Structural Engineering and Mechanics, *Vol. 68, No. 4 (2018) 409-421*
DOI: https://doi.org/10.12989/sem.2018.68.4.409

Bending analysis of functionally graded thick plates with in-plane stiffness variation

Ali Mazari[1], Amina Attia[1, 2], Mohamed Sekkal[1, 3], Abdelhakim Kaci[1, 4], Abdelouahed Tounsi*[1, 5], Abdelmoumen Anis Bousahla[6, 7] and S.R. Mahmoud[8]

[1]Material and Hydrology Laboratory, Faculty of Technology, Civil Engineering Department, University of Sidi Bel Abbes, Algeria
[2]Département de Génie Civil, Institut des Sciences et de la Technologie, Centre Universitaire d'Ain Témouchent, Algérie
[3]Laboratoire de Modélisation et Simulation Multi-échelle, Département de Physique, Faculté des Sciences Exactes, Département de Physique, Université de Sidi Bel Abbés, Algeria
[4]Université Dr Tahar Moulay, Faculté de Technologie, Département de Génie Civil et Hydraulique, BP 138 Cité En-Nasr 20000 Saida, Algérie
[5]Department of Civil and Environmental Engineering, King Fahd University of Petroleum & Minerals, 31261 Dhahran, Eastern Province, Saudi Arabia
[6]Laboratoire de Modélisation et Simulation Multi-échelle, Département de Physique, Faculté des Sciences Exactes, Département de Physique, Université de Sidi Bel Abbés, Algeria
[7]Centre Universitaire Ahmed Zabana de Relizane, Algeria
[8]Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia

(Received May 13, 2018, Revised September 23, 2018, Accepted September 29, 2018)

Abstract. In the present paper, functionally graded (FG) materials are presented to investigate the bending analysis of simply supported plates. It is assumed that the material properties of the plate vary through their length according to the power-law form. The displacement field of the present model is selected based on quasi-3D hyperbolic shear deformation theory. By splitting the deflection into bending, shear and stretching parts, the number of unknowns and equations of motion of the present formulation is reduced and hence makes them simple to use. Governing equations are derived from the principle of virtual displacements. Numerical results for deflections and stresses of powerly graded plates under simply supported boundary conditions are presented. The accuracy of the present formulation is demonstrated by comparing the computed results with those available in the literature. As conclusion, this theory is as accurate as other shear deformation theories and so it becomes more attractive due to smaller number of unknowns. Some numerical results are provided to examine the effects of the material gradation, shear deformation on the static behavior of FG plates with variation of material stiffness through their length.

Keywords: FGM; bending; thick plate

1. Introduction

The concept of functionally graded materials (FGMs) was first introduced by a group of Japanese scientists in 1984 (Yamanouchi 1990, Koizumi 1993). FGMs are a type of advanced composite materials whose properties vary gradually and continuously from one surface to another. Because of this advantage, they have been regarded as one of the advanced inhomogeneous composite materials in many engineering sectors. FGMs are made from a mixture of ceramic and metal. Ceramic constituent provides the high temperature resistance due to its low thermal conductivity and metal constituent resists the failure of the structure. The FGM is widely used in many structural applications such as aerospace, nuclear, civil, nanostructures, and automotive (Janghorban and Zare 2011, Nami and Janghorban 2014, Mouffoki *et al.* 2017, Karami *et al.* 2018a, b, c, Dash *et al.* 2018). The benefit of advanced materials and structures can be demonstrated in other applications such as (Mehar *et al.* 2018, Henderson *et al.* 2018, Hirwani and Panda 2018, Bouadi *et al.* 2018, Cherif *et al.* 2018, Kadari *et al.* 2018, Belabed *et al.* 2018, Hajmohammad *et al.* 2018a, Mehar and Panda 2018, Karami *et al.* 2018d, e, f, g, Yazid *et al.* 2018, Youcef *et al.* 2018, Karami *et al.* 2018h, i, Shahsavari *et al.* 2017, Mehar *et al.* 2017a, b, c, Mahapatra *et al.* 2017a, b, Katariya *et al.* 2017a, Dutta *et al.* 2017, Sahoo *et al.* 2017, Beldjelili *et al.* 2016, Arani and Kolahchi 2016, Ahouel *et al.* 2016, Zemri *et al.* 2015, Bourada *et al.* 2015).

The increase in FGM application requires more accurate plate theories to predict their responses. The classical plate theory (CPT) shows inaccurate results for thick and moderately thick plates as it does not consider shear deformation. The First-order shear deformation theories (FPT) (Mindlin 1951, Reissner 1945) consider the transverse shear deformation effects. However, a shear correction factor is needed to satisfy the zero transverse shear stress boundary conditions at the top and bottom of the plate. The higher-order shear deformation theories (HSDT) (Reddy 1984, 2000, Ren 1986, Touratier 1991, Soldatos 1992, Xiang *et al.* 2009, Akavci 2010, Grover *et al.* 2013, Karama *et al.* 2003, Pradyumna and Bandyopadhyay 2008, Ait Atmane *et al.* 2010, Mantari *et al.* 2012, Kar and Panda 2015a, Karand Panda 2016a, b, Mehar and Panda 2017a, b, Singh and Panda 2017,

*Corresponding author, Professor
E-mail: tou_abdel@yahoo.com

ISSN: 1225-4568 (Print), 1598-6217 (Online)

Kolahchi *et al.* 2017, Mehar *et al.* 2017b, Sekkal *et al.* 2017a, Katariya *et al.* 2017b, Hirwani *et al.* 2018) account for shear deformation effects and satisfy the equilibrium conditions at the top and bottom surfaces of the plate without requiring any shear correction factors. Shear deformation models are also employed to study the mechanical response of nanocomposite structures (Kolahchi and Moniri Bidgoli 2016, Madani *et al.* 2016, Kolahchi *et al.* 2016a, b, Arani and Kolahchi 2016, Bilouei *et al.* 2016, Kolahchi *et al.* 2017a, b, c, Kolahchi and Cheraghbak 2017, Kolahchi *et al.* 2017a, b, c, Zamanian *et al.* 2017, Kolahchi 2017, Shokravi 2017a, b, c, d, Besseghier *et al.* 2017, Hajmohammad *et al.* 2017, 2018b, c, d, e, Zarei *et al.* 2017, Amnieh *et al.* 2018, Golabchi *et al.* 2018). In a number of recent articles, a new refined and robust plate theory for bending response and vibration of simply supported FGM plate with only four unknown functions has been developed (Bourada *et al.* 2012, Bachir Bouiadjra *et al.* 2012, Tounsi *et al.* 2013, Kettaf *et al.* 2013, Ait Amar Meziane *et al.* 2014, Ahmed, 2014, Kolahchi *et al.* 2015, Ait Yahia *et al.* 2015, Kar and Panda, 2015b, Belkorissat *et al.* 2015, Bourada *et al.* 2016, Bouderba *et al.* 2016, Bellifa *et al.* 2016, El-Haina *et al.* 2017, Benadouda *et al.* 2017, Fakhar and Kolahchi 2018, Attia *et al.* 2018, Karami *et al.* 2018f, Fourn *et al.* 2018, Bakhadda *et al.* 2018). Recently, review on refined plate theories can be documented in (Bouderba *et al.* 2013, Tounsi *et al.* 2013, Zidi *et al.* 2014, Larbi Chaht *et al.* 2015, Mahi *et al.* 2015, Karami and Janghorban 2016, Boukhari *et al.* 2016, Bousahla *et al.* 2016, Bounouara *et al.* 2016, Houari *et al.* 2016, Fahsi *et al.* 2017, Khetir *et al.* 2017, Klouche *et al.* 2017, Bellifa *et al.* 2017a, b, Menasria *et al.* 2017, Hachemi *et al.* 2017, Zidi *et al.* 2017, Aldousari 2017, Chikh *et al.* 2017, Shahsavari *et al.* 2018a, Kaci *et al.* 2018, Zine *et al.* 2018, Mokhtar *et al.* 2018, Meksi *et al.* 2019). The thickness stretching effect is neglected in the most of shear deformation theories by considering the transverse displacement as constant. So, this effect should be taken into consideration especially for thick FGM plates (Carrera *et al.* 2011, Sekkal *et al.* 2017b, Younsi *et al.* 2018). To overcome this problem, some quasi-3D theories presented in the literature are developed. Carrera *et al.* (2011) evaluated the effect of thickness stretching in plate/shell structures made by materials which are FGM in the thickness directions. Mantari and Guedes Soares (2013) conducted static analysis of thick FG plates using a novel trigonometric higher-order theory in which stretching effect was considered. Belabed *et al.* (2014) presented an efficient and simple higher order shear and normal deformation theory for FGM plates. Bousahla *et al.* (2014) developed a novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates. Akavci and Tanrikulu (2015) presented two dimensional (2D) and quasi three-dimensional (quasi-3D) shear deformation theories for static and free vibration analysis of single-layer functionally graded (FG) plates using anew hyperbolic shape function. Draiche *et al.* (2016) presented a refined theory with stretching effect for the flexure analysis of laminated composite plates. Benahmed *et al.* (2017) presented a novel quasi-3D hyperbolic shear deformation theory for functionally graded thick rectangular plates resting on elastic foundation. Shahsavari *et al.* (2018b) proposed a novel quasi-3D hyperbolic theory for free vibration of FG plates with porosities resting on Winkler/Pasternak/Kerr foundation. Benchohra *et al*. (2018) employed a new quasi-3D sinusoidal shear deformation theory for FG plates. Abualnour *et al*. (2018) proposed a novel quasi-3D trigonometric plate theory for free vibration analysis of advanced composite plates. Hirwani and Panda (2019) presented nonlinear finite element solutions of thermoelastic deflection and stress responses of internally damaged curved panel structure and two different polynomial types of kinematic theories including the through-thickness stretching effect are employed. Katariya *et al*. (2018) used higher-order kinematic theory including the stretching term effect to study the nonlinear deflection and stress of skew sandwich shell panel. Shahsavari *et al.* (2018c) studied the shear buckling of porous nanoplates using a new size-dependent quasi-3D shear deformation theory. Galerkin method is employed to find the shear buckling forces. Bouhadra *et al*. (2018) proposed an improved HSDT accounting for effect of thickness stretching in advanced composite plates. Zaoui *et al*. (2019) developed a new 2D and quasi-3D shear deformation theories for free vibration of FG plates on elastic foundations.

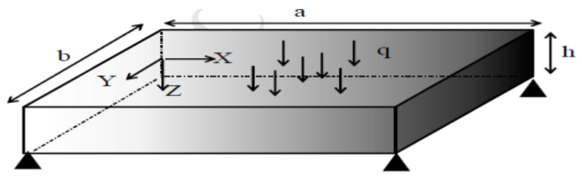

Fig. 1 Schematic representation of FGM plate with distribution of volume fraction along x

It is seen from the above literature analysis that most of the previous studies are related to FG structures whose material properties vary through the thickness only. Thus, there have been a very little number of studies related to FG structures with property variation throughout the length so far.

A novel method has been developed by Amirpour *et al.* (2016) to analyze the elastic deformation of functionally graded thick plates with in-plane stiffness variation using higher order shear deformation theory. For design and analysis of thick FG plates with in- plane stiffness variation, the development of simple theory for bending analysis of thick FG plates with in-plane property variation, including stretching effects, is very relevant and represents the main objective of the present article.

The aim of this work is to analyze bending behavior of thick FG plates with in-plane variation of stiffness (variation of stiffness through the length of the plate) using the higher shear deformation theory including stretching of the thickness. The main challenge in developing accurate models for in-plane property variation compared to the through-the-thickness property variation is that the variation of Young's modulus (material stiffness) through the length (x) leads to mathematically complex five simultaneous governing equations. The solutions of which become relatively difficult as several parameters vary with the length (x). The transverse displacement is apportioned into three components: bending, shear and thickness stretching. Equations of motion are derived from the principle of virtual displacements. The accuracy of obtained solutions is verified by comparing the present results with those predicted by solutions available in the literature.

2. Theoretical developments

Consider a simply supported rectangular FG plate with the length a width b, and thickness h. The x-, y-, and z-coordinates are taken along the length, width, and height of the plate, respectively, as shown in Fig. 1. The formulation is limited to linear elastic material behavior. The FG plate is isotropic with its material properties vary smoothly through the length of the plate.

2.1 Displacement field and strains

The formulation is limited to linear elastic material behavior. The displacement fields of various shear deformation theories are chosen based on following assumptions: (1) The transverse displacement is partitioned into bending and shear and stretching components, (2) the in-plane displacements are partitioned into extension, bending and shear components, (3) the bending parts of the in-plane displacements are similar to those given by the classical plate theory (CPT) and the shear component of axial displacement gives rise to the higher-order variation of shear strain and hence to shear stress through the thickness of the plate in such a way that shear stress vanishes on the top and bottom surfaces. Based on these assumptions, the displacement fields of various higher-orders shear deformation theories are given in a general form as (Hebali *et al.* 2014, Hamidi *et al.* 2015, Bennoun *et al.* 2016, Bouafia *et al.* 2017, Shahsavari *et al.* 2018b)

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w_b}{\partial y} - f(z)\frac{\partial w_s}{\partial y} \\ w(x,y,z) &= w_b(x,y) + w_s(x,y) + g(z)\varphi(x,y) \end{aligned} \tag{1a}$$

The shape function is given as (Zenkour 2013, Bourada *et al.* 2015)

$$f(z) = z - \left[h\sinh\left(\frac{z}{h}\right) - \frac{4}{3}\left(\frac{z^3}{h^2}\right)\cosh\frac{1}{2} \right] \tag{1b}$$

where u_0 and v_0 denote the displacements along the x and y coordinate directions of a point on the mid-plane of the plate, w_b and w_s are the bending and shear components of the transverse displacement, respectively, and the additional displacement φ accounts for the stretching effect. $f(z)$ is a shape function determining the distribution of the transverse shear strain and shear stress through the thickness of the plate. The shape functions $f(z)$ are chosen to satisfy the stress-free boundary conditions on the top and bottom surfaces of the plate, thus a shear correction factor is not required. Note that $g(z)$=0 is required for 2D analysis.

The nonzero linear strains associated with the displacement field in Eq. (1) are

$$\begin{gathered} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z)\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} \\ \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix}, \\ \varepsilon_z = g'(z)\varepsilon_z^0, \end{gathered} \tag{2}$$

Where

$$\begin{gathered} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_b}{\partial x^2} \\ -\dfrac{\partial^2 w_b}{\partial y^2} \\ -2\dfrac{\partial^2 w_b}{\partial x \partial y} \end{Bmatrix}, \\ \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_s}{\partial x^2} \\ -\dfrac{\partial^2 w_s}{\partial y^2} \\ -2\dfrac{\partial^2 w_s}{\partial x \partial y} \end{Bmatrix}, \\ \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial w_s}{\partial y} + \dfrac{\partial \varphi}{\partial y} \\ \dfrac{\partial w_s}{\partial x} + \dfrac{\partial \varphi}{\partial x} \end{Bmatrix}, \quad \varepsilon_z^0 = \varphi(x,y) \end{gathered} \tag{3}$$

and

$$g(z) = 1 - f'(z) \tag{4}$$

2.2 Constitutive relations

The material is assumed to be elastic and inhomogeneous, and the material stiffness vary continuously through the length of the plate and obey a simple power-law distribution of volume fraction of constituents as given by (Amirpour *et al.* 2016)

$$V_c(x) = \left(\frac{x}{a}\right)^p \tag{5}$$

Where p is a parameter that governs the material variation profile through the length of the plate. Since the effect of variation of Poisson's ratio on the response of FG plates is very small (Yang 2005, Kitipornchai 2006), this material parameter is assumed to be constant, and the Young's modulus is considered tobe variable, and can be determined by the rule of mixture as (Tornabene 2009)

$$E(x)=\left(\frac{x}{a}\right)^{p}\left(E_c-E_m\right)+E_m \tag{6}$$

The linear constitutive relations of a FG plate can be written as

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}C_{11}&C_{12}&C_{13}&0&0&0\\ C_{12}&C_{22}&C_{23}&0&0&0\\ C_{13}&C_{23}&C_{33}&0&0&0\\ 0&0&0&C_{44}&0&0\\ 0&0&0&0&C_{55}&0\\ 0&0&0&0&0&C_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{Bmatrix} \tag{7}$$

where (σ_x, σ_y, σ_z, τ_{yz}, τ_{xz}, τ_{xy}) and (ε_x, ε_y, ε_z, γ_{yz}, γ_{xz}, γ_{xy}) are the stress and strain components, respectively. Using the material properties defined in Eq. (6), stiffness coefficients, C_{ij}, can be expressed as

$$C_{11}=C_{22}=C_{33}=\frac{1-\nu}{(1+\nu)(1-2\nu)}E(x), \tag{8a}$$

$$C_{12}=C_{13}=C_{23}=\frac{\nu}{(1+\nu)(1-2\nu)}E(x) \tag{8b}$$

$$C_{44}=C_{55}=C_{66}=\frac{E(x)}{2(1+\nu)}, \tag{8c}$$

If the ε_z=0, then stiffness coefficients, C_{ij}, are,

$$C_{11}=C_{22}=\frac{E(x)}{1-\nu^2(x)}, \tag{9a}$$

$$C_{12}=\nu C_{11}, \tag{9b}$$

$$C_{44}=C_{55}=C_{66}=\frac{E(x)}{2(1+\nu)}, \tag{9c}$$

2.3 Equations of motion

The governing equations of the present theory are derived from the Principle of Virtual Displacements. The internal virtual work is initially formulated as follows

$$\delta U+\delta V=0 \tag{10}$$

Where δU, δV are the variation of the strain energy, the external work by applied forces, respectively, and are described in the following equations

$$\begin{aligned}\delta U&=\int_V\left[\sigma_x\,\delta\varepsilon_x+\sigma_y\,\delta\varepsilon_y+\sigma_z\,\delta\varepsilon_z+\tau_{xy}\,\delta\gamma_{xy}+\tau_{yz}\,\delta\gamma_{yz}+\tau_{xz}\,\delta\gamma_{xz}\right]dV\\ &=\int_A\left[N_x\,\delta\varepsilon_x^0+N_y\,\delta\varepsilon_y^0+N_z\,\delta\varepsilon_z^0+N_{xy}\,\delta\gamma_{xy}^0+M_x^b\,\delta k_x^b+M_y^b\,\delta k_y^b+M_{xy}^b\,\delta k_{xy}^b\right.\\ &\quad\left.+M_x^s\,\delta k_x^s+M_y^s\,\delta k_y^s+M_{xy}^s\,\delta k_{xy}^s+S_{yz}^s\,\delta\gamma_{yz}^s+S_{xz}^s\,\delta\gamma_{xz}^s\right]dA=0\end{aligned} \tag{11}$$

Where A is the top surface and the stress resultants N, M, and S are defined by

$$\begin{Bmatrix}N_x&N_y&N_{xy}\\ M_x^b&M_y^b&M_{xy}^b\\ M_x^s&M_y^s&M_{xy}^s\end{Bmatrix}=\int_{-h/2}^{h/2}\left(\sigma_x,\sigma_y,\tau_{xy}\right)\begin{Bmatrix}1\\ z\\ f(z)\end{Bmatrix}dz \tag{12a}$$

$$\left(S_{xz}^s,S_{xz}^s\right)=\int_{-h/2}^{h/2}\left(\tau_{xz},\tau_{yz}\right)g(z)dz\quad N_z=\int_{-h/2}^{h/2}\sigma_z g'(z)dz \tag{12b}$$

Substituting Eq. (7) into Eq. (12) and integrating through the thickness of the plate, the stress resultants are given as

$$\begin{aligned}M_x^b&=B_{11}d_1u_0+B_{12}d_2v_0-D_{11}d_{11}w_b-D_{12}d_{22}w_b-D_{11}^sd_{11}w_s-D_{12}^sd_{22}w_s+L^2\varphi\\ M_y^b&=B_{12}d_1u_0+B_{22}d_2v_0-D_{12}d_{11}w_b-D_{22}d_{22}w_b-D_{12}^sd_{11}w_s-D_{22}^sd_{22}w_s+L^2\varphi\\ M_{xy}^b&=B_{66}d_2u_0+B_{66}d_1v_0-2D_{66}d_{12}w_b-2d_1D_{66}^sd_{12}w_s\end{aligned} \tag{13b}$$

$$\begin{aligned}M_x^s&=B_{11}^sd_1u_0+B_{12}^sd_2v_0-D_{11}^sd_{11}w_b-D_{12}^sd_{22}w_b-H_{11}^sd_{11}w_s-H_{12}^sd_{22}w_s+L^3\varphi\\ M_y^s&=B_{12}^sd_1u_0+B_{22}^sd_2v_0-D_{12}^sd_{11}w_b-D_{22}^sd_{22}w_b-H_{12}^sd_{11}w_s-H_{22}^sd_{22}w_s+L^3\varphi\\ M_{xy}^s&=B_{66}^sd_2u_0+B_{66}^sd_1v_0-2D_{66}^sd_{12}w_b-2H_{66}^sd_{12}w_s\end{aligned} \tag{13c}$$

$$\begin{aligned}S_{xz}&=A_{55}^sd_1w_s+A_{55}^sd_1\varphi\\ S_{yz}&=A_{44}^sd_2w_s+A_{55}^sd_2\varphi\end{aligned} \tag{13d}$$

The variation of work done by the applied loads can be expressed as

$$\delta V=-\int_A q(\delta w_b+\delta w_s+\delta w_{st})dA \tag{14}$$

Substituting the expressions for δU and δV from Eqs. (11) and (14) into Eq. (10) and integrating by parts, and then collecting the coefficients of δu_0, δv_0, δw_b, δw_s and $\delta\varphi$, the following governing equations of the FG plate with the variation of material property through the length are obtained as follows

$$\begin{aligned}\delta u_0&:\ \frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0\\ \delta v_0&:\ \frac{\partial N_y}{\partial y}+\frac{\partial N_{xy}}{\partial x}=0\\ \delta w_b&:\ \frac{\partial^2 M_x^b}{\partial x^2}+\frac{\partial^2 M_y^b}{\partial y^2}+2\frac{\partial^2 M_{xy}^b}{\partial x\partial y}+q=0\\ \delta w_s&:\ \frac{\partial^2 M_x^s}{\partial x^2}+\frac{\partial^2 M_y^s}{\partial y^2}+2\frac{\partial^2 M_{xy}^s}{\partial x\partial y}+\frac{\partial S_{xz}^s}{\partial x}+\frac{\partial S_{yz}^s}{\partial y}+q=0\\ \delta\varphi&:\ \frac{\partial S_{xz}^s}{\partial x}+\frac{\partial S_{yz}^s}{\partial y}+qg(z)-N_z=0\end{aligned} \tag{15}$$

Introducing Eq. (13) into Eq. (15), equations of motion for FG plate can be obtained as follows

$$\begin{aligned}&A_{11}d_{11}u_0+d_1A_{11}d_1u_0+A_{66}d_{22}u_0+\left(A_{12}+A_{66}\right)d_{12}v_0+d_1A_{12}d_2v_0-d_{111}w_bB_{11}-d_{11}w_bd_1B_{11}\\ &-d_{111}w_sB_{11}^s-d_{11}w_sd_1B_{11}^s-d_{122}w_bB_{12}-d_{22}w_bd_1B_{12}-d_{122}w_sB_{12}^s-d_{22}w_sd_1B_{12}^s\\ &-2d_{122}w_bB_{66}-2d_{122}w_bB_{66}^s+L^1d_1\varphi+d_1L^1\varphi=0\end{aligned} \tag{16a}$$

$$\begin{aligned}&A_{12}d_{12}u_0+A_{22}d_{22}v_0+d_1A_{66}d_1v_0+A_{66}d_{11}v_0+d_1A_{66}d_2u_0+A_{66}d_{12}u_0-2d_{112}w_bB_{66}\\ &-2d_{12}w_bd_1B_{66}-2d_{112}w_sB_{66}^s-2d_{12}w_sd_1B_{66}^s-d_{112}w_bB_{12}-d_{112}w_sB_{12}^s-d_{222}w_bB_{22}\\ &-d_{222}w_sB_{22}^s+L^1d_2\varphi=0\end{aligned} \tag{16b}$$

$$\begin{aligned}&-d_{11}D_{11}d_{11}w_b-2d_1D_{11}d_{111}w_b-D_{11}d_{1111}w_b-d_{11}D_{12}d_{22}w_b-2d_1D_{12}d_{122}w_b-D_{12}d_{1122}w_b\\ &-d_{11}D_{11}^sd_{11}w_s-2d_1D_{11}^sd_{111}w_s-D_{11}^sd_{1111}w_s-d_{11}D_{12}^sd_{22}w_s-2d_1D_{12}^sd_{122}w_s-D_{12}^sd_{1122}w_s\\ &+d_{11}L^2\varphi+2d_1L^2d_1\varphi+L^2d_{11}\varphi-D_{12}d_{1122}w_b-D_{22}d_{2222}w_b-D_{12}^sd_{1122}w_s-D_{22}^sd_{2222}w_s\\ &+L^2d_{22}\varphi-4d_1D_{66}d_{122}w_b-4D_{66}d_{1122}w_b-4d_1D_{66}^sd_{122}w_s-4D_{66}^sd_{1122}w_s+B_{12}d_{112}v_0\\ &+d_{11}B_{12}d_2v_0+2d_1B_{11}d_{22}u_0+B_{11}d_{111}u_0+d_{11}B_{11}d_1u_0+2B_{66}d_{122}v_0+2d_1B_{66}d_{12}v_0\\ &+2B_{66}d_{122}u_0+2d_1B_{66}d_{22}u_0+2d_1B_{12}d_{12}v_0+B_{12}d_{122}u_0+B_{22}d_{222}v_0+q=0\end{aligned} \tag{16c}$$

$$
\begin{aligned}
&- d_{11}D_{11}^s d_{11}w_b - 2d_1 D_{11}^s d_{111}w_b - D_{11}^s d_{1111}w_b - d_{11}D_{12}^s d_{22}w_b - 2d_1 D_{12}^s d_{122}w_b - D_{12}^s d_{1122}w_b \\
&+ d_{11}L^3\varphi + 2d_1 L^3 d_1\varphi + L^3 d_{11}\varphi + L^3 d_{22}\varphi - d_{11}H_{11}^s d_{11}w_s - 2d_1 H_{11}^s d_{111}w_s - H_{11}^s d_{1111}w_s \\
&- d_{11}H_{12}^s d_{22}w_s - 2d_1 H_{12}^s d_{122}w_s - H_{12}^s d_{1122}w_s - H_{12}^s d_{1122}w_s - D_{12}^s d_{1122}w_b - D_{22}^s d_{2222}w_b \\
&- H_{12}^s d_{1122}w_s - H_{22}^s d_{2222}w_s - 4d_1 D_{66}^s d_{122}w_b - 4D_{66}^s d_{1122}w_b - 4d_1 H_{66}^s d_{122}w_s - 4H_{66}^s d_{1122}w_s \\
&+ d_1 A_{55}^s d_{11}w_s + A_{55}^s d_{11}w_s + d_1 A_{55}^s d_1\varphi + A_{55}^s d_{11}\varphi + A_{44}^s d_{22}w_s + A_{44}^s d_{22}\varphi + 2B_{66}^s d_{112}v_0 \\
&+ 2d_1 B_{66}^s d_{22}u_0 + B_{22}^s d_{222}v_0 + 2d_{11}B_{12}^s d_2 v_0 + B_{11}^s d_{111}u_0 + 2d_1 B_{11}^s d_{11}u_0 + d_{11}B_{11}^s d_1 u_0 \\
&+ 2d_1 B_{66}^s d_{12}v_0 + 2B_{66}^s d_{122}u_0 + B_{12}^s d_{122}u_0 + B_{12}^s d_{112}v_0 + 2d_1 B_{12}^s d_{12}v_0 + q = 0
\end{aligned}
\tag{16d}
$$

$$
\begin{aligned}
&d_1 A_{55}^s d_1 w_s + A_{55}^s d_{11}w_s + d_1 A_{55}^s d_1\varphi + A_{55}^s d_{11}\varphi + A_{44}^s d_{22}w_s + A_{44}^s d_{22}\varphi + L^2\left(d_{11}w_b + d_{22}w_b\right) \\
&+ L^3\left(d_{11}w_s + d_{22}w_s\right) - L^4\varphi = 0
\end{aligned}
\tag{16e}
$$

Where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$
d_{ij} = \frac{\partial^2}{\partial x_i \partial x_j},\quad d_{ijl} = \frac{\partial^3}{\partial x_i \partial x_j \partial x_l},\quad d_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m},
$$
$$
d_i = \frac{\partial}{\partial x_i},\quad (i, j, l, m = 1,2).
\tag{17}
$$

And stiffness components are given as

$$
\left(A_{ij}, B_{ij}, D_{ij}, B_{ij}^s, D_{ij}^s, H_{ij}^s\right) = \int_{-h/2}^{h/2} C_{ij}\left(1, z, z^2, f(z), z\,f(z), f^2(z)\right) dz
\tag{18a}
$$

$$
A_{44}^s = A_{55}^s = \int_{-h/2}^{h/2} C_{44}\left[g(z)\right]^2 dz,
\tag{18b}
$$

$$
L^1 = \int_{-h/2}^{h/2} C_{13} g'(z)\, dz \;;\; L^2 = \int_{-h/2}^{h/2} C_{13} z g'(z)\, dz \;;\; L^3 = \int_{-h/2}^{h/2} C_{13} g'(z) f(z)\, dz \;;\; L^4 = \int_{-h/2}^{h/2} C_{33}\left[g'(z)\right]^2 dz
\tag{18c}
$$

3. Analytical solutions

The exact solution of Eq. (16) for the FGM plate under various boundary conditions can be constructed. The boundary conditions for an arbitrary edge with simply supported conditions are:

• Simply supported (S)

$$
\begin{aligned}
&v_0 = w_b = \partial w_b/\partial y = w_s = \partial w_s/\partial y = 0 \qquad at \quad x = 0, a \\
&u_0 = w_b = \partial w_b/\partial x = w_s = \partial w_s/\partial x = 0 \qquad at \quad y = 0, b
\end{aligned}
\tag{19}
$$

The following representation for the displacement quantities, that satisfy the above boundary conditions, is appropriate in the case of our problem

$$
\begin{Bmatrix} u_0 \\ v_0 \\ w_b \\ w_s \\ \varphi \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn} \dfrac{\partial X_m(x)}{\partial x} Y_n(y) \\ V_{mn} X_m(x) \dfrac{\partial Y_n(y)}{\partial y} \\ W_{bmn} X_m(x) Y_n(y) \\ W_{smn} X_m(x) Y_n(y) \\ \varphi_{mn} X_m(x) Y_n(y) \end{Bmatrix}
\tag{20}
$$

For simply supported boundary conditions

$$
X_m(x) = \sin(\lambda x) \qquad ; Y_n(y) = \sin(\mu y)
\tag{21}
$$

The transverse load q is also expanded in the double-Fourier sine series as follows

$$
q(x, y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Q_{mn} \sin(\lambda\, x) \sin(\mu\, y)
\tag{22}
$$

The coefficients Q_{mn} are given below for some typical loads (Zenkour 2006)

$$
Q_{mn} = \frac{4}{ab}\int_0^a\int_0^b q(x, y)\sin(\lambda\, x)\sin(\mu\, y)\,dxdy = \begin{cases} q_0 & \text{for sinusoidal load} \\ \dfrac{16 q_0}{mn\pi^2} & \text{for uniformly distributed load} \end{cases}
\tag{23}
$$

where U_{mn}, V_{mn}, W_{bmn}, W_{smn} and φ_{mn} are arbitrary parameters to be determined. The functions $X_m(x)$ and $Y_n(y)$ are suggested here to satisfy at least the geometric boundary conditions given in Eqs. (21) and represent approximate shapes of the deflected surface of the plate. Noting that $\lambda = m\pi/a$ and $\mu = n\pi/b$.

By substituting Eqs. (20) and (23) into Eq. (16), a set of algebraic equations can be obtained as

$$
\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{bmn} \\ W_{smn} \\ \varphi_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ q_0 \\ q_0 \\ 0 \end{Bmatrix}
\tag{24}
$$

Where the coefficients of the above matrix are given as follows

$$
\begin{aligned}
a_{11} &= \int_0^a\int_0^b (A_{11}\alpha_{12} + \alpha_9 d_1 A_{11} + \alpha_8 A_{33})\alpha_6\, dxdy \\
a_{12} &= \int_0^a\int_0^b (A_{12}\alpha_8 + \alpha_3 d_1 A_{12} + \alpha_8 A_{33})\alpha_6\, dxdy \\
a_{21} &= \int_0^a\int_0^b (A_{33}\alpha_{10} + \alpha_7 d_1 A_{33} + \alpha_{10} A_{12})\alpha_2\, dxdy \\
a_{13} &= \int_0^a\int_0^b (-B_{11}\alpha_{12} - \alpha_9 d_1 B_{11} - \alpha_8 B_{12} - \alpha_3 d_1 B_{12} - 2\alpha_8 B_{66})\alpha_6\, dxdy \\
a_{31} &= \int_0^a\int_0^b (B_{12}\alpha_{11} + 2\alpha_8 d_1 B_{66} + 2\alpha_{11} B_{66} + \alpha_9 d_{11} B_{11} + 2\alpha_{12} d_1 B_{11} + \alpha_{13} B_{11})\alpha_1\, dxdy \\
a_{14} &= \int_0^a\int_0^b (-B_{11}^s\alpha_{12} - d_1 B_{11}^s\alpha_9 - B_{12}^s\alpha_8 - d_1 B_{12}^s\alpha_3 - 2B_{66}^s\alpha_8)\alpha_6\, dxdy \\
a_{41} &= \int_0^a\int_0^b (2d_1 B_{66}^s\alpha_8 + B_{12}^s\alpha_{11} + d_{11} B_{11}^s\alpha_9 + 2B_{66}^s\alpha_{11} + 2d_1 B_{11}^s\alpha_{12} + B_{11}^s\alpha_{13})\alpha_1\, dxdy \\
a_{15} &= \int_0^a\int_0^b (L^1\alpha_6 + d_1 L^1\alpha_1)\alpha_6\, dxdy \\
a_{51} &= \int_0^a\int_0^b (-L^1\alpha_9)\alpha_1\, dxdy \\
a_{22} &= \int_0^a\int_0^b (A_{66}\alpha_{10} + \alpha_7 d_1 A_{66} + \alpha_4 A_{22})\alpha_2\, dxdy \\
a_{23} &= \int_0^a\int_0^b (-\alpha_4 B_{22} - \alpha_{10} B_{12} - 2\alpha_7 d_1 B_{66} - 2\alpha_{10} B_{66})\alpha_2\, dxdy \\
a_{32} &= \int_0^a\int_0^b (2\alpha_8 d_1 B_{12} + \alpha_5 B_{22} + 2\alpha_8 d_1 B_{66} + 2\alpha_{11} B_{66} + \alpha_3 d_{11} B_{12} + \alpha_{11} B_{12})\alpha_1\, dxdy \\
a_{24} &= \int_0^a\int_0^b (-B_{22}^s\alpha_4 - B_{12}^s\alpha_{10} - 2d_1 B_{66}^s\alpha_7 - 2B_{66}^s\alpha_{10})\alpha_2\, dxdy \\
a_{42} &= \int_0^a\int_0^b (2d_1 B_{12}^s\alpha_8 + d_{11} B_{12}^s\alpha_3 + B_{12}^s\alpha_{11} + 2B_{66}^s\alpha_{11} + 2d_1 B_{66}^s\alpha_8 + B_{22}^s\alpha_5)\alpha_1\, dxdy \\
a_{25} &= \int_0^a\int_0^b (L^1\alpha_2)\alpha_2\, dxdy \\
a_{52} &= \int_0^a\int_0^b (-L^1\alpha_3)\alpha_1\, dxdy \\
a_{33} &= \int_0^a\int_0^b (-\alpha_{13} D_{11} - 2d_1 D_{11}\alpha_{12} - 2d_1 D_{12}\alpha_8 - 2D_{12}\alpha_{11} - d_{11} D_{12}\alpha_3 - D_{22}\alpha_5 - d_{11} D_{11}\alpha_9 \\
&\quad - 4d_1 D_{66}\alpha_8 - 4D_{66}\alpha_{11})\alpha_1\, dxdy \\
a_{34} &= \int_0^a\int_0^b (-\alpha_5 D_{22}^s - 2d_1 D_{11}^s\alpha_{12} - 2D_{12}^s\alpha_{11} - d_{11} D_{12}^s\alpha_3 - 2d_1 D_{12}^s\alpha_8 - 4d_1 D_{66}^s\alpha_8 - 4D_{66}^s\alpha_{11} \\
&\quad - d_{11} D_{11}^s\alpha_9 - D_{11}^s\alpha_{13})\alpha_1\, dxdy
\end{aligned}
\tag{25}
$$

$$
\begin{aligned}
a_{34} &= \int_0^a\int_0^b (-\alpha_5 D_{22}^s - 2d_1 D_{11}^s\alpha_{12} - 2D_{12}^s\alpha_{11} - d_{11} D_{12}^s\alpha_3 - 2d_1 D_{12}^s\alpha_8 - 4d_1 D_{66}^s\alpha_8 - 4D_{66}^s\alpha_{11} \\
&\quad - d_{11} D_{11}^s\alpha_9 - D_{11}^s\alpha_{13})\alpha_1\, dxdy
\end{aligned}
$$

$$
\begin{aligned}
a_{43} &= \int_0^a\int_0^b (-\alpha_3 d_{11} D_{12}^s - D_{22}^s\alpha_5 - 4D_{66}^s\alpha_{11} - 4d_1 D_{66}^s\alpha_8 - D_{11}^s\alpha_{13} - 2d_1 D_{11}^s\alpha_{12} - d_{11} D_{11}^s\alpha_9 \\
&\quad - 2d_1 D_{12}^s\alpha_8 - 2D_{12}^s\alpha_{11})\alpha_1\, dxdy \\
a_{35} &= \int_0^a\int_0^b (2d_1 L^2\alpha_6 + L^2\alpha_9 + L^2\alpha_3 + d_{11} L^2\alpha_1)\alpha_1\, dxdy \\
a_{53} &= \int_0^a\int_0^b (L^2\alpha_3 + L^2\alpha_9)\alpha_1\, dxdy \\
a_{44} &= \int_0^a\int_0^b (-\alpha_3 d_{11} H_{12}^s - H_{22}^s\alpha_5 - 4H_{66}^s\alpha_{11} - 4d_1 H_{66}^s\alpha_8 - H_{11}^s\alpha_{13} - 2d_1 H_{11}^s\alpha_{12} - d_{11} H_{11}^s\alpha_9 \\
&\quad - 2d_1 H_{12}^s\alpha_8 - 2H_{12}^s\alpha_{11} + d_1 A_{55}^s\alpha_6 + A_{55}^s\alpha_9 + A_{44}^s\alpha_3)\alpha_1\, dxdy \\
a_{45} &= \int_0^a\int_0^b (d_{11} L^3\alpha_1 + L^3\alpha_3 + L^3\alpha_9 + d_1 A_{55}^s\alpha_6 + A_{44}^s\alpha_3 + A_{55}^s\alpha_9 + 2d_1 L^3\alpha_6)\alpha_1\, dxdy \\
a_{54} &= \int_0^a\int_0^b (L^3\alpha_3 + L^3\alpha_9 + d_1 A_{55}^s\alpha_6 + A_{44}^s\alpha_3 + A_{55}^s\alpha_9)\alpha_1\, dxdy \\
a_{55} &= \int_0^a\int_0^b (-L^4\alpha_1 + d_1 A_{55}^s\alpha_6 + A_{44}^s\alpha_3 + A_{55}^s\alpha_9)\alpha_1\, dxdy
\end{aligned}
$$

Table 1 Material properties used in the functionally graded plates

Material	Properties	
	E	v
Alimunium (Al)	70	0.3
Alumina (Al_2O_3)	380	0.3

Table 2 Comparison of the deflection and stress components of a simply supported homogeneous square plate (*a/h* = 10)

Loading	Theory	ε_z	$\bar{w}(0)$	$\bar{\sigma}_x\left(\frac{h}{2}\right)$	$\bar{\sigma}_y\left(\frac{h}{3}\right)$	$\bar{\tau}_{xz}(0)$	$\bar{\tau}_{yz}\left(\frac{h}{6}\right)$	$\bar{\tau}_{xy}\left(-\frac{h}{3}\right)$
UL	Zenkour (2006)	=0	0.4665	2.8932	1.9103	0.5114	0.4429	1.2850
	Akavci (2015)	=0	0.4665	2.9809	1.9103	0.4988	0.4363	1.2857
	Akavci (2015)	≠0	0.4635	2.9981	1.8925	0.4782	0.4315	1.2578
	Present	≠0	0.4639	2.9064	1.9153	0.4924	0.4376	1.2557
	Present	=0	0.4665	2.8921	1.9106	0.4958	0.4406	1.2857
SL	Zenkour (2006)	=0	0.2960	1.9955	1.3121	0.2462	0.2132	0.7065
	Present	=0	0.2960	1.9943	1.3123	0.2387	0.2121	0.7066
	Present	≠0	0.2942	2.0075	1.3173	0.2386	0.2120	0.7010

Table 3 The non-dimensional displacement and stress components of an Al/Al2O$_3$ FG square plate subjected to sinusoidal load (*a/h* = 10)

p	ε_z	$\bar{w}(0)$	$\bar{\sigma}_x\left(\frac{h}{2}\right)$	$\bar{\sigma}_y\left(\frac{h}{3}\right)$	$\bar{\tau}_{xz}(0)$	$\bar{\tau}_{yz}\left(\frac{h}{6}\right)$	$\bar{\tau}_{xy}\left(-\frac{h}{3}\right)$
1	≠0	0.4965	2.0076	1.3172	0.0742	0.2120	0.2180
	=0	0.5000	1.9943	1.3123	0.0742	0.2121	0.2198
2	≠0	0.6621	1.7570	1.1535	0.0964	0.1805	0.2910
	=0	0.6662	1.7441	1.1481	0.0964	0.1805	0.2934
4	≠0	0.8987	1.4490	0.9518	0.1266	0.1436	0.3958
	=0	0.9043	1.4365	0.9460	0.1266	0.1436	0.3989
10	≠0	1.2173	1.5449	1.0149	0.1701	0.1518	0.5363
	=0	1.2248	1.5311	1.0084	0.1702	0.1519	0.5406

With

$$
\begin{aligned}
&(\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5) = X_m(Y_n, d_2Y_n, d_{22}Y_n, d_{222}Y_n, d_{2222}Y_n)\\
&(\alpha_6,\alpha_7,\alpha_8) = d_1X_m(Y_n, d_2Y_n, d_{22}Y_n)\\
&(\alpha_9,\alpha_{10},\alpha_{11}) = d_{11}X_m(Y_n, d_2Y_n, d_{22}Y_n)\\
&(\alpha_{12},\alpha_{13}) = d_1Y_n(d_{111}X_m, d_{1111}X_m)
\end{aligned}
\tag{26}
$$

4. Numerical results and discussions

In order to verify the accuracy of the present theory in predicting the static response of simply supported FG plates, different examples are solved and compared with the results of various quasi-3D and 2D shear deformation theories. The material properties of the plate are listed in Table 1. The length of the plate is assumed to be 1 m.

The non-dimensional displacement and stress components are

Table 4 The non-dimensional displacement and stress components of an Al/Al_2O_3 FG square plate subjected to sinusoidal load

p	theory	ε_z	$\bar{\sigma}_x\left(\frac{h}{2}\right)$			$\bar{w}(0)$		
			a/h=4	*a/h*=10	*a/h*=100	*a/h*=4	*a/h*=10	*a/h*=100
1	Present	≠0	0.8686	2.0076	19.7752	0.6189	0.4965	0.4736
	present	=0	0.8365	1.9943	19.7594	0.6394	0.5000	0.4736
4	Present	≠0	0.6318	1.4490	14.2524	1.1044	0.8987	0.8592
	present	=0	0.6009	1.4365	14.2408	1.1421	0.9043	0.8592
10	Present	≠0	0.6745	1.5449	15.1914	1.4934	1.2173	1.1642
	present	=0	0.6402	1.5311	15.1790	1.5445	1.2248	1.1643

Table 6 Non-dimensional stress $\bar{\sigma}_x\left(\frac{h}{2}\right)$ of an Al/Al_2O_3 FG plate subjected to sinusoidal distributed load (a/h=10)

b/a	theory	ε_z	p			
			0	1	4	10
1	Present	≠0	2.0075	2.0076	1.4490	1.5449
	Present	=0	1.9943	1.9944	1.4365	1.5311
2	Present	≠0	4.2148	4.2149	3.2089	3.4458
	Present	=0	4.2071	4.2071	3.2017	3.4379
3	Present	≠0	5.1208	5.1209	4.0779	4.4071
	Present	=0	5.1148	5.1149	4.0732	4.4019

$$
\begin{aligned}
&\bar{w} = \frac{10h^3E_c}{q_0a^4} w\left(\frac{a}{2},\frac{b}{2},z\right)\\
&\bar{\sigma}_x(z) = \frac{h}{q_0a}\sigma_x\left(\frac{a}{2},\frac{b}{2},z\right)\\
&\bar{\sigma}_y(z) = \frac{h}{q_0a}\sigma_y\left(\frac{a}{2},\frac{b}{2},z\right)\\
&\bar{\tau}_{xy}(z) = \frac{h}{q_0a}\tau_{xy}(0,0,z)\ \bar{\tau}_{xz}(z) = \frac{h}{q_0a}\tau_{xz}\left(0,\frac{b}{2},z\right)\\
&\bar{\tau}_{yz}(z) = \frac{h}{q_0a}\tau_{yz}\left(\frac{a}{2},0,z\right)
\end{aligned}
\tag{27}
$$

The non-dimensional displacement and stress components for Fig. 6

$$
\begin{aligned}
&\bar{u}(z) = \frac{E_c}{hq_0}u\left(0,\frac{b}{2},z\right)\ \bar{w} = \frac{E_c}{hq_0}w\left(\frac{a}{2},\frac{b}{2},z\right)\ \bar{\sigma}_x(z) = \frac{\sigma_x\left(\frac{a}{2},\frac{b}{2},z\right)}{q_0}\\
&\bar{\sigma}_z(z) = \frac{\sigma_z\left(\frac{a}{2},\frac{b}{2},z\right)}{q_0}\ \bar{\tau}_{xy}(z) = \frac{\tau_{xy}(0,0,z)}{q_0}\\
&\bar{\tau}_{xz}(z) = \frac{\tau_{xz}\left(0,\frac{b}{2},z\right)}{q_0}
\end{aligned}
\tag{28}
$$

In Table 2, the non-dimensional displacement and stresses of a simply supported homogeneous square plate

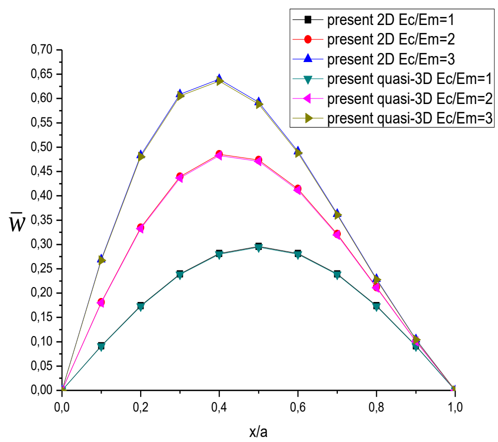

Fig. 2 Deflection of the FG plate with variation of the material stiffness through the length for different E_c/E_m ratios (p=2)

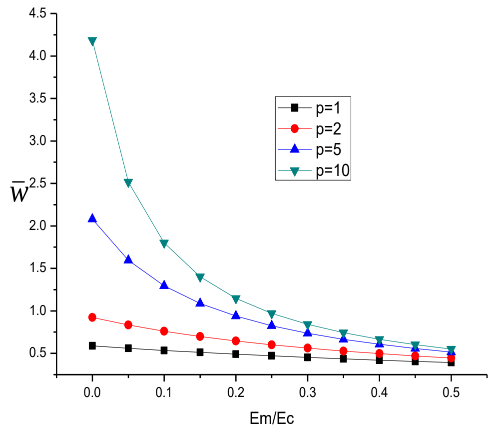

Fig. 3 The effect of material anisotropy on the dimensionless center deflection $\bar{w}$ of an FG plate for different values of p

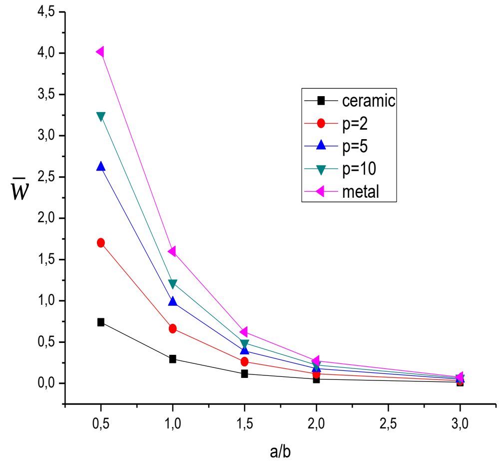

Fig. 4 Dimensionless center deflection $\bar{w}$ as a function of the aspect ratio (a/b) of an FGM plate (a/h=10)

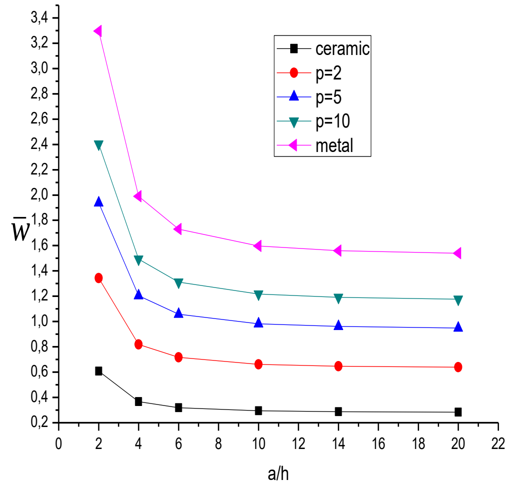

Fig. 5 Dimensionless center deflection $\bar{w}$ as a function of the side to-thickness ratio (a/h) of an FG square plate

subjected to uniformly and sinusoidal distributed loads are presented.

Table 2 shows the calculated results of non-dimensional displacement and stress components of the Homogeneous square plate as compared with the published results of a generalized 2D shear deformation theory by Zenkour (2006) and Akavci (2015), where ε_z=0. It can be seen from the table that the present 2D theory results are in excellent agreement with the 2D theory results of Zenkour (2006) and Akavci (2015). In addition, the present quasi-3D theory yields more accurate results than those obtained by the other two theories.

In Tables 3 and 4, the non-dimensional displacement and stresses of an Al/Al_2O_3 FGM square plate subjected to sinusoidal distributed load are presented for different values of the power-law index. Table 3 shows the calculated results of non-dimensional displacement and stress components of the square FGM plate under uniform load as compared with the results of a generalized 2D shear deformation theory. It is well known that, the quasi-3D theory gives more accurate results than those obtained by the 2D theory where ε_z=0. However, Table 3 demonstrates that these two theories give almost the same results. This is due to the used thickness ratio (a/h=10) which is not appropriate for very thick plates. This table also shows that, the transverse displacement w and the shear stresses $\bar{\tau}_{xz}$ and $\bar{\tau}_{xy}$ increase with the increasing value of power law index p.

Table 4 presents the non-dimensional in-plane stresses $\bar{\sigma}_x$ and non-dimensional transverse displacements $\bar{w}$ of a square plate for different a/h ratios. The present results are compared with the present 2D higher order shear deformation theory. The present quasi-3D that includes both transverse shear and normal deformations give accurate results than those given by the other theory.

The non-dimensional displacements and stresses are presented in Tables 5-6 for various values of aspect ratio b/a, thickness ratio a/h and exponent value p. Table 5 presents the central transverse displacements of the very thick EGM plates. The obtained results are compared with the 2D theory. Since the proposed theory includes the thickness-stretching effect, the results are accurate for thick plates. Meanwhile, 2D theory which do not include the thickness stretching effect overestimate the results. In Table 6, the calculated non-dimensional stresses are presented as

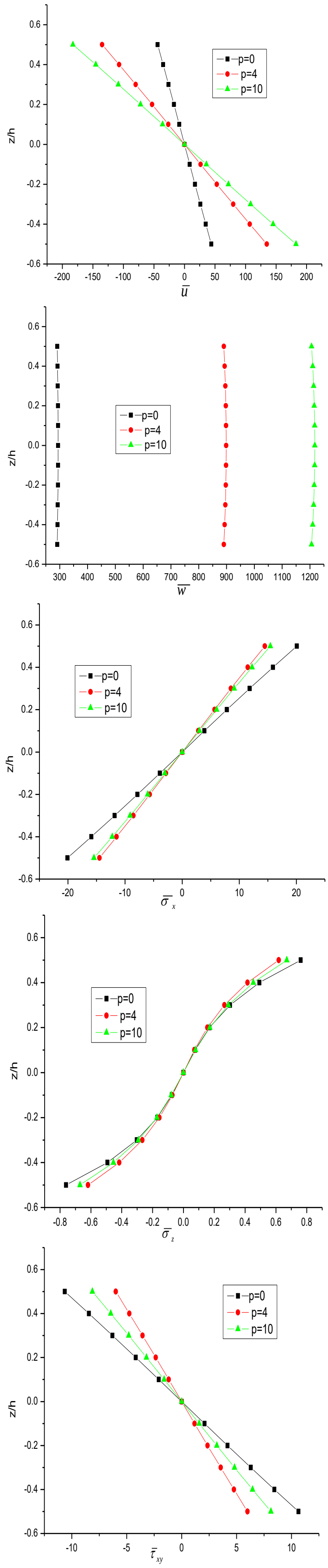

Fig. 6 The distributions of the non-dimensional displacement and stresses of FG plate (a/b=1, a/h=10)

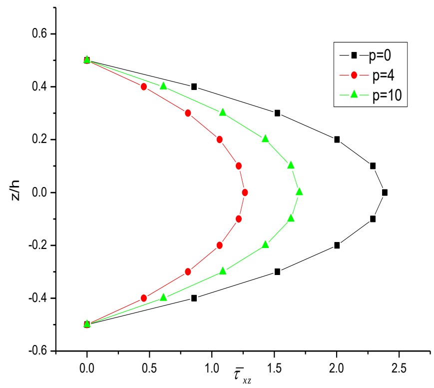

Table 6 Continued

compared with the 2D theory. It is evident from the tables that the present computations are in good agreement with the quasi-3D solutions.

Fig. 2 presents the deflection of the FG plate with variation of the material stiffness through the length for different E_c/E_m ratios. Generally, it is reported that an increase in the stiffness ratio, E_c/E_m, increase the deflection of the FG plate with property variation through the length, which shows the effect of different E_c/E_m ratios on the deflection of the FG plate.

On the whole stiffness of the FG plate is robustly associated with the direction of the material stiffness variation and the correct selection of the modulus ratio E_c/E_m. The modulus ratio also influences the location of maximum deflection. As the E_c/E_m ratio increases, the location of the maximum deflection moves towards the left side of the plate (dominated side). By using this aspect, the location of the maximum deflection, the stiffness of the FG plate can be controlled to meet the desired application with specific performance.

The center deflections of the simply supported FG square plate are compared in Fig. 3 for various ratios of moduli, E_m/E_c (for a given thickness, a/h=10). In other words, the deflections are computed and compared for plates with different ceramic-metal mixtures. It is clear that the deflections decrease smoothly as the volume fraction exponent decreases, and decrease as the ratio of metal-to-ceramic moduli increases.

Figs. 4 and 5 show the variation of the center deflection for various power law exponent p and with different aspect and side-to-thickness ratios, respectively. The FG plate deflection is between those of plate made of ceramic (Al_2O_3) and metal (Al). It can be observed that the deflection of metal rich plates is larger when compared to ceramic rich FGM plates, which can be attributed to the fact that the Young's modulus of ceramic (Al_2O_3, 380 GPa) is higher than that of metal (Al, 70 GPa). Hence for FG plates, the transverse deflection decreases as the power law exponent p decreases. In addition, the deflection of the FGM plate decreases as the aspect ratio increases, whereas it may be unchanged as the side-to-thickness ratio increases.

The stress and displacement distributions through the thickness of Al/Al_2O_3 FGM square plate, under sinusoidal load, are presented in Fig. 6. The results are plotted for various values of power law index p. According to Fig. 6, it is important to note that, the through the thickness distributions of in-plane stresses $\bar{\sigma}_x$ and $\bar{\tau}_{xy}$ are linear for FG plate with in-plane variation of material stiffness along the length. The variation of the in-plane stress $\bar{\sigma}_x$ at the mid-plane (*xy*) of the FG plate with in-plane variation of material stiffness along the length is depicted in Fig. 6. The maximum tensile and compressive stresses occur at the bottom and top surfaces, respectively. It is observed that the stress $\bar{\sigma}_x$ also becomes zero at the mid-surface of the thickness (z=0) for FG plates with property variation through the length, which is the same trend for the case with constant property (homogenous plate).

5. Conclusions

In this work, the bending response of thick FG plates with power-law variation of volume fraction within the length is studied using a quasi-3D hyperbolic shear deformation theory. The exactitude of the present model has been confirmed for bending investigation of a simply supported FG plate under uniformly and sinusoidal distributed loads. From this study, it is observed that contrary to the case of a homogeneous plate, the maximum deflection does not found in the middle of the FG plate with in-plane distribution of material rigidity. It is also noticed that the overall rigidity of the FG plate strongly depends on the direction of the variation of rigidity of the material and the ratio between Young's moduli (E_c/E_m) of the two phases. In addition, the location of the maximum transverse displacement does not depend on the thickness of the FG plate and the maximum deflection increases with an increase in the power index (p). The effect of boundary conditions can be considered in future using the same methodology as is described by Ait Amar Meziane *et al.* (2014) and Abdelaziz *et al.* (2017).

References

Abdelaziz, H.H., Ait Amar Meziane, M., Bousahla, A.A., Tounsi, A., Mahmoud, S.R. and Alwabli, A.S. (2017), “An efficient hyperbolic shear deformation theory for bending, buckling and free vibration of FGM sandwich plates with various boundary conditions”, *Steel Compos. Struct.*, **25**(6), 693-704.

Abualnour, M., Houari, M.S.A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2018), “A novel quasi-3D trigonometric plate theory for free vibration analysis of advanced composite plates”, *Compos. Struct.*, **184**, 688-697.

Ahmed, A. (2014), “Post buckling analysis of sandwich beams with functionally graded faces using a consistent higher order theory”, *Int. J. Civil Struct.Environ.*, **4**(2), 59-64.

Ahouel, M., Houari, M.S.A., Adda Bedia, E.A. and Tounsi, A. (2016), “Size-dependent mechanical behavior of functionally graded trigonometric shear deformable nanobeams including neutral surface position concept”, *Steel Compos. Struct.*, **20**(5), 963-981.

Ait Amar Meziane, M., Abdelaziz, H.H. and Tounsi, A. (2014), “An efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions”, *J. Sandw. Struct. Mater.*, **16**(3), 293-318.

Ait Atmane, H., Tounsi, A. and Adda Bedia, E.A. (2010), “Free vibration analysis of functionally graded plates resting on Winkler-Pasternak elastic foundations using a new shear deformation theory”, *Int. J. Mech. Mater. Des.*, **6**(2), 113-121.

Ait Yahia, S., Ait Atmane, H., Houari, M.S.A. and Tounsi, A. (2015), “Wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories”, *Struct. Eng. Mech.*, **53**(6), 1143-1165.

Akavci, S. (2010), “Two new hyperbolic shear displacement models for orthotropic laminated composite plates”, *Mech. Compos.Mater.*,**46(2)**, 215-226.

Akavci, S.S. and Tanrikulu, A.H. (2015), “Static and free vibration analysis of functionally graded plates based on a new quasi-3D and 2D shear deformation theories”, *Compos. Part B*, **83**, 203-215.

Aldousari, S.M. (2017), "Bending analysis of different material distributions of functionally graded beam”, *Appl. Phys. A*, **123**, 296.

Amirpour, M.R. Das, E.I. and Flores, S. (2016), “Analytical solutions for elastic deformation of functionally graded thick plates with in-plane stiffness variation using higher order shear deformation theory”, *Compos. Part B.*, **94**, 109-121.

Arani, A.J. and Kolahchi, R. (2016), “Buckling analysis of embedded concrete columns armed with carbon nanotubes”, *Comput. Concrete*, **17**(5), 567-578.

Attia, A., Bousahla, A.A., Tounsi, A., Mahmoud, S.R. and Alwabli, A.S. (2018), “A refined four variable plate theory for thermoelastic analysis of FGM plates resting on variable elastic foundations”, *Struct. Eng. Mech.*, **65**(4), 453-464.

Bachir Bouiadjra, M, Houari, M.S.A. and Tounsi, A. (2012), “Thermal buckling of functionally graded plates according to a four-variable refined plate theory”, *J. Therm. Stress.*, **35**, 677-694.

Bakhadda, B., Bachir Bouiadjra, M., Bourada, F., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2018), “Dynamic and bending analysis of carbon nanotube-reinforced composite plates with elastic foundation”, *Wind Struct.*, **27**(5), 311-324.

Belabed, Z., Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2018), “A new 3-unknown hyperbolic shear deformation theory for vibration of functionally graded sandwich plate”, *Earthq. Struct.*, **14**(2), 103-115.

Belabed, Z., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and Anwar Bég, O. (2014), “An efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates”, *Compos. Part B*, **60**, 274-283.

Beldjelili, Y., Tounsi, A. and Mahmoud, S.R. (2016), “Hygro-thermo-mechanical bending of S-FGM plates resting on variable elastic foundations using a four-variable trigonometric plate theory”, *Smart Struct. Syst.*, **18**(4), 755-786.

Belkorissat, I., Houari, M.S.A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2015), “On vibration properties of functionally graded nano-plate using a new nonlocal refined four variable model”, *Steel Compos. Struct.*, **18**(4), 1063-1081.

Bellifa, H., Bakora, A., Tounsi, A., Bousahla, A.A. and Mahmoud, S.R. (2017a), “An efficient and simple four variable refined plate theory for buckling analysis of functionally graded plates”, *Steel Compos. Struct.*, **25**(3), 257-270.

Bellifa, H., Benrahou, K.H., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2017b), “A nonlocal zeroth-order shear deformation theory for nonlinear postbuckling of nanobeams”, *Struct. Eng. Mech.*, **62**(6), 695-702.

Bellifa, H., Benrahou, K.H., Hadji, L., Houari, M.S.A. and Tounsi, A. (2016), “Bending and free vibration analysis of functionally graded plates using a simple shear deformation theory and the concept the neutral surface position”, *J. Braz. Soc. Mech. Sci. Eng.*, **38**(1), 265-275.

Benadouda, M., Ait Atmane, H., Tounsi, A., Bernard, F. and Mahmoud, S.R. (2017), “An efficient shear deformation theory

for wave propagation in functionally graded material beams with porosities", *Earthq. Struct.*, **13**(3), 255-265.
Benahmed, A., Houari, M.S.A., Benyoucef, S., Belakhdar, K. and Tounsi, A. (2017), "A novel quasi-3D hyperbolic shear deformation theory for functionally graded thick rectangular plates on elastic foundation", *Geomech. Eng.*, **12**(1), 9-34.
Benchohra, M., Driz, H., Bakora, A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2018), "A new quasi-3D sinusoidal shear deformation theory for functionally graded plates", *Struct. Eng. Mech.*, **65**(1), 19-31.
Bennoun, M., Houari, M.S.A. and Tounsi, A. (2016), "A novel five variable refined plate theory for vibration analysis of functionally graded sandwich plates", *Mech. Adv. Mater. Struct.*, **23**(4), 423-431.
Besseghier, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), "Free vibration analysis of embedded nanosize FG plates using a new nonlocal trigonometric shear deformation theory", *Smart Struct. Syst.*, **19**(6), 601-614.
Bilouei, B.S., Kolahchi, R. and Bidgoli, M.R. (2016), "Buckling of concrete columns retrofitted with Nano-Fiber Reinforced Polymer (NFRP)", *Comput. Concrete*, **18**(5), 1053-1063.
Bouadi, A., Bousahla, A.A., Houari, M.S.A., Heireche, H. and Tounsi, A. (2018), "A new nonlocal HSDT for analysis of stability of single layer graphene sheet", *Adv. Nano Res.*, **6**(2), 147-162.
Bouafia, K., Kaci, A., Houari, M.S.A., Benzair, A. and Tounsi, A. (2017), "A nonlocal quasi-3D theory for bending and free flexural vibration behaviors of functionally graded nanobeams", *Smart Struct. Syst.*, **19**(2), 115-126.
Bouderba, B., Houari, M.S.A. and Tounsi, A. (2013), "Thermomechanical bending response of FGM thick plates resting on Winkler-Pasternak elastic foundations", *Steel Compos. Struct.*, **14**(1), 85-104.
Bouderba, B., Houari, M.S.A. and Tounsi, A. and Mahmoud, S.R. (2016), "Thermal stability of functionally graded sandwich plates using a simple shear deformation theory", *Struct. Eng. Mech.*, **58**(3), 397-422.
Bouhadra, A., Tounsi, A., Bousahla, A.A., Benyoucef, S. and Mahmoud, S.R. (2018), "Improved HSDT accounting for effect of thickness stretching in advanced composite plates", *Struct. Eng. Mech.*, **66**(1), 61-73.
Boukhari, A., Ait Atmane, H., Houari, M.S.A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2016), "An efficient shear deformation theory for wave propagation of functionally graded material plates", *Struct. Eng. Mech.*, **57**(5), 837-859.
Bounouara, F., Benrahou, K.H., Belkorissat, I. and Tounsi, A. (2016), "A nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates resting on elastic foundation", *Steel Compos. Struct.*, **20**(2), 227-249.
Bourada, F., Amara, K. and Tounsi, A. (2016), "Buckling analysis of isotropic and orthotropic plates using a novel four variable refined plate theory", *Steel Compos. Struct.*, **21**(6), 1287-1306.
Bourada, M., Kaci, A., Houari, M.S.A. and Tounsi, A. (2015), "A new simple shear and normal deformations theory for functionally graded beams", *Steel Compos. Struct.*, **18**(2), 409-423.
Bourada, M., Tounsi, A., Houari, M.S.A. and Adda Bedia, E.A. (2012), "A new four-variable refined plate theory for thermal buckling analysis of functionally graded sandwich plates", *J. Sandw. Struct. Mater.*, **14**, 5-33.
Bousahla, A.A., Benyoucef, S., Tounsi, A. and Mahmoud, S.R. (2016), "On thermal stability of plates with functionally graded coefficient of thermal expansion", *Struct. Eng. Mech.*, **60**(2), 313-335.
Bousahla, A.A., Houari, M.S.A., Tounsi, A. and AddaBedia, E.A. (2014), "A novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates", *Int. J. Comput. Meth.*, **11**(6), 1350082.
Carrera, E, Brischetto, S, Cinefra, M. and Soave, M. (2011), "Effects of thickness stretching in functionally graded plates and shells", *Compos. Part B*, **42**(2), 123-133.
Cherif, R.H., Meradjah, M., Zidour, M., Tounsi, A., Belmahi, H. and Bensattalah, T. (2018), "Vibration analysis of nano beam using differential transform method including thermal effect", *J. Nano Res.*, **54**, 1-14.
Chikh, A., Tounsi, A., Hebali, H. and Mahmoud, S.R. (2017), "Thermal buckling analysis of cross-ply laminated plates using a simplified HSDT", *Smart Struct. Syst.*, **19**(3), 289-297.
Dash, S., Sharma, N., Mahapatra, T.R., Panda, S.K. and Sahu, P. (2018), "Free vibration analysis of functionally graded sandwich flat panel", *IOP Conf. Ser.: Mater. Sci. Eng.*, **377**, 012140.
Draiche, K., Tounsi, A. and Mahmoud, S.R. (2016), "A refined theory with stretching effect for the flexure analysis of laminated composite plates", *Geomech. Eng.*, **11**(5), 671-690.
Dutta, G., Panda, S.K., Mahapatra, T.R. and Singh, V.K. (2017), "Electro-magneto-elastic response of laminated composite plate: A finite element approach", *Int. J. Appl. Comput. Math.*, **3**(3), 2573-2592.
El-Haina, F., Bakora, A., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2017), "A simple analytical approach for thermal buckling of thick functionally graded sandwich plates", *Struct. Eng. Mech.*, **63**(5), 585-595.
Fahsi, A., Tounsi, A., Hebali, H., Chikh, A., Adda Bedia, E.A. and Mahmoud, S.R. (2017), "A four variable refined nth-order shear deformation theory for mechanical and thermal buckling analysis of functionally graded plates", *Geomech. Eng.*, **13**(3), 385-410.
Fakhar, A. and Kolahchi, R. (2018), "Dynamic buckling of magnetorheological fluid integrated by visco-piezo-GPL reinforced plates", *Int. J. Mech. Sci.*, **144**, 788-799.
Fourn, H., Ait Atmane, H., Bourada, M., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2018), "A novel four variable refined plate theory for wave propagation in functionally graded material plates", *Steel Compos. Struct.*, **27**(1), 109-122.
Golabchi, H., Kolahchi, R. and Rabani Bidgoli, M. (2018), "Vibration and instability analysis of pipes reinforced by SiO2 nanoparticles considering agglomeration effects", *Comput. Concrete*, **21**(4), 431-440.
Grover, N., Maiti, D.K. and Singh, B.N. (2013), "A new inverse hyperbolic shear deformation theory for static and buckling analysis of laminated composite and sandwich plates", *Compos. Struct.*, **95**, 667-675.
Hachemi, H., Kaci, A., Houari, M.S.A., Bourada, A., Tounsi, A. and Mahmoud, S.R. (2017), "A new simple three-unknown shear deformation theory for bending analysis of FG plates resting on elastic foundations", *Steel Compos. Struct.*, **25**(6), 717-726.
Hajmohammad, M.H., Azizkhani, M.B. and Kolahchi, R. (2018a), "Multiphase nanocomposite viscoelastic laminated conical shells subjected to magneto-hygrothermal loads: Dynamic buckling analysis", *Int. J. Mech. Sci.*, **137**, 205-213.
Hajmohammad, M.H., Farrokhian, A. and Kolahchi, R. (2018b), "Smart control and vibration of viscoelastic actuator-multiphase nanocomposite conical shells-sensor considering hygrothermal load based on layerwise theory", *Aerosp. Sci. Technol.*, **78**, 260-270.
Hajmohammad, M.H., Kolahchi, R., Zarei, M.S. and Maleki, M. (2018d), "Earthquake induced dynamic deflection of submerged viscoelastic cylindrical shell reinforced by agglomerated CNTs considering thermal and moisture effects", *Compos. Struct.*, **187**, 498-508.
Hajmohammad, M.H., Maleki, M. and Kolahchi, R. (2018c),

"Seismic response of underwater concrete pipes conveying fluid covered with nano-fiber reinforced polymer layer", *Soil Dyn. Earthq. Eng.*, **110**, 18-27.

Hajmohammad, M.H., Maleki, M. and Kolahchi, R. (2018e), "Seismic response of underwater concrete pipes conveying fluid covered with nano-fiber reinforced polymer layer", *Soil Dyn. Earthq. Eng.*, **110**, 18-27.

Hajmohammad, M.H., Zarei, M.S., Nouri, A. and Kolahchi, R. (2017), "Dynamic buckling of sensor/functionally graded-carbon nanotube-reinforced laminated plates/actuator based on sinusoidal-visco-piezoelasticity theories", *J. Sandw. Struct. Mater.*, In Press.

Hamidi, A., Houari, M.S.A., Mahmoud, S.R. and Tounsi, A. (2015), "A sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates", *Steel Compos. Struct.*, **18**(1), 235-253.

Hebali, H., Tounsi, A., Houari, M.S.A., Bessaim, A. and Adda Bedia, E.A. (2014), "A new quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of functionally graded plates", *ASCE J. Eng. Mech.*, **140**(2), 374-383.

Henderson, J.P., Plummer, A. and Johnston, N. (2018), "An electro-hydrostatic actuator for hybrid active-passive vibration isolation", *Int. J. Hydromechatron.*, **1**(1), 47-71.

Hirwani, C.K. and Panda, S.K. (2018), "Numerical and experimental validation of nonlinear deflection and stress responses of pre-damaged glass-fibre reinforced composite structure", *Ocean Eng.*, **159**, 237-252.

Hirwani, C.K. and Panda, S.K. (2019), "Nonlinear finite element solutions of thermoelastic deflection and stress responses of internally damaged curved panel structure", *Appl. Math. Modell.*, **65**, 303-317.

Hirwani, C.K., Biswash, S., Mehar, K. and Panda, S.K. (2018), "Numerical flexural strength analysis of thermally stressed delaminated composite structure under sinusoidal loading", *IOP Conf. Ser.: Mater. Sci. Eng.*, **338**, 012019.

Houari, M.S.A., Tounsi, A., Bessaim, A. and Mahmoud, S.R. (2016), "A new simple three-unknown sinusoidal shear deformation theory for functionally graded plates", *Steel Compos. Struct.*, **22**(2), 257-276.

Janghorban, M. and Zare, A. (2011), "Free vibration analysis of functionally graded carbon nanotubes with variable thickness by differential quadrature method", *Phys. E: Low-Dimens. Syst. Nanostruct.*, **43**(9), 1602-1604.

Kaci, A., Houari, M.S.A., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2018), "Post-buckling analysis of shear-deformable composite beams using a novel simple two-unknown beam theory", *Struct. Eng. Mech.*, **65**(5), 621-631.

Kadari, B., Bessaim, A., Tounsi, A., Heireche, H., Bousahla, A.A. and Houari, M.S.A. (2018), "Buckling analysis of orthotropic nanoscale plates resting on elastic foundations", *J. Nano Res.*, Accepted.

Kar, V.R. and Panda, S.K. (2015a), "Large deformation bending analysis of functionally graded spherical shell using FEM", *Struct. Eng. Mech.*, **53**(4), 661-679.

Kar, V.R. and Panda, S.K. (2015b), "Nonlinear flexural vibration of shear deformable functionally graded spherical shell panel", *Steel Compos. Struct.*, **18**(3), 693-709.

Kar, V.R. and Panda, S.K. (2016a), "Nonlinear thermomechanical deformation behaviour of P-FGM shallow spherical shell panel", *Chin. J. Aeronaut.*, **29**(1), 173-183.

Kar, V.R. and Panda, S.K. (2016b), "Nonlinear thermomechanical behavior of functionally graded material cylindrical/hyperbolic/elliptical shell panel with temperature-dependent and temperature-independent properties", *J. Press. Vess. Technol.*, **138**(6), 061202.

Karama, M., Afaq, K.S. and Mistou, S. (2003), "Mechanical behaviour of laminated composite beam by the new multi-layered laminated composite structures model with transverse shear stress continuity", *Int. J. Sol. Struct.*, **40**(6), 1525-1546.

Karami, B. and Janghorban, M. (2016), "Effect of magnetic field on the wave propagation in nanoplates based on strain gradient theory with one parameter and two-variable refined plate theory", *Mod. Phys. Lett. B.*, **30**(36), 1650421.

Karami, B., Janghorban, M. and Li, L. (2018c), "On guided wave propagation in fully clamped porous functionally graded nanoplates", *Acta Astronaut.*, **143**, 380-390.

Karami, B., Janghorban, M. and Tounsi, A. (2018g), "Nonlocal strain gradient 3D elasticity theory for anisotropic spherical nanoparticles", *Steel Compos. Struct.*, **27**(2), 201-216.

Karami, B., Janghorban, M. and Tounsi, A. (2018i), "Variational approach for wave dispersion in anisotropic doubly-curved nanoshells based on a new nonlocal strain gradient higher order shell theory", *Thin-Wall. Struct.*, **129**, 251-264.

Karami, B., Janghorban, M., Shahsavari, D. and Tounsi, A. (2018h), "A size-dependent quasi-3D model for wave dispersion analysis of FG nanoplates", *Steel Compos. Struct.*, **28**(1), 99-110.

Karami, B., Shahsavari, D. and Janghorban, M. (2018f), "Wave propagation analysis in functionally graded (FG) nanoplates under in-plane magnetic field based on nonlocal strain gradient theory and four variable refined plate theory", *Mech. Adv. Mater. Struct.*, **25**(12), 1047-1057.

Karami, B., Shahsavari, D. and Li, L. (2018b), "Temperature-dependent flexural wave propagation in nanoplate-type porous heterogenous material subjected to in-plane magnetic field", *J. Therm. Stress.*, **41**(4), 483-499.

Karami, B., Shahsavari, D. and Janghorban, M. (2018a), "A size-dependent quasi-3D model for wave dispersion analysis of FG nanoplates", *Steel Compos. Struct.*, **28**(1), 99-110.

Karami, B., Shahsavari, D., Karami, M. and Li, L. (2018e), "Hygrothermal wave characteristic of nanobeam-type inhomogeneous materials with porosity under magnetic field", *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, In Press.

Karami, B., Shahsavari, D., Li, L., Karami, M. and Janghorban, M. (2018d), "Thermal buckling of embedded sandwich piezoelectric nanoplates with functionally graded core by a nonlocal second-order shear deformation theory", *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, In Press.

Katariya, P.V., Hirwani, C.K. and Panda, S.K. (2018), "Geometrically nonlinear deflection and stress analysis of skew sandwich shell panel using higher-order theory", *Eng. Comput.*, In Press.

Katariya, P.V., Panda, S.K. and Mahapatra, T.R. (2017b), "Nonlinear thermal buckling behaviour of laminated composite panel structure including the stretching effect and higher-order finite element", *Adv. Mater. Res.*, **6**(4), 349-361.

Katariya, P.V., Panda, S.K., Hirwani, C.K., Mehar, K. and Thakare, O. (2017a), "Enhancement of thermal buckling strength of laminated sandwich composite panel structure embedded with shape memory alloy fibre", *Smart Struct. Syst.*, **20**(5), 595-605.

Kettaf, F.Z., Houari, M.S.A., Benguediab, M. and Tounsi, A. (2013), "Thermal buckling of functionally graded sandwich plates using a new hyperbolic shear displacement model", *Steel Compos. Struct.*, **15**(4), 399-423.

Khetir, H., Bachir Bouiadjra, M., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), "A new nonlocal trigonometric shear deformation theory for thermal buckling analysis of embedded nanosize FG plates", *Struct. Eng. Mech.*, **64**(4), 391-402.

Kitipornchai, S., Yang, J. and Liew, K.M. (2006), "Random vibration of the functionally graded laminates in thermal environments", *Comput. Meth. Appl. Mech. Eng.*, **195**(9-12), 1075-1095.

Klouche, F., Darcherif, L., Sekkal, M., Tounsi, A. and Mahmoud, S.R. (2017), "An original single variable shear deformation theory for buckling analysis of thick isotropic plates", *Struct. Eng. Mech.*, **63**(4), 439-446.

Koizumi, M. (1993), "The concept of FGM, ceramic transactions", *Function. Grad. Mater.*, **34**, 3-10.

Kolahchi, R. (2017), "A comparative study on the bending, vibration and buckling of viscoelastic sandwich nano-plates based on different nonlocal theories using DC, HDQ and DQ methods", *Aerosp. Sci. Technol.*, **66**, 235-248.

Kolahchi, R. and Moniri Bidgoli, A.M. (2016), "Size-dependent sinusoidal beam model for dynamic instability of single-walled carbon nanotubes", *Appl. Math. Mech.*, **37**(2), 265-274.

Kolahchi, R., Bidgoli, A.M.M. and Heydari, M.M. (2015), "Size dependent bending analysis of FGM nano-sinusoidal plates resting on orthotropic elastic medium", *Struct. Eng. Mech.*, **55**(5), 1001-1014.

Kolahchi, R., Hosseini, H. and Esmailpour, M. (2016a), "Differential cubature and quadrature-Bolotin methods for dynamic stability of embedded piezoelectric nanoplates based on visco-nonlocal-piezoelasticity theories", *Compos. Struct.*, **157**, 174-186.

Kolahchi, R., Keshtegar, B. and Fakhar, M.H. (2017c), "Optimization of dynamic buckling for sandwich nanocomposite plates with sensor and actuator layer based on sinusoidal-visco-piezoelasticity theories using Grey Wolf algorithm", *J. Sandw. Struct. Mater.*, In Press.

Kolahchi, R., Safari, M. and Esmailpour, M. (2016b), "Dynamic stability analysis of temperature-dependent functionally graded CNT-reinforced visco-plates resting on orthotropic elastomeric medium", *Compos. Struct.*, **150**, 255-265.

Kolahchi, R., Zarei, M.S., Hajmohammad, M.H. and Nouri, A. (2017b), "Wave propagation of embedded viscoelastic FG-CNT-reinforced sandwich plates integrated with sensor and actuator based on refined zigzag theory", *Int. J. Mech. Sci.*, **130**, 534-545.

Kolahchi, R., Zarei, M.S., Hajmohammad, M.H. and Oskouei, A.N. (2017a), "Visco-nonlocal-refined Zigzag theories for dynamic buckling of laminated nanoplates using differential cubature-Bolotin methods", *Thin-Wall. Struct.*, **113**, 162-169.

Kolahchi, R. and Cheraghbak, A. (2017), "Agglomeration effects on the dynamic buckling of viscoelastic microplates reinforced with SWCNTs using Bolotin method", *Nonlin. Dyn.*, **90**, 479-492.

Larbi Chaht, F., Kaci, A., Houari, M.S.A., Tounsi, A., Anwar Bég, O. and Mahmoud, S.R. (2015), "Bending and buckling analyses of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect", *Steel. Compos. Struct.*, **18**(2), 425-442.

Madani, H., Hosseini, H. and Shokravi, M. (2016), "Differential cubature method for vibration analysis of embedded FG-CNT-reinforced piezoelectric cylindrical shells subjected to uniform and non-uniform temperature distributions", *Steel Compos. Struct.*, **22**(4), 889-913.

Mahapatra, T.R., Kar, V.R., Panda, S.K. and Mehar, K. (2017a), "Nonlinear thermoelastic deflection of temperature-dependent FGM curved shallow shell under nonlinear thermal loading", *J. Therm. Stress.*, **40**(9), 1184-1199.

Mahapatra, T.R., Mehar, K., Panda, S.K., Dewangan, S. and Dash, S. (2017b), "Flexural strength of functionally graded nanotube reinforced sandwich spherical panel", *IOP Conf. Ser.: Mater. Sci. Eng.*, **178**, 012031.

Mahi, A., Adda Bedia, E.A. and Tounsi, A. (2015), "A new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates", *Appl. Math. Model.*, **39**, 2489-2508.

Mantari, J.L. and GuedesSoares, C. (2013), "A novel higher-order shear deformation theory with stretching effect for functionally graded plates", *Compos. Part B: Eng.*, **45**(1), 268-281.

Mantari, J.L., Oktem, A.S. and GuedesSoares, C. (2012), "A new trigonometric shear deformation theory for isotropic, laminated composite and sandwich plates", *Int. J. Sol. Struct.*, **49**(1), 43-53.

Mehar, K. and Panda, S.K. (2017a), "Thermoelastic analysis of FG-CNT reinforced shear deformable composite plate under various loadings", *Int. J. Appl. Mech.*, **14**(2), 1750019.

Mehar, K. and Panda, S.K. (2017b), "Numerical investigation of nonlinear thermomechanical deflection of functionally graded CNT reinforced doubly curved composite shell panel under different mechanical loads", *Compos. Struct.*, **161**, 287-298.

Mehar, K. and Panda, S.K. (2018), "Thermoelastic flexural analysis of FG-CNT doubly curved shell panel", *Aircr. Eng. Aerosp. Technol.*, **90**(1), 11-23.

Mehar, K., Panda, S.K. and Mahapatra, T.R. (2017a), "Thermoelastic deflection responses of CNT reinforced sandwich shell structure using finite element method", *Scientia Iranic.*, In Press.

Mehar, K., Panda, S.K. and Mahapatra, T.R. (2018), "Large deformation bending responses of nanotube-reinforced polymer composite panel structure: Numerical and experimental analyses", *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*, In Press.

Mehar, K., Panda, S.K. and Patle, B.K. (2017b), "Thermoelastic Vibration and flexural behavior of FG-CNT reinforced composite curved panel", *Int. J. Appl. Mech.*, **9**(4), 1750046.

Mehar, K., Panda, S.K. and Patle, B.K. (2017c), "Stress, deflection, and frequency analysis of CNT reinforced graded sandwich plate under uniform and linear thermal environment: A finite element approach", *Polym. Compos.*, In Press.

Meksi, R, Benyoucef, S., Mahmoudi, A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, SR. (2019), "An analytical solution for bending, buckling and vibration responses of FGM sandwich plates", *J. Sandw. Struct. Mater.*, 1099636217698443.

Menasria, A., Bouhadra, A., Tounsi, A., Bousahla, A.A. and Mahmoud, S.R. (2017), "A new and simple HSDT for thermal stability analysis of FG sandwich plates", *Steel. Compos. Struct.*, **25**(2), 157-175.

Mindlin, R.D. (1951), "Influence of rotatory inertia and shear on flexural motions of isotropic, elastic plates", *J. Appl. Mech.*, **18**(1), 31-38.

Mokhtar, Y., Heireche, H., Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2018), "A novel shear deformation theory for buckling analysis of single layer graphene sheet based on nonlocal elasticity theory", *Smart Struct. Syst.*, **21**(4), 397-405.

Mouffoki, A., Adda Bedia, E.A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), "Vibration analysis of nonlocal advanced nanobeams in hygro-thermal environment using a new two-unknown trigonometric shear deformation beam theory", *Smart Struct. Syst.*, **20**(3), 369-383.

Nami, M.R. and Janghorban, M. (2014), "Resonance behavior of FG rectangular micro/nano plate based on nonlocal elasticity theory and strain gradient theory with one gradient constant", *Compos. Struct.*, **111**, 349-353.

Pradyumna, S. and Bandyopadhyay, J.N. (2008), "Free vibration analysis of functionally graded curved panels using a higher-order finite element formulation", *J. Sound Vibr.*, **318**(1-2), 176-192.

Reddy, J.N. (1984), "A simple higher-order theory for laminated composite plates", *J. Appl. Mech.*, **51**(4), 745-752.

Reddy, J.N. (2000), "Analysis of functionally graded plates", *Int.*

J. Numer. Meth. Eng., **47**(1-3), 663-684.
Reissner, E. (1945), "The effect of transverse shear deformation on the bending of elastic plates", *J. Appl. Mech.*, **12**(2), 69-72.
Ren, J.G. (1986), "A new theory of laminated plate", *Compos. Sci. Technol.*, **26**(3), 225-239.
Sahoo, S.S., Panda, S.K. and Singh, V.K. (2017), "Experimental and numerical investigation of static and free vibration responses of woven glass/epoxy laminated composite plate", *Proceedings of the Institution of Mechanical Engineers, Part L: J. Mater.: Des. Appl.*, **231**(5), 463-478.
Sekkal, M., Fahsi, B., Tounsi, A. and Mahmoud, S.R. (2017a), "A novel and simple higher order shear deformation theory for stability and vibration of functionally graded sandwich plate", *Steel Compos. Struct.*, **25**(4), 389-401.
Sekkal, M., Fahsi, B., Tounsi, A. and Mahmoud, S.R. (2017b), "A new quasi-3D HSDT for buckling and vibration of FG plate", *Struct. Eng. Mech.*, **64**(6), 737-749.
Shahsavari, D., Karami, B., Fahham, H.R. and Li, L. (2018c), "On the shear buckling of porous nanoplates using a new size-dependent quasi-3D shear deformation theory", *Acta Mech.*, In Press.
Shahsavari, D., Karami, B., Janghorban, M. and Li, L. (2017), "Dynamic characteristics of viscoelastic nanoplates under moving load embedded within visco-Pasternak substrate and hygrothermal environment", *Mater. Res. Expr.*, **4**(8), 085013.
Shahsavari, D., Karami, B. and Mansouri, S. (2018a), "Shear buckling of single layer graphene sheets in hygrothermal environment resting on elastic foundation based on different nonlocal strain gradient theories", *Eur. J. Mech.-A/Sol.*, **67**, 200-214.
Shahsavari, D., Shahsavarib, M., Li, L. and Karami, B. (2018b), "A novel quasi-3D hyperbolic theory for free vibration of FG plates with porosities resting on Winkler/Pasternak/Kerr foundation", *Aerosp. Sci. Technol.*, **72**, 134-149.
Shokravi, M. (2017a), "Vibration analysis of silica nanoparticles-reinforced concrete beams considering agglomeration effects", *Comput. Concrete*, **19**(3), 333-338.
Shokravi, M. (2017b), "Buckling analysis of embedded laminated plates with agglomerated CNT-reinforced composite layers using FSDT and DQM", *Geomech. Eng.*, **12**(2), 327-346.
Shokravi, M. (2017c), "Dynamic pull-in and pull-out analysis of viscoelastic nanoplates under electrostatic and Casimir forces via sinusoidal shear deformation theory", *Microelectron. Reliab.*, **71**, 17-28.
Shokravi, M. (2017d), "Buckling of sandwich plates with FG-CNT-reinforced layers resting on orthotropic elastic medium using Reddy plate theory", *Steel Compos. Struct.*, **23**(6), 623-631.
Singh, V.K. and Panda, S.K. (2017), "Geometrical nonlinear free vibration analysis of laminated composite doubly curved shell panels embedded with piezoelectric layers", *J. Vibr. Contr.*, **23**(13), 2078-2093.
Soldatos, K.P. (1992), "A transverse shear deformation theory for homogeneous monoclinic plates", *Acta Mech.*, **94**(3), 195-220.
Tornabene, F. (2009), "Free vibration analysis of functionally graded conical, cylindrical shell and annular plate structures with a four-parameter power-law distribution", *Comput. Meth. Appl. Mech. Eng.*, **198**(37-40), 2911-2935.
Tounsi, A., Houari, M.S.A. and Benyoucef, S. (2013), "A refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates", *Aerosp. Sci. Technol.*, **24**, 209-220.
Touratier, M. (1991), "An efficient standard plate theory", *Int. J. Eng. Sci.*, **29**(8), 901-916.
Xiang, S., Wang, K.M., Ai, Y.T., Sha, Y.D. and Shi, H. (2009), "Analysis of isotropic, sandwich and laminated plates by a meshless method and various shear deformation theories", *Compos.S truct.*, **91**(1), 31-37.
Yamanouchi, M., Koizumi, M., Hirai, T. and Shiota, I. (1990), *Proceedings of the 1st International Symposium on Functionally Gradient Materials*, Sendai, Japan,
Yang, J., Liew, K.M. and Kitipornchai, S. (2005), "Stochastic analysis of compositionally graded plates with system randomness under static loading", *Int. J. Mech. Sci.*, **47**(10), 1519-1541.
Yazid, M., Heireche, H., Tounsi, A., Bousahla, A.A. and Houari, M.S.A. (2018), "A novel nonlocal refined plate theory for stability response of orthotropic single-layer graphene sheet resting on elastic medium", *Smart Struct. Syst.*, **21**(1), 15-25.
Youcef, D.O., Kaci, A., Benzair, A., Bousahla, A.A. and Tounsi, A. (2018), "Dynamic analysis of nanoscale beams including surface stress effects", *Smart Struct. Syst.*, **21**(1), 65-74.
Younsi, A., Tounsi, A, Zaoui, F.Z., Bousahla, A.A. and Mahmoud, S.R. (2018), "Novel quasi-3D and 2D shear deformation theories for bending and free vibration analysis of FGM plates", *Geomech. Eng.*, **14**(6), 519-532.
Zamanian, M., Kolahchi, R. and Bidgoli, M.R. (2017), "Agglomeration effects on the buckling behaviour of embedded concrete columns reinforced with SiO_2nano-particles", *Wind Struct.*, **24**(1), 43-57.
Zaoui, F.Z., Ouinas, D. and Tounsi, A. (2019), "New 2D and quasi-3D shear deformation theories for free vibration of functionally graded plates on elastic foundations", *Compos. Part B*, **159**, 231-247.
Zarei, M.S., Kolahchi, R., Hajmohammad, M.H. and Maleki, M. (2017), "Seismic response of underwater fluid-conveying concrete pipes reinforced with SiO2 nanoparticles and fiber reinforced polymer (FRP) layer", *Soil Dyn. Earthq. Eng.*, **103**, 76-85.
Zemri, A., Houari, M.S.A., Bousahla, A.A. and Tounsi, A. (2015), "A mechanical response of functionally graded nanoscale beam: an assessment of a refined nonlocal shear deformation theory beam theory", *Struct. Eng. Mech.*, **54**(4), 693-710.
Zenkour, A.M. (2013), "A simple four-unknown refined theory for bending analysis of functionally graded plates", *Appl. Math. Model.*, **37**(20-21), 9041-9051.
Zenkour, A.M. (2006), "Generalized shear deformation theory for bending analysis of functionally graded plates", *Appl. Math. Modell.*, **30**, 67-84.
Zidi, M., Houari, M.S.A., Tounsi, A., Bessaim, A. and Mahmoud, S.R. (2017), "A novel simple two-unknown hyperbolic shear deformation theory for functionally graded beams", *Struct. Eng. Mech.*, **64**(2), 145-153.
Zidi, M., Tounsi, A., Houari, M.S.A. and Bég, O.A. (2014), "Bending analysis of FGM plates under hygro-thermo-mechanical loading using a four variable refined plate theory", *Aerosp. Sci. Technol.*, **34**, 24-34.
Zine, A., Tounsi, A., Draiche, K., Sekkal, M. and Mahmoud, S.R. (2018), "A novel higher-order shear deformation theory for bending and free vibration analysis of isotropic and multilayered plates and shells", *Steel Compos. Struct.*, **26**(2), 125-137.

CC