonal struts) is in between the two mean values in the uncorrelated and correlated cases.

For completeness, the PDF of the last-floor displacement (node 4) has also been computed via the uniform distribution assumption for the fluctuations, which implies that all the empirical macro-modelling expressions have an equal probability of occurrence. By looking at Fig. 12, similar trends to those already observed in Fig. 9 for the normal distribution assumption are obtained, and similar conclusions can be drawn. In Fig. 13, the three distributions are compared with each other. It is noted that both the mean value and the characteristic value (95th percentile of the distribution) of the last-floor displacement are lower in the case of uniform distribution as compared to the two normal distributions. In the case of the uniform distribution the dispersion around the mean value is reduced, whereas in the case of normal distribution with correlated fluctuations the dispersion is amplified. Moreover, the deterministic value of u_{x4} in the absence of fluctuations is more or less comprised between the three mean values of the three above mentioned distributions.

Once the displacement vector $\mathbf{u}$ has been characterized probabilistically through the knowledge of the relevant PDF $p_{\mathbf{u}}(\mathbf{u})$, any other response indicator of interest can easily be

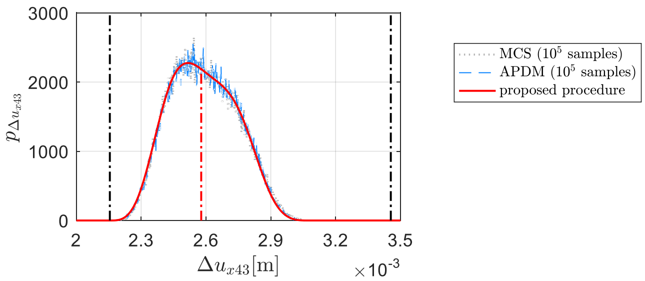


Fig. 14 PDF of the third interstory drift for uniform distribution assumption of the fluctuations: Comparison between the three probabilistic methods

computed as a linear combination of the components of the $\mathbf{u}$ vector, according to the finite element method. As an example, in Fig. 14 the PDF of the third inter-story drift $\Delta u_{x43} = u_{x4} - u_{x3}$ is displayed as computed by the three probabilistic procedures considering that the fluctuations are uniformly distributed between the $\alpha_{\min}$ and $\alpha_{\max}$ values.

In the same graph, we also report the deterministic inter-story drift values calculated by assuming the minimum and maximum values of the w_i discrete set. As expected, since $\alpha_{\min}$ and $\alpha_{\max}$ do not reflect the values of $w_{\min}$ and $w_{\max}$, the deterministic max value does not represent the 100th percentile of the PDF $p_{\Delta u_{x43}}$, which is consistent with what explained in Section 3. The PDF has a slightly asymmetric shape and all the three probabilistic procedure are in good agreement with each other. In Fig. 15, the PDF and CDF of the third inter-story drift $\Delta u_{x43} = u_{x4} - u_{x3}$ is illustrated and compared for the case of normal distribution (uncorrelated and correlated) and uniform distribution of the fluctuations. The results are more or less in line with the previous trends observed for the last-floor displacement u_{x4}: the characteristic value of the distribution is lower in the case of uniform distribution than the normal distributions. However, the mean value from the uniform distribution is in between that of the normal distribution for uncorrelated and correlated assumptions, which is different from the results discussed above for u_{x4}, although the three mean values are very close to each other. The dispersion of the $p_{\Delta u_{x43}}$ is reduced for uniform distribution as compared to normal distribution, whereas it is amplified in the case of normal distribution with correlated fluctuations. The deterministic value

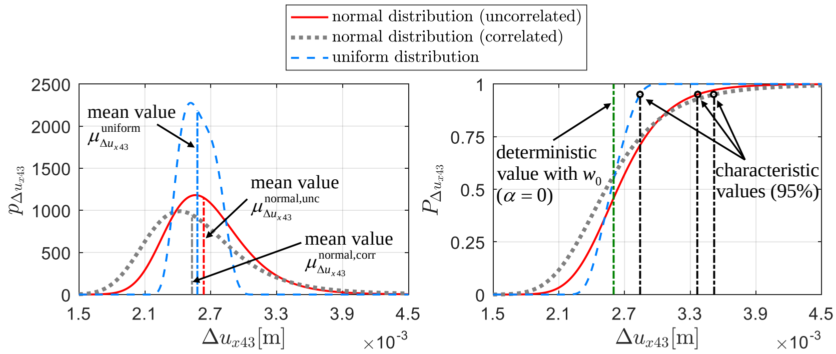


Fig. 15 PDF (left) and CDF (right) of the third interstory drift ($\Delta u_{x43} = u_{x4} - u_{x3}$) for normal (uncorrelated and correlated) and uniform distribution assumption

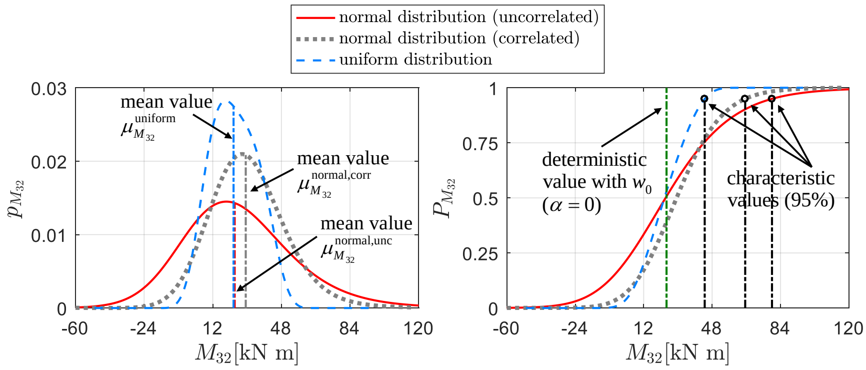


Fig. 16 PDF (left) and CDF (right) of the moment at the top of the column 3-2 (M_{32}) for normal (uncorrelated and correlated) and uniform distribution assumption

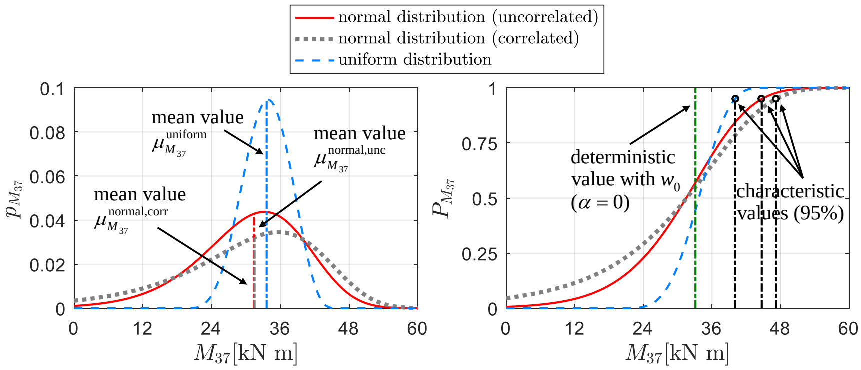


Fig. 17 PDF (left) and CDF (right) of the moment at the top of the column 3-7 (M_{37}) for normal (uncorrelated and correlated) and uniform distribution assumption

obtained with w_0 (in the absence of fluctuations, $\alpha = 0$) lies in between the two mean values calculated with the uniform and normal distributions assumptions of the fluctuations.

It has been observed that the influence of the stiffening contributions offered by the equivalent pin-jointed diagonal struts is more significant for determining the stress and strain in the column elements rather than in the beam elements. This is reasonable, since the masonry infills increase the lateral stiffness of the frame as compared to the case in which they are ignored in the calculation.

To quantify this effect, in Fig. 16, the PDF and CDF of the moment at the top of the column 3-2 (M_{32}) are shown. It is noted that the introduction of the struts with uncertain mechanical properties even produce changes in the signs of the moments for a frame subject to an equivalent distribution of seismic lateral forces as those considered in this example through Eq. (27). In this case, it is also noted that the correlated fluctuations yield a PDF that has less dispersion than in the case of uncorrelated fluctuations. This is in opposite trend as compared to the previous plots. However, from a broader examination of other response quantities (not reported here for the sake of brevity) it was found that the dispersion of such indicators can be higher or lower by comparing the correlated and uncorrelated assumptions. Therefore, no apparent relationship between the correlation of the fluctuations and the shape of the PDF can be inferred, since no clear tendency is observed.

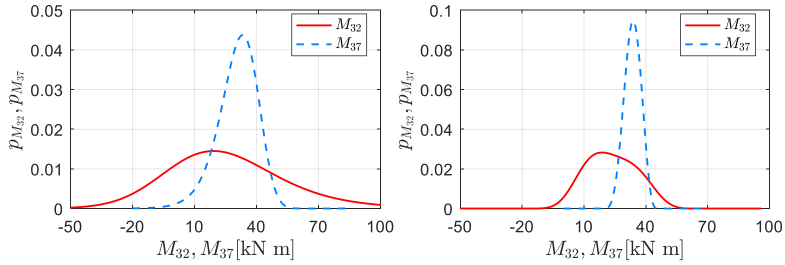


Fig. 18 Comparison of PDF of the moment on the column (M_{32}) and beam (M_{37}) for normal uncorrelated (left) and uniform (right) distribution assumption of the fluctuations

Table 3 List of characteristic parameters of the probabilistic distribution of a set of response quantities (uniform distribution assumption of the fluctuations)

response quantity	response designation	deterministic value for $w = w_0 \quad (\alpha = 0)$	mean value μ	standard deviation σ	COV σ / μ
top-story displacement	u_{x4} [m]	0.0087	0.0086	0.00034	0.039
last interstory drift	Δu_{x43} [m]	0.0026	0.0026	0.00015	0.059
moment on the top of the column	M_{32} [kNm]	24.02	23.38	12.47	0.53
moment on the beam end	M_{37} [kNm]	33.10	33.61	3.97	0.12
moment reaction at the base of the central column	R_{M5} [kNm]	249.5	258.00	60.27	0.23
shear force on the top of the column	V_{32} [kN]	6.10	5.88	7.92	1.35
shear force on the beam end	V_{37} [kN]	81.43	81.64	1.28	0.016

Table 4 List of characteristic parameters of the probabilistic distribution of a set of response quantities (normal uncorrelated distribution of the fluctuations)

response quantity	response designation	deterministic value for $w = w_0 \quad (\alpha = 0)$	mean value μ	standard deviation σ	COV σ / μ
top-story displacement	u_{x4} [m]	0.0087	0.0089	0.0008	0.93
last interstory drift	Δu_{x43} [m]	0.0026	0.0027	0.00038	0.14
moment on the top of the column	M_{32} [kNm]	24.02	26.14	30.36	1.16
moment on the beam end	M_{37} [kNm]	33.10	30.55	9.79	0.32
moment reaction at the base of the central column	R_{M5} [kNm]	249.5	256.15	47.29	0.18
shear force on the top of the column	V_{32} [kN]	6.10	7.12	19.14	2.69
shear force on the beam end	V_{37} [kN]	81.43	80.69	3.18	0.039

On the contrary, the influence of the fluctuations on the stress and strains in the beam elements is less pronounced. As an example, in Fig. 17, the PDF and CDF of the moment at the left side of the beam 3-7 (M_{37}) are illustrated. Unlike the probabilistic characterization of the moment on the column, in this case the values are almost entirely positive (i.e., they do not exhibit sign changes), and the influence of the uncertainty in the masonry panels is less pronounced. In this case, the correlated fluctuations give more dispersed values, in line with other response quantities. Furthermore, the deterministic value of M_{37} identified by an analysis with $w = w_0$ and $\alpha = 0$ for all the struts is again bounded by the mean values of the PDFs for uniform and normal distribution assumptions. In order to highlight the remarkably different influence of the uncertain mechanical properties of the masonry infills on the column and beam elements, in Fig. 18, the PDFs of the moments on the beam and column elements are compared with each other. The two Figs refer to different distribution assumptions of the fluctuations (normal and uniform), but the general qualitative conclusions for the two cases are almost identical. These conclusions are confirmed also for other response quantities, for instance, moments evaluated at other nodes or shear forces.

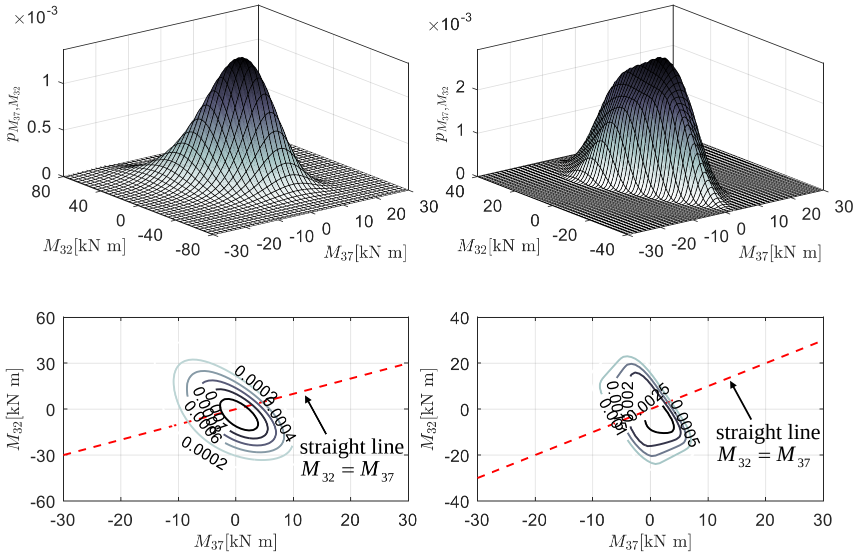


Fig. 19 Joint PDF between the moment on the column (M_{32}) and on the beam (M_{37}) for normal uncorrelated (left) and uniform (right) distribution assumption of the fluctuations

Table 5 List of characteristic parameters of the probabilistic distribution of a set of response quantities (normal correlated distribution of the fluctuations)

response quantity	response designation	deterministic value for $w = w_0 \quad (\alpha = 0)$	mean value μ	standard deviation σ	COV σ/μ
top-story displacement	u_{x4} [m]	0.0087	0.0089	0.0017	0.19
last interstory drift	Δu_{x43} [m]	0.0026	0.0027	0.0005	0.19
moment on the top of the column	M_{32} [kNm]	24.02	26.87	20.58	0.77
moment on the beam end	M_{37} [kNm]	33.10	30.27	14.65	0.48
moment reaction at the base of the central column	R_{M5} [kNm]	249.5	257.59	58.19	0.22
shear force on the top of the column	V_{32} [kN]	6.10	5.24	10.62	2.03
shear force on the beam end	V_{37} [kN]	81.43	80.65	5.2	0.064

Therefore, uncertain mechanical properties of the masonry infills have a great influence on the column stress and strain values, and a reduced influence on the beam response quantities.

The proposed procedure also enables the determination of the joint PDF between two or more response quantities. As an example, in Fig. 19, the joint PDF of the moments on the beam M_{37} and that on the column M_{32} is displayed, namely $p_{M_{32}M_{37}}(M_{32}, M_{37})$. This joint PDF can be interpreted for drawing some general conclusions from a design viewpoint. It is well-known that capacity design establishes a hierarchy of zones among the structural members (Avramidis *et al.* 2015), which is a concept incorporated in seismic provisions (FEMA-274 1997, D.M. LL. PP. 2008). The failure mode of the beam is usually deemed to be more ductile than that of the column.

Therefore, capacity design principles promote failure mechanisms occurring in the beam before those occurring in the column. In a simplified manner, it can be assumed that the design flexural resistance M_{Rd} is related to the design bending moment M_{Ed} calculated in the analysis. Therefore, it is interesting to scrutinize to what extent the typical ratio between moments in beam and column elements is affected by the uncertainty on the masonry panels. To this aim, in the contour plots on the bottom part of Fig. 19 a dashed line has been reported that corresponds to $M_{32} = M_{37}$. This means that points lying above this dashed line in the first quadrant and lying below this dashed line in the third quadrant represent situations in which $|M_{32}| > |M_{37}|$, thus jeopardizing the correct principle of the strength hierarchy underlying the "weak-beam-strong-columns" principle. This, in turn, is likely to produce less ductile collapse mechanism in the masonry infilled RC frame, provided the design resistance is assumed in line with the design bending moments. It is noted that there exists such a probability of occurrence of this phenomenon in both the normal and uniform distribution assumption of the fluctuations. This outcome is affected by the distribution of the stress in the masonry infilled RC frame induced by the presence of the uncertain masonry panels. Although the present analysis is extremely simplified, in the authors' opinion this link is important to provide physical meaning and usefulness of the proposed probabilistic approach of analysis in the case of RC frames with uncertain masonry infills. A summary of the results is reported in Table 3, Tables 4

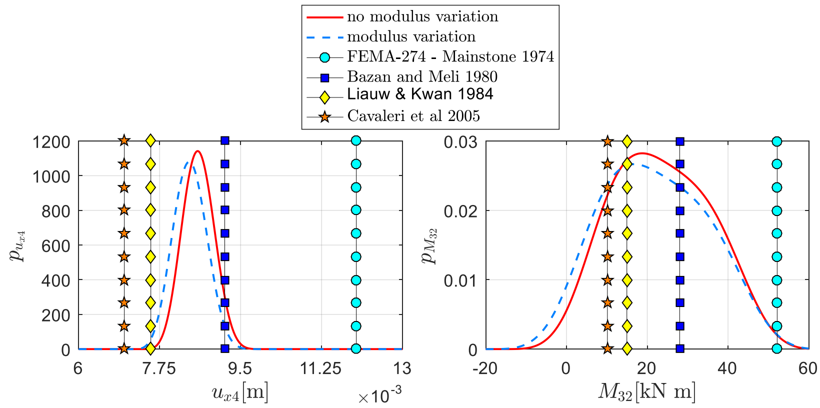


Fig. 20 PDF of the last-floor displacement (u_{x4}) and of the moment on the top of the column (M_{32}) compared to deterministic values predicted by four different deterministic formulations

and 5 for uniform, normal uncorrelated and normal correlated fluctuations, respectively. By inspection of these tables, the following conclusions can be drawn:

- the mean values of several response indicators calculated from the PDF are very close to the deterministic values that can be computed by an analysis of the RC frame in which the diagonal pin-jointed struts are assigned the mean width value $w = w_0$, i.e., with a zero value of the fluctuations according to a mere deterministic analysis;
- the dispersion of the distribution expressed by the standard deviation σ is different from case to case: in general, the sensitivity of the response indicators to the uncertainty of the masonry infills is more pronounced for column-related quantities (e.g., the moment on the top of the column M_{32} or the shear force V_{32}) than for beam-related variables;
- the more pronounced dispersions of the distribution, indicated by the value of the COV, are higher in the cases of column-related quantities, especially the shear forces on the columns.

It is concluded that careful attention must be paid to the values of the stress and strains in a RC frame with masonry infills, especially if the panels are affected by largely scattered results arising from preliminary experiments on the constituting elements and materials. The examples shown in this paper, although carried out on a simple structure and under an equivalent static lateral force procedure, give a preliminary idea of the influence of the masonry infills on a set of response indicators for different modelling assumptions, all related to a macro-modeling approach, and for different distribution assumptions of the relevant equivalent strut widths simulating, in a simplified way, the presence of the stiffening contribution offered by the masonry infills themselves.

Finally, in order to provide insight into the reliability of the proposed predicting expressions for the masonry infill stiffening contribution, a final analysis has been conducted. In this final analysis, the influence of the uncertain nature of the Young's modulus has also been investigated. Instead of assuming a deterministic E value, the sensitivity of the results to the variability of the Young's modulus is here studied. The E value has been sampled by assuming a uniform distribution between $E_{w\min} = 4\text{GPa}$ and $E_{w\max} = 6\text{GPa}$, and the empirical expressions reported in Table 1 have been repeatedly applied in order to have a wider set of statistical data to assess the accuracy of the empirical expressions in comparison with probabilistic results. We here report the PDF of two response indicators: the last-floor displacement u_{x4} and the bending moment on the top of the column M_{32}. Four alternatives of empirical formulae predicting the same response quantities are employed, within a deterministic framework, for comparison purposes. In this way, we can assess the accuracy of four different predicting expressions when compared to a more complete probabilistic analysis. From Fig. 20, it is noted that the influence of the Young's modulus on the probabilistic characterization of the response, at least in the range of E_w explored here, is not particularly significant. Indeed, the obtained PDF with modulus variation is very similar to that obtained with the mean value of E_w. Furthermore, the four deterministic values of the two response quantities u_{x4} and M_{32} predicted with the four considered formulations give very large scatter of results. In particular, it is seen that the Mainstone 1974 expression (Mainstone 1974), also adopted by FEMA-274 and 306 (FEMA-274 1997, FEMA-306 1998), provides extremely large values of the response, on the conservative side. More reasonable estimates of the response mean value as computed by the probabilistic analysis are provided by the other three formulations considered. However, these three formulations (namely, Bazan and Meli 1980 (Bazan and Meli 1990), Liauw and Kwan 1984 (Liauw and Kwan 1984), Cavaleri *et al.* 2005 (Cavaleri *et al.* 2005)) leads to results that are not particularly close to the mean value of the corresponding PDF, especially for the displacement.

It seems that the formula proposed by Bazan and Meli 1980 (Bazan and Meli 1980) is the most accurate one because it is in reasonable agreement with the mean value and it provides conservative estimates of the response.

As a final remark, this comparison definitely highlights the scatter of results that the different formulations may produce, and underlines the importance of probabilistic studies to account for the uncertain nature of the masonry infill mechanical behaviour.

6. Conclusions

The main contents and findings of this research work are summarized as follows:

• A fully probabilistic approach has been proposed for the analysis of the in-plane response of masonry infilled RC frames. More specifically, this paper has been focused on the investigation of the effects of the masonry infills uncertainty on the structural response of RC frames. A macro-modeling approach has been adopted in which the masonry panels are considered via equivalent diagonal pin-jointed struts. The strut widths have been considered as random variables in order to incorporate the stochastic nature of the masonry infills ascribed to their inherent heterogeneous nature and to the large scatter of corresponding experimental results.

• The probabilistic characterization of the complex mechanical behavior of the masonry infills has been based upon an ensemble of empirical expressions proposed in the literature by different authors. For given geometrical and mechanical properties of an assigned masonry infilled RC frame, a procedure for deriving the probabilistic input data of the strut widths has been described, and different modelling assumptions in terms of correlation and shape of the PDF have been explored.

• An effective numerical procedure has been proposed that, unlike Monte-Carlo-based methods, avoids sampling techniques thus implying reduced computational effort, especially for structures with several DOFs. This procedure provides the probabilistic characterization of the system response directly, once the probabilistic characterization of the masonry panels has been established as per the previous bullet point. To the authors' best knowledge, such a direct probability-based procedure has never been considered for the analysis of masonry infilled RC frames, which represents the main novelty of this contribution.

• To demonstrate the kind of results that this procedure can offer, a simple application has been presented, consisting in an equivalent linear analysis of a regular masonry infilled reinforced concrete framed structure. The PDF of a set of response indicators has been determined, and has been compared to the PDF obtained via alternative (more cumbersome) techniques like Monte Carlo method and other strategies earlier proposed in the literature, all requiring sampling operations and providing just an approximation of the PDF as the number of sampling increases.

• The sensitivity of the response to the modeling assumptions, mainly the shape of the PDF and the inherent correlation between the fluctuations of the various strut widths associated with the various masonry panels, has been discussed. For most of the response indicators analyzed, incorporating the correlation of the strut widths in the probabilistic characterization of the input data has led to more dispersed probabilistic characterization of the system response. Furthermore, it has been observed that a deterministic analysis carried out considering the mean values of the strut widths provides reasonable estimates of the mean response as determined by the probabilistic approach. However, as a matter of fact, deterministic analysis cannot give indications on the probability distribution, which is important for reliability-based design.

• As expected, the influence of the uncertainty of the masonry infills is more pronounced for column-related quantities (e.g., the moment, shear forces, etc.) rather than for beam-related variables. Furthermore, it has also observed that, due to the presence of uncertain masonry infills, the weak-beam-strong-columns principle underlying the strength hierarchy criterion might be jeopardized.

• Based on the last two conclusions, it is recommended that conservative safety factors be applied for designing the columns in masonry infilled RC frames in order to take into account, in a simplified way, the randomness of the response due to the stochastic nature of the masonry panels.

• A more specific analysis on four different formulations proposed in the literature has revealed which are the more reliable formulations that are better able to reproduce the mean value of the response as indicated in the present probabilistic study.

Acknowledgments

The financial support from the Italian Ministry of Education, University and Research (PRIN Grant 015TTJN95 and PRIN Grant 2015JW9NJT) is gratefully acknowledged.

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