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Shear buckling analysis of cross-ply laminated plates resting on Pasternak foundation

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Abstract. This paper presents the shear buckling analysis of symmetrically laminated cross-ply plates resting on Pasternak foundation under pure in-plane uniform shear load. The classical laminated plate theory is used for the shear buckling analysis of laminated plates. The Rayleigh-Ritz method with novel plate shape functions is proposed to solve the differential equations and a computer programming is developed to obtain the shear buckling loads. Finally, the effects of the plate aspect ratios, boundary conditions, rotational restraint stiffness, translational restraint stiffness, thickness ratios, modulus ratios and foundation parameters on the shear buckling of the laminated plates are investigated.

Keywords: laminated composite plates; shear buckling; Rayleigh Ritz method; Pasternak foundation

1. Introduction

Laminated composite plates made up of fiber-reinforced layers are frequently used in aerospace, civil, marine, automobile, and other engineering structures. The widespread use of laminate composites during the last three decades is due to their physical properties such as high specific strength and stiffness, resistance to corrosion and fatigue, and lightweight. On the other hand, most of the laminated composite plates are vulnerable to elastic buckling before reaching to the failure strength because of their thin-walled nature. Therefore, it is needed to find some methods for the buckling prediction of the plates.

Several investigations have been carried out on the buckling behavior of laminated composite plates. Jiang *et al.* (2018) studied buckling, postbuckling and nonlinear vibration behaviors of composite laminated trapezoidal plates. Dong *et al.* (2017) investigated the local buckling analysis of an infinite thin rectangular symmetrically laminated composite plate restrained by a tensionless Winkler foundation and subjected to uniform in-plane shear loading. Becheri *et al.* (2016) presented an exact analytical solution for mechanical buckling analysis of symmetrically cross-ply laminated plates including curvature effects. Baseri *et al.* (2016) investigated buckling analysis of an embedded laminated composite plate. Singh and Kumar (2010) studied buckling and postbuckling responses, and the progressive failure of square laminates of symmetric lay-up with a central rectangular cutout under in-plane shear load. Altunsaray and Bayer (2014) investigated the lowest critical value of the compressive force acting in the plane of symmetrically laminated quasi-isotropic thin rectangular plates. Singh *et al.* (2002) used a C0 finite element method for arriving at an eigenvalue problem using higher order shear deformation theory for initial buckling of laminated composite plates. Ashour (2003) employed a finite strip transition matrix technique, a semi analytical method to obtain the buckling loads and the natural frequencies of symmetric cross-ply laminated composite plates with edges elastically restrained against both translation and rotation. Kiani and Mirzaei (2018) studied the shear buckling behaviour of composite skew plates reinforced with aligned single walled carbon nanotubes. Kiani (2016) investigated shear buckling response of carbon nanotube reinforced composite rectangular plates in thermal environment. Kosteletos (1992) investigated the buckling behavior of laminated composite plates with clamped supports in all four-edges under uniform shear load. Shufrin *et al.* (2008) utilized a semi-analytical Kantorovich approach to obtain the buckling condition in the symmetric laminated rectangular plates with different boundary conditions under combined in-plane shear and compressive loading. Loughlan (1999) used the finite strip method to assess the impact of bend-twist coupling on the stability of laminated composite plates under shear loading. Iyengar and Chakraborty (2004) utilized the finite element method to assess the impact of transverse shear on the buckling of composite laminated plates under combined in-plane shear and compressive loading. Qiao and Huo (2011) presented a closed-form solution for local buckling of orthotropic plates under in-plane shear loading. Chen and Qiao (2015) employed Galerkin method to perform the shear buckling analysis of composite laminated plates. Liu *et al.* (2014) used Rayleigh-Ritz method and proposed a new shape function based on plate buckled displacement for buckling analysis of orthotropic plates under combined in-plane shear and axial loads. Biggers and Pageau (1994) evaluated

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the piecewise-uniform approach to tailoring as a means of improving the shear buckling loads of composite plates. Xie *et al.* (2003) investigated shear buckling analysis of asymmetrical angle-ply laminates using the higher-order shear deformation theory.

There are different types of elastic foundation models to describe the interactions between the plate and foundation. One of the simplest models is Winkler, which is one-parameter model with closely spaced independent linear springs (Winkler 1867). In the foundation, the springs which are not situated under the loaded region, are not affected by the load. Pasternak (1954) added a shear layer to Winkler model and assumed a shear interaction between the springs that is performed by joining the ends of springs to a plate. Pasternak model is a two-parameter model which endures only transverse shear deformation. This model has been widely utilized to assess the mechanical behavior of structure–foundation interactions. Recently, the several studies have been carried out to understand the buckling behavior of the plate resting on Pasternak foundation. Rad and Shahraki (2014) investigated buckling responses of cracked functionally graded plates resting on Pasternak foundation under tension. They utilized classical plate theory based on the finite element method. Setoodeh and Karami (2004) studied the buckling behavior of laminated thick composite plates resting on Winkler and Pasternak foundations by employing a three-dimensional elasticity based layer-wise finite element method. Nazarimofrad and Barkhordar (2016) investigated the stability of orthotropic rectangular plate resting on Pasternak foundation for different boundary conditions. Aiello and Ombres (1999) examined buckling loads, free vibrations and vibrations with initial inplane stresses for moderately thick, simply supported rectangular laminates resting on elastic foundations. Kim (2004) investigated the stability and dynamic displacement response of an infinite thin plate resting on a Winkler-type or a two-parameter elastic foundation. Thai *et al.* (2013) proposed a simple refined shear deformation theory for bending, buckling and vibration of thick plates resting on elastic foundation. Dehghan and Baradaran (2011) used a combination of the finite element and differential quadrature methods to solve the buckling and free vibration equations of rectangular thick plates resting on elastic foundations. Khalili *et al.* (2013) used the Lindstedt-Poincare perturbation technique to study the effect of non-ideal boundary conditions on buckling load of laminated plates on elastic foundations.

However, the previous shape functions need to be considered as a function of several terms (similar to Fourier series). It is needed to increase the number of terms to obtain an accurate solution. Therefore, we proposed new different plate shape functions only by one term to obtain an accurate solution for all boundary conditions. On the other hand, in the literature there is no any research on the shear buckling of laminated composite plates resting on Pasternak foundation. The aim of this paper is to investigate the shear buckling analysis of symmetrically laminated cross-ply plates resting on Pasternak foundation under pure in-plane uniform shear load to fill this gap. The classical laminated plate theory is used for the shear buckling analysis of laminated plates. The Rayleigh-Ritz method with novel plate buckled shape functions is proposed to solve the differential equations and a computer programming is developed to obtain the shear buckling loads. Finally, the effects of the plate aspect ratios, boundary conditions, rotational restraint stiffness, translational restraint stiffness, thickness ratios, modulus ratios and foundation parameters on the shear buckling of the laminated plates are investigated.

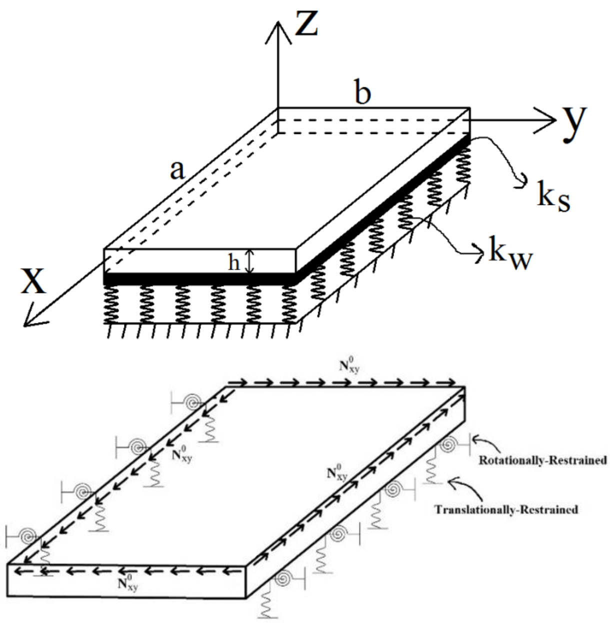


Fig. 1 A laminated composite plate resting on Pasternak foundation

2. Basic equations

In this study, a symmetric laminated composite plate with constant thickness of *h* and dimensions of *a* and*b* resting on Pasternak elastic foundation was considered as shown in Fig. 1.

The displacements of the plate in the (x, y, z) directions are denoted by (u, v, w). Based on the classical plate theory (CLPT), the displacement fields can be assumed as follows

$$u = \{u \quad v \quad w\}^T = \left\{-z\frac{\partial}{\partial x} \quad -z\frac{\partial}{\partial y} \quad 1\right\}^T w_0 \tag{1}$$

where w_0 is the displacements at the mid plane of plate. The strains and stresses of the plate are given by,

$$\varepsilon_p = \left\{-\frac{\partial^2}{\partial x^2} \quad -\frac{\partial^2}{\partial y^2} \quad -2\frac{\partial^2}{\partial x\,\partial y}\right\}^T w_0 \tag{2}$$

$$\sigma_p = \{M_x \quad M_y \quad M_{xy}\}^T \tag{3}$$

where M_x, M_y and M_{xy} are the bending and twisting moments per unit length, respectively. The relations between the strains and stresses can be shown as follows

$$\sigma_p = D\varepsilon_p \tag{4}$$

Due to the assumption of classical plate theory (CLPT) for a laminated composite plate, D can be written as

$$D = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{21} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \tag{5}$$

where D_{ij} is the flexural stiffness matrix and is calculated by Eq. (6).

$$D_{ij} = \frac{1}{3}\sum_{1}^{k} \bar{Q}_{ij}^{(k)}\,(h_k^3 - h_{k-1}^3), \quad (i,j = 1,2,6) \tag{6}$$

The strain energy of a laminated rectangular composite plate resting on Pasternak foundation can be written as follows

$$\Pi_i = \frac{1}{2}\int_0^b\int_0^a \left[M_x\frac{d^2w_0}{dx^2} + M_y\frac{d^2w_0}{dy^2} + M_{xy}\frac{d^2w_0}{dxdy} + k_w{w_0}^2 + k_s\left(\left(\frac{dw_0}{dx}\right)^2 + \left(\frac{dw_0}{dy}\right)^2\right)\right]dxdy \tag{7}$$

The governing strain energy equation (Π_i) of the plate based on Eq. (4) can be written as follows

$$\Pi_i = \frac{1}{2}\int_0^b\int_0^a \left[D_{11}\left(\frac{d^2w_0}{dx^2}\right)^2 + 2D_{12}\frac{d^2w_0}{dx^2}\frac{d^2w_0}{dy^2} + D_{22}\left(\frac{d^2w_0}{dy^2}\right)^2 + 4D_{66}\left(\frac{d^2w_0}{dxdy}\right)^2 + k_w{w_0}^2 + k_s\left(\left(\frac{dw_0}{dx}\right)^2 + \left(\frac{dw_0}{dy}\right)^2\right)\right]dxdy \tag{8}$$

where k_w is the vertical spring modulus of the foundation, k_s is the shear modulus of the foundation. The potential energy of the applied load (Π_e) under the uniform in-plane shear edge load (N_{xy}^0) can be calculated as

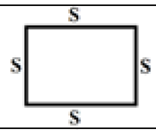
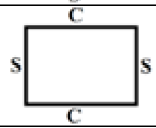
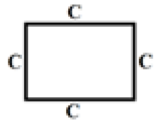
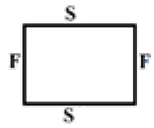
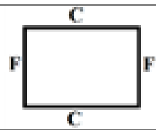
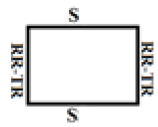
$$\Pi_e = -N_{xy}^0\int_0^b\int_0^a \frac{dw_0}{dx}\frac{dw_0}{dy}dxdy \tag{9}$$

The total elastic potential Π of the plate system can be written as below

$$\Pi = \Pi_i + \Pi_e \tag{10}$$

By substitution of the proper out-of-plane displacement shape function into Eq. (10), the standard eigenvalue problem of buckling can be solved by the Rayleigh-Ritz method. Table 1 shows the proposed shape functions used in this study for different boundary conditions. Wis unknown constant that will remain indeterminate according to the buckling theory. In addition, φ is the skew of the buckling mode and m is the buckling half-wave. In Table 1, the letters S, C, F and RR-TR stand for simply supported, clamped, free and rotational-translational restraint boundary conditions, respectively. β is for adjusting translationally-restrained of edges in both y=0 and y=b which is defined as follows.

Table 1 Proposed shape functions for different boundary conditions

Cases	Shape functions	Boundary conditions
4S	$w_0 = W\sin\left(\frac{\pi(mx-\varphi y)}{a}\right)\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)$	
2S2C	$w_0 = W\sin\left(\frac{\pi(mx-\varphi y)}{a}\right)\sin\left(\frac{\pi x}{a}\right)\left\{\left[1-\cos\left(2\frac{\pi y}{b}\right)\right]\right\}$	
4C	$w_0 = W\sin\left(\frac{\pi(mx-\varphi y)}{a}\right)\left\{1-\cos\left(2\frac{\pi x}{a}\right)\right\}\left\{1-\cos\left(2\frac{\pi y}{b}\right)\right\}$	
2F2S	$w_0 = W\sin\left(\frac{\pi(mx-\varphi y)}{a}\right)\sin\left(\frac{\pi y}{b}\right)$	
2F2C	$w_0 = W\sin\left(\frac{\pi(mx-\varphi y)}{a}\right)\left\{1-\cos\left(2\frac{\pi y}{b}\right)\right\}$	
RR-TR	$w_0 = W\sin\left(\frac{\pi(mx-\varphi y)}{a}\right)\sin\left(\frac{\pi x}{a}\right) * \left\{\beta+\mu\sin\left(\frac{\pi y}{b}\right)+(1-\mu)\left[1-\cos\left(2\frac{\pi y}{b}\right)\right]\right\}$	

$$\beta = \frac{2D_{22}\pi}{2D_{22}\pi + k_t b} \tag{11}$$

where $k_t = 0, k_t = \infty$ and $0 < k_t < \infty$ are for free edge, non-free edge, and translationally-restrained edge, respectively. μ is for adjusting rotationally-restrained of edges in both y=0 and y=b which is defined as follows

$$\mu = \frac{k_r b}{2D_{22}\pi + k_r b} \tag{12}$$

where $k_r = 0, k_r = \infty$ and $0 < k_r < \infty$ are for simply supported edge, clamp supported edge, and rotationally-restrained edge, respectively.

Using the equilibrium condition of the first variational principle of the total potential energy ($\delta\Pi$=0), the buckling condition reduces to the well-known Ritz equation

$$\frac{d\Pi}{dW} = 0 \tag{13}$$

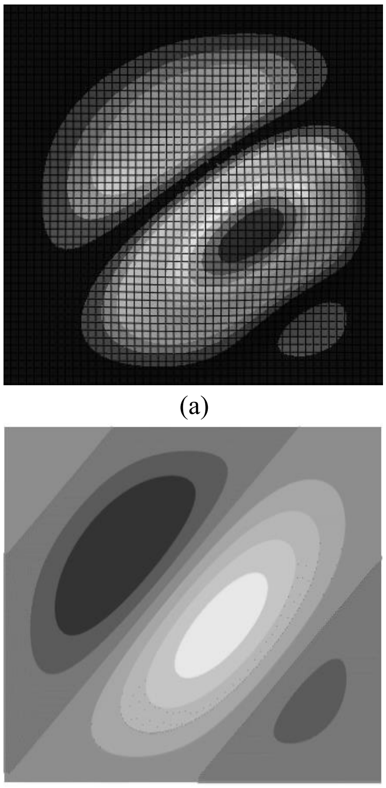
The following relations are used for presentation of the non-dimensional shear buckling load, non-dimensional linear Winkler foundation parameter and non-dimensional Pasternak foundation parameter, respectively

$$\bar{N}_{xy} = \frac{N_{xy}^0 . b^2}{\pi^2\sqrt[4]{D_{11}D_{22}^3}}, K_W = \frac{k_w b^4}{E_2 h^3}, \quad K_S = \frac{k_s b^2}{E_2 h^3} \tag{14}$$

Using the proposed mathematical model and solution

Table 2 Comparisons of the nondimensional shear buckling loads for different boundary conditions and aspect ratios (b/h=250)

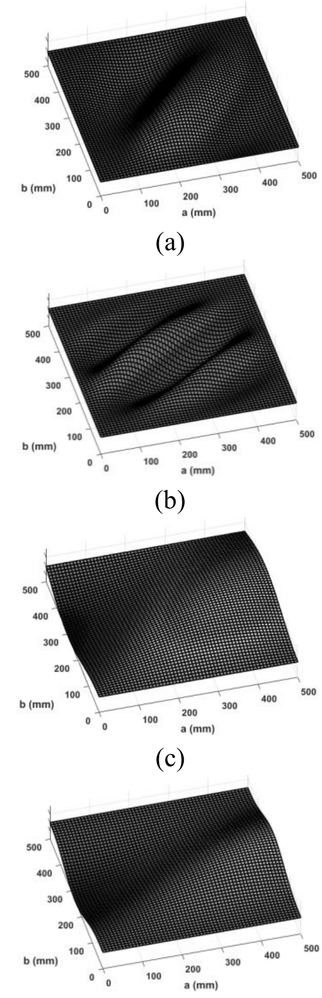
	Present study		Finite element solution	
Boundary conditions	a/b		a/b	
	1	1.8	1	1.8
4S	6.71	3.10	6.29	2.95
2S2C	7.41	4.49	7.13	4.32
4C	14.22	5.95	13.46	5.73
2F2S	2.01	2.11	2.14	1.97
2F2C	3.68	3.55	3.87	3.68



(a)

(b)

Fig. 2 Comparisons of the buckled shapes for simply supported laminated square plate for MATLAB programming (a) and finite element solution (b)



(a)

(b)

(c)

(d)

Fig. 3 The buckled shapes of the laminated square plates for MATLAB programming: (a) 4S, (b) 4C, (c) 2F2S, (d) 2F2C

methodology, a generalized computer program is coded in MATLAB to obtain the shear buckling loads of laminated plates.

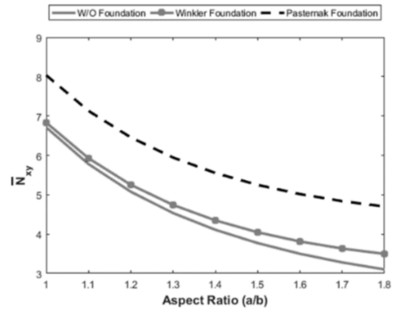
3. Numerical results and discussion

In this study, the validation of the present study with the finite element solution is conducted for symmetric cross-ply $[0\ 90\ 0\ 90\ 0\ 0\ 0\ 90]_s$ laminated plates for different boundary conditions and aspect ratios (a/b). The material properties of elastic lamina are given by: E_1=155.8 GPa, E_2=8.89 GPa, G_{12}=5.14 GPa, v=0.3. As can be seen in Table 2, the nondimensional shear buckling loads of the present study have very good conformity with those of the finite element solution for different boundary conditions and aspect ratios. Fig. 2 shows the buckled shapes of the simply supported (4S) square laminated plate for MATLAB programming and the finite element solution. As seen, the obtained buckled shapes for two methods are consistent to each other.

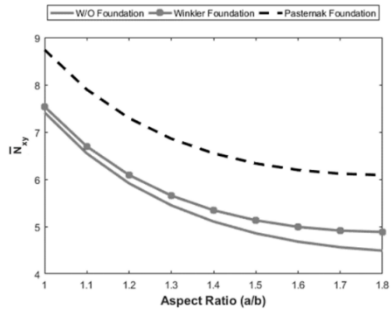
Fig. 3 shows the buckled shapes of the laminated square plates in MATLAB programming for different boundary conditions.

In this study, the effects of the plate aspect ratios, boundary conditions, thickness ratios, modulus ratios, foundation parameters and rotational and translational restraint stiffnesses on the shear buckling loads are investigated.

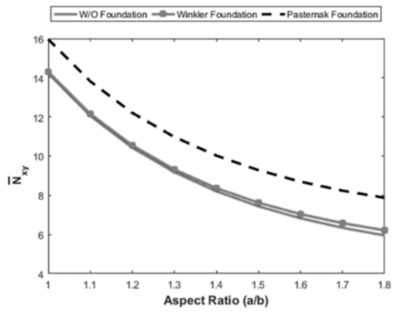
Fig. 4 shows the effect of plate aspect ratios (a/b) on the nondimensional shear buckling load for 4S, 2S2C, 4C, 2F2S and 2F2C boundary conditions without (W/O) foundation, Winkler foundation (K_W=100, K_S=0) and Pasternak foundation (K_W=100, K_S=10, b/h=250). As seen, laminated plates with Pasternak foundation give the highest results for the nondimensional shear buckling load. However, as a/b ratio increases the nondimensional shear



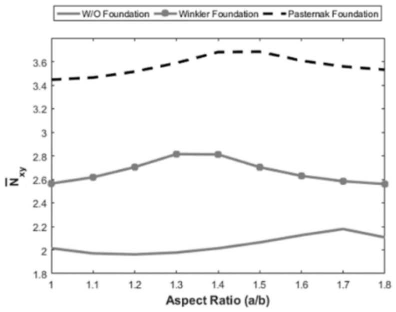
(a)



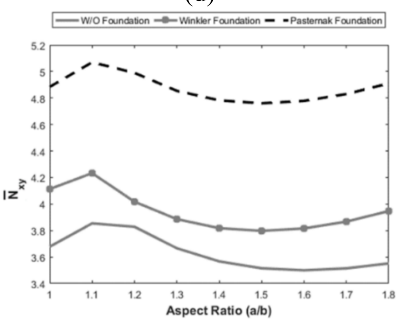
(b)



(c)



(d)



(e)

Fig. 4 Effect of aspect ratios on the nondimensional shear buckling for (a) 4S; (b) 2S2C; (c) 4C; (d) 2F2S; (e) 2F2C boundary conditions without (W/O) foundation, Winkler foundation and Pasternak foundation

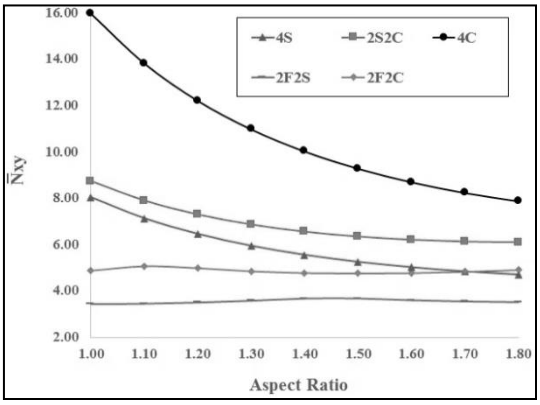


Fig. 5 Effect of plate aspect ratio and boundary conditions on the nondimensional shear buckling load with Pasternak foundation

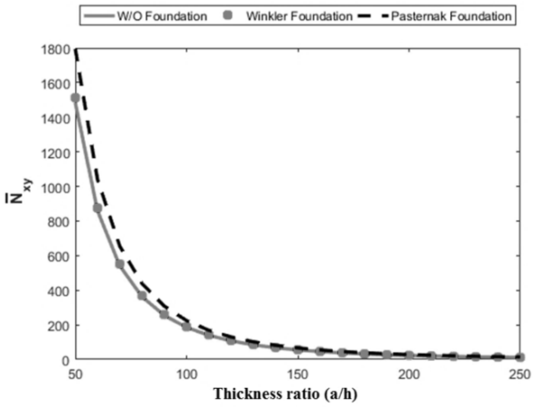


Fig. 6 Effect of thickness ratio (a/h) on the nondimensional shear buckling load for simply supported boundary condition

buckling load generally decreases for different boundary conditions.

Fig. 5 shows the nondimensional shear buckling loads versus plate aspect ratios for Pasternak foundation for all boundary conditions. As seen, by increasing the boundary constraints, the nondimensional shear buckling increases. 4C and 2F2S boundary conditions give the highest and the lowest nondimensional shear buckling loads, respectively.

Fig. 6 shows the effects of thickness ratio (a/h) on the nondimensional shear buckling load for 4S boundary condition for symmetric cross-ply square laminated plates without (W/O) foundation, Winkler foundation and Pasternak foundation. As seen, an increase in the thickness ratio results in a decrease in the shear buckling. However, in the higher thickness ratios, the nondimensional shear buckling load of the plate with Pasternak foundation is similar to the others. Fig. 7 illustrates the effects of the modulus ratio (E_1/E_2) on the nondimensional shear buckling for 4C boundary condition for a/b=1 without (W/O) foundation, Winkler foundation and Pasternak foundation. As can be seen in Fig. 7, as modulus ratio increases the nondimensional shear buckling load increases for without and with Winkler foundations. But, the effect of modulus ratio decreases for larger modulus ratios. On the other hand, as the modulus ratio increases, the

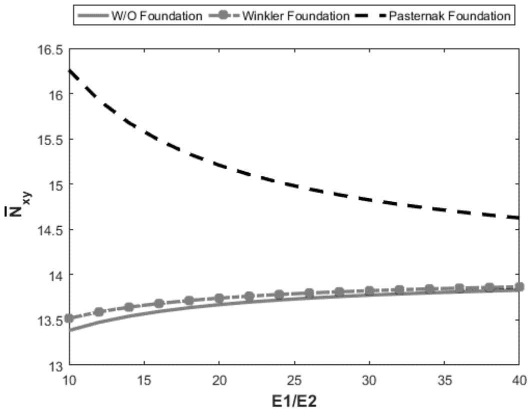


Fig. 7 Effect of the modulus ratio (E_1/E_2) on the nondimensional shear buckling load

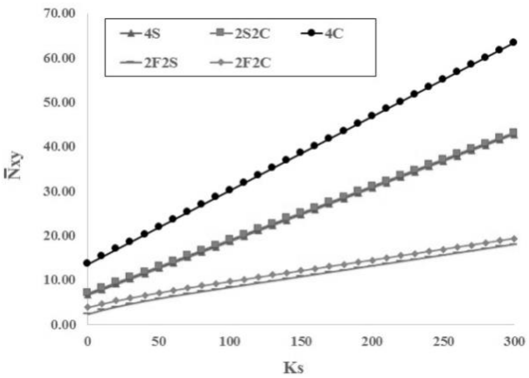


Fig. 8 Effect of the foundation parameter (K_s) on the nondimensional shear buckling load

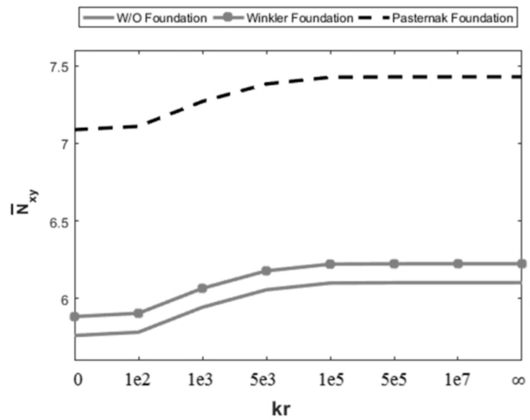


Fig. 9 Effect of rotational restraint stiffnesses on the nondimensional shear buckling load ($k_t = 0$)

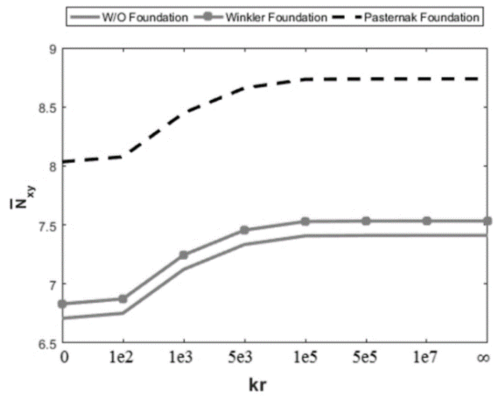


Fig. 10 Effect of rotational restraint stiffnesses on the nondimensional shear buckling load ($k_t = \infty$)

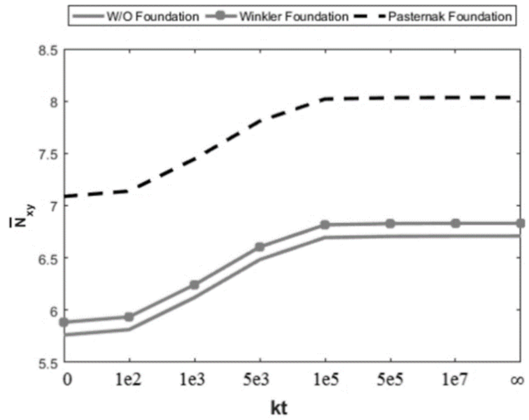


Fig. 11 Effect of translational restraint stiffnesses on the nondimensional shear buckling load ($k_r = 0$)

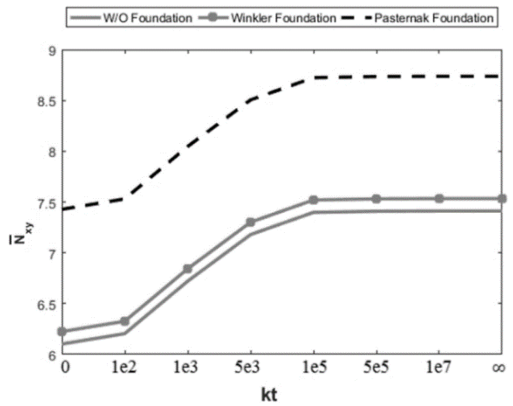


Fig. 12 Effect of translational restraint stiffnesses on the nondimensional shear buckling load ($k_r = \infty$)

nondimensional shear buckling decreases for Pasternak foundation.

Fig. 8 shows the effects of the foundation parameter (K_s) on the nondimensional shear buckling load for Pasternak foundation for different boundary conditions are investigated (a/b=1, K_w=100). As seen, as the foundation parameter increases the nondimensional shear buckling load increases.

In Figs. 9-10, the effect of rotational restraint stiffnesses (k_r) on the nondimensional shear buckling load can be seen. As seen, as rotational restraint stiffness increases the nondimensional shear buckling load increases. But the effect of k_r for larger values on the nondimensional shear buckling load may be ignored.

In Figs. 11-12, the effect of translational restraint stiffnesses (k_t) on the nondimensional shear buckling load can be seen. As seen, as translational restraint stiffness increases the nondimensional shear buckling load increases. But the effect of k_t for larger values on the nondimensional shear buckling load can be ignored.

4. Conclusions

This paper presents the shear buckling analysis of symmetric cross-ply laminated composite plates resting on Pasternak foundation. Different plate buckled shape functions are proposed for various boundary conditions. Finally, the effects of the plate aspect ratios, boundary conditions, thickness ratios, modulus ratios, foundation parameters and rotational and translational restraint stiffnesses on the shear buckling loads of the laminated plates are investigated. Laminated plates with Pasternak foundation give the highest results for the nondimensional shear buckling load. As plate aspect ratio increases the nondimensional shear buckling load generally decreases for different boundary conditions. The plate with clamped boundary condition gives the highest nondimensional shear buckling load of all. As the thickness ratio increases the nondimensional shear buckling load decreases. However, in the higher thickness ratios, the nondimensional shear buckling load of the plate with Pasternak foundation is similar to the others. As the modulus ratio increases, the nondimensional shear buckling load decreases for Pasternak foundation. As the foundation parameter increases the nondimensional shear buckling load increases. As the rotational and translational restraint stiffnesses increase the nondimensional shear buckling load increases. But the effects of rotational and translational restraint stiffnesses for the larger values on the nondimensional shear buckling load may be ignored.

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